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Binary parameters measurement from coalescence event(s) in LIGO

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Purpose: understand the accuracy of parameters determination for a coalescence event

Outline

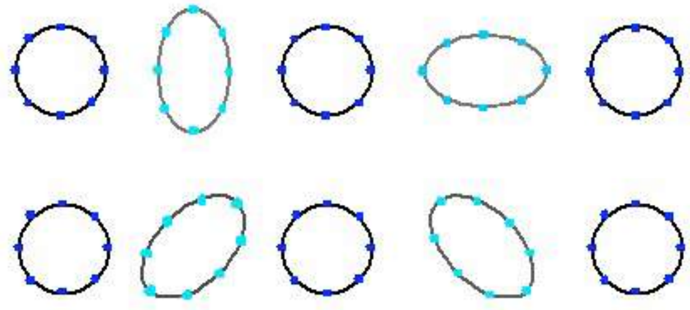
1. binaries coalescences as GW sources
2. a little bit of data analysis
3. GW150914: first event, a peculiar one
4. Conclusions & future prospects:
   a. other events
   b. more detectors

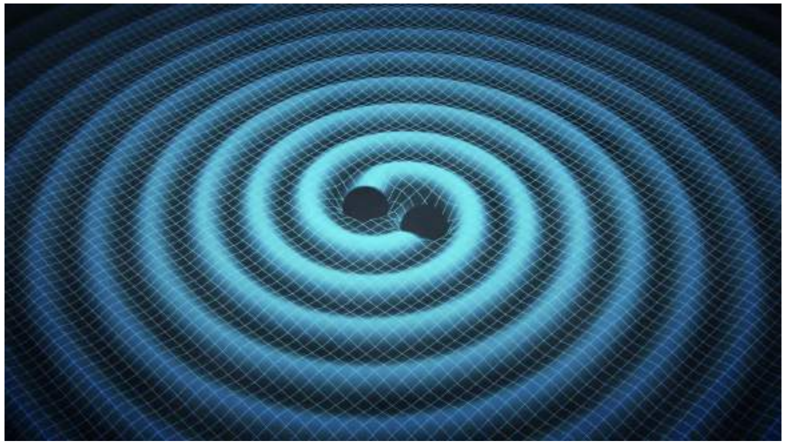
1. Gravitational waves in binary systems

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\Box h_{\mu\nu} = T_{\mu\nu} \qquad h_{ij}^{TT} \simeq \frac{2G}{r}\ddot{Q}_{ij}$$

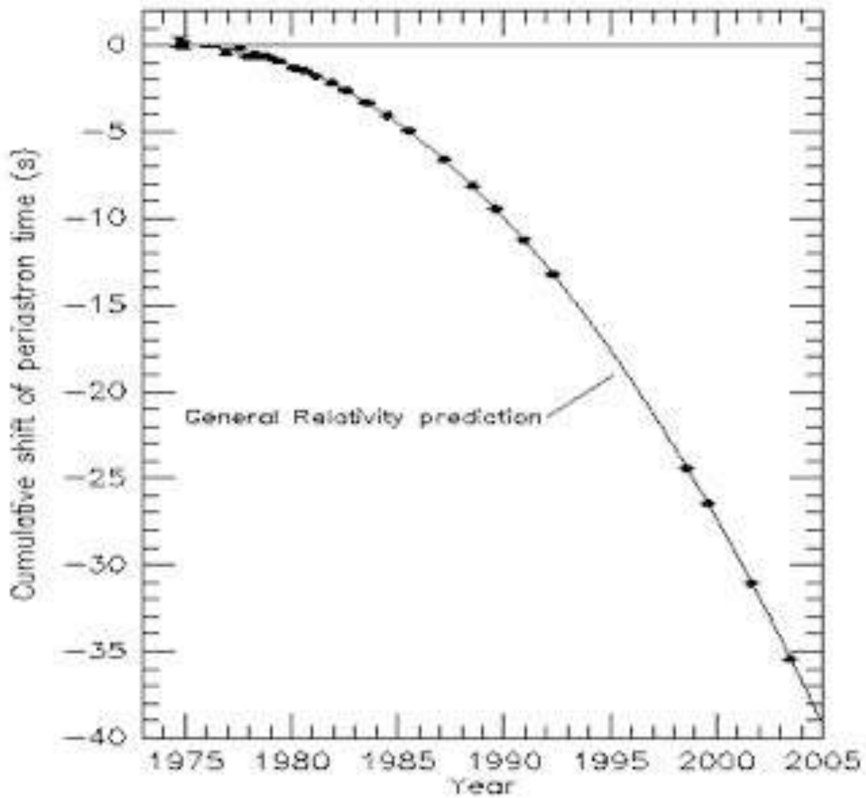
Transverse
and Traceless
(in vacuum)





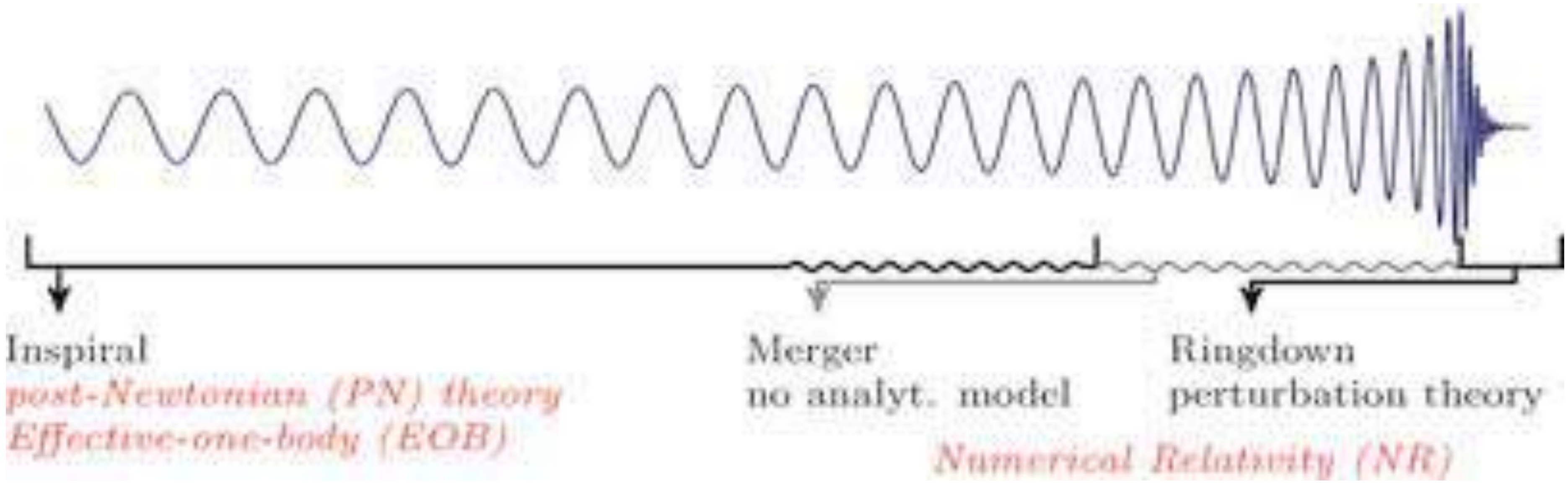
Hulse-Taylor pulsar:
(indirect)
observation of GW

