This research has been co-financed by the European Union (European Social Fund, ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF), under the grants schemes "Funding of proposals that have received a positive evaluation in the 3rd and 4th Call of ERC Grant Schemes" and the EU program "Thales" ESF/NSRF 2007-2013.

European Union
European Social Fund

OPERATIONAL PROGRAMME
EDUCATION AND LIFELONG LEARNING
investing in knowledge society
MINISTRY OF EDUCATION & RELIGIOUS AFFAIRS, CULTURE & SPORTS
MANAGING AUTHORITY

NSRF
2007-2013
programme for development
EUROPEAN SOCIAL FUND

Co-financed by Greece and the European Union

ESF Holograv workshop: "Applied Holography"
7 Martiou 2013

Quantum criticality at finite density, hyperscale violation and symmetry breaking.

Elias Kiritsis

University of Crete APC, Paris

Bibliography

Based on ongoing work with

B. Gouteraux, (Nordita)

and published recent work with

B. Gouteraux (Nordita) arXiv:1212.2625 [hep-th]

B. Gouteraux (Nordita) arXiv:1107.2116 [hep-th]

B. S. Kim and C. Panagopoulos (Crete) arXiv:1012.3464 [cond-mat.str-el]

C. Charmousis, B. Gouteraux (Orsay), B. S. Kim and R. Meyer (Crete)
. arXiv:1005.4690 [hep-th]

Related work:

J. Gath, J. Hartong, N. A. Obers and R. Monteiro, (NBI)
arXiv:1212.3263 [hep-th].

Plan

- Introduction

- Towards mapping the QC landscape

- The arena: Einstein-Maxwell-Dilaton EH theories.

- Generalized Criticality and hyperscaling violations

- Symmetry breaking IR asymptotics and Critical lines

- Outlook

Introduction

There are two roads to the theoretical description of nature:

• Targeted model building driven by experimental data

• Exploration of theoretical possibilities

In QFT the second approach was pioneered by Wilson:

• Specify the symmetry

• Find all theories that are scale invariant (SITs) and respect that symmetry.

• Map the neighborhood of each SIT, by using a local chart of low dimension scaling operators, and determine the local RG flows.

• Fill in the global set of RG Flows, connecting the network of SITs.

Symmetries

• In HEP the basic symmetry required is Poincaré invariance that together with scaling leads (usually) to conformal invariance.

In non-relativistic frameworks (condensed matter) several reductions are possible

• Give up Boosts

• Give up translation invariance

• Give up rotations

• Allowing Lifshitz scaling symmetries

• Allowing more complex symmetries like Schrödinger symmetries.

Fixed point theories

The main "atoms" in the QFT "lego-game" are the Fixed point theories (Scale invariant Theories) for a given symmetry universality class.

• At weak coupling, such theories can be searched for perturbatively. There are VERY FEW examples beyond free-field theories.

• At strong coupling, only very special symmetries (like extended supersymmetry) or the large-N_c expansion can provide a few more examples. (2d is an exception)

♠ All in all, we know VERY FEW Scale invariant Theories in three and more dimensions.

• Since the AdS/CFT correspondence entered the game, many more became known: they are large N_c theories at strong coupling.

• Despite this, we know only a drop in the ocean of SITs.

.

• To go further and map the neighborhood of SITs, we must “solve” them.

• For the first step we need the scaling dimensions.

• For the next step we need OPE coefficients.

• Once we have them we can locally map the neighborhood and draw a flow chart.

♠ The final step , following RG a finite distance away is only possible at weak coupling and in some cases which can be argued on the basis of symmetries and other special info.

Classification of QC theories

• The programm: Classification of SI theories (The Wilsonian approach in AdS/CFT).

• The strategy is to use Effective Holographic Theories (the analogue of effective FT in the holographic case) in order to explore all possible QC holographic scale invariant theories with given symmetries.

Charmousis+Gouteraux+Kim+E.K.+Meyer

To do this we must

1. Select the operators expected to be important for the dynamics

2. Write an effective (gravitational) holographic action that captures the (IR) dynamics by parametrizing the IR asymptotics of interactions .

3. Find the scaling solutions describing extremal saddle points, with given symmetries. Built the $T \to 0$ bh solutions around them

4. Study the physics around each acceptable saddle point.

• This strategy has been applied sporadically so far and started bearing fruit:

• It dealt with various symmetry classes, including Poincaré invariance, Lifshitz symmetries, hyperscaling violation and more general Bianchi-type symmetries.

Charmousis+Gouteraux+Kim+E.K.+Meyer

Perlmutter, Gouteraux+E.K.

Huisje+Sachdev+Swingle

Dong+Harrison+Kachru+Torroba+Wang

Iizuka+Kachru+Kundu+Narayan+Sircar+Trivedi

Donos+Gauntlett, Donos+Gauntlett+Pantelidou

Hartnoll+Huijse, Hartnoll+Shaghoulian, Donos+Hartnoll

Iizuka+Kachru+Kundu+Narayan+Sircar+Trivedi+Wang

Iizuka+Maeda

The ingredients in the classification

♠ We look for QC theories at finite density (single U(1)). We will be for concreteness in 2+1 boundary dimensions. The results are general.

• The minimum number of fields that will be needed in this context are two:

♠ The (conserved) stress tensor $T_{\mu\nu}$ dual to a graviton $g_{\mu\nu}$ in the bulk.

♠ The (conserved) U(1) current J_{μ} dual to a gauge field A_{μ} in the bulk.

• The physics in that case is captured by the Reissner-Nordstrom bh and this has been studied in detail.

• Even this simplest of cases provided for surprises, namely the emergent semilocal criticality in the IR (at finite density) associated to the $AdS_2 \times R^n$ geometry.

• The next step is include the most important scalar operator, dual to a bulk scalar ϕ. The holographic theories become richer and are described by an Einstein-Maxwell-Dilaton theory.

• There are several options that appear in such a case:

• The symmetry: we focus on rotations + 2-d translations +scaling: Hyperscaling geometries:

$$AdS_4 \quad : \quad ds^2 = \frac{dr^2 - dt^2 + d\vec{x}^2}{r^2}$$

$$AdS_2 \times R^2 \quad : \quad ds^2 = \frac{dr^2 - dt^2}{r^2} + d\vec{x}^2$$

$$\mathrm{z-Lifshitz} \quad : \quad ds^2 = -\frac{dt^2}{r^{2z}} + \frac{dr^2 + d\vec{x}^2}{r^2}$$

• With violation of hyperscaling

$$(\theta, \mathrm{z}) - \mathrm{Lifshitz} \quad : \quad ds^2 = r^\theta \left[-\frac{dt^2}{r^{2z}} + \frac{dr^2 + d\vec{x}^2}{r^2} \right]$$

Associated always with a running scalar, towards the edge of field space.

Fractionalized vs cohesive phases

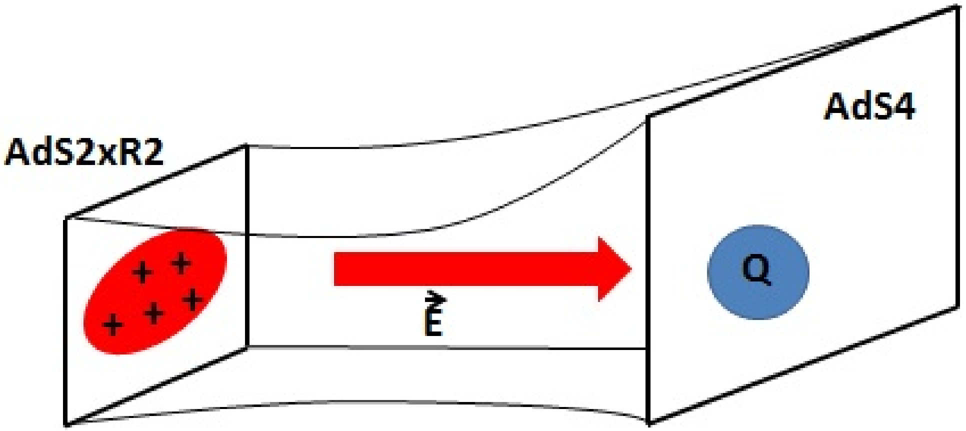
- Event horizons $\Leftrightarrow$ deconfined phases $\Leftrightarrow$ fractionalised dofs.

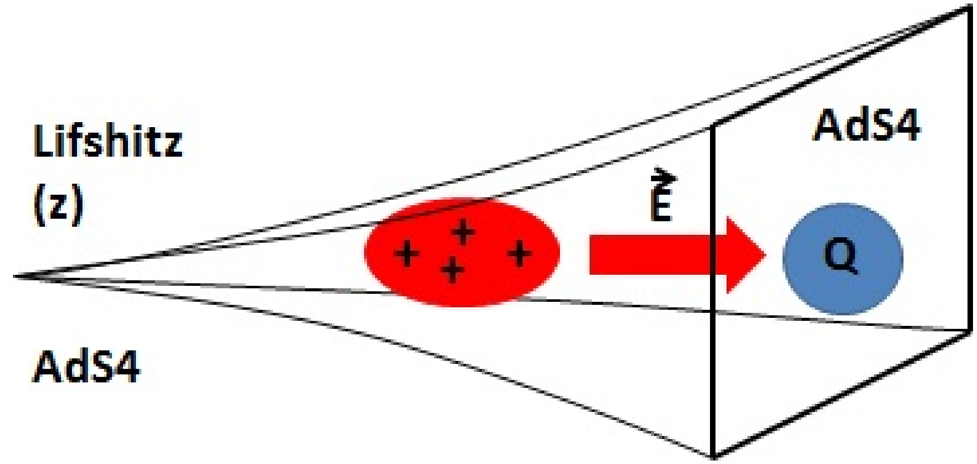
Witten

- fractionalization (condensed matter) = deconfinement (high energy physics)

- The characteristic of the presence of a horizon is the presence of gapless (low energy modes)

- there are separate contributions to the boundary charge density (Hartnoll)





$$Fractionalised\ \ phase\quad :\quad \lim_{r\to\infty}\int\star F\ \simeq Q\neq 0$$

$$Cohesive\ \ phase\quad :\quad \lim_{r\to\infty}\int\star F\ \simeq\ 0$$

The holographic Luttinger theorem is valid in cohesive phases only (Hartnoll).

