Energy-momentum tensor correlators in holography and perturbative QCD

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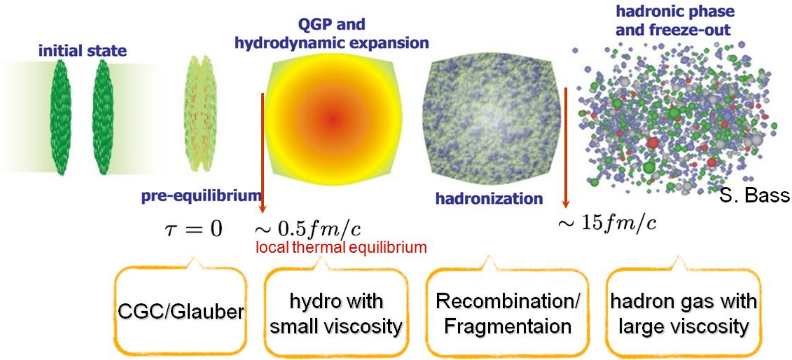
based on

K. Kajantie, M. Krššák., A. Vuorinen, arXiv:1302.1432 [hep-ph].
K. Kajantie, M. Krššák, M. Vepsäläinen, A. Vuorinen, Phys. Rev. D **84**, 086004, arXiv:1104.5352[hep-ph].

Motivation



- Standard lore: RHIC data suggest that strongly coupled quark gluon plasma behaves as an almost ideal liquid with $\eta/s < 0.2$. How to understand this and describe the system?

Transport properties from holography

- Perturbative QCD (weakly coupled): $\frac{\eta}{s} \propto g^4 \gg 1$
- Lattice calculations: indications of small value, but hard to make quantitative conclusions
- AdS/CFT correspondence: for two derivative models universal prediction $\frac{\eta}{s} = \frac{1}{4\pi} \approx 0.08$ — classical argument for holographic description of QGP
- Obvious question: how close is $\mathcal{N} = 4$ SYM to the real world QCD with broken conformality?
  - bulk viscosity is trivial in $\mathcal{N} = 4$ SYM, while in the real world QCD it has non-zero value.
  - If we want to understand strongly coupled QGP using holography, need to be able to break conformal invariance and SUSY!

Improved Holographic QCD

- IHQCD is a non-conformal bottom-up model, designed to mimic properties of large-N_c Yang-Mills theory
  (U. Gursoy, E. Kiritsis, F. Nitti: 0707.1349, 0707.1324)
- Start with background black hole metric

$$ds^2 = b^2(z)\left[-f(z)dt^2 + d\mathbf{x}^2 + dz^2 f^{-1}(z)\right],$$

z- radial coordinate (boundary at $z = 0$, BH horizon at $z = z_h$)

- ...and dilaton gravity action

$$S = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R - \frac{4}{3}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right], \quad \lambda(z) = e^{\phi(z)}.$$

- Conformal invariance broken through introduction of nontrivial potential $V(\phi)$ for the dilaton field

$$\beta = \frac{d\lambda}{db}.$$

To model YM theory, choose potential $V(\lambda)$ according to

$$V(\lambda) = \frac{12}{\mathcal{L}^2}\left[1 + \frac{88}{27}\lambda + \frac{4619}{729}\lambda^2 \frac{\sqrt{1+\ln(1+\lambda)}}{(1+\lambda)^{2/3}}\right], \quad \text{with}$$

1. Coefficients determined by matching holographic beta function to 2-loop perturbative one (in large-N_c YM).
2. Requiring the model to possess a linear glueball spectrum (confinement criterion)
3. Requiring background to be asymptotically AdS

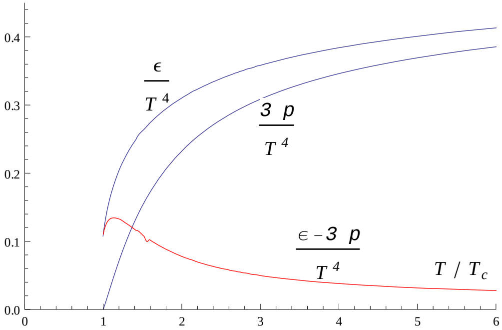
$$b(z) \to \frac{\mathcal{L}}{z}, \qquad z \to 0.$$

