NORDITA

The University of Iceland

Applied Gravity: AdS/CFT and Condensed Matter Physics

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Holograv 2013 workshop
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Applied AdS-CFT

Investigate strongly coupled quantum field theories via classical gravity

- growing list of applications:

  - hydrodynamics of quark gluon plasma
  - holographic QCD
  - quantum critical systems
    - strongly correlated electron systems
    - cold atomic gases
  - out of equilibrium dynamics
  - ....

Bottom-up approach: Look for interesting behavior in simple models

- Assume that classical gravity in (asymptotically) AdS spacetime is dual to some strongly coupled QFT.
- Use AdS/CFT techniques to compute QFT correlation functions.
- Add gauge and matter fields to gravity theory to model interesting physics.
- Back-reaction can modify asymptotic behavior

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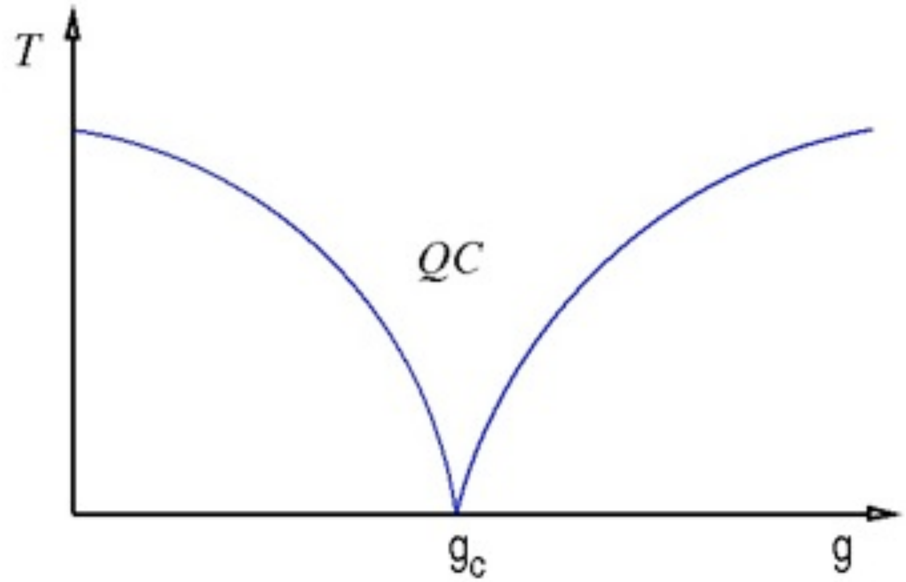
- Back-reaction can modify asymptotic behavior: non AdS - non CFT

Some reviews

- D.T. Son and A. Starinets, *Viscosity, black holes, and quantum field theory*, Ann. Rev. Nucl. Part. Sci. **57** (2007) 95.
- M. Mueller and S. Sachdev, *Quantum criticality and black holes*, arXiv:0810.3005.
- C.P. Herzog, *Lectures on holographic superfluidity and superconductivity,* J. Phys. A: Math. Theor. **42** (2009) 343001.
- S.A. Hartnoll, *Lectures on holographic methods for condensed matter physics*, Class. Quant. Grav. **26** (2009) 224002.
- M. Rangamani, *Gravity and hydrodynamics: Lectures on the fluid-gravity correspondence*, Class. Quant. Grav. **26** (2009) 224003.
- J. McGreevy, *Holographic duality with a view toward many-body physics*, Adv.High Energy Physics **2010** (2010) 723105.
- S. Sachdev, *Condensed matter and AdS/CFT*, arXiv:1002.2947.
- G. Horowitz, *Theory of superconductivity*, Lect. Notes. Phys. **828** (2011) 313.
- S. Hartnoll, *Horizons, holography, and condensed matter*, arXiv:1106.4324.
- V.E. Hubeny, S. Minwalla, and M. Rangamani, *The fluid/gravity correspondence*, arXiv:1107.5780.
- N. Iqbal, H. Liu and M. Mezei, *Lectures on holographic non-Fermi liquids and quantum phase transitions*, arXiv:1110.3814.
- S. Sachdev, *What can gauge-gravity duality teach us about condensed matter physics?,* Annual Review of Condensed Matter Physics **3** (2012) 9.
- V. Keränen and L. Thorlacius, *Holographic geometries for condensed matter applications*, in preparation.

Quantum critical points



Typical behavior at $T = 0$

characteristic energy $\delta \sim (g - g_c)^{z\nu}$

coherence length $\xi \sim (g - g_c)^{-\nu}$

$\delta \sim \xi^{-z}$ z = dynamical scaling exponent

Scale invariant theory at finite T: $\xi = cT^{-1/z}$

Deformation away from fixed pt.: $\lambda_i \sim (\text{length})^{-1}$ QCP has $\lambda_i = 0$

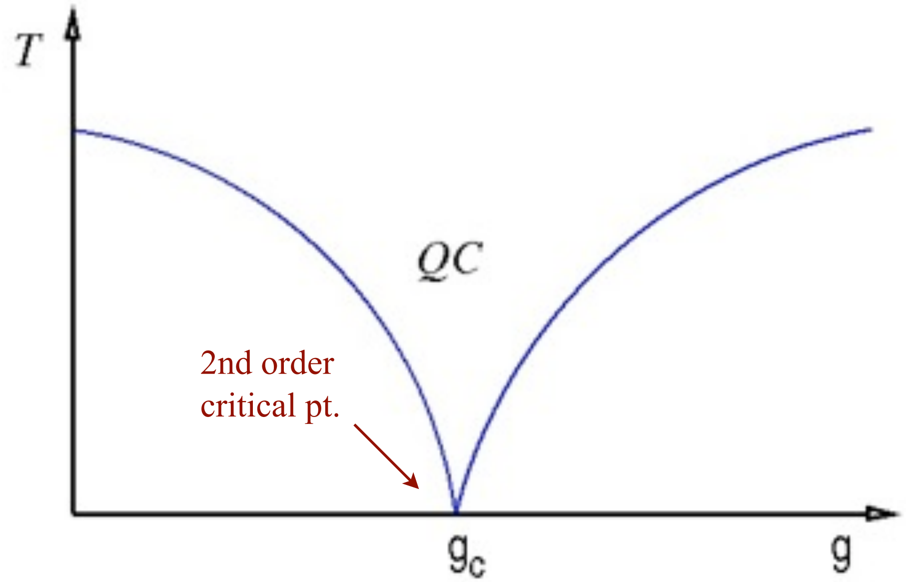
Quantum critical region : $\xi = T^{-1/z}\,\eta(T^{-1/z}\lambda_i)$ $\eta(0) = c$

Physical systems with z = 1, 2, and 3 are known -- non-integer values of z are also possible

$z = 1$ scaling symmetry is part of SO(d+1,1) conformal group = isometries of adS_{d+1}

$z > 1$ scale invariance without conformal invariance - asymptotically Lifshitz spacetime

Quantum critical points



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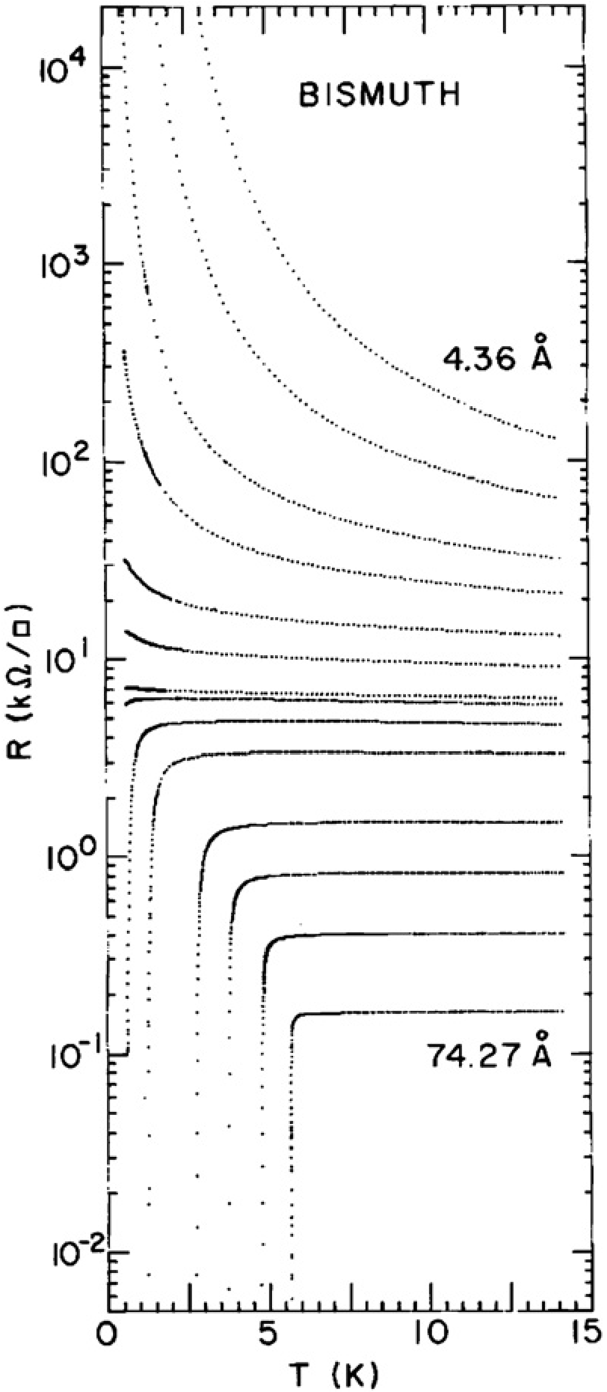


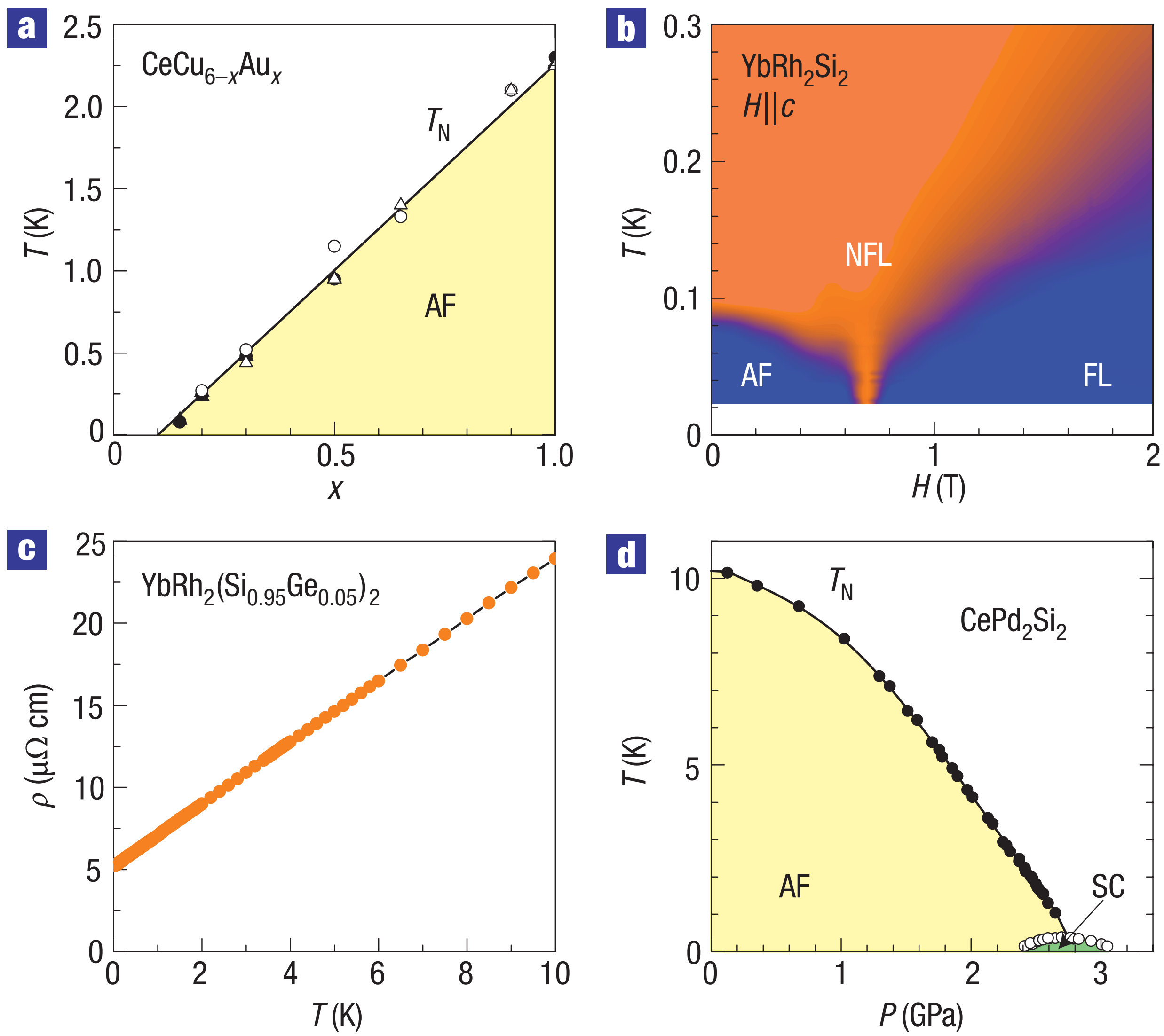
Classic example of a QCP

Resistivity vs. temperature in thin films of bismuth

T = 0 state changes from insulating to superconducting at a critical thickness

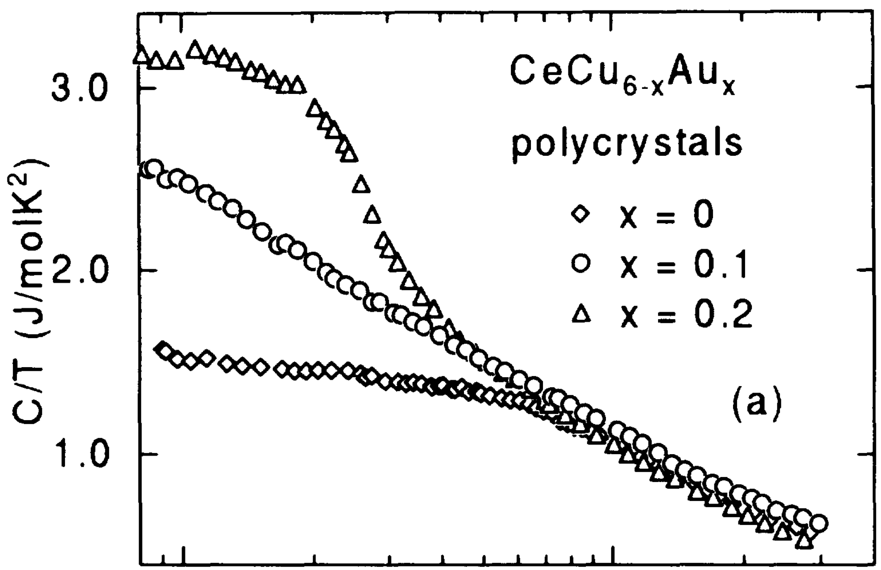
From D.B. Haviland, Y. Liu and A.M. Goldman, Phys. Rev. Lett. **62** (1989) 2180.

