Nonequilibrium QFT approach to leptogenesis

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Outline

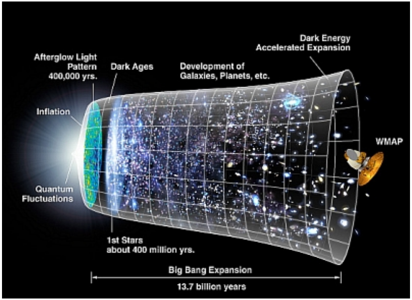
- Introduction
- Part I: Closed Time Path (CTP) formalism of nonequilibrium QFT
- Part II: Application to resonant leptogenesis
- Conclusions

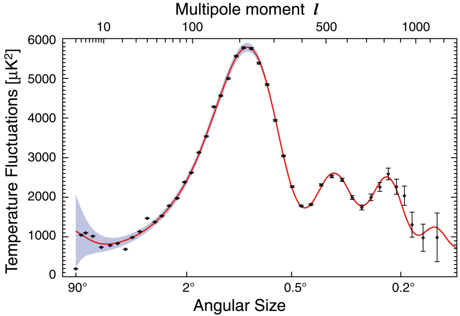
Why baryogenesis?

To explain the excess of matter over antimatter in the Universe:

[WMAP 2010]

$$\frac{n_B}{n_\gamma} = (6.19 \pm 0.15) \times 10^{-10}$$





Sakharov conditions [A. Sakharov (1967)]

Necessary conditions for the generation of a permanent baryon asymmetry:

- Baryon number B violation
- C and CP violation
- Out of equilibrium

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  To make the other conditions effective

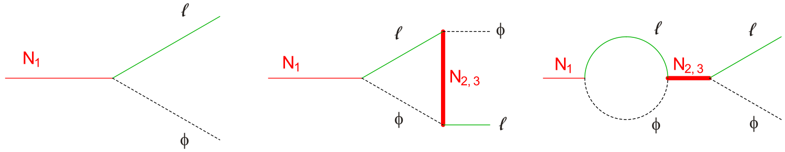
Leptogenesis [Fukugita, Yanagida (1986)]

- The Standard Model is extended by adding heavy right-handed Majorana neutrinos N_i, $i = 1, 2, \dots$ (eg. see-saw models)

$$\mathcal{L} \supset \frac{1}{2}\bar{\psi}_{Ni}\big(\mathrm{i}\partial\!\!\!/ - M_i\big)\psi_{Ni} - Y^*_{ia}\bar{\psi}_{\ell a}(\epsilon\phi)^\dagger\psi_{Ni} + \mathrm{h.c.}$$

- Lepton asymmetry is generated through out-of-equilibrium L- and CP-violating Yukawa decays: $N_1 \to \ell\phi$

- Resonant enhancement of CP-violation for nearly degenerate neutrino masses: $|M_i - M_j| \sim \Gamma$



Baryogenesis via Leptogenesis

- Baryon number B violation
- C violation
- CP violation
- Out of equilibrium

Baryogenesis via Leptogenesis

- Baryon number B violation
  - $B - L$ broken by Majorana masses M_i
  - $B + L$ broken by the EW Sphalerons
- C violation
  - Left-chiral EW interactions
- CP violation
  - Yukawa interactions between ℓ_α and N_i
- Out of equilibrium
  - Expansion of the Universe - decreasing T
  - Inverse decays $\ell\phi \to N_1$ become out of equilibrium when $M_1 \sim T \sim 10^{12}\text{GeV}$ (typically)

Motivation

- Standard approach to leptogenesis calculations is to use Boltzmann equations with vacuum S-matrix elements.

Goal:

First principle description of leptogenesis, including:

- Finite density medium effects
- Flavour coherence effects
- Memory effects, etc.

Questions:

- When Boltzmann description breaks?
- Are finite widths required in the resonant regime: $|M_i - M_j| \sim \Gamma$, even though $M_i \gg \Gamma$?
- Do flavour oscillations in the singlet neutrino sector play any role?

