Efficient calculation of cosmological neutrino clustering

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ARXIV:1510.02907
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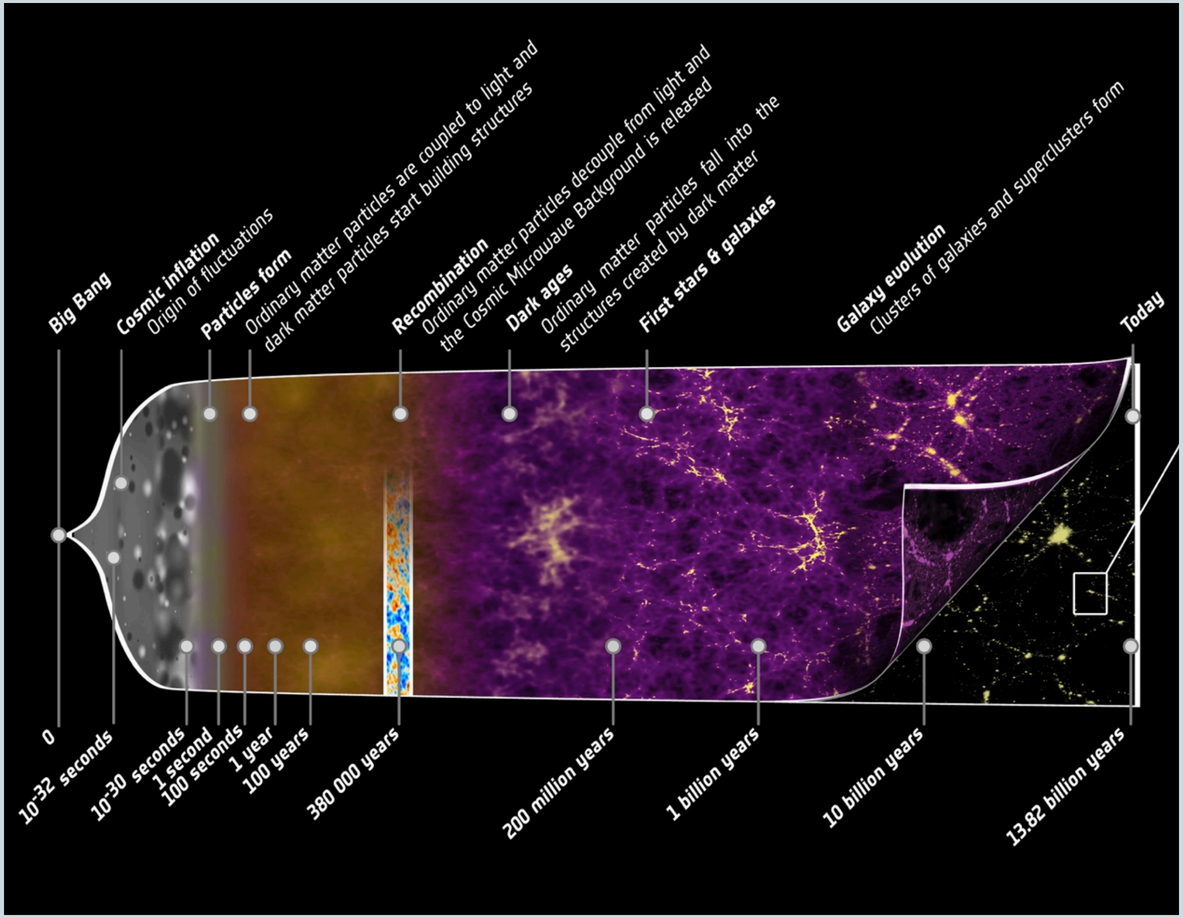
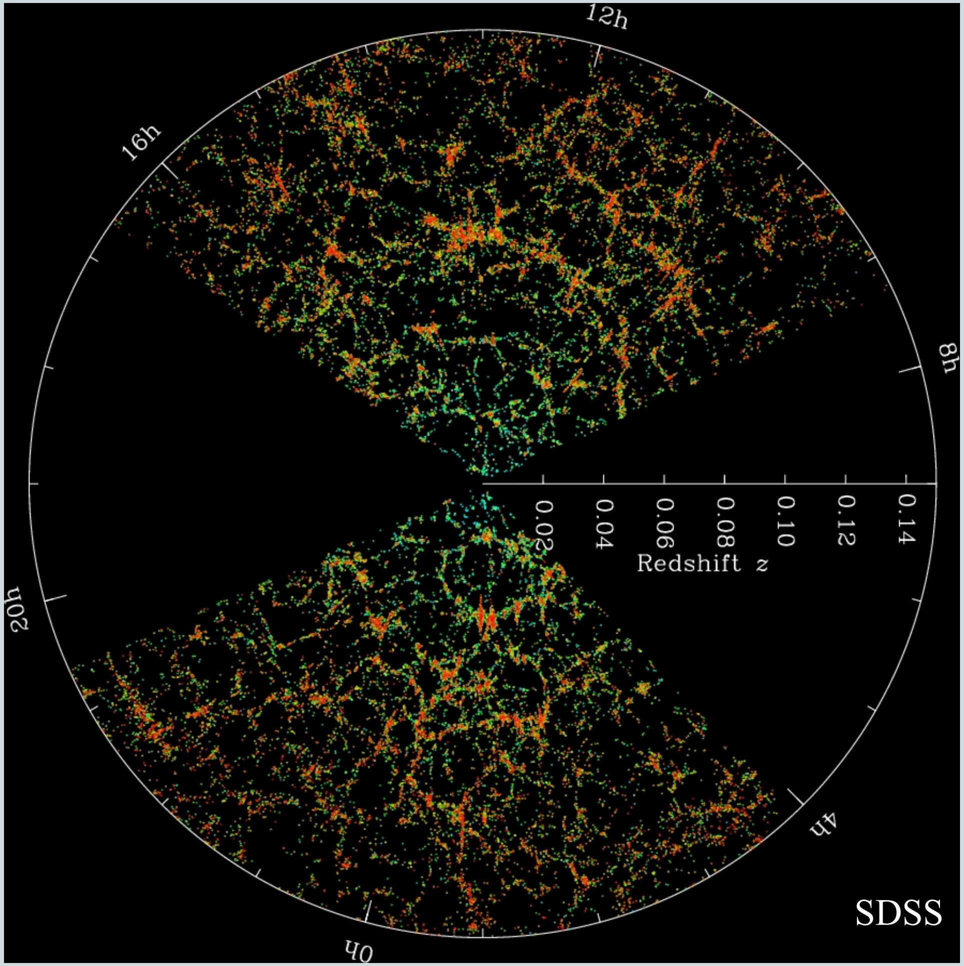
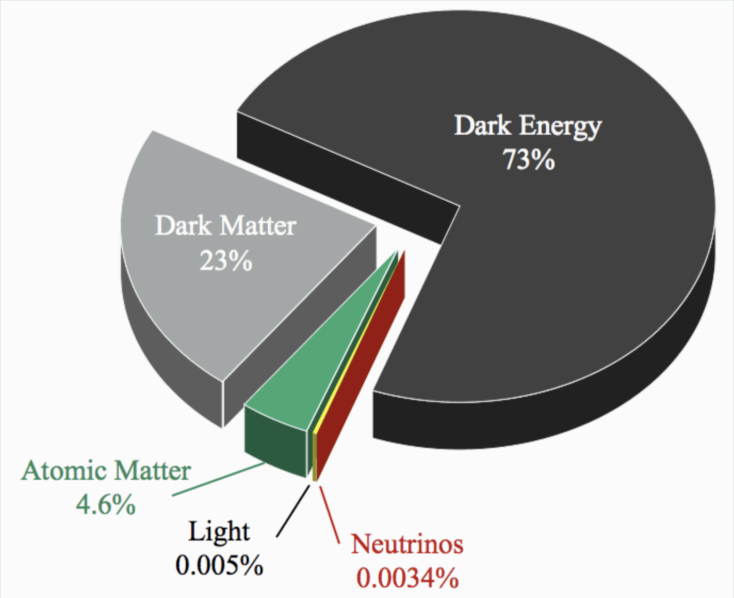
Cosmic history



Cosmic structure



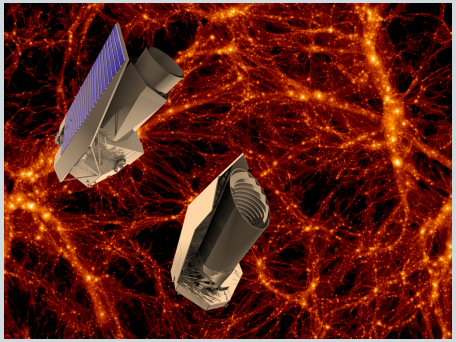
The role of neutrinos



$$\Delta m_{21}^2 = 7.50_{-0.17}^{+0.19} \times 10^{-5} eV^2$$

$$\Delta m_{31}^2 = 2.475_{-0.047}^{+0.047} \times 10^{-3} eV^2$$

Gonzalez-Garcia et al. (2015)



1% accuracy!

