Higgs boson and cosmology – can we learn anything about fundamental physics?

Fedor Bezrukov

The University of Manchester

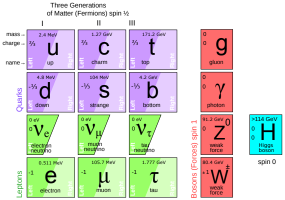
University of Helsinki
October 5, 2016

The University of Manchester

Outline

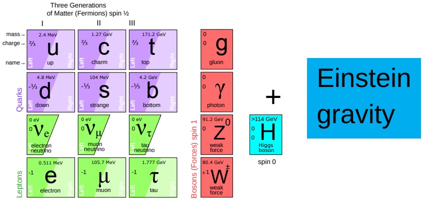
1. Standard Model and the reality of the Universe
   - Standard Model is in great shape!
   - All new physics at low scale–νMSM
   - Top-quark and Higgs-boson masses and vacuum stability
2. Stable Electroweak vacuum
3. Metastable vacuum and Cosmology
   - Safety today
   - Safety at inflation
   - Adding RG corrections
4. Surviving the false vacuum in the Hot Universe

Lesson from LHC so far – Standard Model is good



- SM works in all laboratory/collider experiments (electroweak, strong)
- LHC 2012 – final piece of the model discovered – Higgs boson
  - Mass measured $\sim$ 125 GeV – weak coupling! Perturbative and predictive for high energies
- Add gravity
  - get cosmology
  - get Planck scale $M_P \sim 1.22 \times 10^{19}$ GeV as the highest energy to worry about

Lesson from LHC so far – Standard Model is good



- SM works in all laboratory/collider experiments (electroweak, strong)
- LHC 2012 – final piece of the model discovered – Higgs boson
  - Mass measured $\sim$ 125 GeV – weak coupling! Perturbative and predictive for high energies
- Add gravity
  - get cosmology
  - get Planck scale $M_P \sim 1.22 \times 10^{19}$ GeV as the highest energy to worry about

Many things in cosmology are not explained by SM

Experimental observations

- Dark Matter
- Baryon asymmetry of the Universe
- Inflation (nearly scale invariant spectrum of initial density perturbations)

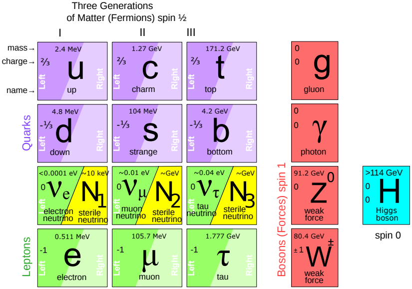
Laboratory also asks for SM extensions

- Neutrino oscillations

Nothing really points to a definite scale above EW

- Neutrino masses and oscillations (absent in SM)
  - Right handed neutrino between 1 eV and 10^{15} GeV
- Dark Matter (absent in SM)
  - Models exist from 10^{-5} eV (axions) up to 10^{20} GeV (Wimpzillas, Q-balls)
- Baryogenesys (absent in SM)
  - Leptogenesys scenarios exist from $M \sim 10$ MeV up to 10^{15} GeV

Possible: New physics only at low scales – νMSM



Role of sterile neutrinos

N_1 $M_1 \sim 1 - 50\text{keV}$: (Warm) Dark Matter,
Note: $M_1 = 7\text{keV}$ has been seen in X-rays?!

$N_{2,3}$ $M_{2,3} \sim$ several GeV:
Gives masses for active neutrinos, Baryogenesys

Asaka, Shaposhnikov'05; Asaka, Blanchet, Shaposhnikov'05

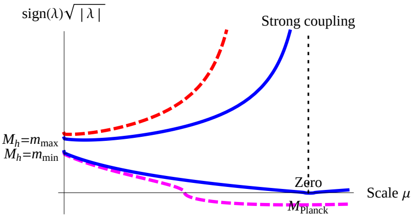
What we are left with?

- Inflationary mechanism required
- Higgs is weakly coupled

but not completely trouble free

Standard Model self-consistency and Radiative Corrections

- Higgs self coupling constant λ changes with energy due to radiative corrections.

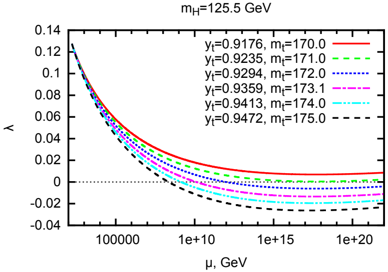


$$(4\pi)^2\beta_\lambda = 24\lambda^2 - 6y_t^4 + \frac{3}{8}(2g_2^4 + (g_2^2 + g_1^2)^2) + (-9g_2^2 - 3g_1^2 + 12y_t^2)\lambda$$

- Behaviour is determined by the masses of the Higgs boson $m_H = \sqrt{2\lambda}v$ and other heavy particles (top quark $m_t = y_t v/\sqrt{2}$)
- If Higgs is heavy $M_H > 170$ GeV – the model enters *strong coupling* at some low energy scale – new physics emerges.

