Thermodynamics of holographic models for QCD in the Veneziano limit

Timo Alho

University of Jyväskylä

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[TA, Järvinen, Kajantie, Kiritsis, Tuominen arXiv:1210.4516]

Overview

- Veneziano QCD
- The model and determining the potentials
- Computing thermodynamics
- Results
- Outlook

Veneziano QCD

Veneziano QCD is a YM theory with N_c colors and N_f fermion flavors, at the limit $N_c, N_f \to \infty$ but $x_f \equiv \frac{N_f}{N_c}$ constant.

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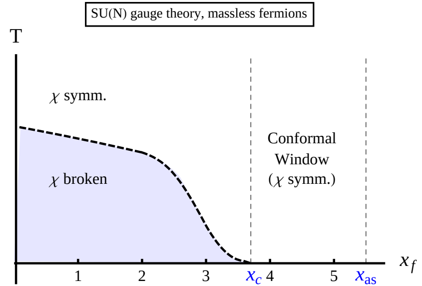
The holographic model and its vacuum structure described in arXiv:1112.1261, and in the previous talk by Järvinen. For studying the thermodynamics, we add a black hole to the bulk. The model stays the same, but the metric ansatz now becomes

$$ds^2 = b^2(r)\left[-f(r)dt^2 + d\mathbf{x}^2 + \frac{dr^2}{f(r)}\right], \tag{1}$$

What to expect

What we expect from QCD -like thermodynamics:



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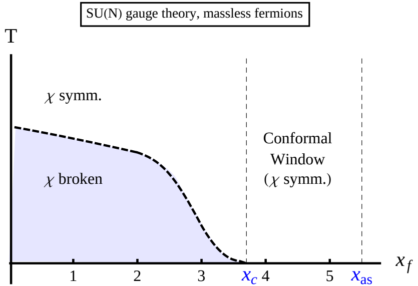

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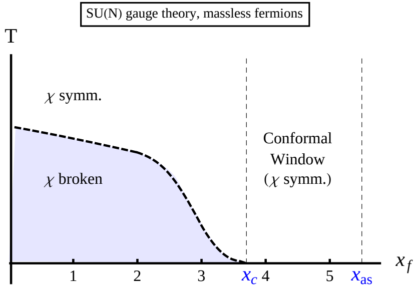

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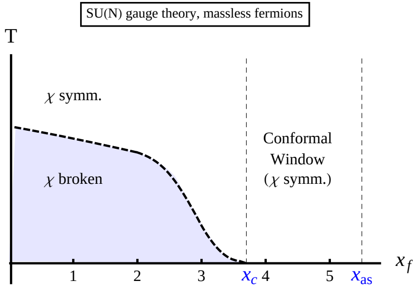

- Conformal window between $x_c \approx 4$ and $x_{as} = 11/2$
- A deconfinement/chiral symmetry restoring transition at $x_f < x_c$
- Miransky scaling at $x_f \lesssim x_c$

Action

To recap the setup, the gravity action is

$$S = M^3 N_c^2 \int d^5x\, \mathcal{L} \equiv \frac{1}{16\pi G_5} \int d^5x\, \mathcal{L}, \tag{2}$$

where

$$\begin{aligned} \mathcal{L} \;&=\; \left[\sqrt{-g}\left(R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda)\right)\right. \\ &\left. - V_f(\lambda,\tau)\sqrt{\det\left(g_{ab} + \kappa(\lambda,\tau)\partial_a\tau\,\partial_b\tau\right)}\right]. \end{aligned} \tag{3}$$

The metric Ansatz is

$$ds^2 = b^2(r)\left[-f(r)dt^2 + d\mathbf{x}^2 + \frac{dr^2}{f(r)}\right], \tag{4}$$

and the two scalar functions, $1/\lambda$ sourcing F^2 and τ sourcing $\langle\bar{q}q\rangle$, are

$$\lambda = \lambda(r) = e^{\phi(r)}, \quad \tau = \tau(r). \tag{5}$$

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- IR limit: confinement and τ divergence

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- The string frame to Einstein frame conversion factor κ may be corrected by a logarithmic factor.