Quantum criticality in heavy fermion materials

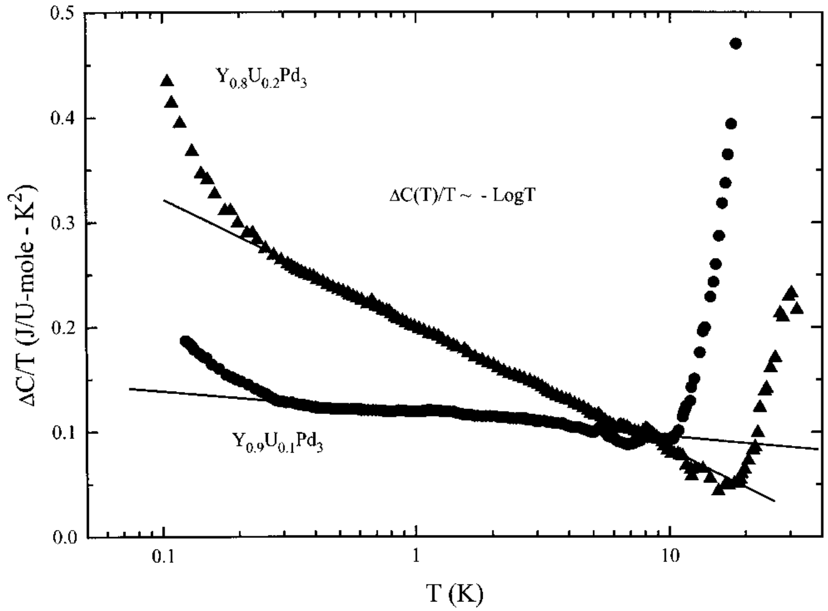


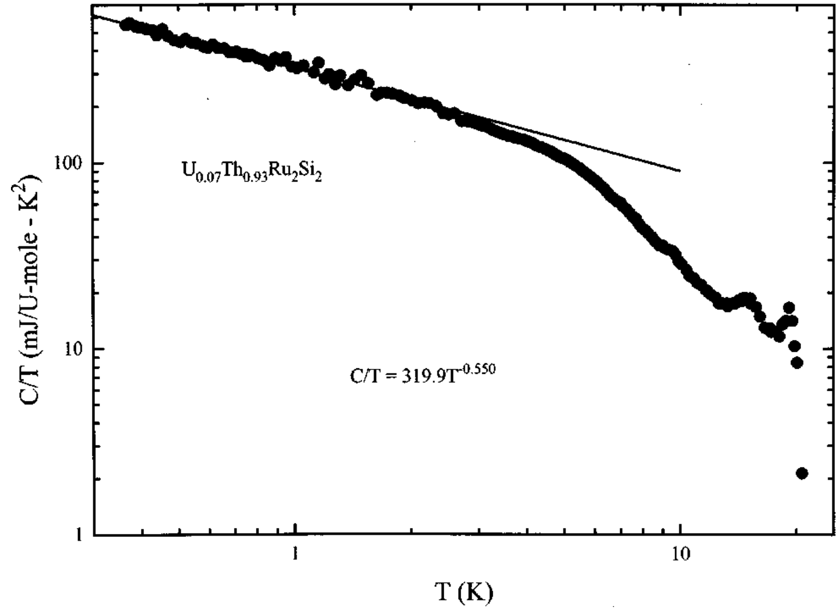
From P. Gegenwart, Q. Si and F. Steglich, Nature Phys. **4** (2008) 186.

Some measured c/T values in heavy fermion metals



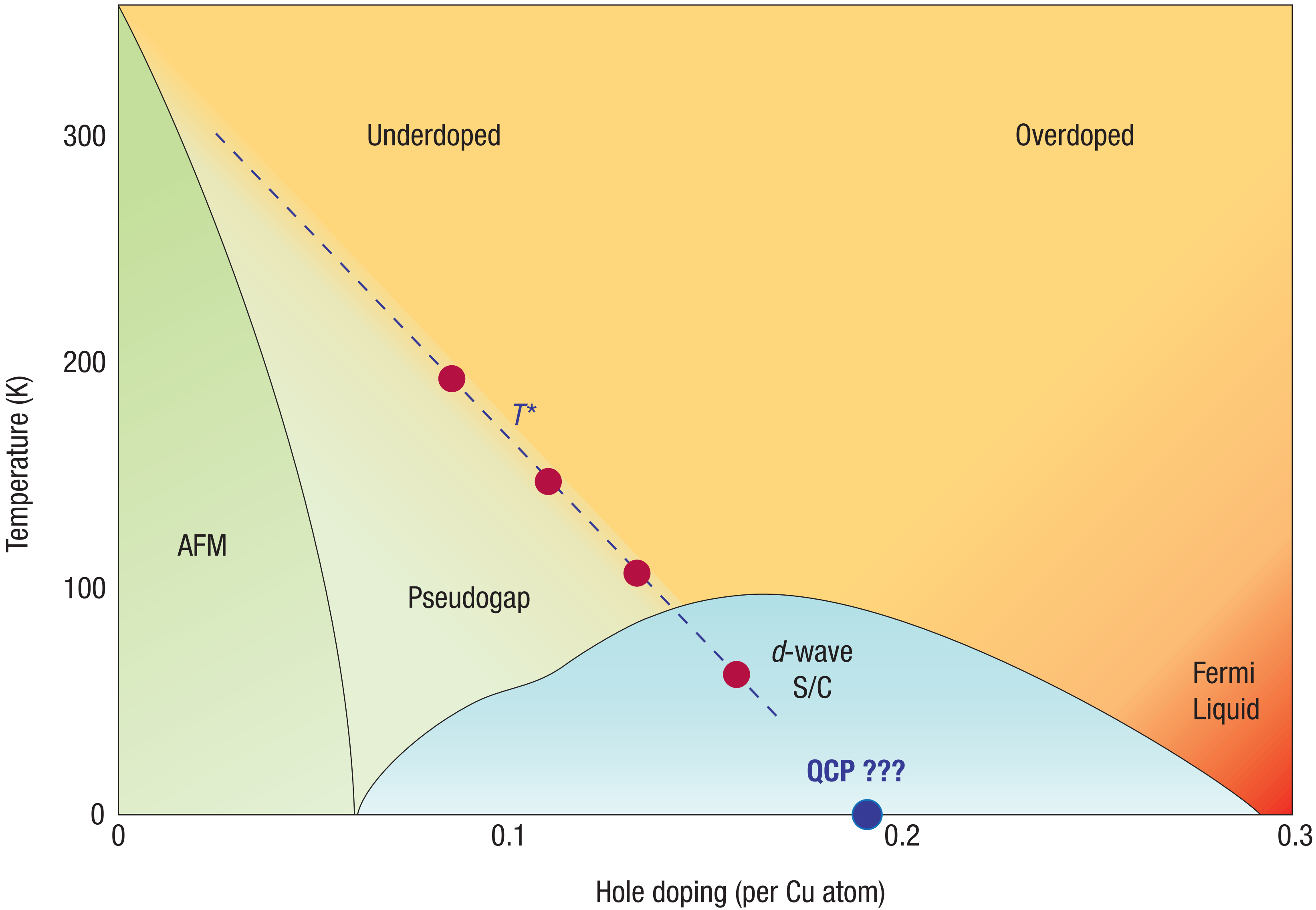
H. Lohneysen et al. *PRL* **72** (1994) 3262.





From G.Stewart, *Rev.Mod.Phys*. **73** (2001) 797.

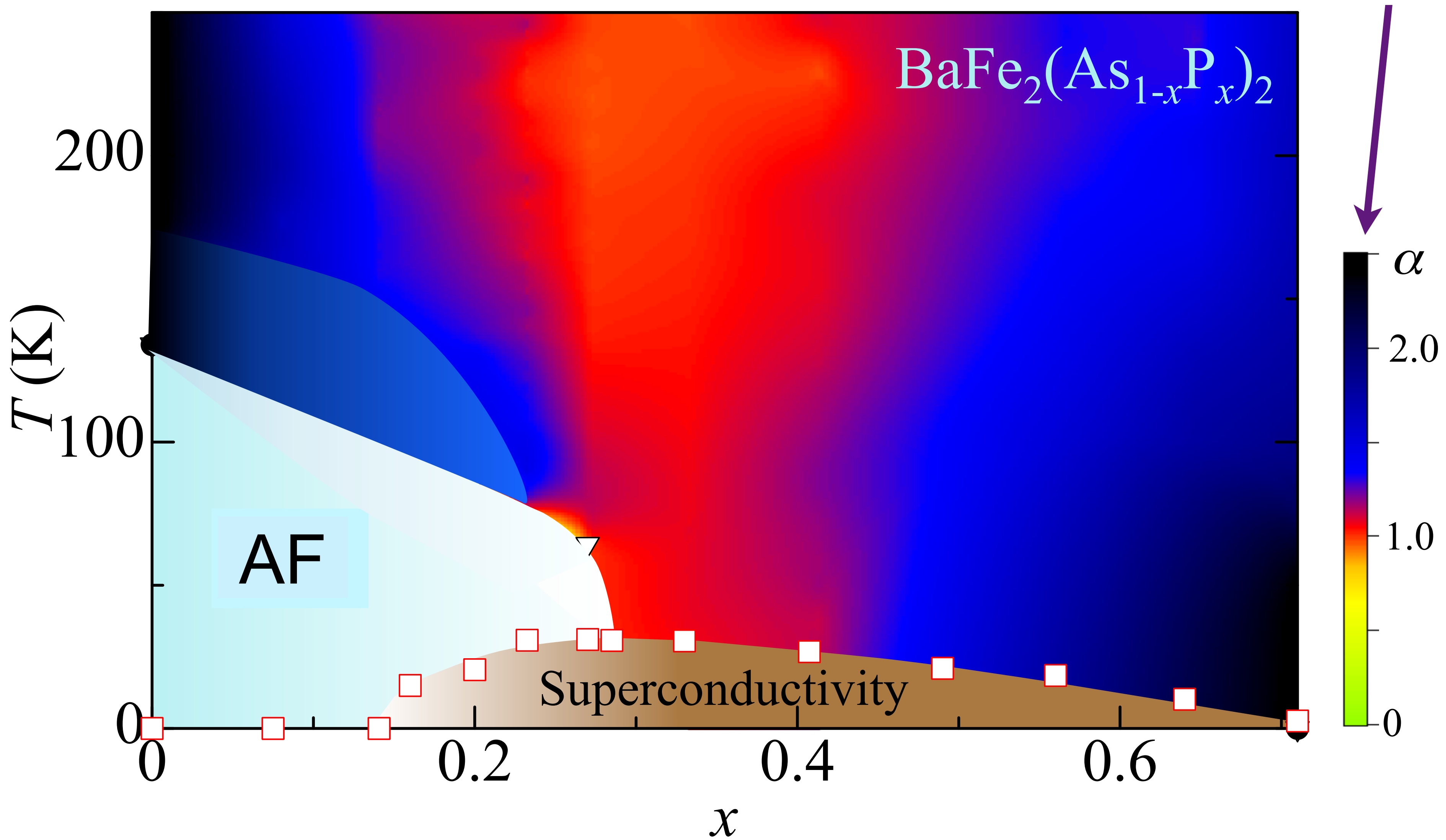
Quantum criticality in high T_c superconductors



From D.M. Broun, Nature Phys. **4** (2008) 170.

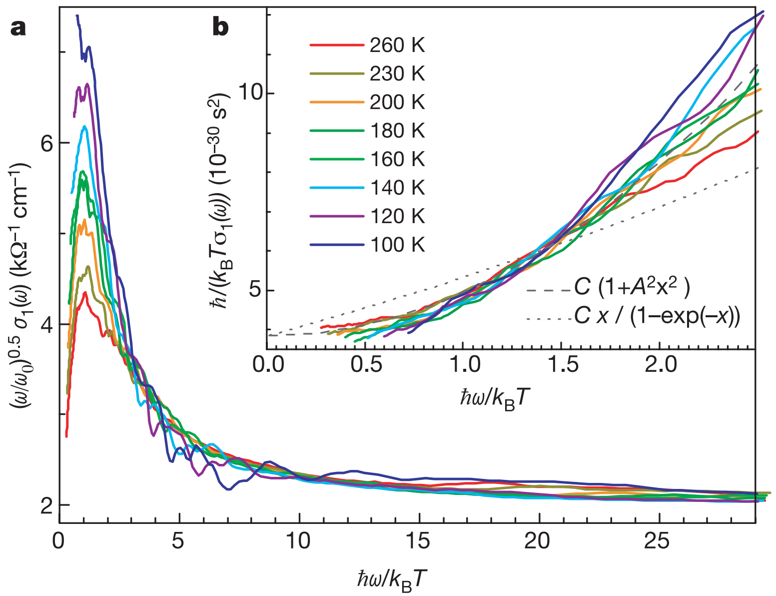
Linear resistivity in strange metal region

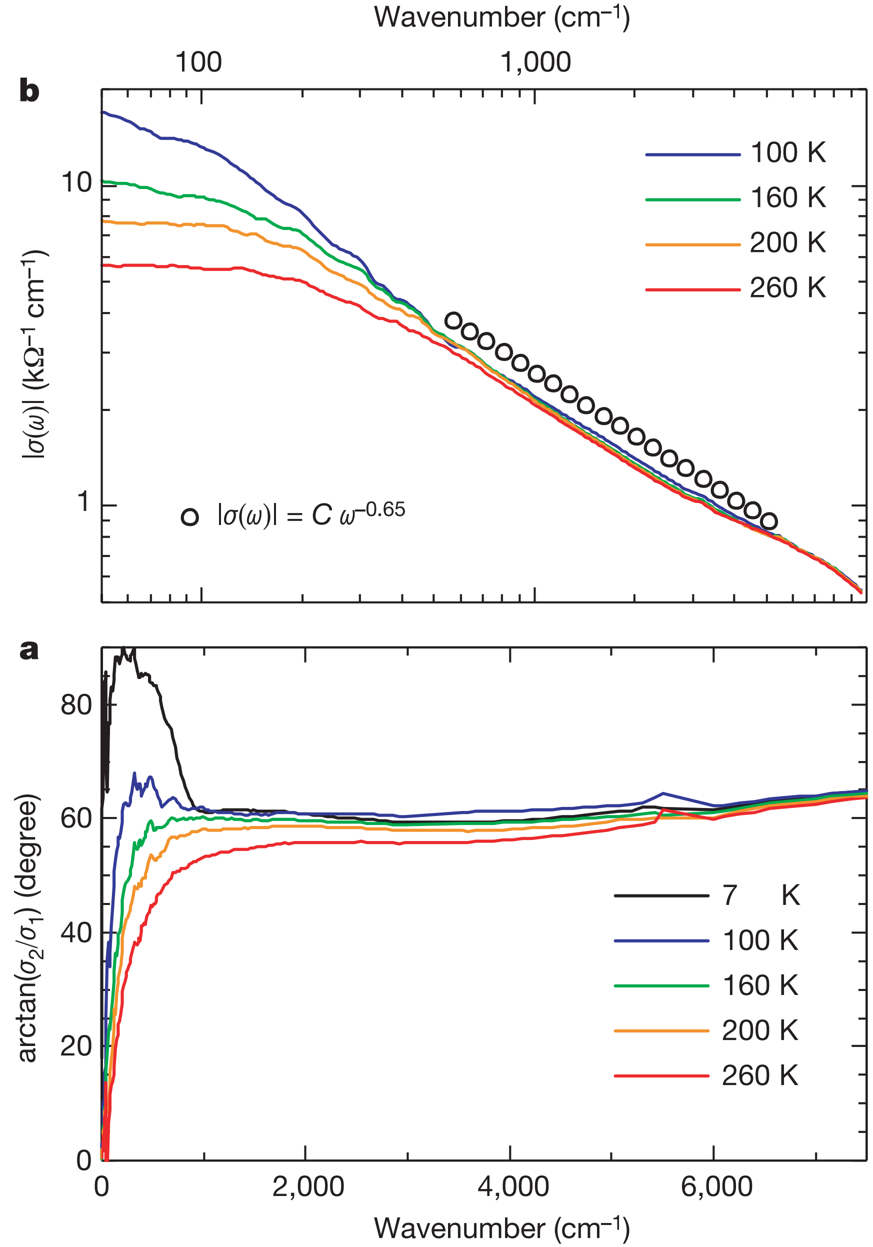
$$\rho \approx \rho_0 + a\,T^{\alpha}$$



From S. Kashara et al., *Phys. Rev. B*. **81** (2010) 184519.

Optical conductivity in $Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$





Drude form at low frequency:

$$\mathrm{Re}\,\sigma(\omega,T) \sim T^{-1}\left(1 + A^2\left(\frac{\omega}{T}\right)^2\right)^{-1}$$

Universal power law at intermediate frequency:

$$\sigma(\omega,T) \approx B\,(-i\omega)^{-2/3}$$

From D. van der Marel et al., *Nature* **425** (2003) 271.