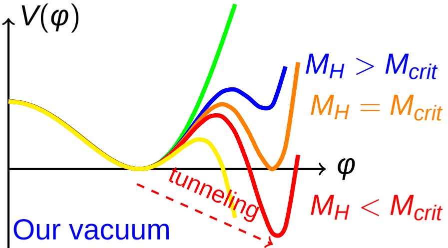
Lower Higgs masses: RG corrections push Higgs coupling to negative values

Coupling λ evolution:



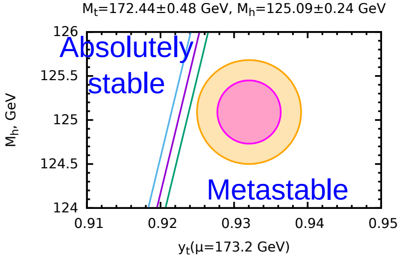
- For Higgs masses $M_H < M_{\text{critical}}$ coupling constant is negative above some scale μ_0.
- The Higgs potential may become negative!
  - Our world is not in the lowest energy state!
  - Problems at some scale $\mu_0 > 10^{10}$ GeV?

Higgs potential $V(\varphi) \simeq \lambda(\varphi)\frac{\varphi^4}{4}$

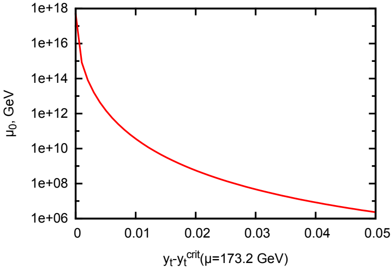


LHC result: SM is definitely perturbative up to Planck scale, and probably has metastable SM vacuum

Experimental values for y_t



Scale μ_0 for $\lambda(\mu_0) = 0$

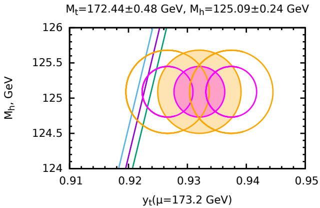


We live close to the metastability boundary – but on which side?!

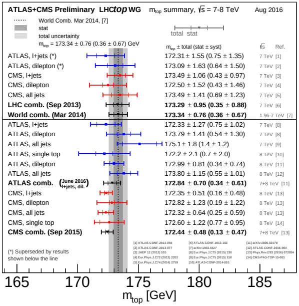
Future measurements of top Yukawa and Higgs mass are essential!

Determination of top quark Yukawa

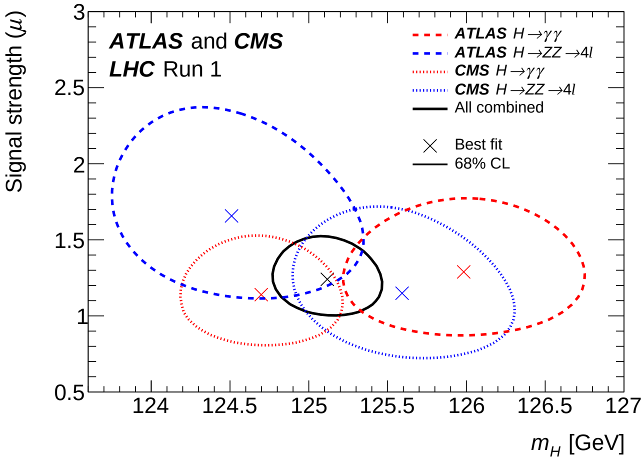
- Hard to determine mass in the events
- Hard to relate the “pole” (the same for “Mont-Carlo”) mass to the $\overline{\text{MS}}$ top quark Yukawa
  - NLO event generators
  - Electroweak corrections – important at the current precision goals!



- Build a lepton collider?
- Improve analysis on a hadron collider?



Higgs boson mass measurements



ATLAS CMS 2015

Vacuum stability – what it means?

- Stable Electroweak vacuum – looks safe
- Metastable – is it ok?

Inflation versus vacuum stability

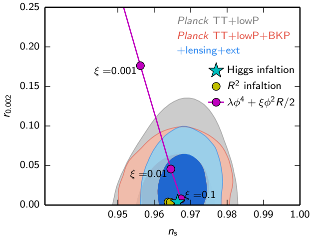
Stable SM vacuum	inflaton & Higgs independent	inflaton & Higgs interacting	inflaton = Higgs
Large r	Yes	Yes	Yes (threshold corr.)
Small r	Yes	Yes	Yes
Planck scale corections	Any	Any	Scale inv.