agreement with GR at 0.1%



Even more precise tests
from double pulsars

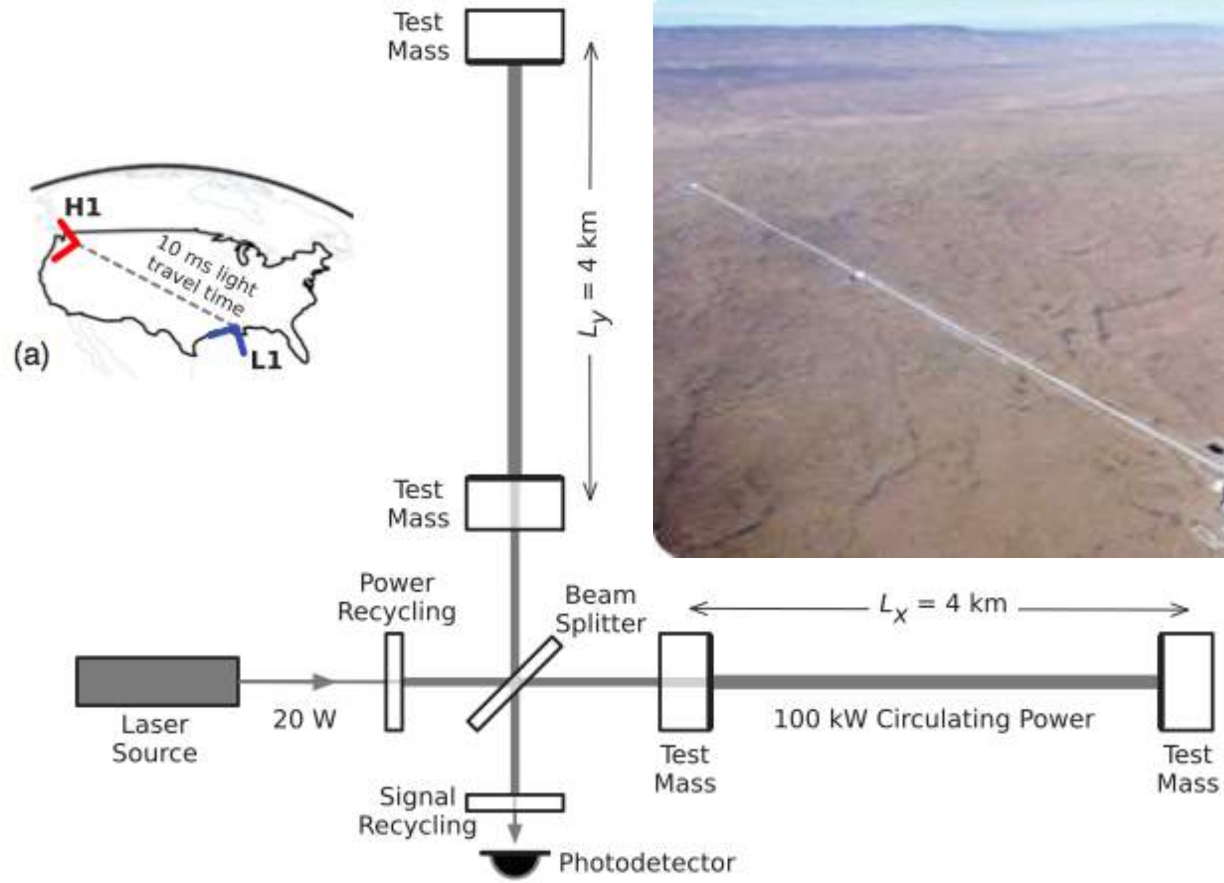
Coalescence signal

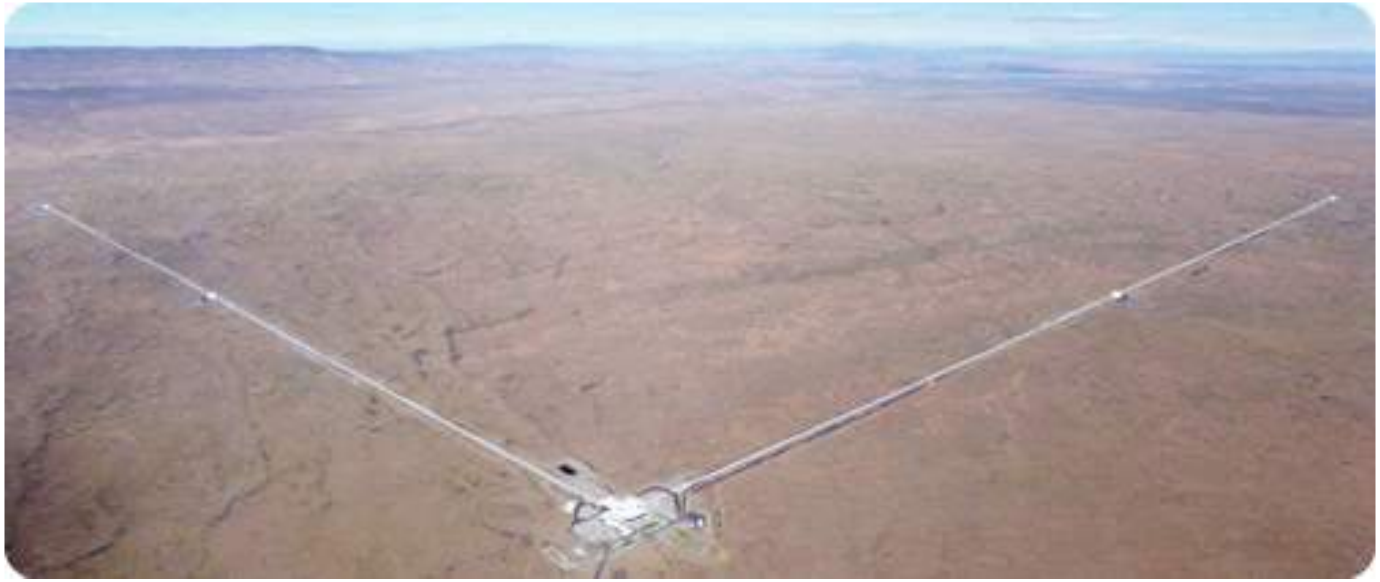


Other interesting sources/kinds of GW's

- Continuos: non axisymmetric rotating NS
- Bursts: supernovae
- Stochastic

2. How to extract the signal from the noise





$$h \simeq \frac{\Delta L}{L} \simeq 10^{-21}$$

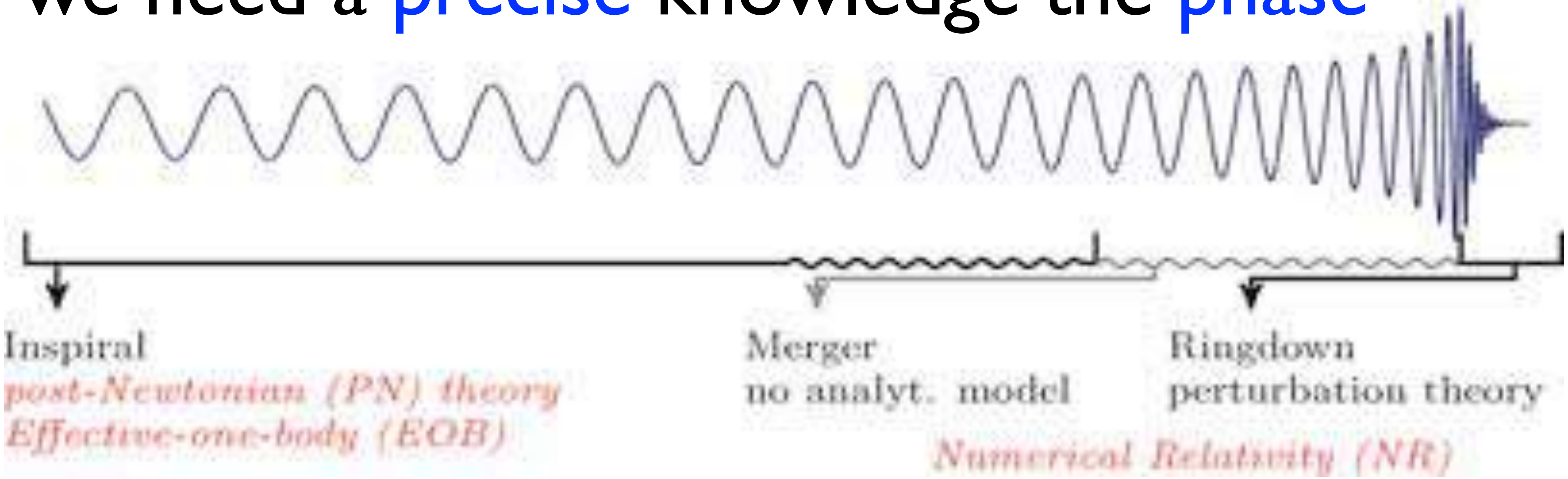
MATCHED FILTERING

out(t)=N(t)+h(t)

$$\int_T out(t)h(t) = \int_T N(t)h(t) + \int_T h^2(t)$$

$$\sim \sqrt{T} \qquad \sim T$$

we need a precise knowledge the phase



(quasi)circular motion

$$\dot{E}[\omega(t)] = -P_{gw}[\omega(t)] \longrightarrow \omega(t) \longrightarrow \Phi_{gw}(t)$$

$$\Phi_{gw}(t) = \int^{t} \omega(\tau)\mathrm{d}\tau = -\int^{v(t)} \omega(v)\frac{1}{P_{gw}(v)}\frac{\mathrm{d}E}{\mathrm{d}v}\mathrm{d}v \simeq \int \frac{\mathrm{d}v}{v^6} \sim v^{-5}$$