Gravity duals at finite temperature

periodic Euclidean time: $\tau \simeq \tau + \beta, \quad \beta = \frac{1}{T}$

β introduces an energy scale: scale symmetry is broken

thermal state in field theory: black hole with $T_{\mathrm{Hawking}} = T_{\mathrm{qft}}$

finite charge density in dual field theory: electric charge on BH

magnetic effects in dual field theory: dyonic BH

$z = 1$: AdS-Reissner-Nordström BH in d+2 dimensions

$z > 1$: charged BH in d+2 dimensional $z > 1$ gravity model

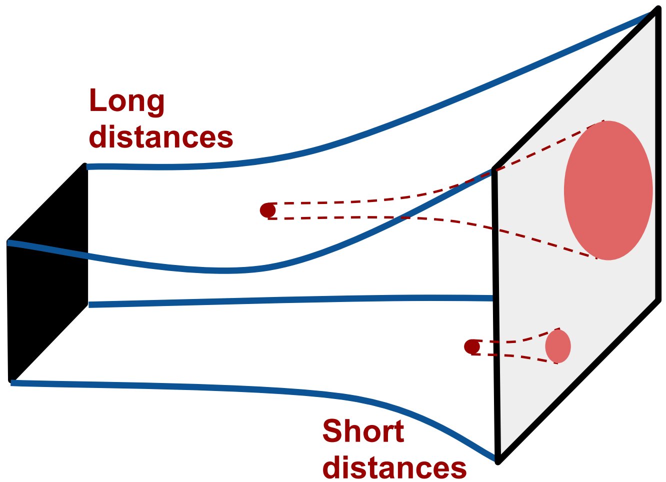
Planar AdS-RN black hole

AdS spacetime (Poincaré coordinates): $$ds^2 = \frac{r^2}{\ell^2}(-dt^2 + d\mathbf{x}^2) + \frac{\ell^2}{r^2}dr^2$$

$$\xi = \frac{\ell^2}{r} \qquad ds^2 = \frac{1}{\xi^2}(-dt^2 + d\mathbf{x}^2 + d\xi^2) \qquad \longleftarrow \quad \ell = 1$$

Planar AdS-RN black hole: $$ds^2 = \frac{1}{\xi^2}\left(-f(\xi)dt^2 + \frac{d\xi^2}{f(\xi)} + d\mathbf{x}^2\right)$$

$$f(\xi) = 1 - \left(1 + \frac{\rho^2}{2}\right)\left(\frac{\xi}{\xi_0}\right)^3 + \frac{\rho^2}{2}\left(\frac{\xi}{\xi_0}\right)^4 \qquad \longleftarrow \quad d = 2$$



From S. Hartnoll, arXiv:1106.4342

Electrical conductivity from AdS/CFT

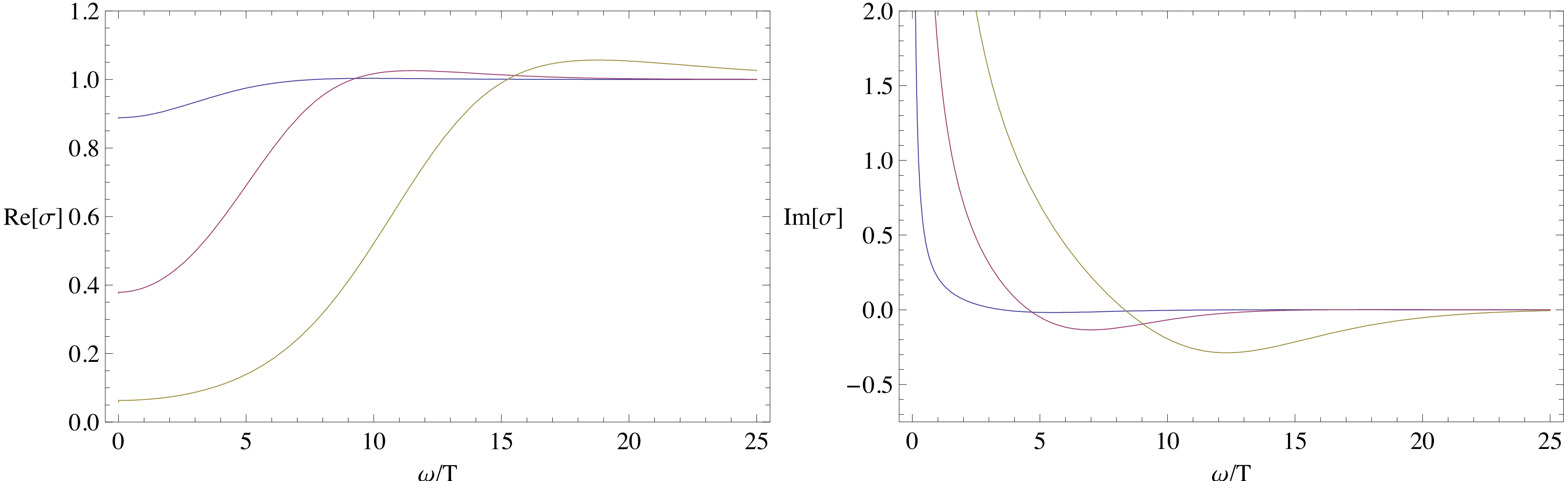
Holographic dictionary: $A_\mu \longleftrightarrow J_\mu$ U(1) current

Solve Maxwell's equations in black hole background

-- with "in-going" boundary conditions at black hole horizon

Asymptotic behavior: $A_x(\omega, \vec{k}, \xi) \approx a_x^{(0)}(\omega, \vec{k}) + a_x^{(1)}(\omega, \vec{k})\xi + \ldots$

Calculation simplifies at $\vec{k} = 0$: $\sigma_{xx}(\omega) = -\frac{i}{\omega} \frac{a_x^{(1)}}{a_x^{(0)}}$



Figures from S. Hartnoll, Class. Quant. Grav. 26 (2009) 224002

Delta function peak in $\mathrm{Re}\ \sigma$ at $\omega = 0$ due to translation invariance

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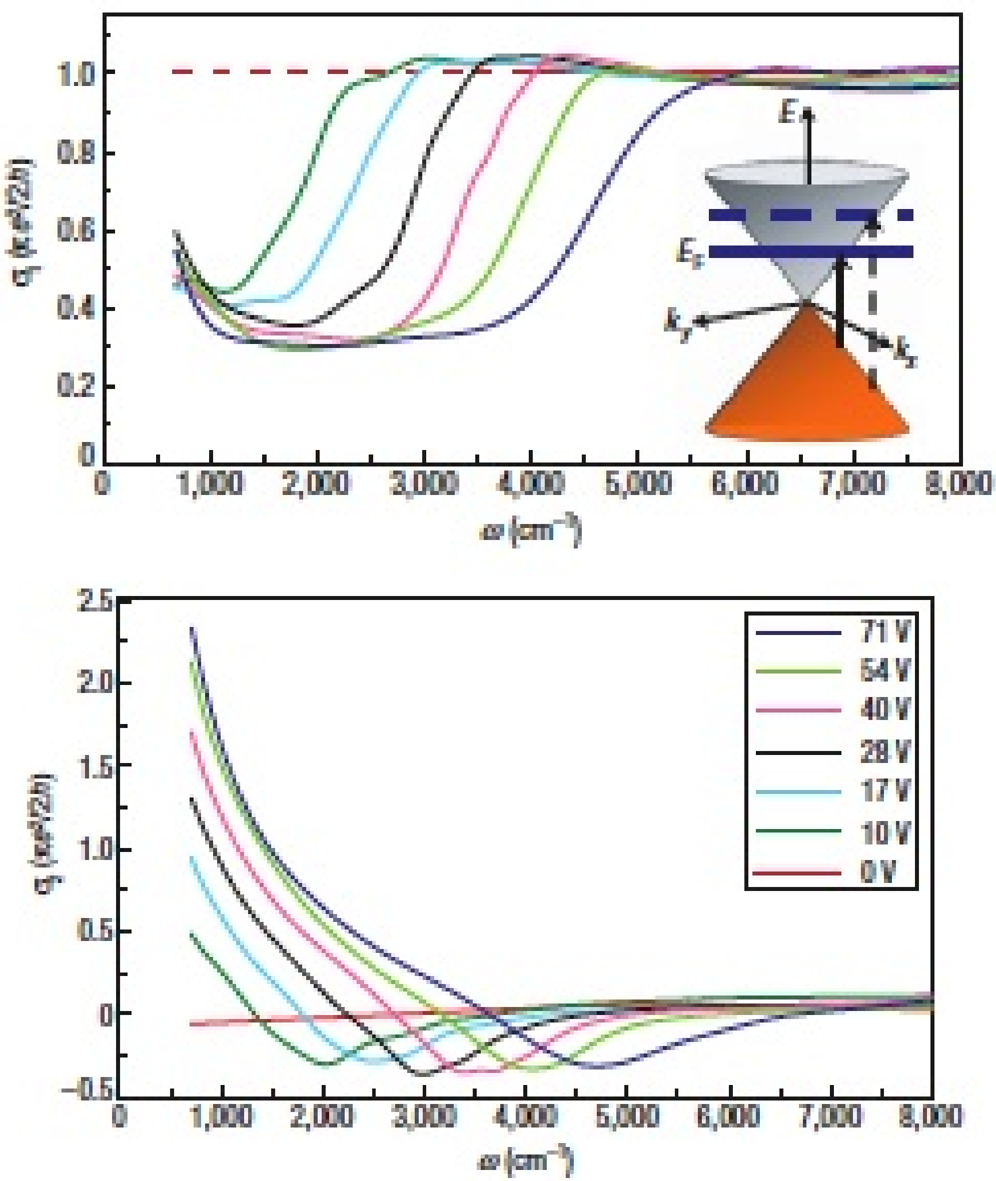
Experimental results in graphene



Figures from S. Sachdev, arXiv:0711.3015

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Holography with anisotropic scaling

Q: Can we give a gravity dual description of a strongly coupled system which exhibits anisotropic scaling?

A: Look for a gravity theory with spacetime metric of the form

$$ds^2 = \ell^2 \left(-r^{2z} dt^2 + r^2 d^2\mathbf{x} + \frac{dr^2}{r^2} \right) \qquad \longleftarrow \ell = 1$$

which is invariant under

$$t \to \lambda^z t, \quad \mathbf{x} \to \lambda \mathbf{x}, \quad r \to \frac{r}{\lambda}$$

Kachru, Liu, & Mulligan '08; Koreteev, Libanov '08

Can have a more general metric that also exhibits hyperscaling violation

$$ds^2 = r^{-2\theta/d} \left(-r^{2z} dt^2 + r^2 d^2\mathbf{x} + \frac{dr^2}{r^2} \right)$$

Holographic models with Lifshitz scaling

I) Einstein-Maxwell-Proca model

Kachru, Liu, & Mulligan '08; Taylor '08; Brynjólfsson et al. '09

$$S_{\text{EMP}} = \int \mathrm{d}^{d+2}x\sqrt{-g}\left(R - 2\Lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - \frac{c^2}{2}\mathcal{A}_\mu\mathcal{A}^\mu\right)$$

II) Einstein-Dilaton-Maxwell model

Taylor '08; Tarrío & Vandoren '11

$$S_{\text{EDM}} = \int \mathrm{d}^{d+2}x\sqrt{-g}\left(R - 2\Lambda - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\sum_{i=1}^{N} e^{\lambda_i\phi}F_i^2\right)$$

we will take N = 1 or 2

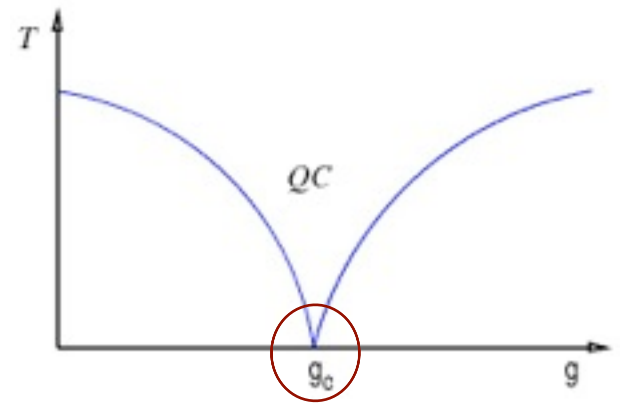
d = 3, 2, or 1 for CM applications

Fixed point metric

The Lifshitz metric

$$\mathbf{x} = (x_1, \ldots, x_d)$$

$$ds^2 = -r^{2z} dt^2 + r^2 d\mathbf{x}^2 + \frac{dr^2}{r^2}$$



is a solution of both models for particular values of couplings and background fields

EMP model:

$$c = \sqrt{z\,d}, \qquad \Lambda = -\frac{z^2 + (d-1)z + d^2}{2}$$

$$\mathcal{A}_t = \sqrt{\frac{2(z-1)}{z}}\, r^z, \quad \mathcal{A}_{x_i} = \mathcal{A}_r = 0 \qquad\qquad A_\mu = 0$$

EDM model:

$$\lambda_1 = -\sqrt{\frac{2d}{z-1}}\,, \qquad \Lambda = -\frac{(d+z)(d+z-1)}{2}\,, \qquad e^{\phi} = \left(\frac{r}{r_0}\right)^{\sqrt{2d(z-1)}}$$

$$F^{(1)}_{rt} = 2 r_0^{z-1} \sqrt{(d+z)(z-1)} \left(\frac{r}{r_0}\right)^{d+z-1}\,, \qquad F^{(2)}_{\mu\nu} = 0$$

Field equations without matter

EMP model:

$$
\begin{aligned}
G_{\mu\nu} + \Lambda g_{\mu\nu} &= T^{\text{Maxwell}}_{\mu\nu} + T^{\text{Proca}}_{\mu\nu} \\
\nabla_\nu F^{\nu\mu} &= 0 \\
\nabla_\nu \mathcal{F}^{\nu\mu} &= c^2 \mathcal{A}^\mu
\end{aligned}
$$

$$
\begin{aligned}
T^{\text{Maxwell}}_{\mu\nu} &= \frac{1}{2}(F_{\mu\lambda}F_\nu{}^\lambda - \frac{1}{4}g_{\mu\nu}F_{\lambda\sigma}F^{\lambda\sigma}) \\
T^{\text{Proca}}_{\mu\nu} &= \frac{1}{2}(\mathcal{F}_{\mu\lambda}\mathcal{F}_\nu{}^\lambda - \frac{1}{4}g_{\mu\nu}\mathcal{F}_{\lambda\sigma}\mathcal{F}^{\lambda\sigma}) + \frac{c^2}{2}(\mathcal{A}_\mu\mathcal{A}_\nu - \frac{1}{2}g_{\mu\nu}\mathcal{A}_\lambda\mathcal{A}^\lambda)
\end{aligned}
$$

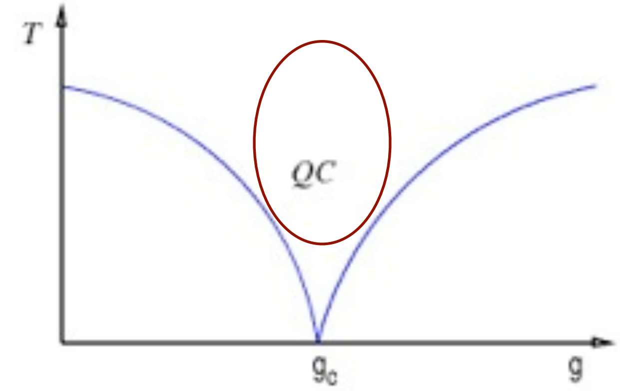
EDM model:

$$
\begin{aligned}
G_{\mu\nu} + \Lambda g_{\mu\nu} &= T^{\phi}_{\mu\nu} + T^{(1)}_{\mu\nu} + T^{(2)}_{\mu\nu} \\
\nabla_\nu \left(e^{\lambda_i \phi} F^{\nu\mu}_{(i)} \right) &= 0 \qquad i = 1, 2 \\
\nabla^2 \phi &= \frac{\lambda_1}{4} e^{\lambda_1 \phi} F^2_{(1)} + \frac{\lambda_2}{4} e^{\lambda_2 \phi} F^2_{(2)}
\end{aligned}
$$

$$
\begin{aligned}
T^{\phi}_{\mu\nu} &= \frac{1}{2}\left(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}(\partial\phi)^2 \right) \\
T^{(i)}_{\mu\nu} &= \frac{e^{\lambda_i\phi}}{2}\left(F_{(i)\mu\lambda}F_{(i)\nu}{}^\lambda - \frac{1}{4}g_{\mu\nu}F^2_{(i)} \right)
\end{aligned}
$$

Quantum critical region

Look for black brane solutions of EMP model



Metric: $$ds^2 = -r^{2z} f(r)^2 dt^2 + r^2 d\mathbf{x}^2 + \frac{g(r)^2}{r^2} dr^2$$

Vielbein: $$e_t^0 = r^z f(r)\,, \quad e_{x_i}^i = r\,, \quad e_r^{d+1} = \frac{g(r)}{r}$$

Vector fields: $$A_M = (\alpha(r), 0, 0, 0)\,, \qquad \mathcal{A}_M = \sqrt{\frac{2(z-1)}{z}}(a(r), 0, 0, 0)$$

Lifshitz geometry: $$f = g = a = b = 1, \qquad \tilde{\rho} = \alpha = 0$$

Field equations:

$d = 2$

$$\begin{aligned}
\dot{\alpha} + \alpha\frac{\dot{f}}{f} &= -z\alpha + \tilde{\rho}e^{-2u}g \\
\dot{a} + a\frac{\dot{f}}{f} &= -za + zgb \\
\dot{b} &= -2b + 2ga \\
\dot{g} + \frac{\dot{f}}{f}g &= (z-1)\left(g^2a^2 - 1\right)g \\
\frac{\dot{f}}{f} &= \frac{g^2}{2}\left((z-1)a^2 - \frac{z(z-1)}{2}b^2 + \frac{(z^2+z+4)}{2} - \frac{\tilde{\rho}^2}{4}e^{-4u}\right) - \frac{(2z+1)}{2}
\end{aligned}$$

$u \equiv \log\left(\frac{r}{r_0}\right)$, $\dot{f} \equiv \frac{df}{du}$

$r = r_0$ event horizon

$\tilde{\rho} = \frac{\rho}{r_0^2}$ charge density

Black brane solutions   EMP model

event horizon: $u = 0$

asymptotic region: $u \to \infty$

U.Danielsson & L.T. '09
R.Mann '09; G.Bertoldi, B.Burrington, & A.Peet '09
E.Brynjolfsson, U.Danielsson, L.T., T. Zingg '09

