Models of inhomogeneity and backreaction: A status report

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J H McKay & DLW: **MNRAS 457 (2016) 3285**

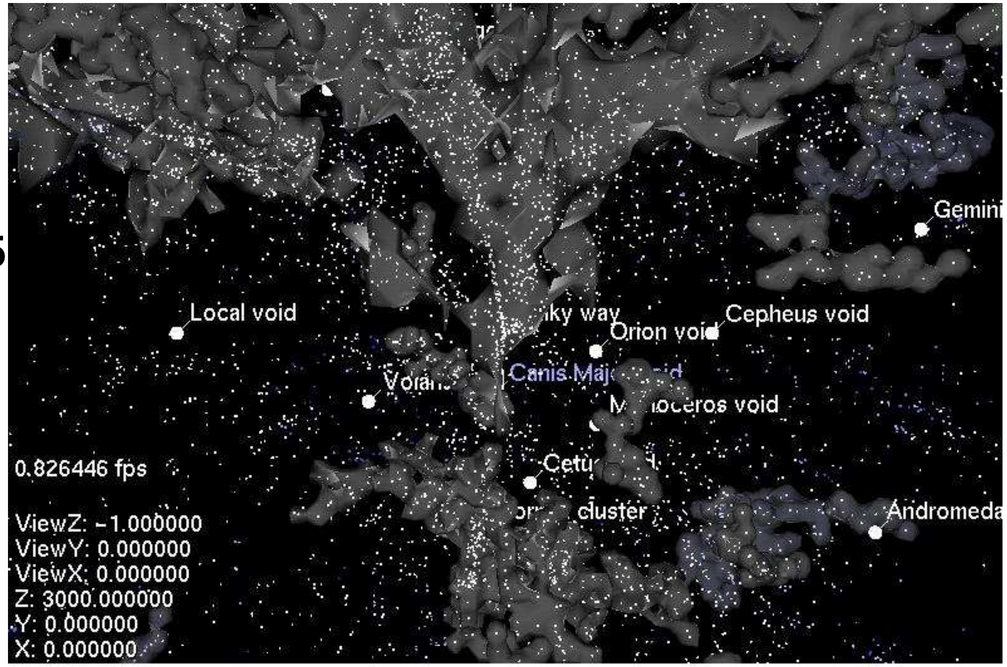
K Bolejko, M A Nazer & DLW:

arXiv:1512.07364

M A Nazer and DLW:

Phys. Rev. D 91 (2013) 063519

DLW – **lecture notes: arXiv:1311.3787**



Outline of talk

- Test and failure of FLRW on $\lesssim 100\,h^{-1}$Mpc
  Result: very strong Bayesian evidence for GR differential expansion on $\lesssim 65\,h^{-1}$Mpc scales, supported by simulations, with proposal to test impact on large angle CMB anomalies
- Concepts of coarse-graining, averaging
- What is dark energy?:

  Dark energy is a misidentification of gradients in quasilocal dilatational kinetic energy

  (in presence of density and spatial curvature gradients on scales $\lesssim 100\,h^{-1}$Mpc – *statistical homogeneity scale (SHS)* – which also alter average cosmic expansion).
- Update on tests of timescape cosmology

Averaging and backreaction

- *Fitting problem* (Ellis 1984):
  On what scale are Einstein's field equations valid?

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- In general $\langle G^{\mu}{}_{\nu}(g_{\alpha\beta})\rangle \neq G^{\mu}{}_{\nu}(\langle g_{\alpha\beta}\rangle)$
- Inhomogeneity in expansion (on $\lesssim 100\, h^{-1}$Mpc scales) may make average non–Friedmann as structure grows
- *Weak backreaction*: Perturb about a given background
- *Strong backreaction*: fully nonlinear
  - Spacetime averages (R. Zalaletdinov 1992, 1993);
  - Spatial averages on hypersurfaces based on a $1+3$ foliation (T. Buchert 2000, 2001).

Cosmic web: typical structures

- Galaxy clusters, 2 – 10 h^{-1}Mpc, form filaments and sheets or “walls” that thread and surround voids
- Universe is void dominated (60–80%) by volume, with distribution peaked at a particular scale (40% of total volume):

Survey	Void diameter	Density contrast
PSCz	$(29.8 \pm 3.5)h^{-1}$Mpc	$\delta_\rho = -0.92 \pm 0.03$
UZC	$(29.2 \pm 2.7)h^{-1}$Mpc	$\delta_\rho = -0.96 \pm 0.01$
2dF NGP	$(29.8 \pm 5.3)h^{-1}$Mpc	$\delta_\rho = -0.94 \pm 0.02$
2dF SGP	$(31.2 \pm 5.3)h^{-1}$Mpc	$\delta_\rho = -0.94 \pm 0.02$

Dominant void statistics in the Point Source Catalogue Survey (PSCz), the Updated Zwicky Catalogue (UZC), and the 2 degree Field Survey (2dF) North Galactic Pole (NGP) and South Galactic Pole (SGP), (Hoyle and Vogeley 2002,2004). More recent results of Pan et al. (2011) using SDSS Data Release 7 similar.

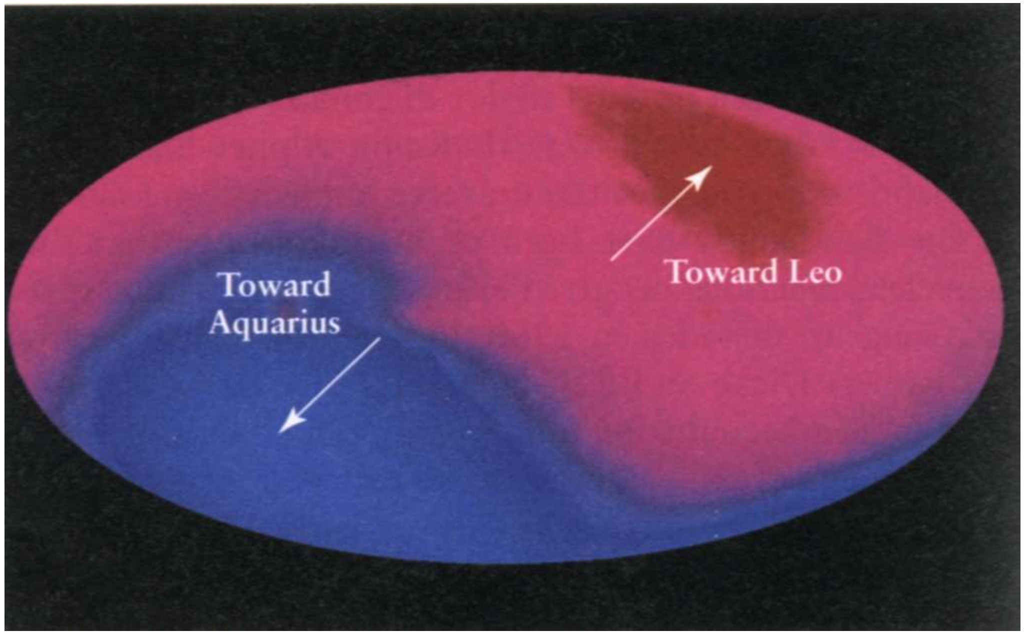
Statistical homogeneity scale (SHS)

- Modulo debate (SDSS Hogg et al 2005, Sylos Labini et al 2009; WiggleZ Scrimgeour et al, 2012), *some notion* of statistical homogeneity reached on 70–100 h^{-1}Mpc scales based on 2–point galaxy correlation function
- Here $\delta\rho/\rho \lesssim 0.07$ on scales $\gtrsim 100\,h^{-1}$Mpc (bounded)
- Why? Initial conditions: initial density perturbations amplified by sound waves below sound horizon at last scattering
- BAO scale close to SHS; in galaxy clustering BAO scale determination is treated in near linear regime in ΛCDM
- No direct evidence for FLRW spatial geometry below SHS

Fundamental cosmological question

- Is space expanding or are we moving?
- General relativity: Relative velocities ("boosts") only defined in Local Inertial Frames
- FLRW expansion differs from a simple Doppler law on large scales
- Standard model cosmology treats effects of inhomogeneity as perturbed FLRW + local boosts
- Here define *differential cosmic expansion* as distance–redshift law not of this type
- Differential cosmic expansion is natural in presence of inhomogeneities (feature of every exact solution of Einstein's equations with inhomogeneous dust source)

Cosmic Microwave Background dipole



- Special Relativity: motion in a thermal bath of photons

$$T' = \frac{T_0}{\gamma(1 - (v/c)\cos\theta')}$$

- $3.37\,$mK dipole: $v_{\text{Sun-CMB}} = 371\,\text{km s}^{-1}$ to $(264.14°, 48.26°)$; splits as $v_{\text{Sun-LG}} = 318.6\,\text{km s}^{-1}$ to $(106°, -6°)$ and $v_{\text{LG-CMB}} = 635 \pm 38\,\text{km s}^{-1}$ to $(276.4°, 29.3°) \pm 3.2°$

Peculiar velocity formalism

- Standard framework, FLRW + Newtonian perturbations, assumes peculiar velocity field

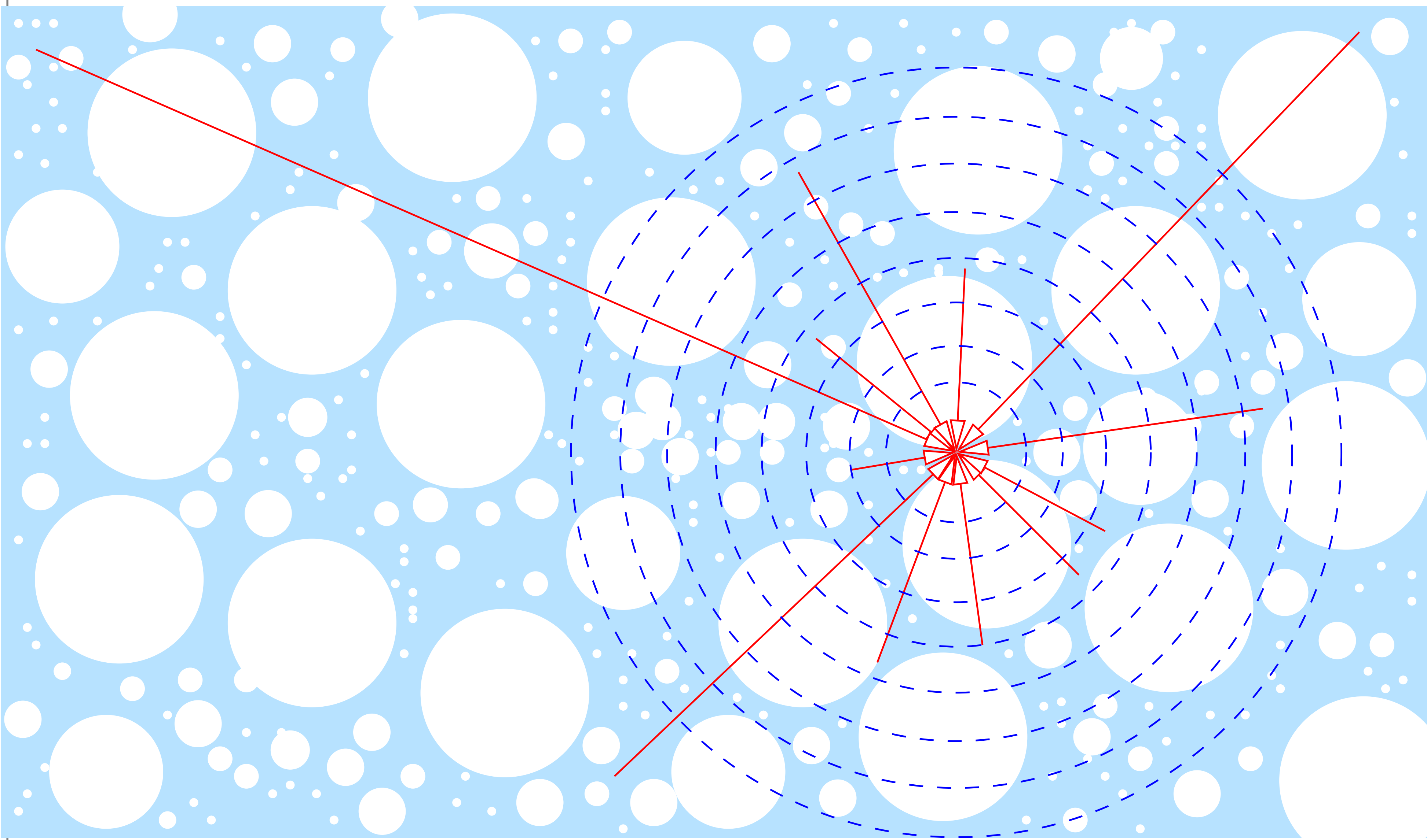
$$v_{\rm pec} = cz - H_0 r$$

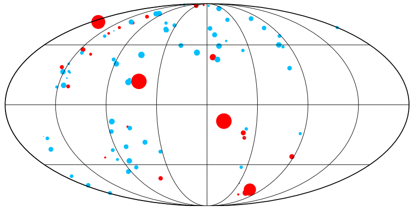
generated by

$$\mathbf{v}(\mathbf{r}) = \frac{H_0 \Omega_{M0}^{0.55}}{4\pi} \int \mathrm{d}^3\mathbf{r}'\, \delta_m(\mathbf{r}') \frac{(\mathbf{r}' - \mathbf{r})}{|\mathbf{r}' - \mathbf{r}|^3}$$

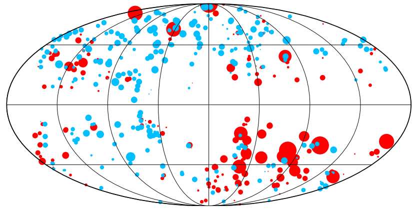
- 3 decades of debate on convergence of $\mathbf{v}(\mathbf{r})$ to velocity of LG w.r.t. CMB frame; Direction agreed, not amplitude or scale (Lavaux et al 2010; Bilicki et al 2011. . .)
- The debate continues: Hess & Kitaura arXiv:1412.7310, Springob et al arXiv:1511.04849, . . .

Apparent Hubble flow variation

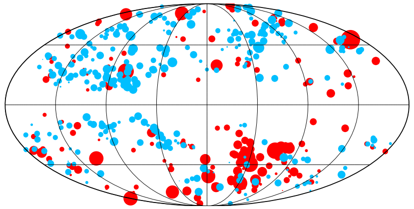




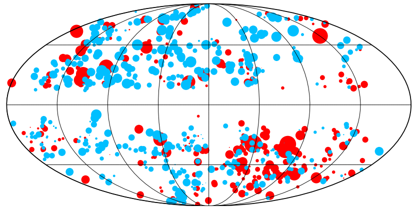
(a) 1: $0 - 12.5\ h^{-1}$ Mpc $N = 92$.



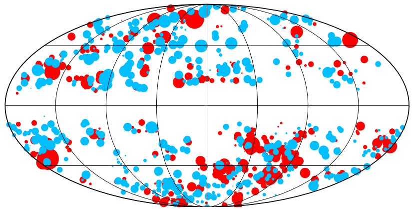
(b) 2: $12.5 - 25\ h^{-1}$ Mpc $N = 505$.



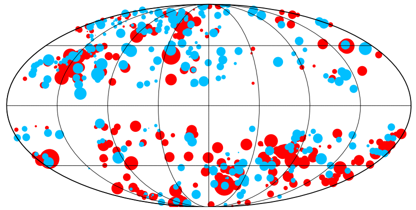
(c) 3: $25 - 37.5\ h^{-1}$ Mpc $N = 514$.



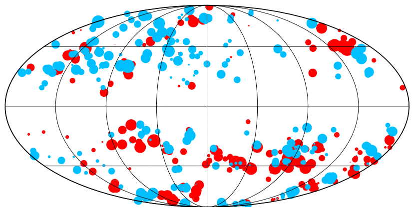
(d) 4: $37.5 - 50\ h^{-1}$ Mpc $N = 731$.



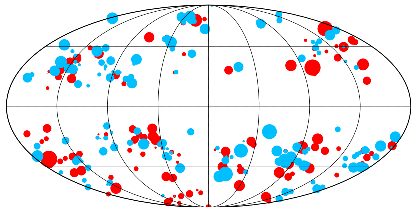
(e) 5: $50 - 62.5\ h^{-1}$ Mpc $N = 819$.



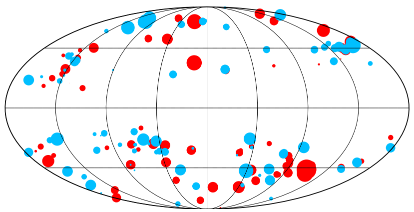
(f) 6: $62.5 - 75\ h^{-1}$ Mpc $N = 562$.



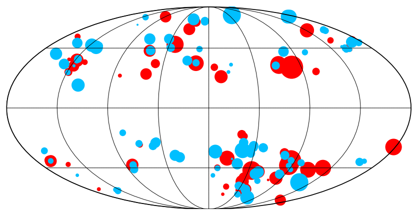
(g) 7: $75 - 87.5\ h^{-1}$ Mpc $N = 414$.



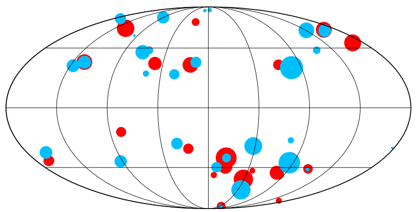
(h) 8: $87.5 - 100\ h^{-1}$ Mpc $N = 304$.