Metastable SM vacuum	inflaton & Higgs independent	inflaton & Higgs interacting	inflaton = Higgs
Large r	No	Yes Model dep.	No
Small r	Yes $r < 10^{-9}$	Yes Model dep.	Yes (threshold corr.)
Planck scale corections	Restricted	Model dep.	Scale inv.

Stable EW vacuum – mostly anything works

Would be a rather dull situation

- No problems throughout the whole thermal evolution of the Universe.
- Adding inflation – many examples
  - R^2 inflation
  - non-minimally coupled Higgs inflation
    - specific CMB predictions
  - Separate scalar inflaton interacting with the Higgs boson
    - Together with requirements of weak coupling and some scale symmetries often predicts hidden light or EW scale scalars



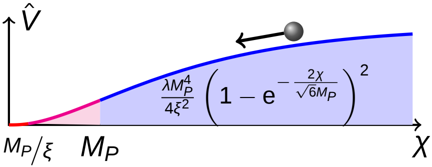
Higgs inflation at tree level

Scalar part of the (Jordan frame) action

$$S_J = \int d^4x\sqrt{-g}\left\{-\frac{M_P^2}{2}R - \xi\frac{h^2}{2}R + g_{\mu\nu}\frac{\partial^\mu h\partial^\nu h}{2} - \frac{\lambda}{4}(h^2-v^2)^2\right\}$$

To get observed $\delta T/T \sim 10^{-5}$

$$\frac{\sqrt{\lambda}}{\xi} = \frac{1}{49000}$$

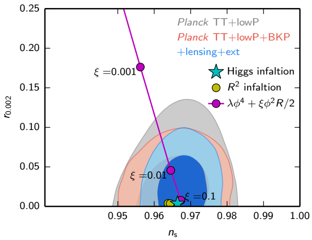


Conformal transformation: $\hat{g}_{\mu\nu} = \sqrt{1+\frac{\xi\varphi^2}{M_P^2}}\, g_{\mu\nu},$

Requirement from UV physics – No corrections $\frac{h^n}{M_P^{4-n}}$ allowed

CMB parameters are predicted

Exactly like preferred by CMB



For large ξ Higgs inflation

spectral index $n \simeq 1 - \frac{8(4N+9)}{(4N+3)^2} \simeq 0.97$

tensor/scalar ratio $r \simeq \frac{192}{(4N+3)^2} \simeq 0.0033$

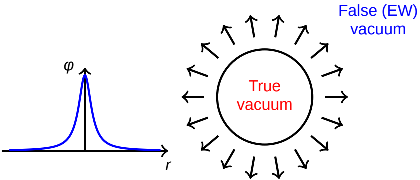
$$\delta T/T \sim 10^{-5} \quad \Longrightarrow \quad \frac{\xi}{\sqrt{\lambda}} \simeq 47000$$

Note: for very near critical top quark/Higgs masses results change and allow for larger r

What if we live in metastable vacuum?

Do not worry! At least not too much

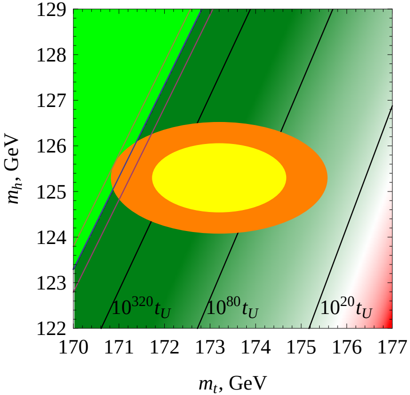
Vacuum decays by creating bubbles of true vacuum, which then expand very fast ($v \to c$)



Tunneling suppression:

$$p_{\text{decay}} \propto e^{-S_{\text{bounce}}} \sim e^{-\frac{8\pi^8}{3\lambda(h)}}$$

Lifetime $\gg$ age of the Universe!



Note on Planck corrections

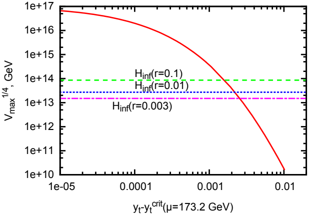
- Critical bubble size $\sim$ Planck scale
- Potential corrections $V_{\text{Planck}} = \pm \frac{\varphi^n}{M_P^{n-4}}$ change lifetime!
  - Only '+' sign is allowed for Planck scale corrections!

As far as we are “safe” now (i.e. at low energies), what about Early Universe?
What happens with the Higgs boson at inflation?