Finally, use Einstein equations to numerically determine $b(z)$, $f(z)$, $\lambda(z)$

$$\begin{aligned} \dot{W} &= 4bW^2 - \frac{1}{f}(W\dot{f} + \frac{1}{3}bV)\,, \qquad & \dot{b} = -b^2 W\,, \\ \dot{\lambda} &= \frac{3}{2}\lambda\sqrt{b\dot{W}}\,, & \ddot{f} = 3\dot{f}\,bW\,. \end{aligned}$$

Thermodynamics

To quantitatively study the predictions of IHQCD, look at thermodynamic quantities, such as energy density or interaction measure



- Last free parameter of the model fixed by matching pressure to weakly coupled large-N_c YM theory

$$\frac{\mathcal{L}^3}{4\pi G_5} = \frac{4N_c^2}{45\pi^2} .$$

Energy momentum correlators in Yang-Mills Theory

In this talk: inspect correlation functions of YM energy momentum tensor

$$
\begin{aligned}
T_{\mu\nu}(x) &= \theta_{\mu\nu}(x) + \frac{1}{4}\delta_{\mu\nu}\theta(x), \\
\theta_{\mu\nu}(x) &= \frac{1}{4}\delta_{\mu\nu}F^a_{\rho\sigma}F^a_{\rho\sigma} - F^a_{\mu\alpha}F^a_{\nu\alpha}, \\
\theta(x) &= \frac{\beta(g)}{2g}F^a_{\rho\sigma}F^a_{\rho\sigma}.
\end{aligned}
$$

In particular, retarded Green's functions in momentum space

$$
G^R_s(\omega, k=0) = -i\int d^4x e^{i\omega t}\theta(t)\left\langle [T_{12}(t,\vec{x}), T_{12}(0,0)] \right\rangle,
$$

$$
G^R_b(\omega, k=0) = -i\int d^4x e^{i\omega t}\theta(t)\left\langle \left[\frac{1}{3}T_{\mu\mu}(t,\vec{x}), \frac{1}{3}T_{\nu\nu}(0,0)\right] \right\rangle.
$$

Motivation: in hydrodynamic limit, energy momentum tensor of viscous liquid

$$
\begin{aligned}
T_{\mu\nu} &= \epsilon u_\mu u_\nu + p P_{\mu\nu} + \sigma_{\mu\nu}, \\
\sigma_{\mu\nu} &= P_\mu{}^\alpha P_\nu{}^\beta \left[\eta \left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3} g_{\alpha\beta} \partial_\lambda u^\lambda \right) + \zeta g_{\alpha\beta} \partial_\lambda u^\lambda \right],
\end{aligned}
$$

where η and ζ are the **shear** and **bulk** viscosities, respectively.

To connect viscosities with correlators

- Define shear and bulk spectral densities

$$
\rho_{s/b}(\omega) = \mathrm{Im}\, G^R_{s/b}(\omega, k = 0),
$$

- ...and use Kubo formula to express the viscosities as the corresponding IR limits

$$
\eta = \lim_{\omega \to 0} \frac{\rho_s(\omega)}{\omega}, \qquad \zeta = \lim_{\omega \to 0} \frac{\rho_b(\omega)}{\omega}.
$$

Of interest: not only transport coefficients, but *comparison of spectral densities and Euclidean correlators with lattice and pQCD*

Holographic shear correlator

To determine the shear spectral density within IHQCD< follow the standard calculation (S. S. Gubser, S. S. Pufu, F. D. Rocha arXiv:0806.0407)

1. Introducing perturbations to background metric

$$g_{12} = \epsilon h_{12},$$

2. Expanding resulting Einstein equations up to 1st order in ϵ

$$\ddot{h}_{12} + \frac{d}{dz}\log(b^3 f)\dot{h}_{12} + \frac{\omega^2}{f^2}h_{12} = 0,$$

   and solve with an infalling boundary condition at the horizon.

3. Evaluating full action on the AdS boundary

$$\rho_s(\omega) = \frac{1}{4\pi}s(T)\frac{\omega}{|h_{12}\,(z \to 0)\,|^2},$$

   where $s(T)$ is the entropy

Holographic bulk correlator

- Consequence of the broken conformal invariance in IHQCD: non-zero bulk viscosity
- To obtain spectral density of the bulk correlator $\langle T_{ii}, T_{jj} \rangle$ introduce metric perturbations

$$g_{ii} = b^2 (1 + \epsilon h_{ii}) ,$$

- And use the fact that at $k = 0$

$$\langle T_{\mu\mu}, T_{\nu\nu} \rangle = \langle T_{ii}, T_{jj} \rangle + (\text{contact terms}).$$