- Note that the two statements:

the current is irrelevant in the IR, (when the U(1) symmetry is broken) and

the flux is zero in the IR,

ARE NOT equivalent.

Broken vs unbroken symmetry

• The U(1) symmetry is unbroken (= the U(1) gauge boson is massless in the bulk). The relevant EMD theory is

$$S_0 = \int d^4x \sqrt{-g} \left[R - \frac{1}{2} \partial\phi^2 - Z(\phi) F^2 + V(\phi) \right]$$

• The U(1) symmetry is broken (= the U(1) gauge boson is massive in the bulk).

There are two types of breakings of the U(1) symmetry:

♠ Explicit: non-trivial coupling for a charged operator in the UV.

♠ Spontaneous: no coupling to a charged operator, only a vev.

• Local criticality in the IR does not distinguish between the two.

The simplest relevant action involves a complex (charged) scalar Ψ,

$$S = M^2 \int d^4x \sqrt{-g} \left[R - \frac{G(|\Psi|)}{2} |D\Psi|^2 + \tilde{V}(|\Psi|) - \frac{\tilde{Z}(|\Psi|)}{4} F^2 \right]$$

with the standard covariant derivative as

$$D_\mu \Psi = \partial_\mu \Psi + iqA_\mu \Psi \,.$$

Fixing the phase of $\Psi = \chi e^{i\theta}$ to zero, and redefining the kinetic terms so that the new scalar ϕ is canonically normalized we obtain

$$S_M = M^2 \int d^4x \sqrt{-g} \left[R - \frac{1}{2} (\partial\phi)^2 + V(\phi) - \frac{Z(\phi)}{4} F^2 - \frac{W(\phi)}{2} A^2 \right].$$

- In the UV

- $W(\phi) = 0$: normal phase, $U(1)$ unbroken

- $W(\phi) \neq 0$: $U(1)$ broken (spontaneously or explicitly)

- In the IR

  - Suppose

$$V_{eff}(\phi) = V(\phi) - Z(\phi)F^2 - W(\phi)A^2 \,, \qquad dV_{eff}(\phi)/d\phi|_{\phi_\star} = 0$$

$\Rightarrow$ solutions with hyperscaling

  - In the IR, suppose that the scalar has a runaway behaviour. Define

$$\gamma = \text{Infimum}\{\gamma_0 \in R : \lim_{\phi\to\infty} e^{-\gamma_0\phi} Z(\phi) > 0\}$$

$$\epsilon = \text{Infimum}\{\epsilon_0 \in R : \lim_{\phi\to\infty} e^{-\epsilon_0\phi} W(\phi) > 0\}$$

$$\delta = \text{Infimum}\{\delta_0 \in R : \lim_{\phi\to\infty} e^{-\delta_0\phi} V(\phi) > 0\}$$

In the deep IR the scalar couplings can be (almost always) approximated by exponentials

$$V(\phi) \sim V_0 e^{\delta\phi}, \quad Z(\phi) \sim Z_0 e^{\gamma\phi}, \quad W(\phi) \sim W_0 e^{\epsilon\phi}, \qquad \phi \to \infty$$

$\Rightarrow$ hyperscaling violation ($\delta \neq 0$)

Generalized QC points

• There are three critical exponents (z, θ, ζ) that characterize the scaling of the theories with a U(1) symmetry

♠ The Lifshitz exponent z defined via the scaling of space-time coordinates

$$(r, x, y) \quad \to \quad \lambda(r, x, y) \quad , \quad t \quad \to \quad \lambda^z \, t$$

♠ The hyperscale-violation exponent θ defined from the transformation of the metric

$$ds^2 \quad \to \quad \lambda^\theta \, ds^2$$

• In all hyperscaling violating solutions known

$$V \, ds^2 \quad \to V \, ds^2$$

This comes from the fact that $R \cdot ds^2$ should scale the same way as V.

• This also explains why $\theta = 0$ if V is a constant (constant scalars).

♠ The U(1) hyperscale-violation exponent ζ is defined from the gauge field one form

$$A \equiv A_t dt \quad \to \lambda^{\zeta}\ A$$

♠ An alternative but equivalent definition is that the charge density Q and chemical potential μ transform as

$$\mu \quad \to \quad \lambda^{-z}\ \mu \quad , \quad Q \quad \to \quad \lambda^{-\zeta}\ Q$$

• The theories with U(1) breaking have three parameters

$$\gamma \quad , \quad \delta \quad , \quad \lim_{\phi\to\infty} \frac{W(\phi)}{Z(\phi)V(\phi)}$$

that allow (z, θ, ζ) to be independent.

Classification of symmetry-breaking QC points

• Solve the equations in the critical regime by enumerating cases depending on the three basic terms:

1. Vector kinetic term

2. Vector mass term

3. Potential term

• The rest of the terms come from the metric and the kinetic term of the scalar. They all scale as $\mathcal{O}(r^{-2})$ for scaling solutions.

• By choosing a subset of these terms to vanish in the critical solution, and solving the rest of the equations we find all possible scaling solutions.

• There are $2^3 - 1$ cases to consider.

• We may study scaling deformations around each solution and learn about its stability or instability.

• Doing this, one finds many possible IR fixed points that are only determined by the asymptotic (IR) behavior of potentials.

• Whether they can actually appear as endpoint of RG flows in a given theory is a different question.

• As we usually solve from the IR, one should start running up from all possible IR scaling theories.

• In may cases, analyses till now were incomplete as there are more than one possible $\langle$vevs$\rangle$ per coupling indicating "competing" solutions.

• All of this happens even in the simplest of EHTs with a single scalar.

Examples in simple cases: zero density

• Consider an EMD at zero charge density. Then the landscape of critical points is determined by:

♠ Finite critical points ϕ_*: $V'(\phi_*) = 0$. If $V''(\phi_*) > 0 \quad \to \quad$ UV fixed point. If $V''(\phi_*) < 0 \quad \to \quad$ IR fixed point.

The geometry at $\phi = \phi_*$ is AdS_{p+1}. UV fixed points are repulsive while IR ones attractive.

♠ Infinite critical points ϕ_∞ (where $\lim_{\phi\to\phi_\infty} V \to \infty$).
The geometry is hyperscaling-violating AdS_{p+1} as ϕ runs to ϕ_∞. The fixed points are attractive or repulsive as a function of the whole diagram.

Examples in simple cases: finite density

Consider a potential with a single minimum for simplicity.

• IR fixed points with $AdS_2 \times R^{p-1}$, constant $\phi = \phi_*$ extremizing

$$V_{eff} = V(\phi) - Z(\phi)F^2$$

• IR fixed points violating hyperscaling with as ϕ runs to ϕ_∞ and a gauge field that has non-trivial IR flux.

• IR fixed points violating hyperscaling with as ϕ runs to ϕ_∞ and a gauge field that has trivial IR flux.

Hyperscaling (constant scalar), neutral IR

$$S = \int d^4x\sqrt{-g}\left[R - \partial\phi^2 - Z(\phi)F^2 - W(\phi)A^2 + V(\phi)\right]$$

$$V_{eff}(\phi) = V(\phi) - Z(\phi)F^2 - W(\phi)A^2\,, \qquad dV_{eff}(\phi)/d\phi|_{\phi_\star} = 0$$

- **Neutral fixed point**: $Q = 0$ at leading order $\Rightarrow$ AdS$_4$ (**cohesive**)

$$\delta A \underset{r\to\infty}{\sim} r^{(2-\Delta_A)}, \qquad \Delta_A = \frac{3}{2} + \sqrt{\frac{1}{4} + \frac{L_\star^2 W_\star}{Z_\star}} > 2$$

If $\Delta_A > 3$, A is dual to an irrelevant operator: scale invariance is unbroken

If $2 < \Delta_A < 3$, A is dual to a relevant operator: flow to some (charged) fixed point,

Gubser+Nellore'09

$$\delta\phi \underset{r\to\infty}{\sim} r^{(3-\Delta_\phi)}, \qquad \Delta_\phi = \frac{3}{2}\left(1 + \sqrt{1 - 4L_\star^2 V_\star''}\right)$$

If $V_\star'' < 0$, $\delta\phi$ is an irrelevant perturbation, while it becomes relevant (and complex) for $V_\star'' > 0$ ($V_\star'' > 9/4L_\star^2$)

Hyperscaling (constant scalar), charged IR

- Charged fixed point ($\phi = \phi_*$): $Q \neq 0$ at leading order

- $W_\star = 0 \Rightarrow AdS_2 \times R^2$: **fractionalised** phase

Two possibly irrelevant deformations ($W'_\star = 0$):

$$\beta_1 = -1, \quad \beta_2 = \frac{1}{2}\left(1 - \sqrt{1-4\lambda}\right), \quad \lambda = \frac{V''_\star}{V_\star} + \frac{W''_\star}{V_\star Z_\star} + \frac{Z''_\star}{Z_\star} - \frac{2V'^2_\star}{V_\star{}^2}$$

β_2 is irrelevant (relevant) if $\lambda < 0$ ($\lambda > 0$).