- if Higgs boson is completely separate from inflation
- if Higgs boson interacts with inflaton/gravitation background
- if Higgs boson drives inflation

Metastable vacuum during inflation *is* dangerous

- Let us suppose Higgs is not at all connected to inflationary physics (e.g. R^2 inflation)
- All fileds have vacuum fluctuation
- Typical momentum $k \sim H_{\text{inf}}$ is of the order of Hubble scale



  - If typical momentum is greater than the potential barrier – SM vacuum would decay if

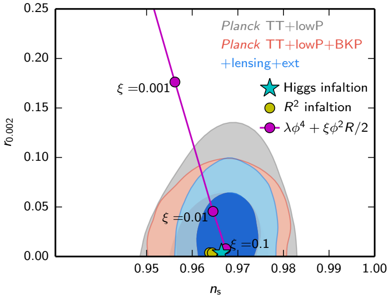
$$H_{\text{inf}} > V_{\text{max}}^{1/4}$$

Most probably, fluctuations at inflation lead to SM vacuum decay…

- Observation of tensor-to-scalar ratio r by CMB polarization missions would mean great danger for metastable SM vacuum!

Measurement of primordial tensor modes determines scale of inflation

$$H_{\rm inf} = \sqrt{\frac{V_{\rm infl}}{3M_P^2}} \sim 8.6 \times 10^{13}\,{\rm GeV}\left(\frac{r}{0.1}\right)^{1/2}$$



Does inflation contradict metastable EW vacuum?

- Higgs interacting with inflation can soothe the problem, but require $h_{\text{beginning of inflation}} \sim 0$
  - Higgs–inflaton (χ) interaction may stabilize the Higgs

$$L_{\text{int}} = -\alpha h^2 \chi^2$$

    (May destabilize at reheating)
  - Higgs-gravity *negative* non-minimal coupling stabilizes Higgs in de-Sitter (inflating) space

$$L_{\text{nm}} = \xi h^2 R$$

    (However, destabilises EW vacuum after inflation)
- New physics below μ_0 may remove Planck scale vacuum and make EW vacuum stable – many examples
  - Threshold effects
  - Additional bosons modify λ running

New physics *above* μ_0 may solve the problem

Requirements

- Minimum at Planck scale should be removed (but can remain near $\mu_0 \sim 10^{10}$ GeV)
- Reheating after inflation should be fast.

No need for new physics at "low" ($< \mu_0$) scales!
Example: Higgs inflation with threshold corrections at M_p/ξ

RG improved effective potential

$$U(\varphi) = \frac{\lambda(\mu)}{4}\varphi^4 + \sum_i \frac{m_i^4(\varphi)}{64\pi^2}\left(\ln\frac{m_i^2(\varphi)}{\mu^2} + \mathrm{const}_i\right) + \cdots$$

with $m_i(\varphi) = g\varphi,\ \frac{y}{\sqrt{2}}\varphi$, so that $m_i^4 \propto \varphi^4$

- U should be independent on non-physical parameter μ – leads to RG equation for λ

$$\frac{\partial\lambda}{\partial\ln\mu} = \beta_\lambda$$

- At the same time, one can choose $\mu \simeq m(\varphi) \simeq y_t\varphi$ to minimize the logarithms

$$U_{\text{RG improved}} \simeq \frac{\lambda(\mu(\varphi))}{4}\varphi^4$$

$$\mu^2 \simeq \alpha^2\frac{y_t}{2}\varphi^2$$

α is of order one

RG improved potential for Higgs inflation

$$U_{\text{RG improved}}(\chi) = \frac{\lambda(\mu)}{4}\frac{M_P^4}{\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

with

$$\mu^2 = \alpha^2 m_t^2(\chi) = \alpha^2 \frac{y_t^2(\mu)}{2}\frac{M_P^2}{\xi}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)$$

- Large λ – slow (logarithmic) running, no noticeable change compared to tree level potential
- Small λ – $\delta\lambda$ significant, may give interesting “features” in the potential (“critical inflation”, large r)
- Most complicated – how really λ behave in HI?

Note on the choice of μ

- μ is the scale appearing in (dimensional) regularization
- No questions asked in the "usual" case of renormalizable theories – only space/field independent choice gives regularization that is not-breaking renormalizability.
- HI is not renormalizable – multiple choices possible

The choice for this talk:

In Jordan frame: $\mu^2 \propto M_P^2 + \xi h^2$

In Einstein frame: $\mu^2 \propto \text{const}$

Adding required counterterms to the action

- In principle – HI is not renormalizable, all counterterms appear at some loop order
- Let us try to add only the *required* counterterms at each order in loop expansion

$$\mathcal{L} = \frac{(\partial\chi)^2}{2} - \frac{\lambda}{4}F^4(\chi) + i\bar{\psi}_t \partial\!\!\!/ \psi_t + \frac{y_t}{\sqrt{2}}F(\chi)\bar{\psi}_t\psi_t$$

