# Relativistic effects on LSS spectrum and bispectrum

Enea Di Dio

ED, Durrer, Marozzi, Montanari [arXiv:1407.0376]
ED, Durrer, Marozzi, Montanari [arXiv:1510.04202]
Irsic , ED, Viel [arXiv:1510.03436]
ED, Montanari, Raccanelli, Durrer, Kamionkowski, Lesgourgues [arXiv:1603.09073]

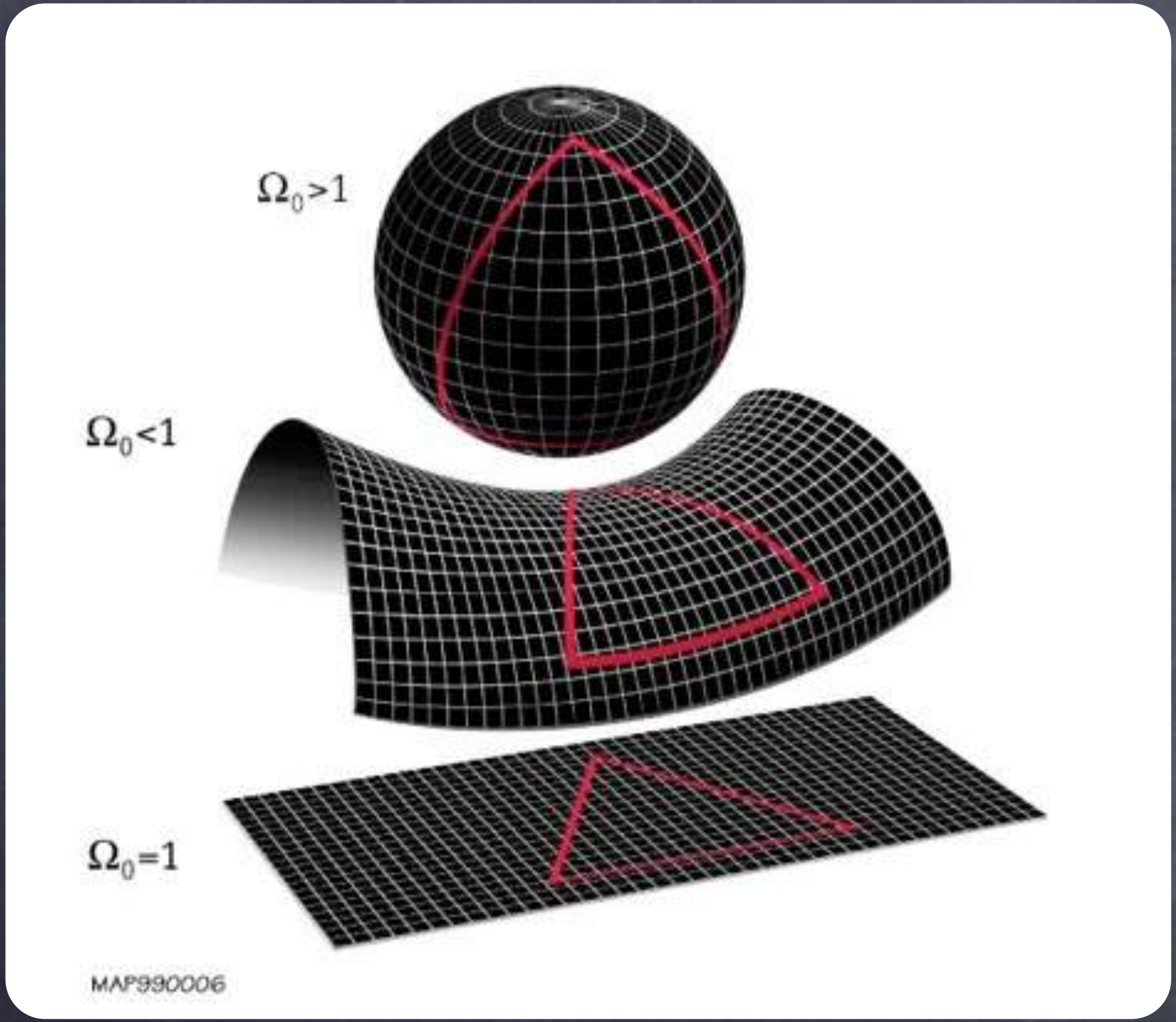
HIP seminar

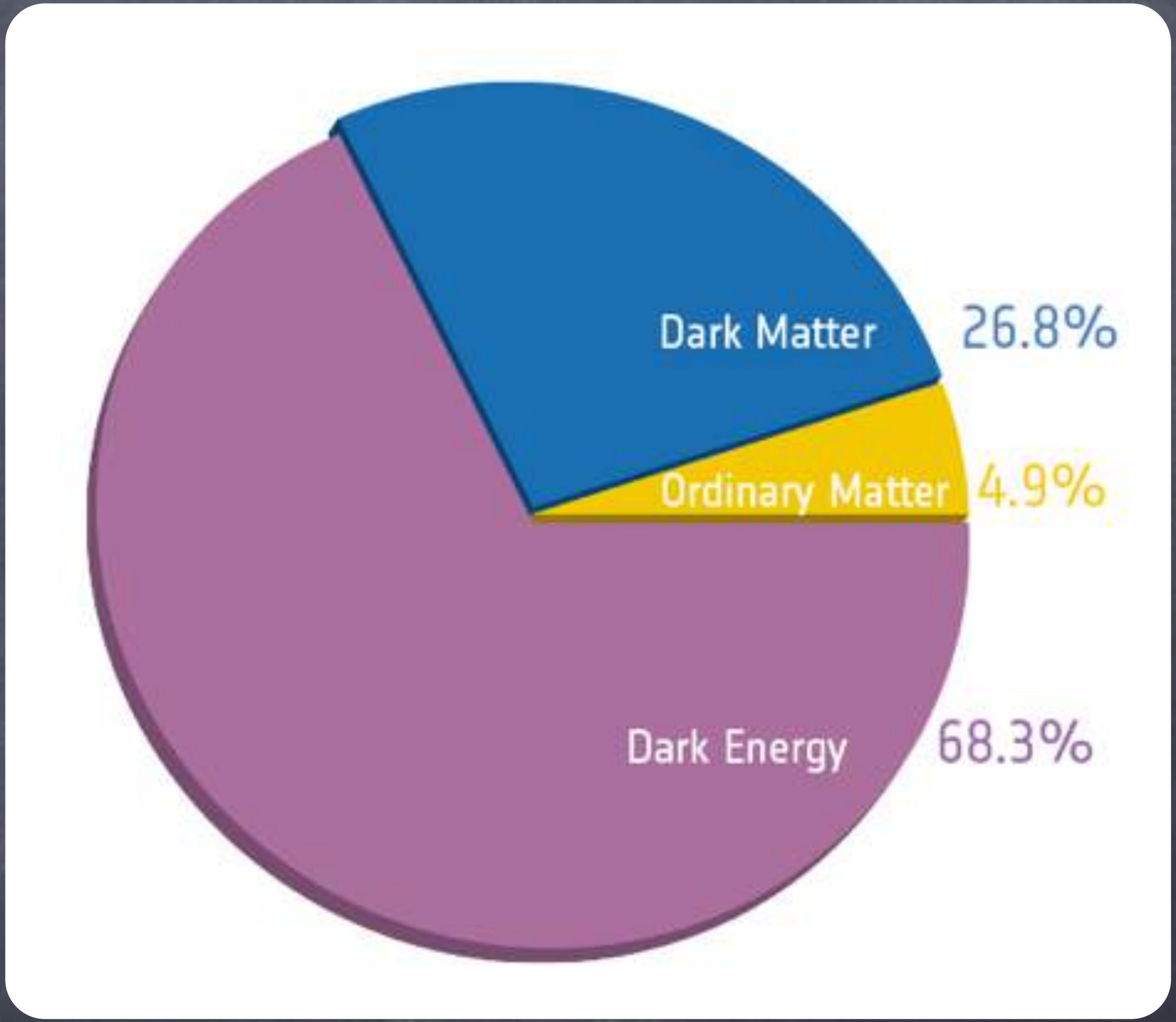
INAF
ISTITUTO NAZIONALE DI ASTROFISICA
NATIONAL INSTITUTE FOR ASTROPHYSICS

Helsinki, 25 May

INFN

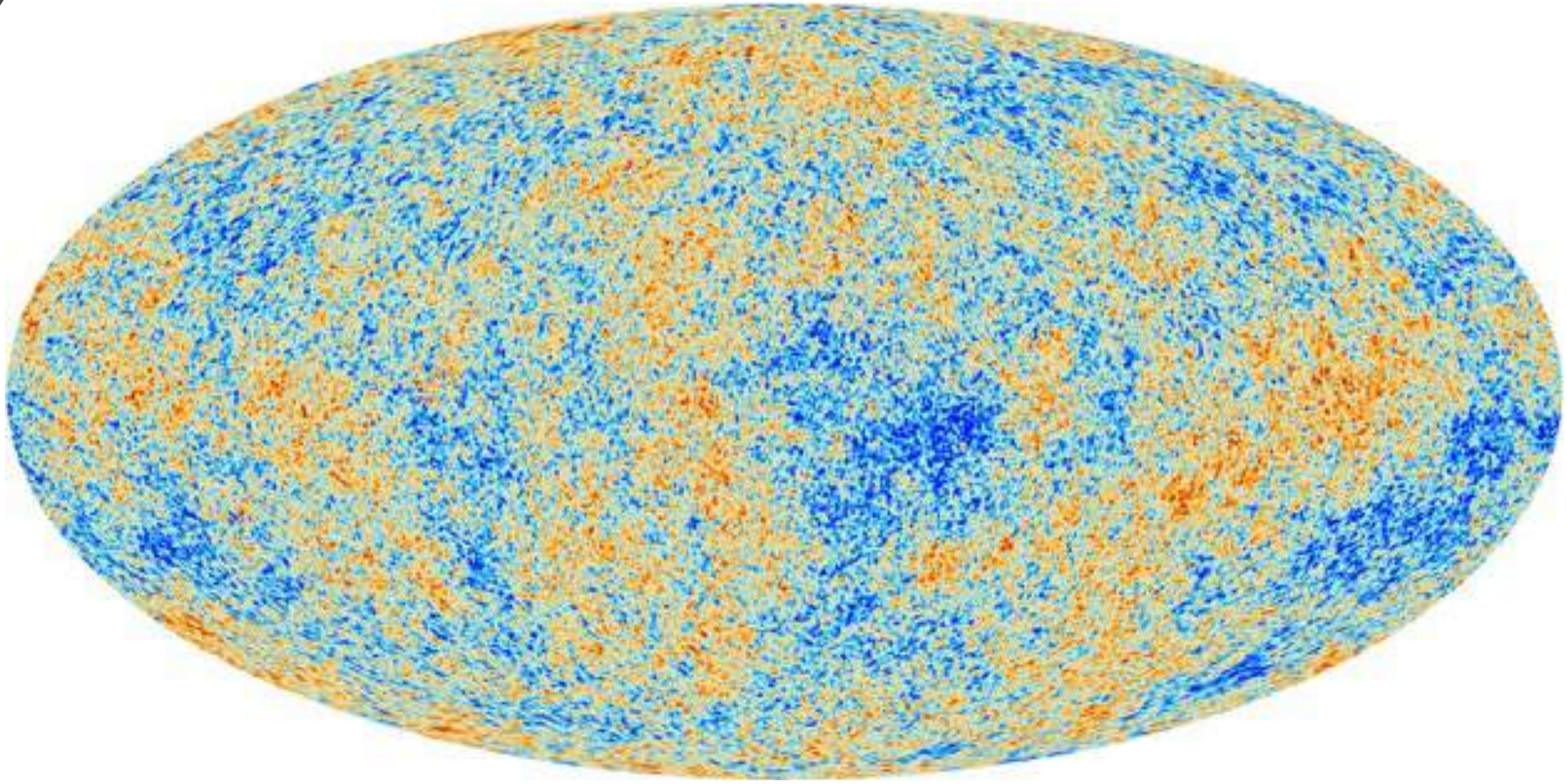
# Standard Cosmological Model



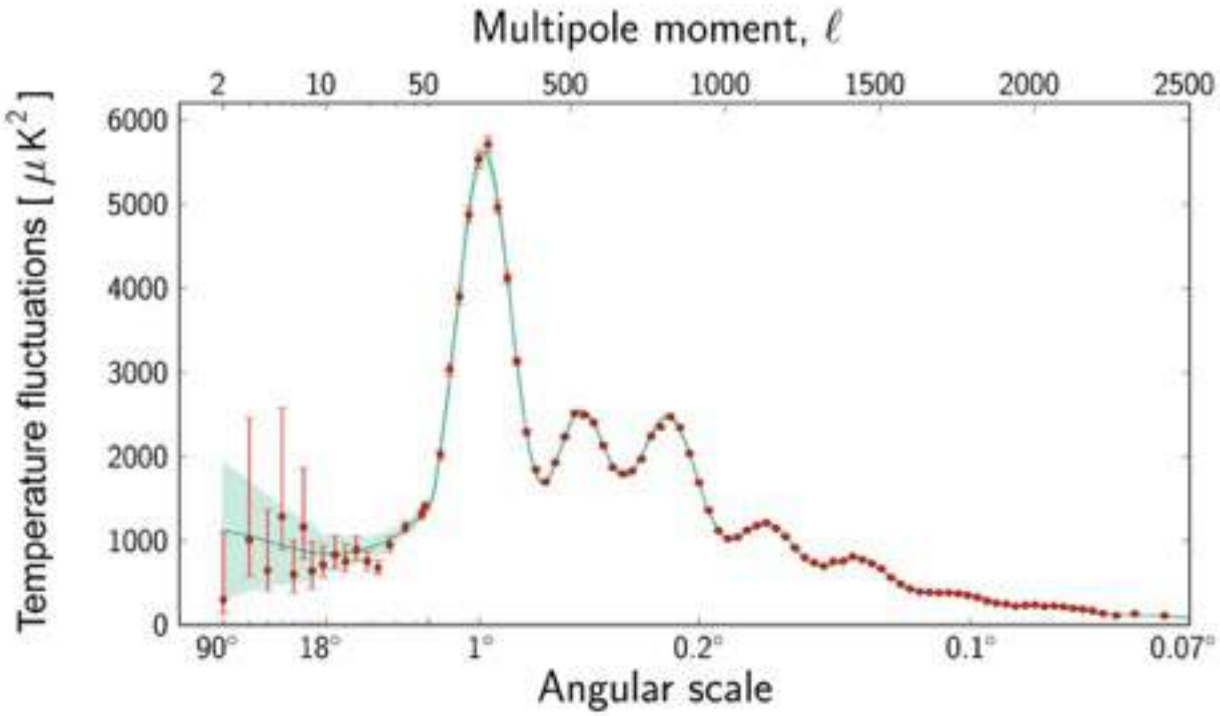




$$G_{\mu\nu} = \kappa T_{\mu\nu}$$

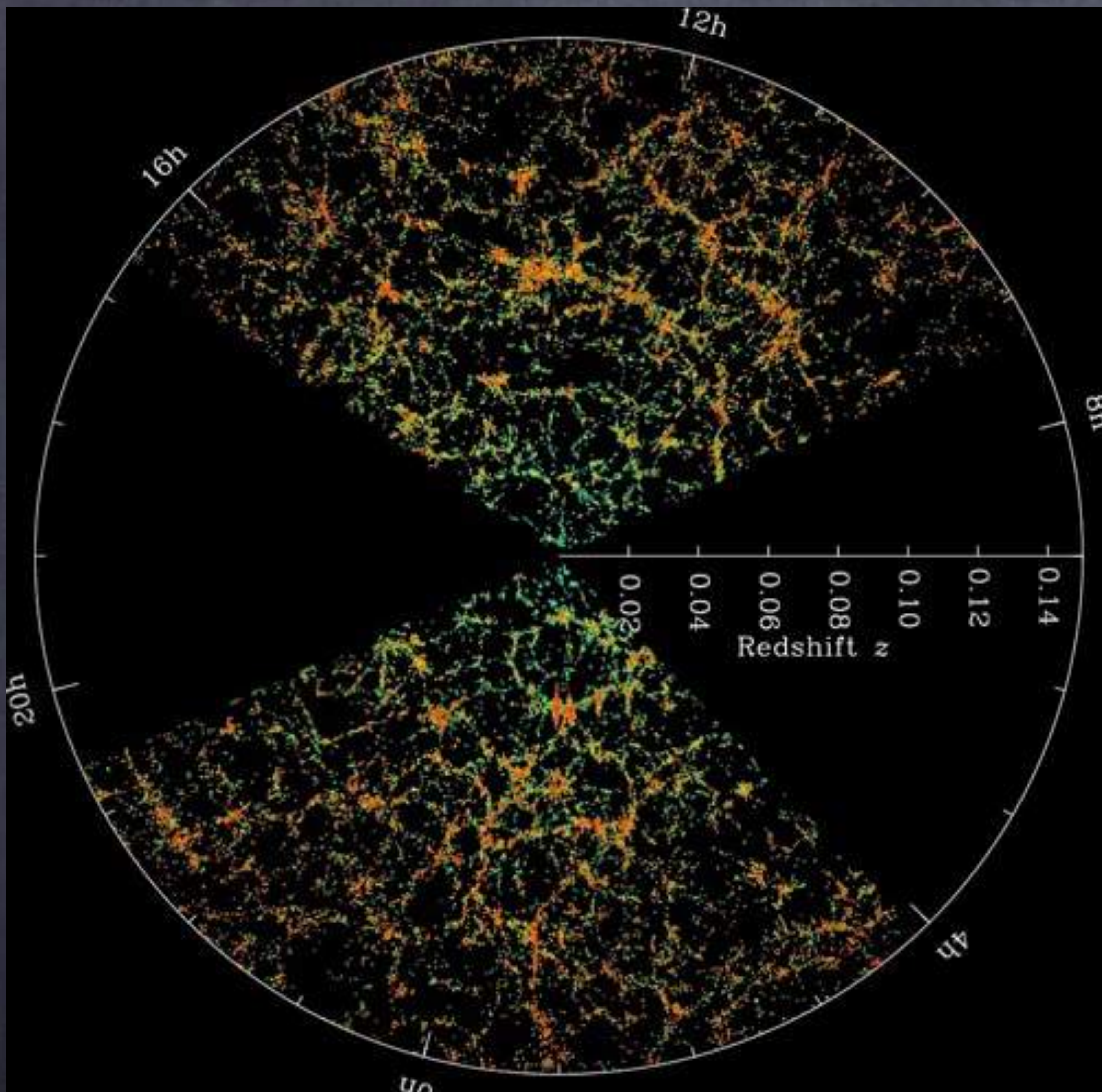
# CMB



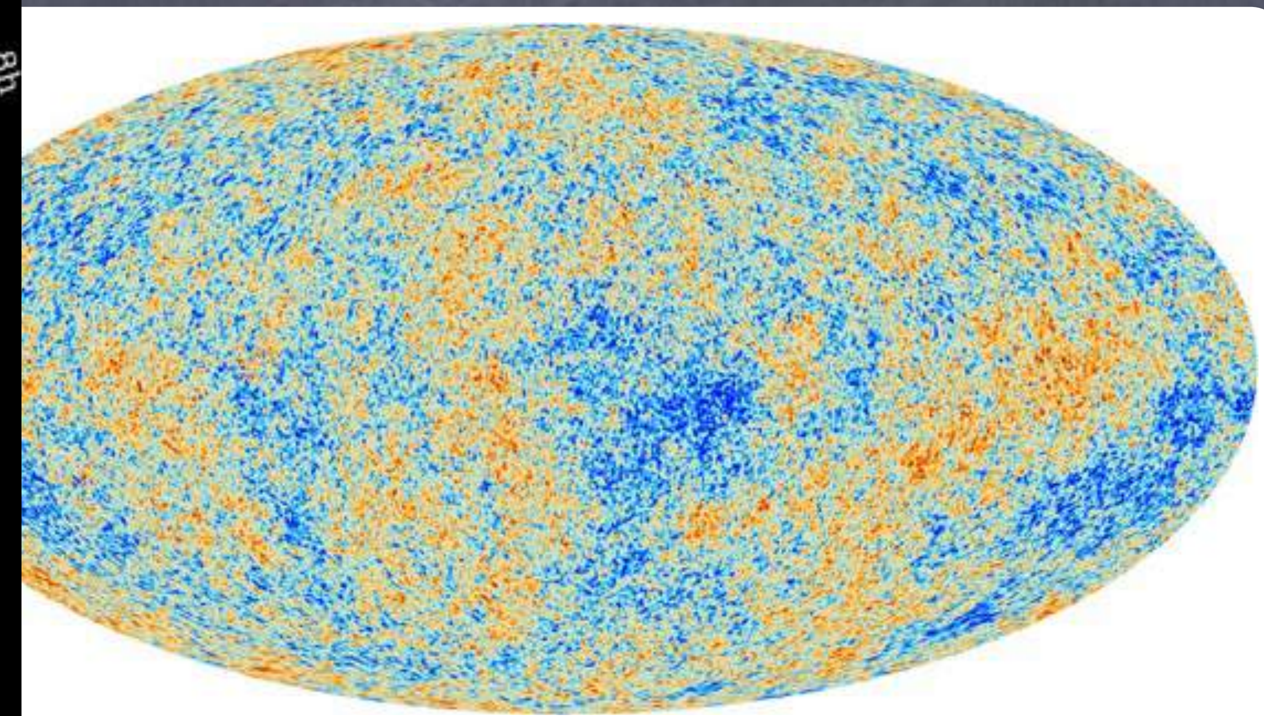
Planck Collaboration



# Large Scale Structures



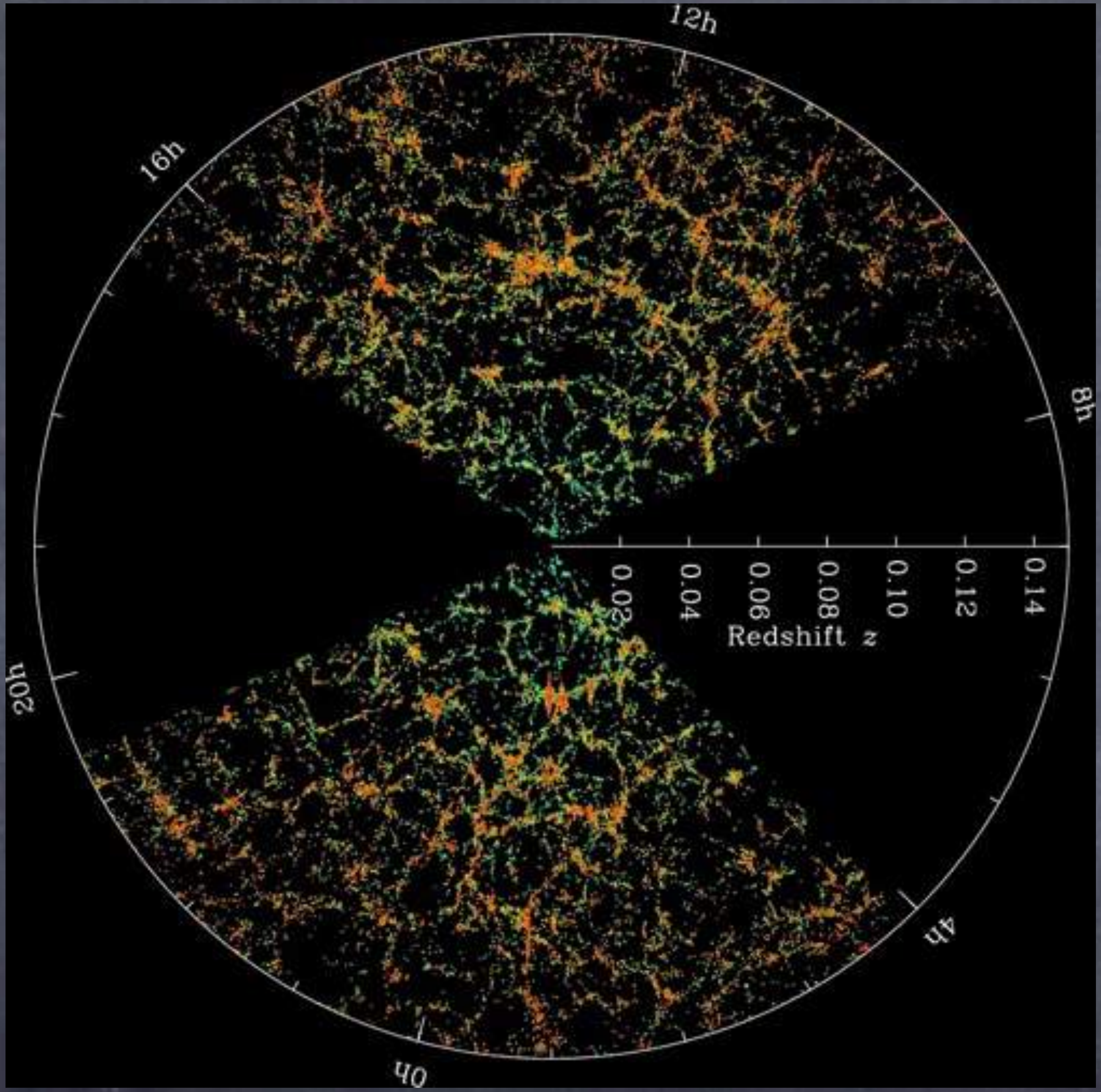

$$2\sum_{\ell=2}^{2500}(2\ell+1)\sim 10^7$$



Planck Collaboration

$$(3000)^3 = 2.7\times 10^{10}$$

# What do we really observe?




Angular position $\mathbf{n}$ Redshift $z$

$$N\left(\mathbf{n}, z\right) d\Omega_{\mathbf{n}} dz$$

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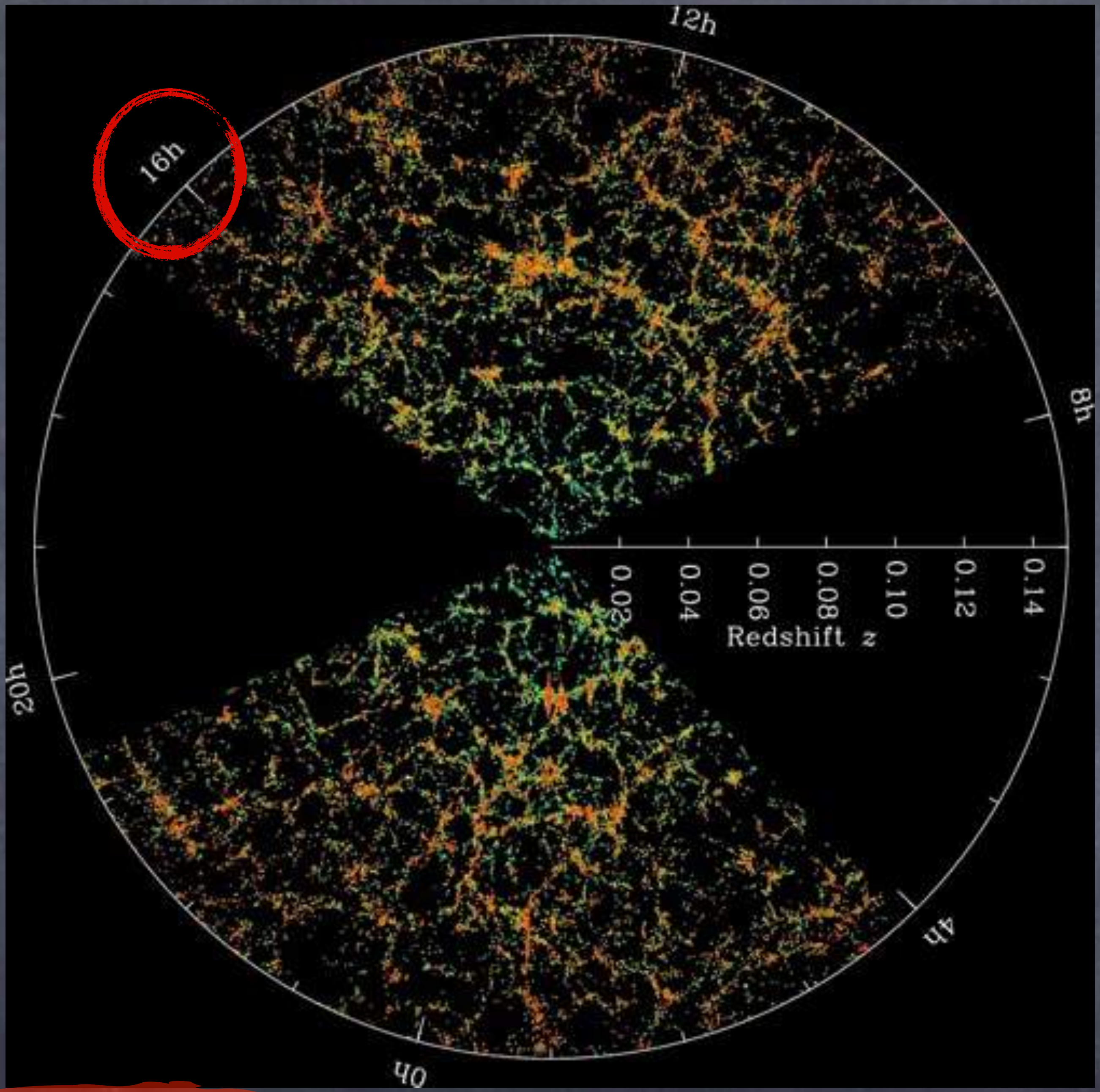