at least 3PN: 3rd order expansion in $v^2 \simeq \frac{GM}{r}$

is required as accuracy for the inspiral phase

3. GW150914:

'live' detection of coalescence of two $\sim 30 M_\odot$ BH's

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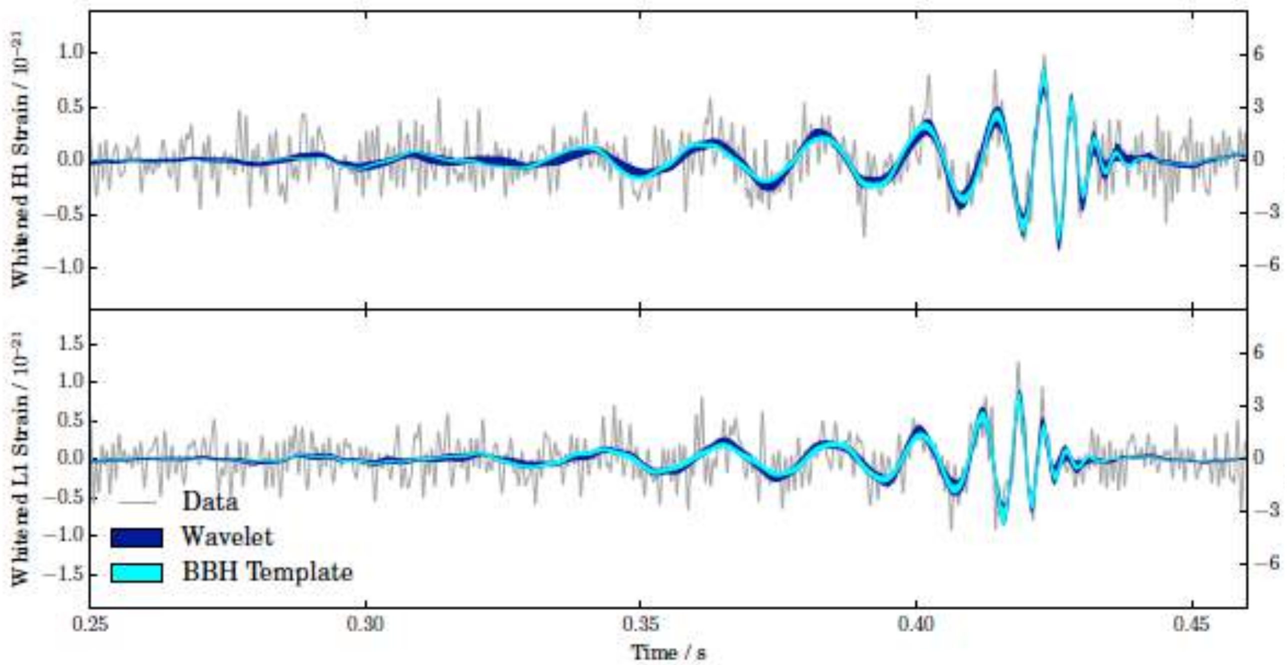
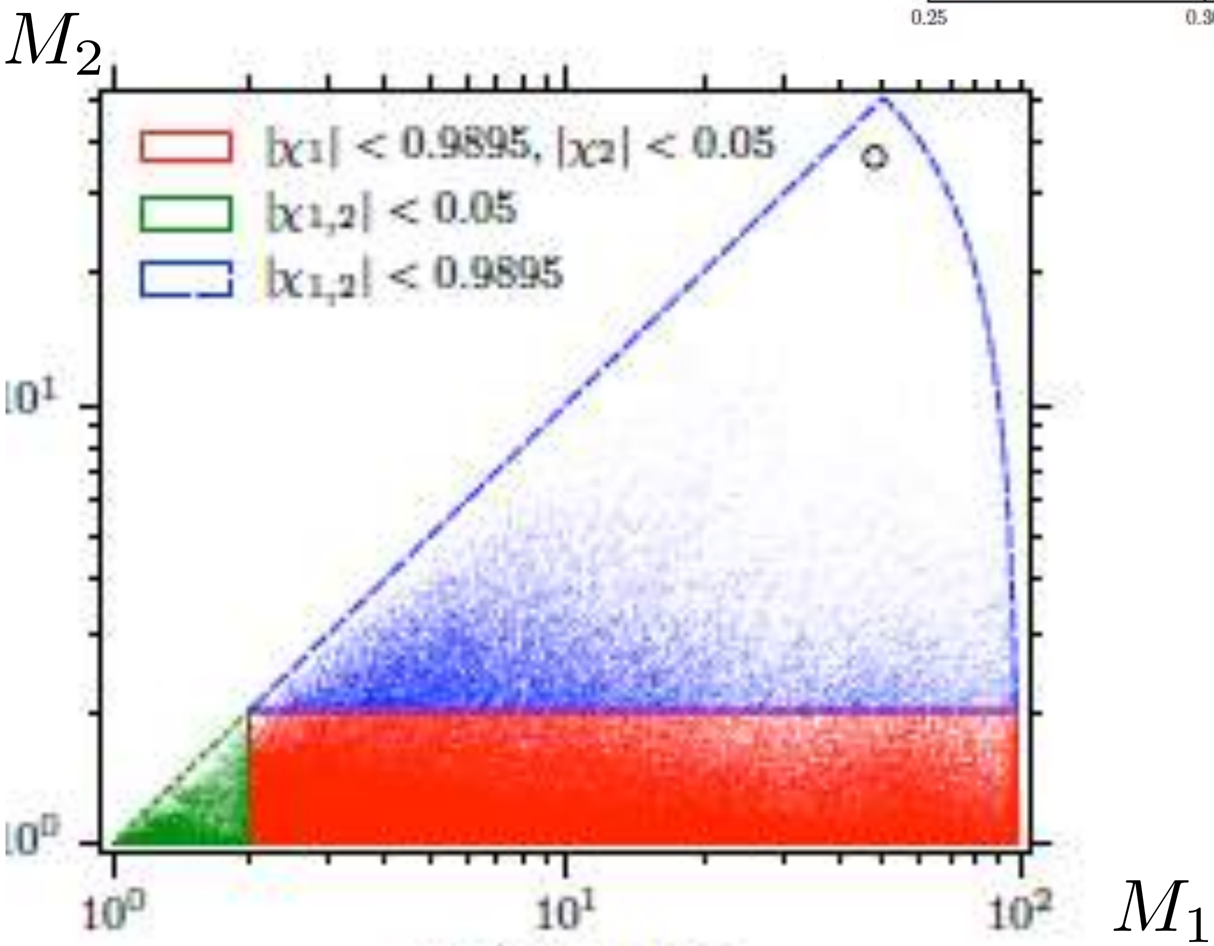
'live' detection of coalescence of two $\sim 30 M_\odot$ BH's

- First GW detection
- First (?) direct observation of a heavy stellar BH
- First observation of collision of two BH's
- First evidence for $\mathcal{O}(10^2) M_\odot$ BH's

High masses: <10 cycles observed in a few milliseconds

(NS-NS: $\mathcal{O}(10^4)$ cycles in ~ 15 minutes)