Known exact solutions:

AdS-Reissner-Nordström: $d = 2,\ z = 1$ (solution exists for any integer $d \geq 1$)

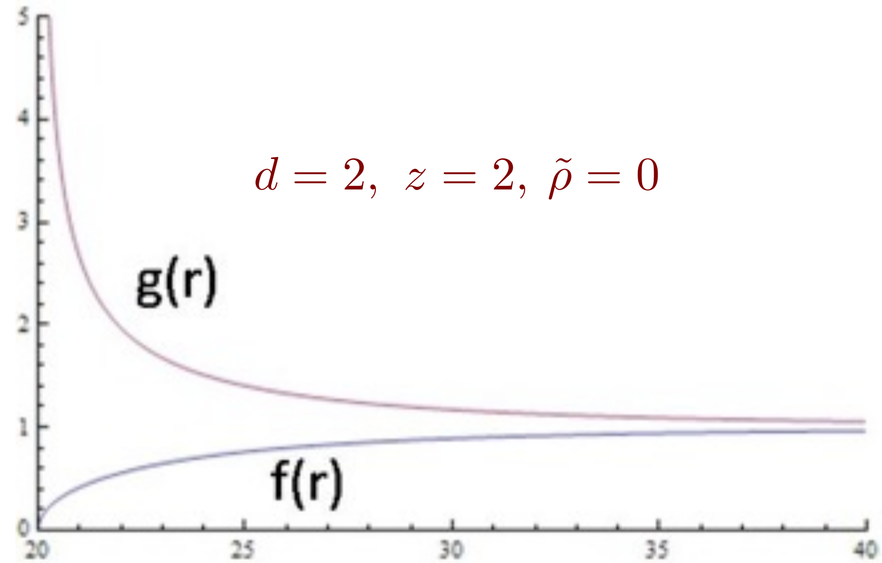
$$f^2 = \frac{1}{g^2} = (1 - e^{-u})(1 + e^{-u} + e^{-2u} - \frac{\tilde{\rho}^2}{4} e^{-3u}), \quad A_t = L r_0 \tilde{\rho}\,(1 - e^{-u}), \quad \mathcal{A}_\mu = 0$$

Exact Lifshitz black brane: $d = 2,\ z = 4,\ \tilde{\rho} = \pm\sqrt{8}$:

$$f^2 = \frac{1}{g^2} = a^2 = 1 - e^{-4u}, \quad b = 1, \quad A_t = \pm\sqrt{2} L r_0^4\,(e^{2u} - 1)$$

can be generalized to solution with $z = 2d$
D.Pang '09

Numerical solutions can be found for any d and z



Asymptotic behavior

EMP model, $d=2$

Linearize e.o.m.'s around Lifshitz point: $g=1+\delta g \quad b=1+\delta b \quad a=1+\delta a$

$$\frac{d}{du}\begin{bmatrix}\delta g\\ \delta b\\ \delta a\end{bmatrix} = M(z)\begin{bmatrix}\delta g\\ \delta b\\ \delta a\end{bmatrix} + \frac{\tilde{\rho}^2}{8}e^{-4u}\begin{bmatrix}1\\ 0\\ 1\end{bmatrix}$$

reduced system involving (*g*,*b*,*a*)

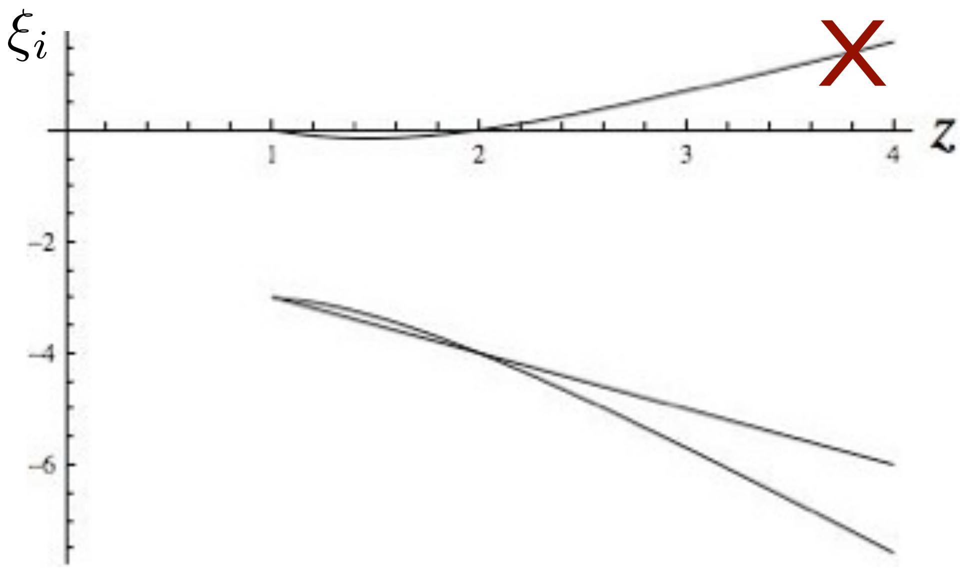
$$M(z) = \begin{bmatrix} -z-1 & \frac{z(z+1)}{2} & -2z+1 \\ 2 & -2 & 2 \\ -3 & \frac{z(z-1)}{2} & z-1 \end{bmatrix}$$

Solve eigenvalue problem: $M(z)\,\vec{F}_i = \xi_i\,\vec{F}_i$

$$\begin{bmatrix}\delta g\\ \delta b\\ \delta a\end{bmatrix} \approx \sum_{i=1}^{3}\alpha_i e^{\xi_i u}\vec{F}_i + \frac{\tilde{\rho}^2}{8}e^{-4u}\vec{F}_0$$

eigenmodes — universal mode

$$\xi_i = \begin{cases} -2-z \\ \frac{1}{2}\left(-2-z\pm\sqrt{9z^2-20z+20}\right) \end{cases}$$



Asymptotically Lifshitz spacetime

Finite energy

U.Danielsson &LT '09
G.Bertoldi et al. '09
S.Ross & O.Saremi '09

Holographic renormalization at z > 1

T. Zingg, '11; S.F. Ross, '11
M. Baggio, J. de Boer, K. Holsheimer '11
R.B. Mann, R. McNees '11; J. Tarrío '12

Black brane solutions in EDM model $(d = 2)$

Tarrío & Vandoren '11

$$ds^2 = -r^{2z}b(r)dt^2 + r^2 d\mathbf{x}^2 + \frac{dr^2}{r^2 b(r)}$$

$$b(r) = 1 - \left(1 + \frac{\tilde{\rho}^2}{4z}\right)\left(\frac{r_0}{r}\right)^{z+2} + \frac{\tilde{\rho}^2}{4z}\left(\frac{r_0}{r}\right)^{2z+2}$$

event horizon at $r = r_0$

$$e^{\phi} = \left(\frac{r}{r_0}\right)^{2\sqrt{z-1}}$$

$$F^{(1)}_{rt} = 2r_0^{z-1}\sqrt{(z+2)(z-1)}\left(\frac{r}{r_0}\right)^{z+1}$$

$$F^{(2)}_{rt} = z\tilde{\rho}\, r_0^{z-1}\left(\frac{r_0}{r}\right)^{z+1}$$

electric charge density

Hawking temperature: $T = \frac{r_0^z}{4\pi}\left(z + 2 - \frac{\tilde{\rho}^2}{4}\right)$

Asymptotic behavior

EDM model, $d = 2$

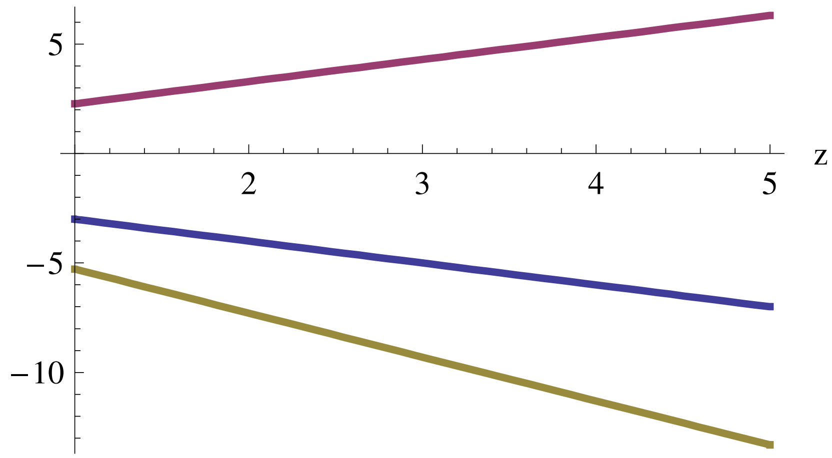
$$ds^2 = -r^{2z} f(r) dt^2 + \frac{dr^2}{r^2 g(r)} + r^2 (dx^2 + dy^2) \,, \qquad e^{\phi(r)} = \left(\frac{r}{r_0}\right)^{2\sqrt{z-1}} e^{\sqrt{z-1}\,\delta\varphi(r)}$$

$$F^{(i)}_{rt} = \rho_i \, r_0^{z-1} \left(\frac{r}{r_0}\right)^{z-3} \sqrt{\frac{f(r)}{g(r)}}\, e^{-\lambda_i \phi(r)} \,, \qquad \lambda_1 = -1/\sqrt{z-1} \,, \qquad \lambda_2 = \sqrt{z-1}$$

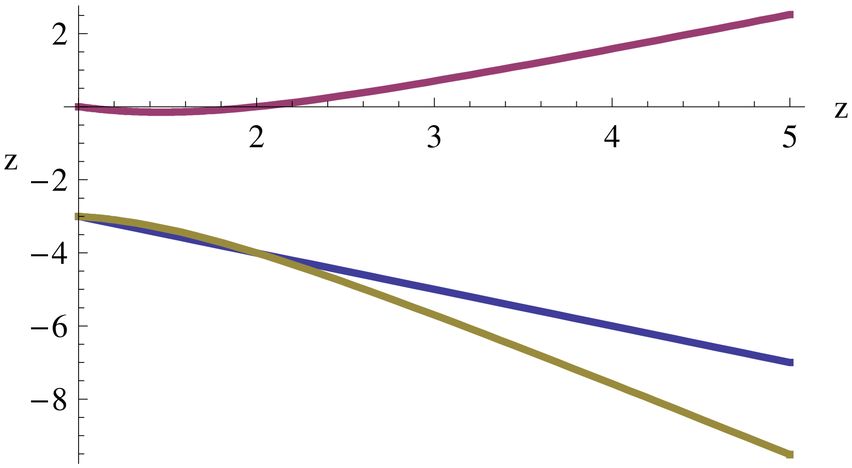
Linearized e.o.m. $$r\frac{d}{dr}\begin{bmatrix} \delta g \\ \delta\varphi \\ \delta\zeta \end{bmatrix} = M(z) \begin{bmatrix} \delta g \\ \delta\varphi \\ \delta\zeta \end{bmatrix} - \frac{\rho_2^2}{4}\left(\frac{r_0}{r}\right)^{2z+2} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad \delta\zeta \equiv r\,\delta\varphi'$$

Eigenvalues of *M(z)* : $$\left\{ -z-2 \,, \; \frac{1}{2}\left(-z-2 \pm \sqrt{(z+2)(9z+10)}\right)\right\}$$

EDM eigenvalues



EMP eigenvalues



Dynamical solutions EDM model, $d = 2$

Keränen, Keski-Vakkuri, & L.T. '11

Rewrite static black brane solution using $dv = dt + \frac{r^{-z-1}}{b(r)} dr$

$$ds^2 = -r^{2z} b(r) dv^2 + 2r^{z-1} dv\, dr + r^2 d\mathbf{x}^2$$

$$e^{\phi} = \left(\frac{r}{r_0}\right)^{2\sqrt{z-1}} \qquad b(r) = 1 - \tilde{m}\left(\frac{r_0}{r}\right)^{z+2} + \frac{\tilde{\rho}^2}{4z}\left(\frac{r_0}{r}\right)^{2z+2}$$

$$F^{(2)}_{rv} = z\tilde{\rho}\, r_0^{z-1}\left(\frac{r_0}{r}\right)^{z+1} \qquad F^{(1)}_{rv} = 2r_0^{z-1}\sqrt{(z+2)(z-1)}\left(\frac{r}{r_0}\right)^{z+1}$$

Lifshitz-Vaidya solution: $\tilde{m} \to \tilde{m}(v)\,, \quad \tilde{\rho} \to \tilde{\rho}(v)$

$\tilde{m}(v)\,,\ \tilde{\rho}(v)$ determined by incoming energy and charge density

Lifshitz-Vaidya geometry in EMP model requires numerical solution for gravitational collapse in asymptotically Lifshitz spacetime

Hawking temperature

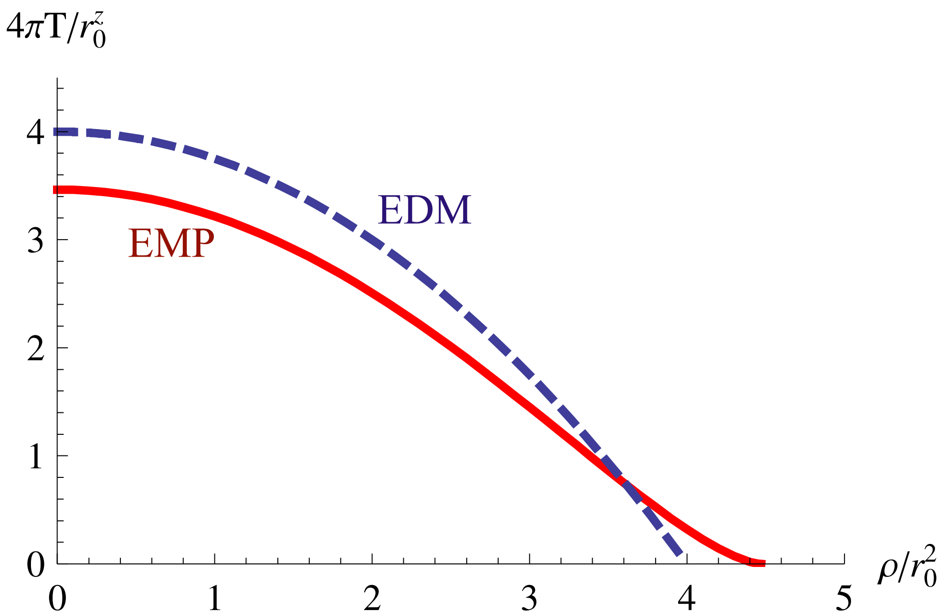
determined numerically in EMP model

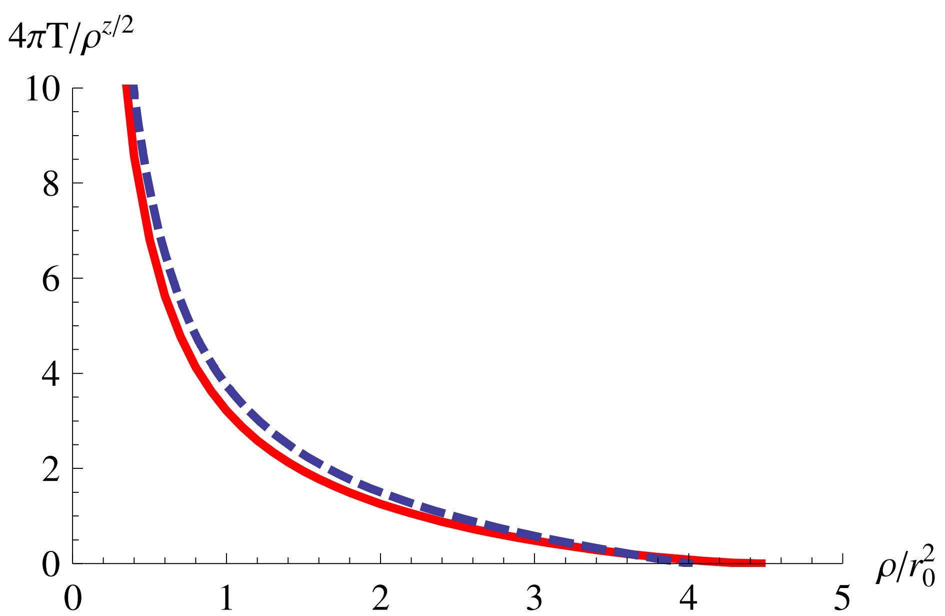
EMP model: $$T_H = \frac{r_0^z}{4\pi}\frac{f_0}{g_0} \equiv \frac{r_0^z}{4\pi}F_z(\tilde{\rho})$$

$$f(u) = \sqrt{u}\,(f_0 + \ldots)$$
$$g(u) = \frac{1}{\sqrt{u}}\,(g_0 + \ldots)$$