(i) 9: $100 - 112.5\ h^{-1}$ Mpc $N = 222$.

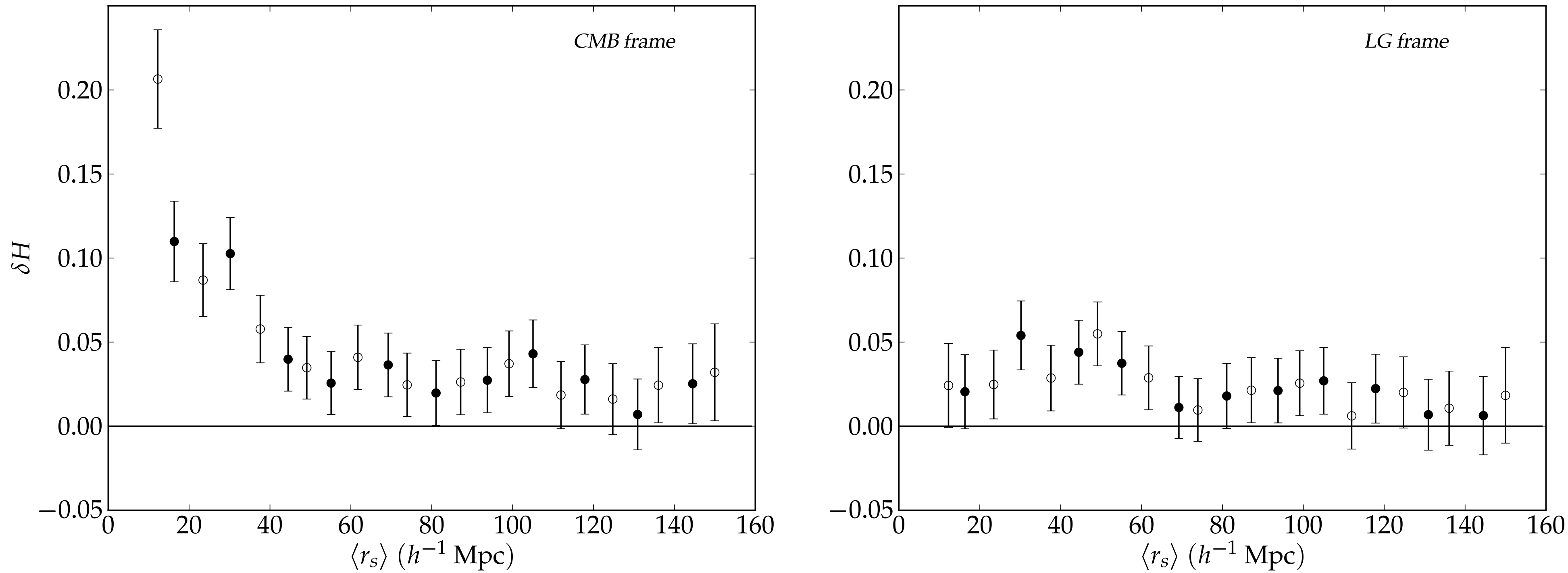


(j) 10: $112.5 - 156.25\ h^{-1}$ Mpc $N = 280$.



(k) 11: $156.25 - 417.4\ h^{-1}$ Mpc $N = 91$.

Radial variation $\delta H_s = (H_s - H_0)/H_0$



- Two choices of shell boundaries (closed and open circles); for each choice data points uncorrelated
- Result: Hubble expansion is very significantly more uniform in LG frame than in CMB frame: $\ln B > 5$.

Boosts and spurious monopole variance

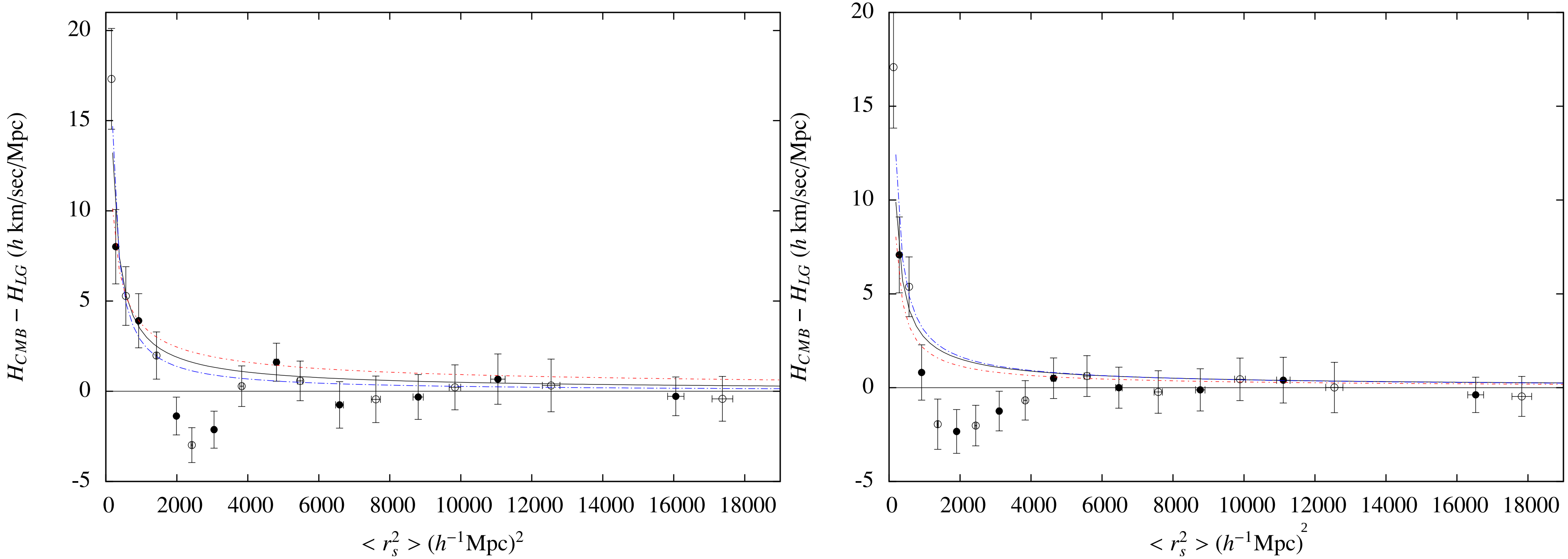
- H_s determined by linear regression in each shell

$$H_s = \left(\sum_{i=1}^{N_s} \frac{(cz_i)^2}{\sigma_i^2}\right)\left(\sum_{i=1}^{N_s} \frac{cz_i r_i}{\sigma_i^2}\right)^{-1},$$

- Any boost $cz_i \to cz_i' = c(\gamma - 1) + \gamma\left[cz_i + v\cos\phi_i(1 + z_i)\right] \simeq cz_i + v\cos\phi_i$, then for uniformly distributed data, linear terms cancel on opposite sides of sky

$$\begin{aligned} H_s' - H_s &\sim \left(\sum_{i=1}^{N_s} \frac{(v\cos\phi_i)^2}{\sigma_i^2}\right)\left(\sum_{i=1}^{N_s} \frac{cz_i r_i}{\sigma_i^2}\right)^{-1} \\ &= \frac{\langle(v\cos\phi_i)^2\rangle}{\langle cz_i r_i\rangle} \sim \frac{v^2}{2H_0\langle r_i^2\rangle} \end{aligned}$$

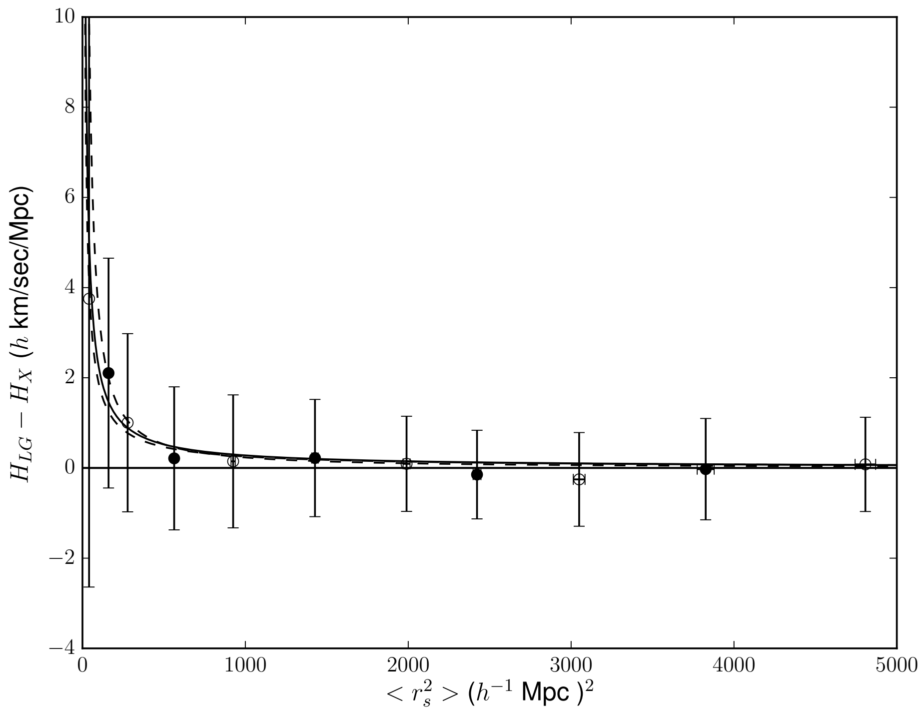
Boost offset and deviation



- $H_{\mathrm{CMB}} - H_{\mathrm{LG}}$: COMPOSITE (left); Cosmicflows-2 (right)
- Fits $\langle r_i^2 \rangle^{-1}$ relation except for $40\,h^{-1} \lesssim r \lesssim 60\,h^{-1}$Mpc (COMPOSITE); or $30\,h^{-1} \lesssim r \lesssim 67\,h^{-1}$Mpc (CF2)
- Broadening of “nonkinematic” region in CF2 consistent with nonremoval of Malmquist distribution biases

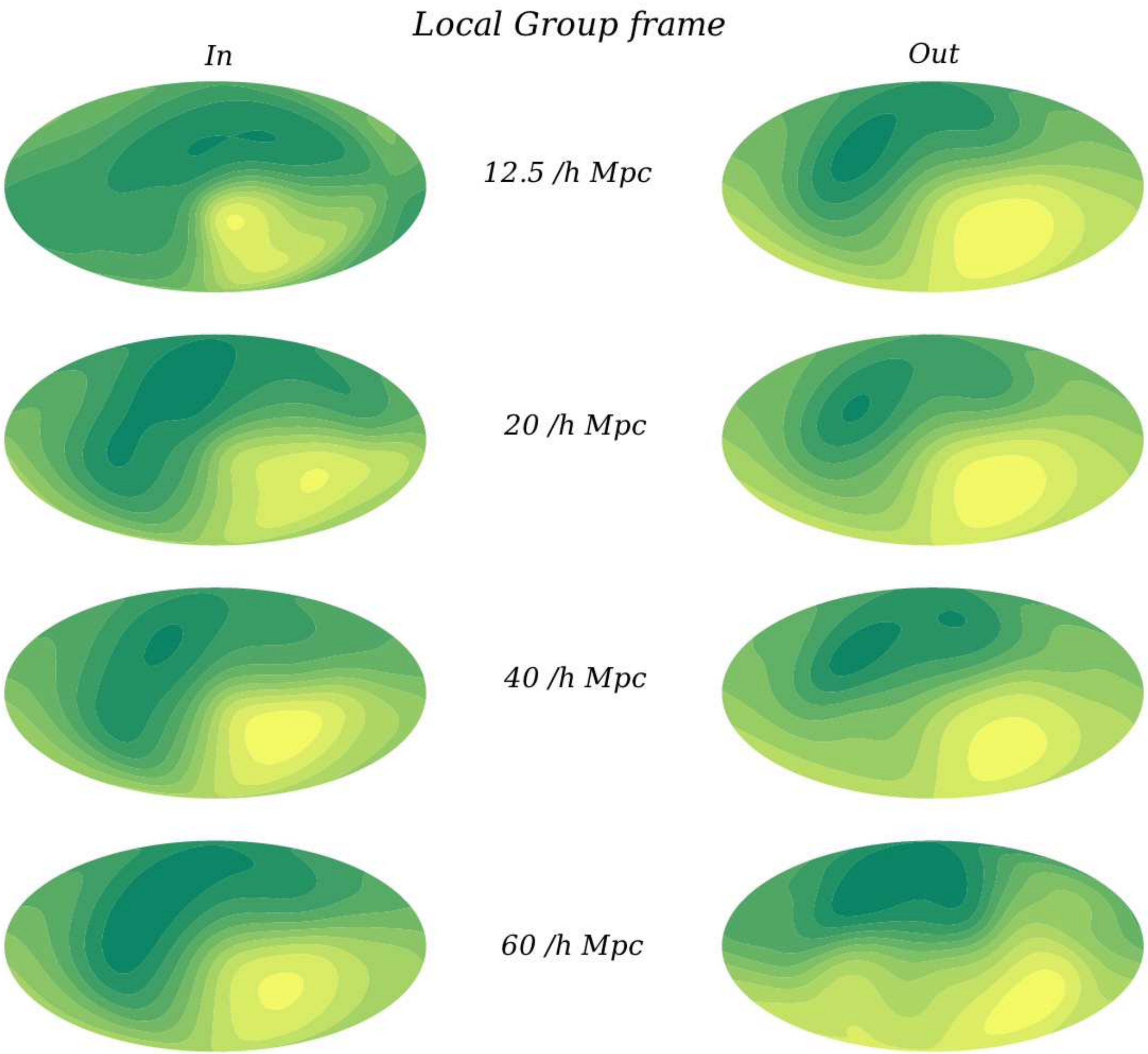
Minimum monopole rest frame?

JH McKay & DLW, arXiv:1503.04192 **= MNRAS 457 (2016) 3285**

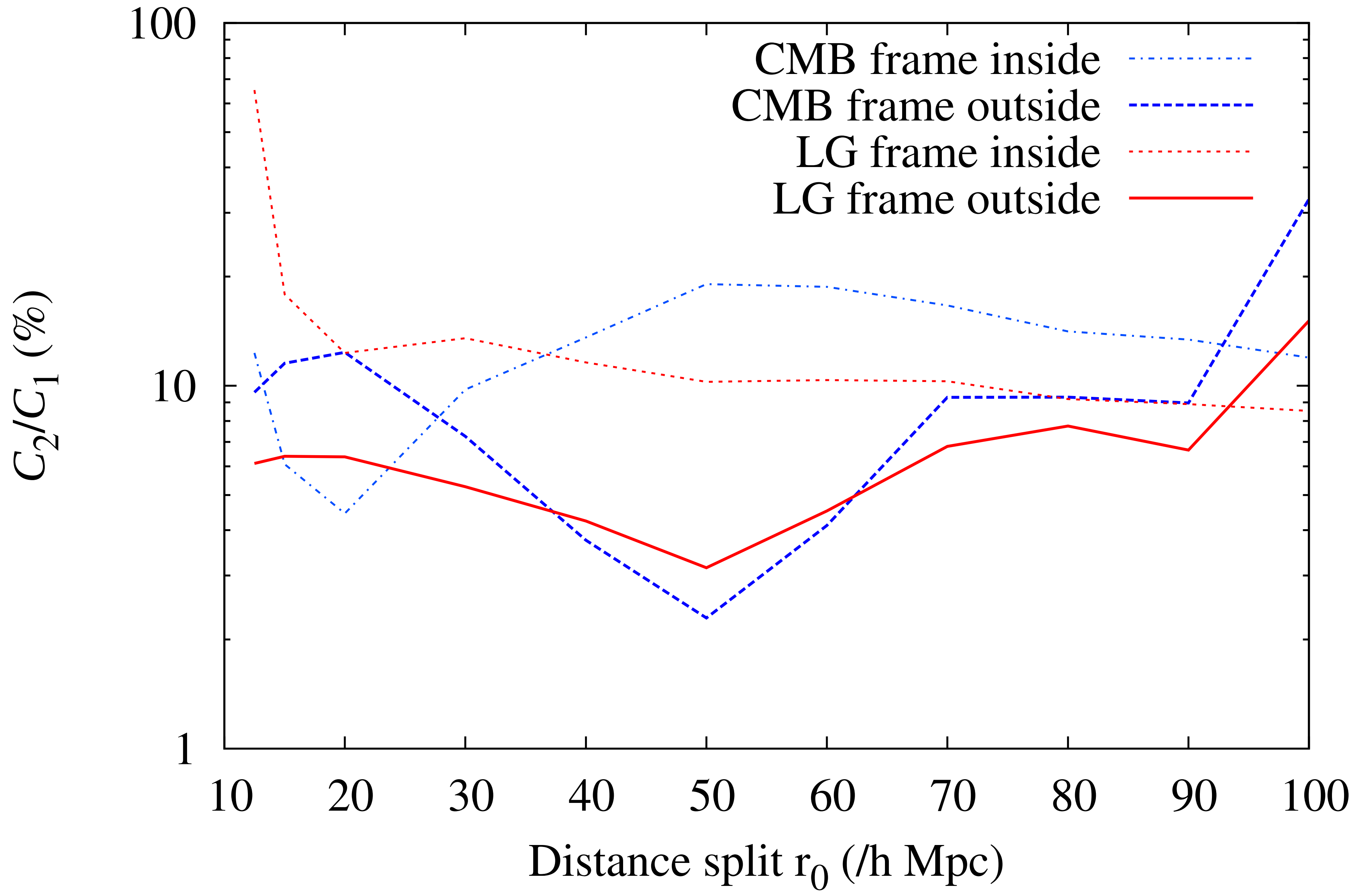


- Determine local boosts with fit to $H_{\rm LG} - H_{\rm X} = A\langle r_i^2 \rangle^p$ giving $p = -1$, $A = v^2/(2H_0)$ within uncertainties
- Minimize $\chi^2_{\rm a}$ within this class; best fit: $v = 122.5\,\text{km s}^{-1}$ to $(\ell, b) = (60^\circ, -4^\circ)$ but also consistent with zero
- Galactic plane: Zone of Avoidance degeneracy