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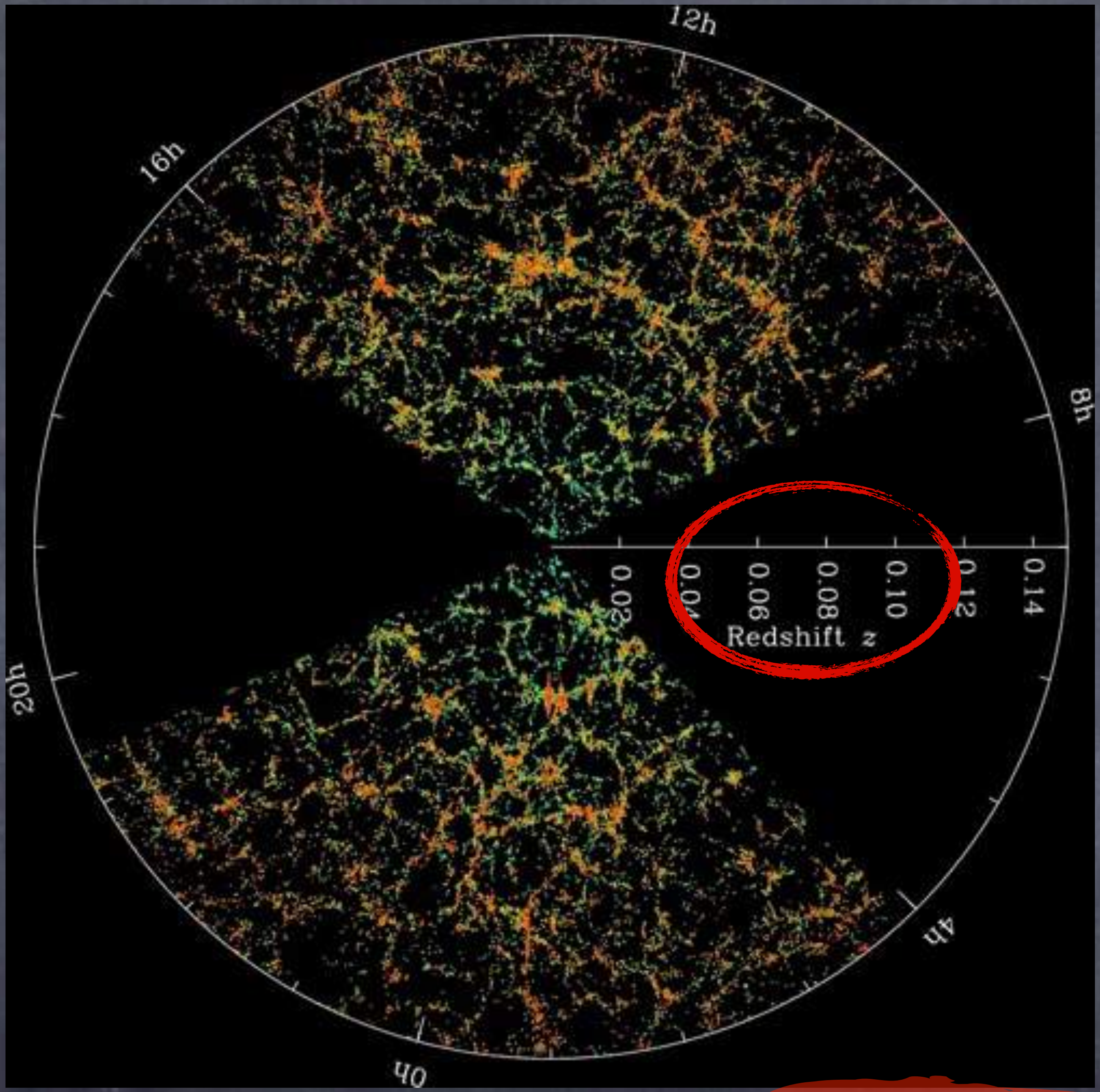


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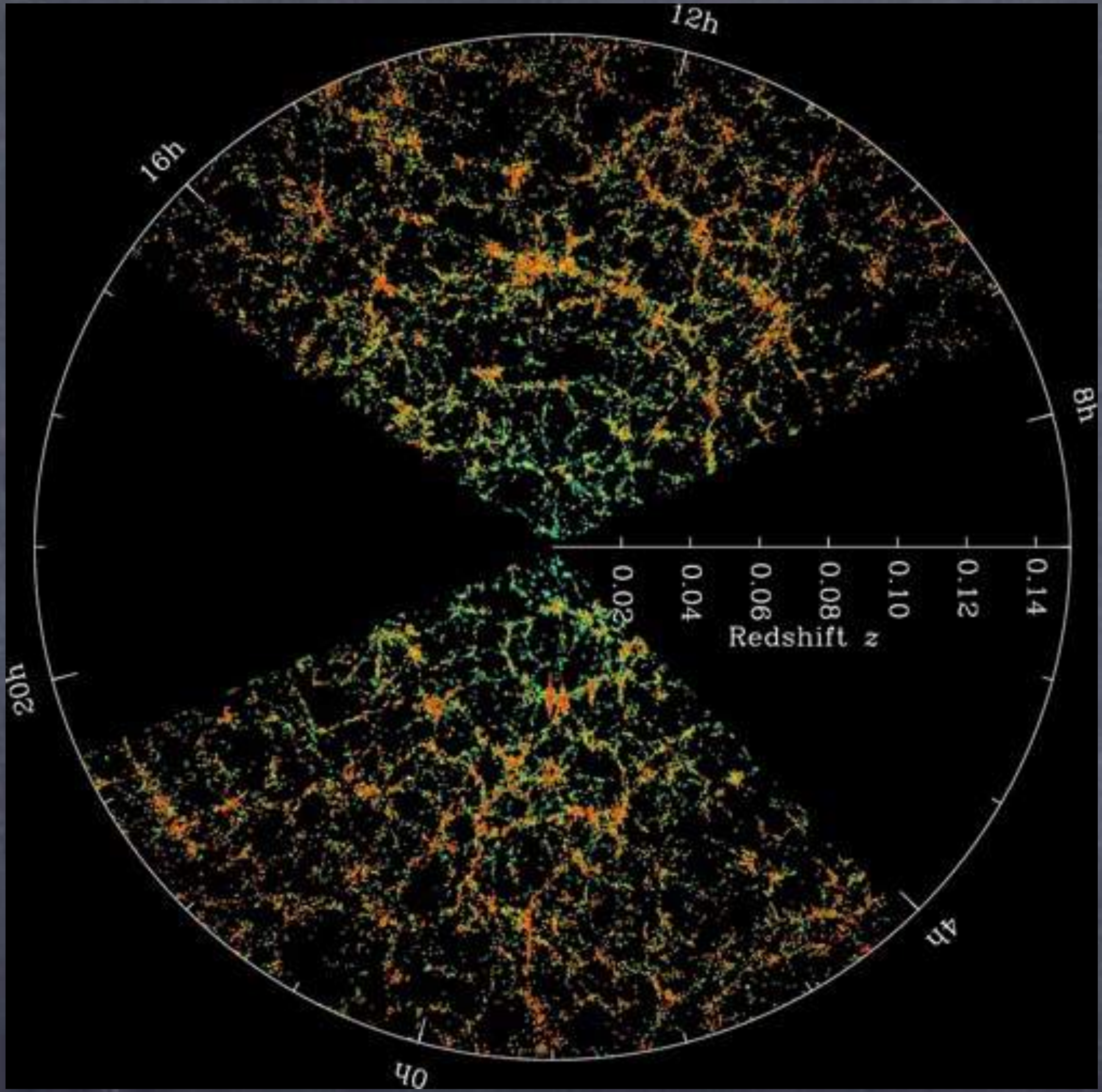


Info about mass, spectral type, ...

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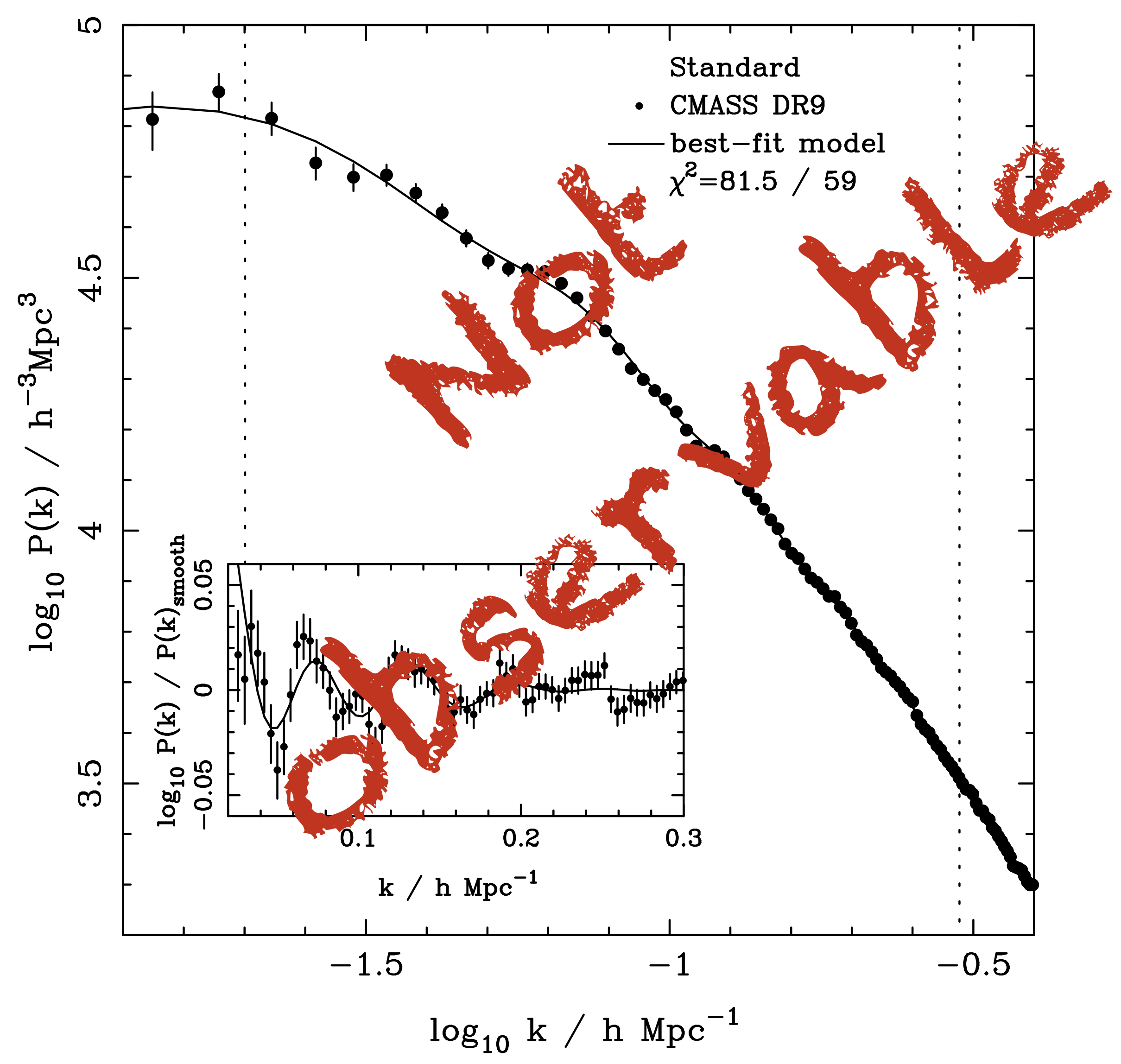
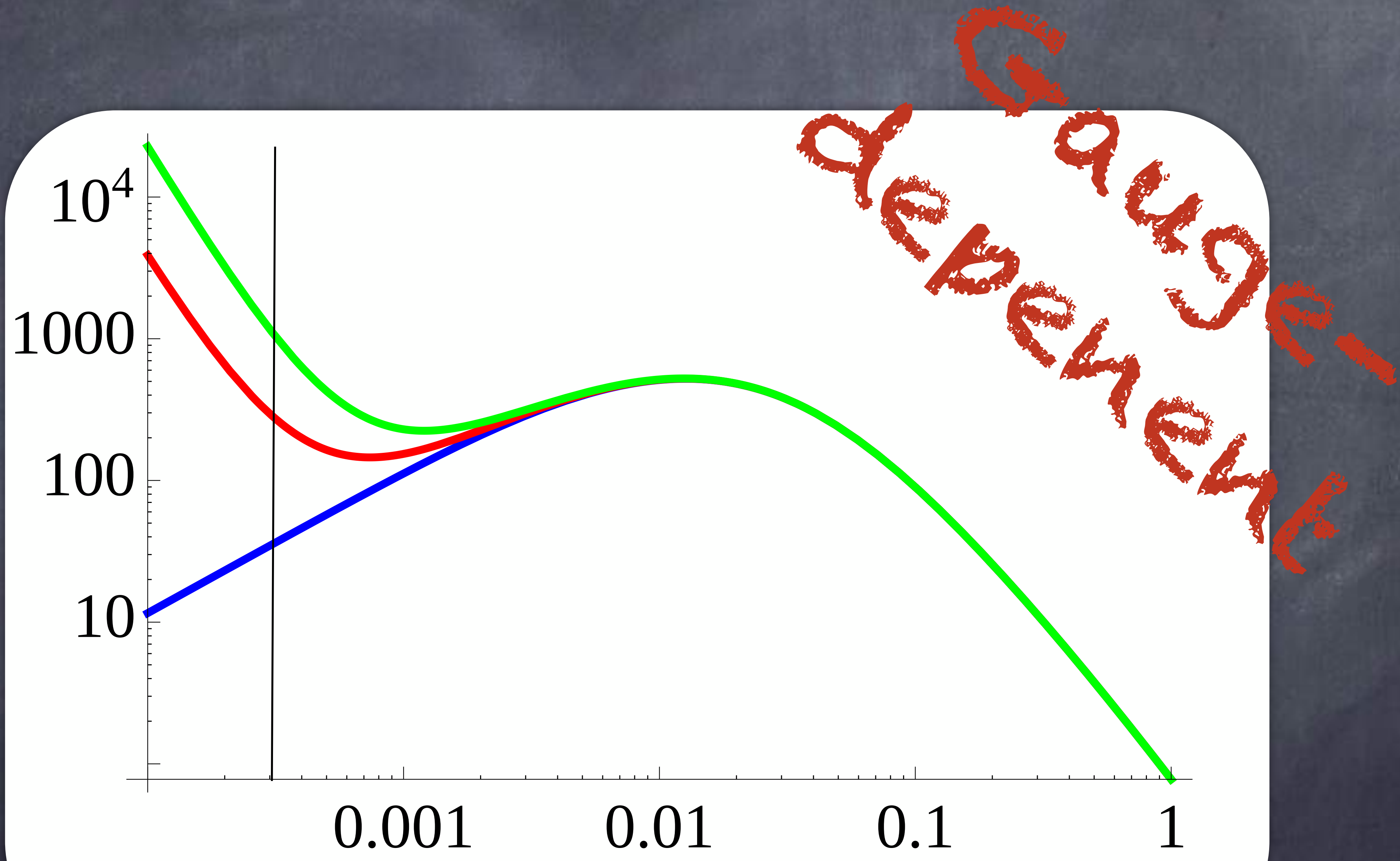
# Large Scale Structures




Anderson et al ’12 [arXiv:1203.6594]

$$P_{\rm obs}\left(k,\mu,z\right)=b\left(z\right)^2\left(1+\beta\left(z\right)\mu^2\right)^2P\left(k,z\right)$$

# Large Scale Structures




Bonvin & Durrer [arXiv:1105.5280]

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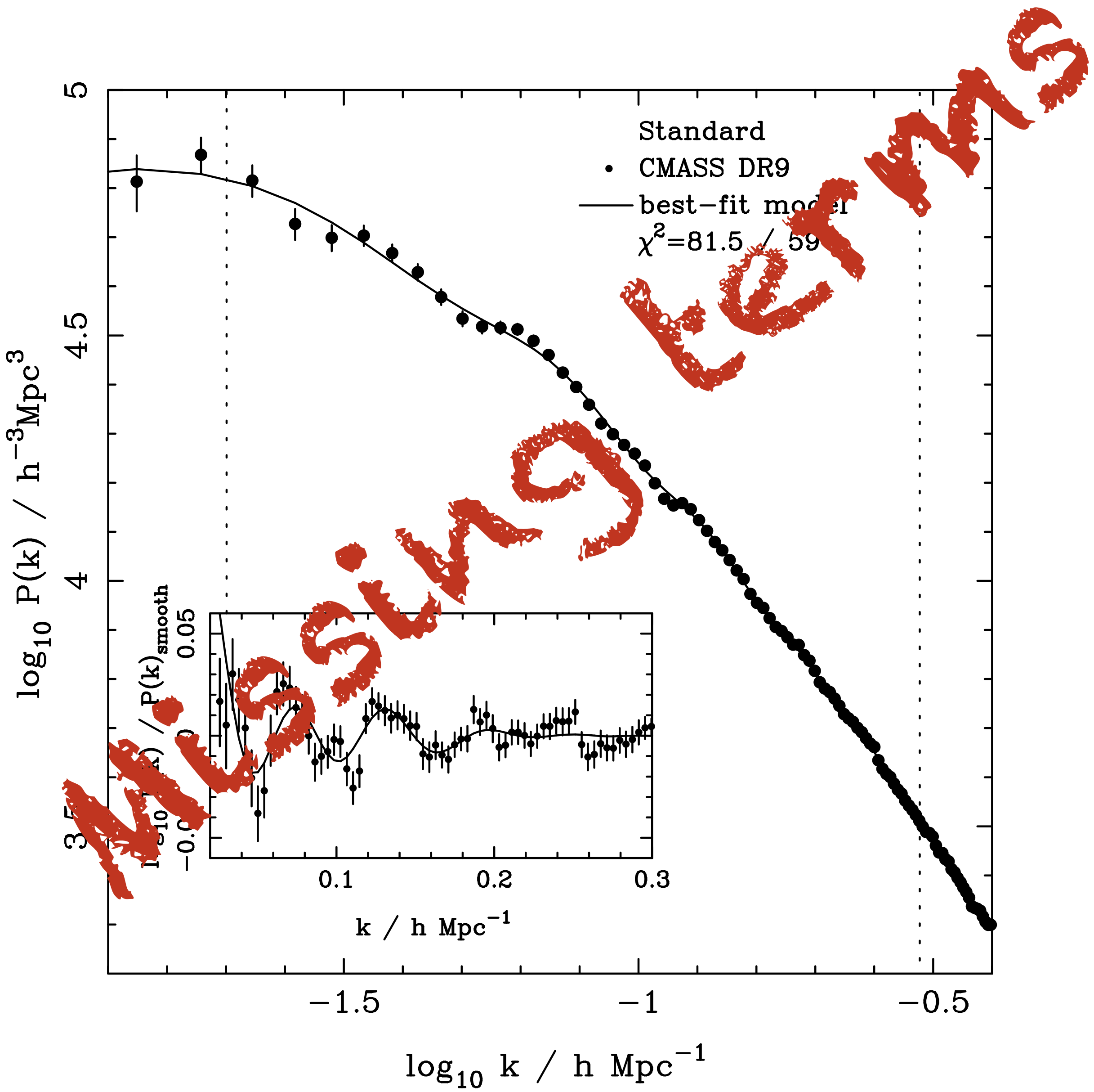


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# Large Scale Structures

$$\Delta_N(\mathbf{n}, z, m_*) = b(z) D_{cm}\left(L > \bar{L}_*\right) + \mathcal{H}^{-1}\partial_r \mathbf{n}\cdot\mathbf{v}$$

Standard

$$- \frac{2-5s}{2}\int_0^r \frac{r-r'}{rr'}\Delta_\Omega(\Psi+\Phi)\,dr'$$
$$+ \left(5s + \frac{2-5s}{\mathcal{H}r} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f^N_{\rm evo}\right)\mathbf{n}\cdot\mathbf{v} + (f^N_{\rm evo}-3)\mathcal{H}v$$
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Relativistic Effects

Bonvin & Durrer [arXiv:1105.5280],
Challinor & Lewis [arXiv:1105.5292],
Yoo [arXiv:1009.3021]

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To compute $\Delta(\mathbf{n}, z) \equiv \dfrac{N(\mathbf{n}, z) - <N>(z)}{<N>(z)}$
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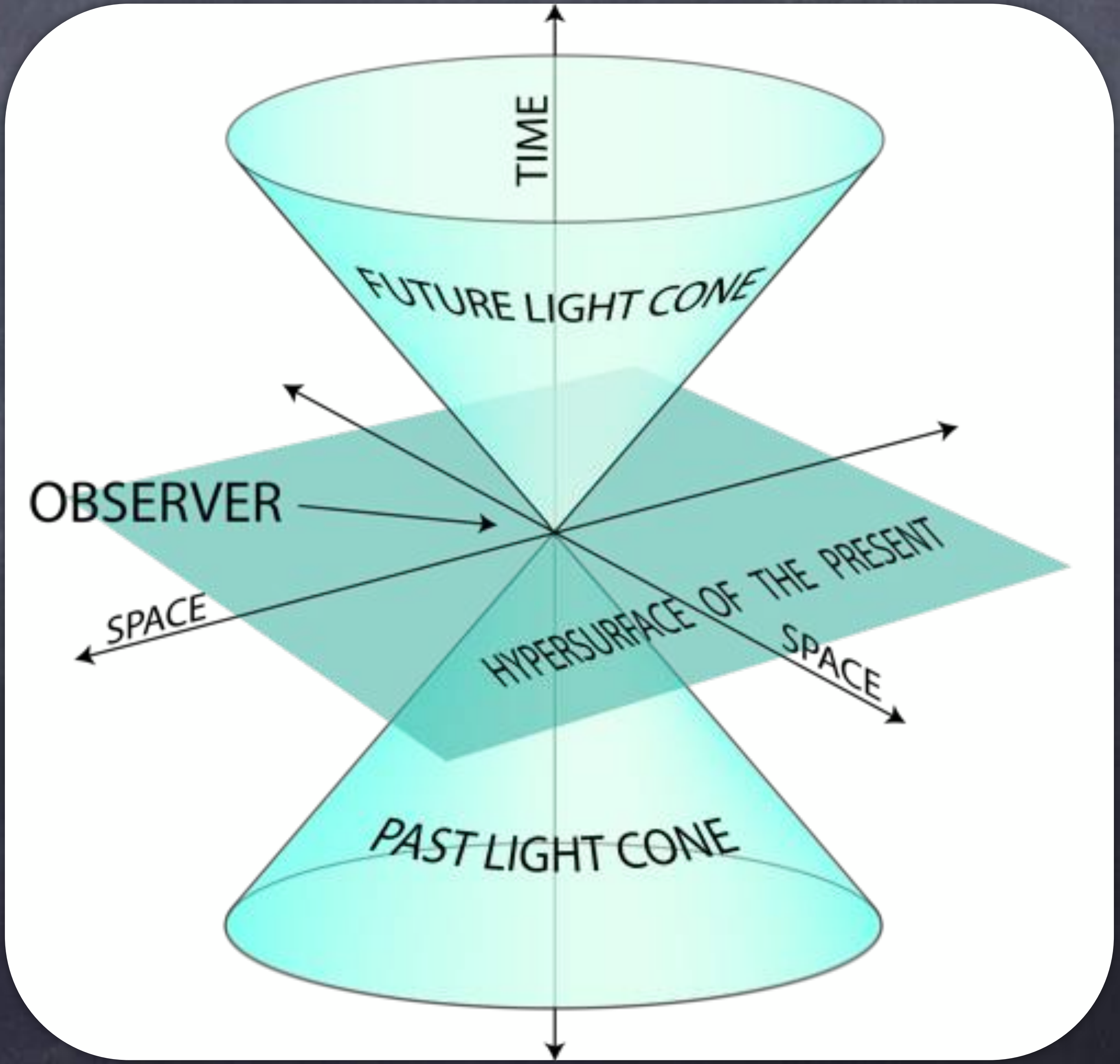

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To compute $\Delta(\mathbf{n}, z) \equiv \frac{N(\mathbf{n}, z) - <N>(z)}{<N>(z)}$
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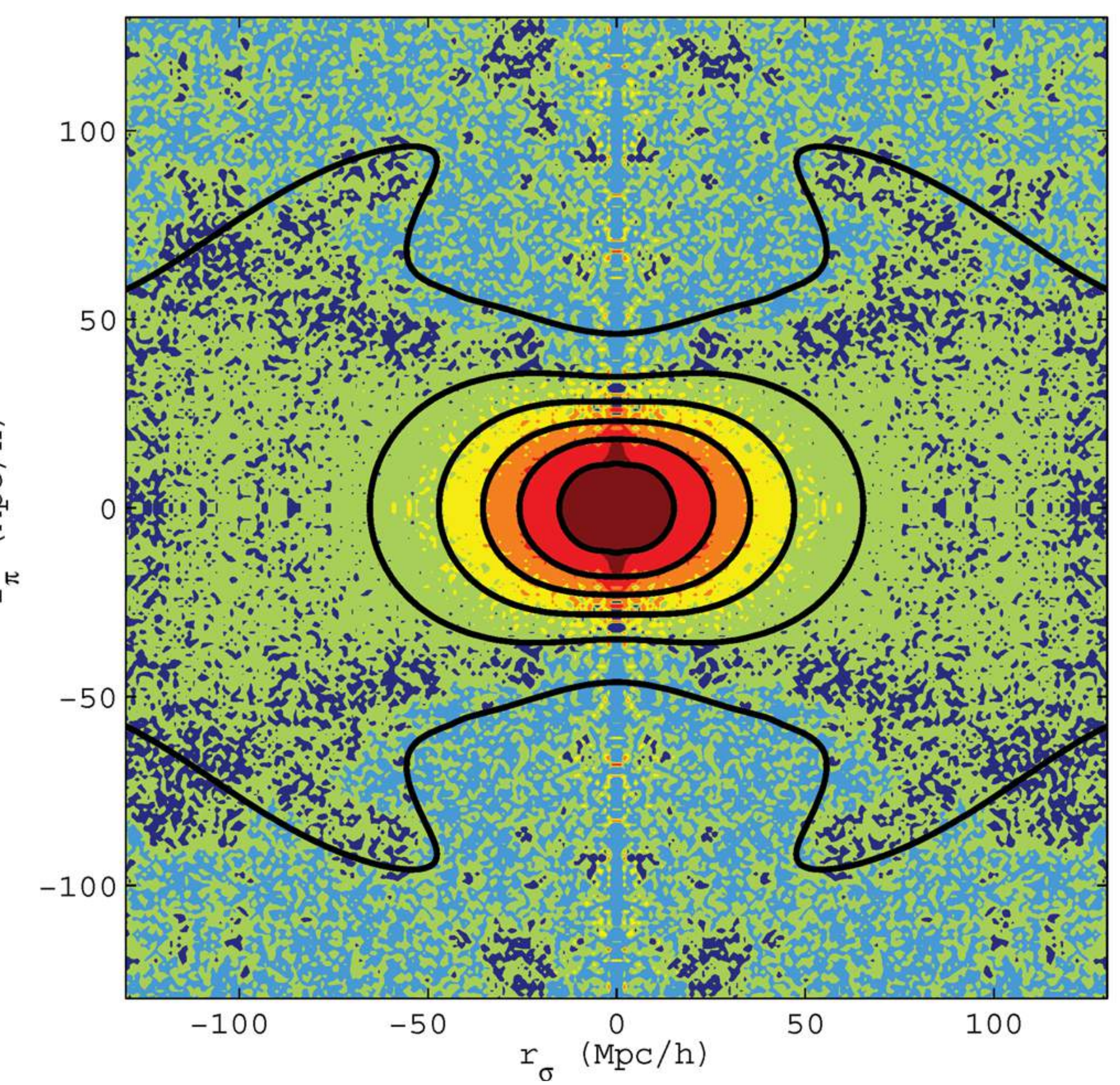


