Decoherence in Inflation

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Contents

- Quantization of cosmological perturbations during inflation
- 'Squeezing' of the vacuum state
- Decoherence and classicalization

Setup

- Background:
  - FRW-model
  - slow-roll inflation with one scalar field φ

$$ds^2 = a^2(\eta)\left(-d\eta^2 + dx^2 + dy^2 + dz^2\right)$$

- Add perturbations to metric and φ
- One scalar degree of freedom: Sasaki-Mukhanov variable ν

Action

- Action:

$$S = \frac{M_{\mathrm{Pl}}^2}{2} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \left(-\frac{1}{2} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi - V(\varphi) \right)$$

- For ν:

$$S = \frac{1}{2} \int d^4x \left(\nu'^2 - \nu_{,i}\nu_{,i} + \frac{z''}{z}\nu^2 \right),$$

$$\frac{z''}{z} = \frac{2 + 9\epsilon_V - 3\eta_V}{\eta^2}$$

Hamiltonian mechanics

- Hamiltonian:

$$H = \int d^3\mathbf{x} \left(\frac{1}{2}p^2 + \frac{1}{2}\nu_{,i}\nu_{,i} - \frac{z''}{2z}\nu^2 \right)$$

- This induces time evolution:

$$\nu' = \frac{\delta H}{\delta p} = \{\nu, H\} = p,$$

$$p' = -\frac{\delta H}{\delta \nu} = \{p, H\} = \nu_{,ii} + \frac{z''}{z}\nu$$

Canonical quantization

- Promote observables to operators, Poisson brackets to commutators:

$$\hat{H} = \frac{1}{2}\int d^3\mathbf{x}\left(\hat{p}^2 + \hat{\nu}_{,i}\hat{\nu}_{,i} - \frac{z''}{z}\hat{\nu}^2\right),$$

$$\hat{\nu}' = -i\left[\hat{\nu}, \hat{H}\right] = \hat{p},$$

$$\hat{p}' = -i\left[\hat{p}, \hat{H}\right] = \hat{\nu}_{,ii} + \frac{z''}{z}\hat{\nu},$$

$$[\hat{\nu}(\eta, \mathbf{x}), \hat{p}(\eta, \mathbf{y})] = i\delta^3(\mathbf{x} - \mathbf{y})$$

Canonical quantization

- Representation:

$$\hat{\nu}(\eta, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left[\hat{c}_{\mathbf{k}} \mu_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{c}^\dagger_{\mathbf{k}} \mu^*_k(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}} \right],$$

$$\hat{p}(\eta, \mathbf{x}) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} \left[\hat{c}_{\mathbf{k}} \mu'_k(\eta) e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{c}^\dagger_{\mathbf{k}} \mu'^*_k(\eta) e^{-i\mathbf{k}\cdot\mathbf{x}} \right]$$

- The annihilation operators define the vacuum state:

$$\hat{c}_{\mathbf{k}} \left|0\right\rangle = 0 \quad \forall\, \mathbf{k} \in \mathbb{R}^3$$

Mode functions

- Have to solve the mode functions $\mu_{\mathbf{k}}$
  - time evolution from eq.s of motion
  - initial conditions corresponding to adiabatic vacuum

$$\mu_k(\eta) = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{k\eta}\right)e^{-ik(\eta-\eta_{\mathrm{ini}})} \tag{1}$$

Vacuum State?

- Assume: ν in its vacuum state during inflation
- We want to further investigate the vacuum state
- To do this, go to the Fourier space and use the Schrödinger picture

To Fourier space

- Finite box with periodic boundary conditions
- Expand:

$$\hat{\nu}(\eta, \mathbf{x}) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \frac{1}{\sqrt{2}} \left(\hat{\nu}_{\mathbf{k}}^{R} + i \hat{\nu}_{\mathbf{k}}^{I} \right) e^{i \mathbf{k} \cdot \mathbf{x}},$$

$$\hat{p}(\eta, \mathbf{x}) = \frac{1}{\sqrt{V}} \sum_{\mathbf{k}} \frac{1}{\sqrt{2}} \left(\hat{p}_{\mathbf{k}}^{R} + i \hat{p}_{\mathbf{k}}^{I} \right) e^{i \mathbf{k} \cdot \mathbf{x}},$$

$$\left[\hat{\nu}_{\mathbf{k}}^{R}, \hat{p}_{\mathbf{k}'}^{R} \right] = \left[\hat{\nu}_{\mathbf{k}}^{I}, \hat{p}_{\mathbf{k}'}^{I} \right] = i \delta_{\mathbf{k}\mathbf{k}'}$$