Lots of recent progress towards more reliable calculational scheme:

- Finite density medium / thermal effects
  - Buchmüller, Fredenhagen (2000)
  - Asaka, Laine, Shaposhnikov (2006)
  - Garny, Hohenegger, Kartavtsev, Lindner (2009 & 2010)
  - Garny, Hohenegger, Kartavtsev (2010 & 2011)
  - Frossard, Garny, Hohenegger, Kartavtsev, Mitrouskas (2012)
  - Beneke, Garbrecht, MH, Schwaller (2010)
  - Garbrecht (2010)
  - Anisimov, Besak, Bödeker (2010)
  - Besak, Bödeker (2012)
  - Kiessig, Plumacher, Thoma (2010)
  - Kiessig, Plumacher (2011)
  - Laine, Schroder (2011)
  - Laine (2012)
- Flavour effects / finite width and memory effects
  - De Simone, Riotto (2007)
  - Beneke, Garbrecht, Fidler, MH, Schwaller (2010)
  - Anisimov, Buchmüller, Drewes, Mendizabal (2010 & 2011)
  - Garny, Hohenegger, Kartavtsev (2011)
  - Garbrecht, MH (2011)

Part I: CTP Formalism of Nonequilibrium QFT

Closed Time Path* (CTP) Formalism

* a.k.a. Schwinger-Keldysh Formalism [Schwinger (1961); Keldysh (1964); Calzetta, Hu (1988)]

- Usually in QFT matrix elements are computed within the in-out framework:

$$\langle \text{out}|\hat{S}|\text{in}\rangle \quad \longleftarrow \quad \text{time-ordered correlators:} \quad \langle T[\psi(x)\bar{\psi}(y)]\rangle$$

- For equilibrium or out-of-equilibrium problems the interesting quantities are expectation values in a finite density medium, e.g.

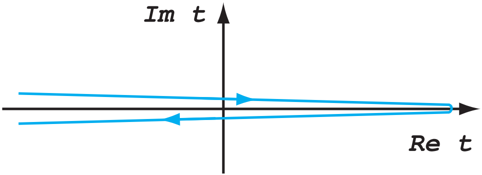
$$\langle j^0(x)\rangle = \langle\bar{\psi}(x)\gamma^0\psi(x)\rangle \equiv \text{tr}\left[\hat{\rho}\,\bar{\psi}(x)\gamma^0\psi(x)\right]$$

  - $\hat{\rho}$ is an (unknown) quantum density operator

- Path-integral formulation of expectation values:

$$\langle \mathcal{O}(x) \rangle = \int D\phi^+ D\phi^- \rho\big[\phi^+(0,\mathbf{x}), \phi^-(0,\mathbf{x})\big] \mathcal{O}(x) \exp\left\{ i\big[S[\phi^+] - S[\phi^-]^*\big]\right\}$$

$\longrightarrow$ In-In generating functional on a Closed Time Path:



$$Z[J^+, J^-] = \int D\phi^+ D\phi^- \rho\big[\phi^+(0,\mathbf{x}), \phi^-(0,\mathbf{x})\big]$$
$$\times \exp\left\{ i\Big[S[\phi^+] - S[\phi^-]^* + (J^+\phi^+ + {J^+}^\dagger {\phi^+}^\dagger) - (J^-\phi^- + {J^-}^\dagger {\phi^-}^\dagger)\Big]\right\}$$

- Path-ordered correlators:

$$iG_{\mathcal{C}}(x,y) = \langle T_{\mathcal{C}}[\phi(x)\phi^\dagger(y)]\rangle$$

  where $T_{\mathcal{C}}$ denotes path-ordering along the CTP.