Reid et al ’12 [arXiv:1203.6641]

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To compute $\Delta(\mathbf{n}, z) \equiv \dfrac{N(\mathbf{n}, z) - <N>(z)}{<N>(z)}$
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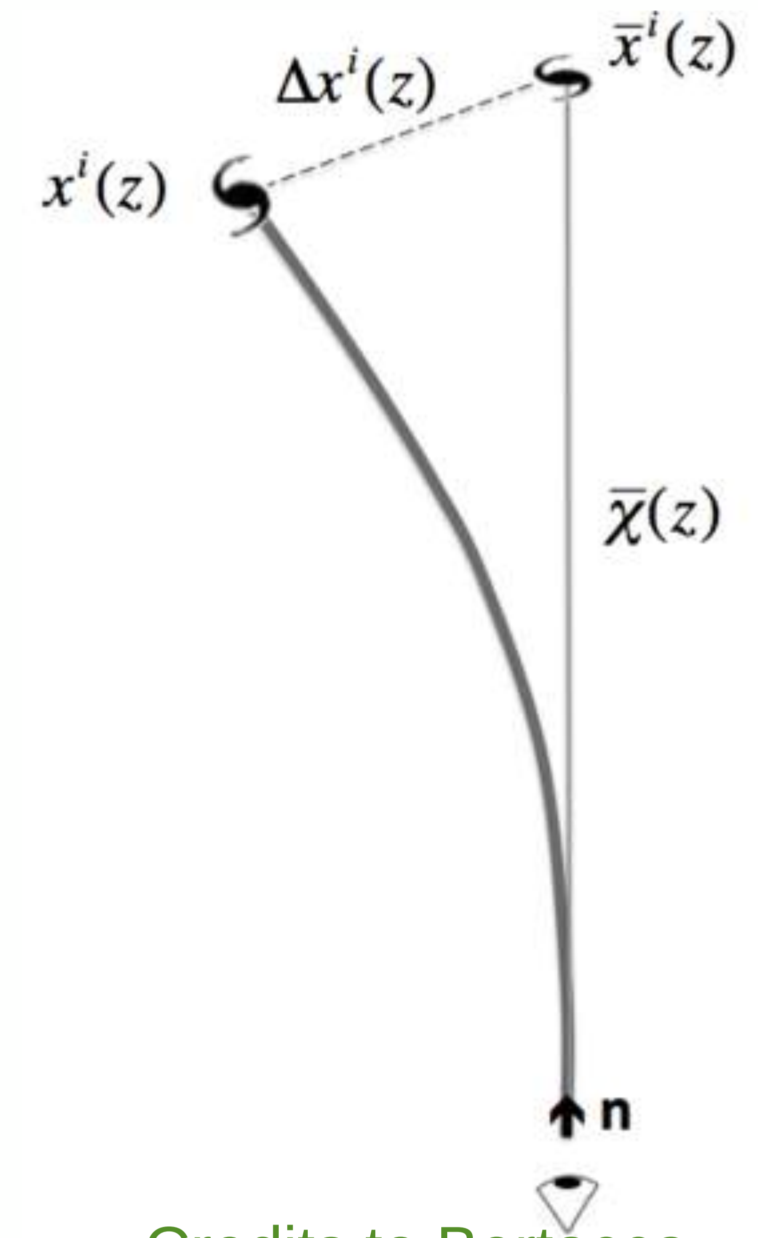
- redshift perturbed by peculiar velocity
- light deflection




Credits to Bertacca

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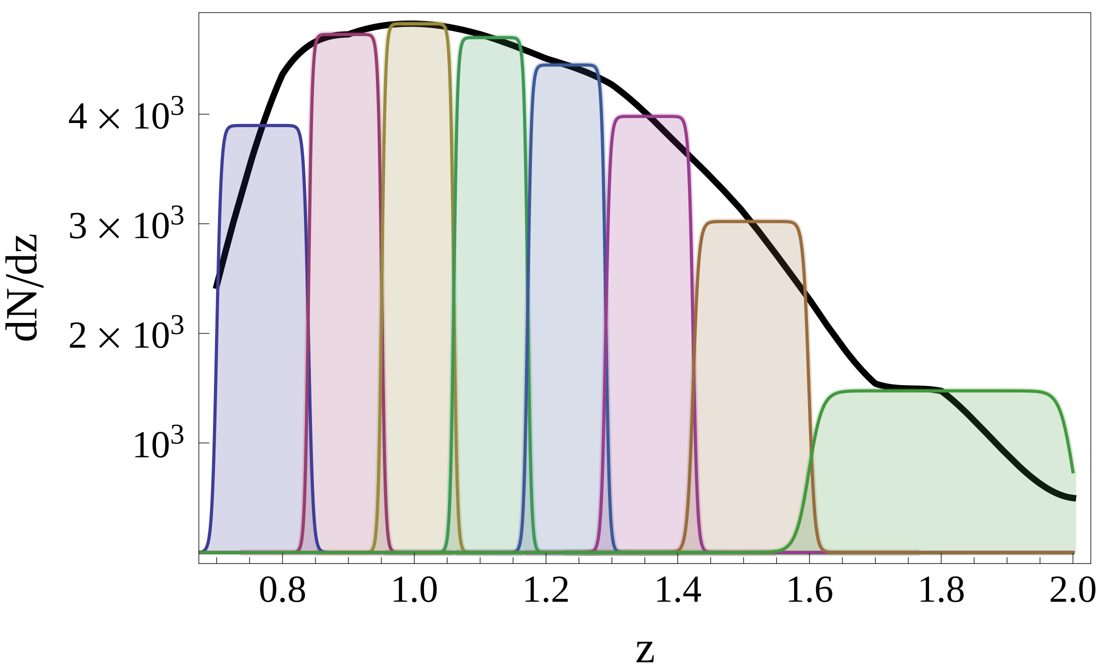
$\rightarrow$ $c_\ell\left(z_1, z_2\right)$

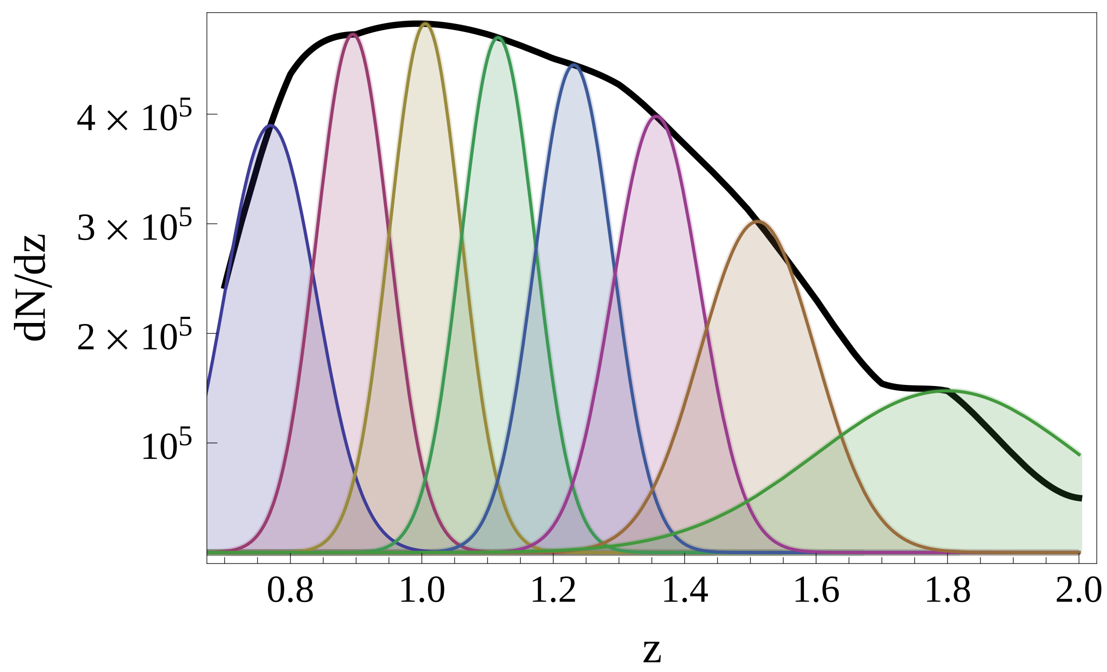
# Binning Strategy

To recover the 3D information we need to split the redshift range in many bins and to consider the cross-correlations between different redshift bins.

Spectroscopic Survey




Photometric Survey

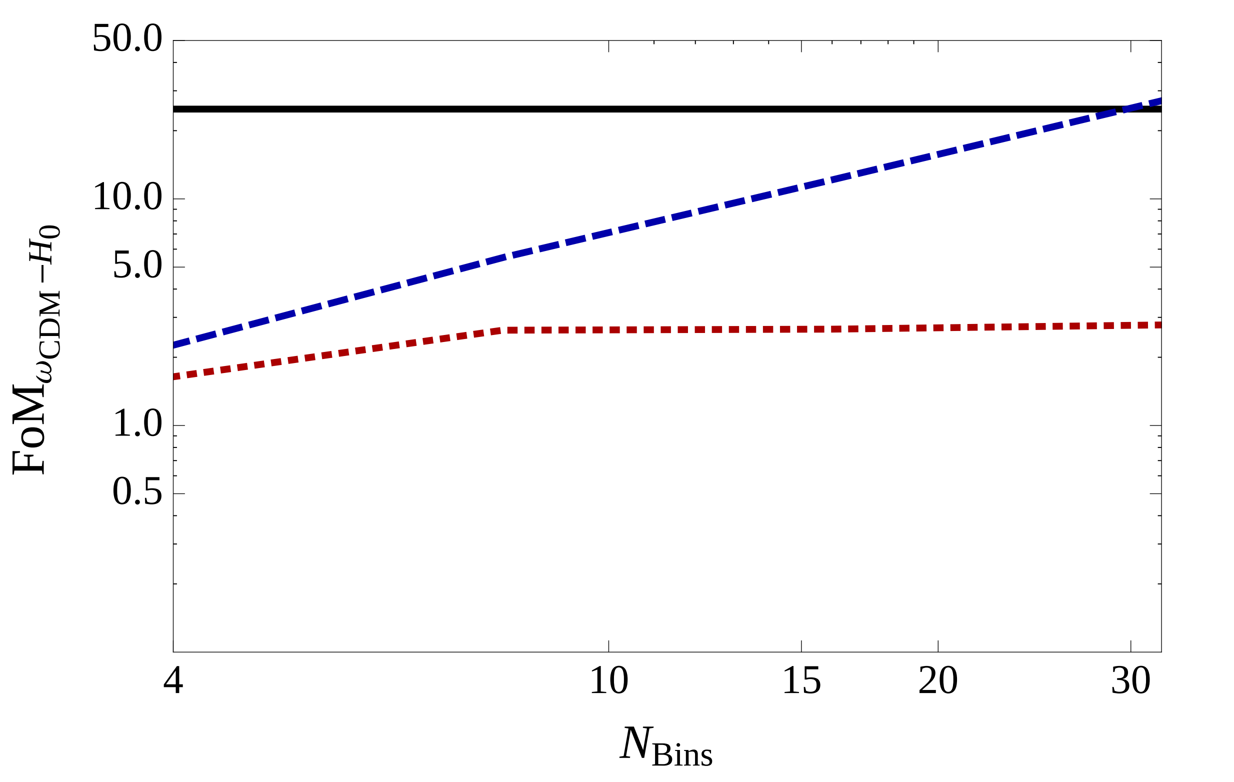



ED, Montanari, Durrer, Lesgourgues [arXiv:1308.6186]

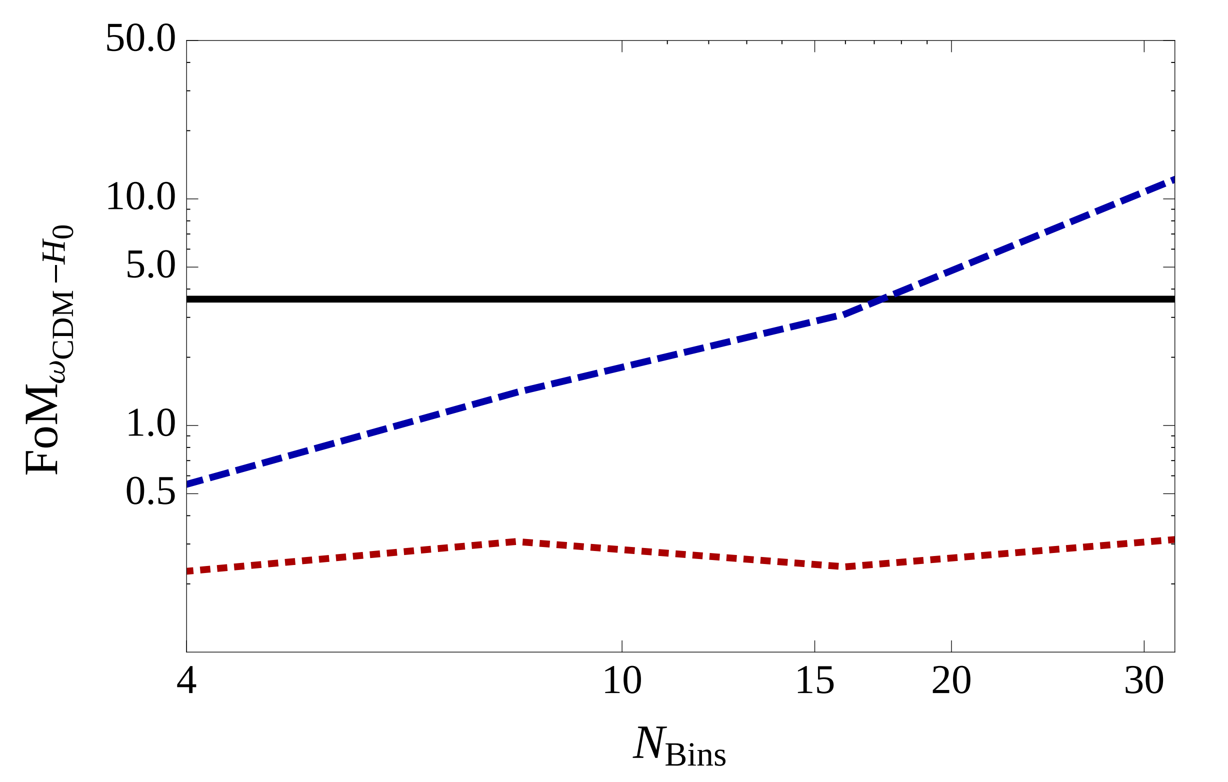
# Cosmological Parameter Forecast

## spectroscopic DES-like

$\lambda_{\min} = 34\ \mathrm{Mpc}/h$




$\lambda_{\min} = 68\ \mathrm{Mpc}/h$




only redshift bins auto-correlations
with redshift bins cross-correlations

ED, Montanari, Durrer, Lesgourgues [arXiv:1308.6186]

# Cosmological Parameter Forecast

## 2D vs 3D

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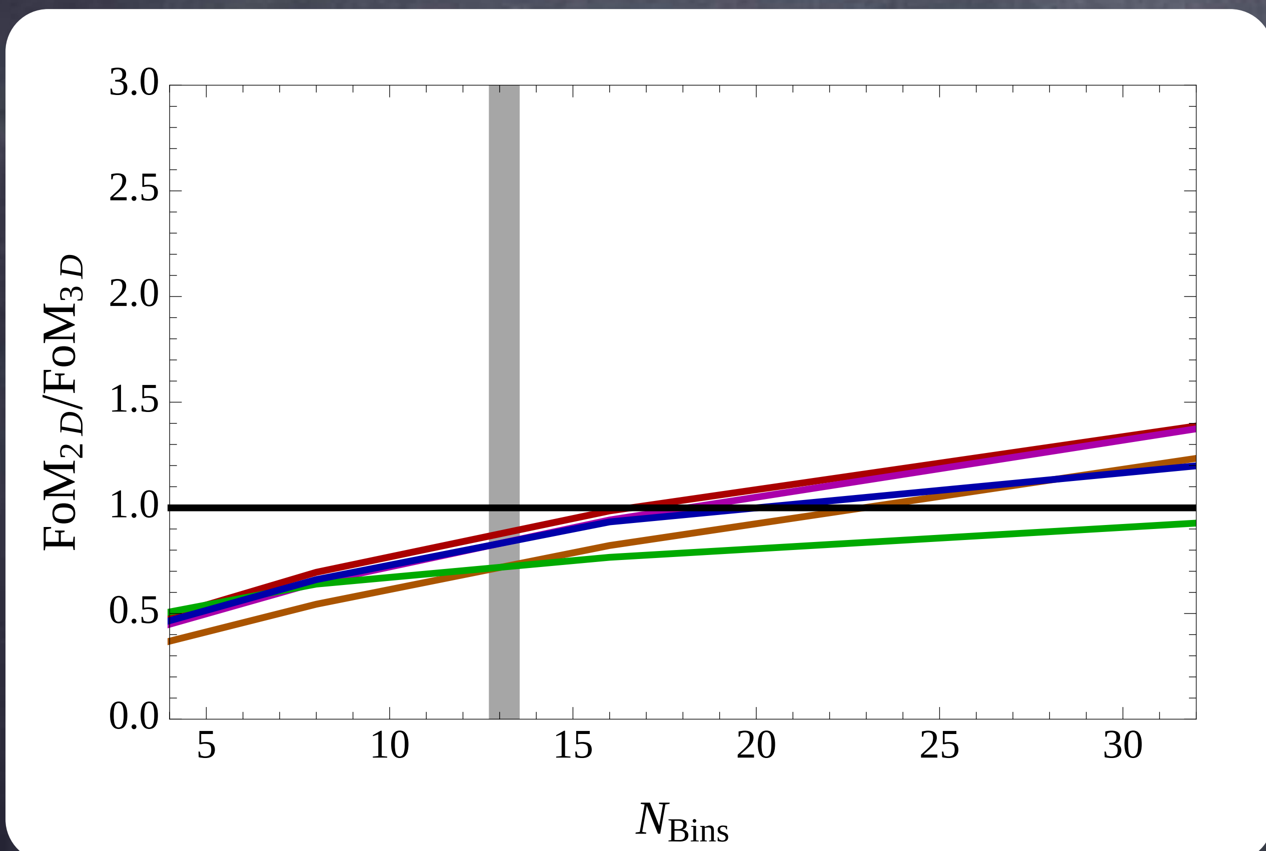


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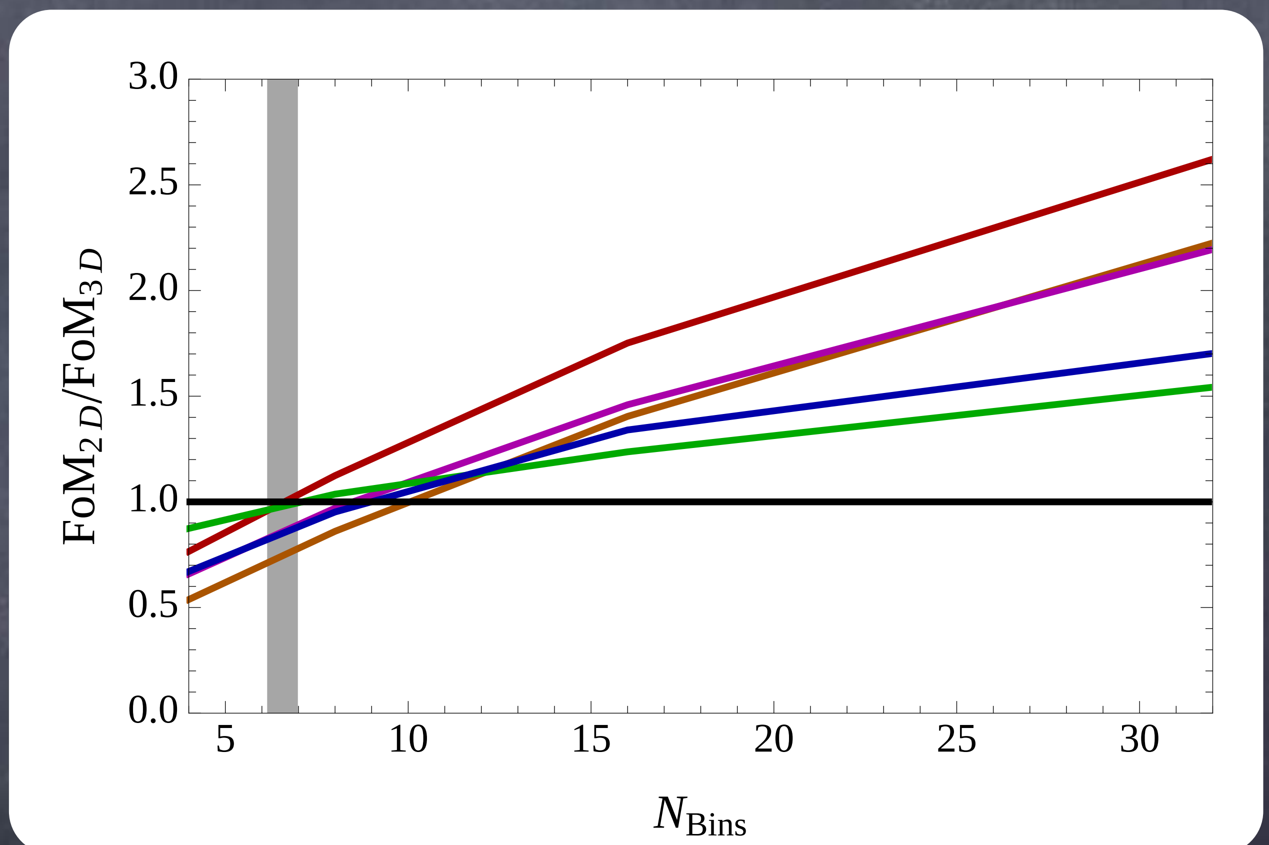


$\omega_b$ $\omega_{\rm CDM}$ $n_s$ $H_0$ $A_s$

ED, Montanari, Durrer, Lesgourgues [arXiv:1308.6186]

# Large Scale Structures

$$\Delta_N(\mathbf{n}, z, m_*) = b(z) D_{cm}\left(L > \bar{L}_*\right) + \mathcal{H}^{-1}\partial_r \mathbf{n}\cdot\mathbf{v} - \frac{2-5s}{2}\int_0^r \frac{r-r'}{rr'}\Delta_\Omega(\Psi+\Phi)\,dr'$$

$\sim D$

$$+ \left(5s + \frac{2-5s}{\mathcal{H}r} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f^N_{\rm evo}\right)\mathbf{n}\cdot\mathbf{v} + (f^N_{\rm evo} - 3)\mathcal{H}v$$

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Bonvin & Durrer [arXiv:1105.5280],
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$$\sim \frac{\mathcal{H}}{k} D \qquad + \left(5s + \frac{2-5s}{\mathcal{H}r} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f^N_{\mathrm{evo}}\right)\mathbf{n}\cdot\mathbf{v} + (f^N_{\mathrm{evo}} - 3)\mathcal{H}v$$

