A universal fermionic analogue of the shear viscosity

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Motivation + Summary

Goal

- Try to find a universal holographic result similar to $\eta/s = 1/4\pi$ from fermionic correlators.

Candidate

- spontaneous SUSY breaking by temperature
- supersound diffusion constant D_s in phonino pole

Result

- explicitly computed for black branes in AdS_{d+1}
- related it to a universal absorption cross section result:

$$\epsilon D_{3/2} = \frac{1}{4\pi G}\,\sigma_{1/2} \quad \leftrightarrow \quad \eta = \frac{1}{16\pi G}\,\sigma_0$$

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Universality of η/s

- One of the key results: universality of $\eta/s = 1/4\pi$ for Einstein gravity, rotational symmetry (Buchel, Liu '03; Kovtun, Son, Starinets '04)
- Kubo formula for shear viscosity:

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int d^4x \, e^{i\omega t} \, \langle [T_{xy}(x), T_{xy}(0)] \rangle$$

- Absorption cross section of graviton h_{xy} coupled to T_{xy} by black brane (Klebanov '97; Gubser, Klebanov, Tseytlin '97):

$$\sigma_{\text{abs}}(\omega) = -\frac{2\kappa^2}{\omega} \text{Im}\, G^R(\omega) = \frac{\kappa^2}{\omega} \int d^4x \, e^{i\omega t} \, \langle [T_{xy}(x), T_{xy}(0)] \rangle$$

Universality of η/s (Kovtun, Son, Starinets '04)

- Therefore (Policastro, Son, Starinets '01)

$$\eta = \frac{\sigma_{\text{abs}}(0)}{2\kappa^2} = \frac{\sigma_{\text{abs}}(0)}{16\pi G}$$

- For minimally coupled massless scalar (like transverse graviton) in spherically symmetric black hole (Das, Gibbons, Mathur '96)

$$\sigma_{\text{abs}}^{l=0}(0) = A$$

- Bekenstein-Hawking entropy $S = \frac{A}{4G} \Rightarrow$

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

- independent of conformality, (non-)confining, SUSY or not, with / without chemical potential

Similar universality results, possibly fermionic?

- Condensed matter applications: Fermi surfaces, (non-)Fermi liquids (Lee '08; Faulkner, Liu, McGreevy, Vegh '09; Cubrovic, Schalm, Zaanen '09)
- spin $1/2 \to$ spin $3/2$: no Fermi-surfaces
  (Belliard, Gubser, Yarom '11; Gauntlett, Sonner, Waldram '11)
- $T_{\mu\nu}$ in same multiplet as supersymmetry current S_μ^α (and R-symmetry current J_μ) (Ferrara, Zumino '75)
- There exists a similar universality result for minimally coupled Dirac fermions. (Das, Gibbons, Mathur '96)
- Try to look at small ω and small k

$$\langle [T_{\mu\nu}(x), T_{\rho\sigma}(0)]\rangle \to \langle \{ S_\mu^\alpha(x), \bar{S}_\nu^{\dot{\alpha}}(0) \} \rangle$$

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SUSY breaking ↔ phonino

- Look at spontaneous SUSY breaking due to temperature.
  (Lebedev, Smilga '89; Kratzert '03)
- can see this from SUSY algebra in thermal state $\langle T_{00}\rangle_T \neq 0$

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2\sigma^\mu_{\alpha\dot{\alpha}} T_{0\mu}$$

- Ward-Takahashi identity

$$\partial_\mu \langle T\,\{S^\mu_\alpha, \bar{S}^\nu_{\dot{\alpha}}\}\rangle = \delta^4(x-y) 2 \langle T^\nu{}_\rho\rangle \sigma^\rho_{\alpha\dot{\alpha}}$$

- $\rightarrow$ phonino mode with pole at $\omega = v_s k - i D_s k^2$ with $v_s = \frac{P}{\epsilon}$

Supersymmetric hydrodynamics

- Describe the IR of a supersymmetric theory with SUSY breaking by temperature ("supersymmetric hydrodynamics") as the effective theory of the phonino and the normal fluid

  (Hoyos, Keren-Zur, Oz '12)
- No classical fermionic charges!
- The constitutive relation (Kovtun, Yaffe '03) with $\rho = S^0$ (first order in the derivative expansion) is not changed by this interpretation!

$$S^i_{\text{diss}} = -D_s \nabla^i \rho - D_\sigma \sigma^{ij} \nabla_j \rho$$

- conformal: $T^\mu_\mu = 0 \ \leftrightarrow \ \gamma^\mu S_\mu = 0 \ \leftrightarrow \ D_s = D_\sigma$
- However it has to be seen as a quantum-mechanical relation where ρ is the quantum phonino field!