- Four propagators with respect to real time variable:

$$\begin{aligned}
iG^{+-}(x,y) &= iG^{<}(x,y) = \langle \phi^\dagger(y)\phi(x)\rangle \\
iG^{-+}(x,y) &= iG^{>}(x,y) = \langle \phi(x)\phi^\dagger(y)\rangle \\
iG^{++}(x,y) &= iG^{T}(x,y) = \langle T(\phi(x)\phi^\dagger(y))\rangle \\
iG^{--}(x,y) &= iG^{\bar{T}}(x,y) = \langle \bar{T}(\phi(x)\phi^\dagger(y))\rangle
\end{aligned}$$

- In diagrammatic Feynman rules each vertex is summed over a CTP branch index:

$$\int d^4x \int d^4y\, G(x,y) \longrightarrow \sum_{a,b=\pm} ab \int d^4x \int d^4y\, G^{ab}(x,y)$$

Thermal equilibrium

- In thermal equilibrium the density operator is given by (canonical ensemble):

$$\hat{\rho} = \frac{1}{Z} e^{-\beta \hat{H}}$$

$\longrightarrow$ Propagators and self energies obey KMS relations:

$$G^{>}(k) = \mathrm{e}^{\beta k_0}\, G^{<}(k)\,, \quad \Sigma^{>}(k) = \mathrm{e}^{\beta k_0}\, \Sigma^{<}(k)$$

$\longrightarrow$ The tree-level propagators are given by

$$\mathrm{i}G^{<}(k) = 2\pi\delta(k^2 - m^2)\mathrm{sign}(k_0) f_B(k_0)\,,$$

$$\mathrm{i}G^{>}(k) = 2\pi\delta(k^2 - m^2)\mathrm{sign}(k_0)(1 + f_B(k_0))\,,$$

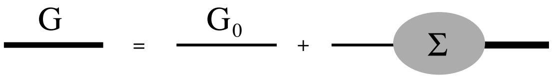
$$\mathrm{i}G^{T}(k) = \frac{\mathrm{i}}{k^2 - m^2 + \mathrm{i}\varepsilon} + 2\pi\delta(k^2 - m^2) f_B(|k_0|)\,,$$

$$\mathrm{i}G^{\bar{T}}(k) = -\frac{\mathrm{i}}{k^2 - m^2 - \mathrm{i}\varepsilon} + 2\pi\delta(k^2 - m^2) f_B(|k_0|)$$

- In equilibrium CTP formalism reduces to real-time formalism of thermal field theory

Out of equilibrium - Kadanoff-Baym Equations

- Generic Schwinger-Dyson equations for (CTP) propagators:



- Kadanoff-Baym equations are the $<,>$ components:[1]

$$(-\partial_x^2 - m^2)G^{<,>} - \Sigma^H \odot G^{<,>} - \Sigma^{<,>} \odot G^H = \frac{1}{2}\left(\Sigma^> \odot G^< - \Sigma^< \odot G^>\right)$$

  - Renormalization, thermal corrections (thermal masses etc.)
  - Finite width effects
  - Collision term

[1] $(A \odot B)(x,y) \equiv \int d^4z\, A(x,z)B(z,y)$ denotes convolution

Kinetic theory limit

- Weak interactions
- Slowly varying background

To separate the external and internal scales, we perform Wigner transform:

$$G(k,X) = \int d^4r\, e^{ik\cdot r}\, G\big(X + \frac{r}{2}, X - \frac{r}{2}\big)\,,$$

where

$X = (x+y)/2$	average coordinate
$r = x - y$	relative coordinate

- Trades the relative coordinate to momentum

- Gradient expansion to the lowest order in
  - X-derivatives: $\partial_X M(X)$, $\partial_X G(k,X)$, $\partial_X \Sigma(k,X)$, etc.
  - coupling constants in $\Sigma(k,X)$

$\longrightarrow$ Quasiparticle picture with singular phase-space structure:

$$iG^{<}(k,X) = 2\pi\delta(k^2 - m^2)\big[\theta(k_0)f_+(\mathbf{k},X) + \theta(-k_0)\big(1 + f_-(\mathbf{k},X)\big)\big]$$