- Again, Einstein equations lead to diff. equation for h_{ii}

$$\ddot{h}_{ii} + \frac{d}{dz} \log(b^3 f X^2) \dot{h}_{ii} + \left(\frac{\omega^2}{f^2} - \frac{\dot{f} \dot{X}}{fX} \right) h_{ii} = 0, \qquad X = \frac{\beta}{3\lambda},$$

- ... giving the bulk spectral density in the form

$$\rho(\omega) = \frac{1}{4\pi} s(T) 6 X^2(z_h) \frac{\omega}{|h_{ii}(z \to 0)|^2}.$$

A few computational details

- Due to complicated logarithmic structure of potential $V(\lambda)$, numerical methods necessary to obtain the full correlators (implemented within Mathematica)
- Numerically stable results achieved for (almost) arbitrary temperatures $T \geq T_c$ and for frequencies up to several thousand T_c by
  - Imposing purely infalling boundary conditions at the horizon through a high order analytic expansion,

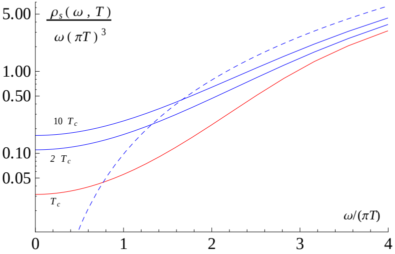
$$h(z \to z_h) = (z - z_h)^{i\omega/\dot{f}_h}[1 + d_1(z - z_h) + d_2(z - z_h)^2 + d_3(z - z_h)^3]\,,$$

  - Using Einstein equations to minimize number of derivatives of background functions f, b and λ in the diff. equations to be solved
- In the high frequency limit, WKB approximation can be used to obtain analytic understanding.

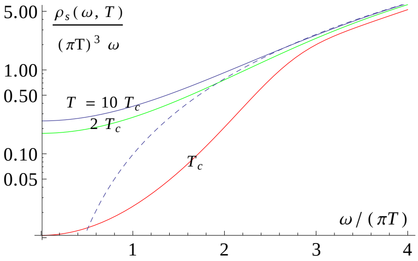
Shear spectral density

The shear spectral function in units of $\mathcal{L}^3/(4\pi G_5)$ both in IHQCD and $\mathcal{N} = 4$ SYM

IHQCD



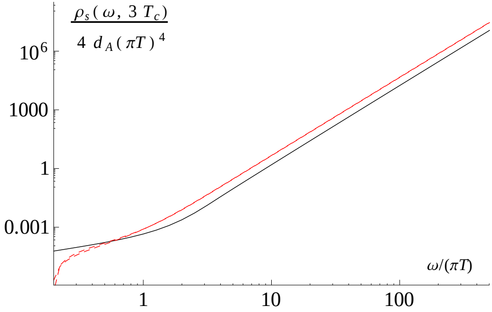
$\mathcal{N} = 4$ SYM



K. Kajantie, M. Vepsalainen, arXiv:1011.5570.

Shear spectral density

Comparison with perturbative QCD prediction at high energies, with $T = 3T_c$ (pQCD: Y. Zhu and A. Vuorinen, arXiv:1212.3818 [hep-ph].)

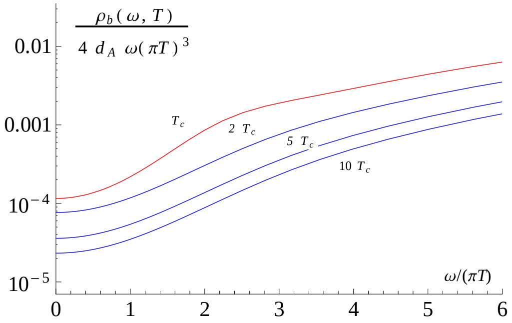


- Functional behaviour of both results $\propto \omega^4$ at large ω; however, as expected overall normalizations do not agree

$$\frac{\rho_s^{pQCD}(\omega \to \infty, T)}{\rho_s^{IHQCD}(\omega \to \infty, T)} = \frac{9}{4}$$

Bulk spectral density

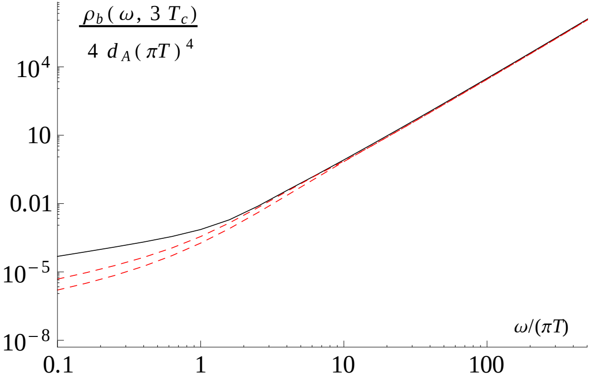
IHQCD bulk spectral density at various temperatures