The longest the measured inspiral phase, the better its parameters can be measured

$$O = h_+ F_+ + h_\times F_\times$$

$$h_+ = h_c \frac{1+\cos^2\iota}{2} \cos\Phi_{GW} \qquad F_+ = \frac{1}{2}\left(1+\cos^2\theta\right)\cos 2\varphi$$

$$h_\times = h_c \cos\iota \ \sin\Phi_{GW} \qquad F_\times = \cos\theta \ \sin 2\varphi$$

$$h_c = \frac{1}{R}\frac{GM_c}{c^2}\left(\frac{\tau}{M_c}\right)^{-1/4} + \ldots$$

$$\tau \equiv \frac{c^3}{5G}(t_c - t) \qquad \nu \equiv \frac{\mu}{M} \qquad M_c \equiv \nu^{3/5} M$$

1PN

$$\Phi_{GW} = -2\left(\frac{\tau}{M_c}\right)^{5/8}\left\{1 + \left(\frac{3715}{8064} + \frac{55}{96}\nu\right)\left(\frac{\nu\tau}{M}\right)^{-1/4} + \left[f\left(\chi_{eff},\nu\right) - \frac{3}{4}\pi\right]\left(\frac{\nu\tau}{M}\right)^{-3/8} + f_{2PN}\left(\nu, S_\perp\right)\left(\frac{\nu\tau}{M}\right)^{-1/2} + \ldots\right\}$$

tail

1.5PN

2PN

$$\chi_{eff} \equiv \frac{c}{G}\left(\frac{\mathbf{S_1}}{m_1} + \frac{\mathbf{S_2}}{m_2}\right)\cdot\frac{\mathbf{L}}{M}$$

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$$\tau \to \tau_o \equiv \tau_e(1+z)$$

$$M_{(c)} \to \mathcal{M}_{(c)} \equiv M_{(c)}(1+z)$$

$$R \to d_L = R(1+z)$$

$$\tau \equiv \frac{c^3}{5G}(t_c - t) \qquad \nu \equiv \frac{\mu}{M} \qquad M_c \equiv \nu^{3/5} M$$

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1602.03840

Properties of the binary black hole merger GW150914

The LIGO Scientific Collaboration and The Virgo Collaboration

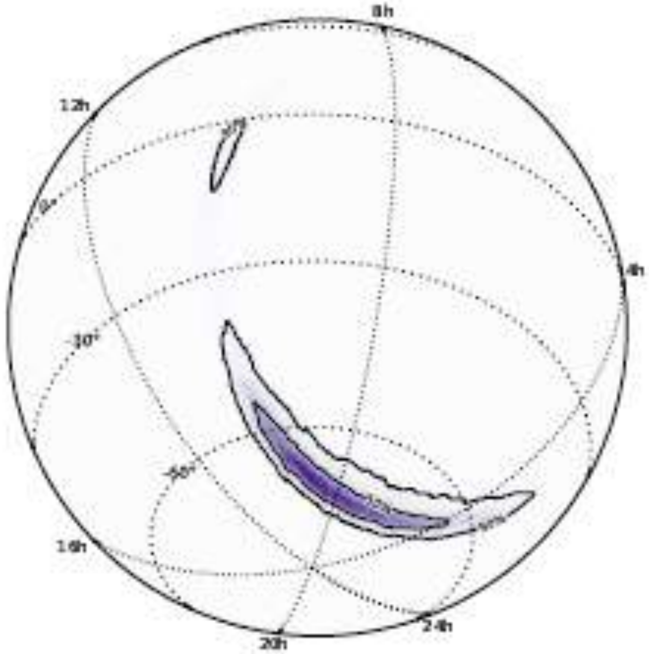
(compiled 12 February 2016)

On September 14, 2015, the Laser Interferometer Gravitational-wave Observatory (LIGO) detected a gravitational-wave transient (GW150914); we characterise the properties of the source and its parameters. The data around the time of the event were analysed coherently across the LIGO network using a suite of accurate waveform models that describe gravitational waves from a compact binary system in general relativity. GW150914 was produced by a nearly equal mass binary black hole of masses $36^{+5}_{-4}\,M_\odot$ and $29^{+4}_{-4}\,M_\odot$ (for each parameter we report the median value and the range of the 90% credible interval). The dimensionless spin magnitude of the more massive black hole is bound to be < 0.7 (at 90% probability). The luminosity distance to the source is 410^{+160}_{-180} Mpc, corresponding to a redshift $0.09^{+0.03}_{-0.04}$ assuming standard cosmology. The source location is constrained to an annulus section of 590 deg^2, primarily in the southern hemisphere. The binary merges into a black hole of mass $62^{+4}_{-4}\,M_\odot$ and spin $0.67^{+0.05}_{-0.07}$. This black hole is significantly more massive than any other known in the stellar-mass regime.