- $W_\star \neq 0 \Rightarrow$ Lifshitz ($z > 1$): **cohesive** phase

Irrelevant deformations ($V'_\star = W'_\star = Z'_\star = 0$):

$$\beta_1 = \frac{1}{2}\left(z + 2 - \sqrt{20 - 20z + 9z^2}\right), \quad \beta_2 = \frac{z+2}{2}\left(1 - \sqrt{1-4\lambda}\right)$$

$$\beta_1 < 0 \quad \text{iff} \quad z > 2\,, \quad \beta_2 < 0 \quad \text{iff} \quad \lambda < 0\,, \quad \lambda = \frac{L^2 V''_\star + \frac{1}{2}L^2 Q^2 W''_\star + \frac{1}{2}z^2 Q^2 Z''_\star}{(z+2)^2}\,.$$

Hyperscaling violating (running scalar), neutral IR

$$V(\phi) \sim V_0 e^{-\delta\phi}, \quad Z(\phi) \sim Z_0 e^{\gamma\phi}, \quad W(\phi) \sim W_0 e^{\epsilon\phi}$$

- Neutral, (cohesive) IR (z=1): to leading order $Z, W \underset{\phi\to\infty}{\longrightarrow} 0$ (power series solution)

$$ds^2 = r^\theta \left(\frac{L^2 dr^2 - dt^2 + d\vec{x}^2}{r^2}\right) + \cdots, \quad \phi = \frac{\theta}{\delta} \log r + \cdots, \quad \theta = \frac{2\delta^2}{\delta^2 - 1}$$

$L^2 = (\theta - 3)(\theta - 2)/V_0$, so the IR is $r \to +\infty$ if $\theta < 0$ or $r \to 0$ if $\theta > 3$

- Note that the symmetry gets restored in the IR solution.

- Two irrelevant deformations

Hyperscaling violating (running), Charged IR (I)

$$V(\phi) \sim V_0 e^{-\delta\phi}, \quad Z(\phi) \sim Z_0 e^{\gamma\phi}, \quad W(\phi) \sim W_0 e^{\epsilon\phi},$$

- Charged, fractionalised fixed point: $Q \neq 0$ but $W \underset{\phi\to\infty}{\longrightarrow} 0$

$$ds^2 = r^\theta \left[-\frac{dt^2}{r^{2z}} + \frac{L^2 dr^2 + d\vec{x}^2}{r^2} \right] + \cdots, \qquad \phi = \frac{\theta}{\delta} \ln r + \cdots \qquad z, \theta = F(\gamma, \delta)$$

$$A_t = Q r^{\theta - z - 2}, \quad L^2(\theta, z), \quad Q(\theta, z)$$

- Allowed parameter range:

$$IR: r \to 0: \quad [2 < \theta \leq 3, z > 1], \quad [\theta > 3, z < 1],$$
$$IR: r \to +\infty: \quad [\theta \leq 0, z > 1], \quad \left[0 < \theta < 2, z > \frac{2+\theta}{2}\right].$$

Always two irrelevant, real deformations, $\delta\phi = \phi_0$ and $\beta(\theta, z)$ in the allowed range.

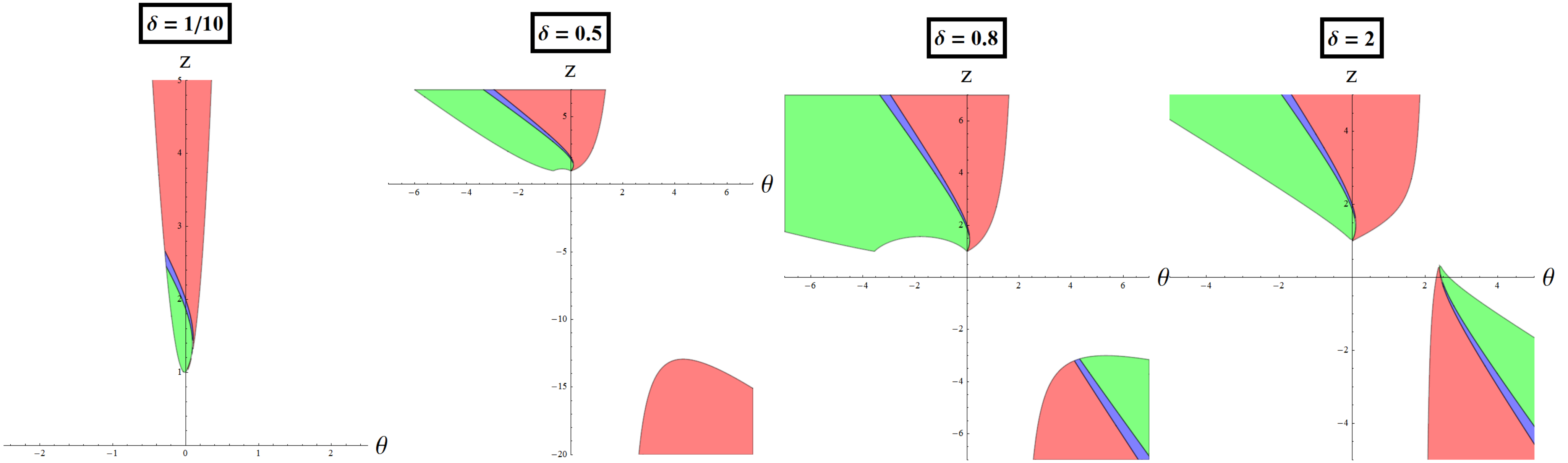
Hyperscaling violating (running), Charged IR (II)

$$V(\phi) \sim V_0 e^{-\delta\phi}, \quad Z(\phi) \sim Z_0 e^{\gamma\phi}, \quad W(\phi) \sim W_0 e^{\epsilon\phi}, \qquad \phi \sim \frac{\theta}{\delta} \ln r \to \infty$$

- Charged, cohesive fixed point: $Q \neq 0$, exact solution:

$$\mathrm{d}s^2 = r^\theta \left[-\frac{\mathrm{d}t^2}{r^{2z}} + \frac{L^2 \mathrm{d}r^2 + \mathrm{d}\vec{x}^2}{r^2} \right], \quad \epsilon = \gamma - \delta\,, \quad (z, \theta, \zeta) \sim F\left(\gamma, \delta, \frac{W_0}{Z_0 V_0} \right)$$

One marginally irrelevant mode $\delta\phi = \phi_0$.



Plots of the allowed parameter space (θ, z) for various values of the exponent δ. The upper left corner is the region where the IR is $r \to +\infty$, the lower right where it is $r \to 0$. In red, we depict the region where β_- is a real irrelevant deformation; in blue, the region where it is real and relevant; in green, the region where it is complex and relevant. In this case, the geometry is dynamically unstable.

QC lines

• The hyperscaling violating solutions correspond to quantum critical critical lines with continuous parameter ϕ_0.

• To leading order in $1/N_c$, the physics is independent of the continuous parameter ϕ_0.

• This is equivalent to the statement that they contain a hyperscaling-violating scale ℓ, and therefore no dimensionless parameter.

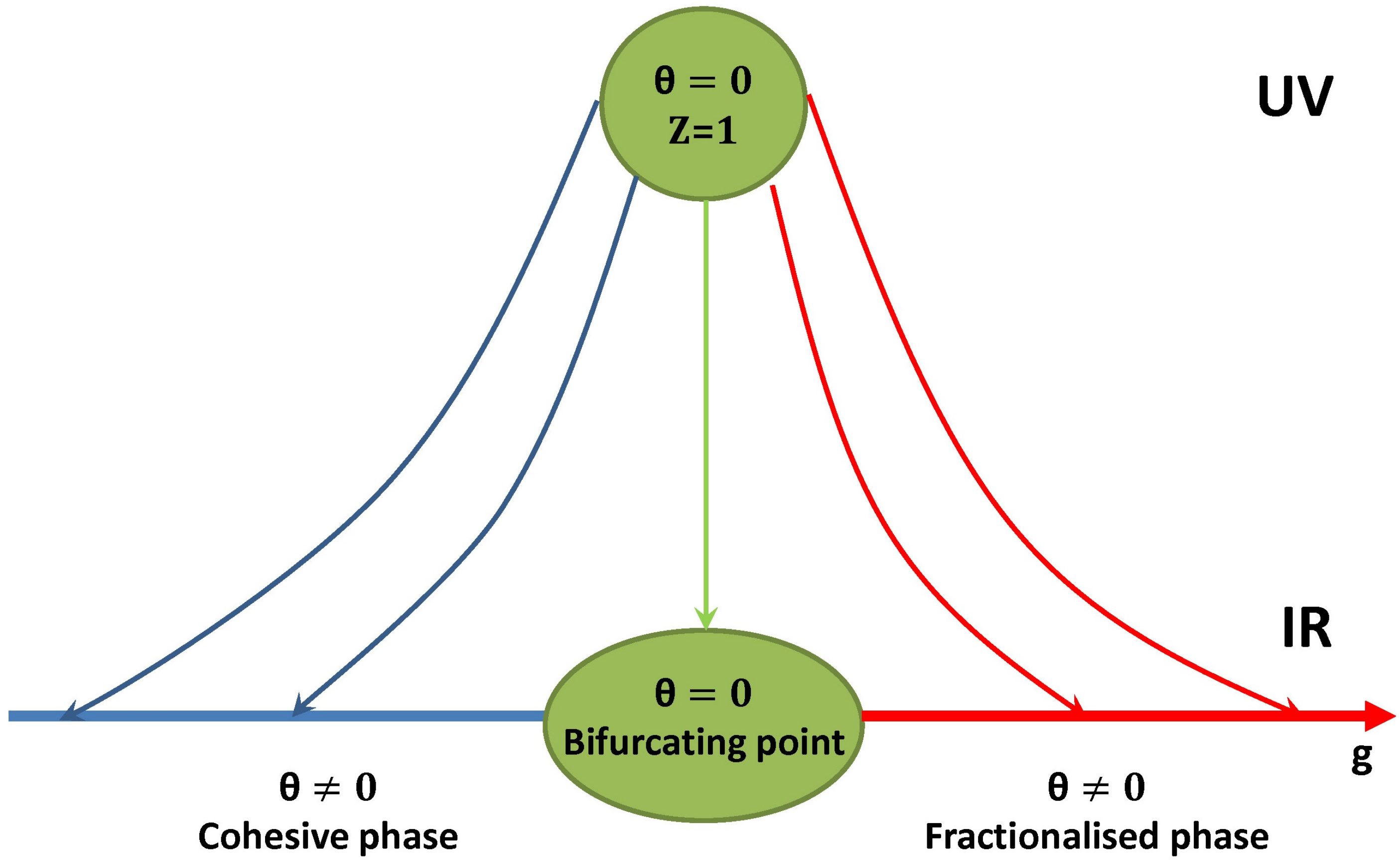
• The situation at $\mathcal{O}(1/N_c^2)$ is expected to generate a bona fide line of points.