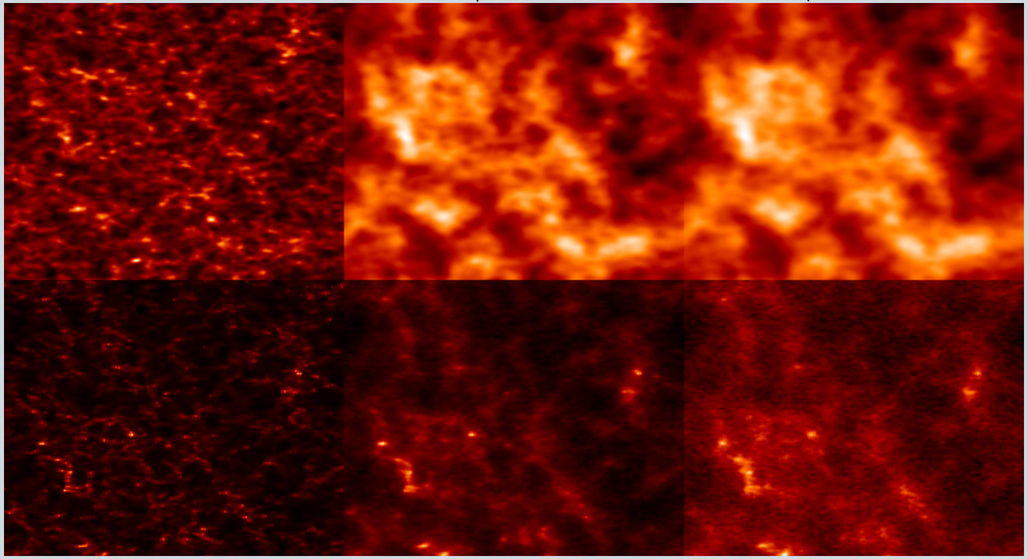
Euclid (2020)

HDM

CDM | $\Sigma m_\nu = 0.6$ eV | $\Sigma m_\nu = 0.3$ eV

z = 4

z = 0



Brandbyge et al. (2008)

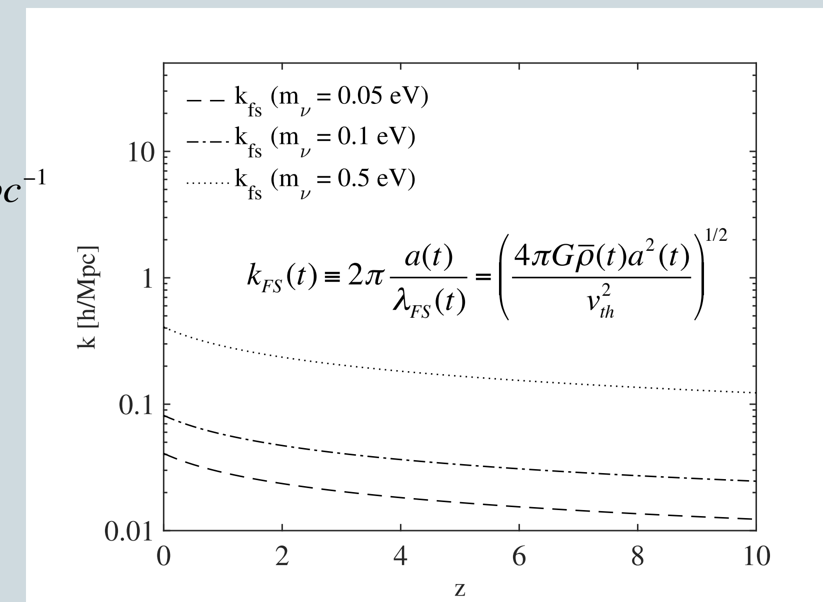
Neutrino free-streaming

$$v_{th} \equiv \frac{\langle p \rangle}{m} \approx \frac{3T_\nu}{m} \approx 150(1+z)\left(\frac{1eV}{m}\right) km\ s^{-1}$$

$$\lambda_{FS}(t) \equiv 2\pi\sqrt{\frac{2}{3}}\frac{v_{th}}{H(t)} \approx 7.7\frac{1+z}{\sqrt{\Omega_\Lambda + \Omega_m(1+z)^3}}\left(\frac{1eV}{m}\right) Mpc\ h^{-1}$$

$$z_{nr,i} \approx 1890\left(\frac{m_{\nu,i}}{1eV}\right)$$

$$k_{nr} \approx 0.01\sqrt{\Omega_m}\left(\frac{m_{\nu,i}}{1eV}\right)^{1/2} hMpc^{-1}$$



Matter power spectrum in the presence of ν

$$P(k,z) = \left\langle \left|\delta_m(k,z)\right|^2 \right\rangle$$

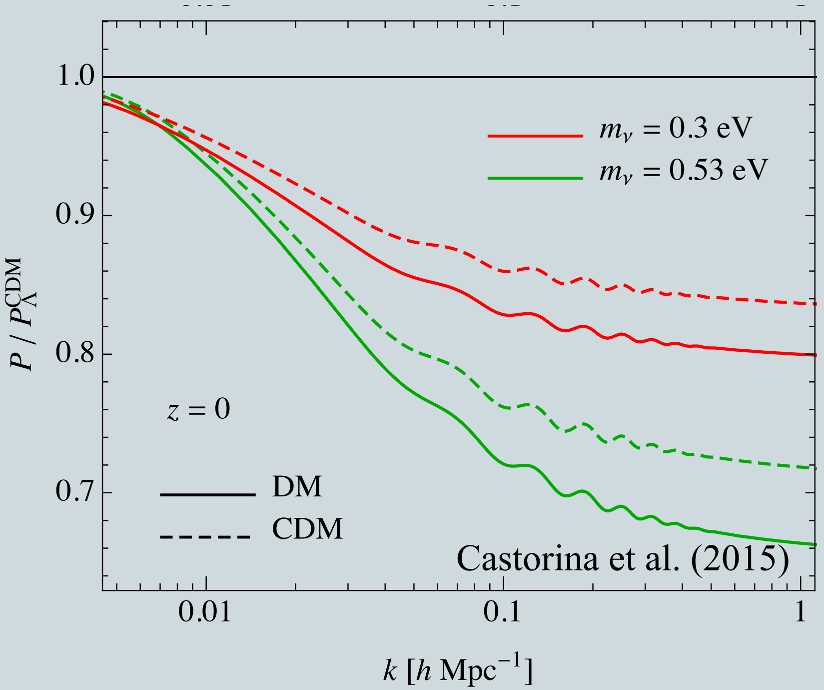
$$\delta_m = \frac{\delta\rho_m}{\bar{\rho}_m}$$

$$\frac{k^2}{a^2}\phi = -4\pi G(\delta\rho_m)$$

$$H^2 = \frac{8\pi G}{3}\left(\rho_\gamma + \rho_b + \rho_{cdm} + \rho_\nu + \rho_\Lambda\right)$$