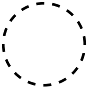
$$F(\chi) \equiv \frac{h(\chi)}{\Omega(\chi)} \approx \left\{ \begin{array}{ll} \chi & , \chi < \frac{M_P}{\xi} \\ \frac{M_P}{\sqrt{\xi}}\left(1 - e^{-\sqrt{2/3}\chi/M_P}\right)^{1/2} & , \chi > \frac{M_P}{\xi} \end{array} \right\}$$

Doing quantum calculations we should add

$$\mathcal{L} + \mathcal{L}_{\text{1-loop}} + \delta\mathcal{L}_{\text{1-loop c.t.}} + \cdots$$

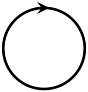
Counterterms: λ modification

Calculating vacuum energy



$$= \frac{1}{2}\,\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right]$$

$$= \frac{9\lambda^2}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} - \ln\frac{\lambda(F^4)''}{4\mu^2} + \frac{3}{2}\right)\left(F'^2 + \frac{1}{3}F''F\right)^2 F^4,$$

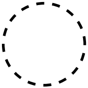


$$= -\,\mathrm{Tr}\ln\left[i\partial\!\!\!/ + y_t F\right]$$

$$= -\frac{y_t^4}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} - \ln\frac{y_t^2F^2}{2\mu^2} + \frac{3}{2}\right)F^4$$

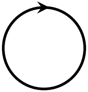
Counterterms: λ modification

Calculating vacuum energy



$$= \frac{1}{2}\,\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right]$$

$$\delta\mathcal{L}_{\mathrm{ct}} = \frac{9\lambda^2}{64\pi^2}\left(\frac{2}{\bar{\epsilon}} + \delta\lambda_{1a}\right)\left(F'^2 + \frac{1}{3}F''F\right)^2 F^4,$$



$$= -\,\mathrm{Tr}\ln\left[i\partial\!\!\!/ + y_t F\right]$$

$$\delta\mathcal{L}_{\mathrm{ct}} = -\frac{y_t^4}{64\pi^2}\left(\frac{2}{\bar{\epsilon}} + \delta\lambda_{1b}\right)F^4$$

Small χ : $F'^4F^4 \sim \chi \sim F^4$

Large χ : $F'^4F^4 \sim e^{-4\chi/\sqrt{6}M_P}$, and $F^4 \sim M_P^4/\xi^2$

$\delta\lambda_{1b}$ – just λ redefinition, while $\delta\lambda_{1a}$ is not!

Modified "evolution" of $\lambda(\mu)$

For RG we should in principle write infinite series

$$\frac{d\lambda}{d\ln\mu} = \beta_\lambda(\lambda, \lambda_1, a \ldots)$$

$$\frac{d\lambda_1}{d\ln\mu} = \beta_{\lambda_1}(\lambda, \lambda_1, \ldots)$$

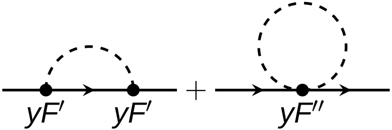
$$\ldots$$

- Assuming δ_i are small and have the same hierarchy, as the loop expansion, we truncate this to just first equation.
- Neglect change of $\delta\lambda_1$ between $\mu \sim M_P/\xi$ and $M_P/\sqrt{\xi}$

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda \left[\left(F'^2 + \frac{1}{3} F'' F \right)^2 - 1 \right],$$

Counterterms: Top Yukawa coupling

Calculating propagation of the top quark in the background χ



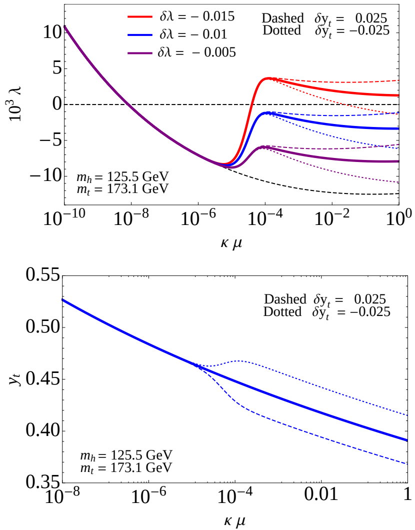
$$\delta\mathcal{L}_{\rm ct} \sim \left(\#\frac{y_t^3}{\bar\varepsilon} + \delta y_{t1}\right) F'^2 F \bar\psi\psi + \left(\#\frac{y_t\lambda}{\bar\varepsilon} + \delta y_{t2}\right) F''(F^4)''\bar\psi\psi$$

$$y_t(\mu) \to y_t(\mu) + \delta y_t \left[F'^2 - 1\right]$$

Threshold effects at M_P/ξ summarized by two new arbitrary constants $\delta\lambda$, δy_t