♠ The argument: map ℓ to internal torus volume.

♠ At tree level string theory is volume independent (this is also true to $\mathcal{O}(N_c^2)$ in QFT à la Eguchi-Kawai).

♠ At one string loop, volume dependence will appear: the dimensionless parameter will be $\frac{\ell}{\ell_s}$.

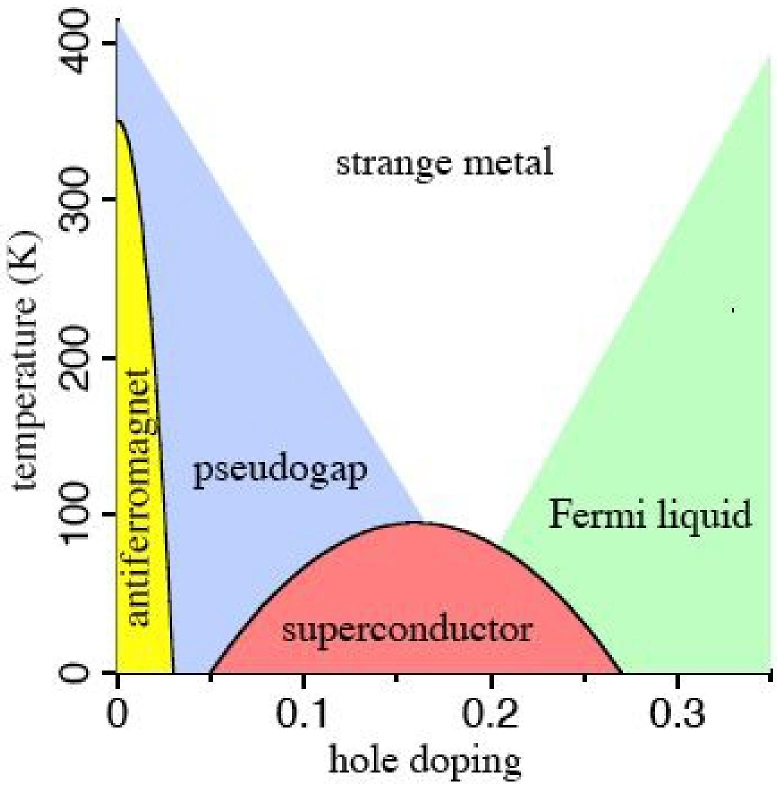
Quantum fractionalisation transitions

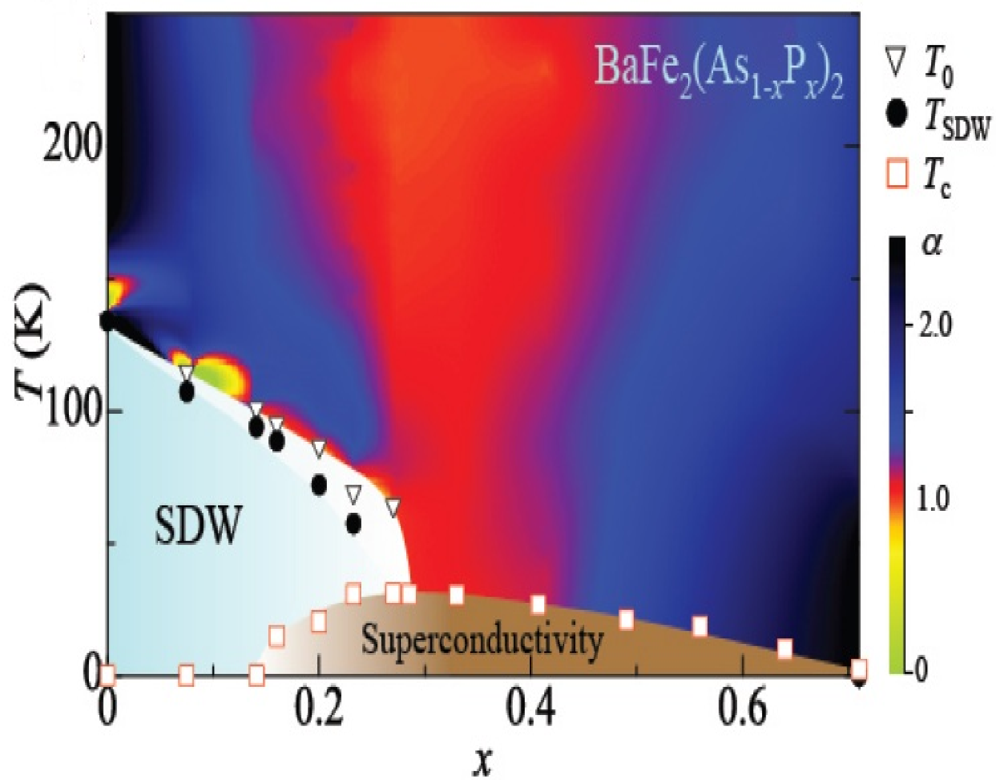


Hartnoll+Huijse'11, Adam+Crampton+Sonner+Withers '12, Goutéraux+E.K '12

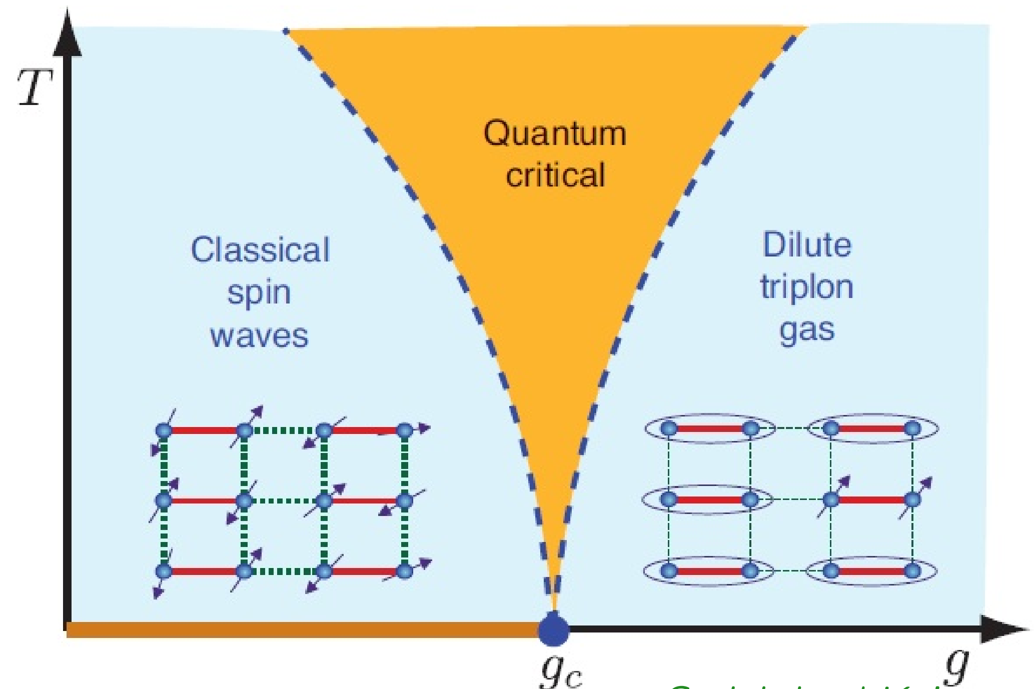
Scale invariant fixed point ($\theta = 0$) with a relevant deformation. To reach this point, the flow must be fine tuned. Away from the critical value, the flow picks up the relevant deformation and lands into hyperscaling violation fixed points: a quantum critical line. The line originates from an extra scaling symmetry: $\phi \to \phi + \phi_0$, $Q \to e^{\#\phi_0} Q$

