(Entanglement) Entropy in three-dimensional higher spin theories

Juan I. Jottar

Based on:

arXiv:1302.0816 + to appear
with Jan de Boer

Higer spin theories

- Broad definition: interacting theories of gravity coupled to a finite (or infinite) number of massless fields of spin $s > 2$.

- Motivation from holography: explore AdS/CFT in a regime where the bulk theory is not just classical (super-)gravity, and the dual theory is not necessarily strongly-coupled:
  - Critical $O(N)$ vector models in $3d$ in the large-N limit dual to higher spin theories of Fradkin-Vasiliev type in AdS_4 (Klebanov, Polyakov 2002; Giombi, Yin 2010-12; Maldacena, Zhiboedov 2011-12)
  - Two-dimensional CFTs with extended (W-)symmetries in the large-N limit dual to higher spin theories in AdS_3 (Gaberdiel, Gopakumar 2010)

- Motivation from GR: singularities, black hole horizons, etc are not invariant under the higher spin gauge symmetries $\Rightarrow$ one must reconsider traditional geometric notions in these setups.

Higer spin theories in AdS_3

- In $3d$ it is possible to truncate the tower of higher spin fields to $s \leq N$. The bulk theory reduces to $SL(N,\mathbb{R}) \times SL(N,\mathbb{R})$ Chern-Simons theory.
- Generalizes the formulation of AdS_3 gravity as an $SL(2,\mathbb{R}) \times SL(2,\mathbb{R})$ Chern-Simons theory (Achucarro, Townsend 1986; Witten 1988)
- In the $N = 2$ case, many universal results recovered from $\mathrm{AdS}_3/\mathrm{CFT}_2$: Cardy entropy formula, entanglement entropy, etc.
- Challenge: extend the holographic dictionary to the $N > 2$ case. The asymptotic symmetry algebra is of W_N type (Henneaux, Rey 2010; Campoleoni et. al. 2010). Black holes, matter probes, partition functions (Gutperle, Kraus 2011; Ammon et. al. 2011; Castro et. al. 2011-12; Gaberdiel, Hartman, Jin 2011-12; Kraus, Perlmutter 2011-12)

Universal results in standard AdS_3/CFT_2

- The BTZ black hole entropy (via Bekenstein-Hawking) and holographic entanglement entropy (via Ryu-Takayanagi) match universal CFT results:

Cardy entropy formula

$$S = 2\pi\sqrt{\frac{c}{6}\left(L_0 - \frac{c}{24}\right)} + 2\pi\sqrt{\frac{c}{6}\left(\bar{L}_0 - \frac{c}{24}\right)}$$

(Single interval) Entanglement entropy at finite temperature $T = \beta^{-1}$

$$S_A = \frac{c}{3}\log\left(\frac{\beta}{\pi a}\sinh\left(\frac{\pi\Delta x}{\beta}\right)\right)$$

where c is the central charge and a the UV cutoff.

- Question: How do we compute (entanglement) entropy in the presence of higher spin charges?

AdS$_3$ Gravity as a Chern-Simons theory

- Take $3d$ gravity with a negative cosmological constant $\Lambda = -1/\ell^2$. Combine dreibein e^a and (dual) spin connection $\omega^a = (1/2!)\epsilon^{abc}\omega_{bc}$ into $SL(2,\mathbb{R})$ connections

$$A = A^a J_a = \omega + \frac{e}{\ell}\,, \qquad \bar{A} = \bar{A}^a J_a = \omega - \frac{e}{\ell}$$

  where the J_a satisfy the $so(2,1) \simeq sl(2,\mathbb{R})$ algebra $[J_a, J_b] = \epsilon_{ab}{}^{c}\, J_c$.