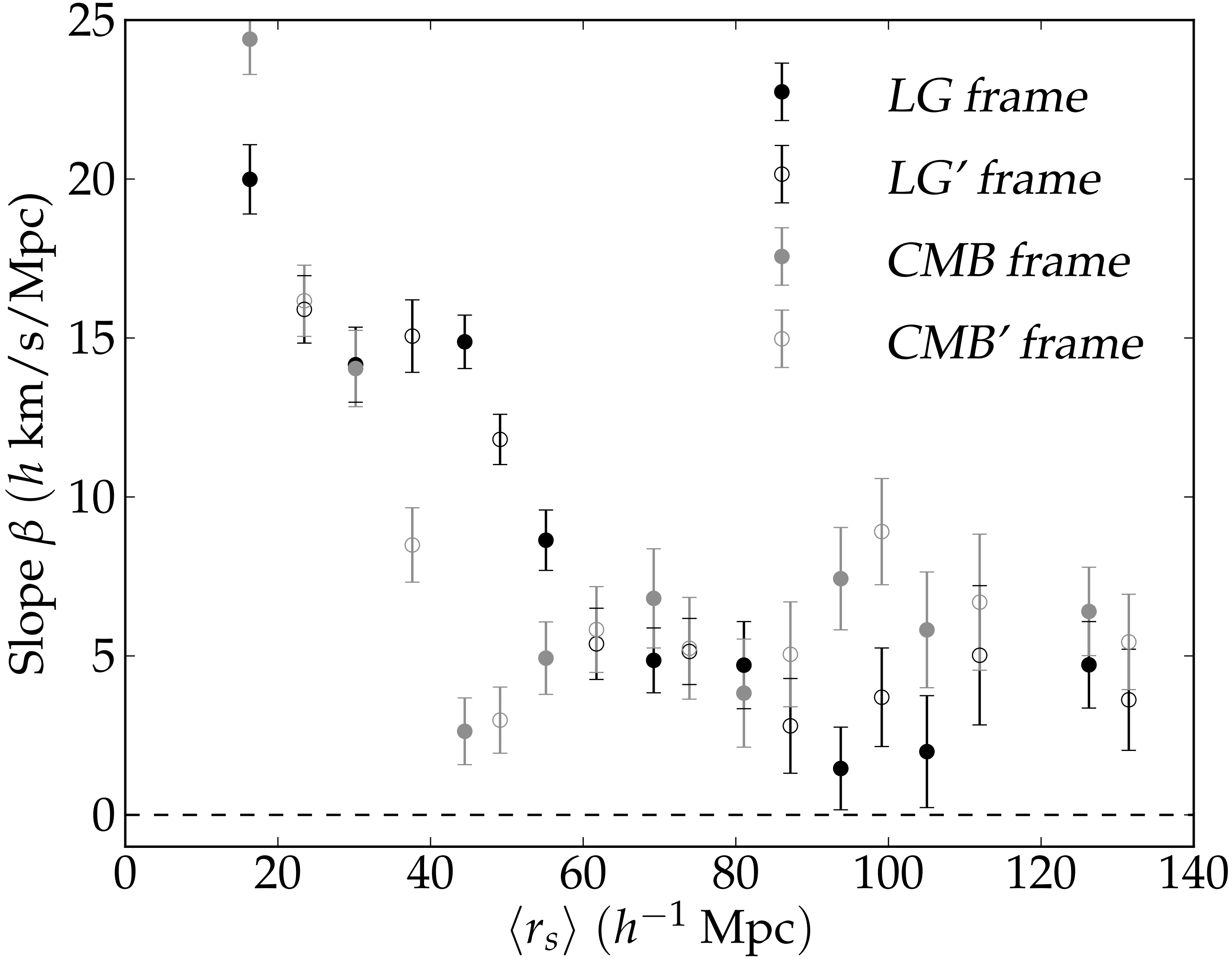
Angular variation: LG frame



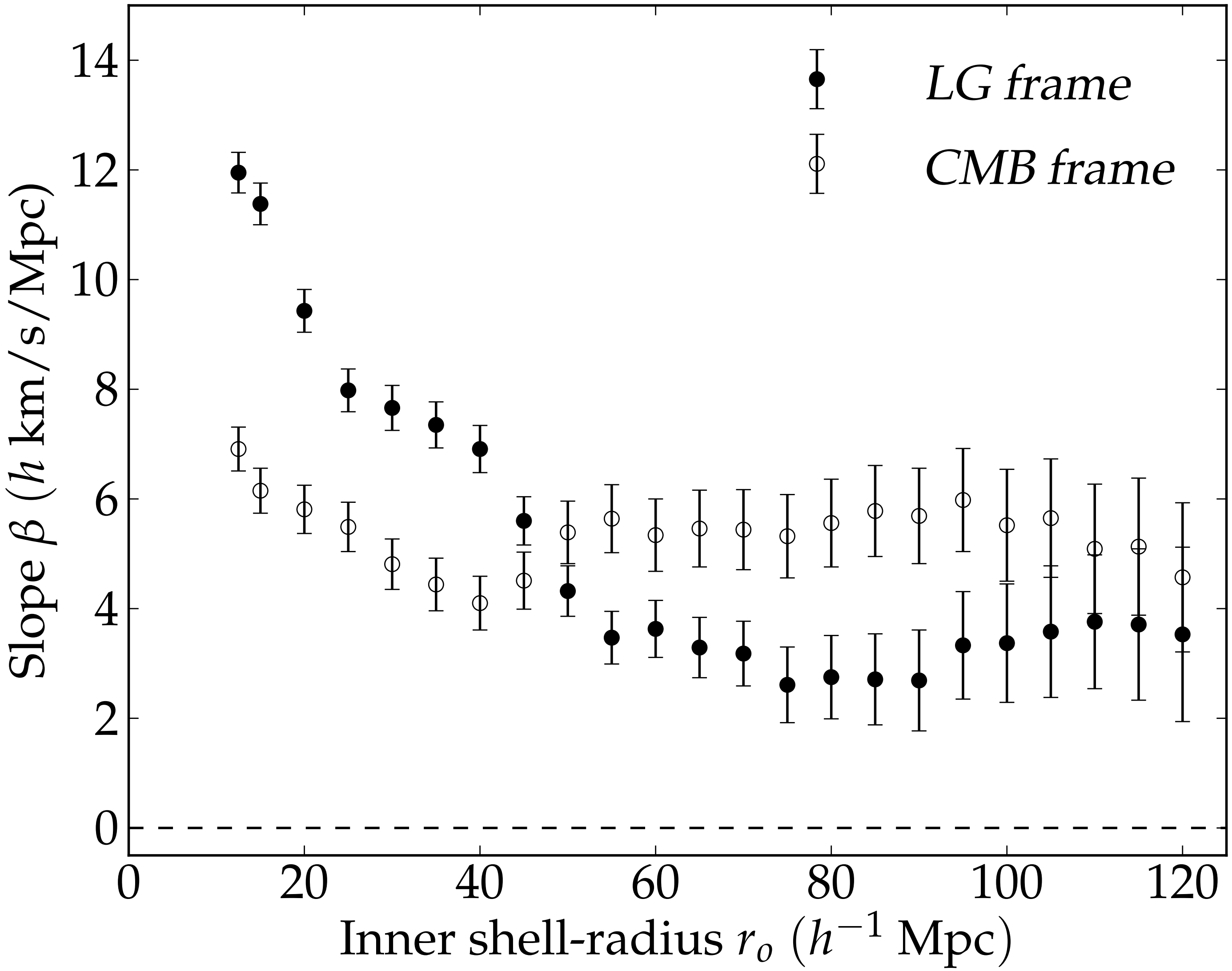
Angular variation quadrupole/dipole ratios



Value of β in $\frac{cz}{r} = H_0 + \beta \cos\phi$



Value of β in $\frac{cz}{r} = H_0 + \beta\cos\phi$



Model small scale differential expansion

- Use exact inhomogeneous solutions of Einstein's equations for structures on $\lesssim 70\,h^{-1}$Mpc scales
- Use asymptotic Planck normalized FLRW model on larger scales: *effective* model for large scale light propagation
- Non-Copernican large void solution for dark energy NOT considered here – large scale FLRW expansion with dark energy taken as effective model for light propagation from CMB
- Use Szekeres model: most general dust solution, reduces to spherically symmetric inhomoneity (Lemaître–Tolman–Bondi (LTB) model) in a limit
- Trace rays from CMB and mock COMPOSITE catalogues

Ray tracing: Szekeres model (1975)

$$
\begin{aligned}
\mathrm{d}s^2 &= c^2\mathrm{d}t^2 - \frac{\left(R' - R\frac{\mathcal{E}'}{\mathcal{E}}\right)^2}{1-k}\mathrm{d}r^2 - \frac{R^2}{\mathcal{E}^2}(\mathrm{d}p^2 + \mathrm{d}q^2), \\
\mathcal{E}(r,p,q) &= \frac{1}{2S}(p^2+q^2) - \frac{P}{S}p - \frac{Q}{S}q + \frac{P^2}{2S} + \frac{Q^2}{2S} + \frac{S}{2}, \\
\dot{R}^2 &= -k(r) + \frac{2M(r)}{R} + \frac{1}{3}\Lambda R^2, \\
t &- t_B(r) = \int_0^R \mathrm{d}\widetilde{R}\left[-k + 2M/\widetilde{R} + \frac{1}{3}\Lambda \widetilde{R}^2\right]^{-1/2}, \\
\kappa\rho &= \frac{2\left(M' - 3M\mathcal{E}'/\mathcal{E}\right)}{R^2\left(R' - R\mathcal{E}'/\mathcal{E}\right)}.
\end{aligned}
$$

where $' \equiv \partial/\partial r$, $\dot{} \equiv \partial/\partial t$, $R = R(t,r)$, $k = k(r) \leq 1$, $S = S(r)$, $P = P(r)$, $Q = Q(r)$, $M = M(r)$. Above eqns satisfied but functions are otherwise arbitrary. We take $t_B(r) = 0$.

Ray tracing: Szekeres model (1975)

Define $\quad p - P = S \cot\frac{\theta}{2}\cos\phi, \qquad q - Q = S\cot\frac{\theta}{2}\sin\phi.$

Then $\quad \mathrm{d}s^2 = c^2\mathrm{d}t^2 - \frac{1}{1-k}\left[R' + \frac{R}{S}\left(S'\cos\theta + N\sin\theta\right)\right]^2 \mathrm{d}r^2$

$$- \left[\frac{S'\sin\theta + N\left(1-\cos\theta\right)}{S}\right]^2 R^2\mathrm{d}r^2 - \left[\frac{\left(\partial_\phi N\right)\left(1-\cos\theta\right)}{S}\right]^2 R^2 dr^2$$

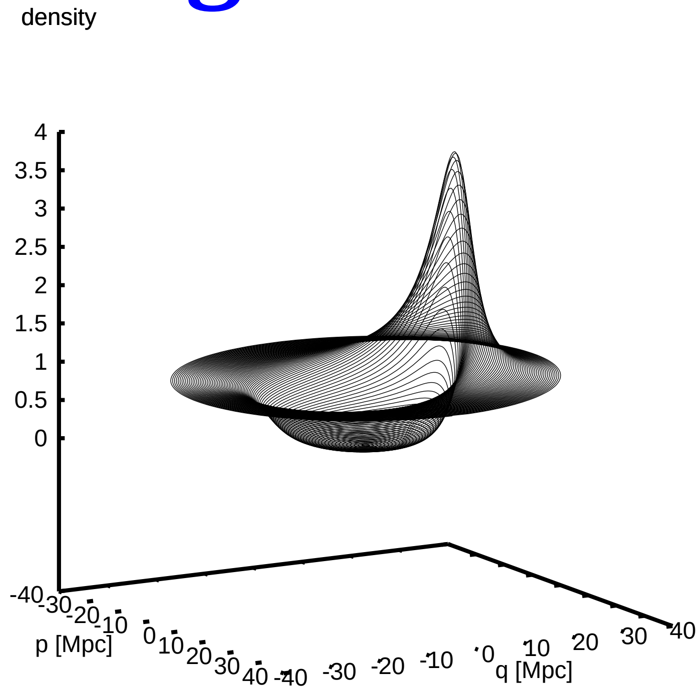
$$+ \frac{2\left[S'\sin\theta + N\left(1-\cos\theta\right)\right]}{S} R^2\mathrm{d}r\,\mathrm{d}\theta$$

$$- \frac{2(\partial_\phi N)\sin\theta\left(1-\cos\theta\right)}{S} R^2\mathrm{d}r\,\mathrm{d}\phi - R^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2),$$

where $\quad N(r,\phi) \equiv \left(P'\cos\phi + Q'\sin\phi\right)$

$$\frac{\mathcal{E}'}{\mathcal{E}} = \frac{-1}{S}\left[S'\cos\theta + N\sin\theta\right]$$

Ray tracing: Szekeres model

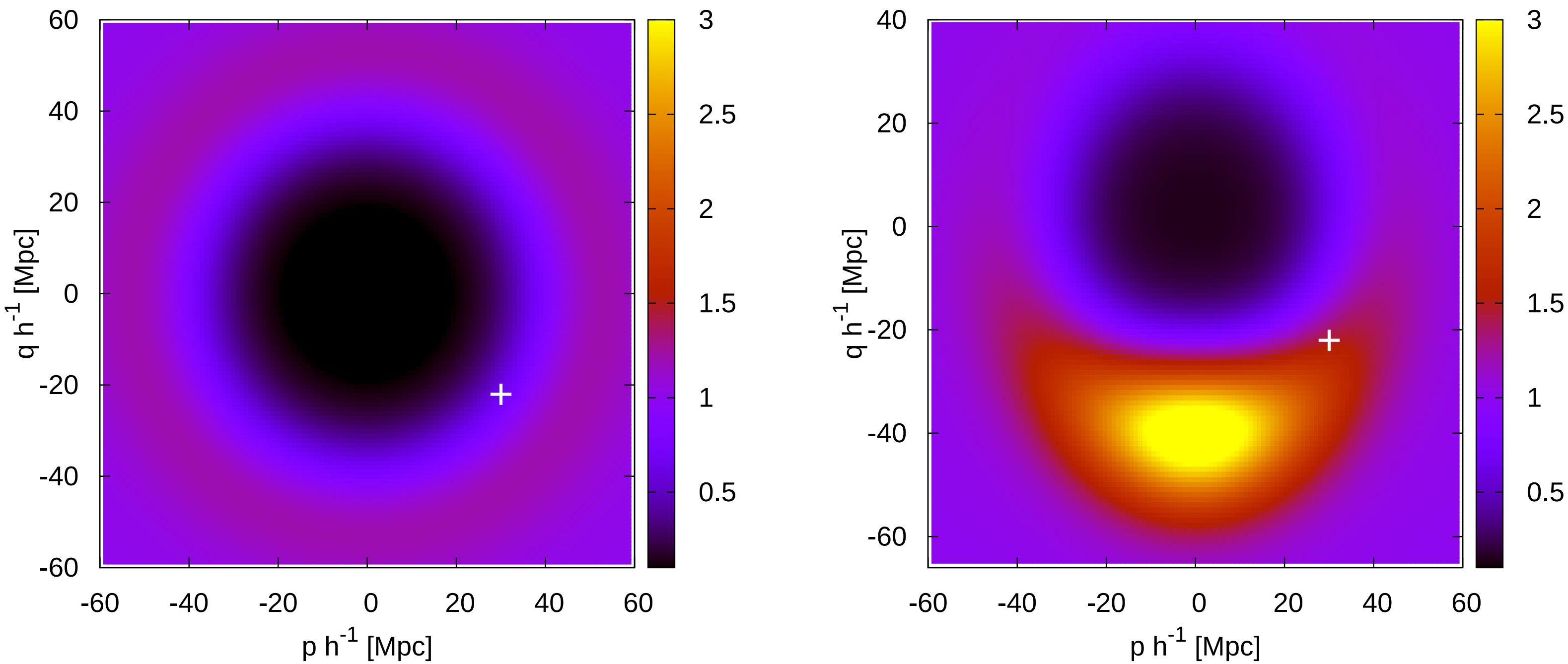


Simplify to $P = Q = 0$; $\quad S = r \left(\frac{r}{1h^{-1}\mathrm{Mpc}} \right)^{\alpha - 1}$

$$\begin{aligned} M &= M_0 r^3 \left[1 + \delta_M(r) \right], \\ \delta_M(r) &= \frac{1}{2}\delta_0 \left(1 - \tanh \frac{r - r_0}{2\Delta r} \right), \end{aligned}$$

$-1 \leq \delta_0 < 0$ underdensity at $r \to 0$; $\delta_M \to 0$ as $r \to \infty$.

LTB and Szekeres profiles



- Fix $\Delta r = 0.1 r_0$, $\varphi_{obs} = 0.5\pi$
- LTB parameters: $\alpha = 0$, $\delta_0 = -0.95$, $r_0 = 45.5\ h^{-1}$ Mpc; $r_{obs} = 28\, h^{-1}$Mpc, $\vartheta_{obs} =$ any
- Szekeres parameters: $\alpha = 0.86$, $\delta_0 = -0.86$; $r_{obs} = 38.5\ h^{-1}$ Mpc; $r_{obs} = 25 h^{-1}$ Mpc, $\vartheta_{obs} = 0.705\pi$.