Critical lines vs critical points in Cuprates



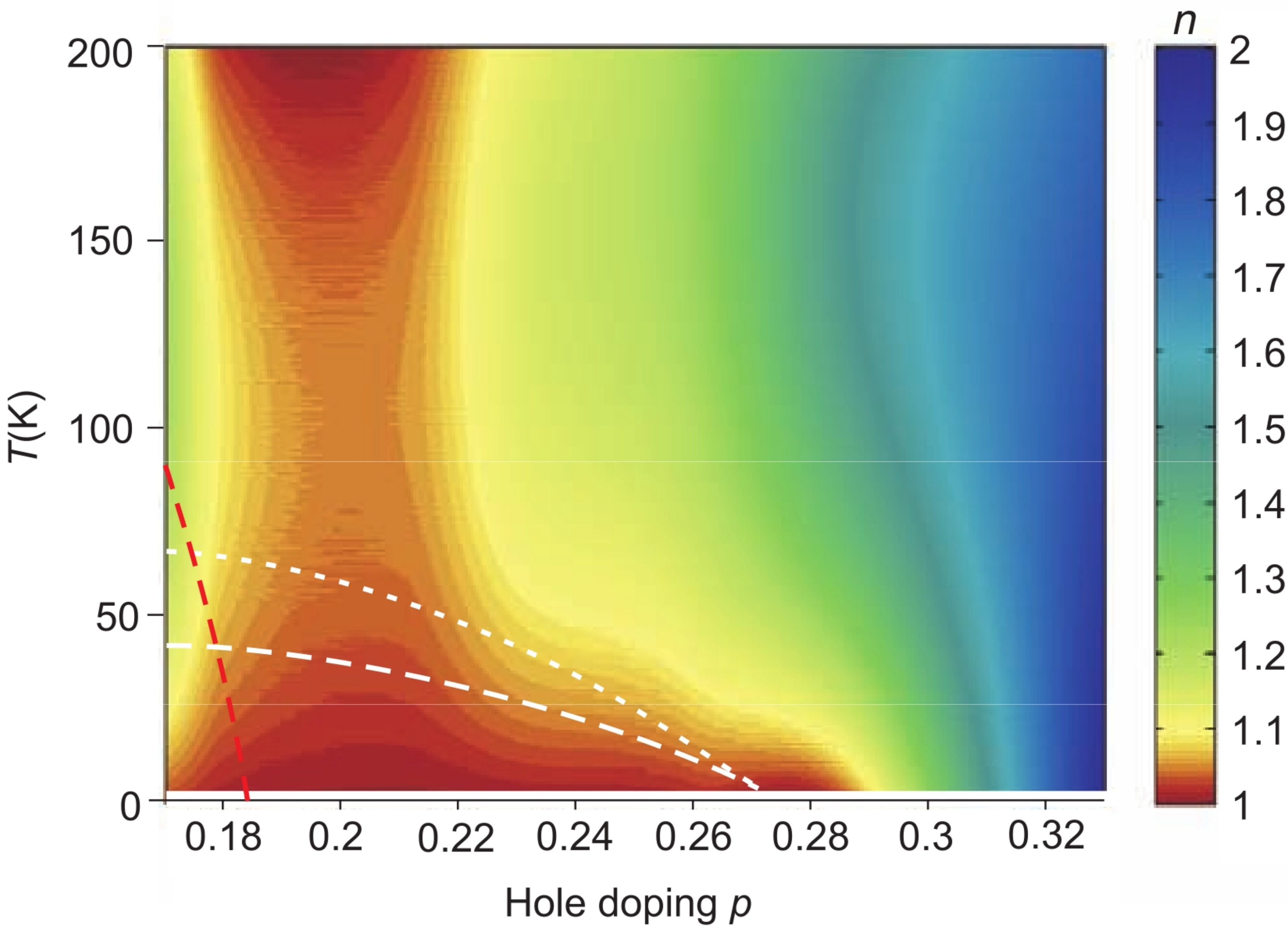


$$H = \Sigma_{\langle ij \rangle} \ J_{ij} \ \vec{S}_i \cdot \vec{S}_j$$
$$J, J/g \rightarrow J_{ij} \quad , \quad g > 1$$



- Quantum phase transition at T=0

- Critical cone above.



$\frac{d \log \rho}{d \log T}$: Anomalous Criticality in the Electrical Resistivity of $La_{2-x}Sr_xCuO_4$.

R. A. Cooper et al., Science **323***, 603 (2009).*

QC systems with Schröndiger symmetry

.

Kim+E.K.+Panagopoulos

• Consider the simplest example: AdS-Schwarzschild Black hole in light-cone coordinates boosted by an arbitrary boost.

$$ds^2 = \frac{\ell^2}{r^2}\left[\frac{(1-f(r))}{4b^2}(dx^+)^2 - (1+f(r))dx^+dx^- + (1-f(r))b^2(dx^-)^2 + dx^2 + dy^2 + \frac{dr^2}{f(r)}\right]$$

• This realizes $z = 2$ non-relativistic Schrödinger symmetry in 2 spatial dimensions.

Golberger (08), Barbon+Fuertes(08), Maldacena+Martelli+Tachikawa (08)

• One can compute the conductivities using the Karch-O'Bannon formalism applied in this context

Kim+Yamada (10)

The conductivity in the absence of magnetic field (but with light-cone electric field) reads

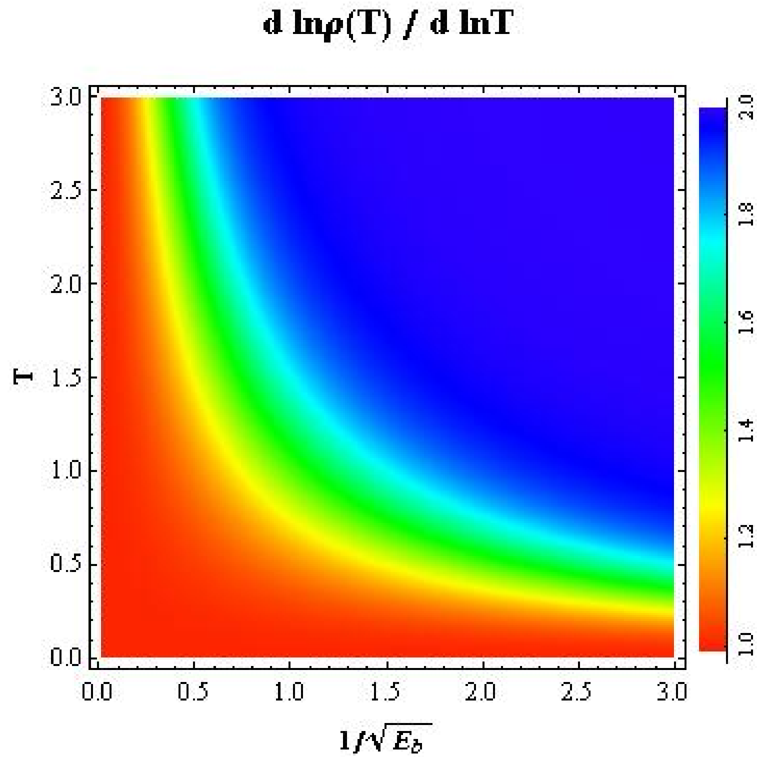
$$\rho = \frac{\rho_0}{\sqrt{\frac{J^2}{t^2 A(t)} + \frac{t^3}{\sqrt{A(t)}}}}\,,\; A(t) = t^2 + \sqrt{1+t^4} \quad , \quad t = \frac{\pi \ell T b}{\sqrt{2b\tilde{E}_b}}\,,\; J^2 = \frac{64\sqrt{2}\langle J^+\rangle^2}{(\tilde{\mathcal{N}} b \cos^3\theta)^2 (2b\tilde{E}_b)^3}.$$

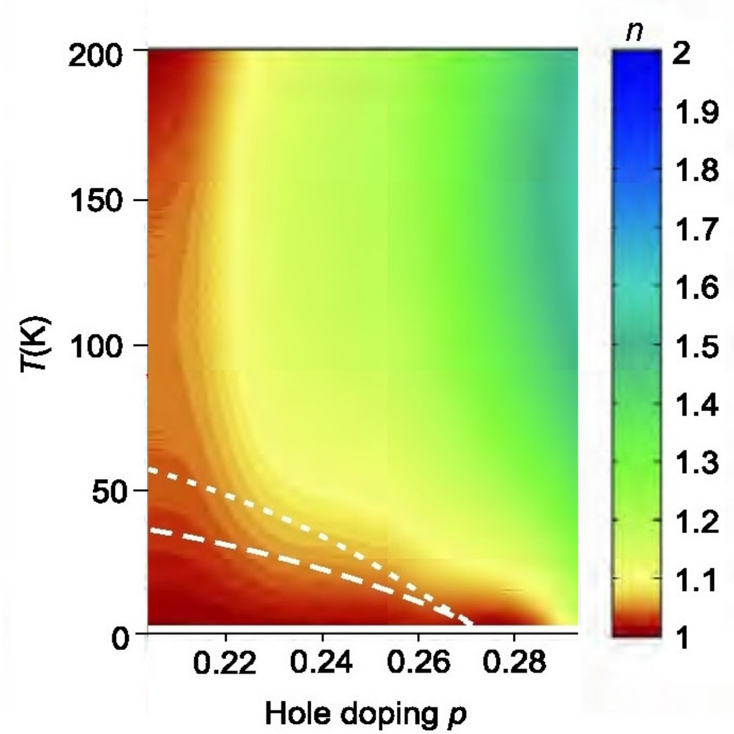
Kim+E.K.+Panagopoulos

When the “drag” term dominates

$$\rho \sim t\sqrt{t^2+\sqrt{1+t^4}}$$

showing a transition from linear to quadratic behavior.