- Defining $CS(A) = A \wedge dA + \frac{2}{3} A \wedge A \wedge A$ one finds ($k \equiv \ell/(4G_3)$)

$$\boxed{g_{\mu\nu} = 2\mathrm{Tr}\left[e_\mu e_\nu\right]}$$

$$\begin{aligned} I_{CS} &\equiv \frac{k}{4\pi}\int_M \mathrm{Tr}\Big[CS(A) - CS(\bar{A})\Big] \\ &= \frac{1}{16\pi G_3}\left[\int_M d^3x\sqrt{|g|}\left(\mathcal{R} + \frac{2}{\ell^2}\right) - \int_{\partial M}\omega^a \wedge e_a\right] \end{aligned}$$

AdS$_3$ Gravity as a Chern-Simons theory

- Take $3d$ gravity with a negative cosmological constant $\Lambda = -1/\ell^2$. Combine dreibein e^a and (dual) spin connection $\omega^a = (1/2!)\epsilon^{abc}\omega_{bc}$ into $SL(2,\mathbb{R})$ connections

$$A = A^a J_a = \omega + \frac{e}{\ell}, \qquad \bar{A} = \bar{A}^a J_a = \omega - \frac{e}{\ell}$$

  where the J_a satisfy the $so(2,1) \simeq sl(2,\mathbb{R})$ algebra $[J_a, J_b] = \epsilon_{ab}{}^c J_c$.

- Defining $CS(A) = A \wedge dA + \frac{2}{3} A \wedge A \wedge A$ one finds $(k \equiv \ell/(4G_3))$

Einstein's equations $\Leftrightarrow$ Flatness

$$F = dA + A \wedge A = 0$$

$$\bar{F} = d\bar{A} + \bar{A} \wedge \bar{A} = 0$$

Boundary conditions in Chern-Simons theory

- Consider a radial coordinate ρ (boundary at $\rho \to \infty$) and boundary coordinates $x^{\pm} = \frac{t}{\ell} \pm \varphi$
- In the Chern-Simons formulation, the Brown-Henneaux b.c. amount to (Coussaert, Henneaux, van Driel 1995):
  1. Impose $A_{-}\big|_{\partial M} \to 0, \; \bar{A}_{+}\big|_{\partial M} \to 0$. The asymptotic symmetries are generated by two copies of an affine algebra

$$\left[J_n^a, J_m^b\right] = i\, f^{ab}{}_c\, J_{n+m}^c + \frac{nk}{2}\delta^{ab}\delta_{n+m,0}$$

  2. Further demand $A - A_{AdS_3} \xrightarrow[\rho\to\infty]{} \mathcal{O}(1)$ (Drinfeld-Sokolov reduction), the asymptotic symmetries reduce to two copies of the Virasoro algebra with central charge $c = 6k = 3\ell/(2G_3)$

$$[L_n, L_m] = (n-m)L_{m+n} + \frac{c}{12} n \left(n^2 - 1\right)\delta_{n+m,0}$$

- Let us denote the $sl(2,\mathbb{R})$ generators by $\Lambda^{\pm}$, Λ^0. Fix radial gauge:

$$A = b^{-1} a(x^+, x^-)\, b + b^{-1} db\,, \quad \bar{A} = b\, \bar{a}(x^+, x^-)\, b^{-1} + b\, db^{-1}$$

with $b = b(\rho) = e^{\rho \Lambda^0}$.

- The space of asymptotically anti-de Sitter solutions with a flat boundary metric can be then parameterized as

$$a = \left(\Lambda^+ - \frac{T(x^+)}{k}\Lambda^-\right) dx^+\,, \quad \bar{a} = \left(-\Lambda^- + \frac{\bar{T}(x^-)}{k}\Lambda^+\right) dx^-$$

with corresponding metrics

$$\frac{ds^2}{\ell^2} = d\rho^2 + \frac{1}{k}\left(T(x^+)\, dx^{+\,2} + \bar{T}(x^-)\, dx^{-\,2}\right) - \left(e^{2\rho} + \frac{T(x^+)\,\bar{T}(x^-)}{k^2}\, e^{-2\rho}\right) dx^+ dx^-$$

T, $\bar{T}$ correspond to the stress tensor. E.g. under a residual gauge transformation that preserves the D-S boundary conditions,

$$\delta T = 2T\partial_+\epsilon + \epsilon\,\partial_+ T + \frac{1}{2}\partial_+^3\epsilon$$

- Let us denote the $sl(2,\mathbb{R})$ generators by $\Lambda^{\pm}$, Λ^0. Fix radial gauge:

$$A = b^{-1} a(x^+, x^-)\, b + b^{-1} db\,, \quad \bar{A} = b\, \bar{a}(x^+, x^-)\, b^{-1} + b\, db^{-1}$$

with $b = b(\rho) = e^{\rho \Lambda^0}$.