		EOBNR	IMRPhenom	Overall
	Detector-frame total mass $M/M_\odot$	$70.3^{+5.3}_{-4.8}$	$70.7^{+3.8}_{-4.0}$	$70.5^{+4.6\pm0.9}_{-4.5\pm1.0}$
6%	Detector-frame chirp mass $\mathcal{M}/M_\odot$	$30.2^{+2.5}_{-1.9}$	$30.5^{+1.7}_{-1.8}$	$30.3^{+2.1\pm0.4}_{-1.9\pm0.4}$
12-14%	Detector-frame primary mass $m_1/M_\odot$	$39.4^{+5.5}_{-4.9}$	$38.3^{+5.5}_{-3.5}$	$38.8^{+5.6\pm0.9}_{-4.1\pm0.3}$
	Detector-frame secondary mass $m_2/M_\odot$	$30.9^{+4.8}_{-4.4}$	$32.2^{+3.6}_{-5.0}$	$31.6^{+4.2\pm0.1}_{-4.9\pm0.6}$
6%	Detector-frame final mass $M_f/M_\odot$	$67.1^{+4.6}_{-4.4}$	$67.4^{+3.4}_{-3.6}$	$67.3^{+4.1\pm0.8}_{-4.0\pm0.9}$
	Source-frame total mass $M^{\rm source}/M_\odot$	$65.0^{+5.0}_{-4.4}$	$64.6^{+4.1}_{-3.5}$	$64.8^{+4.6\pm1.0}_{-3.9\pm0.5}$
6%	Source-frame chirp mass $\mathcal{M}^{\rm source}/M_\odot$	$27.9^{+2.3}_{-1.8}$	$27.9^{+1.8}_{-1.6}$	$27.9^{+2.1\pm0.4}_{-1.7\pm0.2}$
12-14%	Source-frame primary mass $m_1^{\rm source}/M_\odot$	$36.3^{+5.3}_{-4.5}$	$35.1^{+5.2}_{-3.3}$	$35.7^{+5.4\pm1.1}_{-3.8\pm0.0}$
	Source-frame secondary mass $m_2^{\rm source}/M_\odot$	$28.6^{+4.4}_{-4.2}$	$29.5^{+3.3}_{-4.5}$	$29.1^{+3.8\pm0.2}_{-4.4\pm0.5}$
6%	Source-fame final mass $M_f^{\rm source}/M_\odot$	$62.0^{+4.4}_{-4.0}$	$61.6^{+3.7}_{-3.1}$	$61.8^{+4.2\pm0.9}_{-3.5\pm0.4}$
23%	Mass ratio q	$0.79^{+0.18}_{-0.19}$	$0.84^{+0.14}_{-0.21}$	$0.82^{+0.16\pm0.01}_{-0.21\pm0.03}$
	Effective inspiral spin parameter $\chi_{\rm eff}$	$-0.09^{+0.19}_{-0.17}$	$-0.03^{+0.14}_{-0.15}$	$-0.06^{+0.17\pm0.01}_{-0.18\pm0.07}$
100%	Dimensionless primary spin magnitude a_1	$0.32^{+0.45}_{-0.28}$	$0.31^{+0.51}_{-0.27}$	$0.31^{+0.48\pm0.04}_{-0.28\pm0.01}$
	Dimensionless secondary spin magnitude a_2	$0.57^{+0.40}_{-0.51}$	$0.39^{+0.50}_{-0.34}$	$0.46^{+0.48\pm0.07}_{-0.42\pm0.01}$
9%	Final spin a_f	$0.67^{+0.06}_{-0.08}$	$0.67^{+0.05}_{-0.05}$	$0.67^{+0.05\pm0.00}_{-0.07\pm0.03}$
40%	Luminosity distance D_L/Mpc	390^{+170}_{-180}	440^{+140}_{-180}	$410^{+160\pm20}_{-180\pm40}$
	Source redshift z	$0.083^{+0.033}_{-0.036}$	$0.093^{+0.028}_{-0.036}$	$0.088^{+0.031\pm0.004}_{-0.038\pm0.009}$