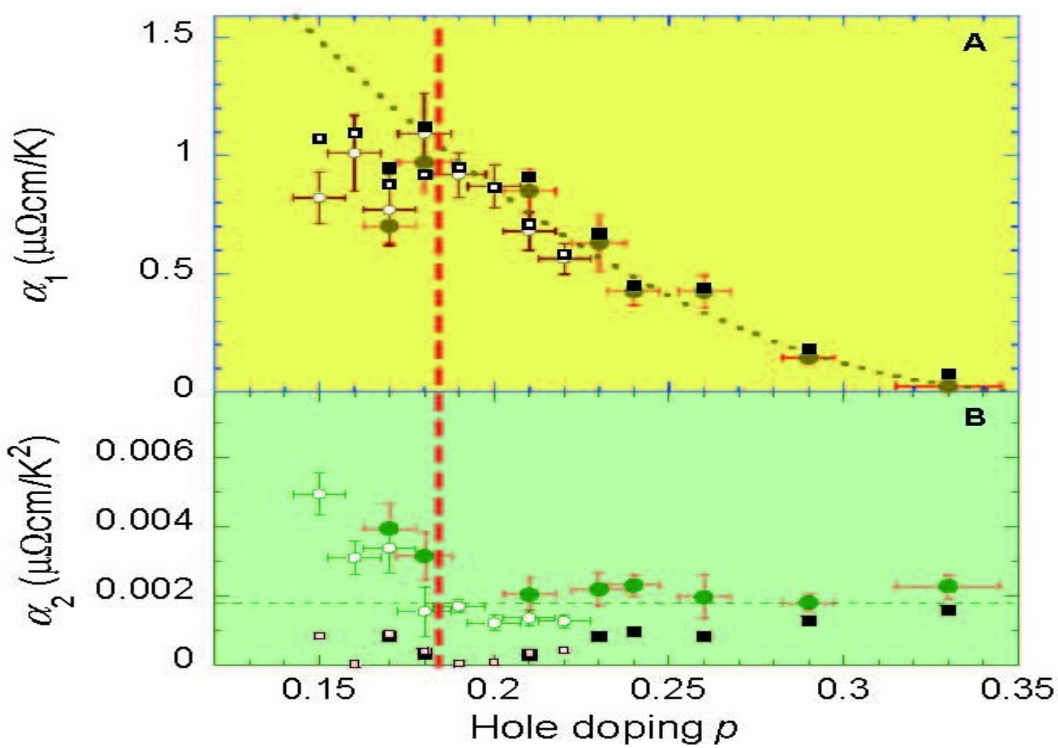


$La_{2-x}Sr_xCuO_4$ in R. A. Cooper et al., Science **323***, 603 (2009).*

• This transition can be achieved by decreasing the light-cone electric field, E_b. It interpolates between AdS and z=2 Lifshitz scaling.

.

- By parametrizing $\rho = a_1 T + a_2 T^2$ we obtain $\alpha_1 \sim \sqrt{E_b}$ and $\rho_2 = constant$.



$La_{2-x}Sr_xCuO_4$ in R. A. Cooper et al., Science **323***, 603 (2009).*

Resistivity at non-zero magnetic field

At finite magnetic field

$$\sigma^{yy} = \sigma_0 \frac{\sqrt{\mathcal{F}_+(t) J^2 + t^4 \sqrt{\mathcal{F}_+(t)} \mathcal{F}_-(t)}}{\mathcal{F}_-(t)} \quad , \quad \sigma^{yz} = \bar{\sigma}_0 \frac{\mathcal{B}}{\mathcal{F}_-(t)}$$

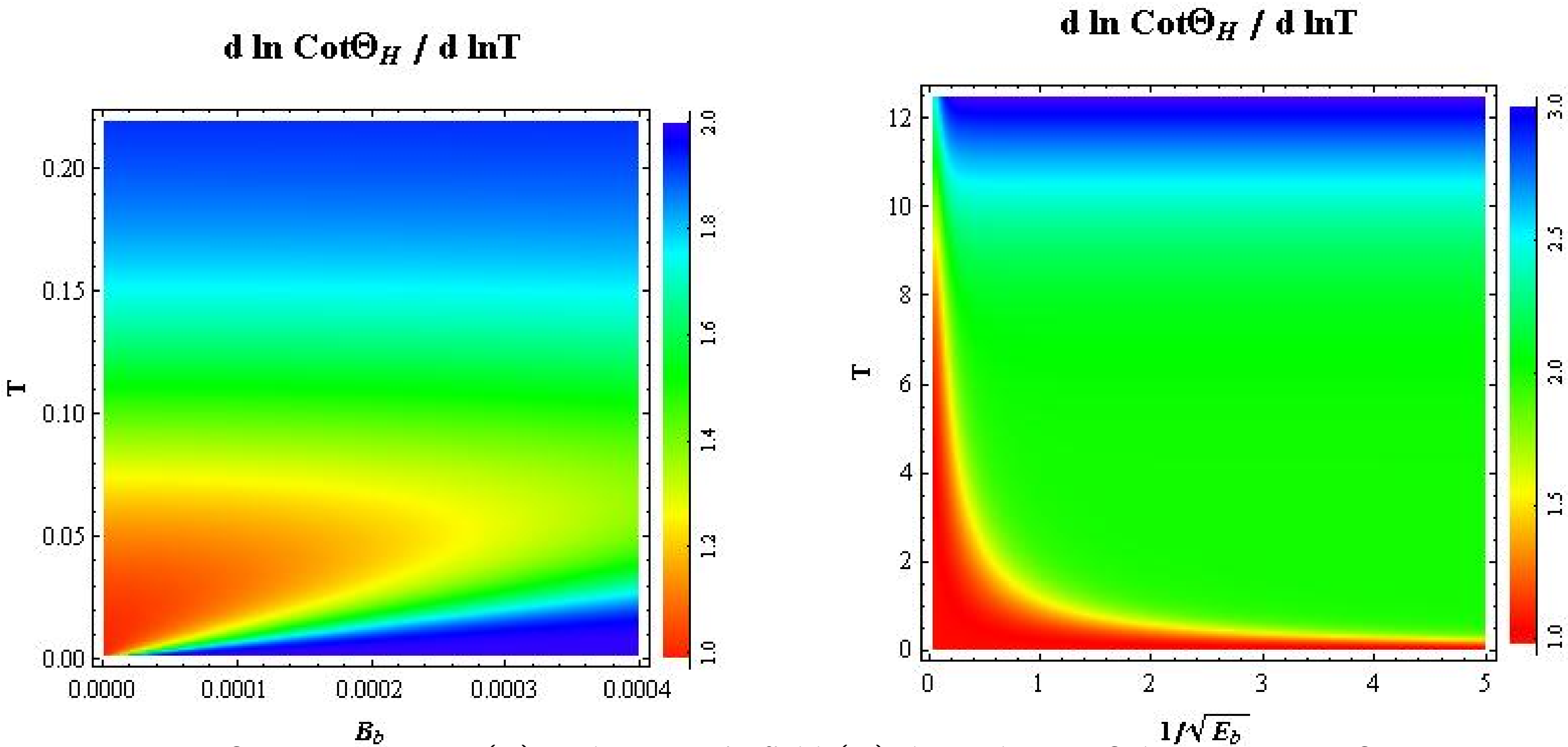
$$\mathcal{F}_\pm = \sqrt{\left(\mathcal{B}^2 + t^4\right)^2 + t^4} \mp \mathcal{B}^2 + t^4 \ , \ \mathcal{B} = \frac{\tilde{B}_b}{2b\tilde{E}_b}$$

• The scaling variable $\mathcal{B} = \frac{\tilde{B}_b}{2b\tilde{E}_b}$ seems to be in agreement with experimental data

*$Tl_2Ba_2CuO_{6+\delta}$ in A. W. Tyler et al., Phys. Rev. B **57**, R278 (1998).*

• The inverse Hall angle is defined as the ratio between Ohmic conductivity and Hall conductivity as

$$\cot \Theta_H = \frac{\sigma^{yy}}{\sigma^{yz}}$$



Left: Temperature (T) and magnetic field (B) dependence of the exponent of

$$\cot\Theta_H \equiv \frac{\sigma^{xx}}{\sigma^{xy}}$$

in the low T, low B regions.

Right: the effective power dependence of $\cot\Theta_H$ at small magnetic field, as a function of temperature and $1/\sqrt{E_b}$.