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Want $m_q = 0$:

- chiral symmetry $\Rightarrow \tau(r) \equiv 0$
- chiral symmetry broken $\Rightarrow \tau(r) \neq 0$, but $m_q = 0$ determines τ_h as a function of λ_h

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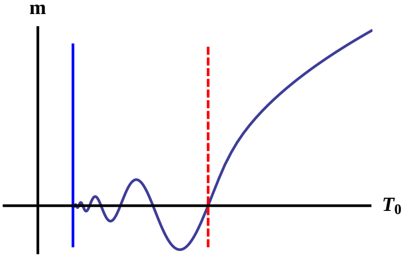

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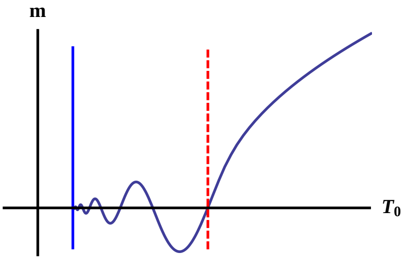

- However, only the one with largest τ_h is thermodynamically relevant (smallest free energy)
- Overall, two separate branches of black hole solutions, one with $\tau \equiv 0$, and one with a dynamic tachyon.

Thermodynamics

$$\begin{aligned}
T &= -\frac{1}{4\pi} f'(r_h(\lambda_h)) \\
s &= \frac{1}{4G_5} b^3(\lambda_h) \\
p &= -F = \frac{1}{4G_5} \int_{\lambda_h}^{\lambda_*} d\lambda_h \left(-\frac{dT}{d\lambda_h} \right) b^3(\lambda_h) + p_0 \\
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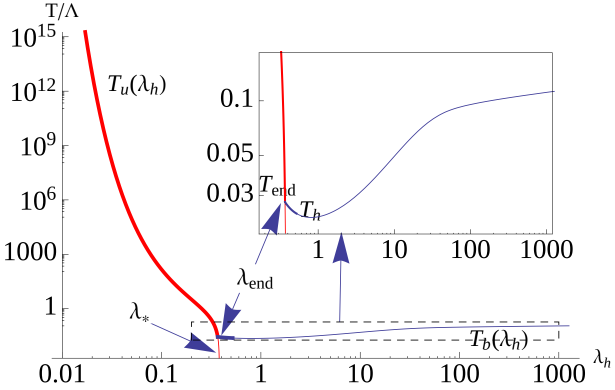
Note that $c_s^2 > 0$ requires that $\frac{dT}{d\lambda_h} < 0$.

The function $T(\lambda_h)$ determines the possible phases of the theory.



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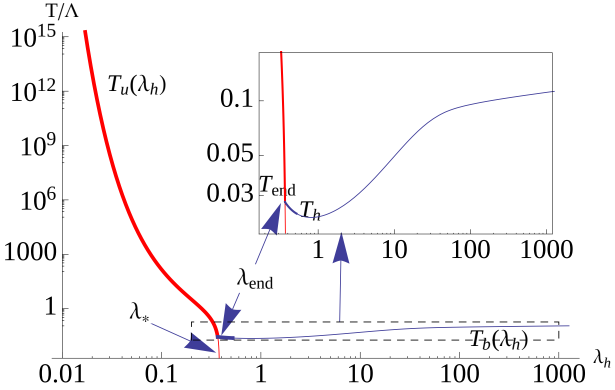

Below $\lambda_h < \lambda_{\rm end}$ the $\tau_h \neq 0$ phase doesn't exist $\Rightarrow$ no chiral symmetry breaking at high T

Computing pressure

To compute the pressure, we need to fix the integration constants in the two branches:

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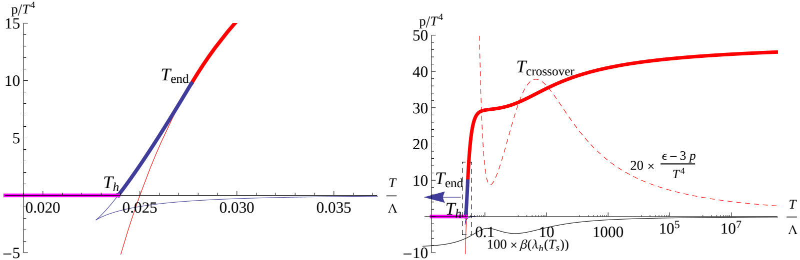
$\Rightarrow$ numerically equivalent to requiring that $p_u(\lambda_{\text{end}}) = p_b(\lambda_{\text{end}})$