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\end{aligned}
$$

$$\sim \left(\frac{\mathcal{H}}{k}\right)^2 D$$

Bonvin & Durrer [arXiv:1105.5280],
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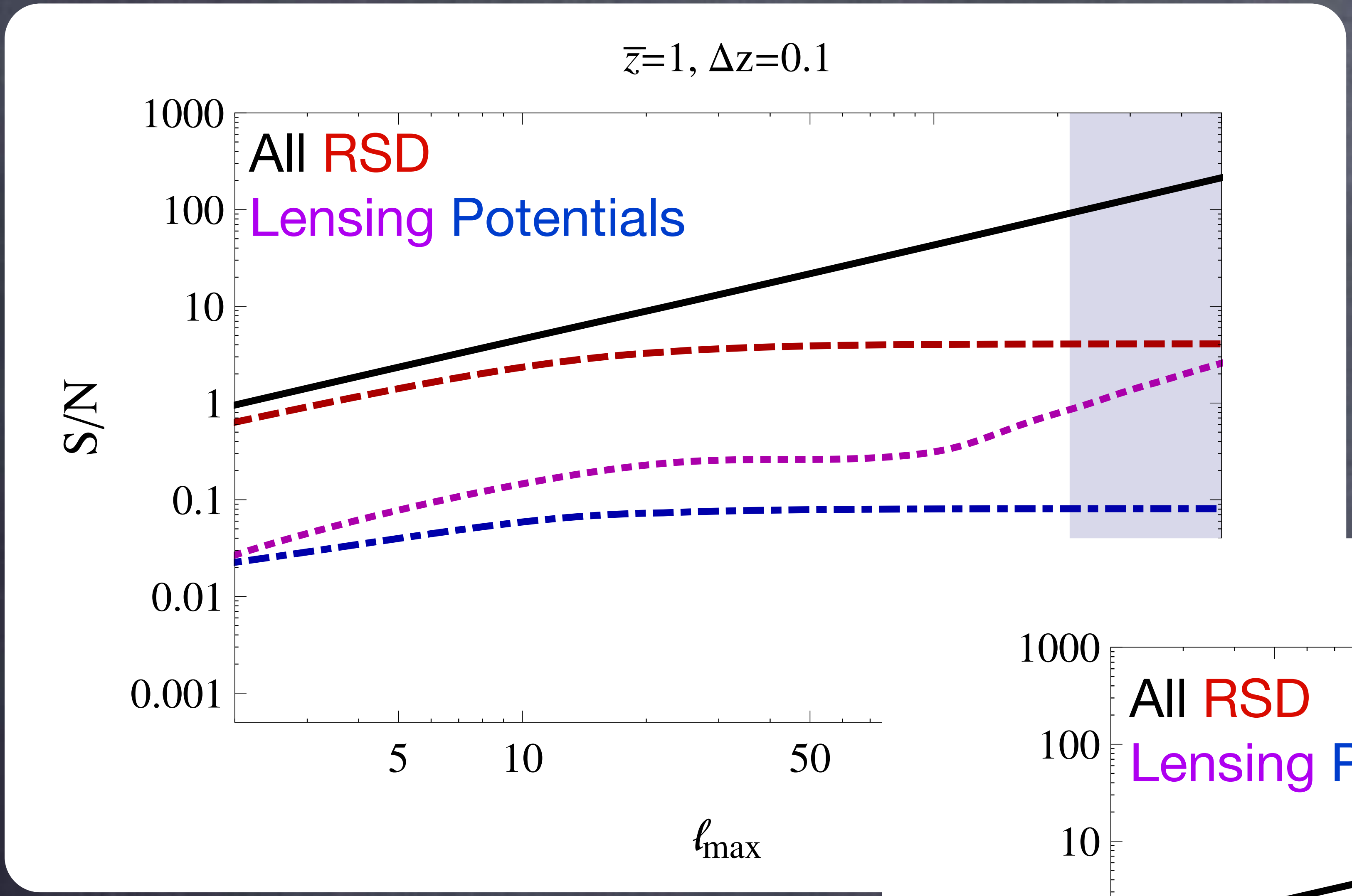
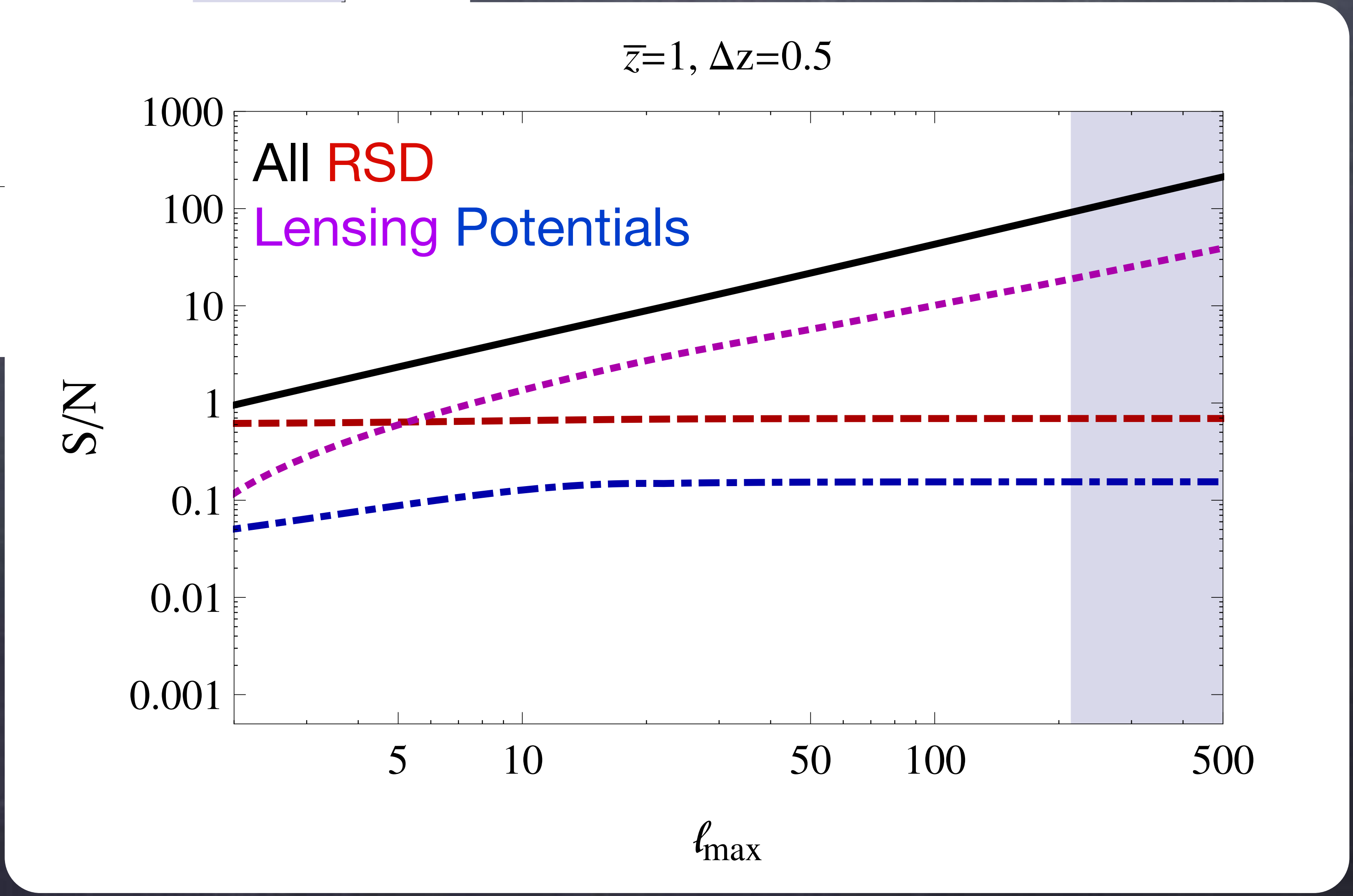
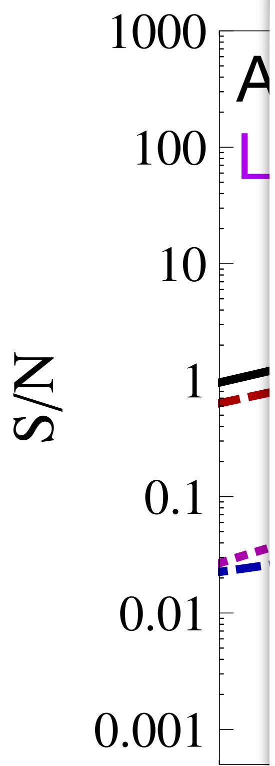
# Lensing Potential

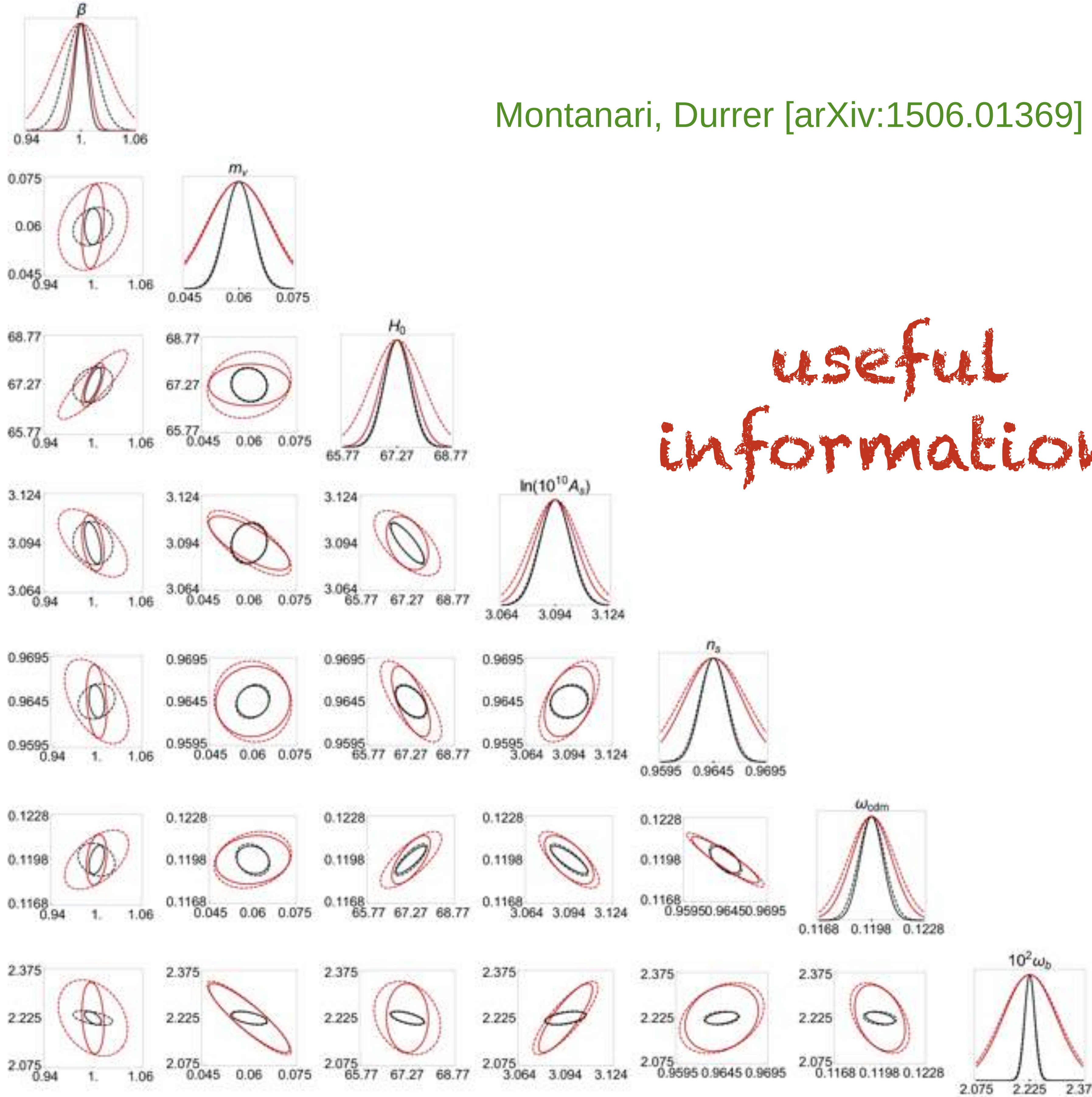



ED, Montanari, Durrer, Lesgourgues, [arXiv:1308.6186]

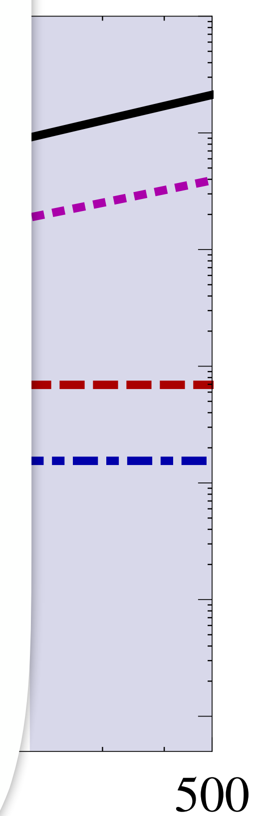


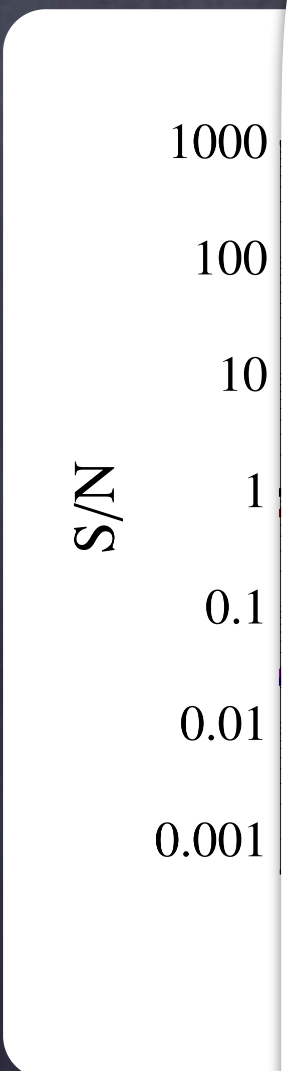

ED, Montan
[arXiv:1308

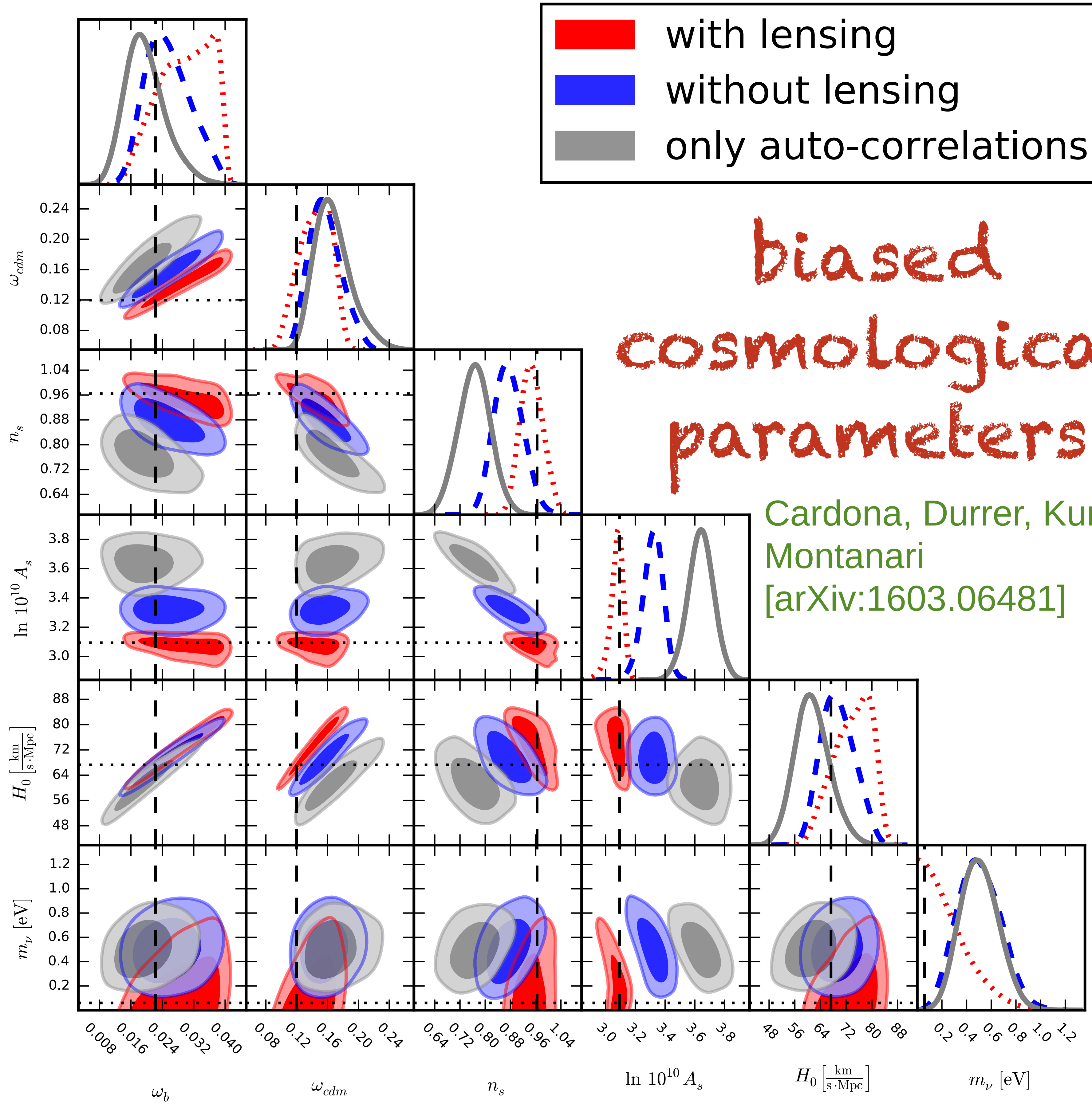


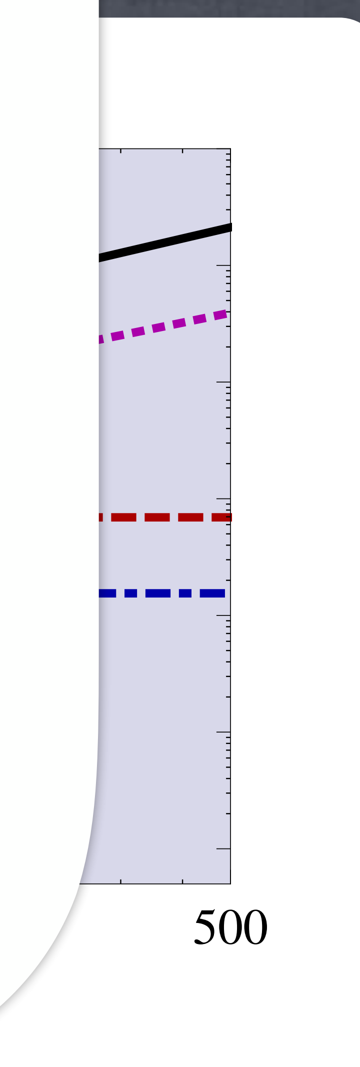

Montanari, Durrer [arXiv:1506.01369]

useful information






**biased cosmological parameters**

Cardona, Durrer, Kunz, Montanari [arXiv:1603.06481]

ED, Mont
[arXiv:13

# Large Scale Structures

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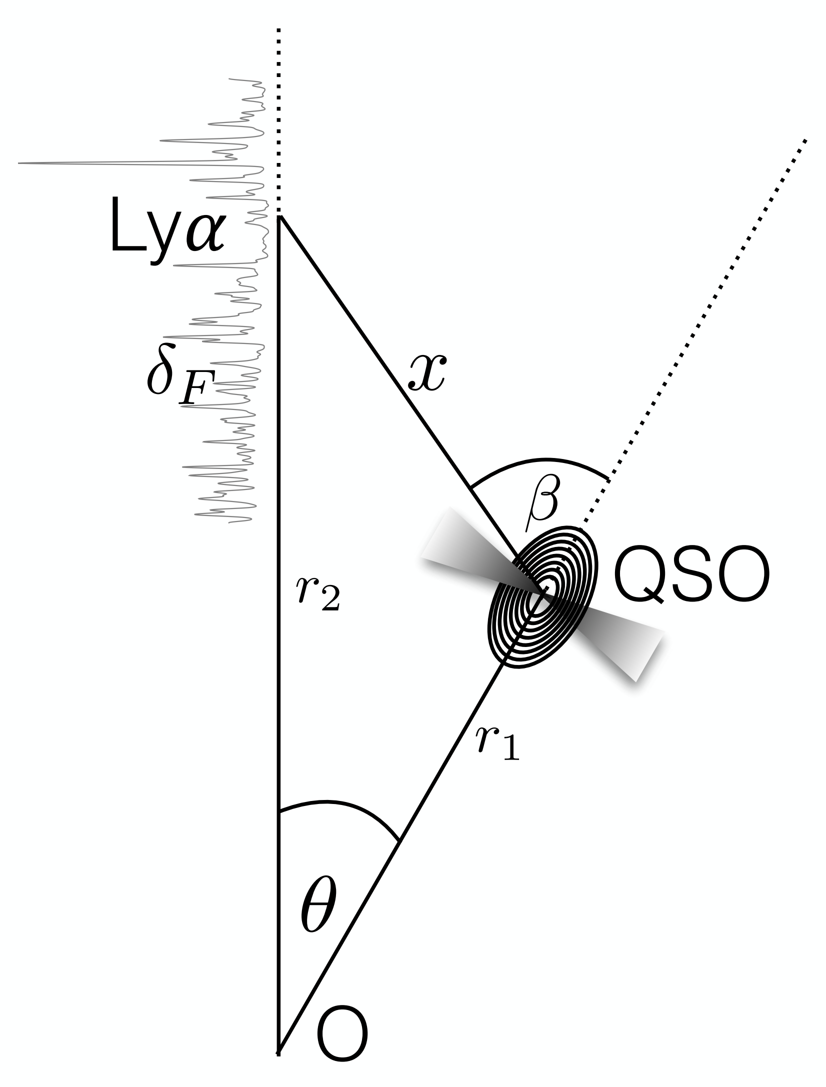
# Quasars x Ly-alpha correlation

Irsic , ED, Viel [arXiv:1510.03436]

$$\xi_{Q\alpha}\left(z_1, z_2, \theta\right) = \left\langle \Delta_Q\left(\mathbf{n}_1, z_1\right) \delta_F\left(\mathbf{n}_2, z_2\right)\right\rangle$$

Large bias factor difference



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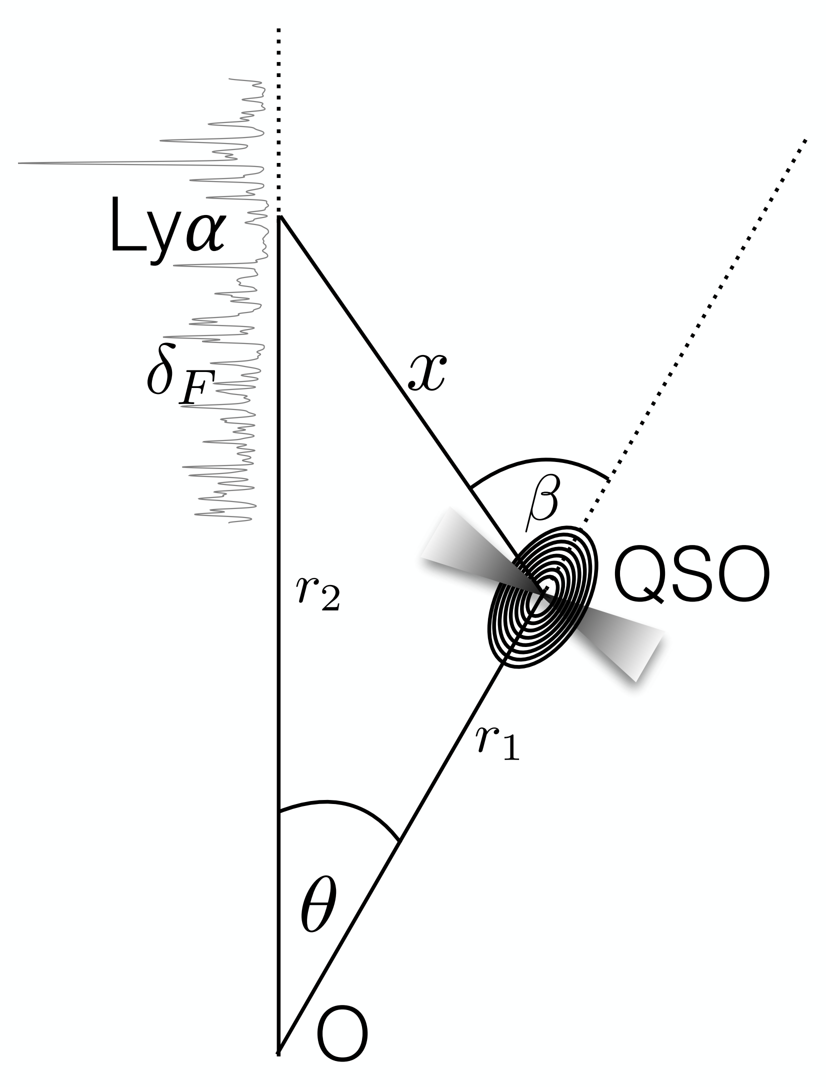


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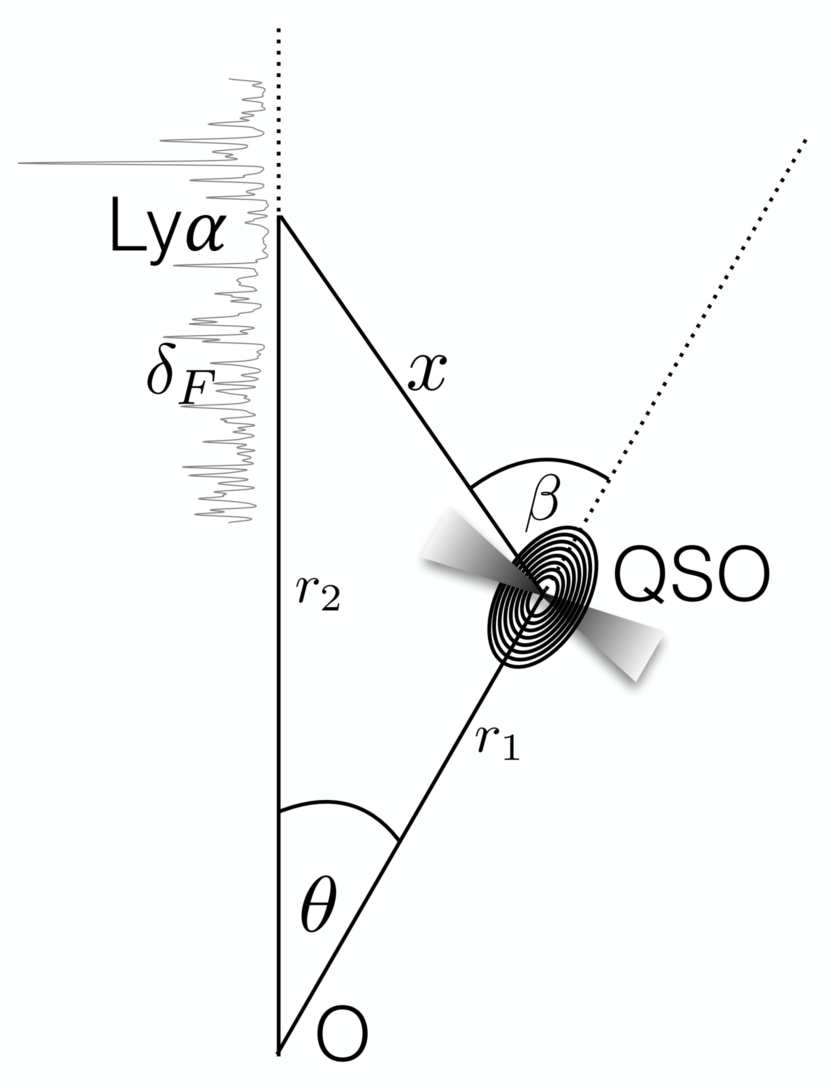
Large bias factor difference

$$\delta_F\left(\mathbf{n}, z\right) = b_\alpha \delta^{\rm sync} + b_v \mathcal{H}^{-1}\partial_r \mathbf{n}\cdot\mathbf{v} - \bar{\tau}\left(z\right)\left[-\left(2+\frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\frac{\delta z}{1+z} + \mathbf{n}\cdot\mathbf{v} + \Psi + \mathcal{H}^{-1}\dot{\Phi} - 3\mathcal{H}v\right]$$

# Quasars x Ly-alpha correlation

Irsic , ED, Viel [arXiv:1510.03436]

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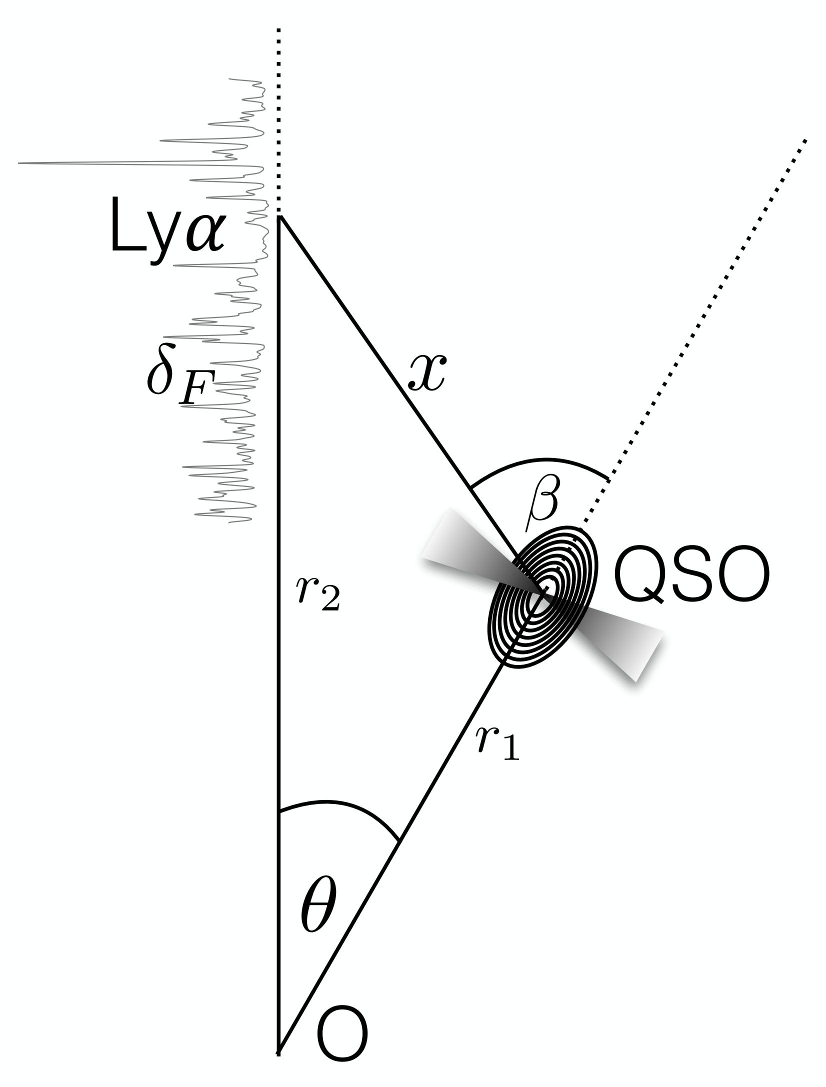
Standard