Szekeres model ray tracing constraints

- Require Planck satellite normalized FLRW model on scales $r \gtrsim 100\, h^{-1}$Mpc; i.e., spatially flat, $\Omega_m = 0.315$ and $H_0 = 67.3$ km/s/Mpc
- CMB temperature has a maximum $T_0 + \Delta T$, where

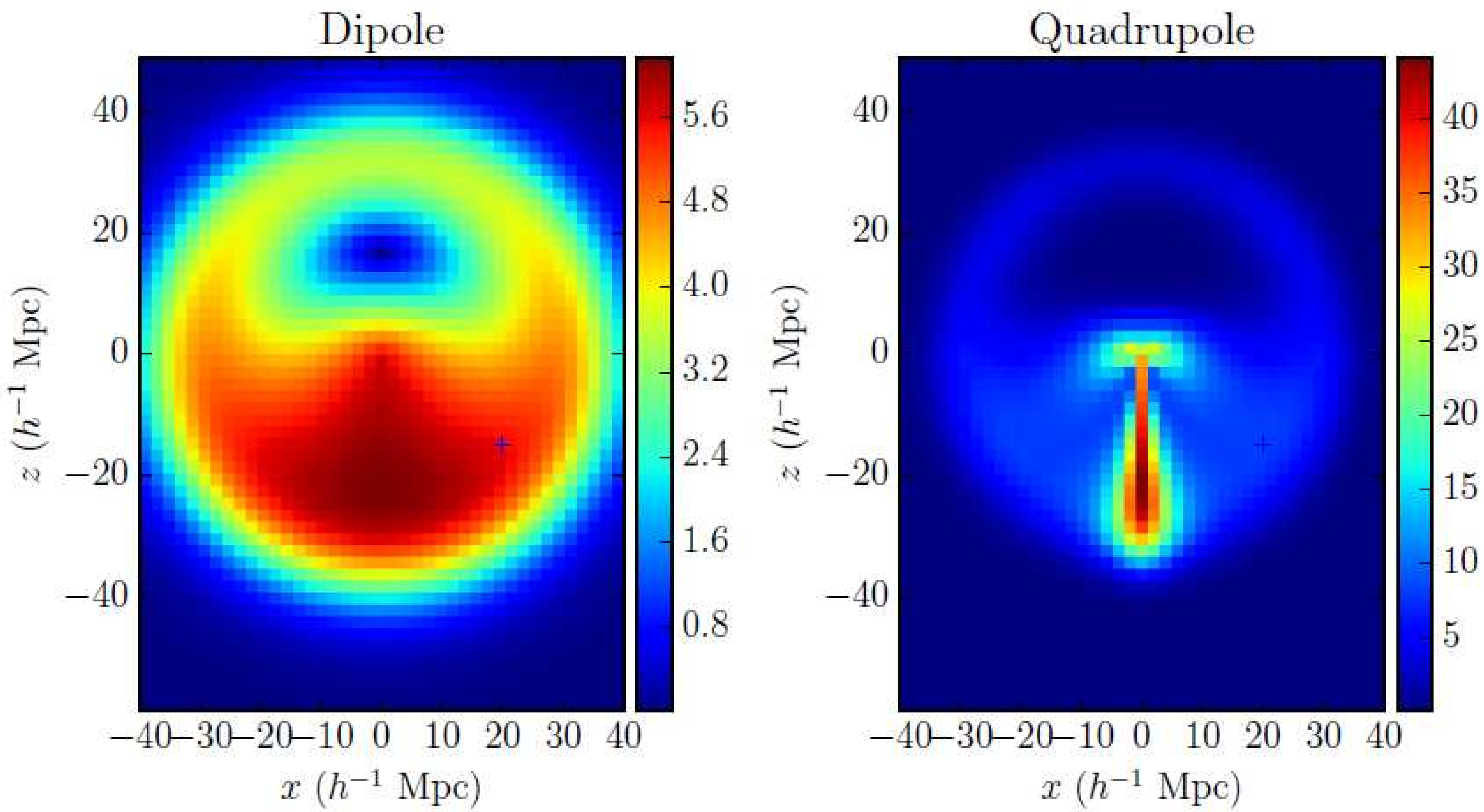
$$\Delta T(\ell = 276.4^\circ, b = 29.3^\circ) = 5.77 \pm 0.36 \text{ mK},$$

  matching dipole amplitude, direction in LG frame
- CMB quadrupole anisotropy lower than observed

$$C_{2,CMB} < 242.2^{+563.6}_{-140.1}\ \mu\text{K}^2.$$

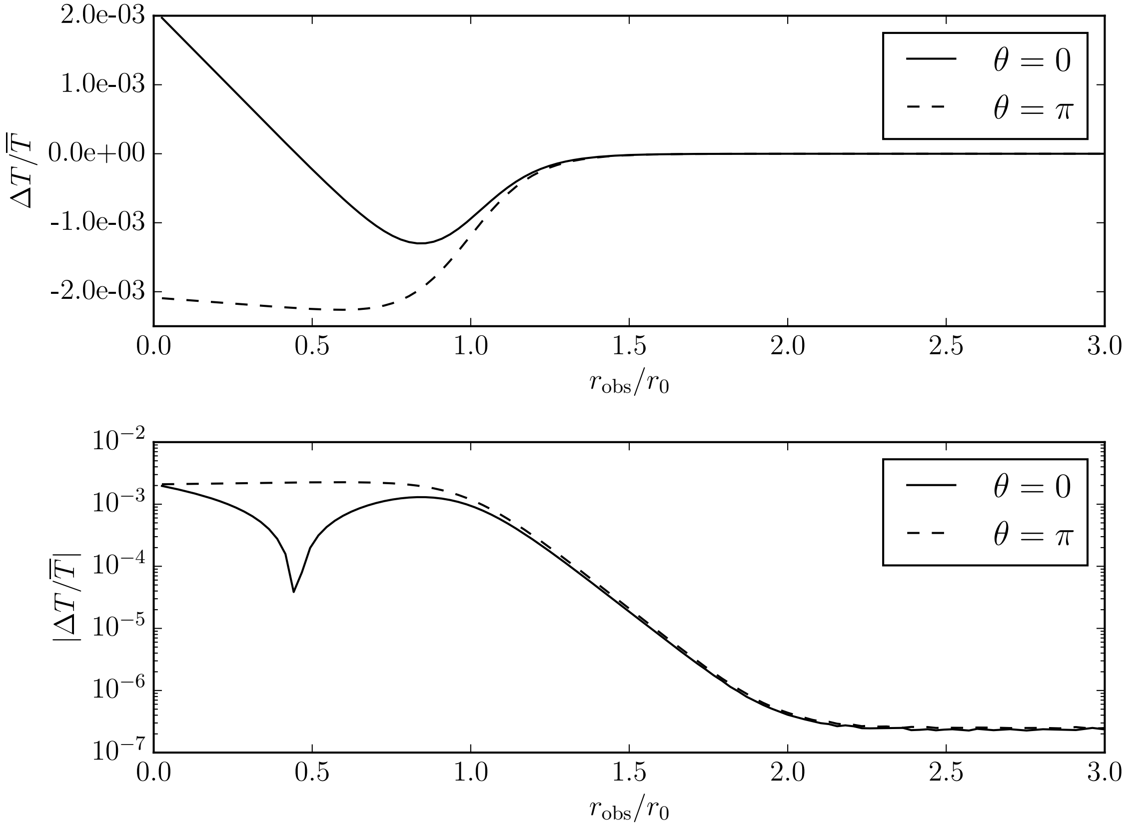
- Hubble expansion dipole (LG frame) matches COMPOSITE one at $z \to 0$, if possible up to $z \sim 0.045$
- Match COMPOSITE quadrupole similarly, if possible

CMB dipole, quadrupole examples



- Generate $z_{\rm ls}(\hat{\mathbf{n}})$ for each gridpoint
- $T = T_{\rm ls}/(1+z_{\rm ls})$; $(T_{\rm max} - T_{\rm min})/2$ left (mK); C_2 right (μK^2)

Peculiar potential not Rees–Sciama



- Rees–Sciama (and ISW) consider photon starting and finishing from *average* point
- Across structure $|\Delta T|/T \sim 2 \times 10^{-7}$
- Inside structure $|\Delta T|/T \sim 2 \times 10^{-3}$

Local expansion variation methodology

$$H_0(\ell, b, z) = \frac{\sum_i H_i w_{d,i} w_{z,i} w_{\theta,i}}{\sum_i w_{d,i} w_{z,i} w_{\theta,i}}, \qquad \langle H_0 \rangle = \frac{1}{4\pi} \int \mathrm{d}\Omega \; H_0(\ell, b, z)$$

$$\zeta_i = z_i + \frac{1}{2}(1 - q_0) z_i^2 - \frac{1}{6}(1 - q_0 - 3q_0^2 + j_0) z_i^3$$

$$H_i = c\zeta_i / d_i, \qquad w_{d,i} = c\zeta_i d_i / (\Delta d_i)^2,$$

$$w_{z,i} = \frac{1}{\sqrt{2\pi}\sigma_z} \exp\left[-\frac{1}{2}\left(\frac{z - z_i}{\sigma_z}\right)^2\right],$$

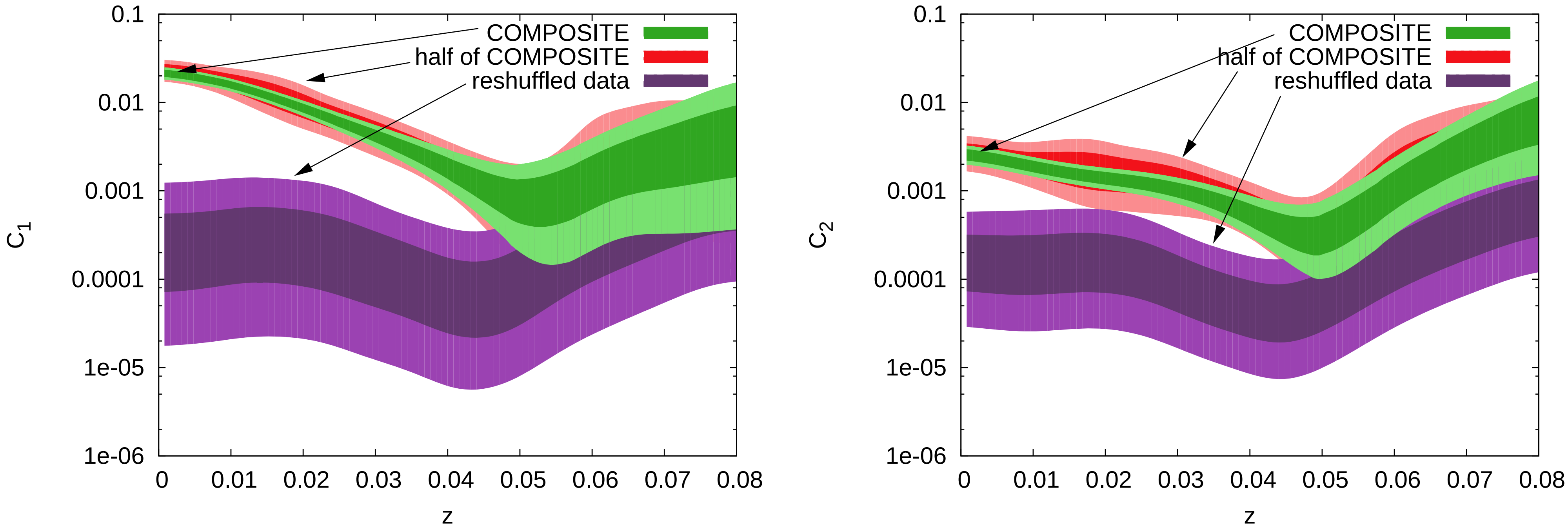
$$w_{\theta,i} = \frac{1}{\sqrt{2\pi}\sigma_\theta} \exp\left[-\frac{1}{2}\left(\frac{\theta_i}{\sigma_\theta}\right)^2\right],$$

$q_0 = -0.5275$, $j_0 = 1$ ($\Omega_m = 0.315$ ΛCDM); $\sigma_z = 0.01$,
$\sigma_\theta = 25^\circ$, $\theta_i =$ angle between each source and boost apex.

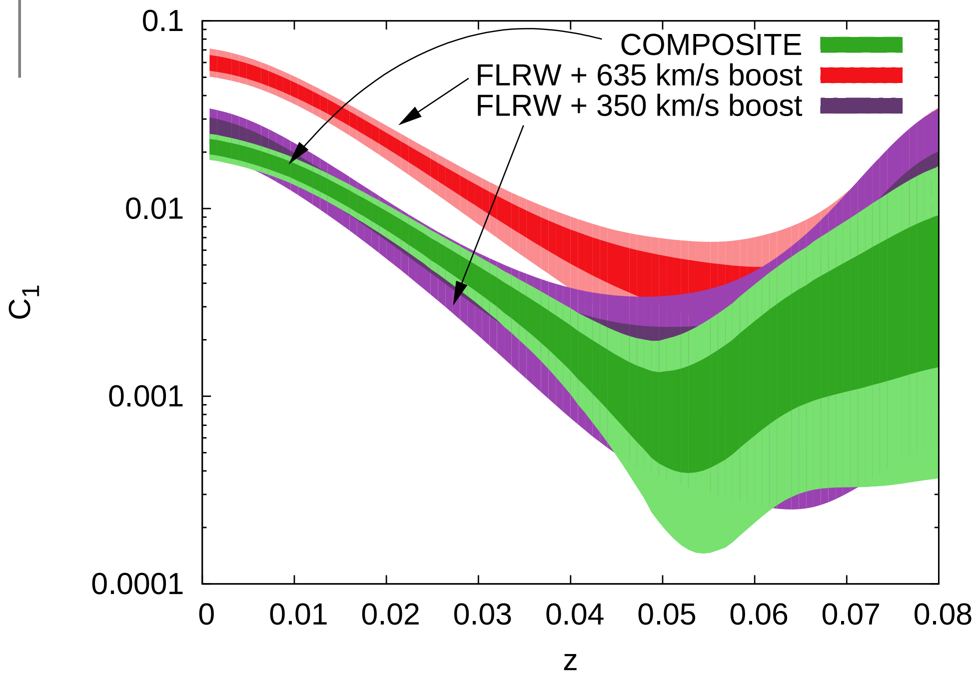
Method and COMPOSITE data

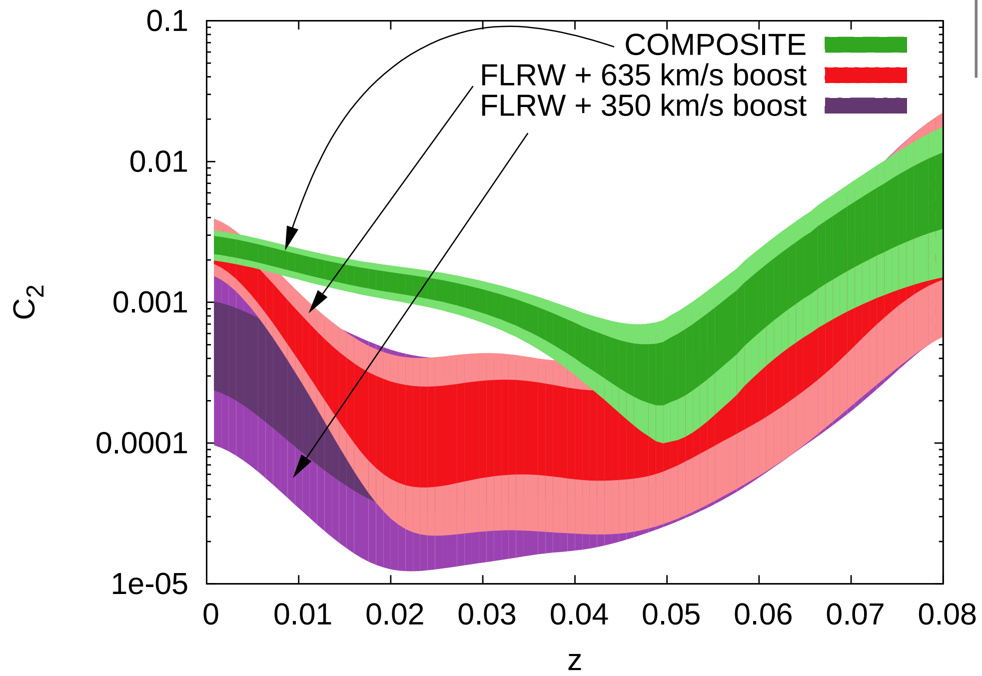
$$\frac{\Delta H_0}{\langle H_0 \rangle} = \frac{H_0(l,b,z) - \langle H_0 \rangle}{\langle H_0 \rangle} = \sum_{l,m} a_{lm} Y_{lm},$$

Expand fractional Hubble expansion variation in multipoles, evaluate angular power spectrum: $C_\ell = \frac{1}{2\ell+1} \sum_m |a_{\ell m}|^2$