Pressure

Typical $p(T)$, with $x_f = N_f/N_c = 3$:



Phase transitions in order of decreasing T:

- A crossover around $T_{\text{crossover}}$
- The second order chiral symmetry breaking transition at T_{end}
- The confining, or hadronization, transition at T_h from the black hole phase to the hadron gas phase

Numerics

From the above, calculate the (x_f, T) phase diagram and other thermodynamic variables for a comprehensive subset of all choices of potential. A numerical code which near-automatically gives the full thermodynamics given the potentials as inputs allowed us to explore ten different potentials with reasonable time and effort.

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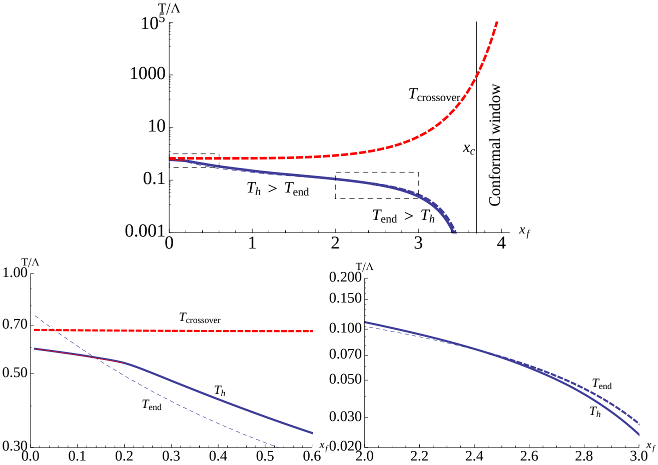
Some examples to follow. Legend:

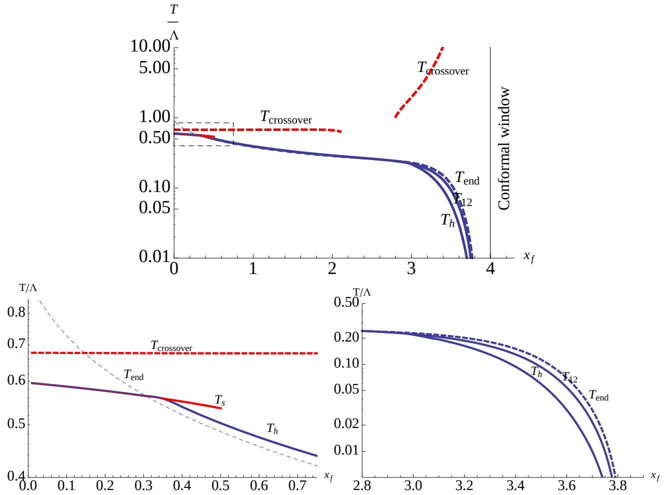
- blue lines: transitions which involve either the $\tau \neq 0$ -branch or the $\tau \neq 0$ vacuum solution
- red lines: transitions which only involve the $\tau = 0$ -solution.
- thick lines: transitions between two stable phases
- thin lines: transitions between two metastable phases

Potential II, W_0 SB-normalized



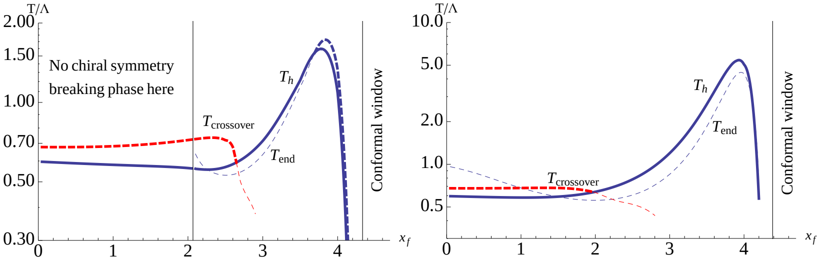
Reasonable phase diagram, but meson masses go as $M_n \sim n$, so not a good QCD model.

Potential I, $W_0 = 12/11$



The most complicated phase diagram we found.