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda \left[\left(F'^2 + \tfrac{1}{3}F''F\right)^2 - 1 \right]$$

$$y_t(\mu) \to y_t(\mu) + \delta y_t \left[F'^2 - 1 \right]$$

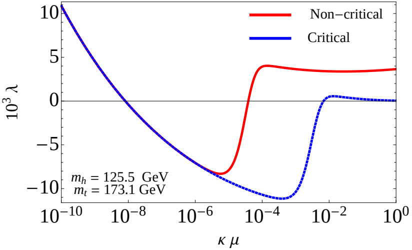


Modified λ evolution can make the potential positive again

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda \left[\left(F'^2 + \tfrac{1}{3} F'' F \right)^2 - 1 \right]$$

$$y_t(\mu) \to y_t(\mu) + \delta y_t \left[F'^2 - 1 \right]$$

Effect on the potential



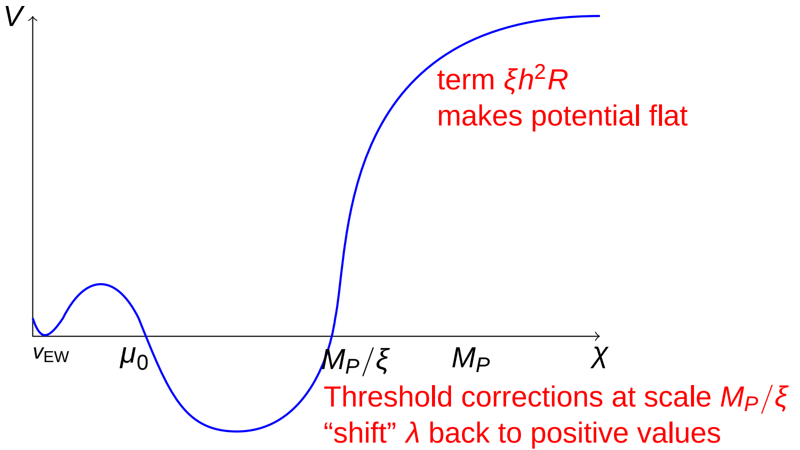
(Red curve: $\xi = 1500$, $\delta y_t = 0.025$, $\delta\lambda = -0.015$)

We should survive after inflation

Naively, we arrive to the true vacuum with large field v.e.v. after inflation. How we ended up in our electroweak vacuum then?

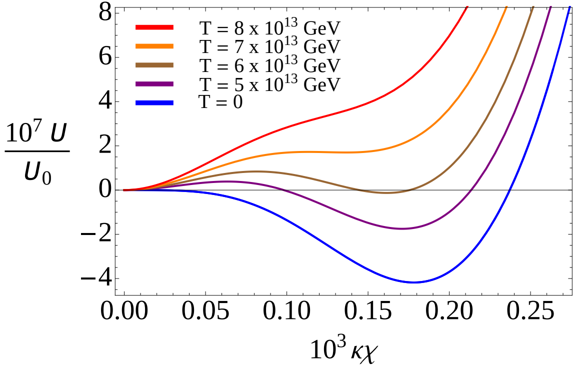
Higgs inflation and radiative corrections

$$S_J = \int d^4x\sqrt{-g}\left\{-\frac{M_P^2}{2}R - \xi\frac{h^2}{2}R + g_{\mu\nu}\frac{\partial^\mu h\partial^\nu h}{2} - \frac{\lambda}{4}(h^2-v^2)^2\right\}$$



(Not really to scale)

In the hot enough Universe only one vacuum remains

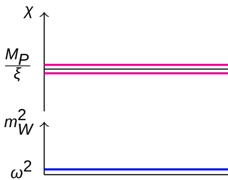


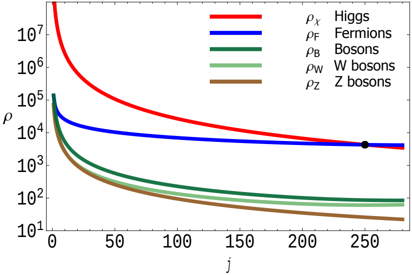
Thermal potential $\Delta V_T = -\frac{1}{6\pi^2} \sum_{\text{particles}} \int_0^\infty \frac{k^4 dk}{\varepsilon_k(m)} \frac{1}{e^{\varepsilon_k(m)/T} \mp 1}$

- Universe has to be reheated to $T_R \gtrsim 10^{14}$ GeV

Preheating is effective via generation of W bosons with its subsequent decay into light fermions

- Background evolution after inflation $\chi < M_P$ $(h < M_P/\sqrt{\xi})$
  - Quadratic potential $U \simeq \frac{\omega^2}{2}\chi^2$ with $\omega = \sqrt{\frac{\lambda}{3}}\frac{M_P}{\xi}$
  - Matter dominated stage $a \propto t^{2/3}$
- Stohastic resonance
  - Particle masses $m_W^2(\chi) \sim g^2 \frac{M_P|\chi|}{\xi}$
  - W bosons are created (non-relativistic)
  - W bosons decay into (light) fermions





Reheating temperature

$$T_{RH} \simeq 1.8 \times 10^{14}\,\text{GeV}$$

Inflation versus vacuum stability

Stable SM vacuum	inflaton & Higgs independent	inflaton & Higgs interacting	inflaton = Higgs
Large *r*	Yes	Yes	Yes (threshold corr.)
Small *r*	Yes	Yes	Yes
Planck scale corections	Any	Any	Scale inv.