- The space of asymptotically anti-de Sitter solutions with a flat boundary metric can be then parameterized as

$$a = \left(\Lambda^+ - \frac{T(x^+)}{k} \Lambda^- \right) dx^+\,, \quad \bar{a} = \left(-\Lambda^- + \frac{\bar{T}(x^-)}{k} \Lambda^+ \right) dx^-$$

with corresponding metrics

$$\frac{ds^2}{\ell^2} = d\rho^2 + \frac{1}{k} \left(T(x^+)\, dx^{+\,2} + \bar{T}(x^-)\, dx^{-\,2} \right) - \left(e^{2\rho} + \frac{T(x^+)\, \bar{T}(x^-)}{k^2}\, e^{-2\rho} \right) dx^+ dx^-$$

Examples: global AdS_3 has $T_{AdS_3} = \bar{T}_{AdS_3} = -k/4$ and BTZ has

$$T_{BTZ} = \frac{1}{2}(M\ell - J) = k\frac{\pi^2 \ell^2}{\beta_-^2} \qquad \bar{T}_{BTZ} = \frac{1}{2}(M\ell + J) = k\frac{\pi^2 \ell^2}{\beta_+^2}$$

Generalizing to $N > 2$: embeddings

- When $N > 2$ one needs to choose an embedding of the "gravitational" $sl(2)$ factor into $sl(N)$. The field content in the bulk and the spectrum of the dual CFT depend on this choice.
- Different embeddings are characterized by the way the fundamental representation of $sl(N)$ decomposes into $sl(2)$ representations (classified by integer partitions of N).
- For concreteness, we will focus on the so-called principal embeddings: the fundamental representation becomes an irreducible rep. of the embedded algebra.
- In the principal embedding, the bulk theory consists of the metric and higher spin fields with $s = 3, \ldots, N$. E.g. $\phi_{\mu\nu\rho} \sim \mathrm{Tr}[e_{(\mu} e_{\nu} e_{\rho)}]$. The dual CFT has, in addition to the stress tensor, conformal primaries of weight $3, \ldots, N$ $\Rightarrow$ Irrelevant operators

W_N algebras

- One can apply Drinfeld-Sokolov boundary conditions in the higher spin theory as well. The Virasoro symmetries are retained, but the full asymptotic algebra is extended by the higher spin currents.
- In the principal embedding, the resulting asymptotic symmetry algebra is a non-linear extension of the Virasoro algebra known as W_N algebra (Henneaux, Rey 2010; Campoleoni et. al. 2010). E.g. W_3:

$$i\,\{\,\mathcal{L}_p\,,\,\mathcal{L}_q\,\} = (p-q)\,\mathcal{L}_{p+q} + \frac{c}{12}\,(p^3-p)\,\delta_{p+q,0}$$

$$i\,\{\,\mathcal{L}_p\,,\,\mathcal{W}_q\,\} = (2p-q)\,\mathcal{W}_{p+q}$$

$$i\,\{\,\mathcal{W}_p\,,\,\mathcal{W}_q\,\} = -\frac{\sigma}{3}\left[(p-q)(2p^2+2q^2-pq-8)\,\mathcal{L}_{p+q} + \frac{96}{c}\,(p-q)\,\Lambda_{p+q} + \frac{c}{12}\,p(p^2-1)(p^2-4)\,\delta_{p+q,0}\right]$$

$$\text{with} \quad \Lambda_p \equiv \sum_{q\in\mathbb{Z}} \mathcal{L}_{p+q}\mathcal{L}_{-q}$$