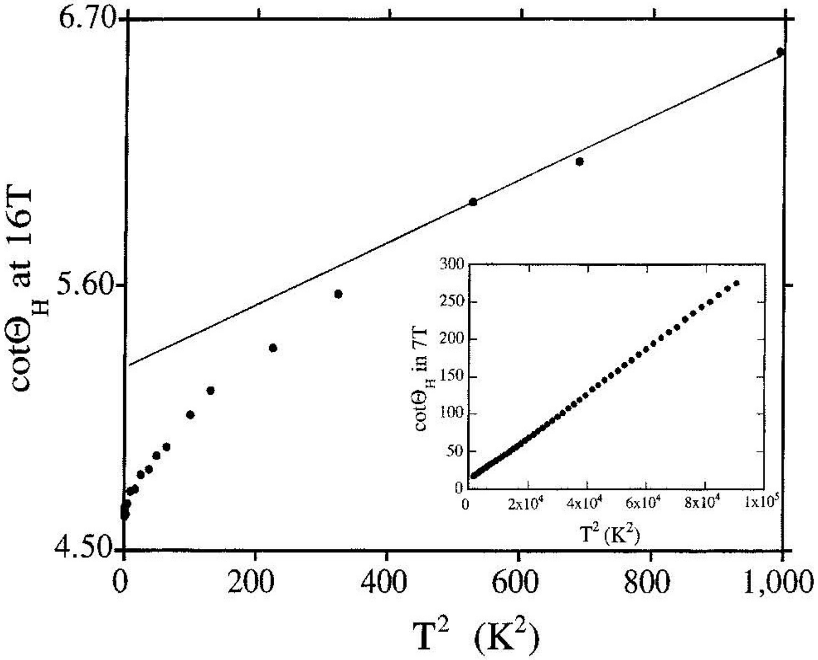
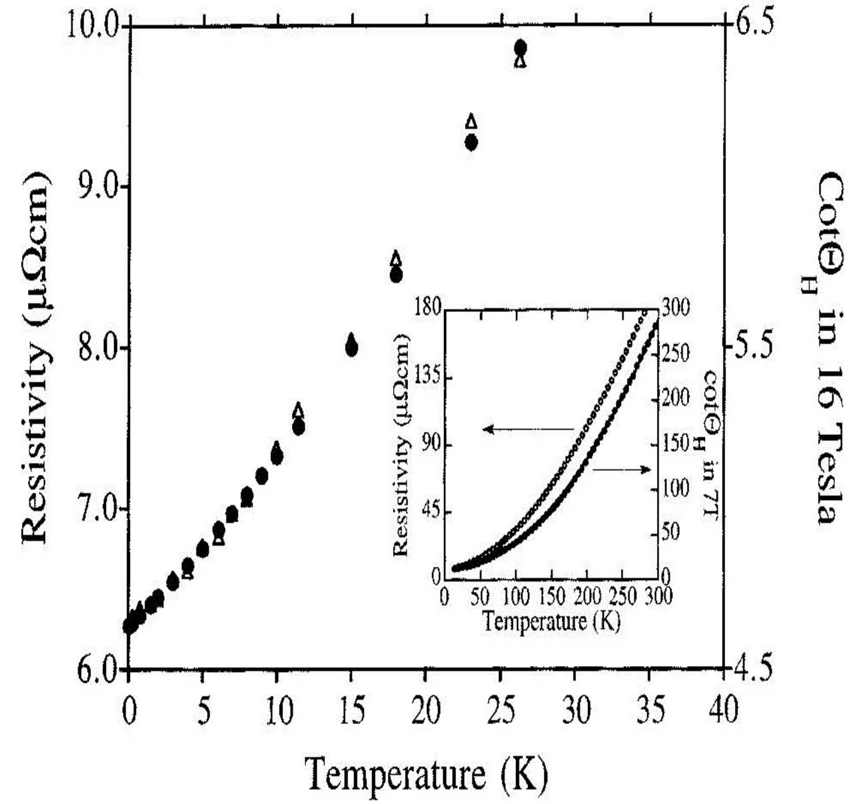
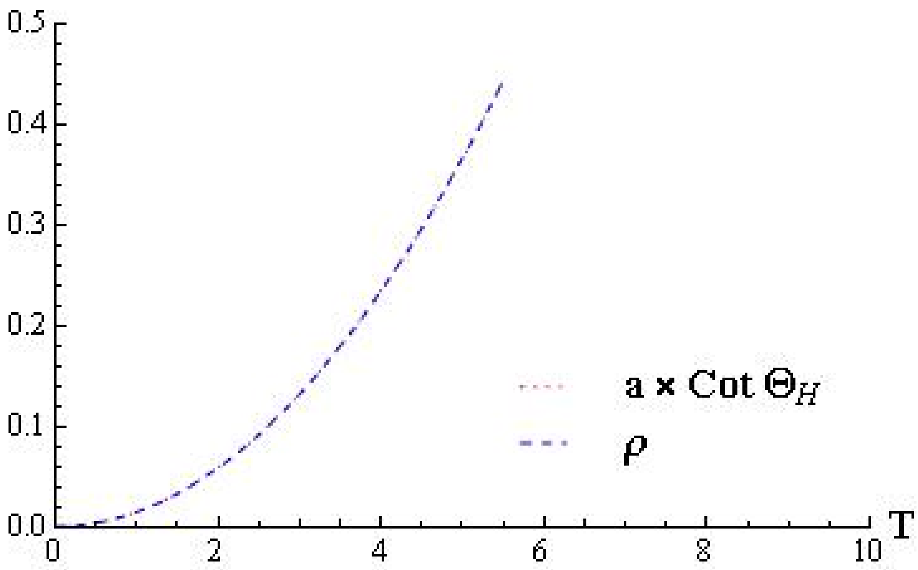


FIG. 8. The cotangent of the Hall angle plotted against T^2 below 30 K. The low-temperature data deviate significantly from the $A+BT^2$ dependence seen at high temperatures (inset), whose extrapolation is shown by the solid line.



The resistivity and $cot\Theta_H$ are correlated at low temperatures in $Tl_2Ba_2CuO_{6+\delta}$

Mackenzie et al. Phys. Rev. B ***53****, 5848 (1996).*



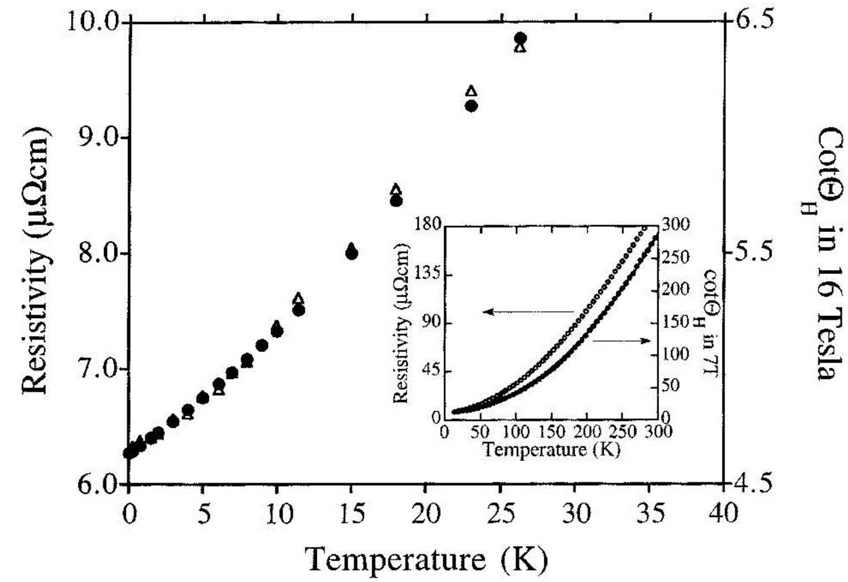


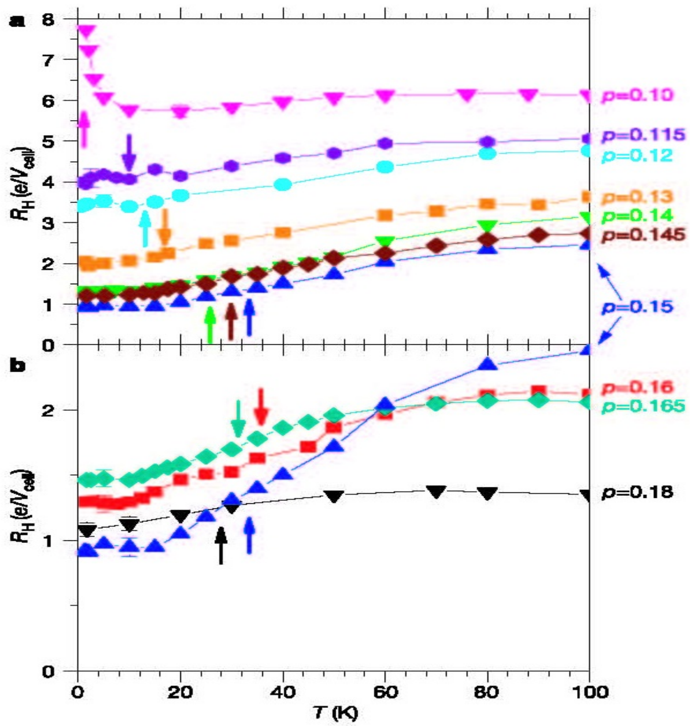
FIG. 9. The cotangent of the Hall angle and the resistivity plotted on linear axes in the low-temperature and (inset) high-temperature regimes. The high-temperature data for cotΘ_H vary as

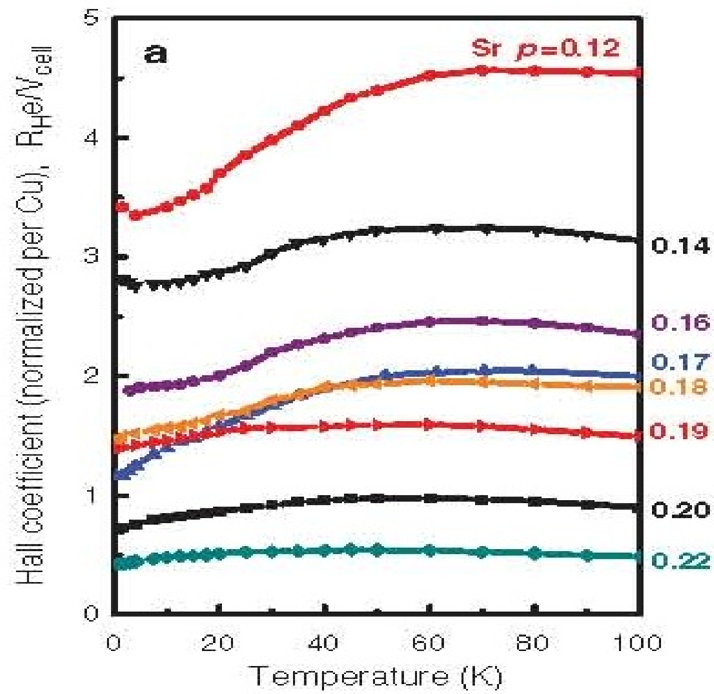
Plot of the resistivity and inverse Hall angle, in the model, for the low-temperature regime with small magnetic field. Note that the inverse Hall angle has been scaled by a constant factor $a = B_b/(32\sqrt{2}\langle J^+\rangle$. This plot is to be compared with left figure from McKenzie et al. Phys. Rev. B **53**, 5848 (1996).