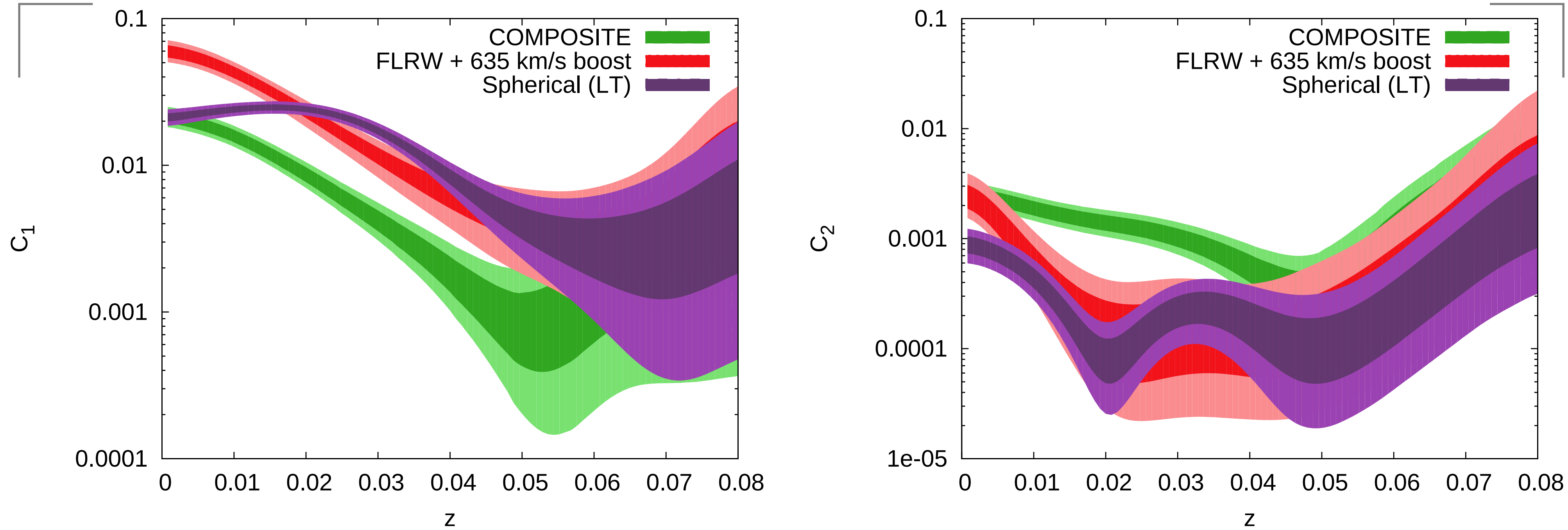
FLRW model in CMB frame + LG boost





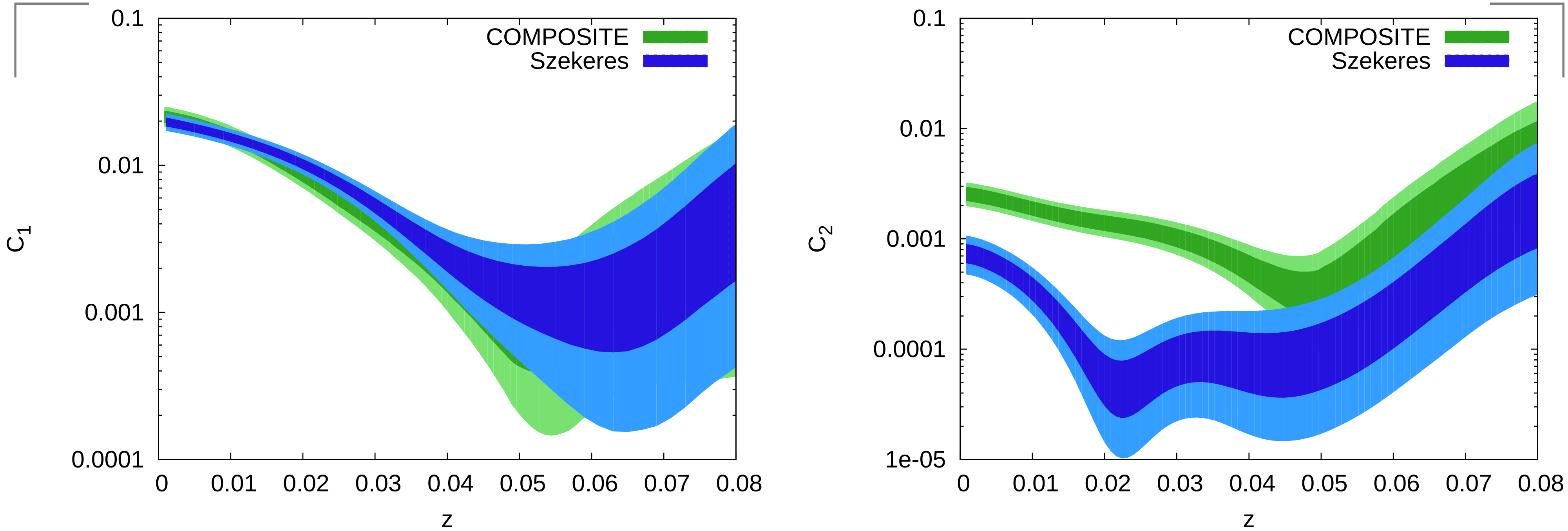
- Result of 100,000 mock COMPOSITE catalogues with same distance uncertainties
- For standard kinematic CMB dipole, H_0 dipole too high over all $z < 0.045$; quadrupole OK only at $z \to 0$
- Dipole result: means bulk flow in standard approach

LTB fit: H dipole, quadrupole



- LTB dipole matches only at $z \to 0$, increases to close to FLRW plus boost case for larger z
- Smaller insignificant quadrupole
- Differential expansion radially only; effective point symmetry on scale larger than inhomogeneity

Best fit Szekeres: H dipole, quadrupole

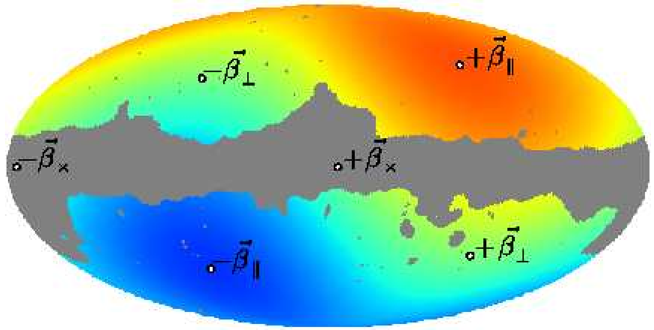


- Szekeres matches dipole on whole $z < 0.045$ range
- Smaller insignificant quadrupole
- Note $C_{2,CMB} = 8.26\ \mu\mathrm{K}^2$ 30 times smaller than observed
- Possible additional smaller amplitudes structures can add quadrupole (future work)

Association with known structures

- Our galaxy is in a local void complex on a filamentary sheet (Tully et al 2008) joined to Virgo cluster. Dominant overdensity $23\,h^{-1}$Mpc wide "Great Attractor":
  - Near side Centaurus, $z_{\rm LG} = 0.0104 \pm 0.0001$, $(\ell, b) = (302.4°, 21.6°)$
  - Far side Norma, $z_{\rm LG} = 0.0141 \pm 0.0002$, $(\ell, b) = (325.3°, -7.3°)$
- Szekeres $\delta\rho/\rho > 2$ ellipsoidal overdense region, spans $0.003 \lesssim z_{\rm LG} \lesssim 0.013$ (or $16\,h^{-1} \lesssim D_L \lesssim 53\,h^{-1}$Mpc) and angles $220° < \ell < 320°$, $-60° < b < 40°$
- Centaurus lies inside; Norma just outside
- Adding structures at larger distances (Perseus–Pisces) will change far side alignment

Systematics for CMB



- Define nonkinematic foreground CMB anisotropies by

$$\Delta T_{\text{nk-hel}} = \frac{T_{\text{model}}}{\gamma_{\text{LG}}(1-\boldsymbol{\beta}_{\text{LG}}\cdot\hat{\mathbf{n}}_{\text{hel}})} - \frac{T_0}{\gamma_{\text{CMB}}(1-\boldsymbol{\beta}_{\text{CMB}}\cdot\hat{\mathbf{n}}_{\text{hel}})}$$

$$T_{\text{model}} = \frac{T_{\text{dec}}}{1+z_{\text{model}}(\hat{\mathbf{n}}_{\text{LG}})}, \quad T_0 = \frac{T_{\text{dec}}}{1+z_{\text{dec}}}$$

$z_{\text{model}}(\hat{\mathbf{n}}_{\text{LG}})$ = anisotropic Szekeres LG frame redshift;
T_0 = present mean CMB temperature

- Constrain $\frac{T_{\text{model}}}{\gamma_{\text{LG}}(1-\boldsymbol{\beta}_{\text{LG}}\cdot\hat{\mathbf{n}}_{\text{hel}})} - T_{\text{obs}}$ by Planck with sky mask

Large angle CMB anomalies?

Anomalies (significance increased after Planck 2013):

- power asymmetry of northern/southern hemispheres
- alignment of the quadrupole and octupole etc;
- low quadrupole power;
- parity asymmetry; . . .

Critical re-examination required; e.g.

- light propagation through Hubble variance dipole foregrounds may differ subtly from Lorentz boost dipole
- dipole subtraction is an integral part of the map-making; is galaxy correctly cleaned?
- Freeman et al (2006): 1–2% change in dipole subtraction may resolve the power asymmetry anomaly.

Planck results arXiv:1303.5087

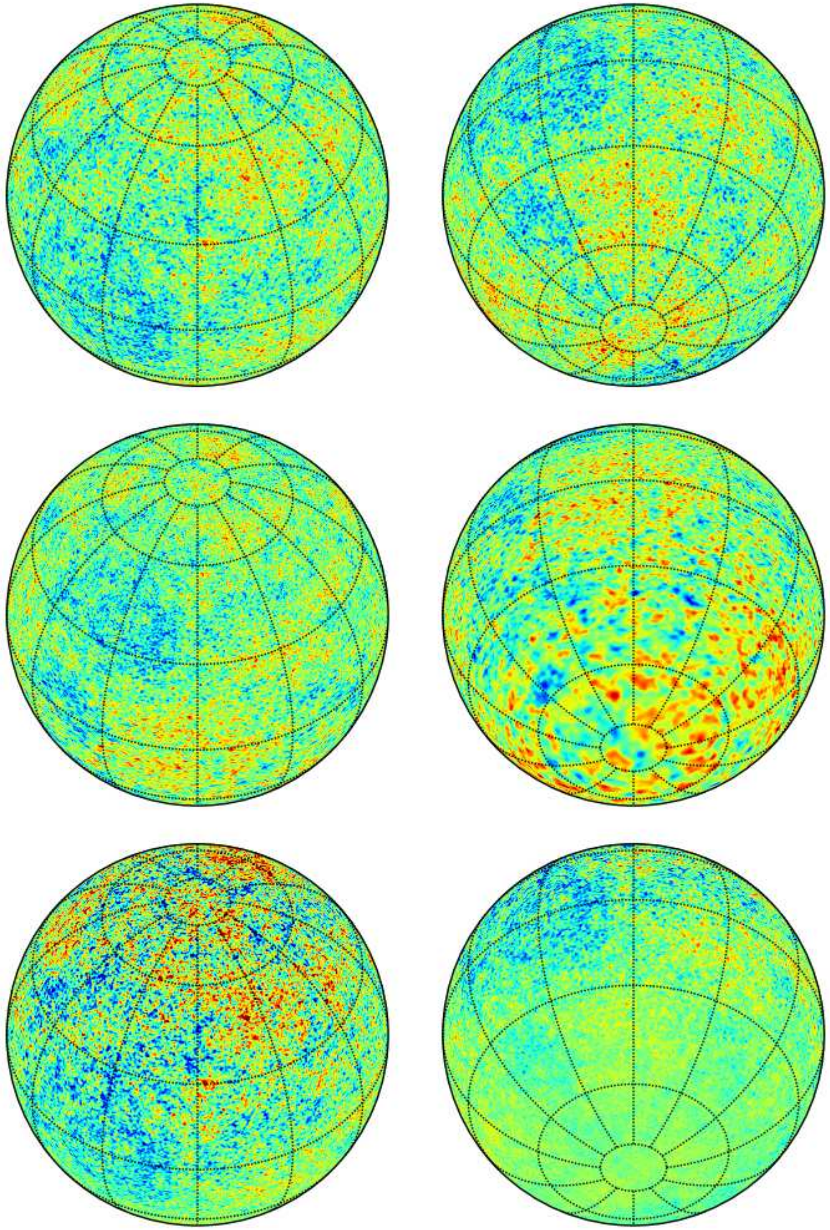
Boost dipole from
second order effects

Original

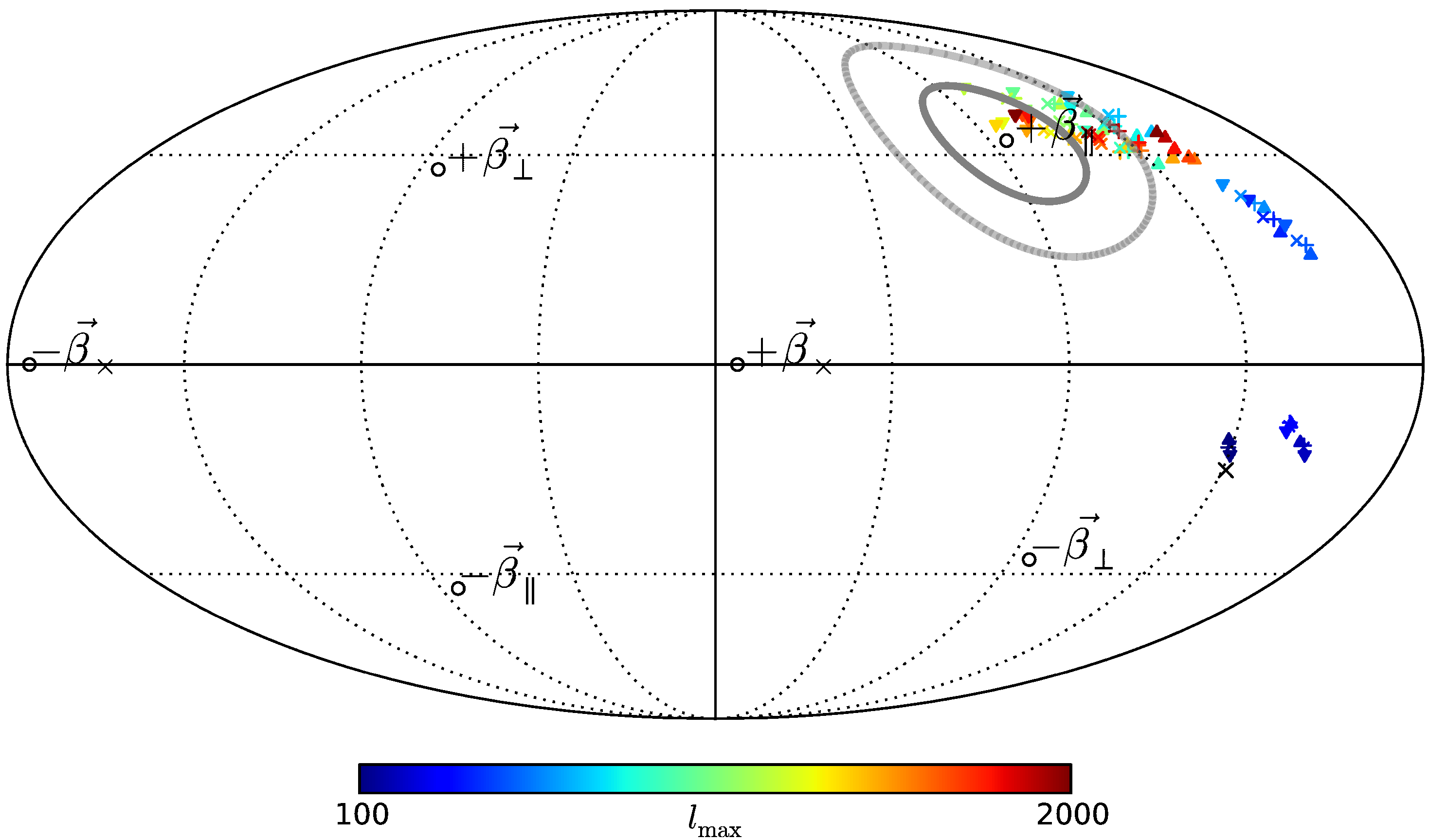
Aberration
(Exaggerated)

Modulation
(Exaggerated)

Eppur si muove?