EDM model: $$T_H = \frac{r_0^z}{4\pi}\left[z + 2 - \frac{\tilde{\rho}^2}{4}\right]$$

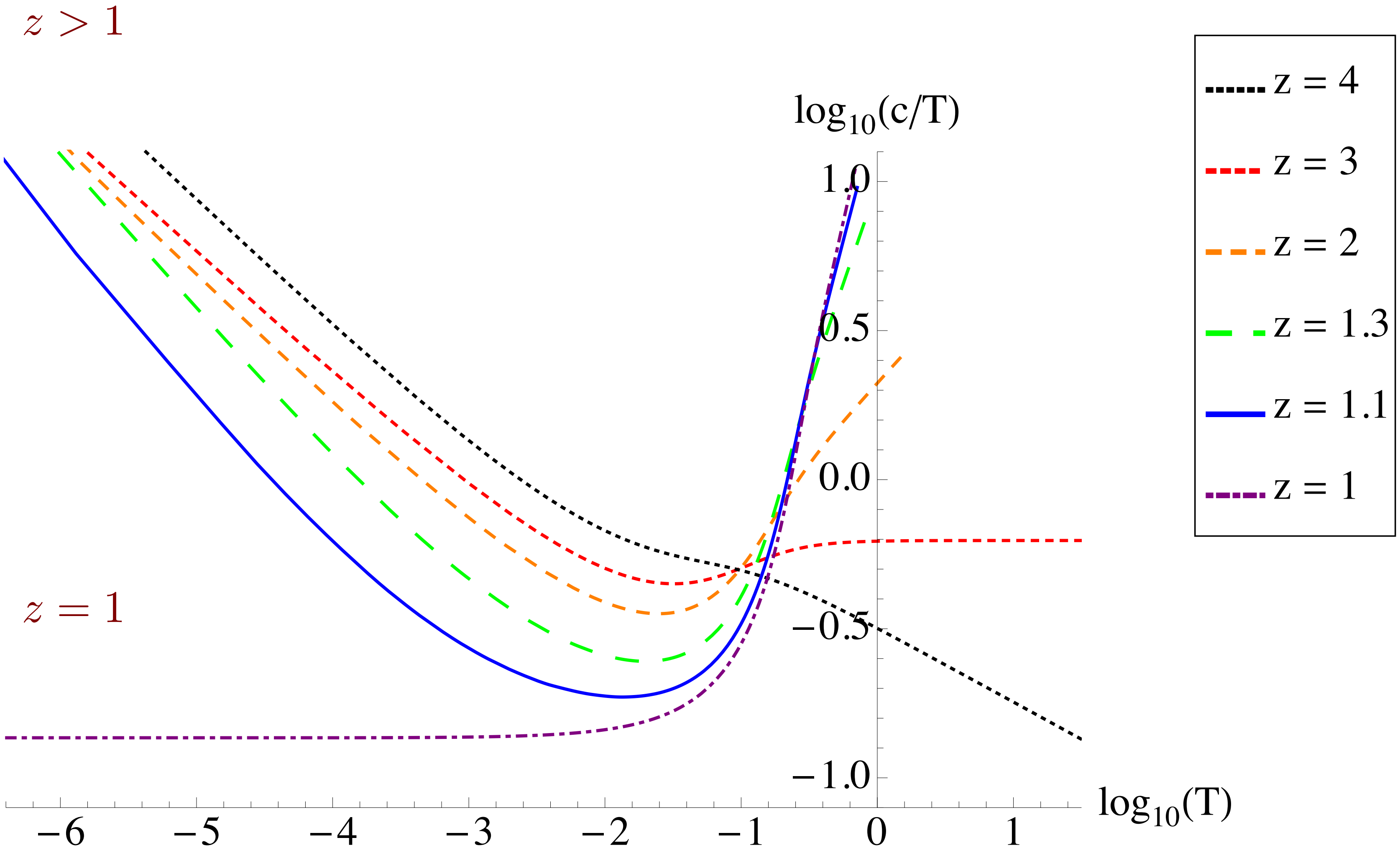
$$\tilde{\rho} = \rho/r_0^2$$



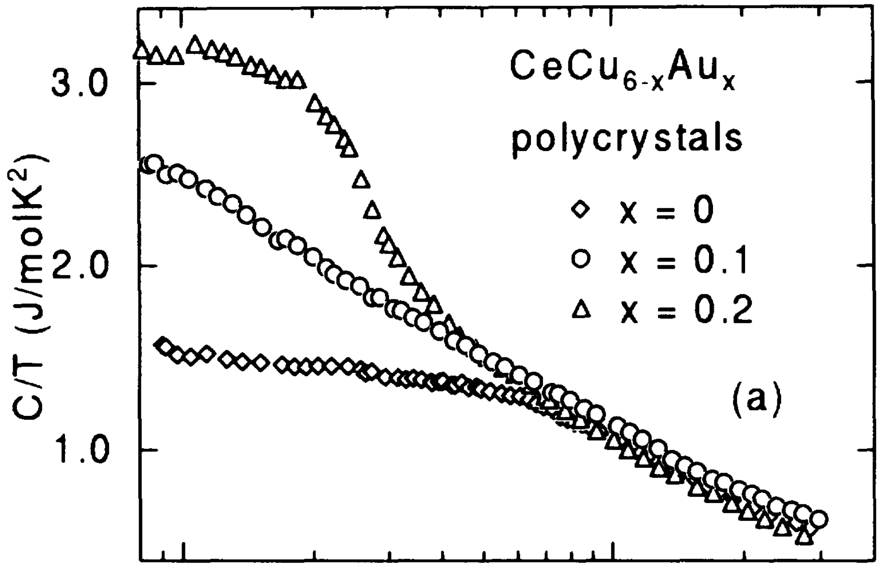


Sommerfeld ratio *vs.* temperature EMP model

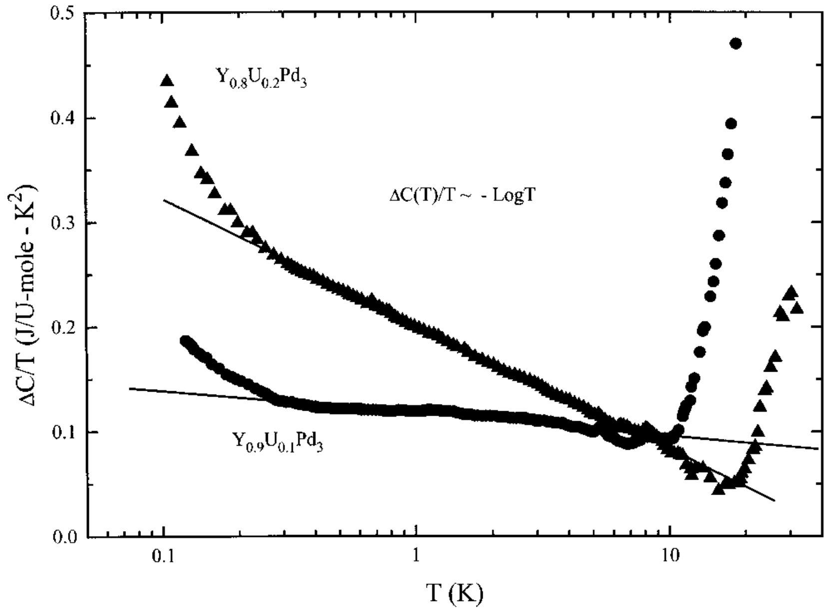
E. Brynjólfsson, U. Danielsson, L.T., T. Zingg, (2010)

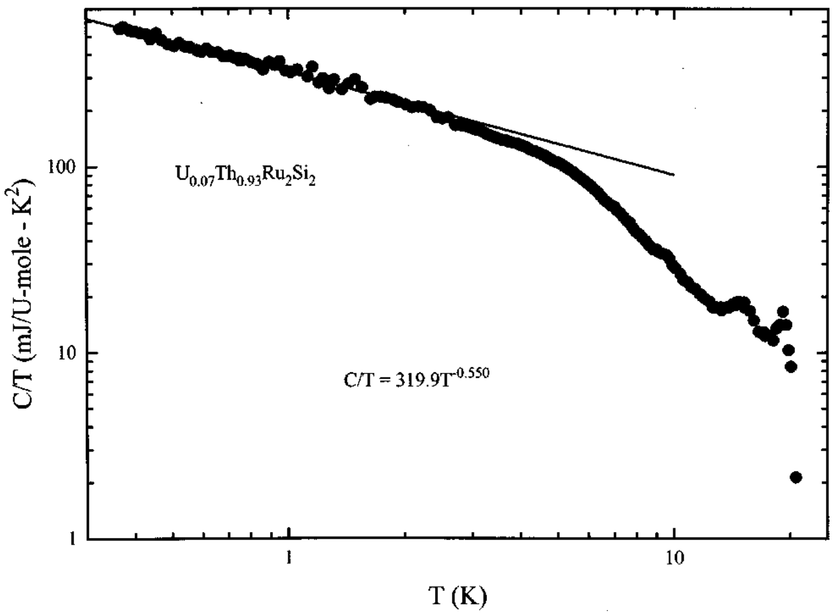


Some measured c/T values in heavy fermion metals



H. Lohneysen et al. *PRL* **72** (1994) 3262.

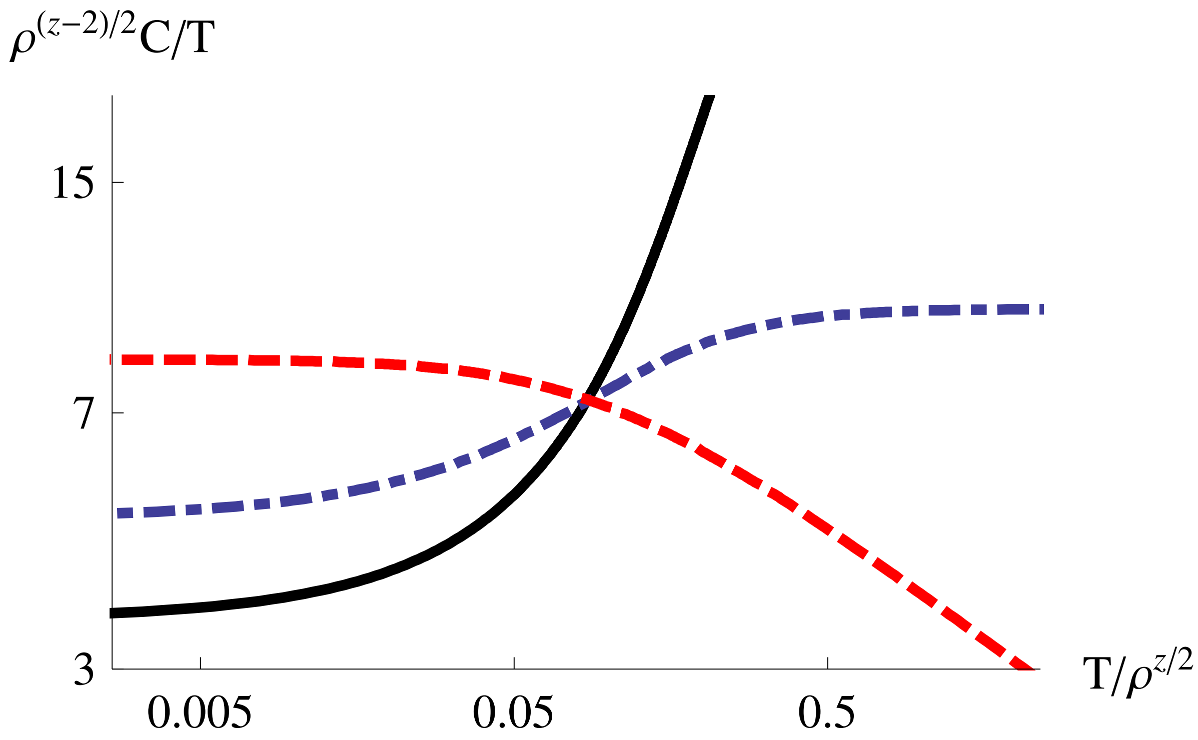




From G.Stewart, *Rev.Mod.Phys*. **73** (2001) 797.

<u>Sommerfeld ratio *vs*. temperature</u> EDM model

V. Keränen, L.T., (2012)



$\frac{c}{T} \rightarrow$ constant at low T for all values of z

Correlation functions of scalar operators

Bulk scalar field: $$S = -\frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\,\left(\partial^\mu\psi\partial_\mu\psi + m^2\psi^2\right)$$

$$r \to \infty \;: \qquad \psi(r) \to c_-\left(r^{-\Delta_-} + \ldots\right) + c_+\left(r^{-\Delta_+} + \ldots\right)$$

$$\Delta_\pm = \frac{z+2}{2} \pm \sqrt{\left(\frac{z+2}{2}\right)^2 + m^2}$$

Calculate 2-pt function of operator dual to ψ in geodesic approximation

$$\langle\mathcal{O}(x)\mathcal{O}(x')\rangle \approx \epsilon^{-2\Delta}e^{-\Delta\int d\tau\sqrt{g_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}}}$$

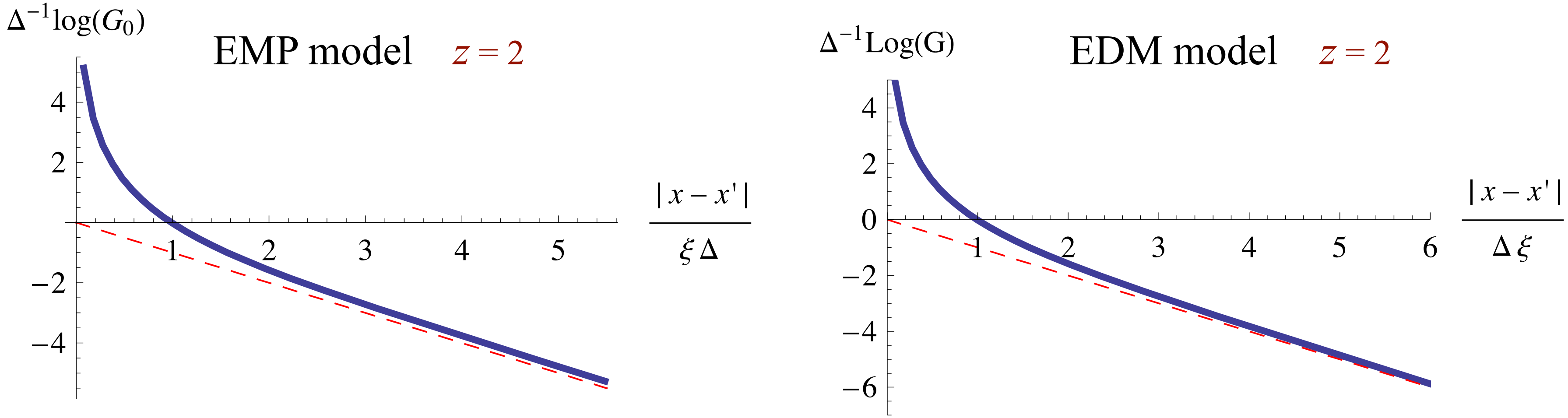
UV cutoff

valid for large $\Delta \approx m$

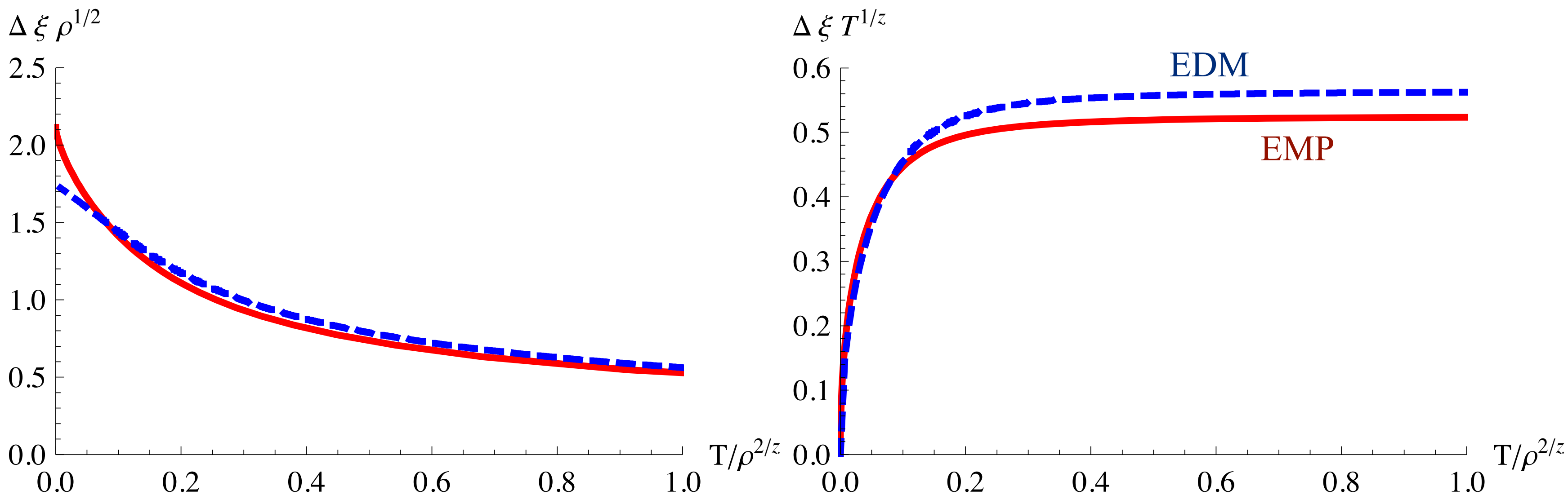
Equal time correlator in Lifshitz geometry $$\langle\mathcal{O}(\mathbf{x},t)\mathcal{O}(\mathbf{x}',t)\rangle \propto \frac{1}{|\mathbf{x}-\mathbf{x}'|^{2\Delta}}$$