# Quasars x Ly-alpha correlation

Irsic , ED, Viel [arXiv:1510.03436]




$$\xi_{Q\alpha}(z_1, z_2, \theta) = \langle \Delta_Q(\mathbf{n}_1, z_1)\, \delta_F(\mathbf{n}_2, z_2) \rangle$$

Large bias factor difference

$$\delta_F(\mathbf{n}, z) = b_\alpha \delta^{\rm sync} + b_v \mathcal{H}^{-1} \partial_r \mathbf{n}\cdot\mathbf{v} - \bar{\tau}(z)\left[-\left(2 + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\frac{\delta z}{1+z} + \mathbf{n}\cdot\mathbf{v} + \Psi + \mathcal{H}^{-1}\dot{\Phi} - 3\mathcal{H}v\right]$$

Relativistic

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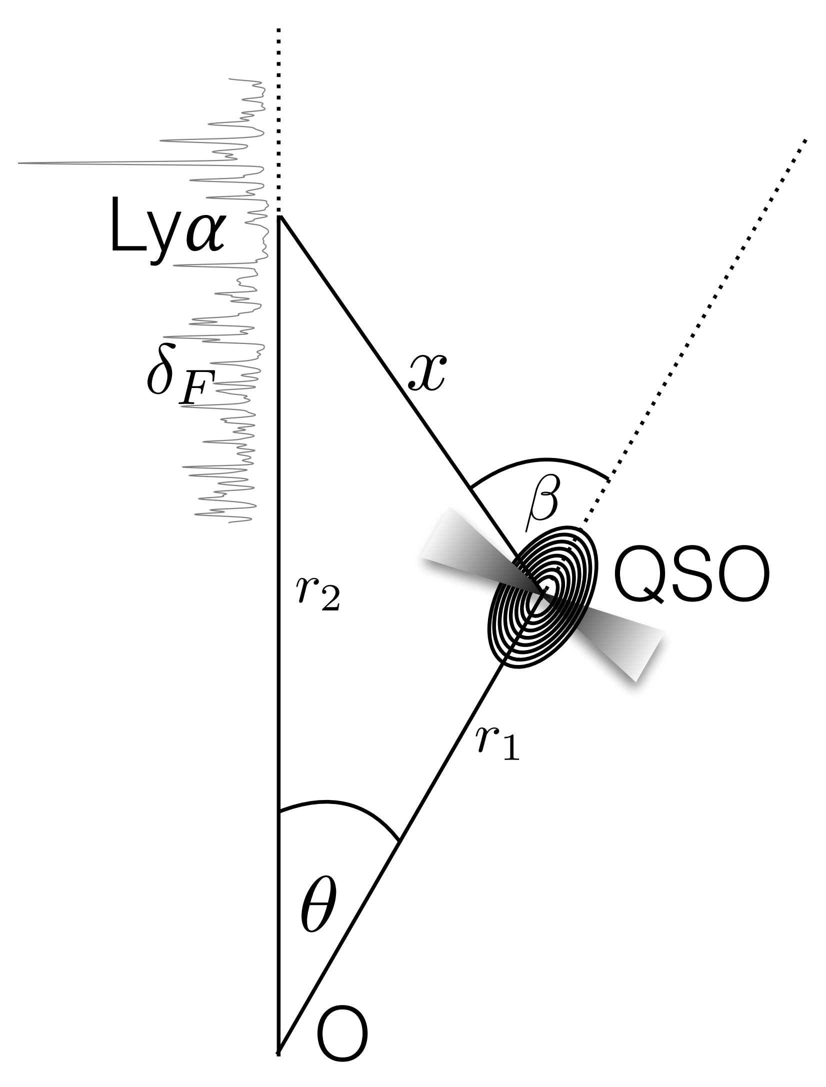
$$\xi_{Q\alpha}\left(z_1, z_2, \theta\right) = \left\langle \Delta_Q\left(\mathbf{n}_1, z_1\right)\delta_F\left(\mathbf{n}_2, z_2\right)\right\rangle$$

$$\xi_{Q\alpha} = \xi_{Q\alpha}^{\mathrm{newt}} + \xi_{Q\alpha}^{\mathrm{magnification}} + \xi_{Q\alpha}^{\mathrm{relativistic}}$$



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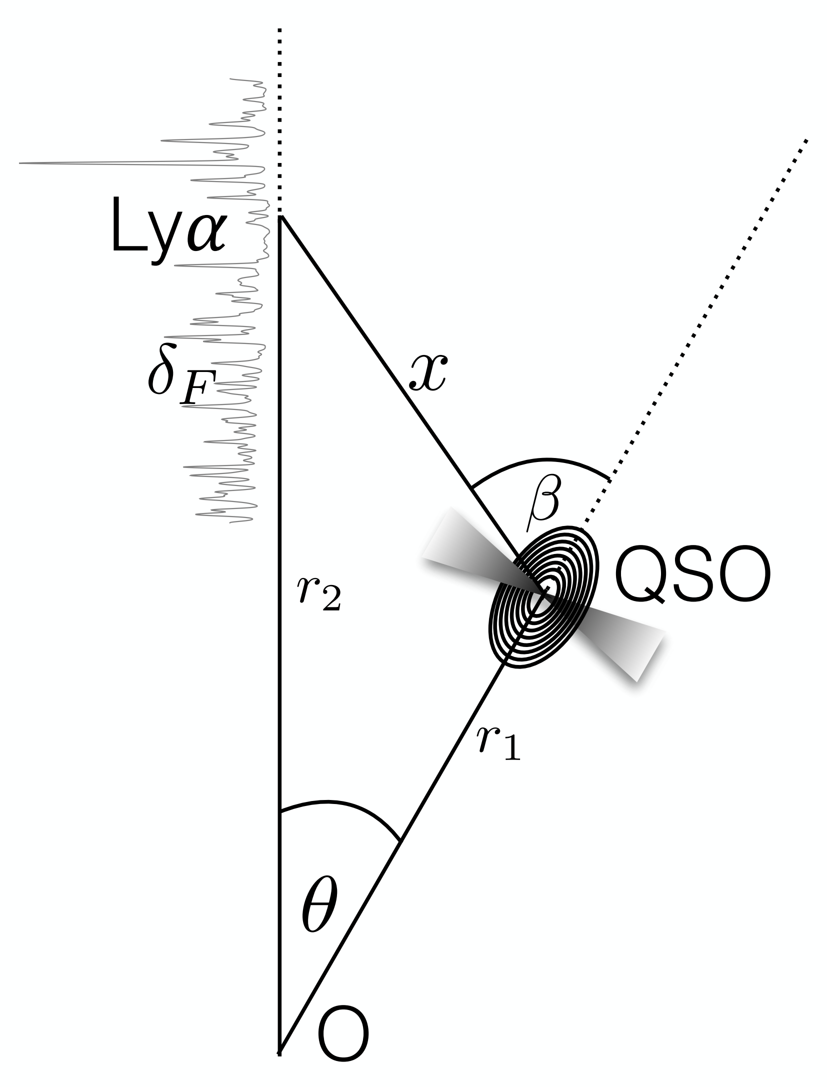


$$\xi_{Q\alpha}\left(z_1, z_2, \theta\right) = \left\langle \Delta_Q\left(\mathbf{n}_1, z_1\right) \delta_F\left(\mathbf{n}_2, z_2\right)\right\rangle$$

$$\xi_{Q\alpha} = \xi_{Q\alpha}^{\rm newt} + \xi_{Q\alpha}^{\rm magnification} + \xi_{Q\alpha}^{\rm relativistic}$$

$$\begin{aligned}\xi_{Q\alpha}^{\rm newt} \sim\ & b_Q b_\alpha \int \frac{dk}{2\pi^2} k^2 P\left(k\right) j_0\left(kx\right) \\ & + b_v \int \frac{dk}{2\pi^2} k^2 f^2 P\left(k\right) \left[\frac{1}{5} j_0\left(kx\right) - \frac{4}{7} j_2\left(kx\right) + \frac{8}{35} j_4\left(kx\right)\right] \\ & + \left(b_Q b_v + b_\alpha\right) \int \frac{dk}{2\pi^2} k^2 f P\left(k\right) \left[\frac{1}{3} j_0\left(kx\right) - \frac{2}{3} j_2\left(kx\right)\right]\end{aligned}$$

Order $\mathcal{O}(1)$

Even spherical Bessel functions

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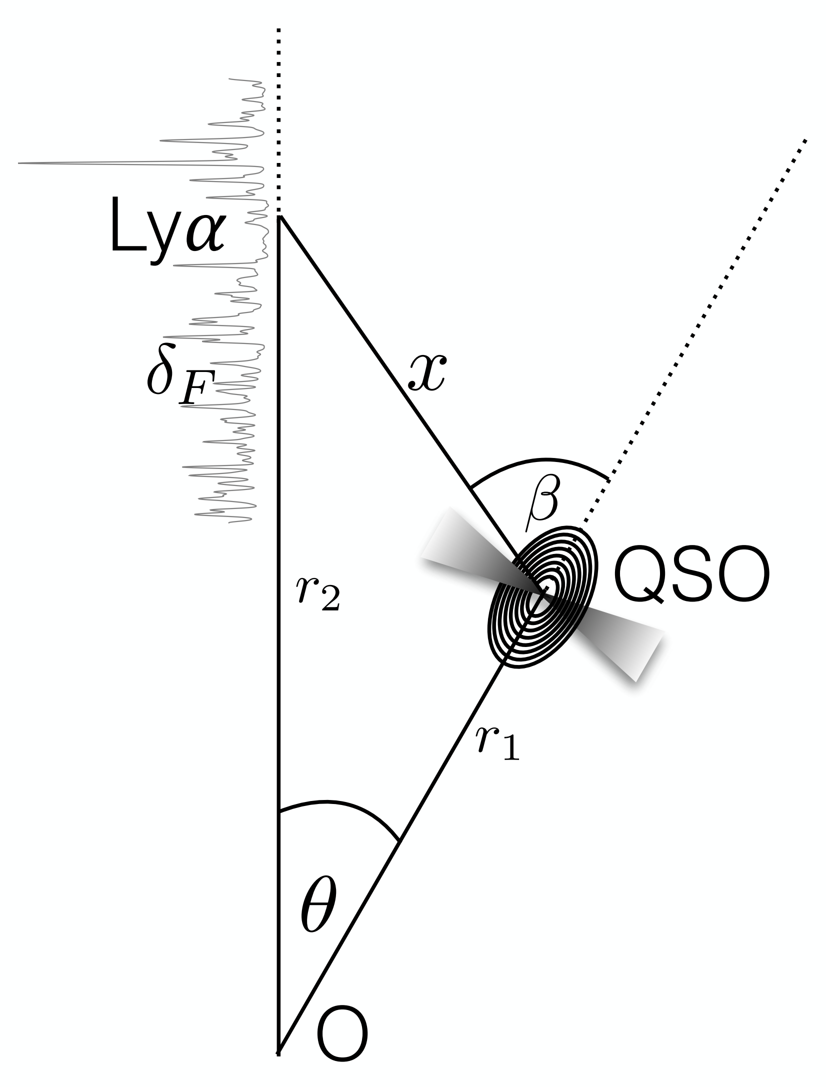


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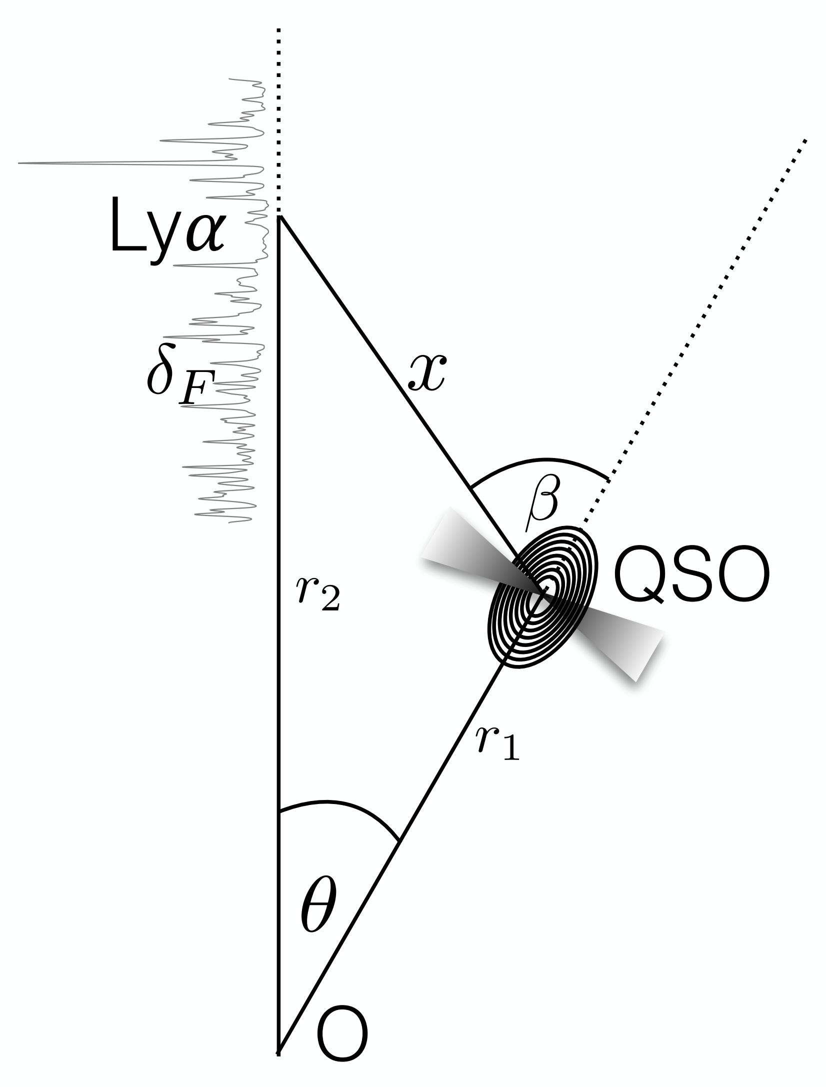
$$\xi_{Q\alpha}^{\text{Doppler}} \sim \left(-b_Q\mathcal{R}_\alpha + \mathcal{R}_Q b_\alpha\right)\int \frac{dk}{2\pi^2} k^2 f P\left(k\right) j_1\left(kx\right)\frac{\mathcal{H}}{k}$$
$$+\left(-\mathcal{R}_\alpha + \mathcal{R}_Q b_v\right)\int \frac{dk}{2\pi^2} k^2 f^2 P\left(k\right)\left[\frac{3}{5}j_1\left(kx\right) - \frac{2}{5}j_3\left(kx\right)\right]\frac{\mathcal{H}}{k}$$
$$+\mathcal{R}_\alpha\mathcal{R}_Q\int \frac{dk}{2\pi^2} k^2 f^2 P\left(k\right)\left[\frac{1}{3}j_0\left(kx\right) - \frac{2}{3}j_2\left(kx\right)\right]\left(\frac{\mathcal{H}}{k}\right)^2$$

Order $\mathcal{O}\left(\mathcal{H}/k\right)$

Odd spherical Bessel functions

# Quasars x Ly-alpha correlation

Irsic , ED, Viel [arXiv:1510.03436]

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single tracer

$$\xi_{Q\alpha}^{\text{Doppler}} \sim \left(-b_Q\mathcal{R}_\alpha + \mathcal{R}_Q b_\alpha\right)\int \frac{dk}{2\pi^2}k^2 f P\left(k\right) j_1\left(kx\right)\frac{\mathcal{H}}{k}$$

$$+\left(-\mathcal{R}_\alpha + \mathcal{R}_Q b_v\right)\int \frac{dk}{2\pi^2}k^2 f^2 P\left(k\right)\left[\frac{3}{5}j_1\left(kx\right) - \frac{2}{5}j_3\left(kx\right)\right]\frac{\mathcal{H}}{k}$$

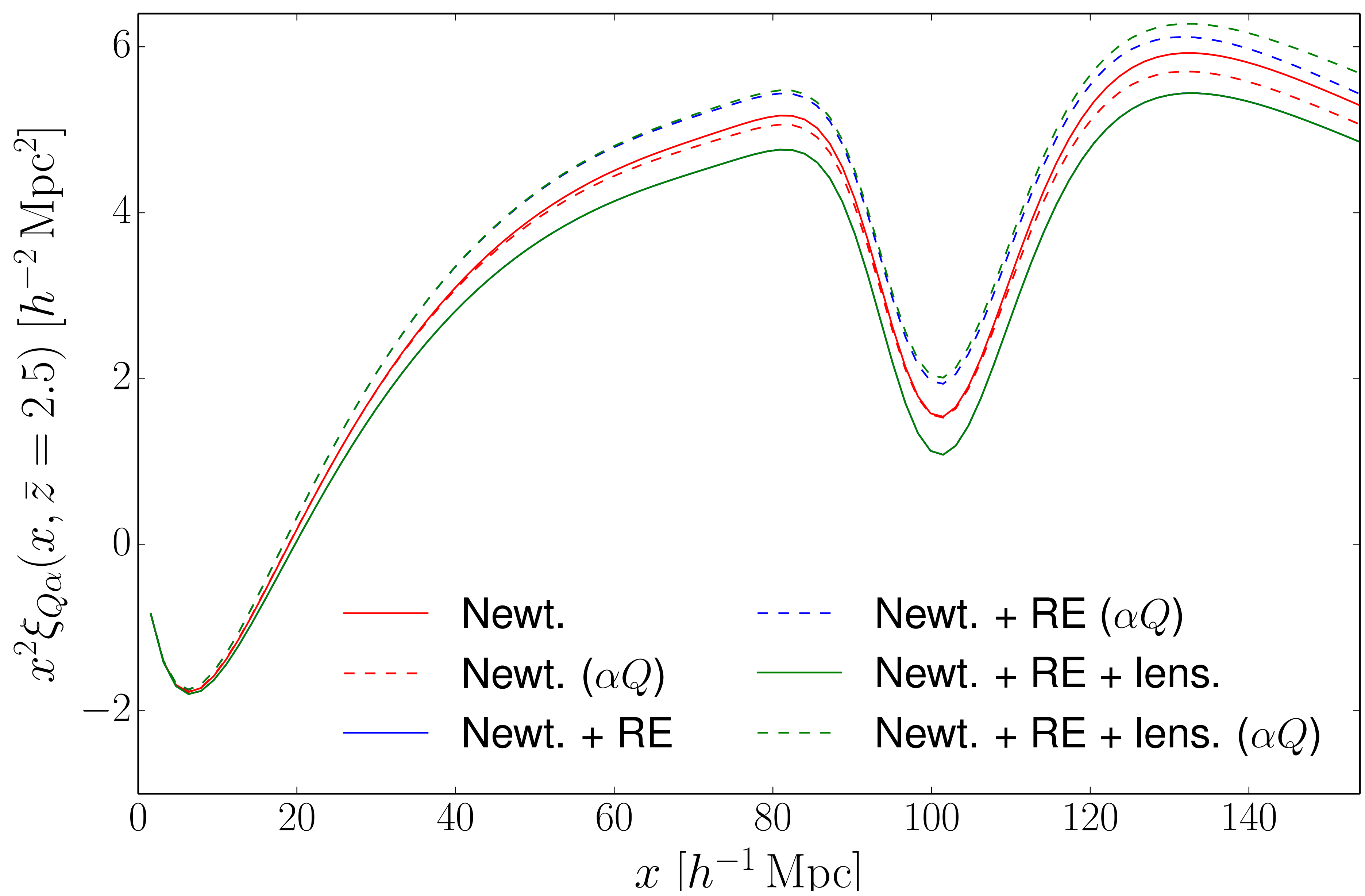
$$+\mathcal{R}_\alpha\mathcal{R}_Q\int \frac{dk}{2\pi^2}k^2 f^2 P\left(k\right)\left[\frac{1}{3}j_0\left(kx\right) - \frac{2}{3}j_2\left(kx\right)\right]\left(\frac{\mathcal{H}}{k}\right)^2$$

Order $\mathcal{O}\left(\mathcal{H}/k\right)$ $\mathcal{O}\left(\mathcal{H}^2/k^2\right)$

even

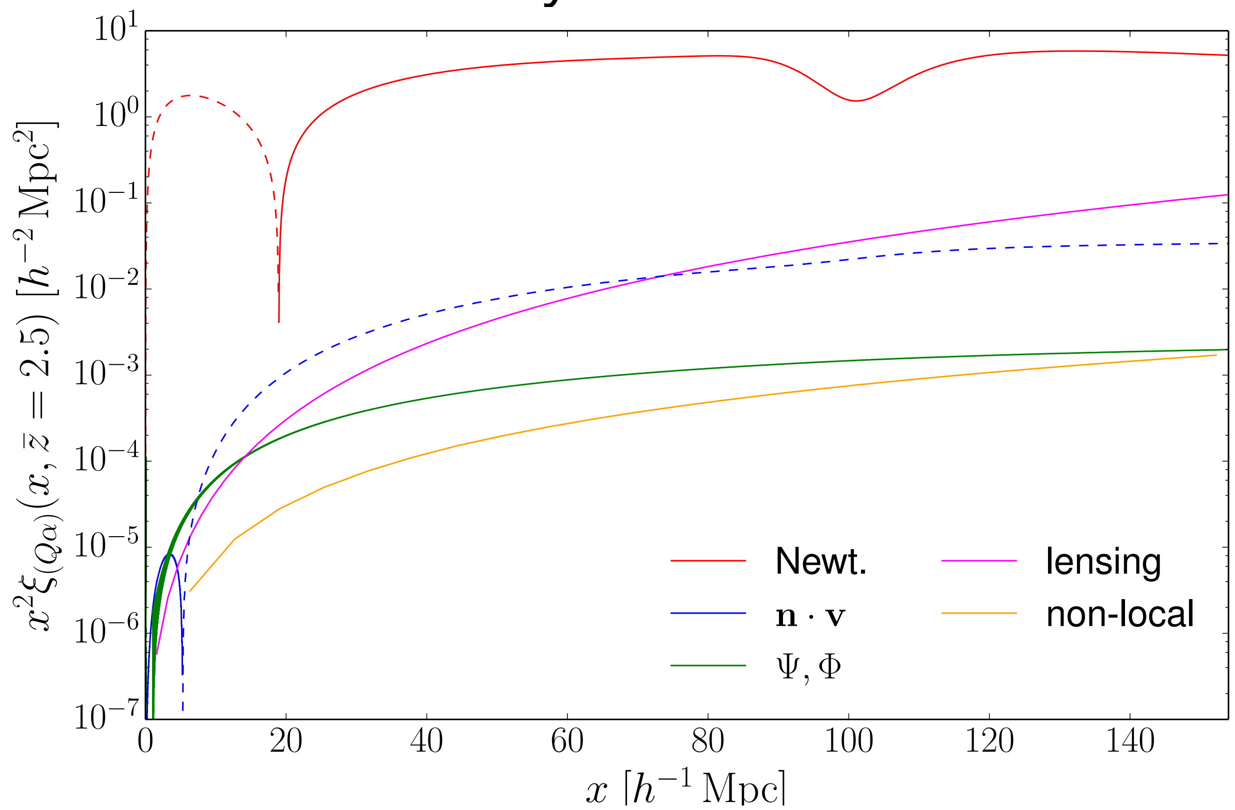
Odd spherical Bessel functions

# Relativistic effects on Lyman-$\alpha$ forest

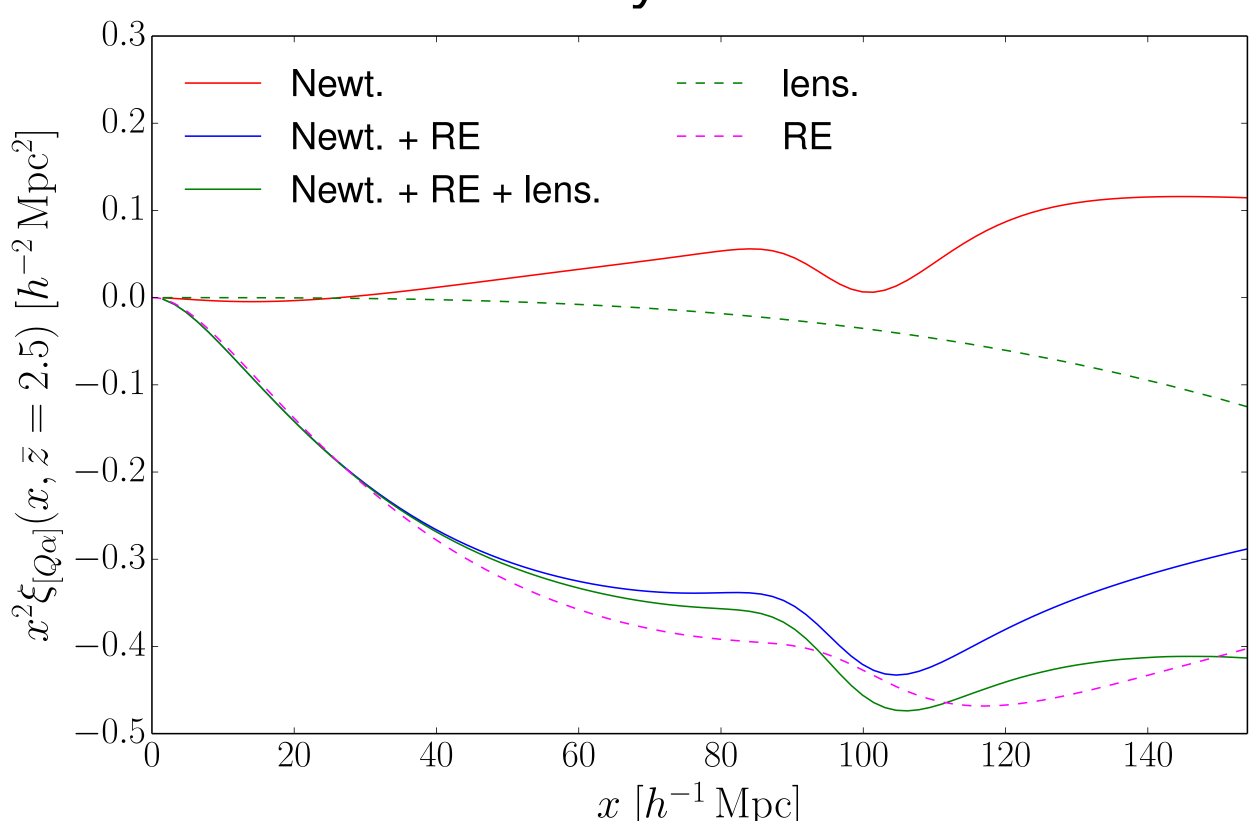



Irsic , ED, Viel [arXiv:1510.03436]