- Slow convergence - numerically more challenging in large-ω region
- Bulk viscosity decreases with increasing temperature (as expected, in conformal limit $\zeta = 0$)

Bulk spectral density

Comparison with perturbative QCD prediction at high energies, with $T = 3T_c$

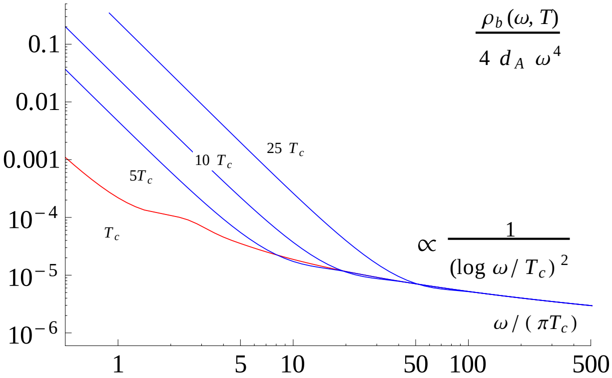


pQCD: M. Laine, A. Vuorinen and Y. Zhu, arXiv:1108.1259 [hep-ph].

- In large-ω region find perfect numerical matching

Large-ω limit

Closer look at large frequencies: ρ_b/ω^4 vs. $\omega/(\pi T_c)$



Large-ω limit

- Numerical fact: in large-ω region,

$$\frac{\rho_b(\omega \to \infty, T)}{\omega^4} \to \frac{1}{(\log \omega / T_c)^2},$$

- In pQCD, this behavior understood as coming from running of g, cf.

$$T^\mu_\mu = \frac{\beta(g)}{2g} F^a_{\mu\nu} F^a_{\mu\nu} \,,$$

- where

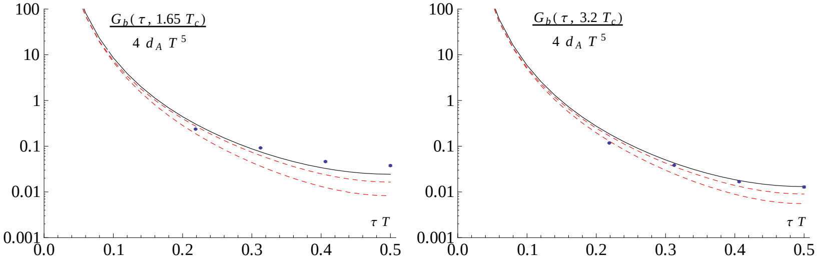
$$\frac{\beta(g)}{2g} \propto g^2 \propto \frac{1}{(\log \omega / T_c)},$$

- In contrast, in our holographic calculation both $\beta(g)$ and g are independent of ω — logarithmic behavior entirely from $h_{ii}(z \to 0)$

Euclidean correlators and lattice QCD

Simplest quantity to measure on the lattice: Euclidean imaginary time correlator

$$G_b(\tau, T) = \int_0^\infty \frac{d\omega}{\pi} \rho_b(\omega, T) \frac{\cosh\left[\left(1 - 2T\tau\right)\frac{\pi}{2}\omega\right]}{\sinh\left(\frac{\pi}{2}\omega\right)}.$$



lattice: H. B. Meyer, JHEP 1004 (2010) 099 [arXiv:1002.3343 [hep-lat]].

- Lattice seems to favour IHQCD over pQCD, though difference decreases with increasing temperature

Conclusions

- We have used IHQCD to calculate correlators in both shear and bulk channels of large-N_c Yang-Mills theory
  - Results subsequently compared with both pQCD and lattice QCD predictions
- In shear channel, a strong effect of non-conformality observed for temperatures close to T_c
- Bulk channel results fundamentally new: in $\mathcal{N} = 4$ SYM, result vanishes due to conformal invariance
- For large ω, functional behavior of spectral densities in IHQCD agrees with perturbative predictions
  - In bulk case, perfect numerical agreement with pQCD in the UV
- Lattice data for Euclidean imaginary time correlators better described by IHQCD than NLO pQCD