At finite temperature and charge density $$\langle\mathcal{O}(\mathbf{x},t)\mathcal{O}(\mathbf{x}',t)\rangle \propto e^{-|\mathbf{x}-\mathbf{x}'|/\xi}$$

2-pt correlator as a function of $|\mathbf{x} - \mathbf{x}'|$



Thermal length as a function of T/μ



General scaling dimensions

Scalar wave equation: $$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\psi) - m^2\psi = 0$$

Mode expansion: $$\psi(\mathbf{x},u,\tau) = \sum_n \int \frac{d^2k}{(2\pi)^2} e^{-i\omega_n\tau + i\mathbf{k}\cdot\mathbf{x}}\psi_n(u,k) \qquad u \equiv 1/r$$

$$u^{3+z}\sqrt{\frac{g}{f}}\partial_u(u^{-1-z}\sqrt{fg}\partial_u\psi_n) - \left(\frac{u^{2z}}{f}\omega_n^2 + u^2k^2 + m^2\right)\psi_n = 0$$

Asymptotic behavior: $$\psi_n(u,k) = \psi_n^{(-)}u^{\Delta_-} + \psi_n^{(+)}u^{\Delta_+} + \ldots \qquad u \to 0$$

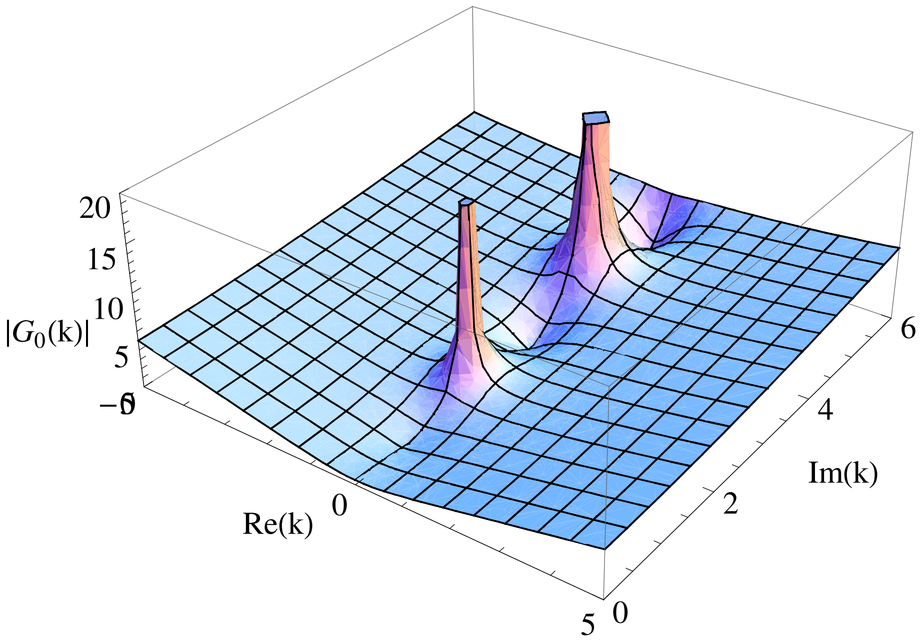
Regularity at horizon: $$\psi_n(u,k) \approx \psi_n^{(0)} \exp\left[-u_0^{z-1}\omega_n \int^u \frac{du'}{\sqrt{f(u')g(u')}}\right]$$

Holographic dictionary: $$e^{-S_E} = \left\langle \exp\left[\beta\sum_n\int \frac{d^2k}{(2\pi)^2}\mathcal{O}_n(k)\psi_{-n}^{(-)}(-k)\right]\right\rangle$$

Two-point function: $$\langle \mathcal{O}_n(k)\mathcal{O}_{n'}(k')\rangle = T\delta_{n+n',0}\delta^2(k+k')(2\pi)^2 G_n(k)$$

$$G_n(k) = 2\sqrt{(z+2)^2/4 + m^2}\,\frac{\psi_n^{(+)}}{\psi_n^{(-)}}$$

Characteristic length scale

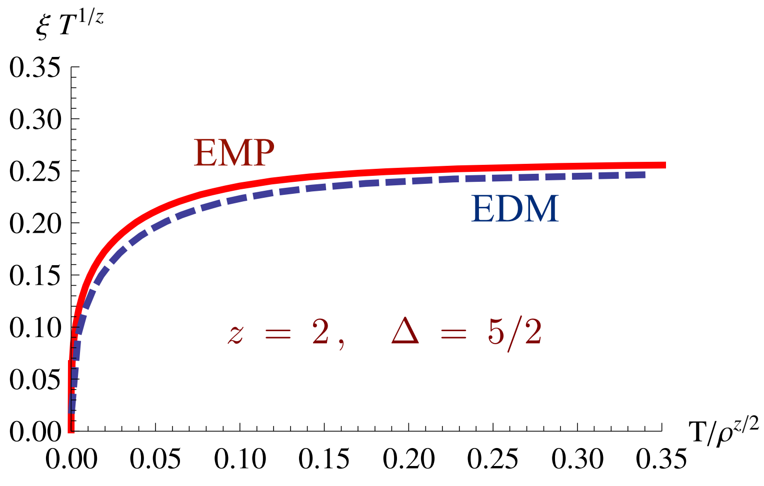


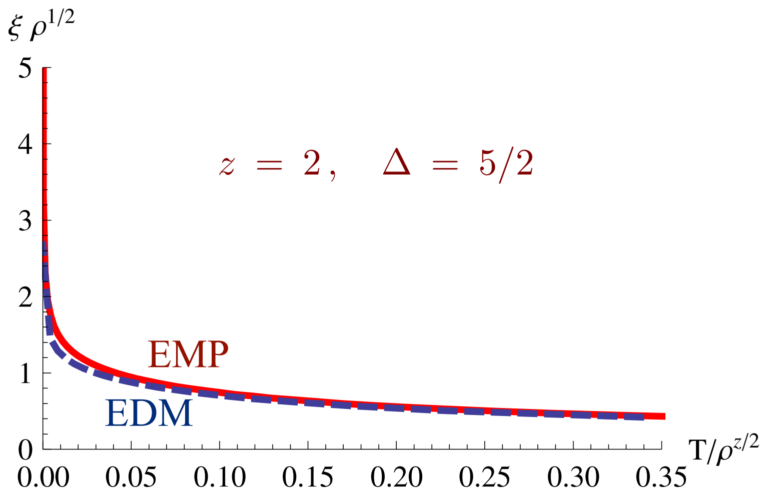
$$G(\mathbf{x},\mathbf{x}') \propto e^{-k_*|\mathbf{x}-\mathbf{x}'|}$$

k_* = pole closest to real axis

$G_0(k)$ = lowest Matsubara mode

$$\xi = 1/k_*$$





Holographic superconductors

S.Gubser, *Phys. Rev.* D**78** (2008) 065034
S.A. Hartnoll, C.P. Herzog, G.T. Horowitz, *Phys. Rev. Lett.* **101** (2008) 031601

Couple a charged scalar field to gravitational system

instability at low T : black brane with scalar "hair"

AdS/CFT prescription: hair corresponds to sc condensate

transport properties: solve classical wave equation in bh background

add magnetic field: dyonic black hole -- holographic sc is type II

conformal system: start from AdS-RN exact solution

z > 1 systems: work with numerical Lifshitz black branes

E.Brynjolfsson, U.Danielsson, L.T., T.Zingg,
J. Phys. A: Math. Theor. **43** (2010) 065401

Holographic superconductors with Lifshitz scaling

Add electric charge to the scalar field:

$$S_\psi = -\frac{1}{2}\int d^4x\sqrt{-g}\,\left(g^{\mu\nu}(\partial_\mu\psi^* + iq\mathcal{A}_\mu\psi^*)(\partial_\nu\psi - iq\mathcal{A}_\nu\psi) + m^2\psi^*\psi\right)$$

Two independent solutions: $\psi(x^\mu) = c_+\psi_+(x^\mu) + c_-\psi_-(x^\mu)$

Asymptotic behavior: $\psi_\pm(x^\mu) \to r^{-\Delta_\pm}\tilde{\psi}_\pm(\tau,\theta,\varphi) + \ldots$ $\quad \Delta_\pm = \frac{z+2}{2} \pm \sqrt{\left(\frac{z+2}{2}\right)^2 + m^2L^2}$

Finite Euclidean action: $L^2\,m^2 > -\frac{(z+2)^2}{4}$ analog of BF bound

Only ψ_+ falls off sufficiently rapidly as $r\to\infty$ if $L^2\,m^2 > -\frac{(z+2)^2}{4} + 1$

ψ is then dual to an operator O_+ of dimension Δ_+ in the dual field theory

Two choices if $-\frac{(z+2)^2}{4} + 1 > L^2\,m^2 > -\frac{(z+2)^2}{4}$

$\psi = \psi_+$ dual to O_+ of dim Δ_+ OR $\psi = \psi_-$ dual to O_- of dim Δ_-

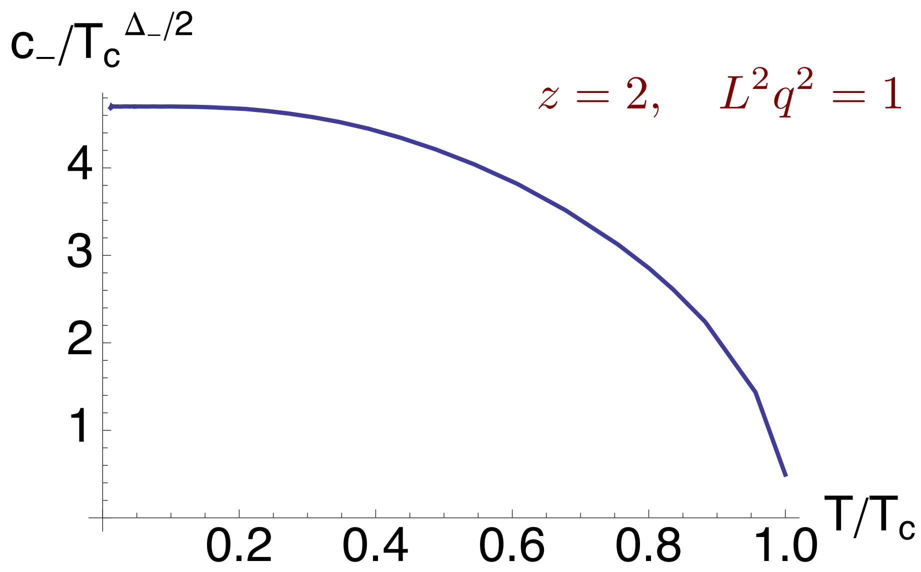
Superconducting phase

We work with $L^2\, m^2 = -\dfrac{(z+2)^2}{4} + \dfrac{1}{4}$ so that $\Delta_\pm = \dfrac{z+2}{2} \pm \dfrac{1}{2}$

A holographic superconductor in the superconducting phase is then dual to a hairy black hole with either

$$c_+ = 0, \quad \langle O_- \rangle \propto c_- \qquad \text{or} \qquad c_- = 0, \quad \langle O_+ \rangle \propto c_+$$

Numerical results for superconducting condensate:



Zero temperature entropy

Low temperature limit is described by a near extremal black brane

$z = 1:$ Extremal RN black brane has non-vanishing entropy

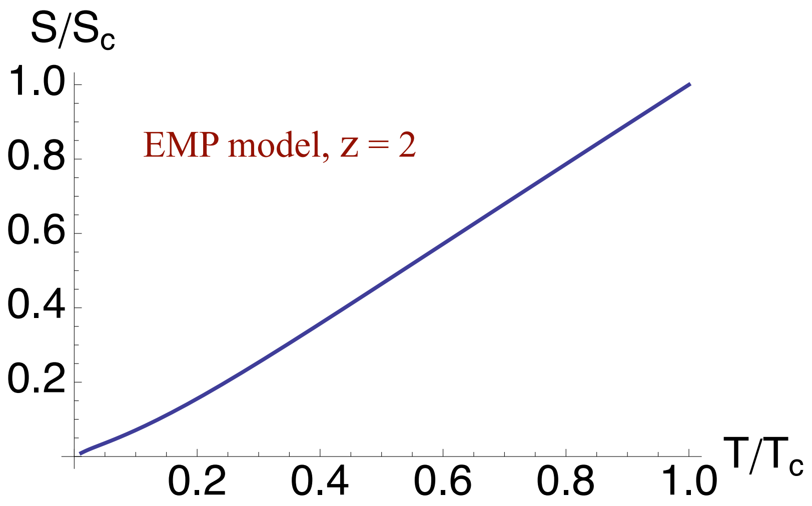
BUT

black brane with charged scalar hair has vanishing entropy density in extremal limit

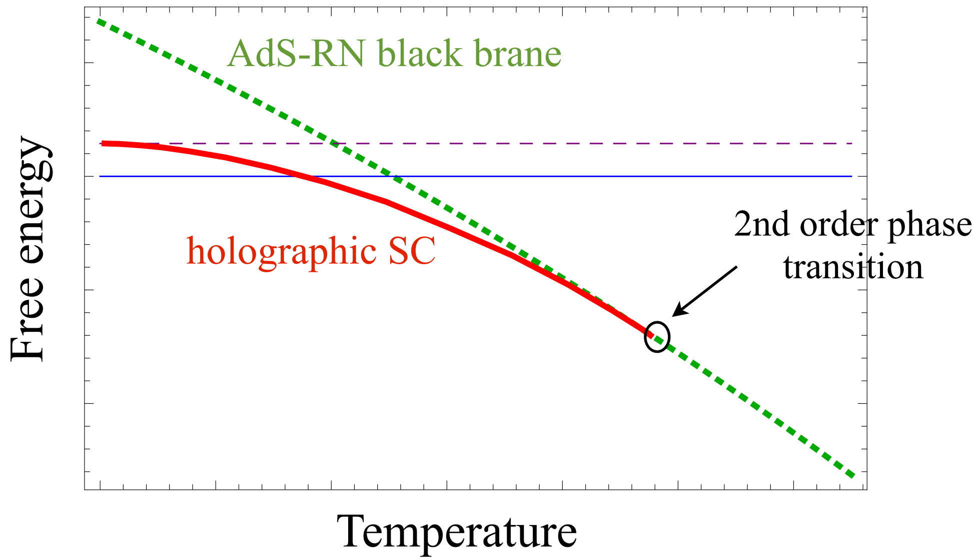
G.Horowitz and M.Roberts (2009)

$z > 1:$ Lifshitz black brane without hair has non-vanishing entropy in extremal limit

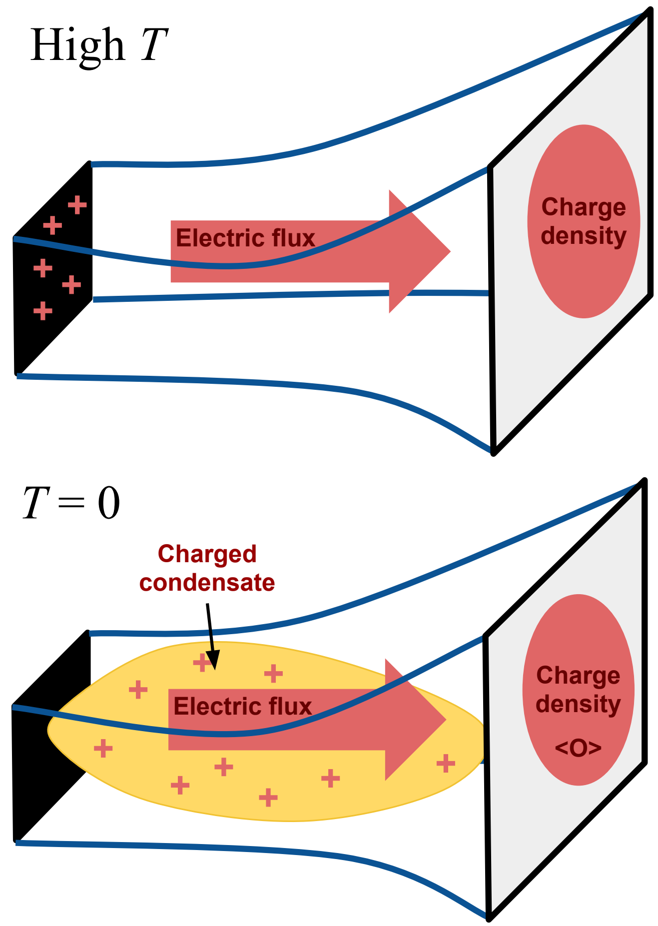
E. Brynjólfsson, U. Danielsson, L.T., T. Zingg, (2010)