So far within AdS / CFT:

- In 4d, $\mathcal{N} = 4$ SYM the retarded correlator $\langle S_\mu \bar{S}_\nu \rangle$ has been computed. (Policastro '08; Kontoudi, Policastro '12)

$$2\pi T D_s = \frac{4}{9}\sqrt{2}$$

- It was further studied numerically for $\mu \neq 0$ in STU black hole.
- In 3d, this was studied numerically (AdS_4 gauged SUGRA). (Gauntlett, Sonner, Waldram '11)

$$2\pi T D_s^{3d} \neq 2\pi T D_s^{4d}$$

- $\Rightarrow$ calculate in d dimensions for non-dilatonic AdS_{d+1} black brane theories (D3, M2, M5) for $\mu = 0$ and search for universality

Constitutive relation

- In arbitrary space-time dimension d, reorder the constitutive relation according to representations of $O(d-1)$:

$$S^i_{\text{diss}} = -D_{3/2} \underbrace{\left(\delta^i_j - \frac{1}{d-1}\gamma^i\gamma_j\right)}_{\gamma^i \text{ irreducible } \leftrightarrow \text{ spin } 3/2} \nabla^j\rho - D_{1/2} \underbrace{\gamma^i \nabla\!\!\!/ \rho}_{\gamma^i \text{ ``trace''}}$$

- completely analogous to

$$T^{ij}_{\text{diss}} = -\eta \underbrace{\left(\delta^{ik}\delta^{jl} + \delta^{jk}\delta^{il} - \frac{2}{d-1}\delta^{ij}\delta^{kl}\right)}_{\text{symmetric traceless } \leftrightarrow \text{ spin } 2} \nabla^k u^l - \zeta\delta^{ij} \underbrace{(\nabla\cdot u)}_{\text{trace}}$$

- conformal: $T^\mu_\mu = 0 \leftrightarrow \zeta = 0$ & $\gamma^\mu S_\mu = 0 \leftrightarrow D_{1/2} = 0$
- expectation: $D_{3/2}$, rather than D_s, universal as η ?!

Kubo formula

One may derive a new Kubo formula for $D_{3/2}$ assuming a Dirac spinor S^μ_α (for even d, a corresponding Weyl version looks similar):

$$\epsilon D_{3/2} = \frac{1}{\mathrm{Tr}\,(-\gamma^0\gamma^0)} \lim_{\omega,k\to 0} \mathrm{Tr}\left(-\gamma^0\ \mathrm{Im}\int d^d x\, e^{i\omega t}\,\langle\, S^x_T(x)\bar{S}^x_T(0)\,\rangle\right)\,,$$

where the spin 3/2 part of S^j is $S^i_T = \left(\delta^i_j - \frac{1}{d-1}\gamma^i\gamma_j\right) S^j$.

- Recall that η proceeds from $\langle T_{xy} T_{xy}\rangle$ in the same way.
- One may also relate this to the (polarization averaged) absorption cross section of a minimally coupled massless fermion by a black hole background.

Universal absorption cross sections (Das, Gibbons, Mathur '96)

- Take a spherically symmetric, asymptotically flat, non-extremal black hole background:

$$ds^2 = -f(r)dt^2 + g(r)\left(dr^2 + r^2 d\Omega_p^2\right)$$

- Note, that at the horizon $f(r_H) = 0$ but $g(r_H) \neq 0$.
- Then for minimally coupled massless s-wave scalars in the low-energy limit $\omega \to 0$:

$$\sigma_0 = A$$

- Similarly, for minimally coupled massless Dirac fermions:

$$\sigma_{1/2} = 2g(r_H)^{-p/2} A$$

- This is twice the area of the horizon in a conformally related spatially flat space-time $ds^2 = dr^2 + r^2 d\Omega_p^2$.

Transverse gravitino

- Dual bulk field to the supersymmetry current on the boundary: gravitino ($\leftrightarrow$ supergravity dual) e.g. in AdS$_{d+1}$

$$S = \int d^{d+1}x\sqrt{-g}\bar{\Psi}_\mu \left(\Gamma^{\mu\nu\rho}D_\nu - m\Gamma^{\mu\rho}\right)\Psi_\rho \,,$$

- For $mL = \frac{d-1}{2}$, Ψ_μ has d.o.f. of massless spin 3/2 field

  (Townsend '77; Deser, Zumino '77)

- In simple Rarita-Schwinger equation, the transverse gravitino $\Psi^i_T = \left(\delta^i_j - \frac{1}{d-1}\gamma^i\gamma_j\right)\Psi^j$ has spin 1/2 equation of motion!
- This seems to be analogous to the transverse graviton h_{xy} obeying scalar equation of motion $\Box h^y_x = 0$.