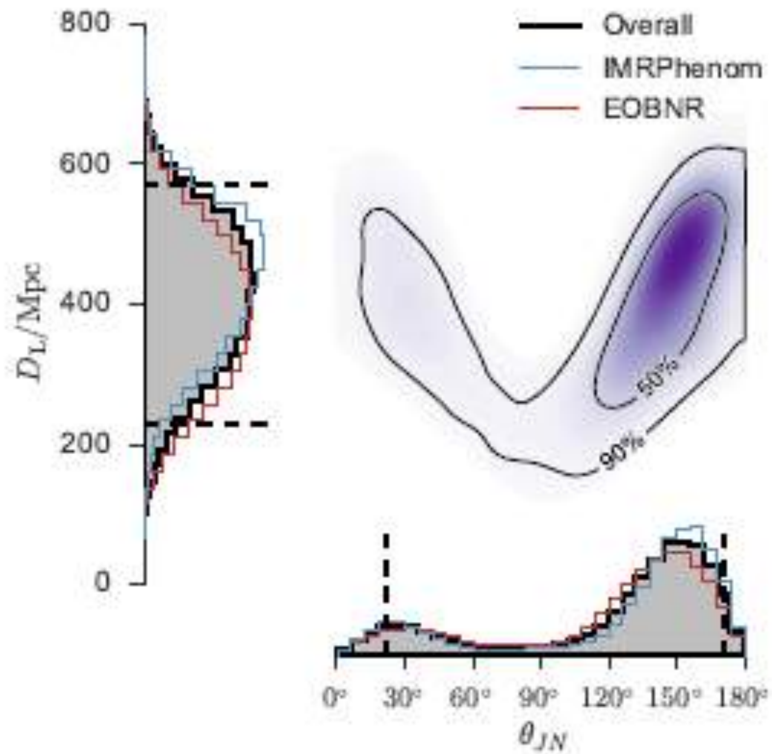
$$\chi \equiv \frac{cS}{Gm^2}$$

17% $\Delta M_{GW} = 3.0 \pm 0.5 M_\odot$



arrival direction:
somewhere in the
southern emisphere

orbital plane
roughly (anti)aligned
with line of sight

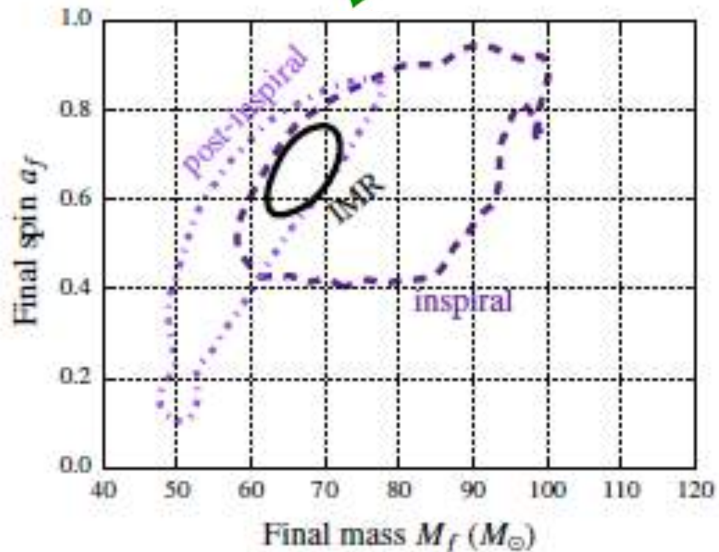


1602.03841

Tests of general relativity with GW150914

The LIGO detection of GW150914 provides an unprecedented opportunity to study the two-body motion of a compact-object binary in the large velocity, highly nonlinear regime, and to witness the final merger of the binary and the excitation of uniquely relativistic modes of the gravitational field. We carry out several investigations to determine whether GW150914 is consistent with a binary black-hole merger in general relativity. We find that the final-remnant's mass and spin, determined from the inspiral and post-inspiral phases of the signal, are mutually consistent with the binary black-hole solution in general relativity. The data following the peak of GW150914 are consistent with the least-damped quasi-normal-mode inferred from the mass and spin of the remnant black hole. By using waveform models that allow for parameterized general-relativity violations during the inspiral and merger phases, we perform quantitative tests on the gravitational-wave phase in the dynamical regime and, bound, for the first time several high-order post-Newtonian coefficients. We constrain the graviton Compton wavelength in a hypothetical theory of gravity in which the graviton is massive and place a 90%-confidence lower bound of 10^{13} km. Within our statistical uncertainties, we find no evidence for violations of general relativity in the genuinely strong-field regime of gravity.

- Power excess after subtraction ✓
- Graviton mass $\lambda_c \geq 10^{13} km$
- Polarization x
- Consistency inspiral vs. merger vs. ringdown ✓



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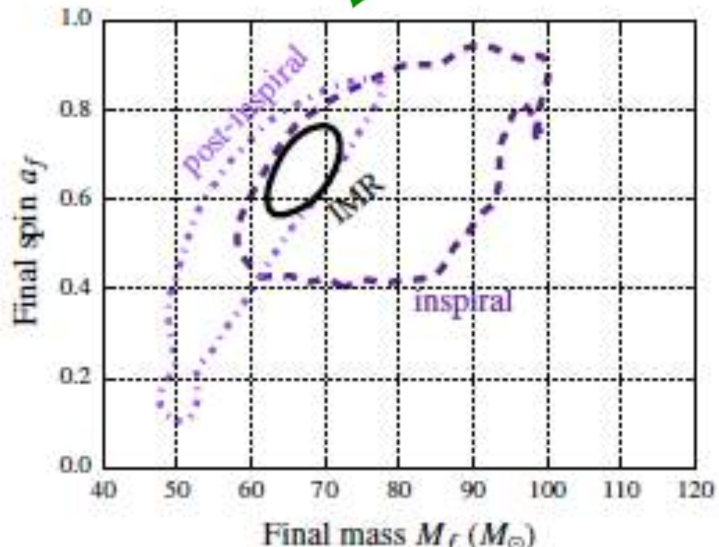
- Graviton mass $\lambda_c \geq 10^{13} km$
- Polarization x
- Consistency inspiral vs. merger vs. ringdown ✓
- PN parameter deviations



- PN parameter deviations:

0.5PN, 1PN, 1.5PN, 2PN

$$\Phi_{GW} = -2\left(\frac{\tau}{M_c}\right)^{5/8}\left\{1 + 0\cdot\left(\frac{\nu\tau}{M}\right)^{-1/8} + \left(\frac{3715}{8064}+\frac{55}{96}\nu\right)\left(\frac{\nu\tau}{M}\right)^{-1/4} + \left[f\left(\chi_{eff},\nu\right) - \frac{3}{4}\pi\right]\left(\frac{\nu\tau}{M}\right)^{-3/8} + f_{2PN}\left(\nu,S_\perp\right)\left(\frac{\nu\tau}{M}\right)^{-1/2} + \dots\right\}$$

$(1+\delta\varphi_{xPN})$ → applied to each term (leading, 0.5PN, 1PN, 1.5PN, 2PN)

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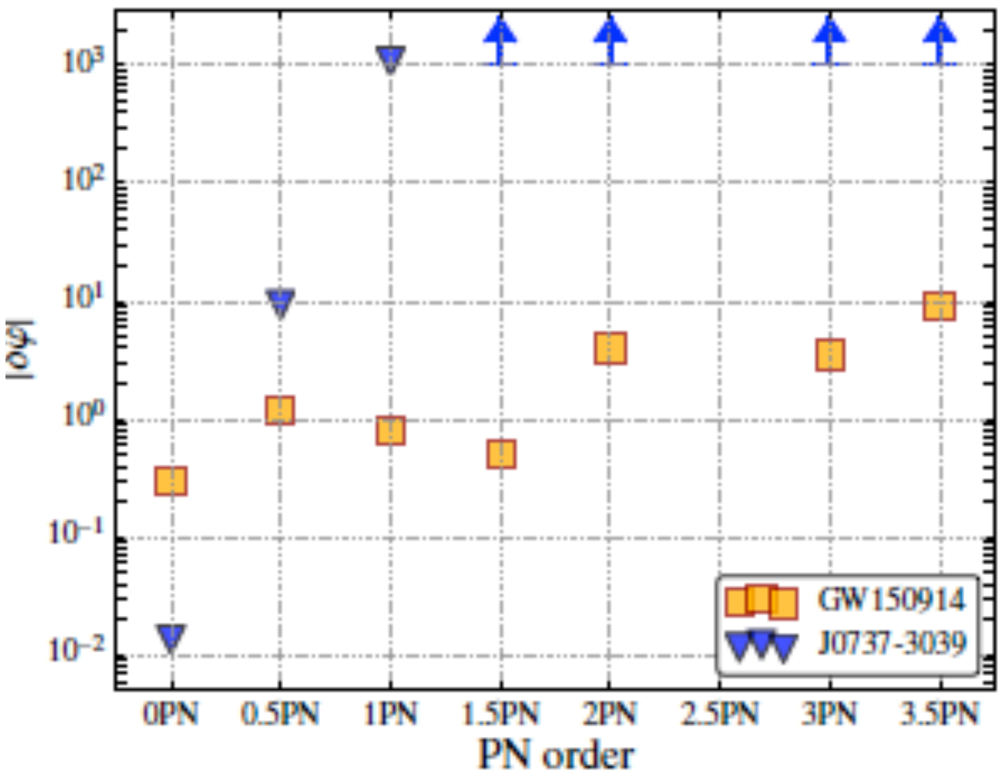
0.5PN, 1PN, 1.5PN, 2PN

$$(1 + \delta\varphi_{xPN})$$



OK, but not that exciting so far:

- poor PN expansion
- single event degeneracy

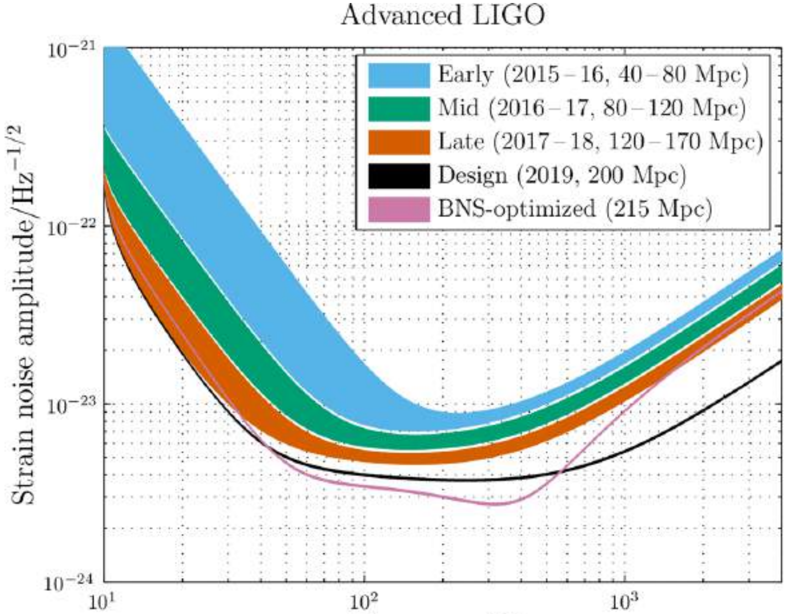
4. Conclusions and future prospects I: other events

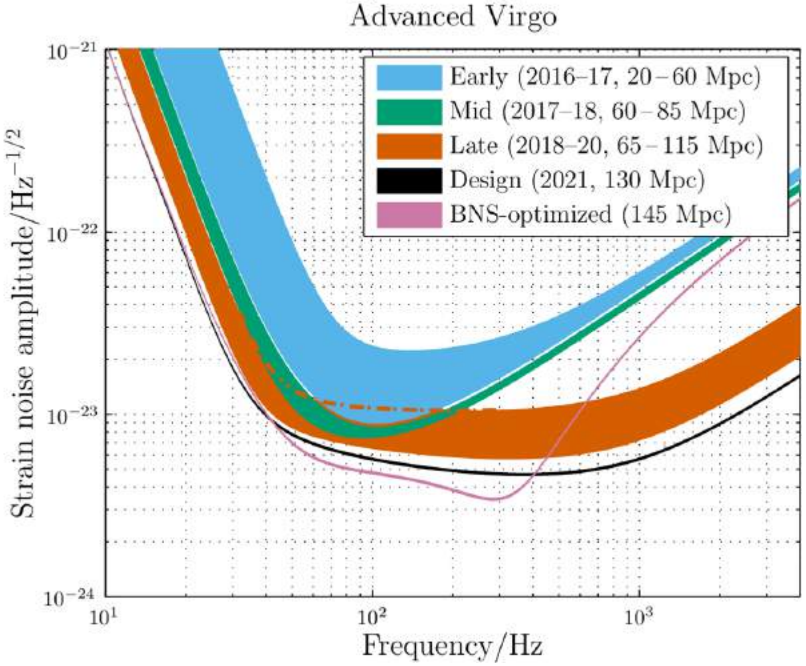
- more cycles: more precision in parameters determination.
- spin orientation: possible precession effects

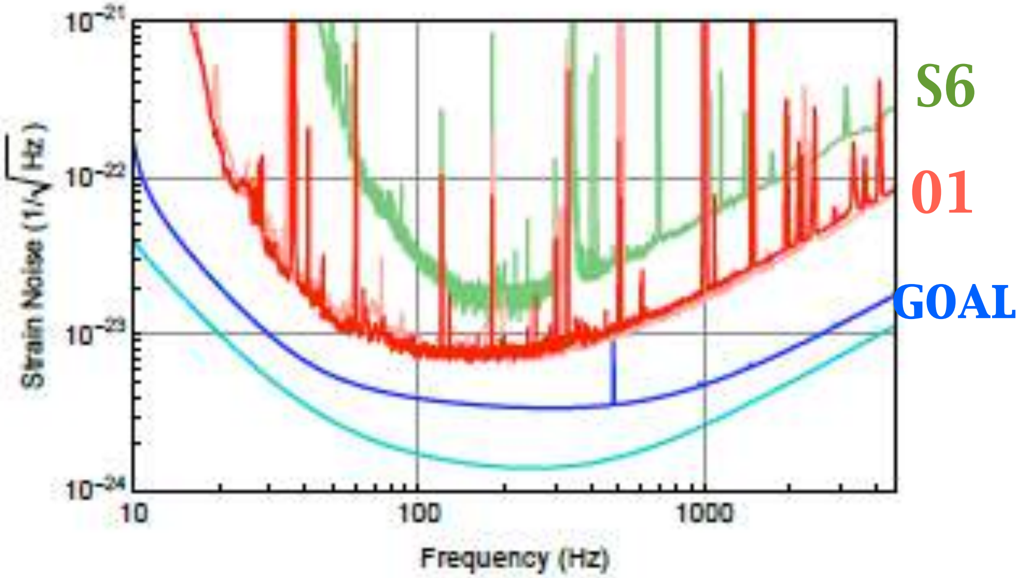
4. Conclusions and future prospects II: more detectors

- higher detection rate, good also for constraining PN deviations
- arrival direction
- degeneracy breaking in measured amplitude:
  - distance
  - polarisation

- Full 01 analysis released soon
- 02 with VIRGO starting fall 2016







- KAGRA operational around 2020
- LIGO-India approved