Metastable SM vacuum	inflaton & Higgs independent	inflaton & Higgs interacting	inflaton = Higgs
Large *r*	No	Yes Model dep.	No
Small *r*	Yes $r < 10^{-9}$	Yes Model dep.	Yes (threshold corr.)
Planck scale corections	Restricted	Model dep.	Scale inv.

Conclusions: Higgs potential stability

what is good and what is bad?

Bad

Predictions depend on high scale physics

Conclusions: Higgs potential stability

what is good and what is bad?

Bad

Predictions depend on high scale physics

Good

Predictions depend on high scale physics

Inflation versus vacuum stability

Stable SM vacuum	inflaton & Higgs independent	inflaton & Higgs interacting	inflaton = Higgs
Large *r*	Yes	Yes	Yes (threshold corr.)
Small *r*	Yes	Yes	Yes
Planck scale corections	Any	Any	Scale inv.

Metastable SM vacuum	inflaton & Higgs independent	inflaton & Higgs interacting	inflaton = Higgs
Large *r*	No	Yes Model dep.	No
Small *r*	Yes $r < 10^{-9}$	Yes Model dep.	Yes (threshold corr.)
Planck scale corections	Restricted	Model dep.	Scale inv.

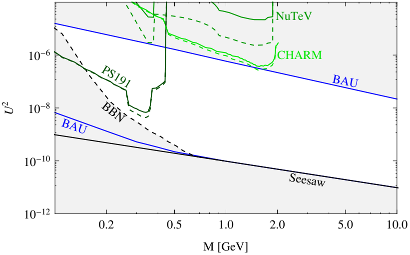
Backup

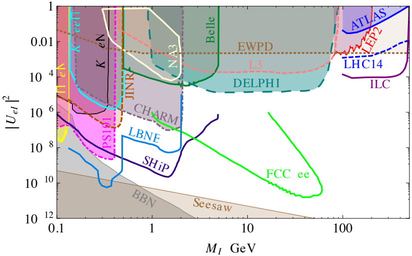
Search for $N_{2,3}$ is possible

- Leptogenesys by $N_{2,3}$

  $\Delta M / M \sim 10^{-3}$

- Experimental searches
  - $N_{2,3}$ production in hadron decays (LHCb):
    - Missing energy in K decays
    - Peaks in Dalitz plot
  - $N_{2,3}$ decays into SM
    - Beam target: SHiP
    - High luminosity lepton collider at Z peak





Note: Other related models (e.g. scalars for DM generation, light inflaton) also show up in such experiments

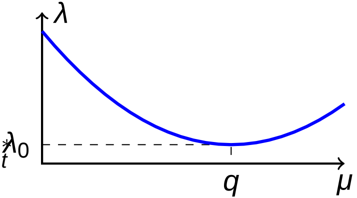
RG running indicates small λ at Planck scale

Renormalization evolution of the Higgs self coupling λ

$$\lambda \simeq \lambda_0 + b \ln^2 \frac{\mu}{q}$$

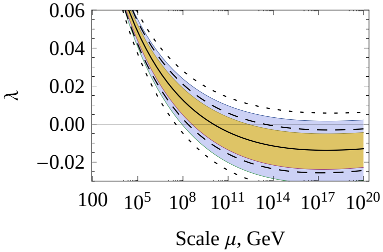
$b \simeq 0.000023$

λ_0 – small
q of the order M_p } depend on M_h^*, m_t^*



$$(4\pi)^2 \frac{\partial \lambda}{\partial \ln \mu} = 24\lambda^2 - 6y_t^4 + \frac{3}{8}(2g_2^4 + (g_2^2 + g_1^2)^2) + (-9g_2^2 - 3g_1^2 + 12y_t^2)\lambda$$

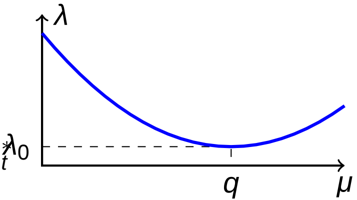
Higgs mass M_h=125.3±0.6 GeV



RG running indicates small λ at Planck scale

Potentials in different regimes

$$\lambda \simeq \lambda_0 + b \ln^2 \frac{\mu}{q}$$

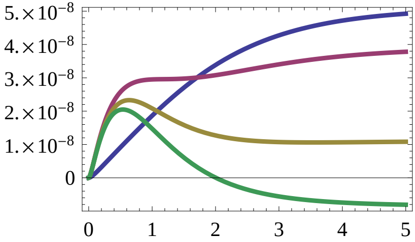


$b \simeq 0.000023$

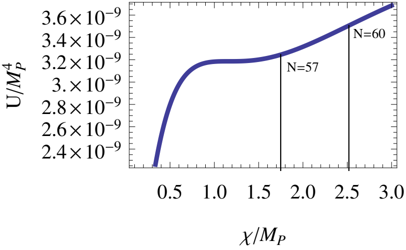
λ_0 – small, q of the order M_p } depend on M_h^*, m_t^*