Potentials I_* and II_*



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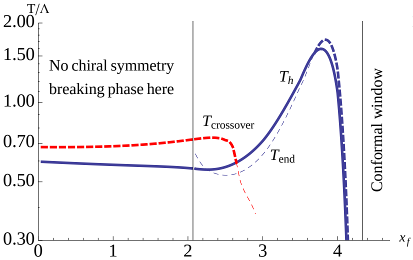
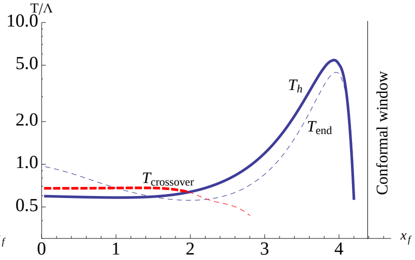
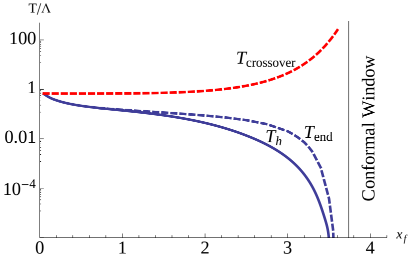




I_* has no $\chi_S B$ at low-x_f, II_* has the same spectrum problem as II.

Potential I with log-modified κ



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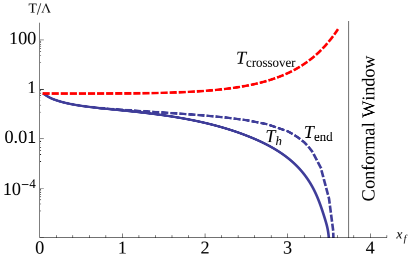
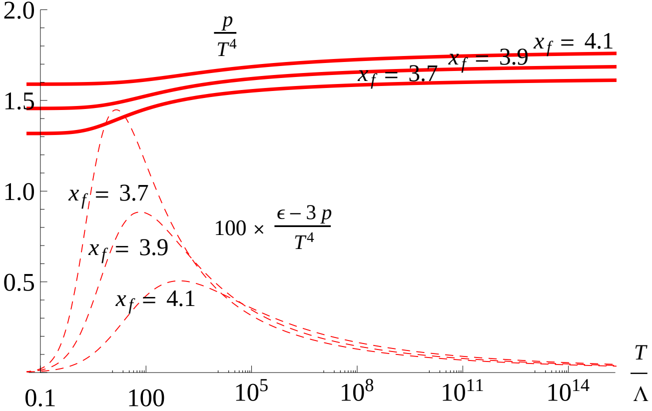


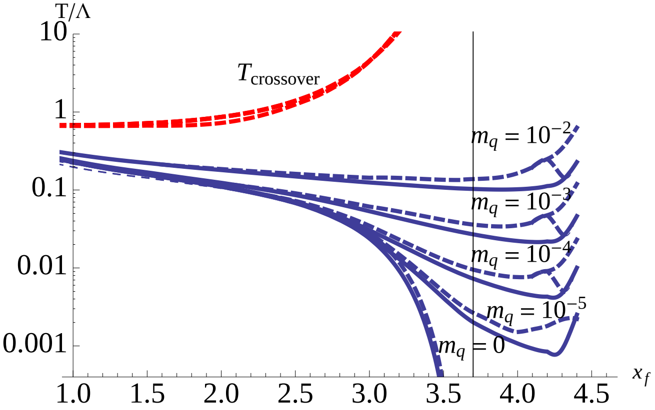
- phase diagram compatible with expectations for QCD-like theory
- meson trajectory is Regge-like
- $\Rightarrow$ best candidate for modeling QCD-like theories (for now, Kiritsis et. al. working further on this)

The conformal window



No $\tau \neq 0$ solution in the conformal window $\Rightarrow$ thermodynamics in the conformal window is qualitatively independent of the choice of potential.

Finite mass (potential II, W_0 SB -normalized)



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- A region with a walking coupling constant, i.e. quasi-conformality, when x_f is slightly below x_c
- at small x_f, chiral symmetry restoration always coincides with deconfinement.

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- non-degenerate quark masses

That's all, folks! Thank you!