To Fourier space

- Hamiltonian:

$$\hat{H} = \sum_{\mathbf{k}\in\mathbb{R}^3_+} \Big[\hat{H}_{\mathbf{k},R}(\eta) + \hat{H}_{\mathbf{k},I}(\eta)\Big],$$

$$\hat{H}_{\mathbf{k},A}(\eta) \equiv \frac{1}{2}\hat{p}_{\mathbf{k}}^{A\,2} + \frac{\omega_k^2(\eta)}{2}\hat{\nu}_{\mathbf{k}}^{A\,2},$$

$$\omega_k^2(\eta) \equiv k^2 - \frac{z''}{z}$$

- Different modes decouple and evolve independently

To the Schrödinger picture

- Independent wave functions:

$$\psi_{\mathbf{k}}^{A}(\nu_{\mathbf{k}}^{A},\eta) \equiv \left\langle \nu_{\mathbf{k}}^{A} \middle| \psi_{\mathbf{k},A}(\eta) \right\rangle$$

- Operator representations:

$$\hat{\nu}_{\mathbf{k}}^{A}\psi_{\mathbf{k}}^{A}(\nu_{\mathbf{k}}^{A},\eta) = \nu_{\mathbf{k}}^{A}\psi_{\mathbf{k}}^{A}(\nu_{\mathbf{k}}^{A},\eta),$$
$$\hat{p}_{\mathbf{k}}^{A}\psi_{\mathbf{k}}^{A}(\nu_{\mathbf{k}}^{A},\eta) = -i\frac{\partial}{\partial\nu_{\mathbf{k}}^{A}}\psi_{\mathbf{k}}^{A}(\nu_{\mathbf{k}}^{A},\eta)$$

- Time evolution in the Schrödinger picture:

$$\psi_{\mathbf{k}}^{\prime A}(\nu_{\mathbf{k}}^{A},\eta) = -i\left(-\frac{1}{2}\frac{\partial^2}{\partial(\nu_{\mathbf{k}}^{A})^2} + \frac{\omega_k^2(\eta)}{2}\nu_{\mathbf{k}}^{A\,2}\right)\psi_{\mathbf{k}}^{A}(\nu_{\mathbf{k}}^{A},\eta)$$

Vacuum Wave Function

- Condition for vacuum:

$$\hat{c}_{\mathbf{k}} \left|0\right\rangle = 0$$
$$\Rightarrow \left(-i\mu_k'^* \nu_{\mathbf{k}}^A + \mu_k^* \frac{\partial}{\partial \nu_{\mathbf{k}}^A}\right) \psi_{\mathbf{k}}^A\left(\nu_{\mathbf{k}}^A, \eta\right) = 0$$

- Solution:

$$\psi_{\mathbf{k}}^A\left(\nu_{\mathbf{k}}^A, \eta\right) = C(\eta) e^{-\Omega_k(\eta)\nu_{\mathbf{k}}^{A2}}$$

Vacuum Wave Function

- With Schrödinger equation and normalization:

$$\Omega_k(\eta) = -\frac{i}{2}\frac{\mu_k'^*}{\mu_k^*},$$

$$C(\eta) = N(\eta)e^{i\theta(\eta)},$$

$$N(\eta) = \left(\frac{2\,\mathrm{Re}\,\Omega_k(\eta)}{\pi}\right)^{\frac{1}{4}},$$

$$\theta(\eta) = -\int_{\eta_{\mathrm{ini}}}^{\eta} d\eta'\,\mathrm{Re}\,\Omega_k(\eta')$$

Vacuum Wave Function

- Uncertainties of $\hat{\nu}_{\mathbf{k}}^{A}$ and $\hat{p}_{\mathbf{k}}^{A}$ grow without limit as time goes on, with

$$\Delta\nu_{\mathbf{k}}^{A}\Delta p_{\mathbf{k}}^{A} = \frac{1}{2}\sqrt{1 + \frac{1}{\eta^6 k^6}}$$