# Symmetric

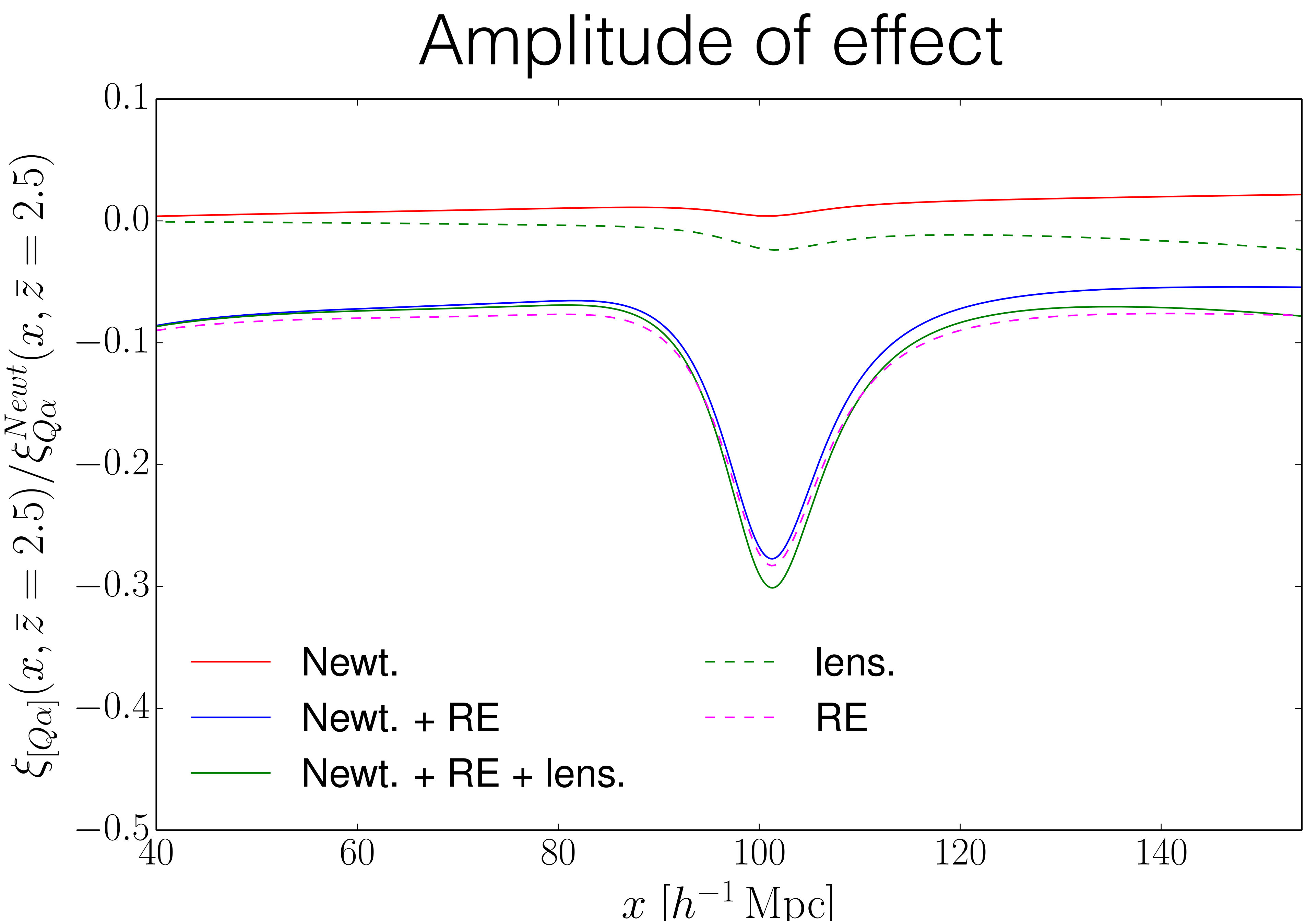



# Anti-Symmetric




Irsic , ED, Viel [arXiv:1510.03436]

# Relativistic effects on Lyman-$\alpha$ forest




Irsic , ED, Viel [arXiv:1510.03436]

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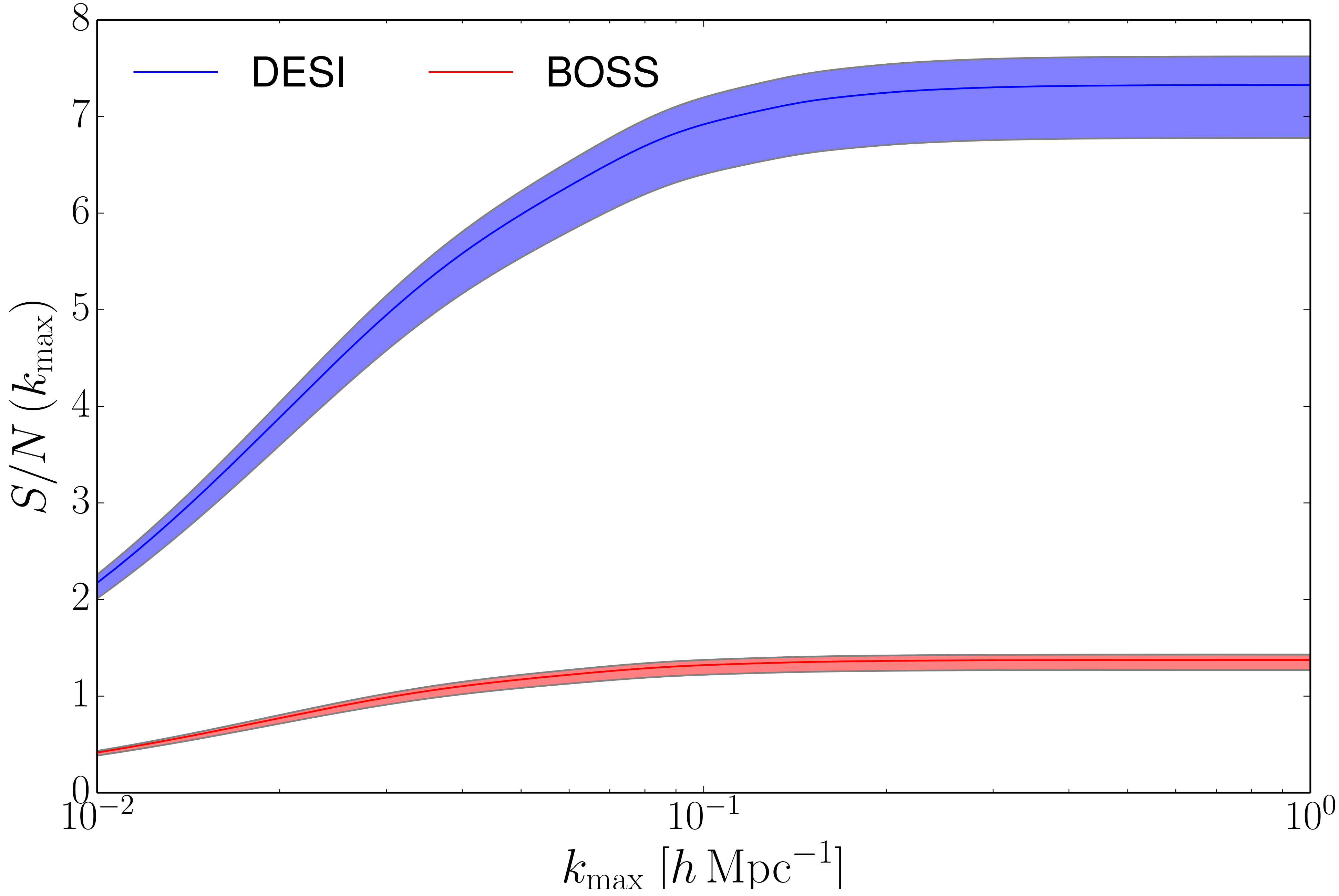



Irsic , ED, Viel [arXiv:1510.03436]

# Single tracer vs multi-tracers

| | single | multi |
|---|---|---|
| Leading order correction | $\mathcal{O}\left(\mathcal{H}^2/k^2\right)$ | $\mathcal{O}\left(\mathcal{H}/k\right)$ |
| Parity | even | odd |
| Relevant scales | super-Hubble | all |
| Limited by | cosmic variance | shot noise |
| Forecasted detection | No | Yes |

# Large Scale Structures

$$\Delta_N(\mathbf{n}, z, m_*) = b(z) D_{cm}\left(L > \bar{L}_*\right) + \mathcal{H}^{-1}\partial_r \mathbf{n}\cdot\mathbf{v}$$

$$- \frac{2-5s}{2}\int_0^r \frac{r-r'}{rr'}\Delta_\Omega(\Psi+\Phi)\,dr'$$

$$+ \left(5s + \frac{2-5s}{\mathcal{H}r} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f^N_{\text{evo}}\right)\mathbf{n}\cdot\mathbf{v} + (f^N_{\text{evo}} - 3)\mathcal{H}v$$

$$+ \left(5s + \frac{2-5s}{\mathcal{H}r} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f^N_{\text{evo}}\right)\int_0^r \left(\dot{\Psi}+\dot{\Phi}\right)dr'$$

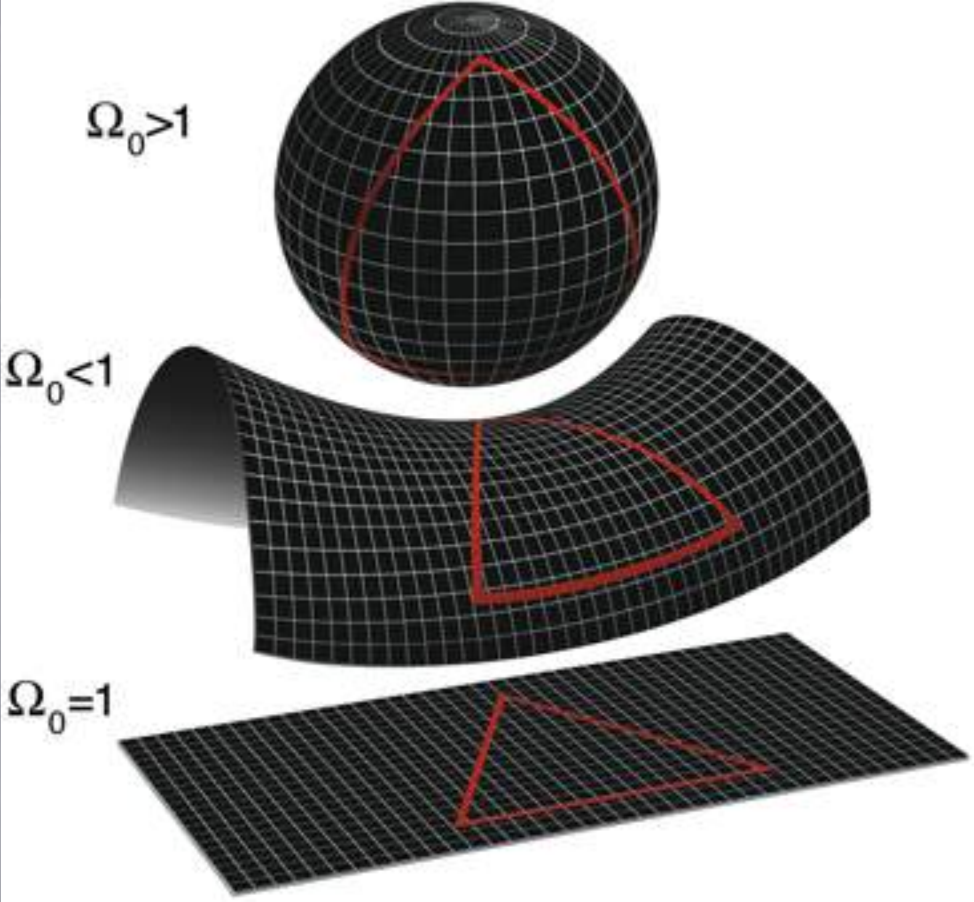
$$+ \left(5s + \frac{2-5s}{\mathcal{H}r} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f^N_{\text{evo}}\right)\Psi$$

$$+ \frac{2-5s}{r}\int_0^r (\Psi+\Phi)\,dr$$

$$+ (5s-2)\,\Phi + \Psi + \mathcal{H}^{-1}\dot{\Phi}$$

Bonvin & Durrer [arXiv:1105.5280],
Challinor & Lewis [arXiv:1105.5292],
Yoo [arXiv:1009.3021]

# Curvature constraints from LSS

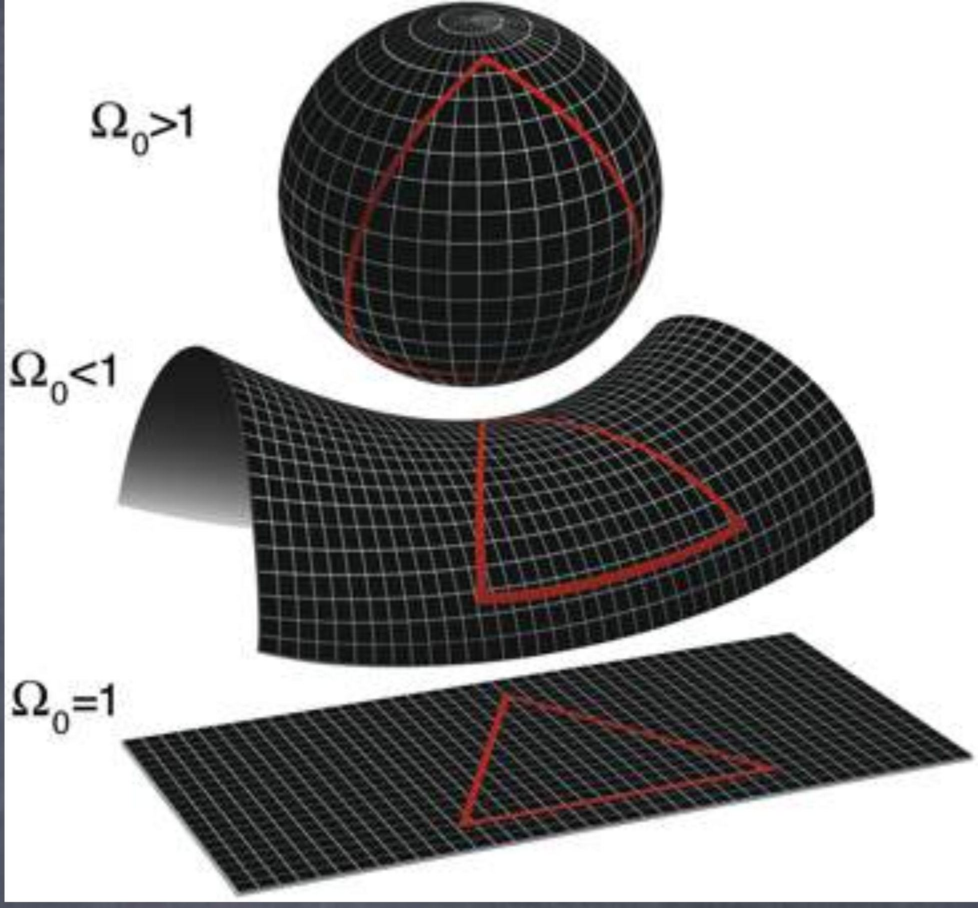



Tightest constraints $|\Omega_K| \leq 0.003$

Predicted limit of detectability $\Omega_K \sim \mathcal{O}\left(10^{-4}\right)$

Cosmic variance limit $\Omega_K \sim \mathcal{O}\left(10^{-5}\right)$

# Curvature constraints from LSS

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Cosmic variance limit $\Omega_K \sim \mathcal{O}\left(10^{-5}\right)$

$$d\tilde{s}^2 = a^2\left(-\left(1+2\Psi\right)d\tau^2 + \left(1-2\Phi\right)\gamma_{ij}dx^i dx^j\right)$$

where $\gamma_{ij}dx^i dx^j = \left[dr^2 + \{S_K^2\left(r\right)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]$

and $S_K\left(r\right) = \begin{cases} \frac{1}{\sqrt{K}}\sin\left(\sqrt{K}r\right) & \text{for } K > 0 \\ r & \text{for } K = 0 \\ \frac{1}{\sqrt{|K|}}\sinh\left(\sqrt{|K|}r\right) & \text{for } K < 0 \end{cases}$

# Curvature constraints from LSS

$$\Delta(\mathbf{n}, z, m_*) = bD_{cm} + \frac{1}{\mathcal{H}}\partial_r(\mathbf{n}\cdot\mathbf{v})$$

$$- \frac{2-5s}{2}\int_\tau^{\tau_o} \frac{S_K(r-\tilde{r})}{S_K(r)\,S_K(\tilde{r})}\Delta_\Omega(\Psi+\Phi)\,d\tilde{\tau}$$

$$+ \left(5s + \frac{2-5s}{\mathcal{H}}\frac{S'_K}{S_K} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} - f_{\mathrm{evo}}\right)$$

$$\times\left(\Psi + \mathbf{n}\cdot\mathbf{v} + \int_\tau^{\tau_o}\left(\dot{\Psi}+\dot{\Phi}\right)d\tilde{\tau}\right)$$

$$+ (5s-2)\,\Phi + \Psi + \frac{1}{\mathcal{H}}\dot{\Phi} + (f_{\mathrm{evo}}-3)\mathcal{H}v$$

$$+ (2-5s)\,\frac{S'_K}{S_K}\int_\tau^{\tau_o}(\Psi+\Phi)\,d\tilde{\tau}$$

ED, Montanari, Raccanelli, Durrer, Kamionkowski, Lesgourgues [arXiv:1603.09073]

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CLASS v.2.5

http://class-code.net

$$+ \left(5s-2\right)\Phi + \Psi + \frac{1}{\mathcal{H}}\dot{\Phi} + \left(f_{\rm evo}-3\right)\mathcal{H}v$$

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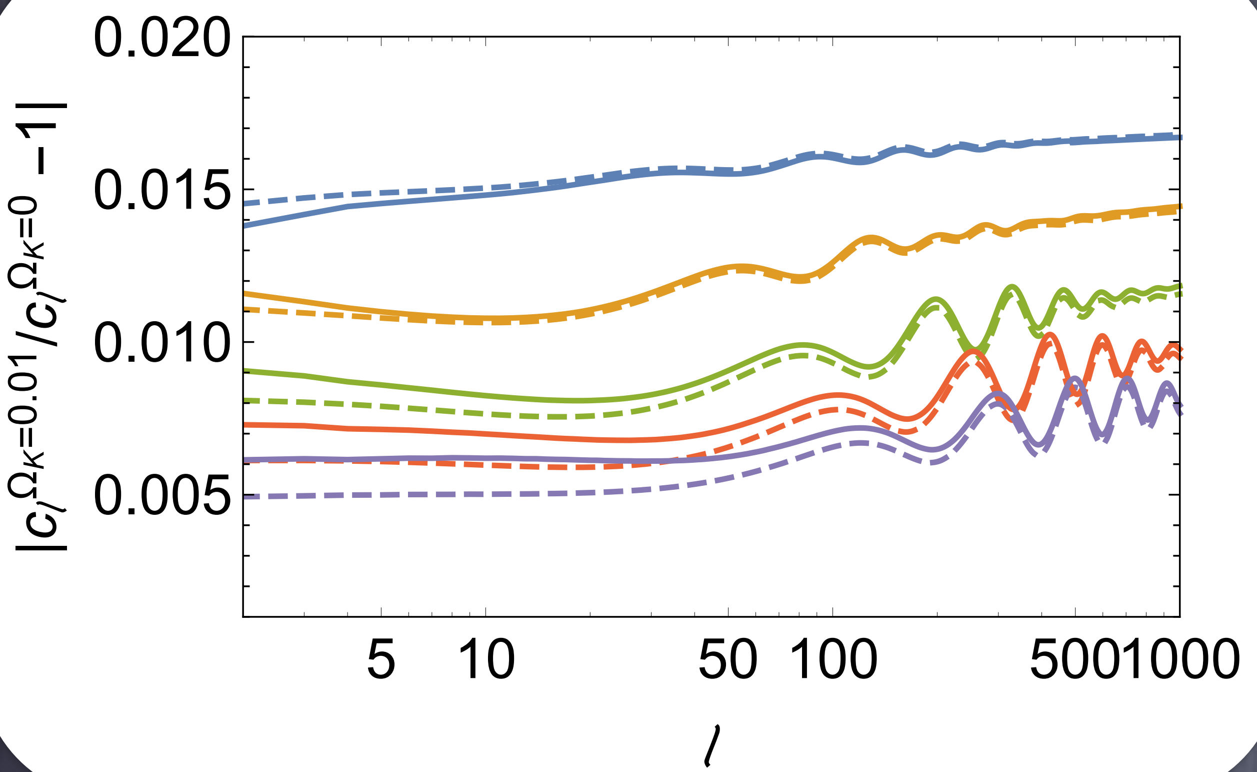


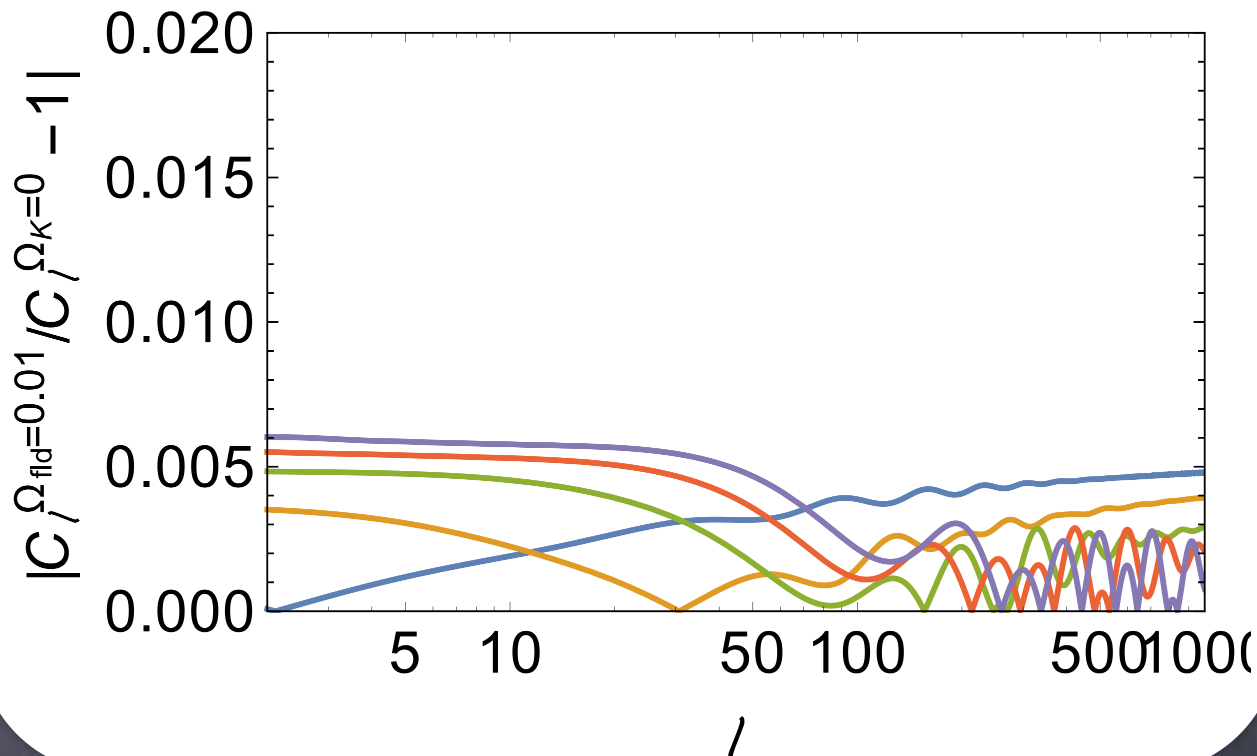


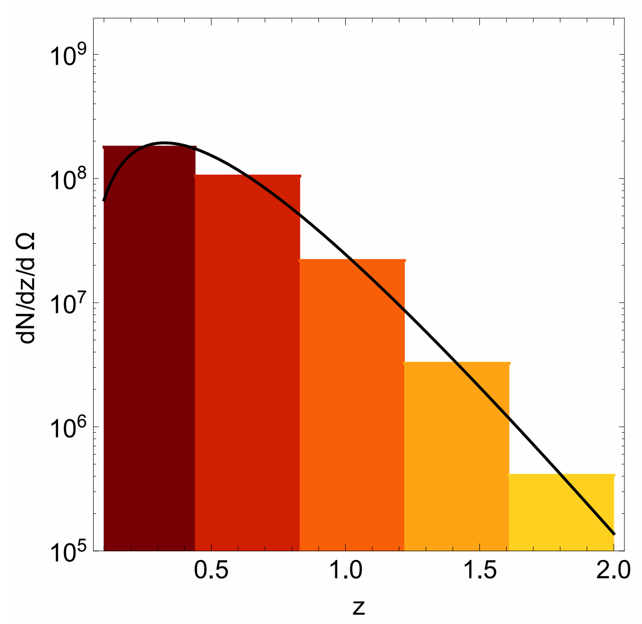


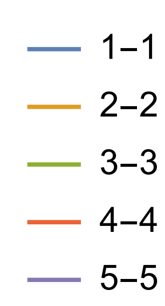


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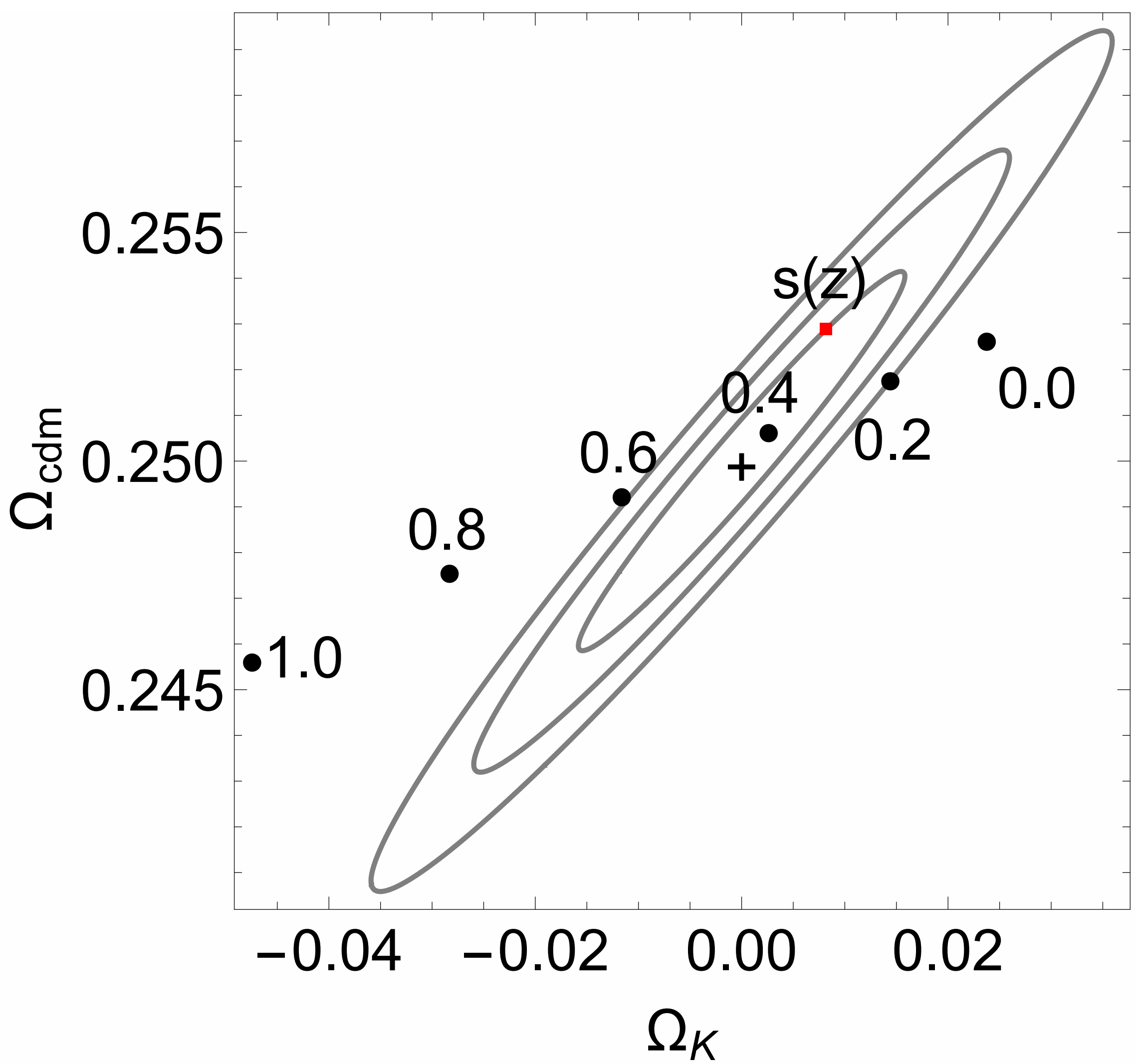


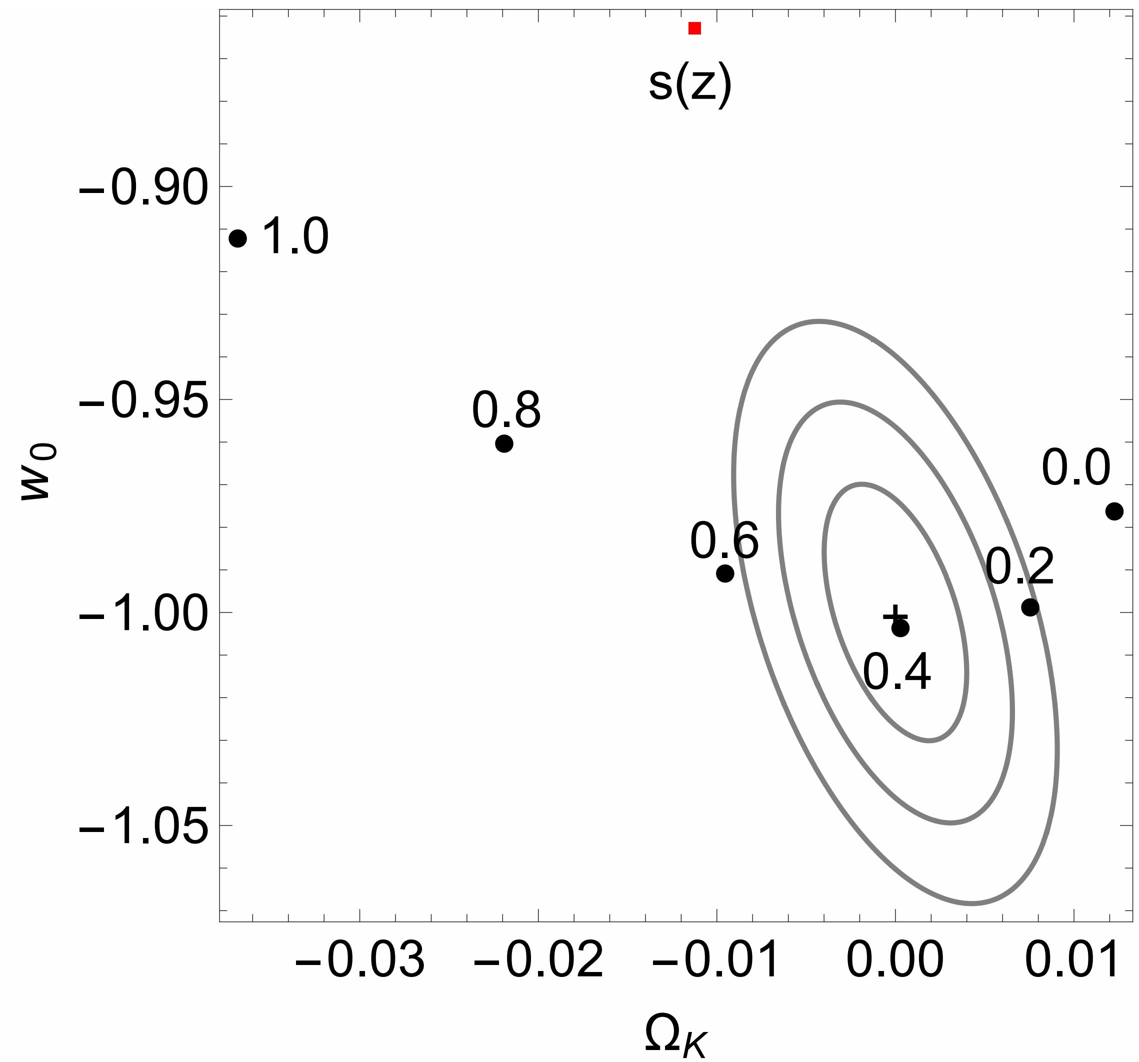


Spatial curvature, and other cosmological parameters, are strongly biased if relativistic effects (mainly cosmic magnification) are neglected.