- The (particle/antiparticle) distribution functions $f_\pm(\mathbf{k},X)$ obey quantum Boltzmann-type kinetic equations:

$$\partial_t f_\pm + \mathbf{v}\cdot\partial_{\mathbf{x}} f_\pm + \mathbf{F}\cdot\partial_{\mathbf{k}} f_\pm = (\partial_t f_\pm)_{\text{coll}} \equiv \mathcal{C}_{\text{coll}\pm}$$

Coherent quasiparticle approximation (cQPA)

[MH, Kainulainen, Rahkila (2008-2011); Fidler, MH, Kainulainen, Rahkila (2011)]

- In cQPA (fermionic) propagators $S(k,t)$ are NOT assumed to be nearly time translation invariant ($\partial_t S(k,t)$ are not necessarily small)

KB-equations $\longrightarrow$ flavour-mixing Constraint and Kinetic Equations:

$$\text{(CE)} \qquad 2k_0 \bar{S}^{<} = \{H, \bar{S}^{<}\} + \left(e^{\frac{i}{2}\partial_{k_0}^{\Sigma}\cdot\partial_t^{S}} \bar{\Sigma}^{<} \bar{S}^{h} - i e^{\frac{i}{2}\partial_{k_0}^{\Sigma}\cdot\partial_t^{S}} \bar{\Sigma}^{\mathcal{A}} (\bar{S}^{<} - \bar{S}^{<}_{\rm eq}) + \text{h.c.} \right)$$

$$\text{(KE)} \qquad i\partial_t \bar{S}^{<} = [H, \bar{S}^{<}] + \left(e^{\frac{i}{2}\partial_{k_0}^{\Sigma}\cdot\partial_t^{S}} \bar{\Sigma}^{<} \bar{S}^{h} - i e^{\frac{i}{2}\partial_{k_0}^{\Sigma}\cdot\partial_t^{S}} \bar{\Sigma}^{\mathcal{A}} (\bar{S}^{<} - \bar{S}^{<}_{\rm eq}) - \text{h.c.} \right)$$

where

$$\bar{S}^{<}(k,t) \equiv i S^{<}(k,t)\gamma^0 , \qquad H_{ij} \equiv \left(\mathbf{k}\cdot\alpha + M_i\gamma^0\right)\delta_{ij}$$

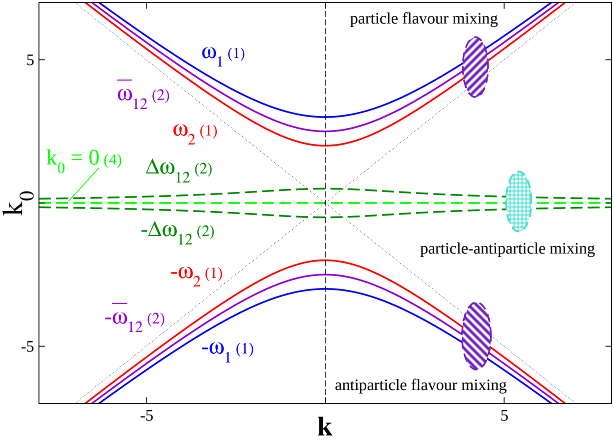
$\longrightarrow$ new oscillatory solutions encoding quantum (flavour) coherence[2]:

$$iS^{<}_{ij}(k,t) = 2\pi \sum_{h\pm} \frac{1}{4\omega_i\omega_j} P_h(\not{k}_{i\pm} + m_i)\Big[(\not{k}_{j\pm} + m_j) f^{m}_{ijh\pm}(\mathbf{k},t)\delta(k_0 \mp \bar{\omega}_{ij}) - (\not{k}_{j\mp} + m_j) f^{c}_{ijh\pm}(\mathbf{k},t)\delta(k_0 \mp \Delta\omega_{ij})\Big]$$