Turning on sources

- Black hole solutions carrying higher spin charges have been constructed (Ammon, Gutperle, Kraus, Perlmutter 2011; Castro, Hijano, Lepage-Jutier, Maloney 2011)
- Since black holes represent states in thermodynamic equilibrium, they must carry chemical potentials which are the thermodynamic conjugate of the higher spin charges.
- The structure of the solutions is

$$a = \Big(\Lambda^+ + Q\Big)dx^+ + \Big(M + \ldots\Big)dx^-$$

$$\big[\Lambda^-, Q\big] = 0\,, \qquad Q: \text{ VEVs}$$
$$\big[\Lambda^+, M\big] = 0\,, \qquad M: \text{ sources}$$

Turning on sources

- Black hole solutions carrying higher spin charges have been constructed (Ammon, Gutperle, Kraus, Perlmutter 2011; Castro, Hijano, Lepage-Jutier, Maloney 2011)
- Since black holes represent states in thermodynamic equilibrium, they must carry chemical potentials which are the thermodynamic conjugate of the higher spin charges.
- The structure of the solutions is

$$a = \Big(\Lambda^+ + Q\Big) dx^+ + \Big(M + \dots\Big) dx^-$$

$$a = \begin{pmatrix} 0 & \frac{Q_2}{2} & Q_3 \\ 1 & 0 & \frac{Q_2}{2} \\ 0 & 1 & 0 \end{pmatrix} dx^+ + \begin{pmatrix} -\frac{\mu_3 Q_2}{6} & \frac{\mu_2 Q_2}{2} + \mu_3 Q_3 & \mu_2 Q_3 + \frac{\mu_3 Q_2^2}{4} \\ \mu_2 & \frac{\mu_3 Q_2}{3} & \frac{\mu_2 Q_2}{2} + \mu_3 Q_3 \\ \mu_3 & \mu_2 & -\frac{\mu_3 Q_2}{6} \end{pmatrix} dx^-$$

Turning on sources

- Black hole solutions carrying higher spin charges have been constructed (Ammon, Gutperle, Kraus, Perlmutter 2011; Castro, Hijano, Lepage-Jutier, Maloney 2011)
- Since black holes represent states in thermodynamic equilibrium, they must carry chemical potentials which are the thermodynamic conjugate of the higher spin charges.
- The structure of the solutions is

$$a = \Big(\Lambda^+ + Q\Big)dx^+ + \Big(M + \dots\Big)dx^-$$

$$\begin{aligned} \big[\Lambda^-, Q\big] &= 0\,, \qquad Q:\ \text{VEVs} \\ \big[\Lambda^+, M\big] &= 0\,, \qquad M:\ \text{sources} \end{aligned}$$

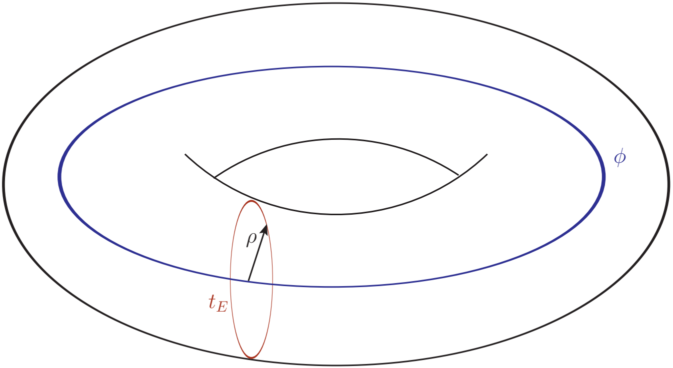
Ward identities

- Momentarily going back to $N = 2$ for simplicity:
$$a = \begin{pmatrix} 0 & T_{++} \\ 1 & 0 \end{pmatrix} dx^+ + \begin{pmatrix} -\frac{1}{2}\partial_+\mu & \mu T_{++} - \frac{1}{2}\partial_+^2\mu \\ \mu & \frac{1}{2}\partial_+\mu \end{pmatrix} dx^-$$
Flatness (bulk EOM) is equivalent to
$$\partial_- T_{++} = 2T_{++}\,\partial_+\mu + \mu\,\partial_+ T_{++} - \frac{1}{2}\partial_+^3\mu$$
which is the stress tensor Ward identity in the presence of a coupling $\int d^2x\,\mu T_{++}$.