Planck Doppler boosting 1303.5087

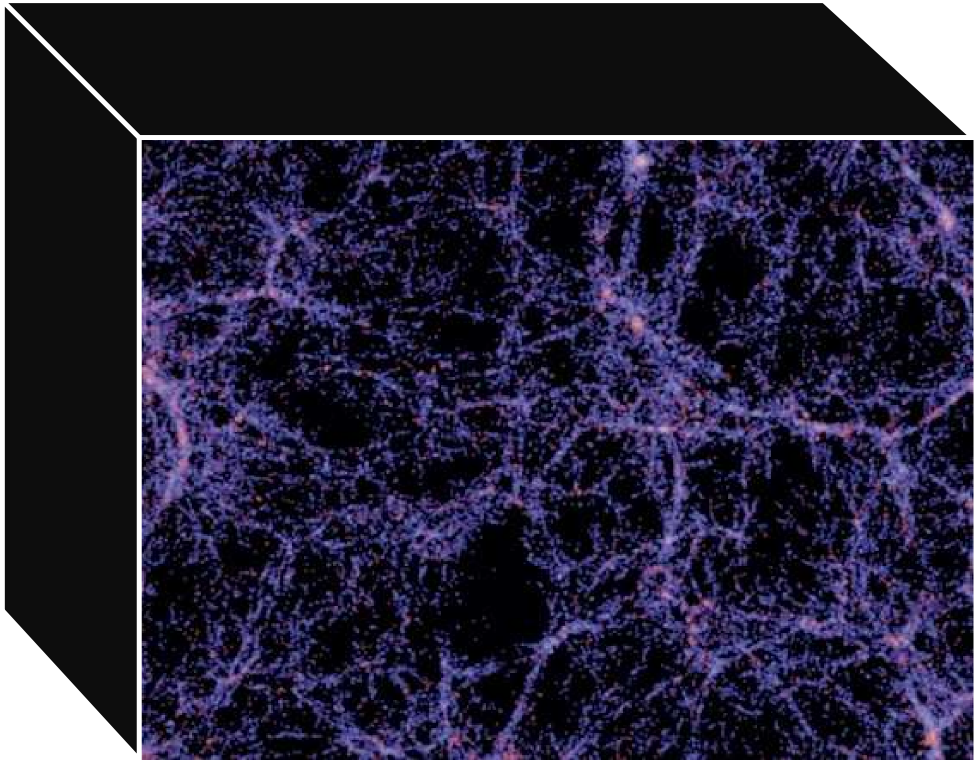


- Dipole direction consistent with CMB dipole $(\ell, b) = (264^\circ, 48^\circ)$ for small angles, $l_{\rm min} = 500 < l < l_{\rm max} = 2000$
- When $l < l_{\rm max} = 100$, shifts to WMAP power asymmetry modulation dipole $(\ell, b) = (224^\circ, -22^\circ) \pm 24^\circ$

Non-kinematic dipole in radio surveys

- Effects of aberration and frequency shift also testable in large radio galaxy surveys (number counts)
- Rubart and Schwarz, arXiv:1301.5559, have conducted a careful analysis to resolve earlier conflicting claims of Blake and Wall (2002) and Singal (2011)
- Rubart & Schwarz result: kinematic origin of radio galaxy dipole ruled out at 99.5% confidence
- Our smoothed Hubble variance dipole in LG frame $(180+\ell_d, -b_d) = (263^\circ \pm 6^\circ, 39^\circ \pm 3^\circ)$ for $r > r_o$ with $20\,h^{-1} \lesssim r_o \lesssim 45\,h^{-1}$Mpc, or $(\text{RA}, \text{dec}) = (162^\circ \pm 4^\circ, -14^\circ \pm 3^\circ)$, lies within error circle of NVSS survey dipole found by Rubart & Schwarz, $(\text{RA}, \text{dec}) = (154^\circ \pm 21^\circ, -2^\circ \pm 21^\circ)$

Back to backreaction...



- Need to consider relative position of observers over scales of tens of Mpc over which $\delta\rho/\rho \sim -1$.
- GR is a local theory: gradients in spatial curvature and gravitational energy can lead to calibration differences between our rulers & clocks and volume average ones

What is a cosmological particle (dust)?

- In FLRW one takes observers "comoving with the dust"
- Traditionally galaxies were regarded as dust. However,
  - Neither galaxies nor galaxy clusters are homogeneously distributed today
  - Dust particles should have (on average) invariant masses over the timescale of the problem
- Must coarse-grain over expanding fluid elements larger than the largest typical structures [voids of diameter $30\,h^{-1}$Mpc with $\delta_\rho \sim -0.95$ are $\gtrsim 40\%$ of $z = 0$ universe]

$$\left.\begin{array}{c} g_{\mu\nu}^{\text{stellar}} \to g_{\mu\nu}^{\text{galaxy}} \to g_{\mu\nu}^{\text{cluster}} \to g_{\mu\nu}^{\text{wall}} \\ \vdots \\ g_{\mu\nu}^{\text{void}} \end{array}\right\} \to g_{\mu\nu}^{\text{universe}}$$

Dilemma of gravitational energy…

- In GR spacetime carries *energy* & *angular momentum*

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- On account of the strong equivalence principle, $T_{\mu\nu}$ contains localizable energy–momentum only
- Kinetic energy and energy associated with spatial curvature are in $G_{\mu\nu}$: variations are “quasilocal”!
- Newtonian version, $T - U = -V$, of Friedmann equation

$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G\rho}{3}$$

where $T = \frac{1}{2} m\dot{a}^2 x^2$, $U = -\frac{1}{2} kmc^2 x^2$, $V = -\frac{4}{3}\pi G\rho a^2 x^2 m$; $\mathbf{r} = a(t)\mathbf{x}$.

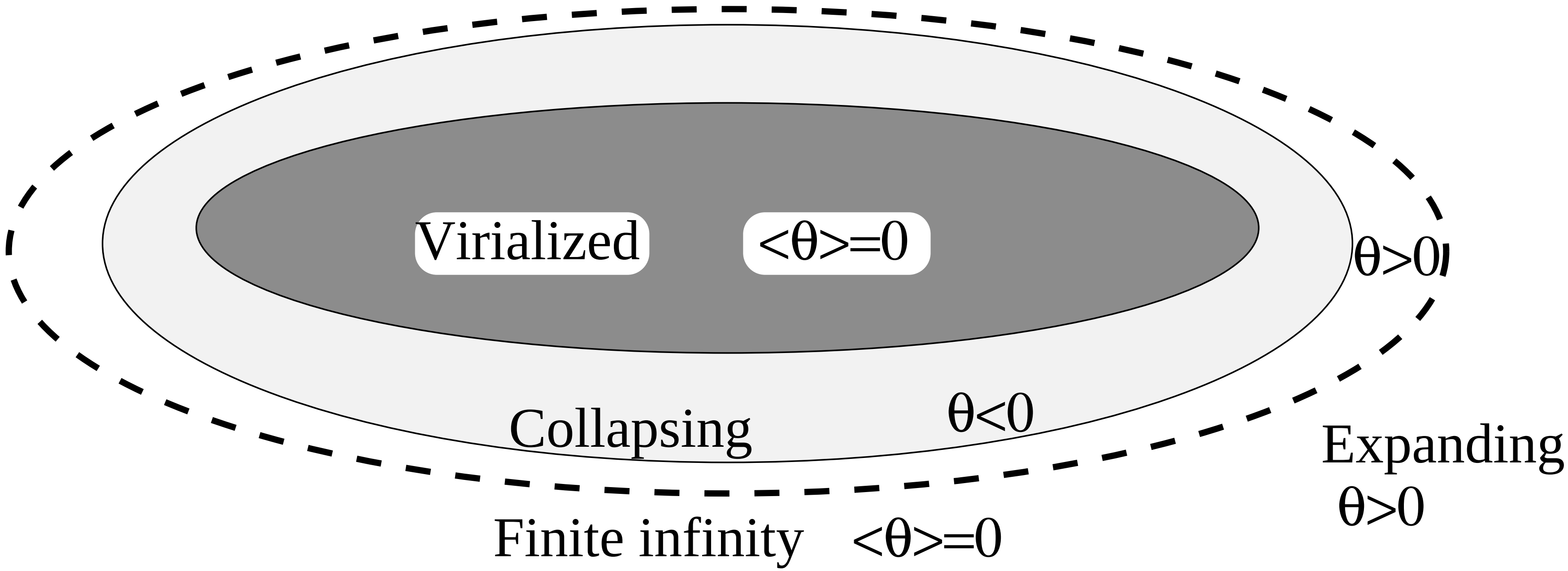
Cosmological Equivalence Principle

- *In cosmological averages it is always possible to choose a suitably defined spacetime region, the cosmological inertial region, on whose boundary average motions (timelike and null) can be described by geodesics in a geometry which is Minkowski up to some time-dependent conformal transformation*,

$$\mathrm{d}s^2_{\mathrm{CIR}} = a^2(\eta)\left[-\mathrm{d}\eta^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right],$$

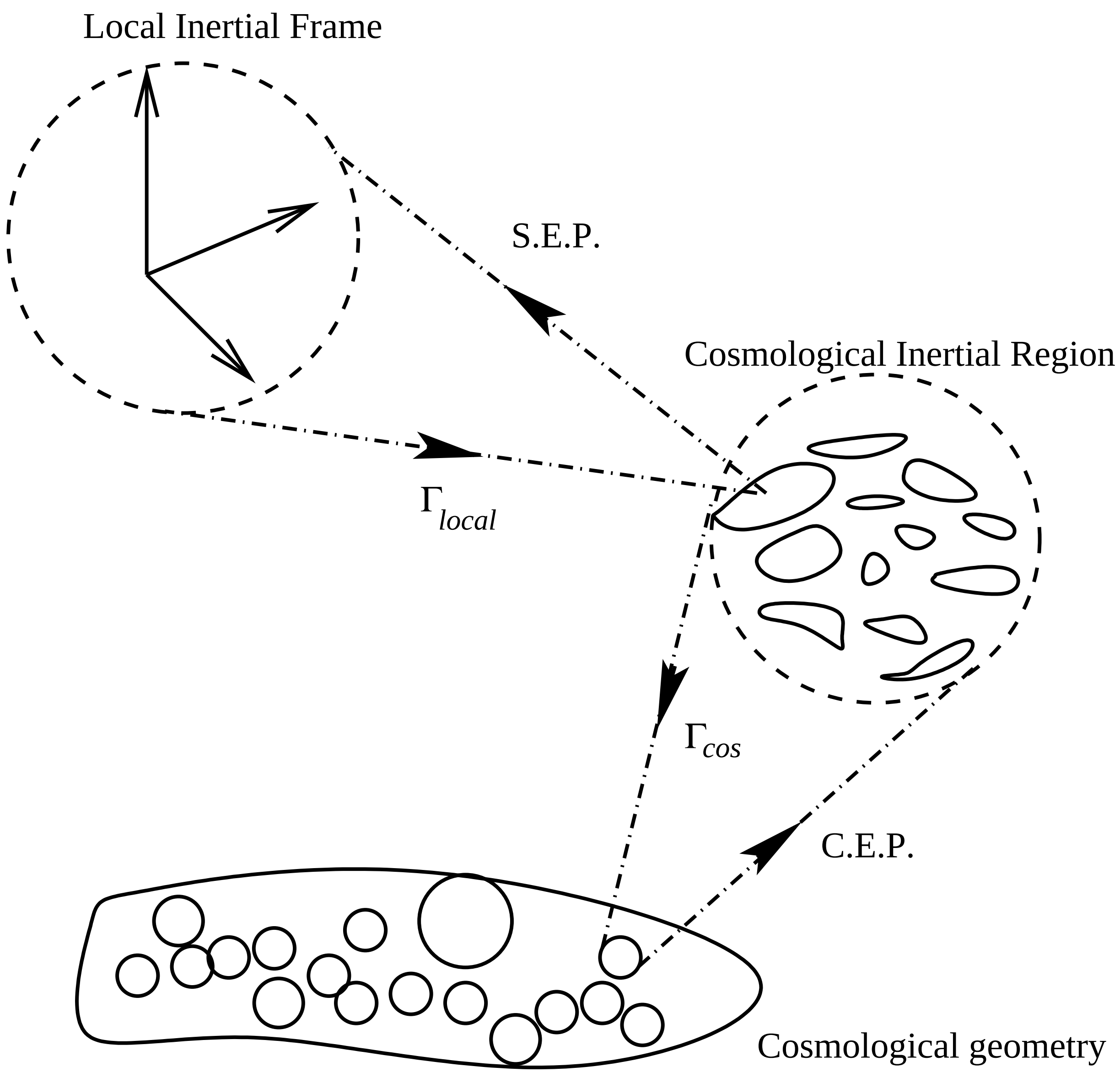
- Defines Cosmological Inertial Region (CIR) in which *regionally isotropic* volume expansion is equivalent to a velocity in special relativity
- Such velocities integrated on a bounding 2-sphere define *“kinetic energy of expansion”*: globally it has gradients

Finite infinity



- Define *finite infinity*, "*fi*" as boundary to *connected* region within which *average expansion* vanishes $\langle\vartheta\rangle = 0$ and expansion is positive outside.
- Shape of *fi* boundary irrelevant (minimal surface generally): could typically contain a galaxy cluster.

Statistical geometry...



Why is ΛCDM so successful?

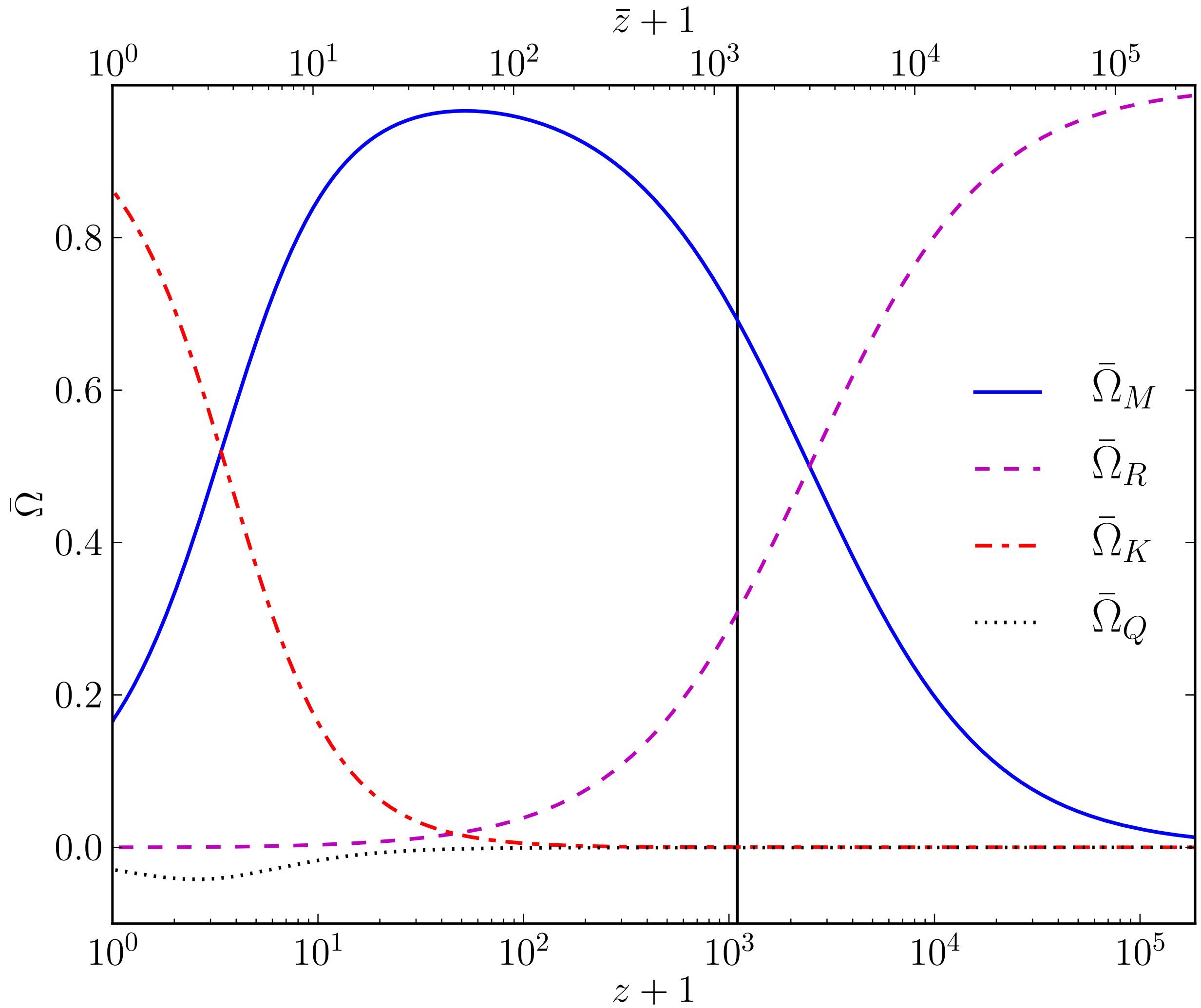
- The early Universe was extremely close to homogeneous and isotropic
- Finite infinity geometry (2 – $15\,h^{-1}$Mpc) is close to spatially flat (Einstein–de Sitter at late times) – N–body simulations successful *for bound structure*
- At late epochs there is a simplifying principle – Cosmological Equivalence Principle
- Hubble parameter (first derivative of statistical metric; i.e., connection) is to some extent a “gauge choice”
  - Affects ‘local’/global H_0 issue
  - Has contributed to fights (e.g., Sandage vs de Vaucouleurs) H_0 depends on measurement scale
- *Even on small scales there is a notion of uniform Hubble flow at expense of calibration of rulers AND CLOCKS*

Timescape phenomenology

$$ds^2 = -(1+2\Phi)c^2 dt^2 + a^2(1-2\Psi)g_{ij}dx^i dx^j$$

- Global statistical metric by Buchert average not a solution of Einstein equations
- Solve for Buchert equations for ensemble of void and finite infinity (wall) regions; conformally match radial null geodesics of finite infinity and statistical geometries, fit to observations
- Relative regional volume deceleration integrates to a substantial difference in clock calibration of bound system observers relative to volume average over age of universe
- Difference in *bare* (statistical or volume–average) and *dressed* (regional or finite–infinity) parameters

Bare cosmological parameters



J.A.G. Duley, M.A. Nazer & DLW, CQG 30 (2013) 175006: full numerical solution with matter, radiation

Apparent cosmic acceleration

- Volume average observer sees no apparent cosmic acceleration

$$\bar{q} = \frac{2\,(1 - f_{\rm v})^2}{(2 + f_{\rm v})^2}\,.$$

As $t \to \infty$, $f_{\rm v} \to 1$ and $\bar{q} \to 0^+$.