- $f^{m}_{i\neq j}$-functions mediate particle-particle and antiparticle-antiparticle flavour coherence
- f^{c}-functions mediate particle-antiparticle (flavour) coherence

[2] $P_h = \frac{1}{2}(1 + h\,\hat{\mathbf{k}}\cdot\gamma^0\vec{\gamma}\gamma^5)$, with $h = \pm 1$ are helicity projectors, $k^{\mu}_{i\pm} \equiv (\pm\omega_i, \mathbf{k})$, $\bar{\omega}_{ij} \equiv \frac{1}{2}(\omega_i + \omega_j)$ and $\Delta\omega_{ij} \equiv \frac{1}{2}(\omega_i - \omega_j)$

- Phase space shell structure:



- The distribution functions $f^{m,c}$ obey coherent quantum Boltzmann equations (flavor indices are suppressed):

$$\partial_t f^m_{h\pm} = \mp\, i2\Delta\omega f^m_{h\pm} + \sum_\alpha \Phi'_{\pm\alpha} f_\alpha + \mathcal{C}^m_\pm[f_\alpha]$$

$$\partial_t f^c_{h\pm} = \mp\, i2\bar{\omega} f^c_{h\pm} + \sum_\alpha \Phi'_{c\pm\alpha} f_\alpha + \mathcal{C}^c_\pm[f_\alpha]$$

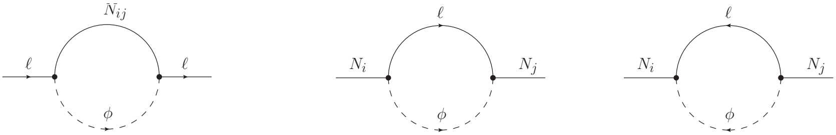
  - Φ' involve the first order gradients $\partial_t m_i$, $U\partial_t U^\dagger$ and $V\partial_t V^\dagger$.
  - $\mathcal{C}[f_\alpha]$ are the collision integrals involving the self-energies Σ.

- To zeroth order in $\partial_t m$ and Σ:
  $f^m_{h\pm}$ and $f^c_{h\pm}$ are simply oscillatory with frequencies $\Delta\omega$ and $\bar{\omega}$.

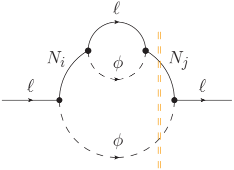
Part II: Application to resonant leptogenesis

Self-energies

- 2PI-type approach: one-loop self-energies, N_{ij} is independent variable [Garbrecht, MH (2011); Garny, Hohenegger, Kartavtsev (2011); MH, Kainulainen, Rahkila (in progress)]



perturbatively
$\longrightarrow$



- Wave-function type CP-violation arises from N_i flavour mixing

Approximations

Relevant scales:

- External scale: Hubble expansion rate $H \sim T^2/M_{\rm pl}$
- Internal scale: momentum $k \sim T$

$$\Longrightarrow \quad H \ll k$$

- In addition: For weak coupling the interaction rates:

$$\Gamma \ll k$$

During the leptogenesis at $T > \Lambda_{\rm EW}$:

$$k \gg \Gamma_g \gg \Gamma_Y \sim H$$

- ℓ and ϕ fields remain in kinetic equilibrium due to fast gauge interactions

$\longrightarrow$ Due to scale hierarchy kinetic theory limit is applicable

$\longrightarrow$ KB-equations in cQPA scheme for flavour-mixing neutrinos N_i

In addition:

- Kinetic equation for lepton asymmetry:

$$\partial_t \left(n_\ell - \bar{n}_\ell\right) = -\frac{1}{2}\int \frac{d^4k}{(2\pi)^4} \mathrm{tr}\left[\mathrm{i}\Sigma_\ell^{>}(k) P_\mathrm{L} \mathrm{i}S_\ell^{<}(k) - \mathrm{i}\Sigma_\ell^{<}(k) P_\mathrm{L} \mathrm{i}S_\ell^{>}(k)\right] + h.c.$$

$$\equiv -\mathcal{W}\left(n_\ell - \bar{n}_\ell\right) + \mathcal{S}_{CP}$$

Non-resonant regime: $|M_i - M_j| \gg \Gamma$

- Rapidly oscillating flavour-coherence solutions can be neglected

- Perturbative solution to the constraint equation:

$$i\delta S^{<}_{ii}(k,t) = -\,2\pi(\not{k} + M_i)\delta f_i(\mathbf{k},t)\delta(k^2 - M_i^2)$$

$$i\delta S^{<}_{ij}(k,t) = \frac{i}{M_i^2 - M_j^2}\Big[(\not{k} + M_i)\Sigma^{\mathcal{A}}_{ij}(k) i\delta S^{<}_{jj}(k,t) + i\delta S^{<}_{ii}(k,t)\Sigma^{\mathcal{A}}_{ij}(k)(\not{k} + M_j)\Big]$$

- CP-violating source involves finite density corrections: [Beneke, Garbrecht, MH, Schwaller (2010)]

$$\mathcal{S}^{\rm wf}_{CP} = 8\,{\rm Im}[Y_1^2 Y_2^{*\,2}]\frac{M_1 M_2}{M_1^2 - M_2^2}\sum_{i=1,2}\int\frac{d^3k}{(2\pi)^3\,2\omega_i}\Sigma_\mu(k_i)\,\Sigma^\mu(k_i)\,\delta f_i(\mathbf{k},t)$$

Resonant regime: $|M_i - M_j| \lesssim \Gamma$

- Flavour-coherence important!

- Lowest order solution to the constraint equation:

$$i\delta S^{<}_{ij}(k,t) = -2\pi \sum_{h=\pm} P_h(\not{k} + \bar{M}_{ij})\delta f_{h\,ij}(\mathbf{k},t)\delta(k^2 - \bar{M}^2_{ij})$$

- Neutrino flavour-mixing dynamics from local kinetic equation:

$$\partial_t \delta f_h = -\frac{i}{2\omega}\left[M^2, \delta f_h\right] - \left\{\Gamma_h, \delta f_h\right\} - \partial_t f_{\rm eq}$$

- *CP*-violating source:

$$\mathcal{S}_{CP} = \sum_{i,j}\sum_{h=\pm} Y_i^* Y_j \int \frac{d^3k}{(2\pi)^3\, 2\omega}\left[k^\mu(\delta f_{h\,ij} - \delta f^*_{h\,ij}) + h\tilde{k}^\mu(\delta f_{h\,ij} + \delta f^*_{h\,ij})\right]\Sigma_\mu(k)$$

$\bar{M}_{ij} \equiv (M_i + M_j)/2\,, \qquad P_h \equiv \frac{1}{2}(1 + h\,\hat{\mathbf{k}}\cdot\gamma^0\gamma\gamma^5)\,, \qquad \Gamma_{h\,ij}(\mathbf{k}) \equiv \left(\mathrm{Re}[Y_iY_j^*]\,\frac{k^\mu}{\omega} + ih\,\mathrm{Im}[Y_iY_j^*]\,\frac{\tilde{k}^\mu}{\omega}\right)\Sigma_\mu(k)$

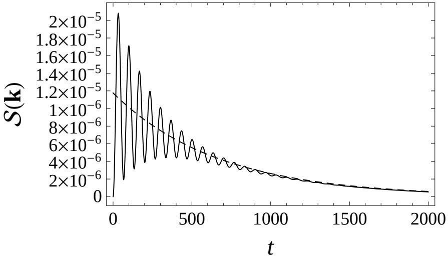
Toy model: constant background

- Initial conditions (N_1 vacuum, N_2 thermal):