- But a certain linear combination becomes well defined: squeezing

Wigner function

- Wigner function is a way to visualize correlations between a canonical variable $\hat{x}$ and the corresponding canonical momentum $\hat{p}$

$$W_\psi(x, p) \equiv \frac{1}{\pi} \int dy \, \psi^*(x + y)\psi(x - y)e^{2ipy}$$

- 'Phase space probability density'

Wigner function

- Properties:

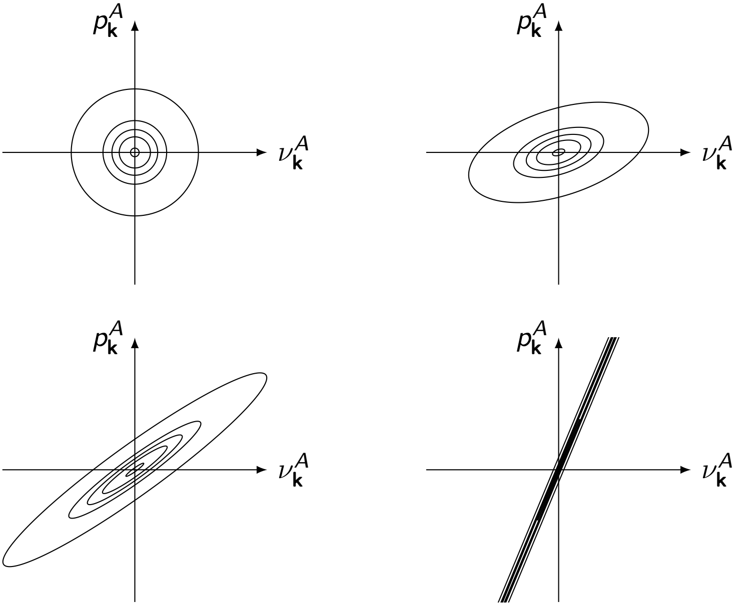
$$\int dx\,dp\,W_\psi(x,p) = 1,$$
$$\int dp\,W_\psi(x,p) = |\psi(x)|^2,$$
$$\int dx\,W_\psi(x,p) = |\psi(p)|^2,$$
$$2\pi \int dx\,dp\,W_\psi(x,p) W_\phi(x,p) = |\langle\psi|\phi\rangle|^2$$

Wigner function of the Vacuum State

- For our vacuum state:

$$W_0(\nu_{\mathbf{k}}^A, p_{\mathbf{k}}^A) = \frac{\left|\psi_{\mathbf{k}}^A(\nu_{\mathbf{k}}^A)\right|^2}{\sqrt{2\pi \operatorname{Re}\Omega_k}} \exp\left[-\frac{1}{2\operatorname{Re}\Omega_k}\left(p_{\mathbf{k}}^A + 2\nu_{\mathbf{k}}^A \operatorname{Im}\Omega_k\right)^2\right]$$
$$\xrightarrow{\eta\to 0} \left|\psi_{\mathbf{k}}^A(\nu_{\mathbf{k}}^A)\right|^2 \delta\left(p_{\mathbf{k}}^A + 2\nu_{\mathbf{k}}^A \operatorname{Im}\Omega_k\right)$$

- Uncertainty of the combination $p_{\mathbf{k}}^A + 2\nu_{\mathbf{k}}^A \operatorname{Im}\Omega_k$ becomes small


$p_{\mathbf{k}}^A$
$\nu_{\mathbf{k}}^A$
$p_{\mathbf{k}}^A$
$\nu_{\mathbf{k}}^A$
$p_{\mathbf{k}}^A$
$\nu_{\mathbf{k}}^A$
$p_{\mathbf{k}}^A$
$\nu_{\mathbf{k}}^A$

Wigner function of the Vacuum State

- Back in the Heisenberg picture, the operators obey

$$\hat{\nu}_{\mathbf{k}}^{A}(\eta) \approx \frac{1}{k^2\eta}\hat{p}_{\mathbf{k}0}^{A} - \frac{1}{3}k^2\eta^2\hat{\nu}_{\mathbf{k}0}^{A},$$

$$\hat{p}_{\mathbf{k}}^{A}(\eta) \approx -\frac{1}{k^2\eta^2}\hat{p}_{\mathbf{k}0}^{A} - \frac{2}{3}k^2\eta\hat{\nu}_{\mathbf{k}0}^{A}$$

- At late times, then:

$$\hat{p}_{\mathbf{k}}^{A}(\eta) \approx -\frac{1}{\eta}\hat{\nu}_{\mathbf{k}}^{A}(\eta),$$

$$\hat{\nu}_{\mathbf{k}}^{A}(\eta_1) \approx \frac{\eta_2}{\eta_1}\hat{\nu}_{\mathbf{k}}^{A}(\eta_2)$$

Classical or Not?