Thermodynamic stability

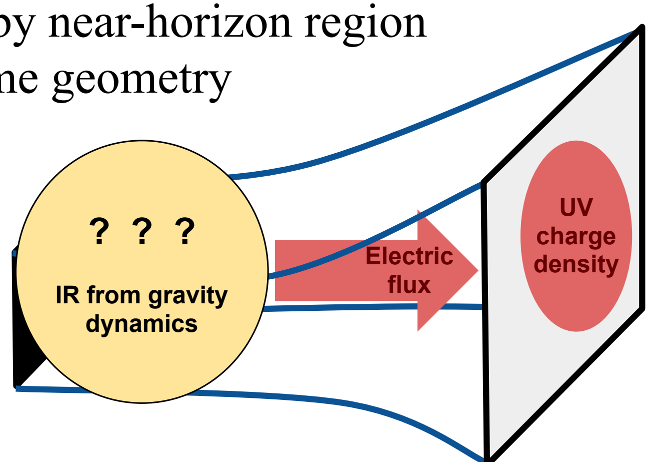


(from G. Horowitz and B. Way, arXiv:1007.3714)



(from S. Hartnoll, arXiv:1106.4342)

Low T dynamics at finite density is governed by near-horizon region in spacetime geometry



(from S. Hartnoll, arXiv:1106.4342)

Holographic metals $d = 2$

Include charged fermions in the bulk: $S_{\mathrm{matter}} = -\int \mathrm{d}^4x\sqrt{-g}\ \{\bar{\Psi}\not{D}\Psi + m\bar{\Psi}\Psi\}$

Fermion probe calculations: S.-S. Lee (2008); H.Liu, J.McGreevy, D.Vegh (2009); T. Faulkner, H. Liu, J. McGreevy, D.Vegh (2009); M.Cubrovic, J.Zaanen, K.Schalm (2009)

Dirac equation: $(\not{D} + m)\Psi = 0$ $\quad D_M = \partial_M + \frac{1}{4}\omega_{abM}\Gamma^{ab} - iqA_M$

Boundary fermions: $\psi_\pm(t,\vec{x}) = \lim_{r\to\infty}\Psi_\pm(t,\vec{x},r)$ $\quad \Gamma^3\Psi_\pm = \pm\Psi_\pm$

$$\Psi_\pm(t,\vec{x},r) = \frac{1}{(2\pi)^3}\int d\omega\, d^2k\tilde{\Psi}_\pm(\omega,\vec{k},r)e^{-i\omega t + i\vec{k}\cdot\vec{x}}$$

Adapt AdS/CFT prescription to compute $G_R(\omega,k)$

Single fermion spectral function $\quad A(\omega,k) = \frac{1}{\pi}\mathrm{Im}\left(\mathrm{Tr}\left[i\sigma^3 G_R(\omega,k)\right]\right)$

can be directly compared to ARPES data.

Holographic Fermi surface

$$G_R(\omega, k)^{-1} \sim \omega - v_F(k - k_F) - i\Gamma + \ldots$$

$$v_F,\ k_F \sim \mu \qquad \Gamma \sim \omega^{2\nu} \qquad \nu = \sqrt{m^2 - q^2 + \frac{k_F^2}{\mu^2}}$$

Depending on the probe parameters we can have:

$\nu > \frac{1}{2}$ long-lived quasiparticles — Landau Fermi liquid: $\Gamma \sim \omega^2$

$\nu < \frac{1}{2}$ no stable quasiparticles

$\nu = \frac{1}{2}$ log suppressed quasiparticle residue — marginal Fermi liquid

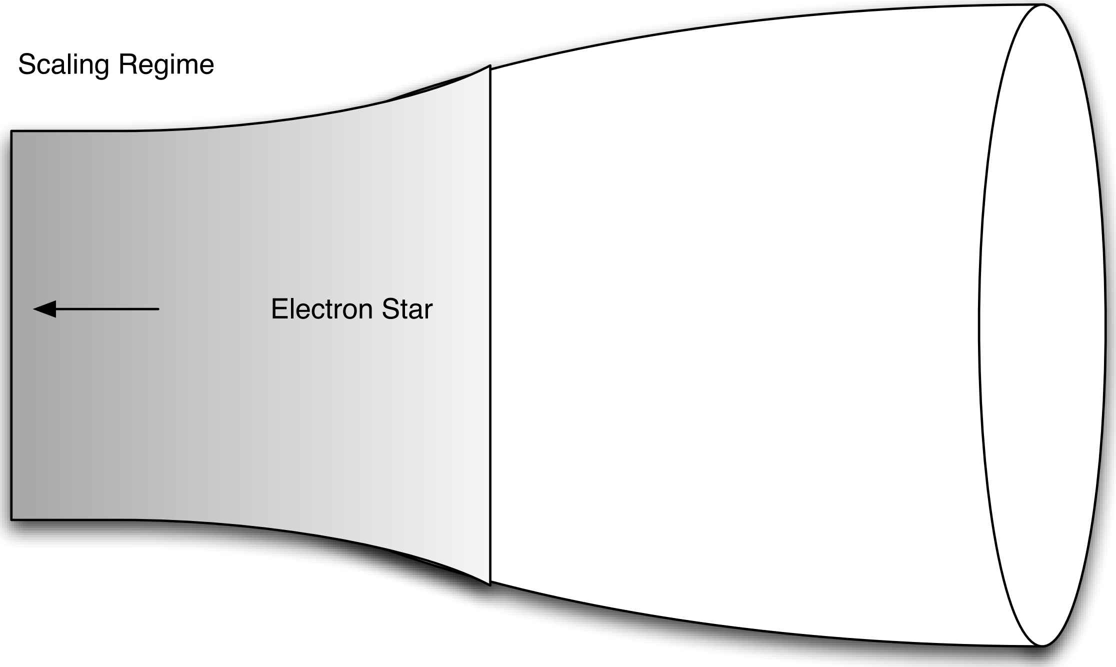
Going beyond fermion probe approximation

Fermion many-body problem in AdS_4 No easier than original problem!

Thomas-Fermi approximation: Treat fermions as a continuous charged fluid

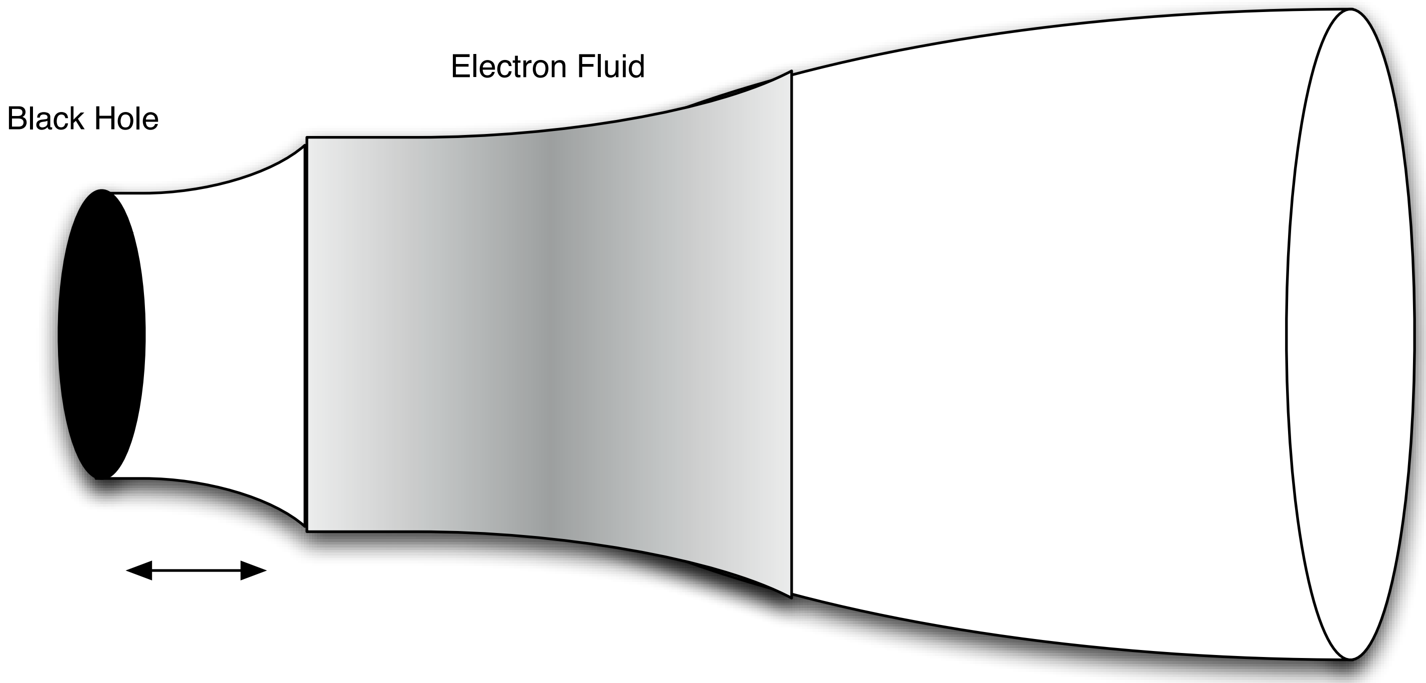
S. Hartnoll, J. Polchinski, E. Silverstein, D. Tong (2009)
S. Hartnoll, A. Tavanfar (2010)

T = 0 configuration is an *electron star*



Electron stars at finite temperature

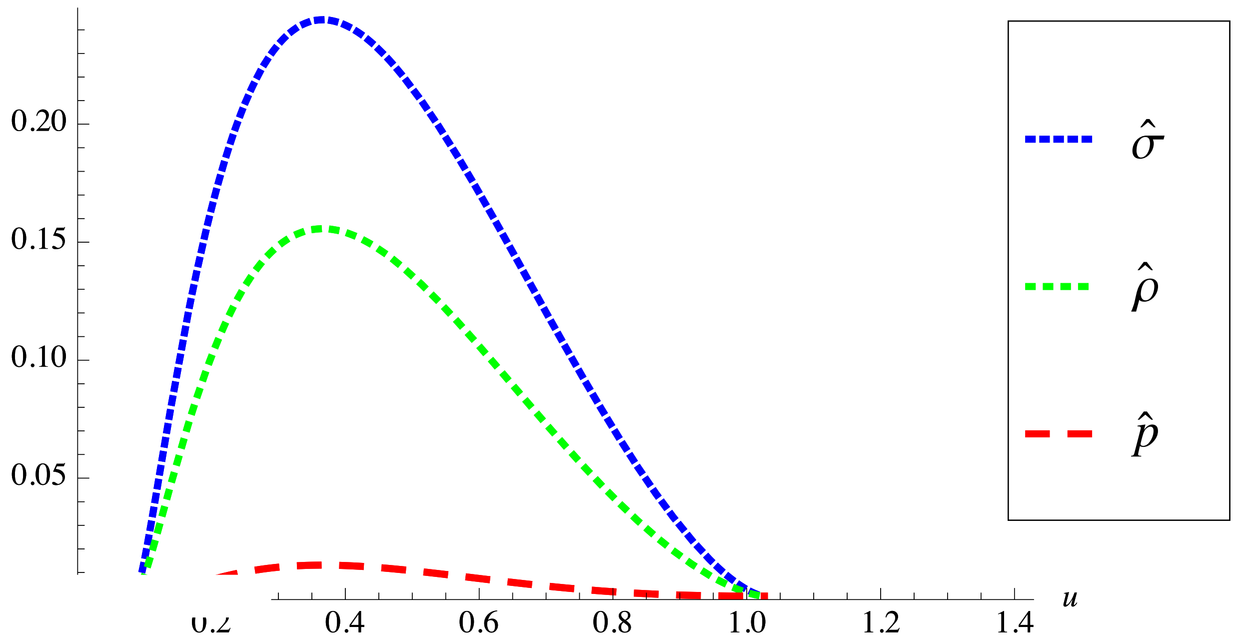
S. Hartnoll, P. Petrov (2010);
V. Giangreco Puletti, S. Nowling, L.T., T. Zingg (2010)



NORDITA

Electron stars at finite temperature

S. Hartnoll, P. Petrov (2010);
V. Giangreco Puletti, S. Nowling, L.T., T. Zingg (2010)



NORDITA

Field equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \frac{3}{L^2}g_{\mu\nu} = \kappa^2(T^{\rm Maxwell}_{\mu\nu} + T^{\rm fluid}_{\mu\nu})\,, \qquad \nabla^\nu F_{\mu\nu} = e^2\, J^{\rm fluid}_\mu\,.$$

$$T^{\rm Maxwell}_{\mu\nu} = \frac{1}{e^2}\left(F_{\mu\lambda}F_\nu{}^\lambda - \frac{1}{4}g_{\mu\nu}F_{\lambda\sigma}F^{\lambda\sigma}\right),$$

$$T^{\rm fluid}_{\mu\nu} = (\rho + p)u_\mu u_\nu + p\, g_{\mu\nu}\,,$$

$$J^{\rm fluid}_\mu = \sigma u_\mu\,,$$

Fluid description

$$e^2 \sim \frac{\kappa}{L} \ll 1$$

$$mL \gg 1$$

Ansatz

$$ds^2 = -f(v)dt^2 + g(v)dv^2 + \frac{1}{v^2}(dx^2 + dy^2)\,, \qquad A = \frac{e}{\kappa}h(v)\,dt\,,$$

$$\frac{d\hat f}{du} + \frac{\hat k^2}{2} + \hat f(1 - 3e^{-2u}\hat g) = e^{-2u}\hat f\hat g\hat p\,, \qquad u = -\log(v/v_0)$$

$$\frac{d\hat k}{du} + \hat k = e^{-2u}\left(\frac{1}{2}\hat h\hat k + \hat f\right)\frac{\hat g\hat\sigma}{\sqrt{\hat f}}\,,$$

$$\frac{1}{\hat f}\frac{d\hat f}{du} + \frac{1}{\hat g}\frac{d\hat g}{du} - 4 = e^{-2u}\frac{\hat g\hat h\hat\sigma}{\sqrt{\hat f}}\,,$$

local chemical potential

$$\hat\mu = \frac{\hat h}{\sqrt{\hat f}}$$

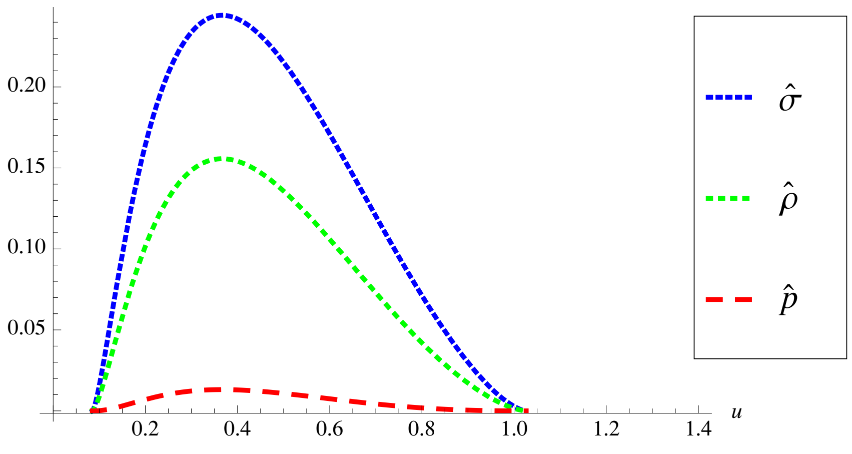
Equation of state

$$\hat\sigma = \hat\beta\int_{\hat m}^{\hat\mu} d\varepsilon\,\varepsilon\sqrt{\varepsilon^2 - \hat m^2}\,, \qquad \hat\rho = \hat\beta\int_{\hat m}^{\hat\mu} d\varepsilon\,\varepsilon^2\sqrt{\varepsilon^2 - \hat m^2}\,, \qquad -\hat p = \hat\rho - \hat\mu\hat\sigma\,,$$