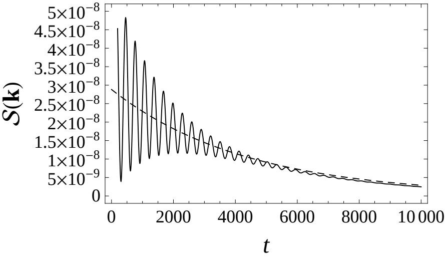
$$\delta f_{h\,11}(\mathbf{k},0) = -f_{\rm eq}(\mathbf{k})\,, \qquad \delta f_{h\,22}(\mathbf{k},0) = \delta f_{h\,12}(\mathbf{k},0) = \delta f_{h\,21}(\mathbf{k},0) = 0$$

- Non-resonant regime: $|M_2 - M_1| \gg \Gamma$

$$M_1 = T\,, \qquad M_2 = 1.1\,T\,, \qquad Y_1 = 0.1\,, \qquad Y_2 = 0.2 + 0.1{\rm i}$$



Non-relativitic case: $|\mathbf{k}| = 0.5\,T$



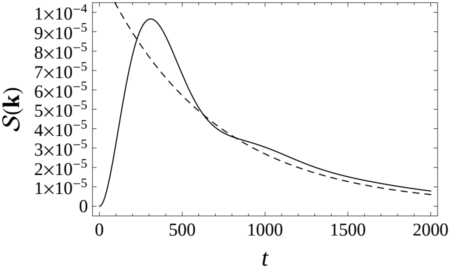
Relativitic case: $|\mathbf{k}| = 5\,T$

— full kinetic equations

--- peturbative result

- Resonant regime: $|M_2 - M_1| \sim \Gamma$

$$M_1 = T\,, \qquad M_2 = 1.01\,T\,, \qquad Y_1 = 0.1\,, \qquad Y_2 = 0.2 + 0.1\mathrm{i}$$



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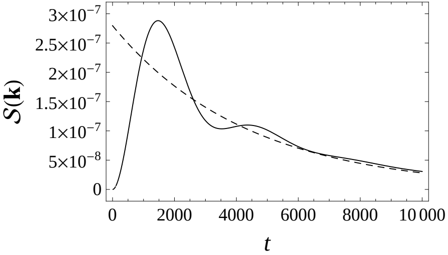


Relativitic case: $|\mathbf{k}| = 5\,T$

- In resonant regime the correction to perturbative result can be sizeble

- In the limit $|\Gamma_{12}| \ll |\Gamma_{11} - \Gamma_{22}|$ and $|\Gamma_{12}| \lesssim |M_1 - M_2|$, we get:

$$\mathcal{S}_{CP} = -\,8\,\mathrm{Im}[Y_1^2 Y_2^{*2}] \int \frac{d^3k}{(2\pi)^3\, 2\omega} \frac{M_1 M_2 (M_1^2 - M_2^2)}{(M_1^2 - M_2^2)^2 + \omega^2(\Gamma_{11} - \Gamma_{22})^2} \Sigma_\mu(k)\Sigma^\mu(k)$$
$$\times \left[\mathrm{e}^{-\Gamma_{11} t} - \left(\cos(\Delta\omega\, t) + \frac{\Gamma_{11} - \Gamma_{22}}{2\Delta\omega} \sin(\Delta\omega\, t) \right) \mathrm{e}^{-(\Gamma_{11}+\Gamma_{22})t/2} \right] f_{\mathrm{eq}}(\mathbf{k})$$

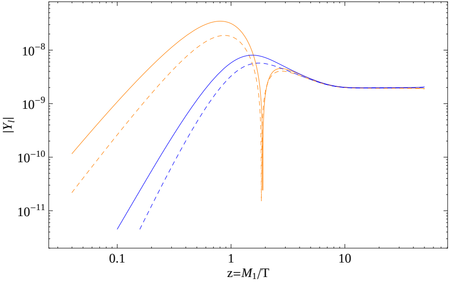
where $\Delta\omega(\mathbf{k}) \equiv |M_1^2 - M_2^2|/(2\omega(\mathbf{k}))$