- Important subtlety: μ corresponds to a non-trivial boundary metric; in the Euclidean formulation, globally-defined black hole solutions have constant T_{++} , and (a constant) μ can be incorporated via the modular parameter of the boundary torus instead $\Rightarrow$ for $N = 2$ we can still have black hole solutions with $a_- = 0$. For $N > 2$ we source irrelevant operators and $a_- \neq 0$.

Euclidean continuation: smoothness conditions

- Consider the analytical continuation $x^{+} \to z\,,\, x^{-} \to -\bar{z}\,.$ In the Euclidean formulation the topology of the bulk manifold is that of a solid torus, and the boundary torus is defined by the identifications $z \simeq z + 2\pi \simeq z + 2\pi\tau$ (e.g. $\tau_{BTZ} = i\beta(1+\Omega)/(2\pi)$)



Euclidean continuation: smoothness conditions

- Consider the analytical continuation $x^+ \to z\,,\ x^- \to -\bar{z}\,.$ In the Euclidean formulation the topology of the bulk manifold is that of a solid torus, and the boundary torus is defined by the identifications $z \simeq z + 2\pi \simeq z + 2\pi\tau$ (e.g. $\tau_{BTZ} = i\beta(1+\Omega)/(2\pi)$)
- The holonomy under $z \simeq z + 2\pi\tau$ is

$$\mathrm{Hol}_{\tau,\bar{\tau}}(A) = b^{-1} e^{h}\, b\,, \qquad \mathrm{Hol}_{\tau,\bar{\tau}}(\bar{A}) = b\, e^{\bar{h}} b^{-1}$$

where the matrices h and $\bar{h}$ are defined as

$$h = 2\pi\left(\tau a_z + \bar{\tau} a_{\bar{z}}\right)\,, \qquad \bar{h} = 2\pi\left(\tau \bar{a}_z + \bar{\tau}\bar{a}_{\bar{z}}\right).$$

- Demanding that the holonomy around the contractible cycle is trivial provides a gauge-invariant characterization of a smooth black hole horizon (Gutperle, Kraus 2011).

Entropy from Euclidean variational principle

- In the saddle-point approximation (large T and c) the CFT partition function is obtained from the Euclidean on-shell action as

$$\ln Z = -I^{(E)}_{os} = -\left(I^{(E)}_{CS} + I^{(E)}_{Bdy}\right)\Big|_{os}$$

where

$$I^{(E)}_{CS} = \frac{ik_{cs}}{4\pi}\int_M \mathrm{Tr}\Big[CS(A) - CS(\bar{A})\Big]$$

and I_{Bby} is a boundary term chosen such that

$$\delta \ln Z \sim T\delta\tau - \bar{T}\delta\bar{\tau} + \sum_{j=3}^{N}\left(Q_j\delta\mu_j - \bar{Q}_j\delta\bar{\mu}_j\right)$$

- The entropy can then be obtained by performing a Legendre transform. As a function of the charges, S satisfies

$$\delta S \sim -\tau\delta T + \bar{\tau}\delta\bar{T} + \sum_{j=3}^{N}\left(-\mu_j\delta Q_j + \bar{\mu}_j\delta\bar{Q}_j\right)$$

Higher spin black hole entropy

- We constructed boundary terms suited to black hole solutions in Drinfeld-Sokolov form.
- Evaluating the on-shell action (free energy) and Legendre-transforming we found (de Boer, J.I.J., 2013)

$$S = -2\pi i k_{cs} \,\mathrm{Tr}\Big[\left(a_z + a_{\bar{z}}\right)\left(\tau a_z + \bar{\tau} a_{\bar{z}}\right) - \left(\bar{a}_z + \bar{a}_{\bar{z}}\right)\left(\tau \bar{a}_z + \bar{\tau} \bar{a}_{\bar{z}}\right)\Big]$$