$\delta_{cdm} \propto a$ only cold dark matter

$\delta_{cdm} \propto a^{1-3/5 f_\nu}$ in the presence of ν
cdm+hdm=mdm



$$\frac{P(k)^{\Lambda MDM}}{P(k)^{\Lambda CDM}} \approx 1 - 8 f_\nu$$

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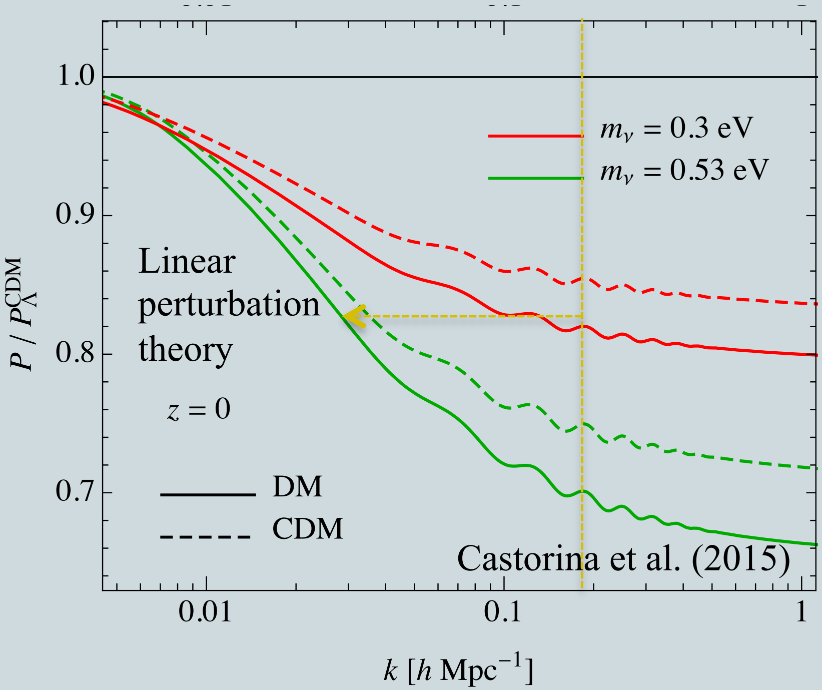

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Linear perturbation theory

$$f(x^i, P_j, \tau) = f_0(q)\left[1 + \Psi(x^i, q, n_j, \tau)\right]$$

Phase-space distribution

$$\frac{\partial f}{\partial \tau} + \frac{\partial f}{\partial x^i}\frac{\partial x^i}{\partial \tau} + \frac{\partial f}{\partial q}\frac{\partial q}{\partial \tau} + \frac{\partial f}{\partial n^i}\frac{\partial n^i}{\partial \tau} = 0$$

Collisionless Boltzmann equation

$$\frac{\partial \Psi}{\partial \tau} + i\frac{q}{\varepsilon}(\vec{k}\cdot\hat{n})\Psi + \frac{d\ln f_0}{d\ln q}\left[\dot{\eta} - \frac{\dot{h} + 6\dot{\eta}}{2}(\vec{k}\cdot\hat{n})^2\right] = 0$$

Synchronous gauge, Fourier space

$$T_{\mu\nu} = \int dP_1\, dP_2\, dP_3 (-g)^{-1/2} \frac{P_\mu P_\nu}{P^0} f(x^j, P_j, \tau)$$

Energy-momentum tensor

Perturbed components:

$$\delta\rho = a^{-4}\int \varepsilon q^2\, dq\, d\Omega f_0(q)\Psi$$

Energy density

$$\delta p = \frac{1}{3}a^{-4}\int \frac{q^4}{\varepsilon}\, dq\, d\Omega f_0(q)\Psi$$

Pressure

$$\delta T_i^0 = a^{-4}\int q^3\, dq\, d\Omega n_i f_0(q)\Psi$$

Energy flux $\quad (\bar{\rho} + \bar{p})\theta \equiv ik^i \delta T_i^0$

$$\Sigma_j^i = a^{-4}\int \frac{q^4}{\varepsilon}\, dq\, d\Omega \left(n_i n_j - \frac{1}{3}\delta_{ij}\right) f_0(q)\Psi$$

Shear stress $\quad (\bar{\rho} + \bar{p})\sigma \equiv -\left(n_i n_j - \frac{1}{3}\delta_{ij}\right)\Sigma_j^i$

Neutrino perturbations

$$\Psi(\vec{k},\hat{n},q,\tau)=\sum_{l=0}^{\infty}(-i)^l(2l+1)\Psi_l(\vec{k},q,\tau)P_l(\vec{k}\cdot\hat{n})$$

Legendre expansion

$$\delta\rho=4\pi a^{-4}\int \varepsilon q^2\,dq f_0(q)\Psi_0$$

$$\delta p=\frac{4\pi}{3}a^{-4}\int\frac{q^4}{\varepsilon}dq f_0(q)\Psi_0$$

$$(\bar{\rho}+\bar{p})\theta=4\pi k a^{-4}\int q^3\,dq f_0(q)\Psi_1$$

$$(\bar{\rho}+\bar{p})\sigma=\frac{8\pi}{3}a^{-4}\int\frac{q^4}{\varepsilon}dq f_0(q)\Psi_2$$

$$\dot{\Psi}_0=-k\frac{q}{\varepsilon}\Psi_1+\frac{1}{6}\dot{h}\frac{d\ln f_0}{d\ln q}$$

$$\dot{\Psi}_1=k\frac{q}{3\varepsilon}(\Psi_0-2\Psi_2)$$

Energy-momentum conservation

Note: Ψ_0 and Ψ_1 are gauge dependent

$$\dot{\Psi}_2=k\frac{q}{5\varepsilon}(2\Psi_1-3\Psi_3)-\left(\frac{1}{15}\dot{h}+\frac{2}{5}\dot{\eta}\right)\frac{d\ln f_0}{d\ln q}$$

Neutrino free-streaming

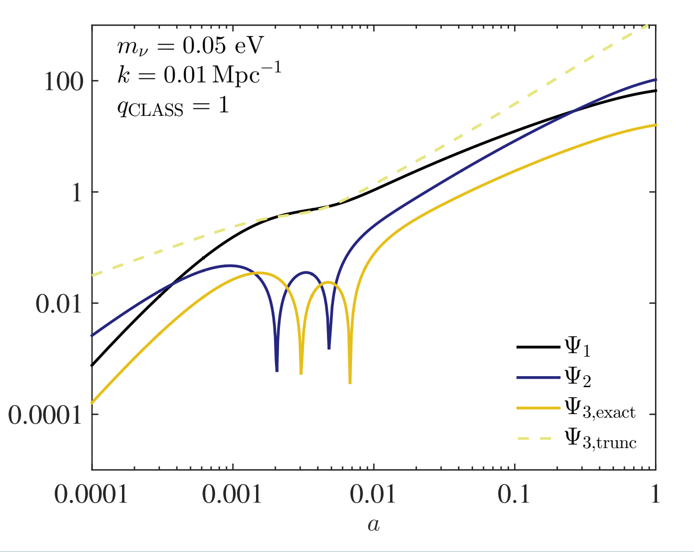
$$\dot{\Psi}_l=k\frac{q}{(2l+1)\varepsilon}(l\Psi_{l-1}-(l+1)\Psi_{l+1}),\ \ l\geq 3$$

The moment hierarchy truncation

$$\dot{\Psi}_l = k\frac{q}{(2l+1)\varepsilon}\left(l\Psi_{l-1} - (l+1)\Psi_{l+1}\right),\ \ l \geq 3$$

In the absence of any gravitational source term, or for very high l $\Psi_l(k\tau) \propto j_l(k\tau)$

$$\Psi_{l+1} = \frac{2l+1}{k\tau}\Psi_l - \Psi_{l-1}$$



$l = 2$

$$\dot{\Psi}_2 = k\frac{q}{5\varepsilon}(2\Psi_1 - 3\Psi_3) - \left(\frac{1}{15}\dot{h} + \frac{2}{5}\dot{\eta}\right)\frac{d\ln f_0}{d\ln q}$$

$$\Psi_3 \approx \frac{5\varepsilon}{k\tau}\Psi_2 - \Psi_1$$

Note: gauge dependent,

while the real one is gauge invariant

Archidiacono & Hannestad, arXiv:1510.02907

A new moment hierarchy truncation

In a non-expanding Universe and in the absence of gravity, the Boltzmann hierarchy:

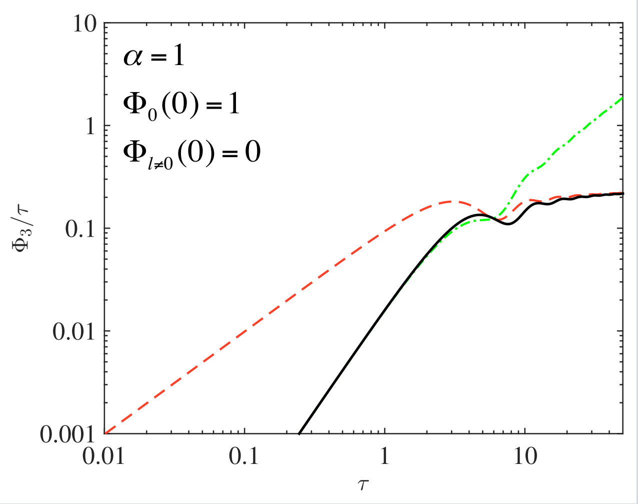
$$\dot{\Psi}_l = \frac{\alpha}{(2l+1)}(l\Psi_{l-1} - (l+1)\Psi_{l+1})$$

with solutions $\Psi_l \propto j_l(\alpha\tau)$

$$\dot{\Phi}_l = \frac{\alpha}{(2l+1)}(l\Phi_{l-1} - (l+1)\Phi_{l+1}) + f(\tau)(\delta_{l0} + \delta_{l2})$$

$$\Phi_l = \frac{g(\tau)}{\sqrt{2l+1}} \qquad \alpha\tau >> 1 \rightarrow \Phi_3 \approx \sqrt{\frac{5}{7}}\Phi_2$$

$$\Phi_l = \frac{\alpha\tau}{2l+1}\Phi_{l-1} \qquad \alpha\tau \leq 1 \rightarrow \Phi_3 \approx \frac{\alpha\tau}{7}\Phi_2$$

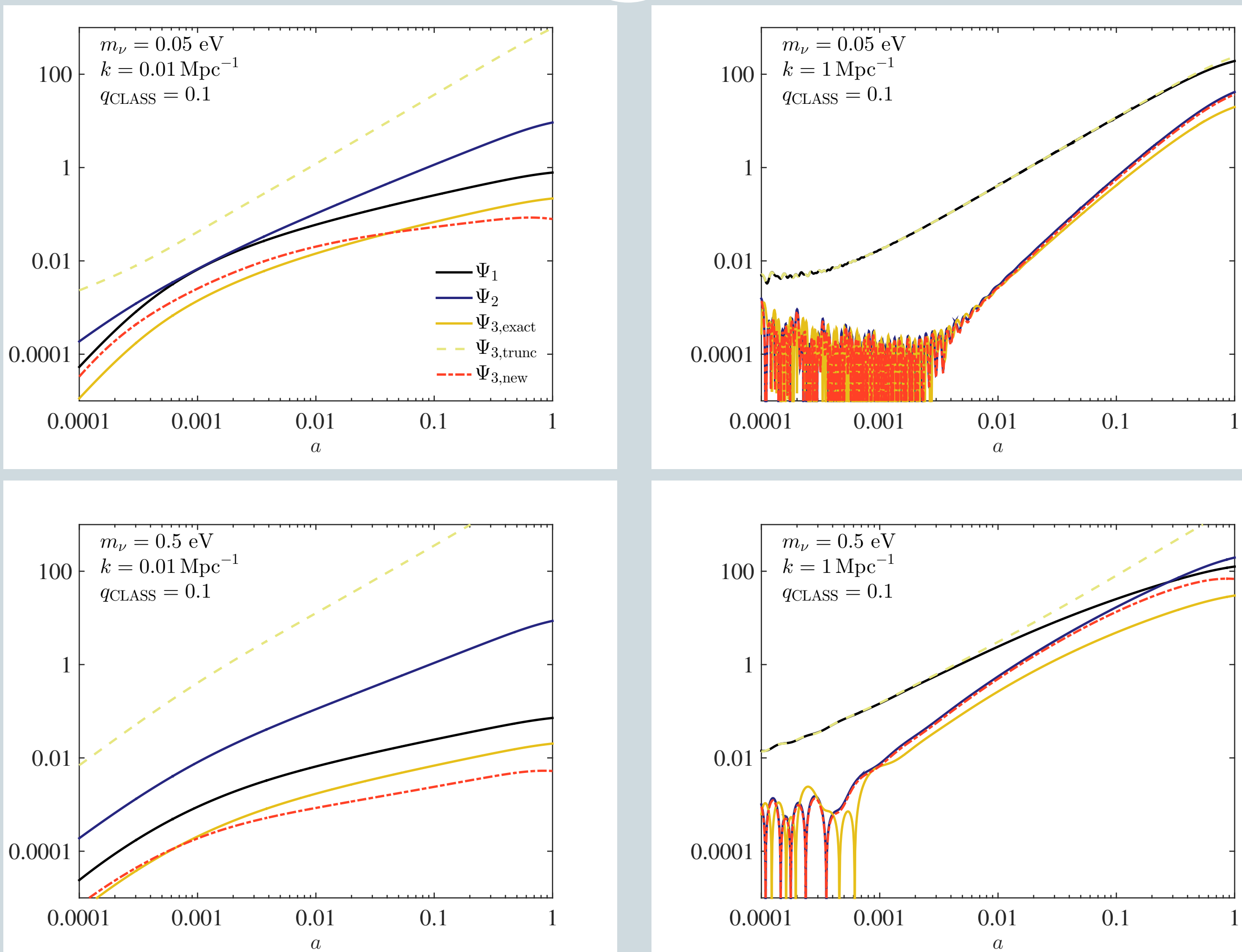


Ansatz:

$$\Psi_3 \approx \left(\frac{\frac{x}{7}\frac{\beta}{x} + \sqrt{\frac{5}{7}}\frac{x}{\beta}}{\frac{\beta}{x} + \frac{x}{\beta}} \right)\Psi_2, \qquad x = \frac{qk\tau}{\varepsilon}$$