- The 'resonant' structure of the denominator is in agreement with previous works [Buchmuller, Plumacher (1997); Anisimov, Broncano, Plumacher (2005); Garny, Hohenegger, Kartavtsev (2011)]

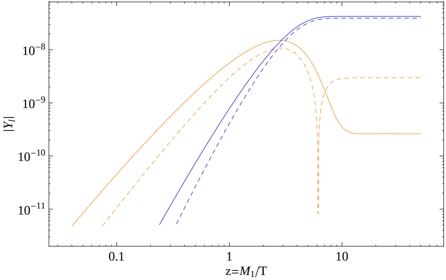
- The new terms arise due to singlet neutrino flavour oscillations

  $\longrightarrow$ Important in the resonant regime: $\Delta\omega \sim \Gamma$

Finite density effects [Beneke, Garbrecht, MH, Schwaller (2010)]



Strong washout



Weak washout

- Lepton Asymmetry: $Y_\ell = \frac{n_\ell - \bar{n}_\ell}{s}$
  - —— full solution
  - - - - no thermal corrections in loops
- Finite density corrections are sizable for weak washout

Conclusions

- CTP formalism is a powerful tool to study out-of-equilibrium phenomena of the early Universe:
  - First principle approach providing a controlled approximation scheme
  - Naturally incorporates finite density medium effects, flavour coherence effects etc.
- CTP approach to resonant leptogenesis:
  - LO dynamics can be described by local transport equations within quasiparticle picture
  - Resonant enhancement is obtained through N_i flavour-mixing dynamics (observed for mixing scalar fields in [Liu, Segre (1993); Covi, Roulet (1996)])
  - In resonant regime: $|M_i - M_j| \lesssim \Gamma$, neutrino N_i flavour oscillations can be important
  - Finite density corrections are sizable for weak washout

Extra: One-loop self-energies

- Neutrino N_i self-energy:

$$\mathrm{i}\Sigma_{ij}^{<,>}(k) = g_w \int \frac{d^4k'}{(2\pi)^4}\frac{d^4k''}{(2\pi)^4}(2\pi)^4\delta^{(4)}(k-k'-k'') \times \left\{ Y_i Y_j^* P_\mathrm{L} \mathrm{i}S_\ell^{<,>}(k') P_\mathrm{R} \mathrm{i}\Delta_\phi^{ab}(k'') - Y_i^* Y_j P_\mathrm{R} C \mathrm{i}S_\ell^{>,<T}(-k') C^\dagger P_\mathrm{L} \mathrm{i}\Delta_\phi^{>,<}(-k'') \right\}$$

$\longrightarrow$

$$\Sigma_{ij}^{\mathcal{A}}(k) = \frac{1}{2}(\mathrm{i}\Sigma_{ij}^{>}(k) - \mathrm{i}\Sigma_{ij}^{<}(k)) \equiv (Y_j^* Y_i P_\mathrm{L} + Y_i^* Y_j P_\mathrm{R})\gamma^\mu \Sigma_\mu(k)$$

$$\Sigma^\mu(k) = \int \frac{d^3p}{(2\pi)^3 2|\mathbf{p}|}\frac{d^3q}{(2\pi)^3 2|\mathbf{q}|}(2\pi)^4\delta^4(k-p-q)\, p^\mu \left(1 - f_\ell^{\mathrm{eq}}(\mathbf{p}) + f_\phi^{\mathrm{eq}}(\mathbf{q})\right)$$

- Charged lepton ℓ self-energy (lowest order in gradients):

$$\mathrm{i}\Sigma_\ell^{<,>}(k,t) = \int \frac{d^4k'}{(2\pi)^4}\frac{d^4k''}{(2\pi)^4}(2\pi)^4\delta^4(k-k'-k'') Y_i^* Y_j P_\mathrm{R} \mathrm{i}S_{ij}^{<,>}(k',t) P_\mathrm{L} \mathrm{i}\Delta_\phi^{>,<}(-k'')$$