- In the BTZ branch the smoothness conditions can be encoded as $\mathrm{spec}\Big(2\pi\left(\tau a_z + \bar{\tau} a_{\bar{z}}\right)\Big) = \mathrm{spec}\left(2\pi i \Lambda^0\right)$. Using this it is easy to prove that our entropy formula satisfies the first law (integrability).
- Moreover, it is valid for both static and rotating higher spin black holes, in any embedding (generalizes (Canto, Bañados, Theisen, 2012))

More fun with the entropy

- For constant connections, $a_\varphi = (a_z + a_{\bar z})$ and $(\tau a_z + \bar\tau a_{\bar z})$ commute by the e.o.m. Together with $(\tau a_z + \bar\tau a_{\bar z}) = u^{-1}\left(i\Lambda^0\right)u$ from the holonomy conditions this implies

$$S = 2\pi k_{cs} \mathrm{Tr}\left[\left(\lambda_\varphi - \bar\lambda_\varphi\right)\Lambda^0\right]$$

where λ_φ and $\bar\lambda_\varphi$ are the diagonal matrices whose entries contain the eigenvalues of a_φ and $\bar a_\varphi$.

- This is the higher-spin generalization of the Cardy formula for the pure gravity case. Recall:

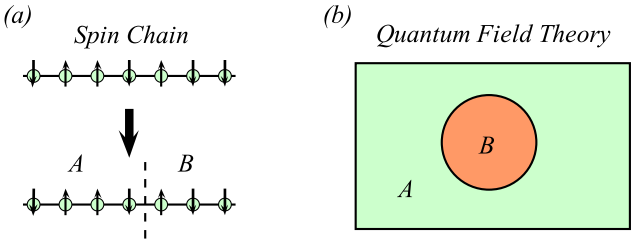
$$S_{BTZ} = 2\pi\left(\sqrt{\frac{c}{6}\mathcal{T}} + \sqrt{\frac{c}{6}\bar{\mathcal{T}}}\right) \qquad a_\varphi = u^{-1}\begin{pmatrix}\sqrt{\frac{\mathcal{T}}{k}} & 0 \\ 0 & -\sqrt{\frac{\mathcal{T}}{k}}\end{pmatrix}u$$

$$\boxed{c = 6k}$$

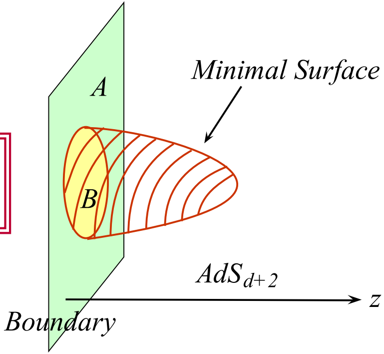
Entanglement

- Consider a quantum system in a pure (or mixed) state, with density operator $\rho = |\Psi\rangle\langle\Psi|$ (or $\rho = e^{-\beta H}$).
- If we partition the Hilbert space as $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$ ($B = A^c$), the reduced density matrix for subsystem A is defined as $\rho_A = \mathrm{Tr}_B\,\rho$.
- The entanglement entropy S_A associated with A is then given by the Von Neumann entropy of ρ_A: $S_A = -\mathrm{Tr}_A\,\rho_A \log \rho_A$.
- It is a non-local measure of the correlation between subsystems A and B, and can be defined at $T = 0$ (as opposed to $S_{thermal}$).
- It can be useful to characterize different phases in the absence of spontaneous symmetry breaking and classical order parameters.