The Hall Conductivity $R_H = \left.\frac{\rho_{yz}}{B}\right|_{B=0}$ is constant in the two different regimes (linear and quadratic)

$$R_H \simeq \frac{\bar{\sigma}_0}{\sigma_0^2 J^2} \sim E_b$$

and decreases with doping.

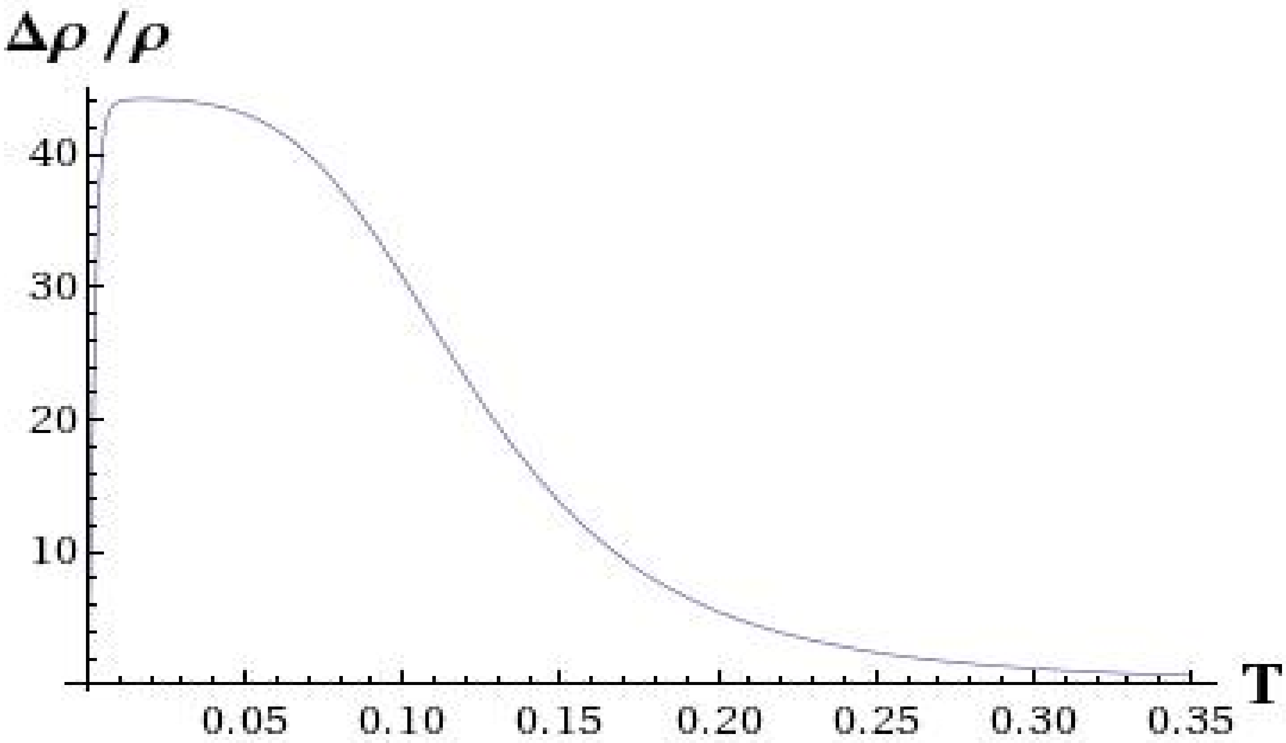




$Bi_2Sr_{2-x}La_xCuO_{6+\delta}$ from F. F. Balakirev et al., NATURE 424 (2003) 912; Phys. Rev. Lett. **102**, *017004 (2009).*

• The magnetoresistance

$$\frac{\Delta\rho}{\rho} = \frac{\rho_{yy}(B) - \rho_{yy}(0)}{\rho_{yy}(0)}$$



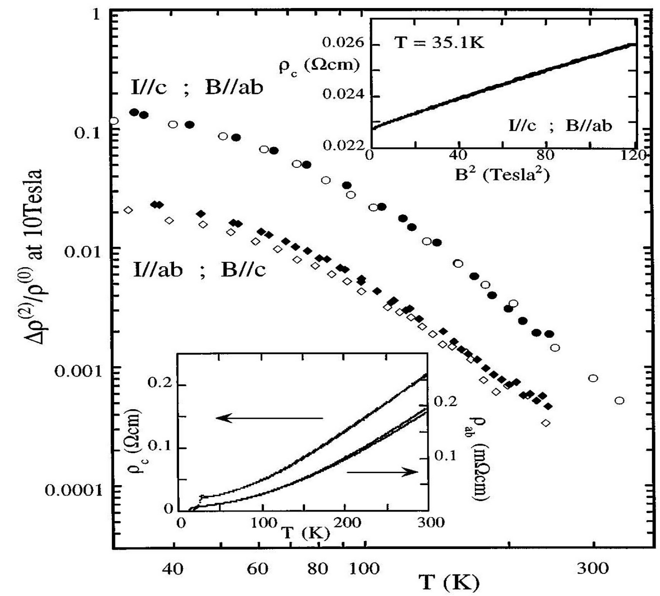


FIG. 1. T dependences of the B^2 terms $\Delta\rho^{(2)}/\rho^{(0)}$ at 10 T for c-axis MR (circles) and a-b plane MR (diamonds) in overdoped $Tl_2Ba_2CuO_6$. Data for two crystals are shown in each case. Bottom inset: Zero-field $\rho_c(T)$ and $\rho_{ab}(T)$ for the crystals shown in the main figure. Top inset: MR field sweep at 35.1 K for $\mathbf{I} \parallel c$, $\mathbf{B} \parallel ab$.

N. E. Hussey et al., Phys. Rev. Lett, ***76****, 122 (1996).*

- We find that the modified Köhler rule

$$\tilde{K} = (\cot\Theta_H)^2 \frac{\Delta\rho}{\rho} \simeq \text{temperature} \quad \text{independent}$$

is valid in regions (linear+quadratic), as demanded by data,

J. M. Harris et al., Phys. Rev. Lett, **75**, *1391 (1995).*

- We also find that the Köhler rule

$$K = \rho^2 \frac{\Delta\rho}{\rho} \simeq \text{temperature} \quad \text{independent}$$

is approximately valid in the same regions.

This is not supported by the data at high temperatures but is valid at low temperatures.

Outlook

• We have used the concept of EHT to classify QC points in EMD theories with or without unbroken symmetry

• This is a part of an EHT program that is currently extended to more general situations: more symmetries, CP-odd interactions, more scalars and U(1)'s etc.

• We characterize all QC geometries with U(1) operator in terms of three critical exponents (z, θ, ζ)

• The behaviors we find are rich and calculable. They are the first step into completing a phase diagram.

• The method is general and applicable to all gravitational theories.

• The observables, like current-current correlators, as well as condensate correlators should be computed.

• General results characterizing the critical exponents may be derived (work in progress)

.

THANK YOU

Detailed plan of the presentation

- Title page 1 minutes
- Bibliography 2 minutes
- The plan 3 minutes
- Introduction 4 minutes
- Symmetries 5 minutes
- Fixed Point Theories 7 minutes
- Classification of QC theories 9 minutes
- The ingredients of the classification 12 minutes
- Fractionalized vs cohesive phases 16 minutes
- Broken vs unbroken symmetry. 21 minutes
- Classification of symmetry-breaking QC points 23 minutes
- Examples in simple cases: zero density 25 minutes
- Examples in simple cases: finite density 26 minutes

- Hyperscaling (constant scalar), neutral IR 27 minutes
- Hyperscaling (constant scalar), charged IR 28 minutes
- Hyperscaling violating (running scalar), neutral IR 29 minutes
- Hyperscaling violating (running), Charged IR (I) 30 minutes
- Hyperscaling violating (running), Charged IR (II) 33 minutes
- Quantum fractionalisation transitions 34 minutes
- Critical lines vs critical points 38 minutes
- QC systems with Schröndiger symmetry 42 minutes
- Resistivity at non-zero magnetic field 47 minutes
- Outlook 49 minutes