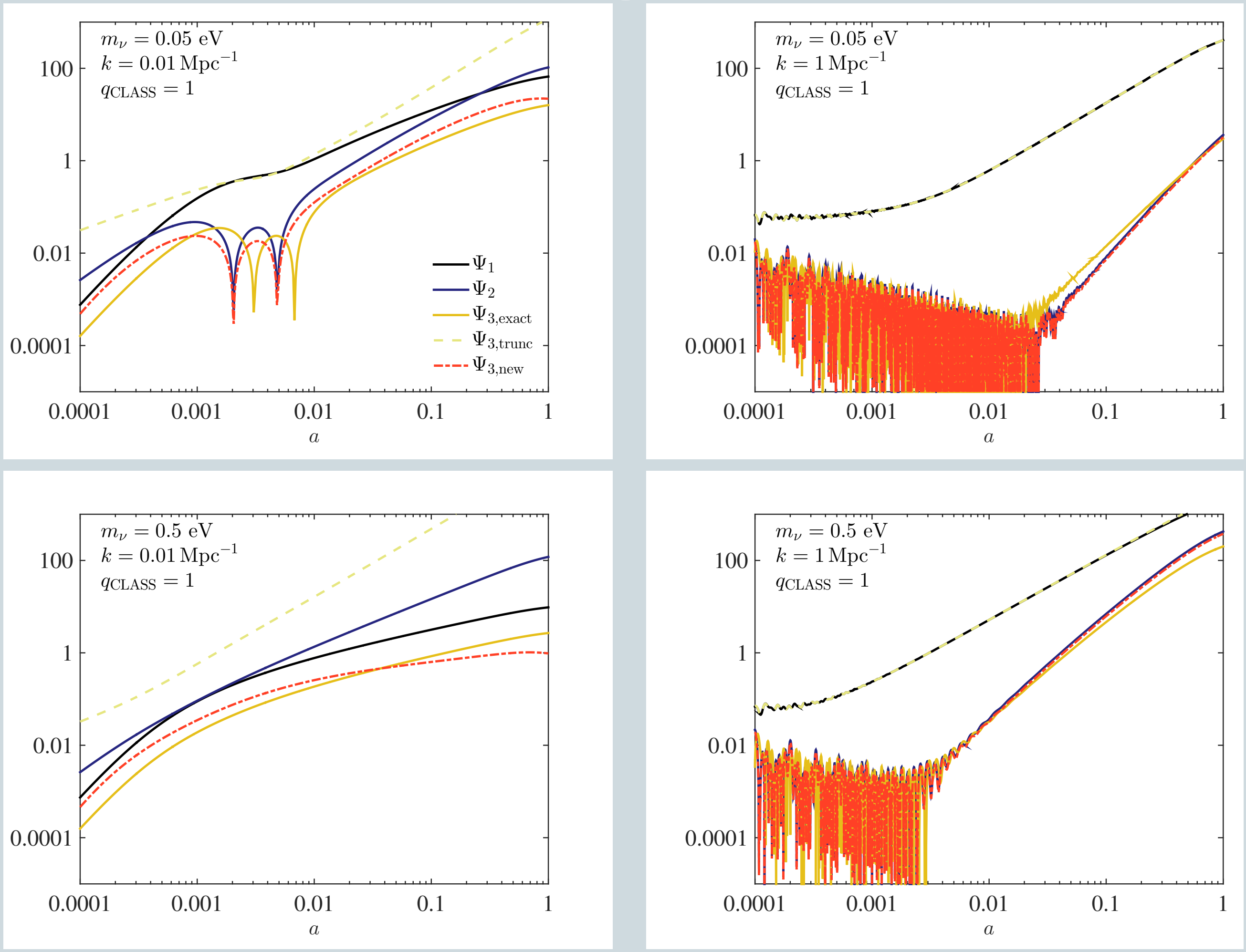
$$\Psi_3 \approx \left(\frac{\frac{1}{7} + \sqrt{\frac{5}{7}}x}{\frac{1}{x} + x} \right)\left[\frac{k}{1h/Mpc}\right]^{0.12}\Psi_2$$

Comparison



Archidiacono & Hannestad, arXiv:1510.02907

Comparison



Archidiacono & Hannestad, arXiv:1510.02907

The fluid approximation

Lesgourgues & Tram (2011)

Writing the hierarchy in terms of fluid quantities (δ, θ, σ)

Differentiate $(\bar{\rho}+\bar{p})\sigma = \frac{8\pi}{3} a^{-4} \int \frac{q^4}{\varepsilon} dq f_0(q) \Psi_2$

Same ansatz $\Psi_3 \approx \frac{5\varepsilon}{k\tau} \Psi_2 - \Psi_1$

$$\dot{\sigma} = -3\left(\frac{1}{\tau} + \frac{\dot{a}}{a}\left[\frac{2}{3} - c_g^2 - \frac{1}{3}\frac{\tilde{p}}{p}\right]\right)\sigma + \frac{4}{3}\frac{c_{vis}^2}{1+w}\left[2\theta + \dot{h}\right]$$

$$c_{vis}^2 = 3wc_g^2$$

$$c_g^2 = \frac{\dot{p}}{\dot{\rho}} = \frac{w}{3(1+w)}\left(5 - \frac{\tilde{p}}{p}\right)$$

$$\tilde{p} = \frac{4\pi}{3} a^{-4} \int_0^\infty \frac{q^6}{\varepsilon^3} dq f_0(q)$$

CLASS built-in approximation

Pros: no momentum integration!

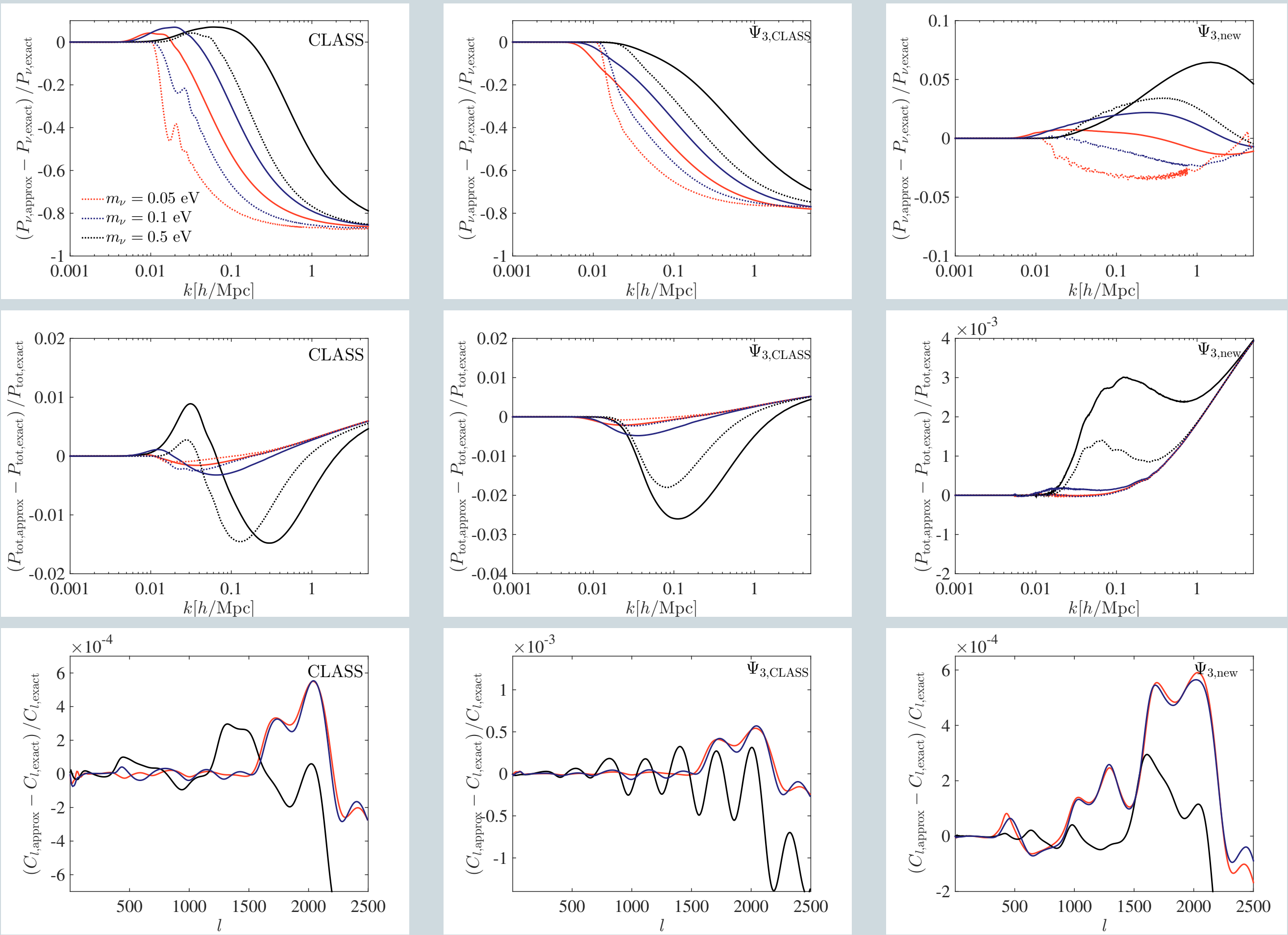
Comparison: observables

$k\tau \leq 31$

$l = 17$

$k\tau > 31$

approximation
or
truncation



z=0
z=10

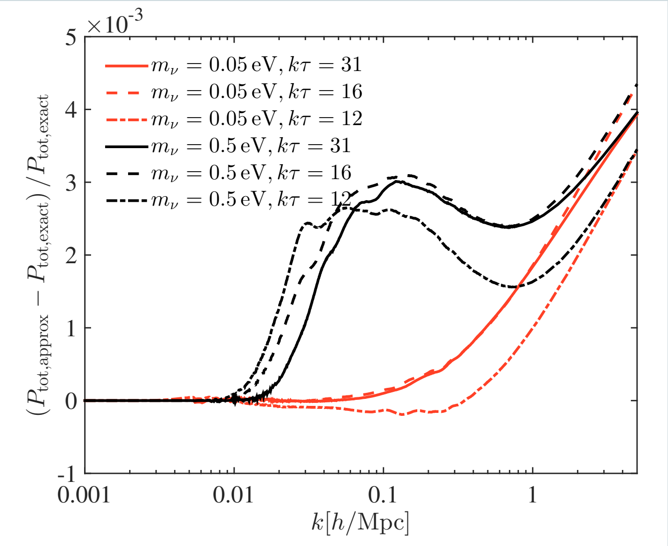
(< 1% Euclid)