Ryu and Takayanagi (2006)



$$\boxed{S_A = \frac{\mathrm{Area}(\gamma_A)}{4G_N}}$$



Entanglement entropy for higher spin theories

- Our starting observation is that the geodesic distance in AdS_3 can be written as

$$\cosh d(P,Q) = \frac{1}{2}\mathrm{Tr}_{\mathcal{R}}\left[\mathcal{P}\exp\left(\int_Q^P \bar{A}\right)\mathcal{P}\exp\left(\int_P^Q A\right)\right]$$

- Since ent ent in AdS_3 is related to the geodesic distance via the Ryu-Takayanagi prescription, we propose

$$S_{ent} = k_{cs}\log\left[\lim_{\rho_0\to\infty} W(P,Q)\Big|_{\rho_P=\rho_Q=\rho_0}\right]$$

where

$$W(P,Q) \equiv \mathrm{Tr}_{\mathcal{R}}\left[\mathcal{P}\exp\left(\int_Q^P \bar{A}\right)\mathcal{P}\exp\left(\int_P^Q A\right)\right]$$

Test 1: Recover known CFT results in the absence of higher spin charges

- For $N = 2$ we get:

$$S_{PAdS_3} = \frac{c}{3} \log \left[\frac{x_P - x_Q}{a} \right]$$

$$S_{AdS_3} = \frac{c}{3} \log \left[\frac{\ell}{a} \sin \left(\frac{\varphi_P - \varphi_Q}{2} \right) \right]$$

$$S_{BTZ} = \frac{c}{6} \log \left\{ \frac{\beta_+ \beta_-}{\pi^2 a^2} \sinh \left[\frac{\ell}{\beta_+} \pi \left(\varphi_P - \varphi_Q \right) \right] \sinh \left[\frac{\ell}{\beta_-} \pi \left(\varphi_P - \varphi_Q \right) \right] \right\}$$

in agreement with CFT results (Holzhey, Larsen, Wilczek 1994; Calabrese, Cardy 2004) and holographic calculations (Ryu,Takayanagi, 2006; Hubeny, Rangamani, Takayanagi, 2007)

Test 2: Thermal entropy

- At finite T, when system A grows and approaches the full system one should recover the thermal entropy.
- In R-T this result is recovered because the geodesic then wraps the horizon, effectively computing its area.
- In our proposal the Wilson lines become loops in the ϕ direction, and

$$S_{EE} \to k_{cs} \log \left[\lim_{\rho_0 \to \infty} \mathrm{Tr} \left(\mathcal{P} e^{-\oint \bar{A}_\varphi \, d\varphi} \; \mathcal{P} e^{\oint A_\varphi \, d\varphi} \right) \Big|_{\rho=\rho_0} \right]$$
$$= k_{cs} \log \left[\lim_{\rho_0 \to \infty} \mathrm{Tr} \Big(\mathrm{Hol}_\varphi(\bar{A}) \mathrm{Hol}_\varphi(A) \Big) \Big|_{\rho=\rho_0} \right]$$

- For constant connections $\mathrm{Hol}_\varphi(A) = b^{-1} e^{2\pi a_\varphi} b$, and (choosing the representation appropriately) we recover

$$S = 2\pi k_{cs} \mathrm{Tr}\Big[\left(\lambda_\varphi - \bar{\lambda}_\varphi \right) \Lambda^0 \Big]$$

Test 3: Strong subadditivity

- An important property of entanglement entropy is that it is strongly subadditive

$$S_{EE}(A) + S_{EE}(B) \geq S_{EE}(A \cup B) + S_{EE}(A \cap B)$$

- We do not have a generic proof that our prescription satisfies this property, but we have verified it numerically in different examples, including the spin-3 black hole.

Outlook

- Higher spin theories are an interesting arena to explore holography, and may offer a window into the dynamics of string theories in certain limits.
- Many simplifications in three dimensions: truncation to finite number of higher spin fields, Chern-Simons formulation, some concrete checks possible with $2d$ CFT machinery.
- Usual $\mathrm{AdS}_3/\mathrm{CFT}_2$ recovers several universal results such as Cardy entropy formula, entanglement entropy at finite and zero temperature, etc. We have discussed the generalization of these results to higher spins in AdS_3.
- It would be interesting to have a first principles (field-theoretical) derivation of entaglement entropy in CFTs deformed by higher spin operators.