- Canonical variables commute; classical regime?
- No: the state is still very much quantum mechanical!
- But: expectation values practically indistinguishable from classical case ('decoherence without decoherence')

Classical or Not?

- We wish to have an ensemble of classical states, described by a density operator:

$$\hat{\rho} = \sum_i w_i \, |\psi_i\rangle\langle\psi_i|$$

- Achieved through environment induced decoherence

A Laboratory Example

- Direct product Hilbert space:
  system $\times$ environment

$$
\begin{aligned}
|\Psi\rangle &= |\psi\rangle \otimes |E\rangle \\
&= \left[\sum_i c_i \, |\psi_i\rangle\right] \otimes |E\rangle
\end{aligned}
$$

- Interaction between these two:

$$\hat{H}_{\mathrm{int}} = \sum_i |\psi_i\rangle\langle\psi_i| \otimes \hat{A}_i$$

Decoherence: A Laboratory Example

- Time evolution:

$$|\Psi(t)\rangle = \sum_i c_i \,|\psi_i\rangle \otimes \exp\Big(-i\hat{A}_i t\Big)\,|E\rangle \equiv \sum_i c_i\,|\psi_i\rangle \otimes |E_i(t)\rangle$$

- Take trace over the orthogonal environmental states:

$$\begin{aligned}\hat{\rho}_{\rm red} &= \sum_{i,j,k} c_i c_j^* \,|\psi_i\rangle\langle\psi_j| \otimes \underbrace{\langle E_k|E_i\rangle}_{\delta_{ki}}\underbrace{\langle E_j|E_k\rangle}_{\delta_{jk}} \\ &= \sum_i |c_i|^2\,|\psi_i\rangle\langle\psi_i|\end{aligned}$$

Decoherence: A Laboratory Example

- Reduced density matrix $\hat{\rho}_{\mathrm{red}}$ describes the system for the purposes of observing its state
- Decoherence: from superposition to a classical ensemble
  - loss of coherence, information of phases

Environment in Cosmology

- What is the environment for the cosmological perturbations?
  - other fields?
  - other Fourier modes?

Master Equation

- A fairly general case described by a master equation:

$$\hat{\rho}'_{\mathrm{red}} = -i\left[\hat{H}_S, \hat{\rho}_{\mathrm{red}}\right] - \frac{1}{2}\sum_k \left(\hat{L}_k^\dagger \hat{L}_k \hat{\rho}_{\mathrm{red}} + \hat{\rho}_{\mathrm{red}} \hat{L}_k^\dagger \hat{L}_k - 2\hat{L}_k \hat{\rho}_{\mathrm{red}} \hat{L}_k^\dagger\right)$$

- Assumptions:
  - environment is ‘large’
  - correlation time is short: process is Markovian

Master Equation

- Cosmological perturbations:

$$\hat{L} = \Lambda \hat{\nu}_{\mathbf{k}}^{A}$$

- To accommodate this, introduce the density matrix:

$$\hat{\rho}_{\mathrm{red}} = \int d\nu_{\mathbf{k}}^{A} d\nu_{\mathbf{k}}^{\prime A} \left| \nu_{\mathbf{k}}^{A} \right\rangle \rho_{\mathrm{red}} \left(\nu_{\mathbf{k}}^{A}, \nu_{\mathbf{k}}^{\prime A} \right) \left\langle \nu_{\mathbf{k}}^{\prime A} \right|$$

Master Equation

- Lindblad equation becomes:

$$\rho_{\rm red}\left(\nu_{\mathbf{k}}^{A},\nu_{\mathbf{k}}'^{A},\eta\right)' = \left[\frac{i}{2}\left(\frac{\partial^2}{\partial(\nu_{\mathbf{k}}^{A})^2} - \frac{\partial^2}{\partial(\nu_{\mathbf{k}}'^{A})^2}\right) - i\frac{\omega_k^2}{2}\left(\nu_{\mathbf{k}}^{A\,2} - \nu_{\mathbf{k}}'^{A\,2}\right) - \frac{\Lambda}{2}\left(\nu_{\mathbf{k}}^{A} - \nu_{\mathbf{k}}'^{A}\right)^2\right]\rho_{\rm red}\left(\nu_{\mathbf{k}}^{A},\nu_{\mathbf{k}}'^{A},\eta\right)$$