Archidiacono & Hannestad, arXiv:1510.02907

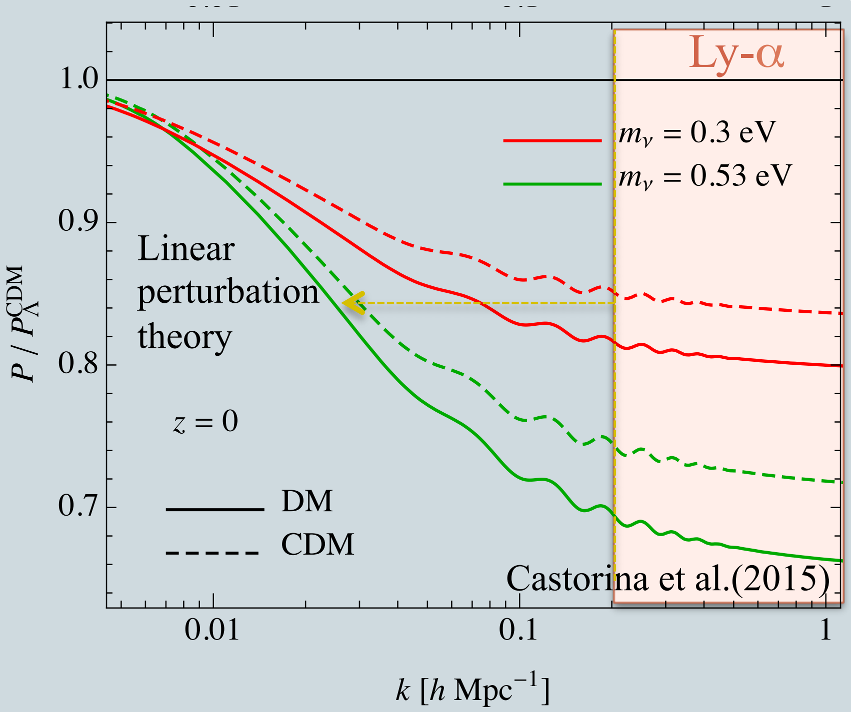
Comparison: runtime

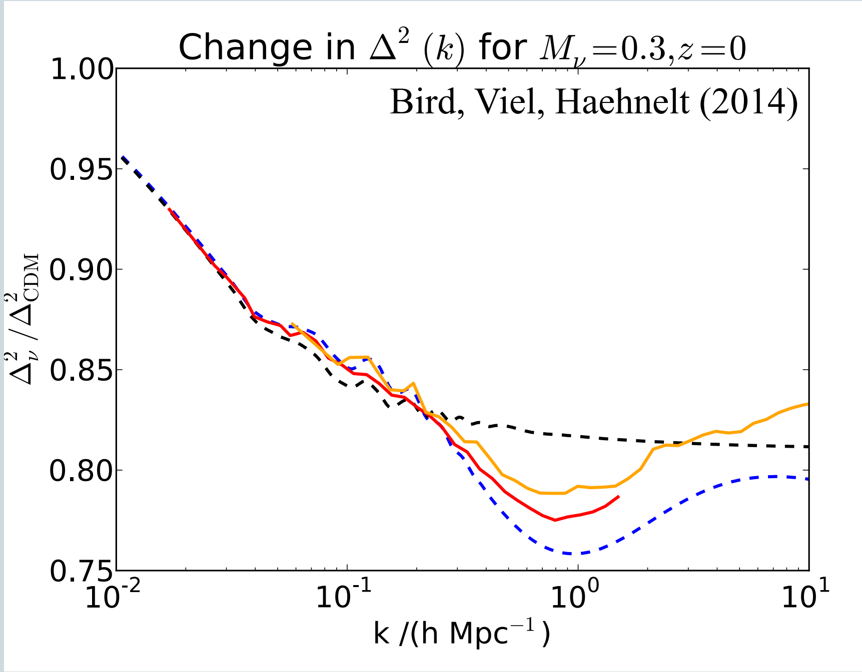
Only one thread, no OpenMPI

	CLASS ($k\tau$ > 31)	New Ψ_3 ($k\tau$ > 31)	New Ψ_3 ($k\tau$ > 12)
CPUt/ CPUt(l=17)	0.24	0.35	0.13



The non-linear regime





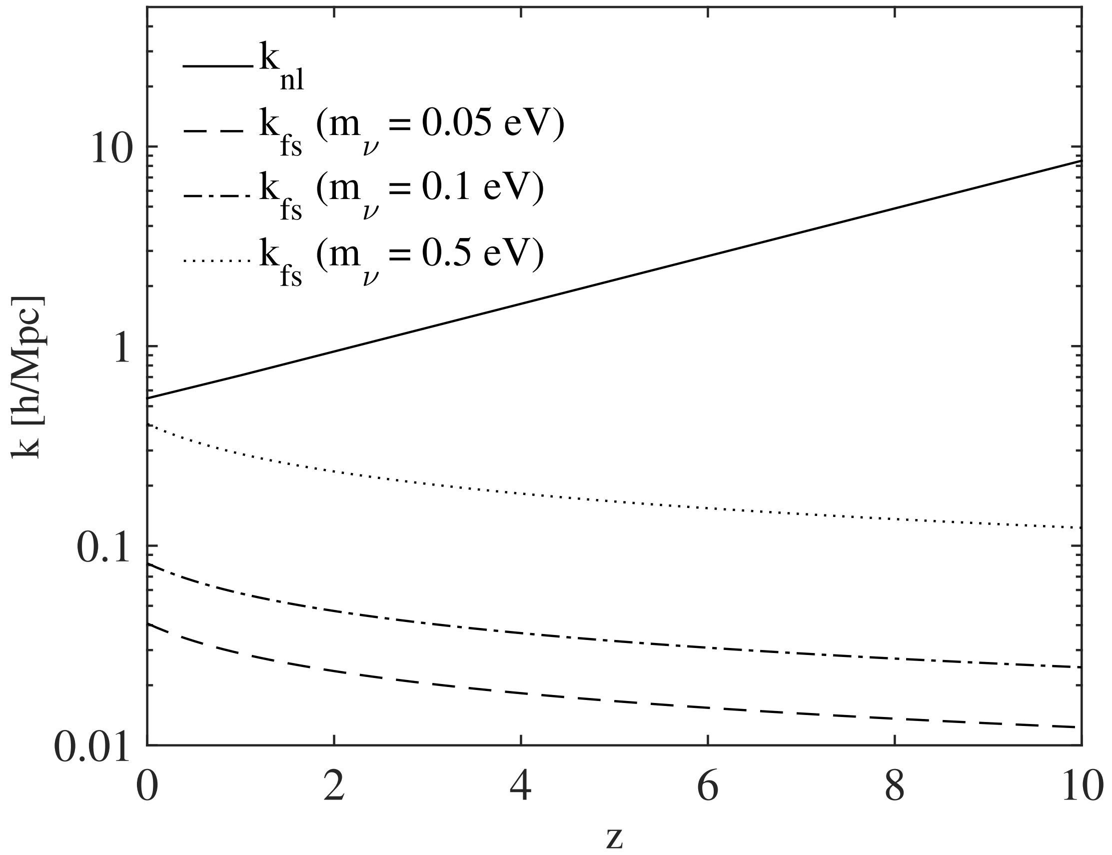
Non-linear methods

- Beyond linear order perturbation theory
  - Fuhrer & Wong (2014)
  - Blas, Garny, Konstandin, Lesgourgues (2014)
  - Dupuy & Bernardeau (2014)