# Second Order

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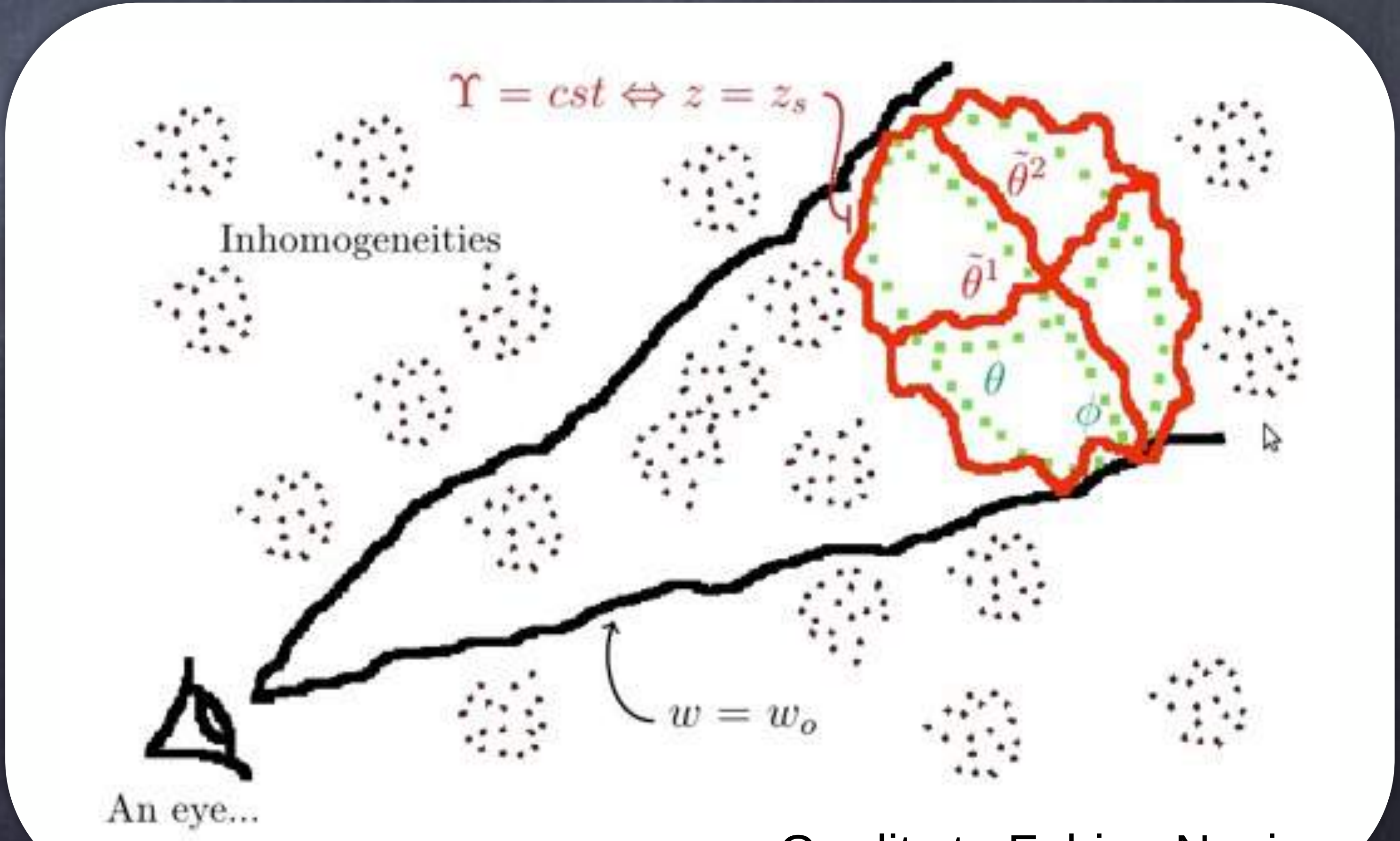
Geodesic light-cone gauge



Credits to Fabien Nugier

Gasperini, Marozzi, Nugier, Veneziano [arXiv:1104.1167]

# Second Order

$$\Delta(\mathbf{n}, z) \equiv \frac{N(\mathbf{n}, z) - <N>(z)}{<N>(z)}$$

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$$N(\mathbf{n}, z) = \rho(\mathbf{n}, z) V(\mathbf{n}, z)$$

Density perturbation

$$\rho(\mathbf{n}, z) = \bar{\rho}\left(1 + \delta^{(1)} + \delta^{(2)}\right)$$

Volume perturbation

$$V(\mathbf{n}, z) = \bar{V}\left(1 + \frac{\delta V^{(1)}}{\bar{V}} + \frac{\delta V^{(2)}}{\bar{V}}\right)$$

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$$\Delta(\mathbf{n}, z) = \left[\delta^{(1)} + \frac{\delta V^{(1)}}{\bar{V}} + \delta^{(1)}\frac{\delta V^{(1)}}{\bar{V}} + \delta^{(2)} + \frac{\delta V^{(2)}}{\bar{V}} - \langle\delta^{(1)}\frac{\delta V^{(1)}}{\bar{V}}\rangle - \langle\delta^{(2)}\rangle - \langle\frac{\delta V^{(2)}}{\bar{V}}\rangle\right]$$

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1-order

# Second Order

$$\Delta(\mathbf{n}, z) \equiv \frac{N(\mathbf{n}, z) - < N > (z)}{< N > (z)}$$

$$N(\mathbf{n}, z) = \rho(\mathbf{n}, z) V(\mathbf{n}, z)$$

Density perturbation

$$\rho(\mathbf{n}, z) = \bar{\rho}\left(1 + \delta^{(1)} + \delta^{(2)}\right)$$

Volume perturbation

$$V(\mathbf{n}, z) = \bar{V}\left(1 + \frac{\delta V^{(1)}}{\bar{V}} + \frac{\delta V^{(2)}}{\bar{V}}\right)$$

$$\Delta(\mathbf{n}, z) = \left[\delta^{(1)} + \frac{\delta V^{(1)}}{\bar{V}} + \delta^{(1)}\frac{\delta V^{(1)}}{\bar{V}} + \delta^{(2)} + \frac{\delta V^{(2)}}{\bar{V}} - \langle \delta^{(1)}\frac{\delta V^{(1)}}{\bar{V}} \rangle - \langle \delta^{(2)} \rangle - \langle \frac{\delta V^{(2)}}{\bar{V}} \rangle\right]$$

2-order

$$
\begin{aligned}
\Sigma_{IS} = & \Big(-\frac{2}{\mathcal{H}_s r_s}-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\Big\{-v_{||s}^{(2)}-\frac{1}{2}\phi_s^{(2)}-\frac{1}{2}\int_{\eta_s}^{\eta_0}d\eta'\partial_{\eta'}\Big[\phi^{(2)}\left(\eta'\right)+\psi^{(2)}\left(\eta'\right)\Big]+\frac{1}{2}\left(v_{||s}\right)^2 \\
& +\frac{1}{2}\left(\psi_s^I\right)^2+\left(-v_{||s}-\psi_s^I\right)\Big(-\psi_s^I-2\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)\Big)+\frac{1}{2}v_{\perp s}^a v_{\perp a\,s} \\
& +2a\,v_{\perp s}^a\partial_a\int_{\eta_s}^{\eta_o}d\eta'\psi^I\left(\eta'\right)+4\int_{\eta_s}^{\eta_0}d\eta'\Big[\psi^I\left(\eta'\right)\partial_{\eta'}\psi^I\left(\eta'\right)+\partial_{\eta'}\psi^I\left(\eta'\right)\int_{\eta'}^{\eta_o}d\eta''\partial_{\eta''}\psi^I\left(\eta''\right) \\
& +\psi^I\left(\eta'\right)\int_{\eta'}^{\eta_o}d\eta''\partial_{\eta''}^2\psi^I\left(\eta''\right)-\gamma_0^{ab}\partial_a\Big(\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)\Big)\partial_b\Big(\int_{\eta'}^{\eta_o}d\eta''\partial_{\eta''}\psi^I\left(\eta''\right)\Big)\Big] \\
& +2\partial_a\left(v_{||s}+\psi_s^I\right)\int_{\eta_s}^{\eta_o}d\eta'\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right) \\
& +4\int_{\eta_s}^{\eta_o}d\eta'\partial_a\left(\partial_{\eta'}\psi^I\left(\eta'\right)\right)\int_{\eta'}^{\eta_o}d\eta''\gamma_0^{ab}\partial_b\int_{\eta''}^{\eta_o}d\eta'''\psi^I\left(\eta'''\right)\Big\} \\
& +\Big[\frac{1}{2}\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}+\frac{3}{2}\Big(\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)^2-\frac{1}{2}\frac{\mathcal{H}_s''}{\mathcal{H}_s^3}+\frac{1}{\mathcal{H}_s r_s}\Big(1+3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}+\frac{1}{\mathcal{H}_s r_s}\Big)\Big]\Big[\left(v_{||s}\right)^2+(\psi_s^I)^2+2\psi_s^I v_{||s} \\
& +4\left(v_{||s}+\psi_s^I\right)\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)+4\Big(\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)\Big)^2\Big]-\psi_s^{(2)}+\frac{1}{2}\phi_s^{(2)}+\frac{1}{2\mathcal{H}_s}\partial_\eta\psi_s^{(2)} \\
& +\frac{1}{\mathcal{H}_s}\partial_r v_{||s}^{(2)}-\frac{1}{2}\frac{1}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\Big[\psi^{(2)}+\phi^{(2)}\Big]\left(\eta'\right)+\frac{1}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\Big[\psi^{(2)}+\phi^{(2)}\Big]\left(\eta'\right) \\
& +2\Big(1-\frac{1}{\mathcal{H}_s r_s}\Big)\Big\{-\frac{2}{\mathcal{H}_s}v_{||s}\partial_r v_{||s}-(v_{||s})^2-v_{\perp a\,s}v_{\perp\,s}^a+\Big[-\frac{1}{\mathcal{H}_s}\partial_\eta\psi_s^I-2\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right) \\
& -\frac{2}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')+\frac{2}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\psi^I\left(\eta'\right)\Big]v_{||s}+\Big[-2\psi_s^I-4\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right) \\
& -2\mathcal{H}_s\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')\Big]\frac{1}{\mathcal{H}_s}\partial_r v_{||s}-av_{\perp\,s}^a\partial_a\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')+\big[\partial_r\psi_s^I+2\partial_\eta\psi_s^I \\
& +2\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}^2\psi^I\left(\eta'\right)\Big]\Big(-2\int_{\eta_s}^{\eta_o}d\eta'\psi^I\left(\eta'\right)\Big) \\
& -\Big(-\psi_s^I-2\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)\Big)\Big[\frac{2}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\psi^I\left(\eta'\right)-\frac{2}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\psi^I\left(\eta'\right) \\
& -\frac{1}{\mathcal{H}_s}\partial_\eta\psi_s^I-\psi_s^I\Big]\Big\}+\frac{3}{2}v_{\perp a\,s}v_{\perp\,s}^a-\frac{2}{\mathcal{H}_s}av_{\perp\,s}^a\partial_a v_{||s}+\Big(\frac{5}{2}+\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)(v_{||s})^2 \\
& +\Big(5+3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\frac{1}{\mathcal{H}_s}v_{||s}\partial_r v_{||s}+\frac{1}{\mathcal{H}_s^2}\Big[v_{||s}\partial_r^2 v_{||s}+\left(\partial_r v_{||s}\right)^2\Big]+\Big[-\frac{1}{\mathcal{H}_s^2}\left(\partial_r^2\psi_s^I+\partial_\eta^2\psi_s^I-\partial_\eta\partial_r\psi_s^I\right) \\
& -\frac{1}{\mathcal{H}_s}\partial_r\psi_s^I-\frac{3}{\mathcal{H}_s}\Big(-1-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}-\frac{2}{3}\frac{1}{\mathcal{H}_s r_s}\Big)\partial_\eta\psi_s^I-\frac{4}{r_s}\Big(-1-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta') \\
& +\Big(-2-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\frac{2}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\psi^I\left(\eta'\right)-2\Big(-2-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right) \\
& +\frac{4}{\mathcal{H}_s r_s}\psi_s^I-\frac{2}{\mathcal{H}_s r_s^2}\int_{\eta_s}^{\eta_o}d\eta'\Delta_2\psi^I\left(\eta'\right)\Big]v_{||s}+\Big[2\Big(2+\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}+\frac{2}{\mathcal{H}_s r_s}\Big)\partial_r v_{||s} \\
& +\frac{2}{\mathcal{H}_s}\partial_r^2 v_{||s}\Big]\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')+\Big[\frac{2}{\mathcal{H}_s}\Big(5+3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\partial_r v_{||s}+\frac{2}{\mathcal{H}_s^2}\partial_r^2 v_{||s}\Big]\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right) \\
& -\frac{2}{\mathcal{H}_s}\partial_r v_{||s}\frac{1}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\psi^I\left(\eta'\right)+\frac{2}{\mathcal{H}_s^2}\partial_\eta\psi_s^I\partial_r v_{||s}+\frac{1}{\mathcal{H}_s}\Big[\frac{1}{\mathcal{H}_s}\partial_r^2 v_{||s} \\
& +\Big(6+3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\partial_r v_{||s}\Big]\psi_s^I+\frac{2}{\mathcal{H}_s r_s}av_{\perp\,s}^a\partial_a\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')-\frac{1}{\mathcal{H}_s}av_{\perp\,s}^a\partial_a\psi_s^I \\
& -\frac{4}{\mathcal{H}_s}\gamma_{0s}^{ab}\partial_a v_{||s}\partial_b\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')+\frac{4}{r_s^2}\Big(\int_{\eta_s}^{\eta_o}d\eta'\psi^I\left(\eta'\right)\Big)^2+\Big\{\Big[2\Big(-2-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\psi_s^I \\
& +4\Big(-2-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)-\frac{2}{\mathcal{H}_s}\partial_\eta\psi_s^I\Big]\frac{1}{r_s}+2\Big(-2-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}^2\psi^I\left(\eta'\right) \\
& +2\Big(-2-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\partial_\eta\psi_s^I+\Big(-1-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\partial_r\psi_s^I-\frac{1}{\mathcal{H}_s}\partial_\eta\partial_r\psi_s^I\Big\}\Big(-2\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')\Big) \\
& +\Big[\frac{3}{\mathcal{H}_s}\Big(-1-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}-\frac{2}{3}\frac{1}{\mathcal{H}_s r_s}\Big)\partial_\eta\psi_s^I+\frac{1}{\mathcal{H}_s}\partial_r\psi_s^I-\Big(2-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\psi_s^I \\
& +\frac{1}{\mathcal{H}_s^2}\left(\partial_r^2\psi_s^I+\partial_\eta^2\psi_s^I-\partial_\eta\partial_r\psi_s^I\right)\Big]\Big(-2\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)\Big)+\Big[\Big(-2-2\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}-\frac{2}{\mathcal{H}_s r_s}\Big)\psi_s^I \\
& +4\Big(-2-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}-\frac{1}{\mathcal{H}_s r_s}\Big)\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)-\frac{10}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta') \\
& -\frac{2}{\mathcal{H}_s}\partial_\eta\psi_s^I\Big]\frac{1}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\psi^I\left(\eta'\right)+2\Big(\frac{1}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\psi^I\left(\eta'\right)\Big)^2 \\
& +\Big(-\frac{1}{2}-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\left(\psi_s^I\right)^2+\frac{1}{\mathcal{H}_s^2}\left(\partial_\eta\psi_s^I\right)^2+\Big[-\frac{1}{\mathcal{H}_s^2}\left(\partial_r^2\psi_s^I+\partial_\eta^2\psi_s^I-\partial_\eta\partial_r\psi_s^I\right) \\
& +\frac{1}{\mathcal{H}_s}\Big(4+3\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}+\frac{2}{\mathcal{H}_s r_s}\Big)\partial_\eta\psi_s^I-\frac{1}{\mathcal{H}_s}\partial_r\psi_s^I\Big]\psi_s^I \\
& +\frac{4}{\mathcal{H}_s}\gamma_{0\,s}^{ab}\partial_a\Big(\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')\Big)\partial_b\Big(\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)\Big)+2\partial_a\Big[\Big(1-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)v_{||s}-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\psi_s^I \\
& -\frac{1}{\mathcal{H}_s}\left(\partial_\eta\psi_s^I+\partial_r v_{||s}\right)\Big]\int_{\eta_s}^{\eta_o}d\eta'\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right) \\
& +4\Big(1-\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\int_{\eta_s}^{\eta_o}d\eta'\partial_a\left(\partial_{\eta'}\psi^I(\eta')\right)\int_{\eta'}^{\eta_o}d\eta''\gamma_0^{ab}\partial_b\int_{\eta''}^{\eta_o}d\eta'''\psi^I\left(\eta'''\right) \\
& +2av_{\perp\,s}^a\partial_a\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')+\frac{4}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\Big[\psi^I\left(\eta'\right)\Big(-\psi^I\left(\eta'\right)-2\int_{\eta'}^{\eta_o}d\eta''\partial_{\eta''}\psi^I\left(\eta''\right)\Big) \\
& +\gamma_0^{ab}\partial_a\Big(\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)\Big)\partial_b\Big(\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)\Big)\Big]+4\partial_a\psi_s^I\int_{\eta_s}^{\eta_o}d\eta'\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right) \\
& +\frac{4}{r_s}\Big[\int_{\eta_s}^{\eta_o}d\eta'\partial_a\Big(\frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\psi^I\left(\eta'\right)-2\psi^I\left(\eta'\right)\Big)\int_{\eta'}^{\eta_o}d\eta''\gamma_0^{ab}\partial_b\int_{\eta''}^{\eta_o}d\eta'''\psi^I\left(\eta'''\right)\Big] \\
& +2\partial_a\Big(\int_{\eta_s}^{\eta_o}d\eta'\psi^I\left(\eta'\right)\Big)\Big[-\gamma_{0s}^{ab}\partial_b\int_{\eta_s}^{\eta_o}d\eta'\psi^I\left(\eta'\right)-2\int_{\eta_s}^{\eta_o}d\eta'\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o}d\eta''\partial_{\eta''}\psi^I\left(\eta''\right)\Big] \\
& +2\partial_a\Big(\int_{\eta_s}^{\eta_o}d\eta'\gamma_0^{bc}\partial_c\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)\Big)\partial_b\Big(\int_{\eta_s}^{\eta_o}d\eta'\gamma_0^{ad}\partial_d\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)\Big) \\
& -4\Big(\int_{\eta_s}^{\eta_o}d\eta'\psi^I\left(\eta'\right)\Big)\int_{\eta_s}^{\eta_o}d\eta'\Big[-\frac{1}{(\eta_o-\eta')^3}\int_{\eta'}^{\eta_o}d\eta''\Delta_2\psi^I\left(\eta''\right)+\frac{1}{(\eta_o-\eta')^2}\Big(\frac{1}{2}\Delta_2\psi^I\left(\eta'\right) \\
& +\int_{\eta'}^{\eta_o}d\eta''\partial_{\eta''}\left(\Delta_2\psi^I\left(\eta''\right)\right)\Big)\Big]+\frac{2}{(\sin\theta_o)^2}\Big[\frac{1}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\frac{\eta'-\eta_s}{\eta_o-\eta'}\partial_{\theta_o}\psi^I\left(\eta'\right)\Big]^2 \\
& +\frac{2}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\Big[\psi^I\left(\eta'\right)\Big(-\psi^I\left(\eta'\right)-2\int_{\eta'}^{\eta_o}d\eta''\partial_{\eta''}\psi^I\left(\eta''\right)\Big) \\
& +\gamma_0^{ab}\partial_a\Big(\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)\Big)\partial_b\Big(\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)\Big)\Big] \\
& +2\int_{\eta_s}^{\eta_o}d\eta'\Big\{-2\psi^I\left(\eta'\right)\frac{1}{\eta_o-\eta'}\int_{\eta'}^{\eta_o}d\eta''\frac{\eta''-\eta'}{\eta_o-\eta''}\Delta_2\partial_{\eta''}\psi^I\left(\eta''\right) \\
& +2\gamma_0^{ab}\partial_b\Big(\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)\Big)\frac{1}{\eta_o-\eta'}\int_{\eta'}^{\eta_o}d\eta''\frac{\eta''-\eta'}{\eta_o-\eta''}\partial_a\Delta_2\psi^I\left(\eta''\right) \\
& -\Big(-2\psi^I\left(\eta'\right)-2\int_{\eta'}^{\eta_o}d\eta''\partial_{\eta''}\psi^I\left(\eta''\right)\Big)\frac{1}{(\eta_o-\eta')^2}\int_{\eta'}^{\eta_o}d\eta''\Delta_2\psi^I\left(\eta''\right) \\
& -2\partial_a\psi^I\left(\eta'\right)\int_{\eta'}^{\eta_o}d\eta''\gamma_0^{ab}\partial_b\int_{\eta''}^{\eta_o}d\eta'''\partial_{\eta'''}\psi^I\left(\eta'''\right) \\
& +2\partial_a\Big[\gamma_0^{db}\partial_b\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)\Big]\int_{\eta'}^{\eta_o}d\eta''\partial_d\Big[\gamma_0^{ac}\partial_c\int_{\eta''}^{\eta_o}d\eta'''\psi^I\left(\eta'''\right)\Big] \\
& +2\gamma_0^{ab}\partial_a\Big(\psi^I\left(\eta'\right)+\int_{\eta'}^{\eta_o}d\eta''\partial_{\eta''}\psi^I\left(\eta''\right)\Big)\partial_b\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)\Big\} \\
& +\Big[\Big(\frac{2}{\mathcal{H}_s r_s}+\frac{\mathcal{H}_s'}{\mathcal{H}_s^2}\Big)\Big(v_{||s}+\psi_s^I+2\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)\Big)-3v_{||s}+\frac{1}{\mathcal{H}_s}\partial_r v_{||s}-4\psi_s^I \\
& -6\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)+\frac{4}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')-\frac{2}{r_s}\int_{\eta_s}^{\eta_o}d\eta'\frac{\eta'-\eta_s}{\eta_o-\eta'}\Delta_2\psi^I\left(\eta'\right)+\frac{1}{\mathcal{H}_s}\partial_\eta\psi_s^I\Big]\delta_\rho^{(1)} \\
& +\Big[\frac{1}{\bar\rho}\partial_\eta\left(\bar\rho\,\delta_\rho^{(1)}\right)-\partial_r\delta_\rho^{(1)}\Big]\frac{1}{\mathcal{H}_s}\Big(-v_{||s}-\psi_s^I-2\int_{\eta_s}^{\eta_o}d\eta'\partial_{\eta'}\psi^I\left(\eta'\right)\Big) \\
& +2\partial_r\delta_\rho^{(1)}\int_{\eta_s}^{\eta_o}d\eta'\psi^I(\eta')-2\partial_a\delta_\rho^{(1)}\int_{\eta_s}^{\eta_o}d\eta'\gamma_0^{ab}\partial_b\int_{\eta'}^{\eta_o}d\eta''\psi^I\left(\eta''\right)+\delta_\rho^{(2)}\,. \qquad (4.43)
\end{aligned}
$$

# Second Order

$$\Delta(\mathbf{n}, z) \equiv \frac{N(\mathbf{n}, z) - <N>(z)}{<N>(z)}$$

## Leading terms

$$\begin{aligned}\Sigma^{(2)}(\mathbf{n}, z) = &\ \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right) \\ & - 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a\psi\right] + 2\kappa^2 \\ & - 2\nabla_b\kappa\nabla^b\psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z) - r}{r}\Delta_2\left(\nabla^b\Psi_1\nabla_b\Psi_1\right) - 2\int_0^{r(z)}\frac{dr}{r}\nabla^a\Psi_1\nabla_a\kappa\end{aligned}$$

ED, Durrer, Marozzi, Montanari [arXiv:1510.04202]

# Second Order

$$\Delta(\mathbf{n}, z) \equiv \frac{N(\mathbf{n}, z) - <N>(z)}{<N>(z)}$$

## Leading terms

density-RSD

$$\Sigma^{(2)}(\mathbf{n}, z) = \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right)$$
$$- 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a \psi\right] + 2\kappa^2$$
$$- 2\nabla_b \kappa \nabla^b \psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z) - r}{r}\Delta_2\left(\nabla^b \Psi_1 \nabla_b \Psi_1\right) - 2\int_0^{r(z)} \frac{dr}{r}\nabla^a \Psi_1 \nabla_a \kappa$$

ED, Durrer, Marozzi, Montanari [arXiv:1510.04202]