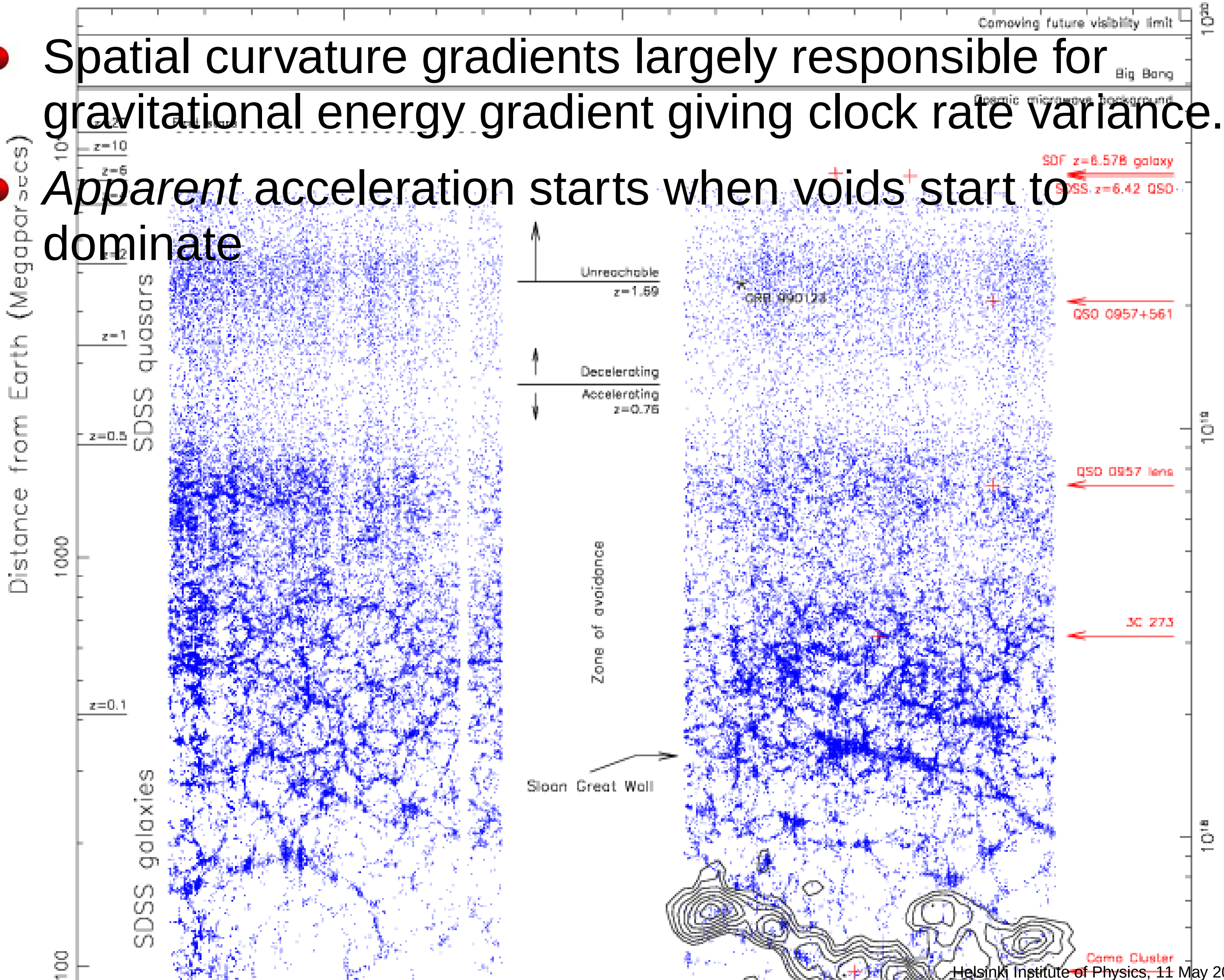
- A wall observer registers apparent cosmic acceleration

$$q = \frac{-\,(1 - f_{\rm v})\,(8{f_{\rm v}}^3 + 39{f_{\rm v}}^2 - 12f_{\rm v} - 8)}{\left(4 + f_{\rm v} + 4{f_{\rm v}}^2\right)^2}\,,$$

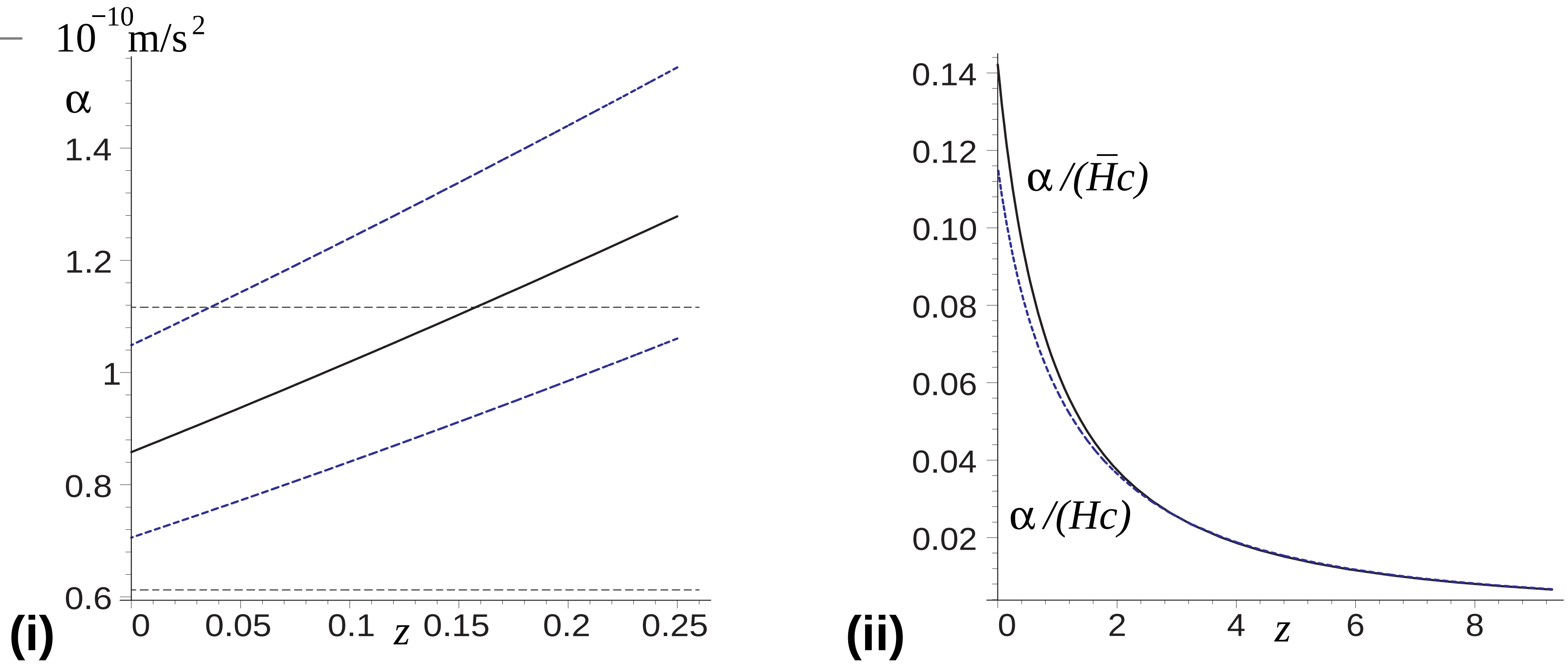
Effective deceleration parameter starts at $q \sim \frac{1}{2}$, for small $f_{\rm v}$; changes sign when $f_{\rm v} = 0.5867\ldots$, and approaches $q \to 0^-$ at late times.

Cosmic coincidence problem solved

- Spatial curvature gradients largely responsible for gravitational energy gradient giving clock rate variance.
- *Apparent* acceleration starts when voids start to dominate



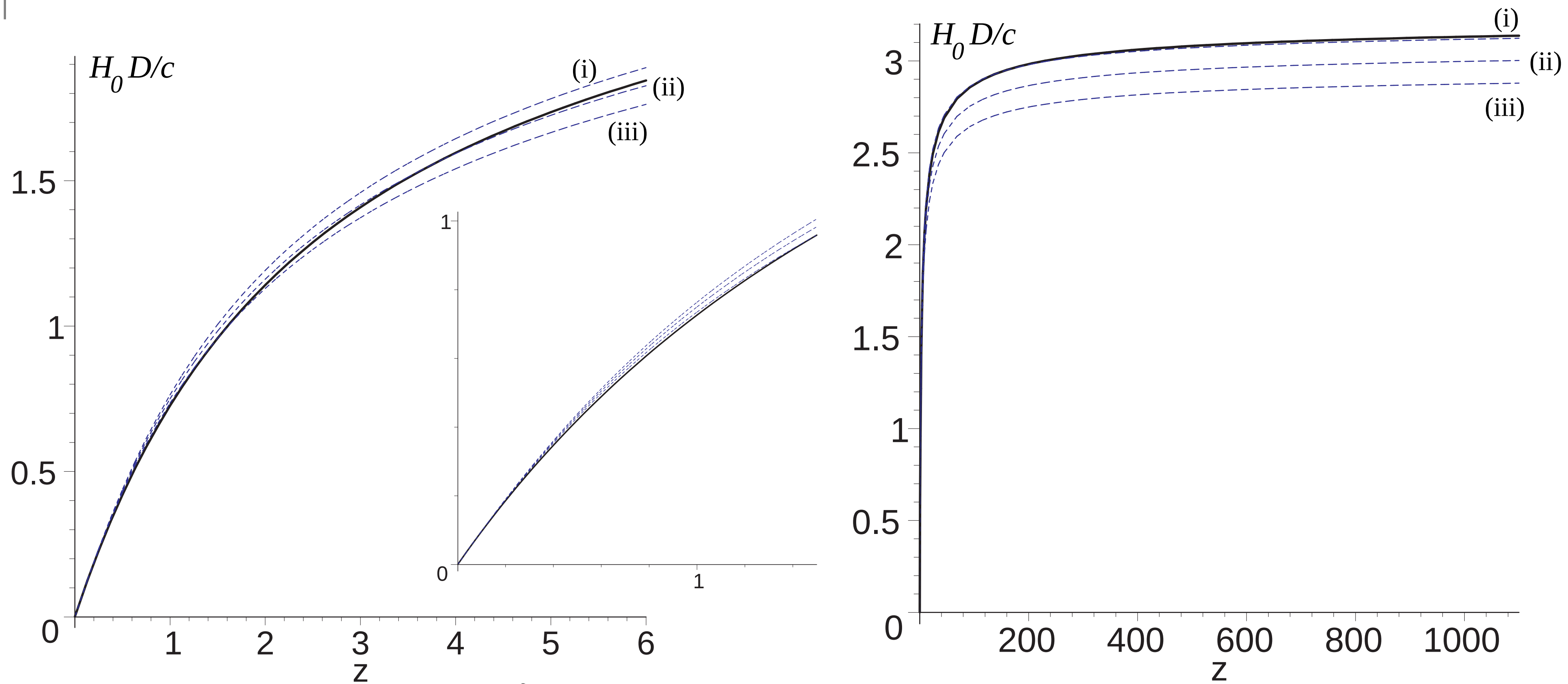
Relative deceleration scale



By cosmological equivalence principle the instantaneous relative deceleration of backgrounds gives an instantaneous 4-acceleration of magnitude $\alpha = H_0 c\bar{\gamma}\dot{\bar{\gamma}}/(\sqrt{\bar{\gamma}^2 - 1})$ beyond which *weak field cosmological general relativity* will be changed from Newtonian expectations: **(i)** as absolute scale nearby; **(ii)** divided by Hubble parameter to large z.

- Relative *volume* deceleration of expanding regions of different local density/curvature, leads cumulatively to canonical clocks differing by $\mathrm{d}t = \bar{\gamma}_{\rm w}\,\mathrm{d}\tau_{\rm w}$ ($\rightarrow\sim 35\%$)

Dressed "comoving distance" $D(z)$

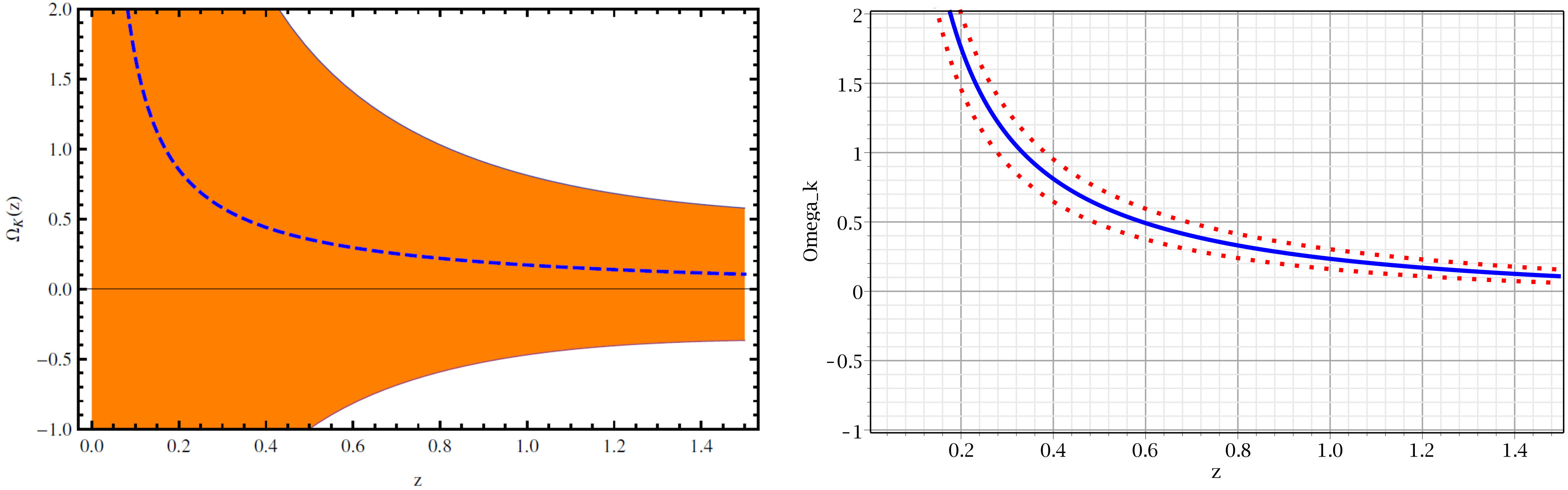


TS model, with $f_{v0} = 0.695$, **(black)** compared to 3 spatially flat ΛCDM models (blue): **(i)** $\Omega_{M0} = 0.3175$ (best-fit ΛCDM model to Planck); (ii) $\Omega_{M0} = 0.35$; **(iii)** $\Omega_{M0} = 0.388$.

Clarkson Bassett Lu test $\Omega_k(z)$

- For Friedmann equation a statistic constant for all z

$$\Omega_{\rm k0} = \Omega_k(z) = \frac{[c^{-1}H(z)D'(z)]^2 - 1}{[c^{-1}H_0 D(z)]^2}$$

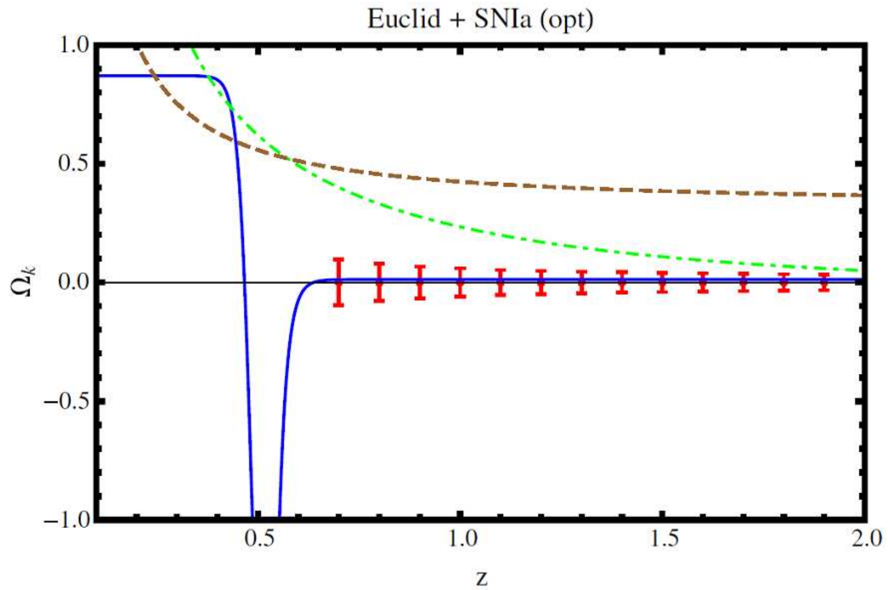


Left panel: CBL statistic from Sapone, Majerotto and Nesseris, PRD 90, 023012 (2015) Fig 8,

using existing data from SneIa (Union2) and passively evolving galaxies for $H(z)$.

Right panel: TS prediction, with $f_{\rm v0} = 0.695^{+0.041}_{-0.051}$.

Clarkson Bassett Lu test with *Euclid*



- Projected uncertainties for ΛCDM model with *Euclid* $+$ 1000 SneIa, Sapone *et al*, arXiv:1402.2236v2 Fig 10
- Timescape prediction (green), compared to non-Copernican Gpc void model (blue), and *tardis* cosmology, Lavinto *et al* arXiv:1308.6731 (brown).
- Timescape prediction becomes greater than uncertainties for $z \lesssim 1.5$. (Falsfiable.)