- N-body simulations
  - Hybrid methods: Brandbyge & Hannestad (2009 & 2010)
  - Semi-linear methods: Ali-Haimoud & Bird (2012)

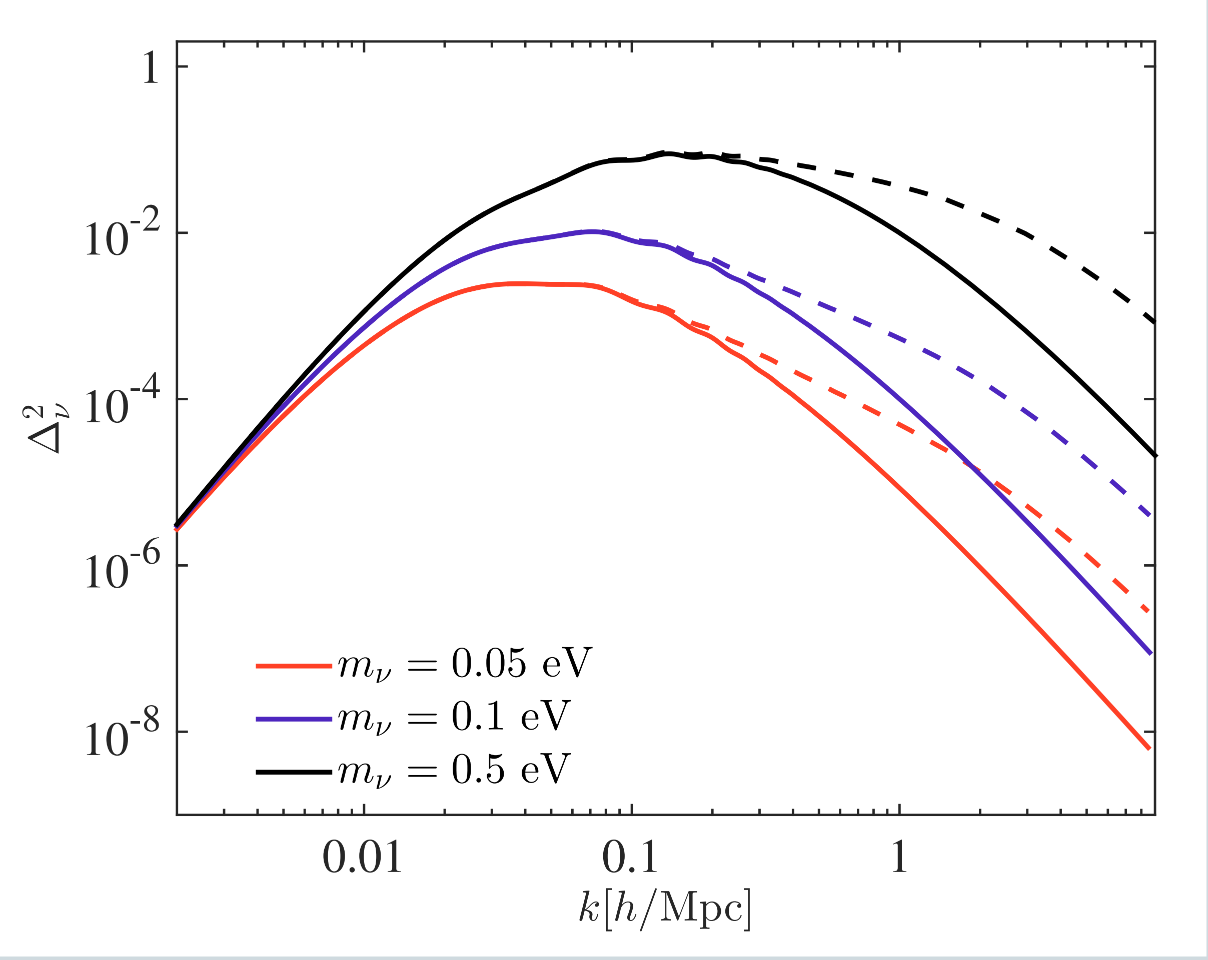
- Our approach: using HALOFIT, we account for the non-linear growth of cold dark matter overdensities and gravitational potential, then we evolve linear neutrino perturbations in the “non-linear” gravitational potential. The entire computation is in Fourier k space.

Free-streaming scale vs non-linear scale



Archidiacono & Hannestad, arXiv:1510.02907

Neutrino power spectrum



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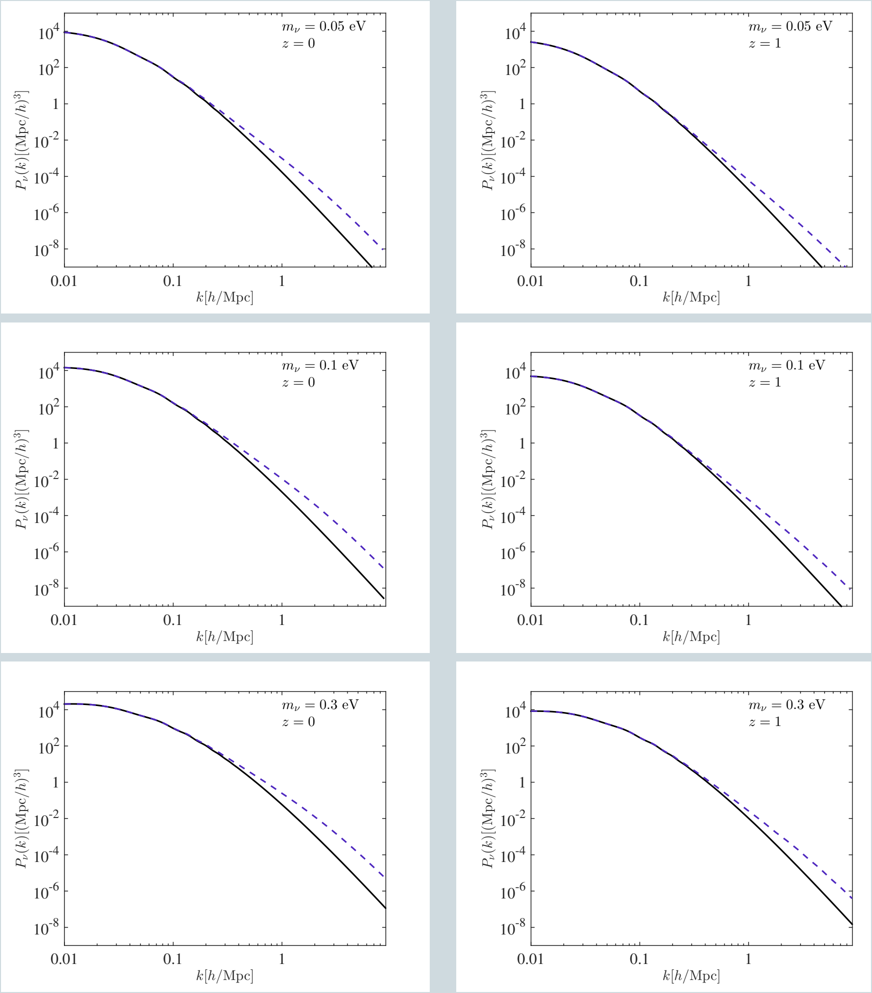