# Second Order

$$\Delta(\mathbf{n}, z) \equiv \frac{N(\mathbf{n}, z) - <N>(z)}{<N>(z)}$$

$$Z_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{b_2}{2} + b_1 F_2(\mathbf{k}_1, \mathbf{k}_2) + f\mu^2 G_2(\mathbf{k}_1, \mathbf{k}_2) + \frac{f\mu k}{2}\left[\frac{\mu_1}{k_1}\left(b_1 + f\mu_2^2\right) + \frac{\mu_2}{k_2}\left(b_1 + f\mu_1^2\right)\right]$$

density-RSD

$$\Sigma^{(2)}(\mathbf{n}, z) = \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right)$$
$$- 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a \psi\right] + 2\kappa^2$$
$$- 2\nabla_b \kappa \nabla^b \psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z) - r}{r}\Delta_2\left(\nabla^b \Psi_1 \nabla_b \Psi_1\right) - 2\int_0^{r(z)} \frac{dr}{r}\nabla^a \Psi_1 \nabla_a \kappa$$

ED, Durrer, Marozzi, Montanari [arXiv:1510.04202]

# Second Order

$$\Delta(\mathbf{n}, z) \equiv \frac{N(\mathbf{n}, z) - <N>(z)}{<N>(z)}$$

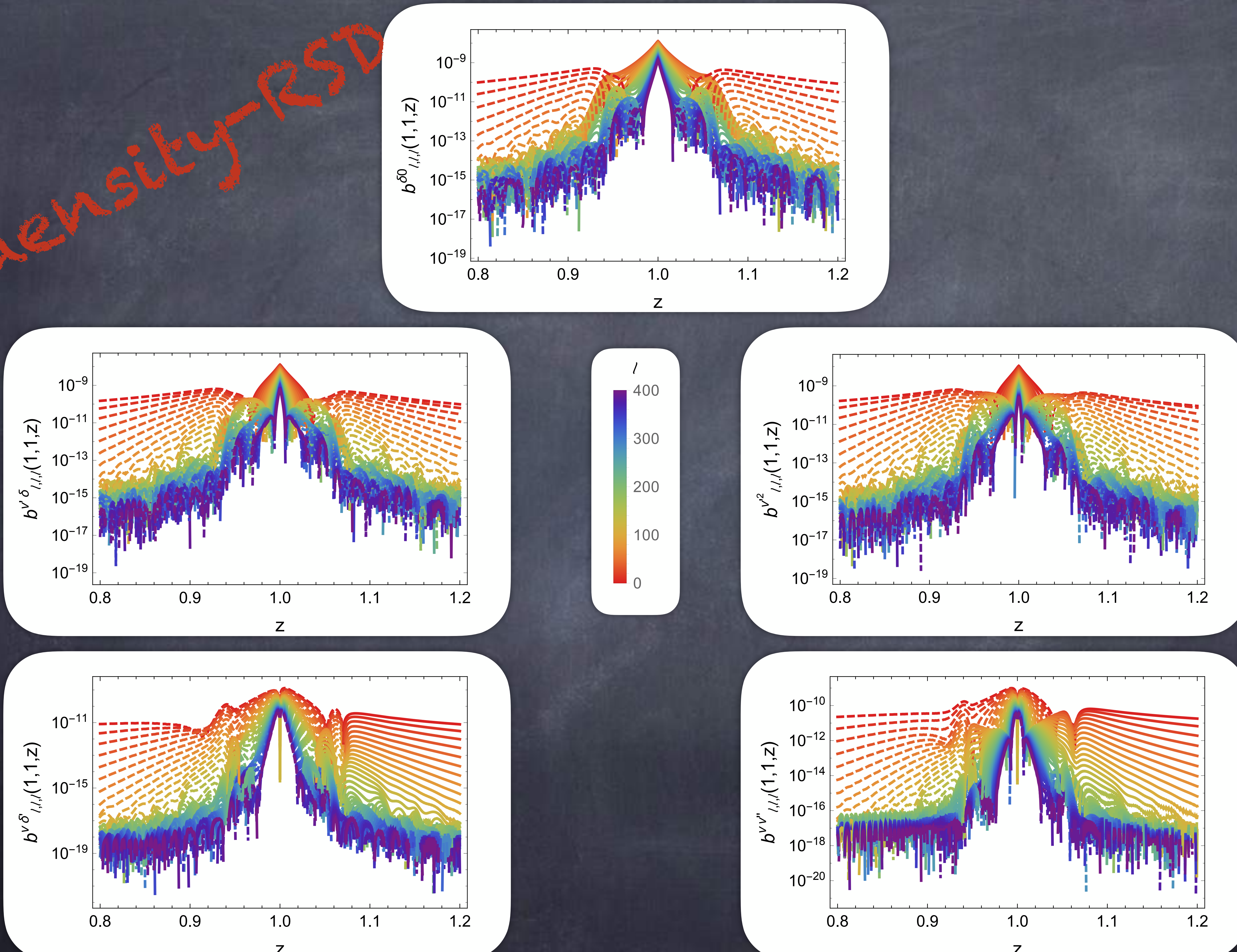
## Leading terms

$$\Sigma^{(2)}(\mathbf{n}, z) = \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v + \mathcal{H}^{-1}\left(\partial_r v \partial_r \delta + \partial_r^2 v\, \delta\right)$$
$$- 2\kappa^{(2)} - 2\delta\kappa + \nabla_a \delta \nabla^a \psi + \mathcal{H}^{-1}\left[-2(\partial_r^2 v)\kappa + \nabla_a(\partial_r^2 v)\nabla^a \psi\right] + 2\kappa^2$$
$$- 2\nabla_b \kappa \nabla^b \psi - \frac{1}{2r(z)}\int_0^{r(z)} dr \frac{r(z) - r}{r}\Delta_2\left(\nabla^b \Psi_1 \nabla_b \Psi_1\right) - 2\int_0^{r(z)} \frac{dr}{r}\nabla^a \Psi_1 \nabla_a \kappa$$

like-lensing terms

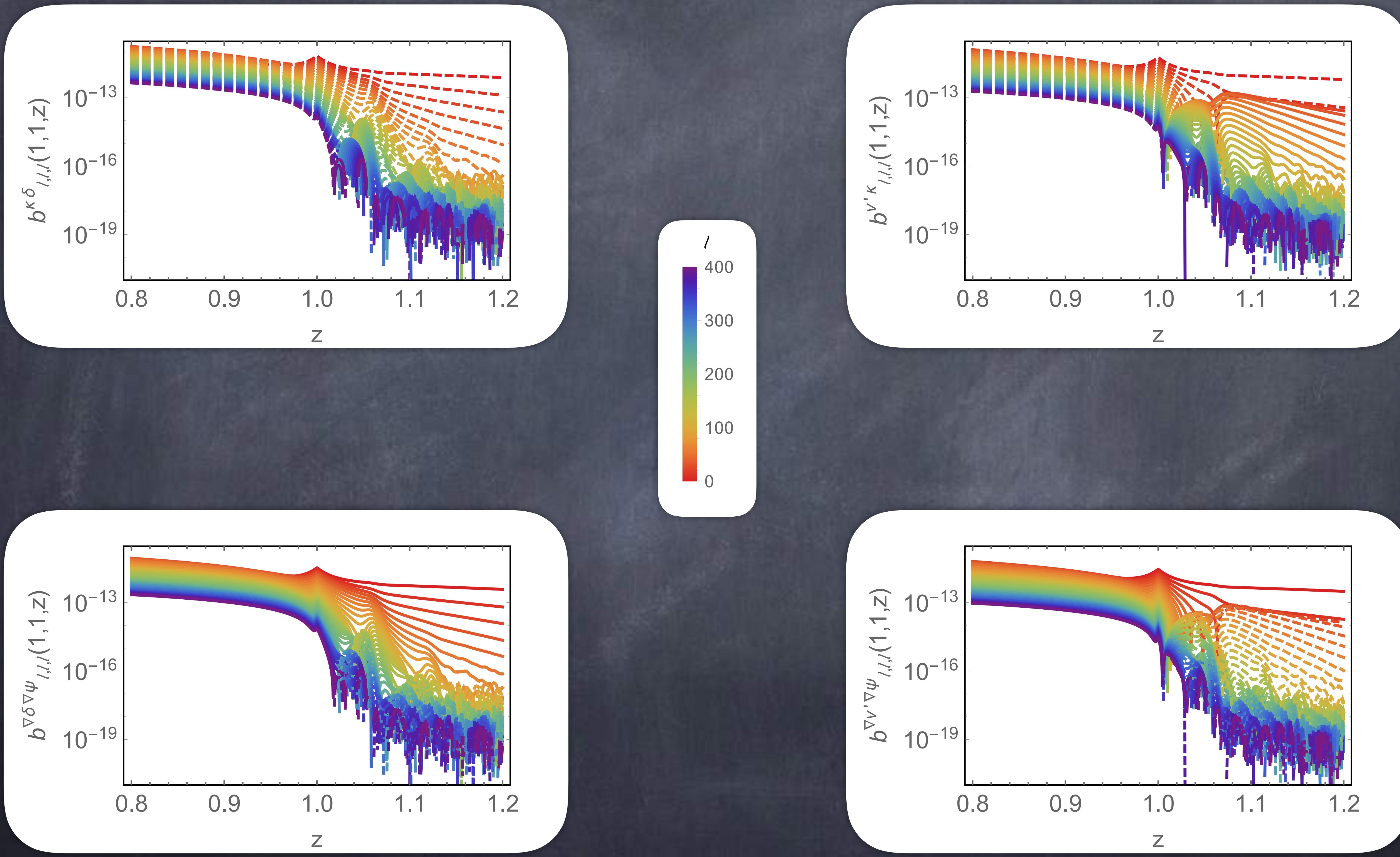
ED, Durrer, Marozzi, Montanari [arXiv:1510.04202]

density-RSD



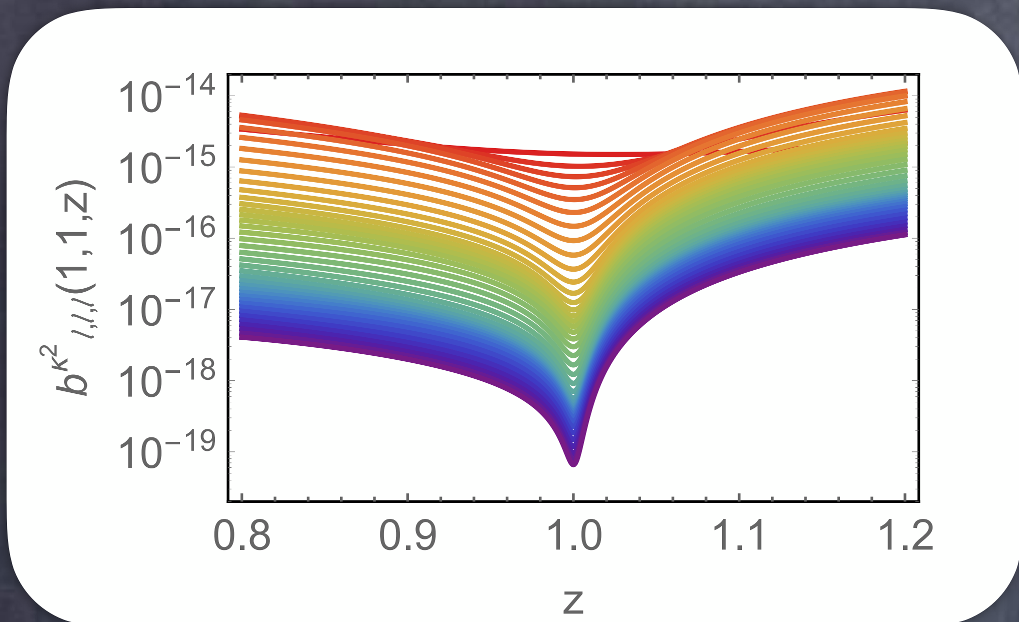

ED, Durrer, Marozzi, Montanari [arXiv:1510.04202]

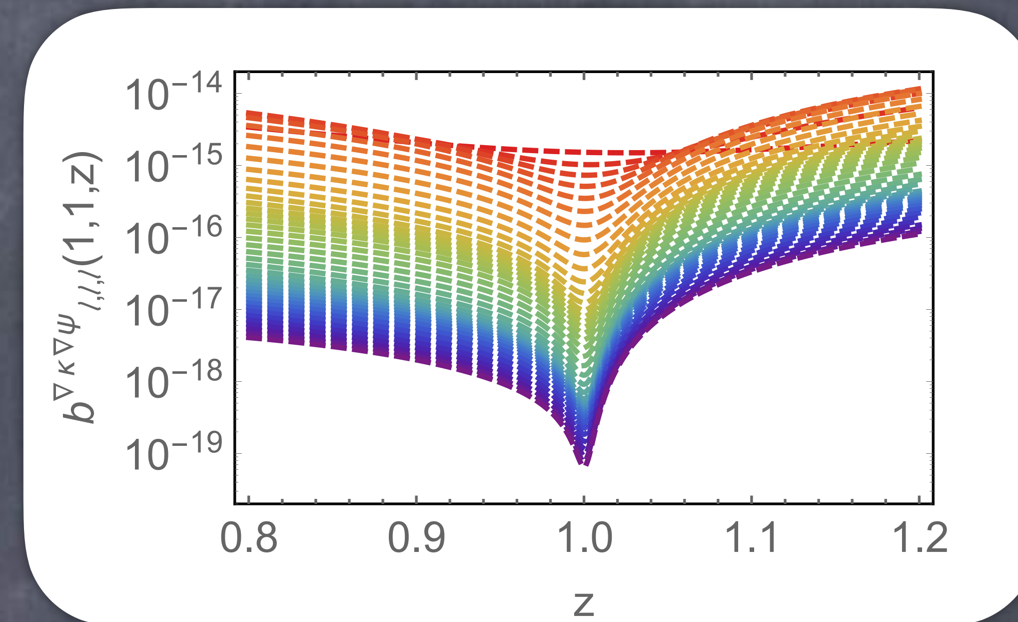
# lensing x newtonian

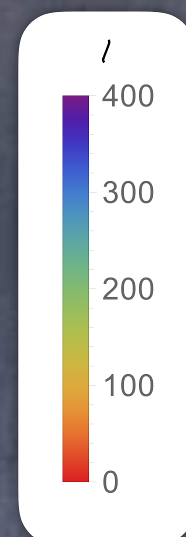


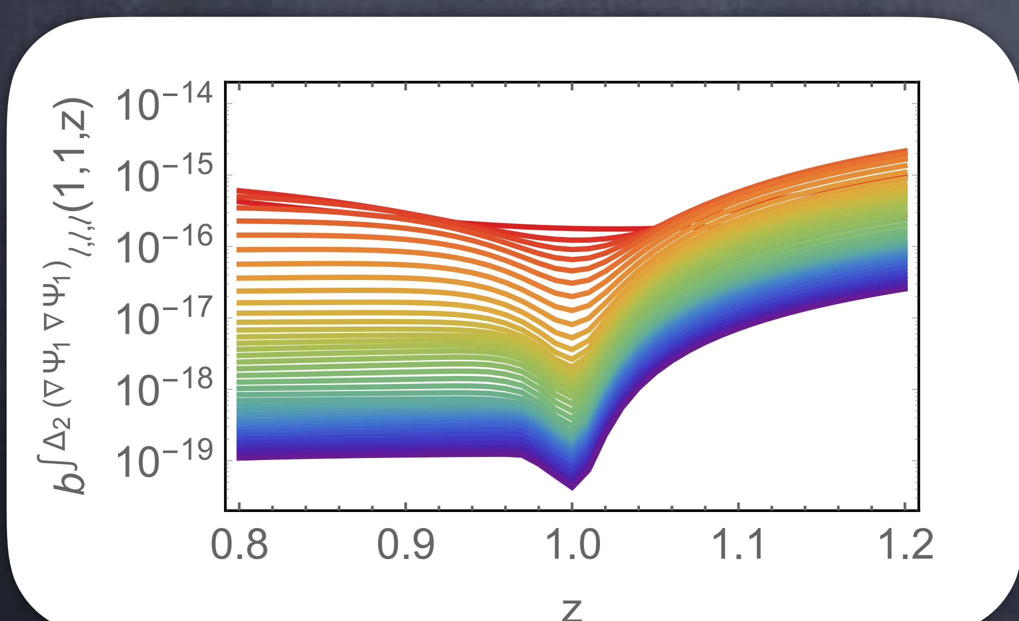

ED, Durrer, Marozzi, Montanari [arXiv:1510.04202]

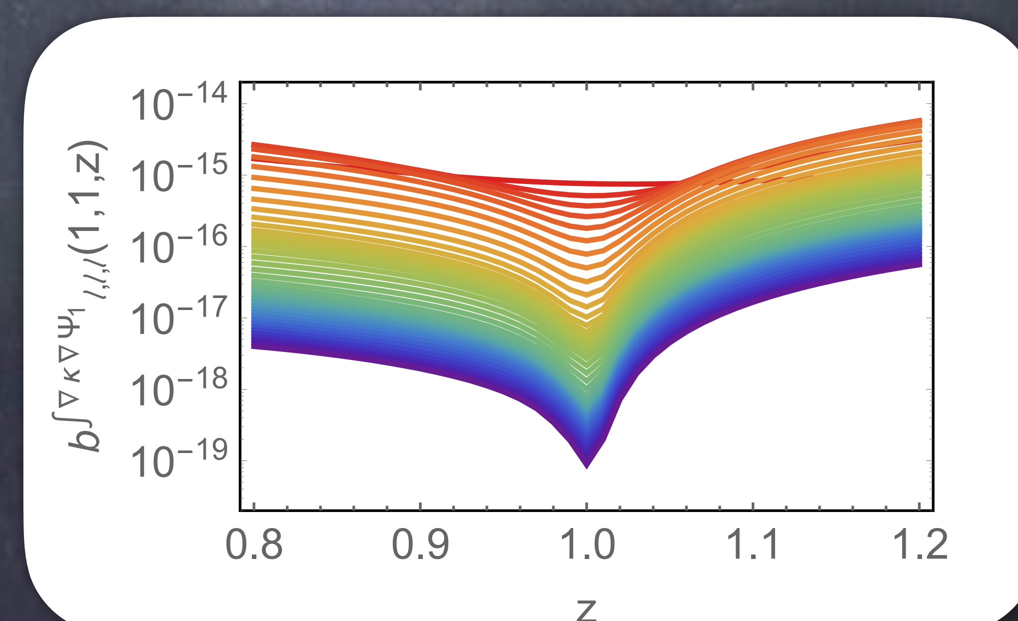
# Lensing



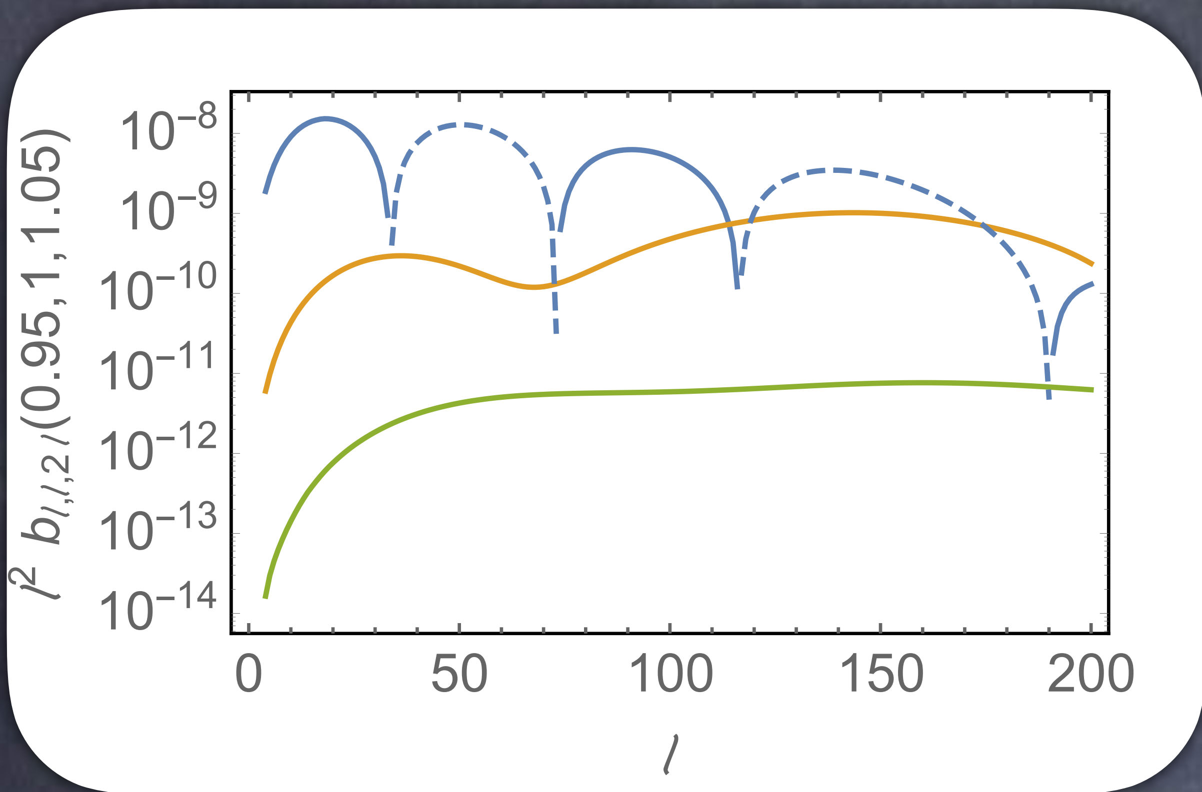



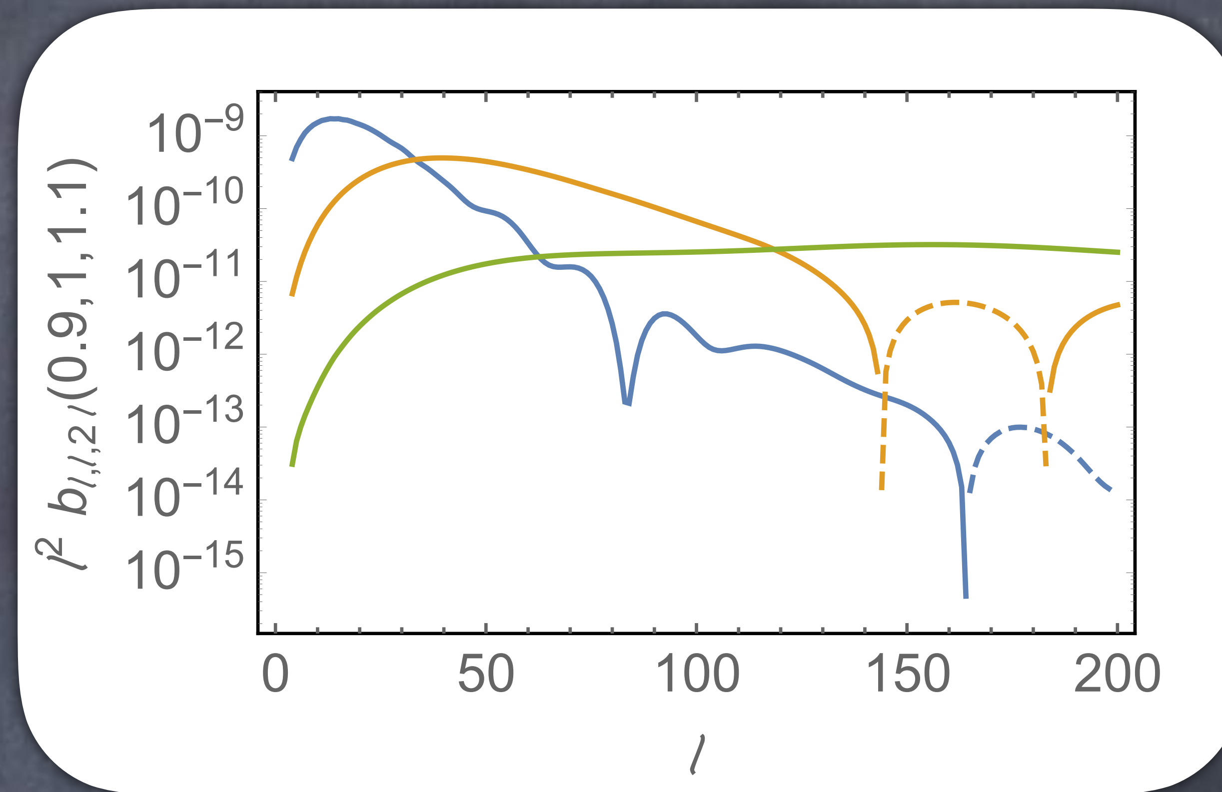



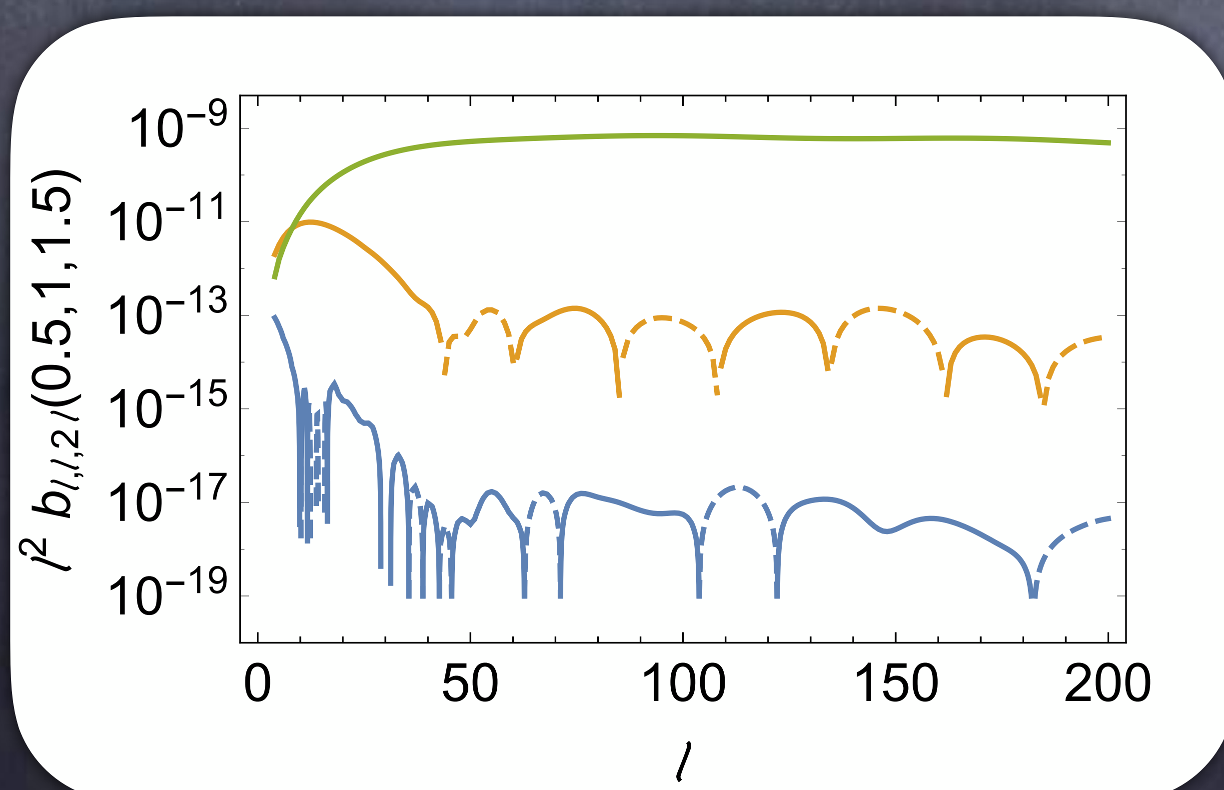

Beyond Born-Approximation

ED, Durrer, Marozzi, Montanari [arXiv:1510.04202]

# Bispectrum





ED, Durrer, Marozzi, Montanari [arXiv:1510.04202]

# Conclusions

- The observable quantity includes several relativistic corrections
  - which encode useful information
  - and may bias the parameter estimation, if neglected
- Multi-tracers technique allows to isolate some relativistic correction. Future surveys should be able to measure them
- relativistic number counts can be generalized to non-flat universes
  - neglecting relativistic effects may bias the curvature parameter as well
- Many new contributions to the bispectrum
  - how do they affect $f_{\rm NL}$?