Planck constraints $D_A + r_{drag}$

- Dressed Hubble constant $H_0 = 61.7 \pm 3.0$ km/s/Mpc
- Bare Hubble constant $H_{\mathrm{w0}} = \bar{H}_0 = 50.1 \pm 1.7$ km/s/Mpc
- Local max Hubble constant $H_{\mathrm{v0}} = 75.2^{+2.0}_{-2.6}$ km/s/Mpc
- Present void fraction $f_{\mathrm{v0}} = 0.695^{+0.041}_{-0.051}$
- Bare matter density parameter $\bar{\Omega}_{\mathrm{M0}} = 0.167^{+0.036}_{-0.037}$
- Dressed matter density parameter $\Omega_{\mathrm{M0}} = 0.41^{+0.06}_{-0.05}$
- Dressed baryon density parameter $\Omega_{\mathrm{B0}} = 0.074^{+0.013}_{-0.011}$
- Nonbaryonic/baryonic matter ratio $\Omega_{\mathrm{C0}}/\Omega_{\mathrm{B0}} = 4.6^{+2.5}_{-2.1}$
- Age of universe (galaxy/wall) $\tau_{\mathrm{w0}} = 14.2 \pm 0.5$ Gyr
- Age of universe (volume-average) $t_0 = 17.5 \pm 0.6$ Gyr
- Apparent acceleration onset $z_{\mathrm{acc}} = 0.46^{+0.26}_{-0.25}$

CMB acoustic peaks, $\ell > 50$, full fit

- Use FLRW model prior to last scattering best matched to timescape equivalent parameters
- Use Vonlanthen, Räsänen, R. Durrer (2010) procedure to map timescape model d_A to FLRW reference d'_A

$$\begin{aligned} C_l &= \sum_{\tilde{l}} \frac{2\tilde{l}+1}{2} C'_{\tilde{l}} \int_0^\pi \sin\theta \, \mathrm{d}\theta \, P_{\tilde{l}} \left[\cos(\theta \, d_A / d'_A)\right] P_\ell(\cos\theta) \\ &\approx \left(\frac{d'_A}{d_A}\right)^2 C'_{\frac{d'_A}{d_A}\ell} , \qquad \ell > 50 \end{aligned}$$

- Ignore $\ell < 50$ in fit (late ISW effect may well differ)
- Fit FLRW model that decelerates by same amount from last scattering til today (in volume-average time) – systematic uncertainties depending on method adopted

Matching average expansion history

- Determine FLRW scale factor (hatted) to match volume–average timescape one (barred) at all epochs

$$\frac{\hat{a}_0}{\hat{a}} = \frac{\bar{a}_0}{\bar{a}} = \frac{\bar{T}}{\bar{T}_0} = 1 + \bar{z}$$

BUT $\hat{H} \neq \bar{H}, \qquad \hat{\Omega}_{\rm M} \neq \bar{\Omega}_M, \qquad \hat{\Omega}_{\rm R} \neq \bar{\Omega}_R$

for most $\bar{z}$. However, for SOME FLRW solution set

$$\begin{aligned} \hat{H}_0 &= \bar{H}_0 \\ \hat{\Omega}_{\rm M0} &= \bar{\Omega}_{\rm M0} \qquad \Rightarrow \; \hat{\Omega}_{\rm B0} = \bar{\Omega}_{\rm B0} \\ \hat{\Omega}_{\rm R0} &= \bar{\Omega}_{\rm R0} = \frac{32\sigma_{\rm B}\pi G}{3c^3\bar{H}_0^2}\left[1 + \tfrac{7}{8}\left(\tfrac{4}{11}\right)^{4/3} N_{\rm eff}\right]\bar{T}_0{}^4 \end{aligned}$$

- Ensures matter–radiation equality occurs at the same (bare) redshift, $\bar{z}$

Model matching detail

- For general FLRW model $\hat{\Omega}_{\rm M} + \hat{\Omega}_{\rm R} + \hat{\Omega}_{\rm k} + \hat{\Omega}_{\Lambda} = 1$,

$$\Rightarrow \hat{\Omega}_{\Lambda 0} = 1 - \hat{\Omega}_{\rm k0} - \bar{\Omega}_{\rm M0} - \bar{\Omega}_{\rm R0}.$$

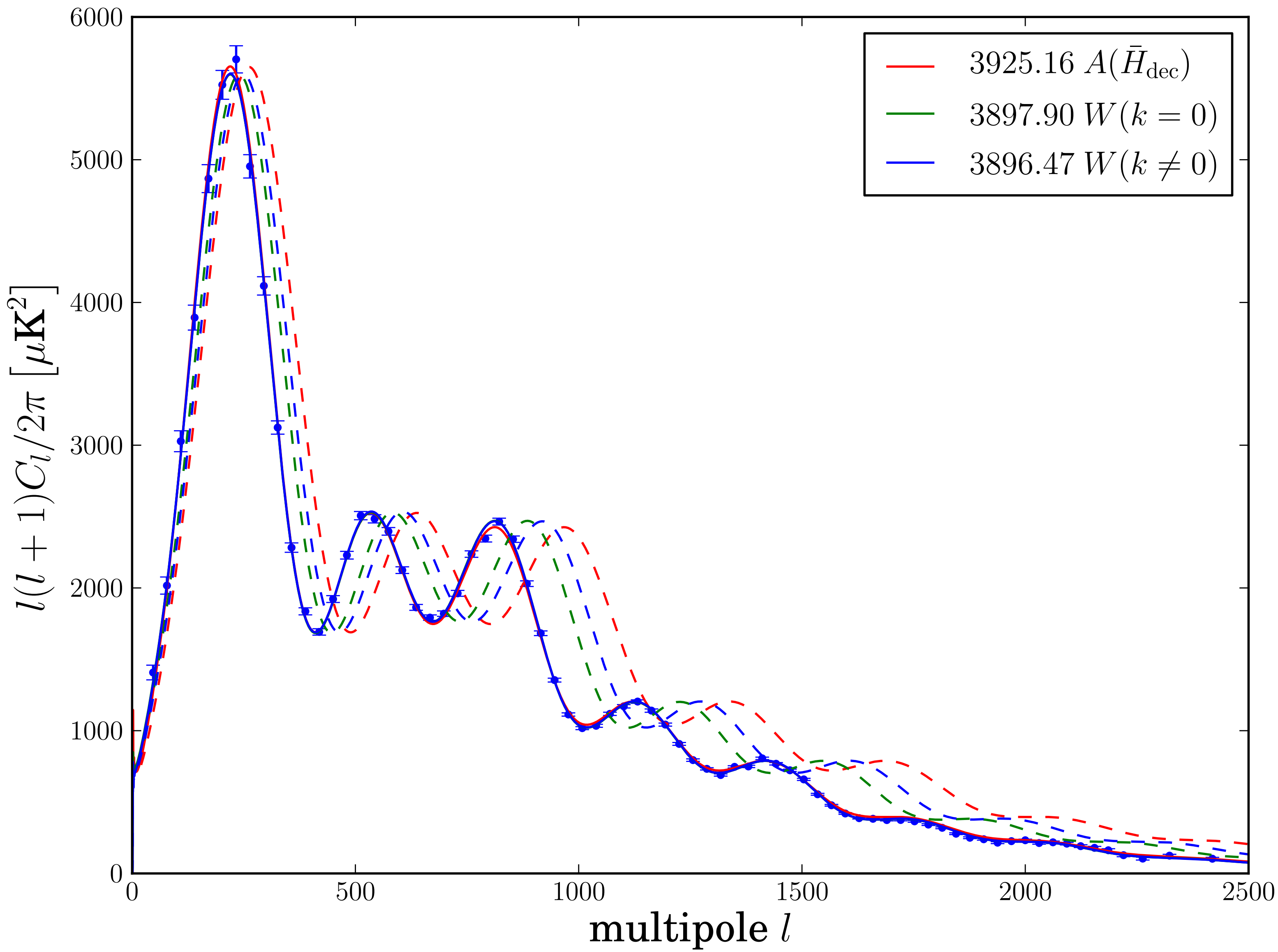
- One parameter left to constrain – at early times $\hat{H} \simeq \bar{H}$, $\quad \hat{\Omega}_{\rm M} \simeq \bar{\Omega}_{M}$, $\quad \hat{\Omega}_{\rm R} \simeq \bar{\Omega}_{R}$ BUT many choices with $\delta\hat{\Omega} \lesssim 10^{-5}$ in matched density parameters at $t_{\rm dec}$

1. A($\bar{H}_{\rm dec}$): Match Hubble parameter match at $t_{\rm dec}$
2. A($\bar{r}_{\mathcal{H}}$): Match comoving particle horizon scale
3. A($\bar{\eta}_0$): Match of bare conformal time age of the Universe
4. A(t_0): Match of the bare age of the Universe
5. A($\hat{\Omega}_{\Lambda 0} = 0$): FLRW without Λ

Matching wall expansion history

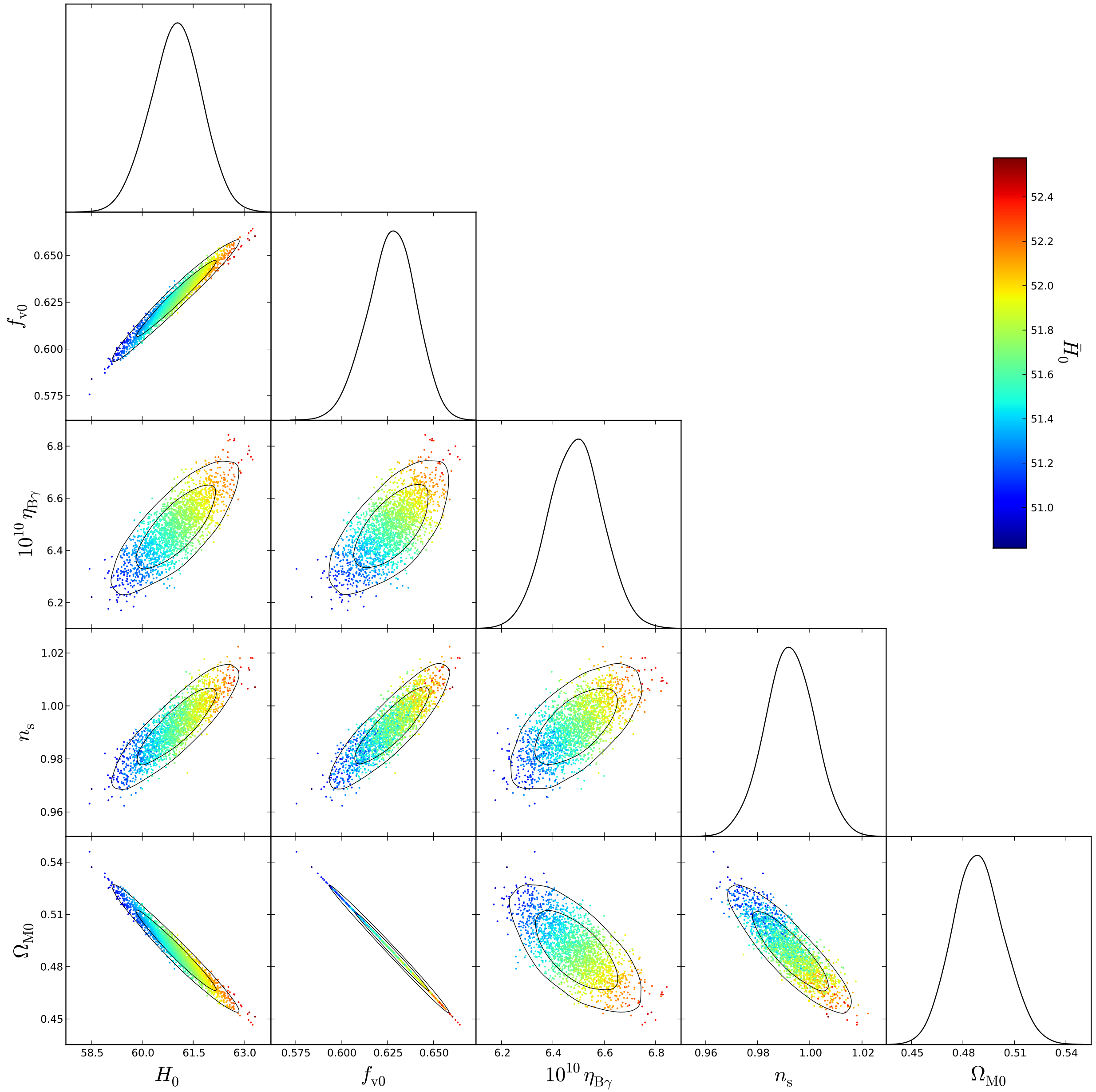
- In timescape model geometry below finite infinity scales close to Einstein-de Sitter
- Alternative matching procedures based on matching wall geometry only
- Likely to give a better match for parameters affecting bound structures – baryon–to–photon ratio $\eta_{B\gamma}$, spectral index n_s – but not average expansion history
- Two methods analysed
  1. W($k = 0$): Spatial curvature zero
  2. W($k \neq 0$): Initial (tiny) FLRW (negative) spatial curvature

CMB acoustic peaks, full Planck fit



MCMC coding by M.A. Nazer, adapting *CLASS*

M.A. Nazer + DLW, arXiv:1410.3470

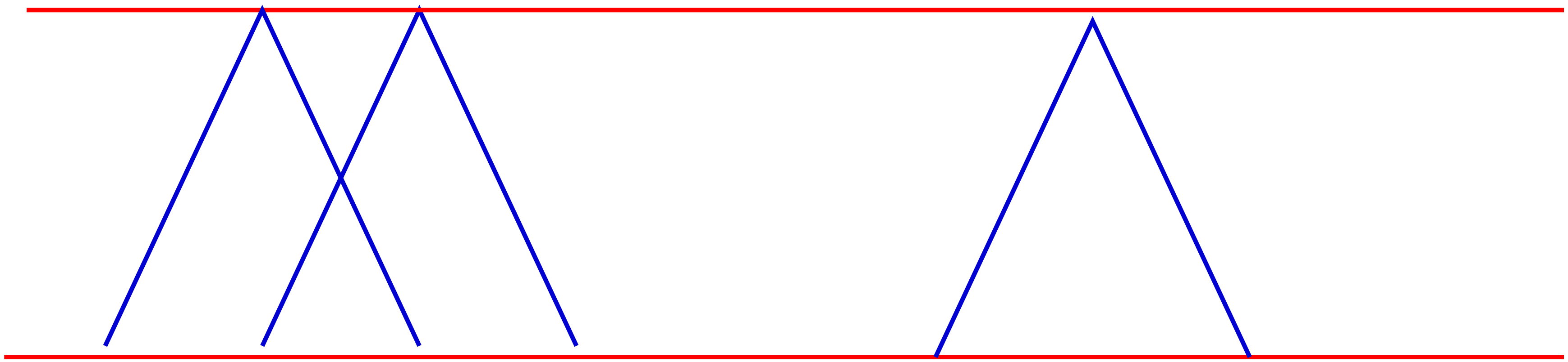


CMB acoustic peaks: arXiv:1410.3470

- Likelihood $-\ln \mathcal{L} = 3925.16$, 3897.90 and 3896.47 for A($\bar{H}_{\rm dec}$), W($k = 0$) and W($k \neq 0$) methods respectively on $50 \leq \ell \leq 2500$, c.f., ΛCDM: 3895.5 using `MINUIT` or 3896.9 using `CosmoMC`.
- $H_0 = 61.0$ km/s/Mpc (± 1.3% stat) (± 8% sys); $f_{\rm v0} = 0.627$ (± 2.33% stat) (± 13% sys).
- Previous D_A + r_{drag} constraints give concordance for baryon–to–photon ratio $10^{10}\eta_{B\gamma} = 5.1 \pm 0.5$ with no primordial ^{7}Li anomaly, $\Omega_{\rm C0}/\Omega_{\rm B0}$ possibly 30% lower.
- Full fit – driven by 2nd/3rd peak heights, $\Omega_{\rm C0}/\Omega_{\rm B0}$, ratio – gives $10^{10}\eta_{B\gamma} = 6.08$ (± 1.5% stat) (± 8.5% sys).
- With bestfit values, primordial ^{7}Li anomalous and BOSS $z = 2.34$ result in tension at level similar to ΛCDM

Back to the early Universe

- BUT backreaction in primordial plasma neglected
- Backreaction of similar order to density perturbations (10^{-5}); little influence on background but may influence growth of perturbations
- First step: add pressure to new “relativistic Lagrangian formalism” (Buchert & coworkers, 2012-15)
- Dimensional reduction to 2 dimensions at high energy in many approaches to quantum gravity. Spacetime is relational structure: when all relations lightlike spacetime melts (mathematical challenge!)



Conclusion/Outlook

- A global FLRW geometry is a conceptual prison holding back most cosmologists
- Claim: FLRW geometry plus boosts falsified below SHS
- A 0.5% nonkinematic anisotropy on $\lesssim 65\,h^{-1}$Mpc scales has profound implications for cosmology
- On $> 100\,h^{-1}$Mpc scales viable phenomenological models of backreaction – timescape – are possible
- New tests vis–à–vis ΛCDM; new challenges
- Quasilocal gravitational energy is one of the biggest unresolved mysteries of general relativity. Can we coarse grain Weyl to Ricci curvature quasilocally (building on N Uzun, arXiv:1602.07861)?
- "Modified Geometry" rather than "Modified Gravity"