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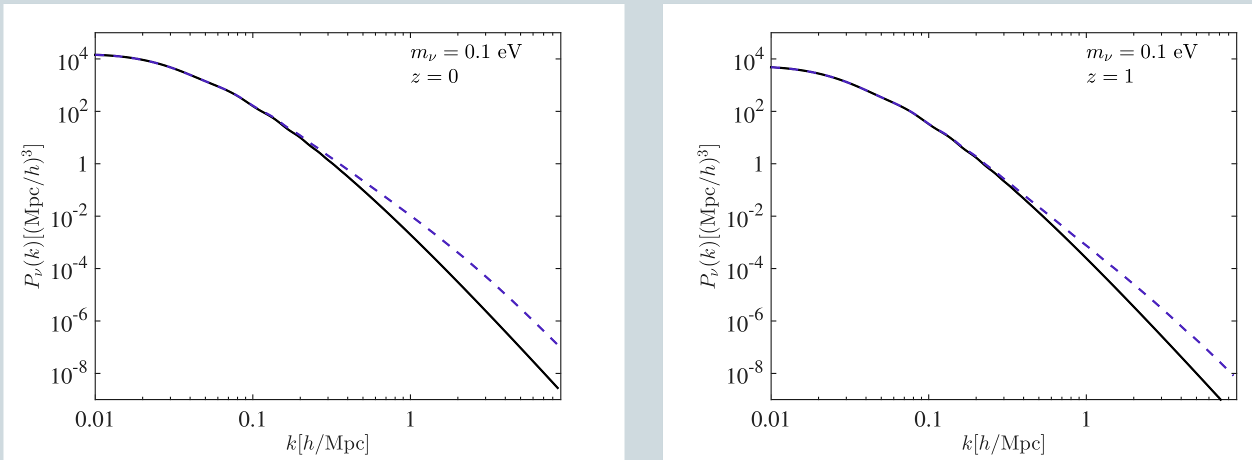

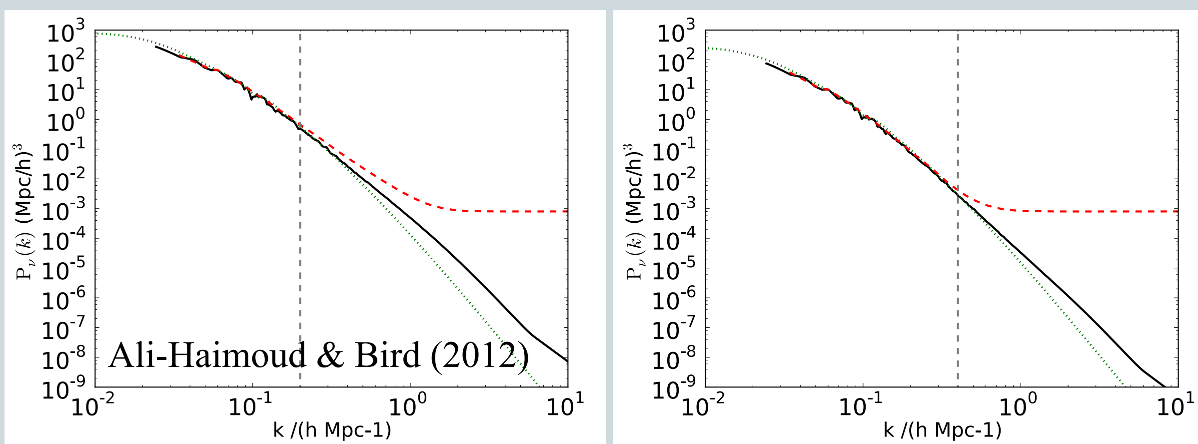

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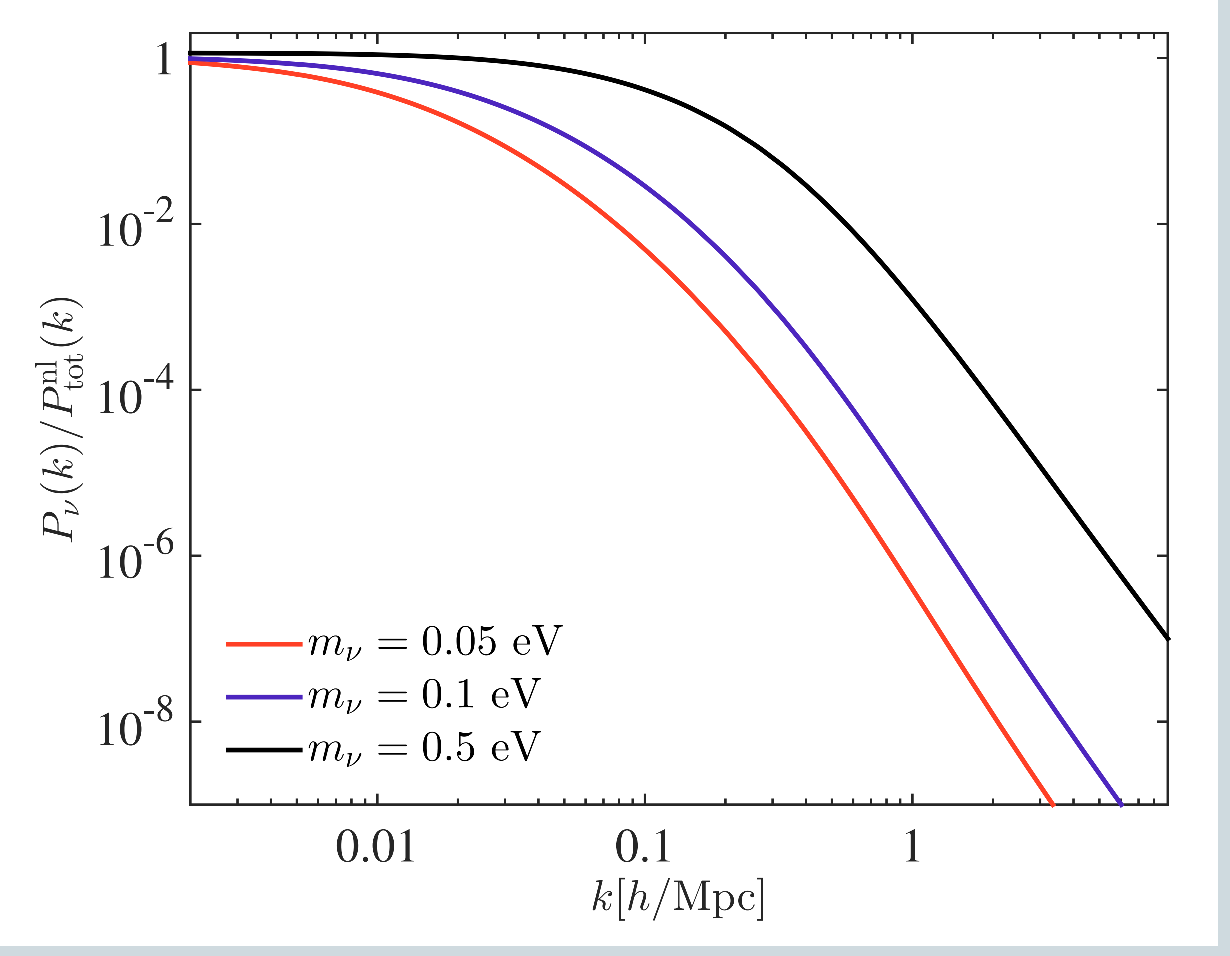
Conclusions

- We have demonstrated that the neutrino evolution hierarchy can be solved very accurately even if truncated at $l = 2$. Our approximation for the $l = 3$ term allowed us to reliably calculate the neutrino power spectrum to better than ~5% precision for masses up to 1.5 eV. The matter power spectrum has a precision of better than 0.5% because of the relatively small direct contribution of neutrinos to this quantity. The new approximation to Ψ_3 is significantly more precise than previously used once.

- We showed how the neutrino power spectrum can be calculated using the full non-linear gravitational potential, but keeping the entire computation in k-space. The results obtained using this technique are completely consistent with those from N-body simulations implementing neutrinos in Fourier-space. However, in our case the neutrino power spectrum can be obtained in a few seconds whereas the N-body technique requires far bigger computational resources.

Backup



$P_\nu(k)/P_{\rm tot}^{\rm nl}(k)$
1
10^{-2}
10^{-4}
10^{-6}
10^{-8}
$m_\nu = 0.05$ eV
$m_\nu = 0.1$ eV
$m_\nu = 0.5$ eV
0.01
0.1
1
$k[h/{\rm Mpc}]$