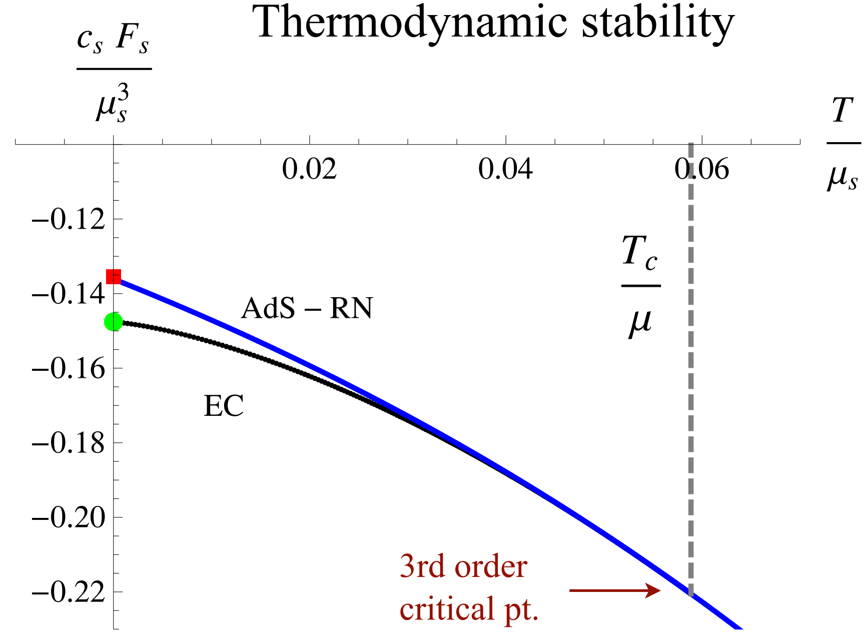
Fermion fluid

$$\hat\mu^2 = \frac{\hat h^2(u)}{\hat f(u)} > \hat m^2$$

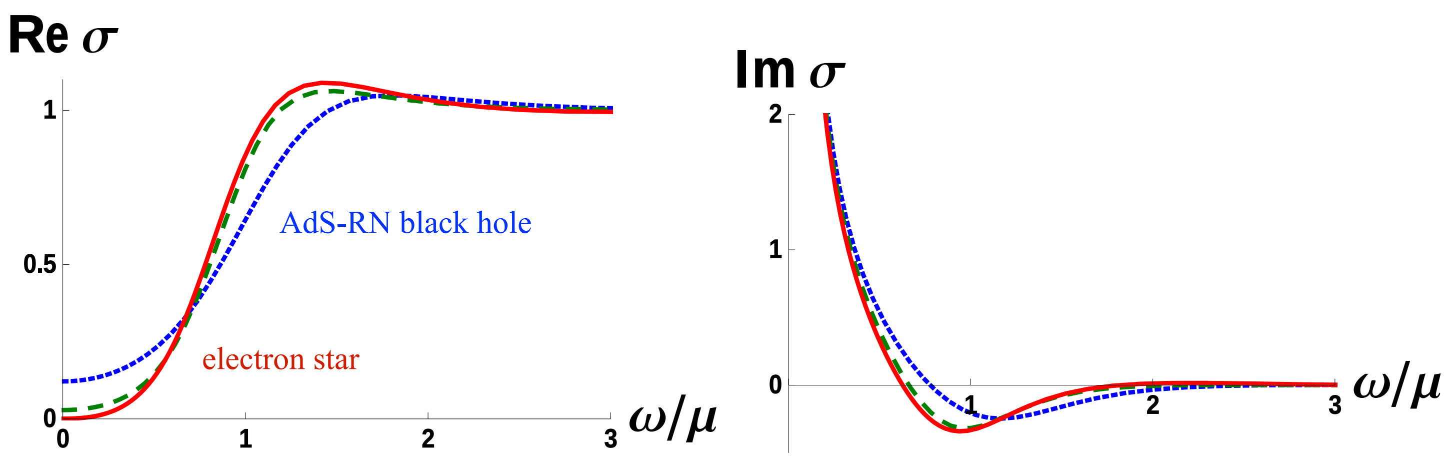
Radial profiles of fluid variables



Thermodynamic stability



Electrical conductivity



Fermi surface in boundary theory?

- Fermion probe calculations — S. Hartnoll, D.M. Hofman and D. Vegh, *JHEP* **1108** (2011) 096
- Friedel oscillations — V. Giangreco Puletti, S. Nowling, L.T., T. Zingg, *JHEP* **1201** (2012) 073
- Magnetic oscillations — S. Hartnoll, D.M. Hofman and A. Tavanfar, *EPL* **95** (2011) 31002
  V. Giangreco Puletti, S. Nowling, L.T., T. Zingg, in preparation

Going beyond Thomas-Fermi approximation

- Confined Fermi liquid — S. Sachdev, *PRD* **84** (2011) 066009
- Quantum electron star — A. Allais, J. McGreevy and S.J. Suh, *PRL* **108** (2012) 231602
- Dirac hair — M. Cubrovic, J. Zaanen, K. Schalm, *JHEP* **1110** (2011) 017
- WKB fluid — M. Medvedyeva, E. Gubankova, M. Cubrovic, K. Schalm and J. Zaanen
  arXiv:1302.5149 [hep-th]

Holographic lattice

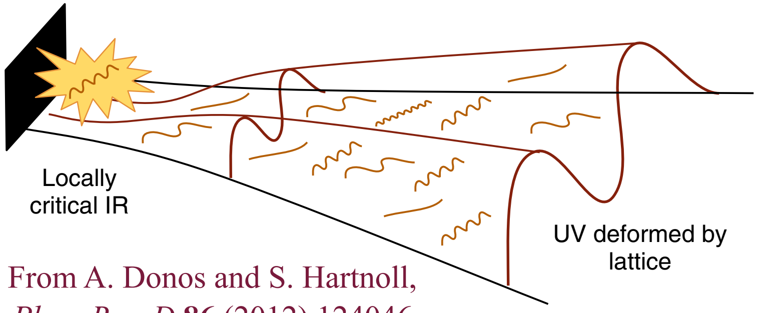
R. Flauger, E. Pajer and S. Papanikolaou '10
K. Maeda, T. Okamura and J.-i. Koga '11
S. A. Hartnoll and D. M. Hofman '12
G. Horowitz, J. Santos and D. Tong '12
A. Donos and S. Hartnoll '12

Break translation symmetry via UV boundary conditions

- Scalar field lattice: $\psi \to \psi_1(x,y)\,\frac{1}{r} + \psi_2(x,y)\,\frac{1}{r^2} + \dots$

$$\psi_1 = A_0 \cos(k_0 x)$$

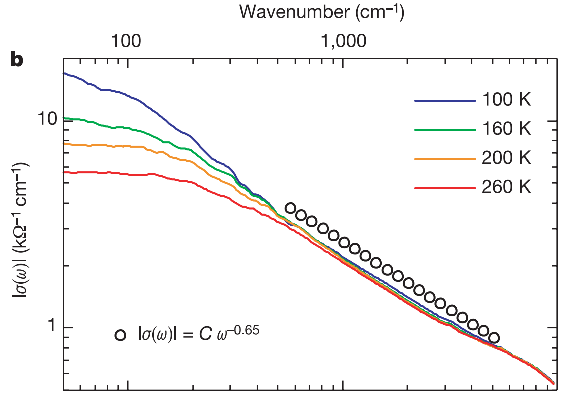
- Ionic lattice: $A_t \to \mu\,(1 + A_0 \cos(k_0 x)) + \dots$



From A. Donos and S. Hartnoll, *Phys. Rev. D* **86** (2012) 124046

Optical conductivity G. Horowitz, J. Santos and D. Tong, *JHEP* **1207** (2012) 168

- low frequency: $\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$
- intermediate frequency: $|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$
- high frequency: $\sigma(\omega) \to \text{constant}$



Linear resistivity from BKT quantum critical point A. Donos and S. Hartnoll, *PRD* **86** (2012) 124046

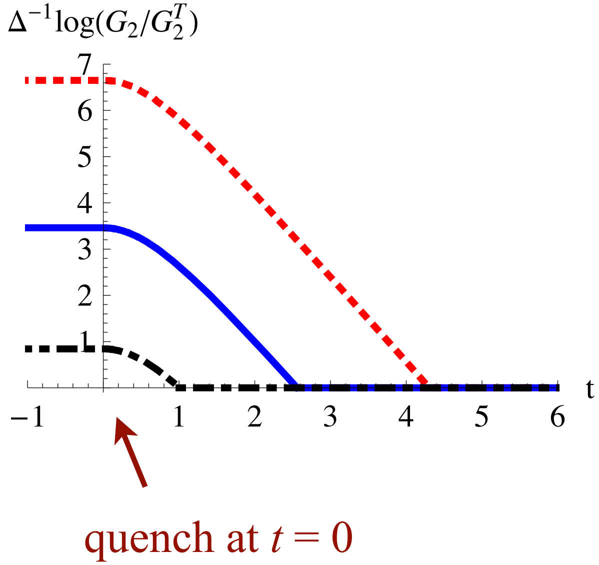
Thermalization after a holographic quench

Lifshitz-Vaidya solution: $$ds^2 = -r^{2z} b(r,v) dv^2 + 2r^{z-1} dv\, dr + r^2 d\mathbf{x}^2$$

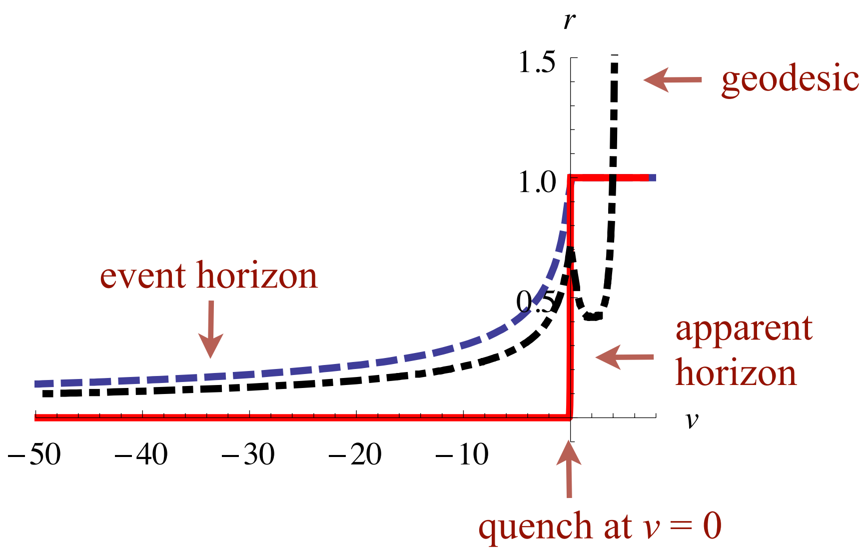
$$b(r) = 1 - \tilde{m}(v) \left(\frac{r_0}{r}\right)^{z+2} \qquad \tilde{\rho}(v) = 0$$

$$\tilde{m}(v) = \frac{1}{2}\left(1 - \tanh\left(v/v_0\right)\right)$$

Logarithm of the equal-time 2-point correlator, with its thermal value subtracted, for three different values of $|\mathbf{x} - \mathbf{x}'|$



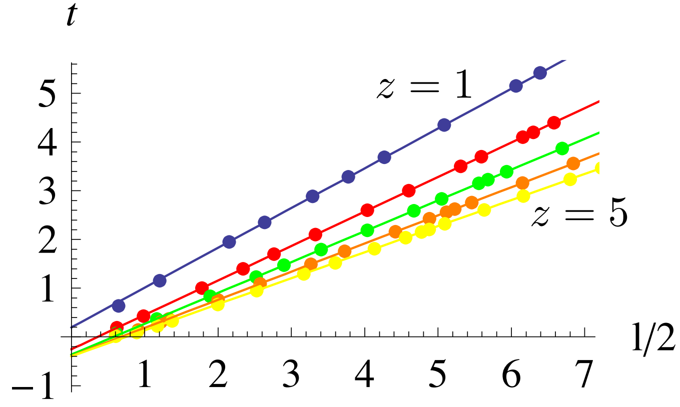
The 2-point function deviates from its thermal value if the geodesic passes through the brane horizon in the bulk



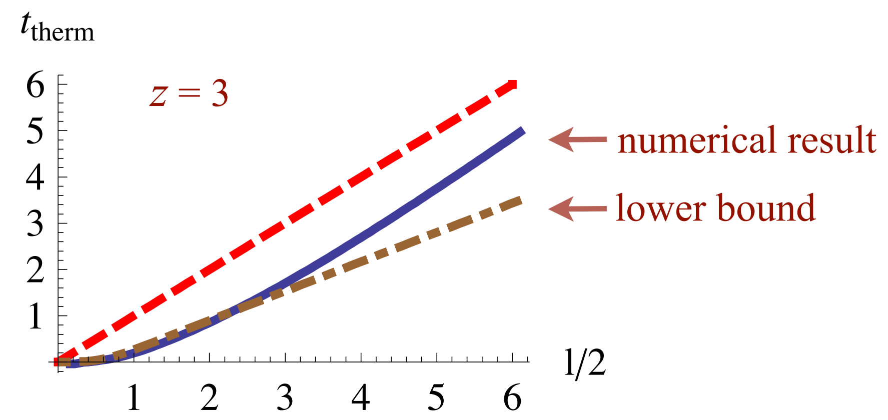
Horizon effect

Lower bounds on thermalization times from geometry of geodesics

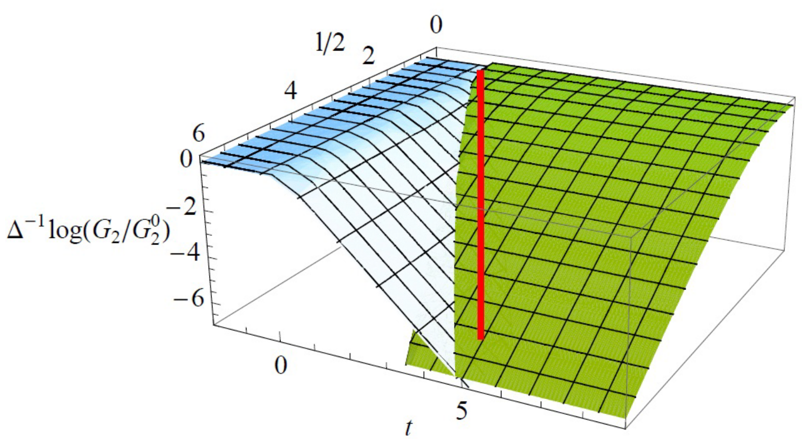
$$v_{\rm th} \propto \sqrt{1+\frac{z}{2}}$$



Thermalization time for $z = 3$ in geodesic approximation (numerical calculation)



Logarithm of quench correlator for z = 3 with vacuum value subtracted



Holographic entanglement entropy

Ryu & Takayanagi '06
Hubeny, Rangamani, & Takayanagi '07

$$S_{ent} = \frac{1}{4}\int d^2\sigma \sqrt{|\det \partial_\alpha X^\mu \partial_\beta X_\mu|}$$

The entanglement entropy is given by the area of a minimal surface *S* that ends on a curve $C = \partial S$ at AdS boundary

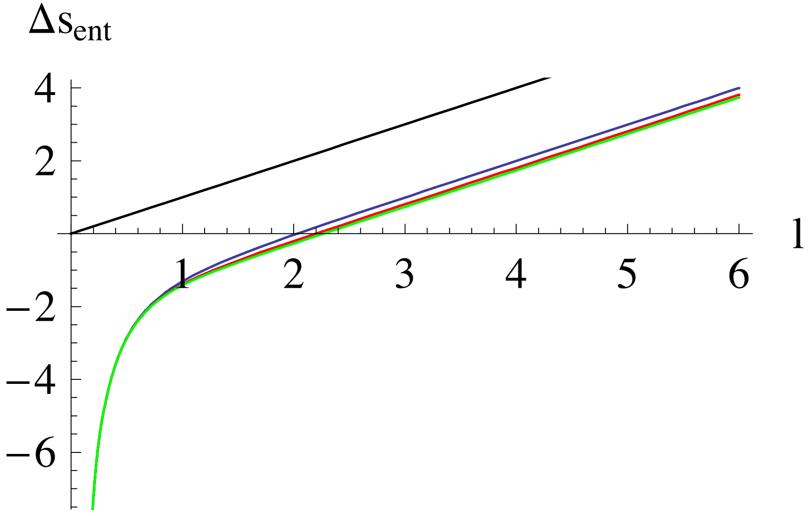
Assume that geometric formula applies in asymptotically Lifshitz spacetime and calculate entropy density on an infinite strip of width *l*

In the Lifshitz background one obtains the same result as in AdS spacetime

$$s_{ent} = \frac{2}{\epsilon} + \frac{1}{l}\frac{\pi\Gamma(-1/4)\Gamma(3/4)}{\Gamma(1/4)^2}$$

Thermal equilibrium

Finite part of entanglement entropy



Quench