Main result

- however with mass $m \sim \frac{1}{L}$ which comes from KK reduction / consistent truncation of e.g. transverse sphere.
- In higher dimension: massless $\Rightarrow$ can use theorem

$$\boxed{\epsilon D_{3/2} = \frac{1}{4\pi G}\,\sigma_{\text{abs},1/2}(0) = \frac{1}{2\pi G}\,g(r_H)^{-p/2}A}$$

- idependent of the boundary spinor (odd / even d) being Dirac, Weyl or Majorana

Application

- Take non-dilatonic black p-branes with $\mathrm{AdS}_{p+2}\times S^{D-p-2}$ near-horizon geometry (Gibbons, Horowitz, Townsend '95)

$$ds^2 = -f(r)dt^2 + \frac{r^2}{L_{\mathrm{AdS}}^2} d\vec{x}_p^2 + f(r)^{-1} dr^2 + L_{\mathrm{Sph}}^2 d\Omega_{D-p-2}^2 \,,$$

where

$$f(r) = \frac{r^2}{L_{\mathrm{AdS}}^2} - \left(\frac{R}{L_{\mathrm{AdS}}}\right)^2 \left(\frac{R}{r}\right)^{p-1}$$

- Evaluate $\sigma_{1/2}$ before sphere reduction and use necessary consistent KK sphere reduction condition (Cvetic, Lu, Pope '00)

$$(D-p-5)(p-1) = 4$$

Application result

- We get
$$\frac{\sigma}{A} = \frac{1}{4}\, 2^{2/d}\,,$$
which we also computed by first extending the spinor theorem to finite mass.
- From this we also get the supersound diffusion constant
$$2\,\pi\, T D_s = \frac{2^{2/d}\, d(d-2)}{2(d-1)^2}$$
- in agreement with $\mathcal{N}=4$ SYM result for $d=4$ ✓

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Computation (1)

- Alternatively we may directly compute D_s following

  (Policastro '08; Kontoudi, Policastro '12)

- Solve (gauge-fixed) Rarita-Schwinger equation in AdS_{d+1} black brane background:

$$(\not\nabla + m)\Psi_\mu = 0$$

- actually

$$\gamma^d \Psi_0' - \frac{i\omega}{f}\gamma^0\Psi_0 - \frac{f'}{2f}\gamma^0\Psi_d + \frac{f'}{4f}\gamma^d\Psi_0 + \frac{ikl}{r\sqrt{f}}\gamma^1\Psi_0 + \frac{d-1}{2r}\gamma^d\Psi_0 + \frac{m}{\sqrt{f}}\Psi_0 = 0\,,$$

$$\gamma^d \Psi_d' - \frac{i\omega}{f}\gamma^0\Psi_d - \frac{f'}{2f}\gamma^0\Psi_0 + \frac{f'}{4f}\gamma^d\Psi_d + \frac{ikl}{r\sqrt{f}}\gamma^1\Psi_d + \frac{1}{r}\left(\frac{d+1}{2}\gamma^d\Psi_d + \gamma^0\Psi_0\right) + \frac{m}{\sqrt{f}}\Psi_d = 0\,,$$

$$\gamma^d \Psi_j' - \frac{i\omega}{f}\gamma^0\Psi_j + \frac{f'}{4f}\gamma^d\Psi_j + \frac{ikl}{r\sqrt{f}}\gamma^1\Psi_j + \frac{1}{r}\gamma^j\Psi_d + \frac{d-1}{2r}\gamma^d\Psi_j + \frac{m}{\sqrt{f}}\Psi_j = 0\,,$$

Computation (2)

- solve perturbatively up to first order in ω and k
- impose ingoing boundary conditions at the horizon (Son, Starinets '03)

$$\psi_d \sim (r-R)^{-3/4-\frac{i\omega}{4\pi T}}\psi_{d,0}$$

- identify source terms $\propto r^{\Delta-d} = r^{-1/2}$ for dimension $\Delta = \frac{1}{2}(d+2|m|)$ dual operators
- evaluate diffusive pole of retarded supersymmetry current Green's function ($\leftrightarrow$ phonino pole) and read off

$$\omega = v_s k - iD_s k^2$$

Results

- Supersound velocity

$$v_s = \frac{1}{d-1} = \frac{P}{\epsilon} = v_{\text{sound}}^2 \quad \text{as expected for CFT}_d \ \checkmark$$

- Main result: supersound diffusion constant

$$2\pi T D_s = \frac{2^{2/d} d(d-2)}{2(d-1)^2}$$

- in exact agreement with universality considerations ✓
- Also possible:
  - solve to zero'th order in ω, k and use Kubo formula
  - main complication: boundary term normalization
  - use higher-dimensional SUSY algebra

Conclusion / Outlook

Main results

- Connection to universal absorption cross section:

$$\epsilon D_{3/2} = \frac{1}{4\pi G}\,\sigma_{1/2} \quad \leftrightarrow \quad \eta = \frac{1}{16\pi G}\,\sigma_0$$

- Explicitly computed for non-dilatonic AdS_{d+1} black branes

Outlook

- Is there a quantity one should divide by?
- The transverse gravitino is generically not minimally coupled anymore for $\mu \neq 0$ (Pauli terms). Extension possible or generic limitation to $\mu \to 0$?