- Concentrate on the interaction part:

$$\rho_{\rm red}\left(\nu_{\mathbf{k}}^{A},\nu_{\mathbf{k}}'^{A},\eta\right)' = -\frac{\Lambda}{2}\left(\nu_{\mathbf{k}}^{A} - \nu_{\mathbf{k}}'^{A}\right)^2\rho_{\rm red}\left(\nu_{\mathbf{k}}^{A},\nu_{\mathbf{k}}'^{A},\eta\right)$$

$$\Rightarrow \rho_{\rm red}\left(\nu_{\mathbf{k}}^{A},\nu_{\mathbf{k}}'^{A},\eta\right) = \rho_{\rm red}\left(\nu_{\mathbf{k}}^{A},\nu_{\mathbf{k}}'^{A},\eta_{\rm ini}\right)\exp\left\{-\frac{\Lambda}{2}\left(\nu_{\mathbf{k}}^{A} - \nu_{\mathbf{k}}'^{A}\right)^2(\eta-\eta_{\rm ini})\right\}$$

Master Equation

- This leads to decoherence:

$$\hat{\rho}_{\text{red}} \xrightarrow{\eta\to 0} \frac{1}{\delta(0)} \int d\nu_{\mathbf{k}}^{A} \left|\nu_{\mathbf{k}}^{A}\right\rangle \rho_{\text{red}}\left(\nu_{\mathbf{k}}^{A}, \nu_{\mathbf{k}}^{A}\right) \left\langle \nu_{\mathbf{k}}^{A}\right|$$

$$= \frac{1}{\delta(0)} \int d\nu_{\mathbf{k}}^{A} \left|\nu_{\mathbf{k}}^{A}\right\rangle \left|\psi_{\mathbf{k}}^{A}\left(\nu_{\mathbf{k}}^{A}\right)\right|^{2} \left\langle \nu_{\mathbf{k}}^{A}\right|$$

Pointer Basis

- This suggests that the field value basis is the right pointer basis
- Proof that $\hat{\nu}^A_{\mathbf{k}}$ is a proper pointer variable:

$$\left[\hat{\nu}^A_{\mathbf{k}}, \hat{H}_{\mathrm{int}}\right] = 0,$$
$$\left[\hat{\nu}^A_{\mathbf{k}}(\eta_1), \hat{\nu}^A_{\mathbf{k}}(\eta_2)\right] = 0$$

- (Latter comes from squeezing)

Classicalization

- Decoherence brings the squeezed vacuum state into an ensemble of states with well-defined ν-values
- These correspond to different classical universes, which evolve classically onwards
  - classical universes stay classical

A Word about the Measurement Problem

- How is the observed universe singled out?

Detecting Decoherence

- Expectation values:

$$\left\langle F\left(\nu_{\mathbf{k}}^{A}, p_{\mathbf{k}}^{A}\right)\right\rangle = \mathrm{tr}\left[\hat{\rho} F\left(\hat{\nu}_{\mathbf{k}}^{A}, \hat{p}_{\mathbf{k}}^{A}\right)\right]$$
$$= \int d\nu_{\mathbf{k}}^{A} \left\langle \nu_{\mathbf{k}}^{A}\middle|\hat{\rho} F\left(\hat{\nu}_{\mathbf{k}}^{A}, -\frac{1}{\eta}\hat{\nu}_{\mathbf{k}}^{A}\right)\middle|\nu_{\mathbf{k}}^{A}\right\rangle$$
$$= \int d\nu_{\mathbf{k}}^{A} \left\langle \nu_{\mathbf{k}}^{A}\middle|\hat{\rho}\middle|\nu_{\mathbf{k}}^{A}\right\rangle F\left(\nu_{\mathbf{k}}^{A}, -\frac{1}{\eta}\nu_{\mathbf{k}}^{A}\right)$$

- These are insensitive to decoherence
- Decoherence w.r.t another basis would produce different results

Summary

- Primordial scalar perturbations get squeezed during inflation
- Decoherence in field value basis helps understand the transition from quantum to classical physics

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