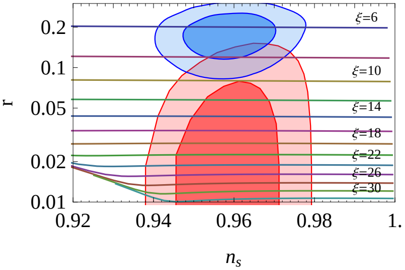
$$U(\chi) \simeq \frac{\lambda(\mu) M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

$$\mu^2 = \alpha^2 \frac{y_t(\mu)^2}{2}\frac{M_P^2}{\xi}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)$$



Interesting inflation near to the critical point

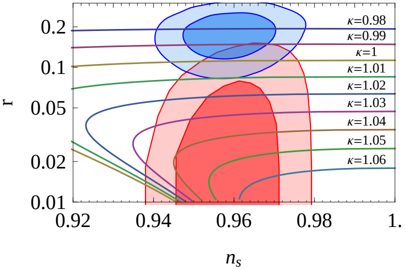




Parameters in
particle physics: λ_0, q, ξ
cosmology: $\mathcal{P}_R$, r, n_s

$\kappa \sim q\frac{\sqrt{\xi}}{M_P}\frac{\sqrt{2}}{y_t}$

For given r (or ξ) very small change of κ (or M_h^*) gives any n_s



Modifying the gravity action gives inflation

Another way to get inflation in the SM

The first working inflationary model

Starobinsky'80

The gravity action gets higher derivative terms

$$S_J = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2} R + \frac{\zeta^2}{4} R^2 \right\} + S_{SM}$$

Conformal transformation

conformal transformation (change of variables)

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \; , \qquad \Omega^2 \equiv \exp\left(\frac{\chi(x)}{\sqrt{6}M_P}\right)$$

$\chi(x)$ – new field (d.o.f.) “scalaron”

Resulting action (Einstein frame action)

$$S_E = \int d^4x \sqrt{-\hat{g}} \left\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - \frac{M_P^4}{4\zeta^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2 \right\}$$

Cut off scale today

Let us work in the Einstein frame for simplicty

Change of variables: $\frac{d\chi}{dh} = \frac{M_P\sqrt{M_P^2+(\xi+6\xi^2)h^2}}{M_P^2+\xi h^2}$ leads to the higher order terms in the potential (expanded in a power law series)

$$V(\chi) = \lambda\frac{h^4}{4\Omega^4} \simeq \lambda\frac{h^4}{4} \simeq \lambda\frac{\chi^4}{4} + \#\frac{\chi^6}{(M_P/\xi)^2} + \cdots$$

Unitarity is violated at tree level

in scattering processes (eg. $2 \to 4$) with energy above the "cut-off"

$$E > \Lambda_0 \sim \frac{M_P}{\xi}$$

Hubble scale at inflation is $H \sim \lambda^{1/2}\frac{M_P}{\xi}$ – not much smaller than the today cut-off Λ_0 :(

Burgess:2009ea,Barbon:2009ya,Hertzberg:2010dc

”Cut off” is background dependent!

Classical background — Quantum perturbations

$$\chi(x,t) = \bar{\chi}(t) + \delta\chi(x,t)$$

leads to background dependent suppression of operators of dim $n > 4$

$$\frac{\mathcal{O}_{(n)}(\delta\chi)}{[\Lambda_{(n)}(\bar{\chi})]^{n-4}}$$

Example

Potential in the inflationary region $\chi > M_P$:

$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

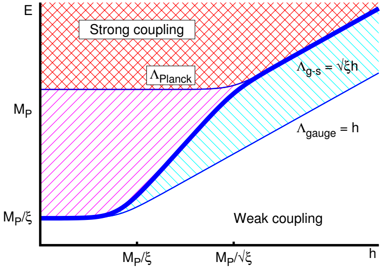
leads to operators of the form: $\frac{\mathcal{O}_{(n)}(\delta\chi)}{[\Lambda_{(n)}(\bar{\chi})]^{n-4}} = \frac{\lambda M_P^4}{\xi^2} e^{-\frac{2\bar{\chi}}{\sqrt{6}M_P}} \frac{(\delta\chi)^n}{M_P^n}$

Leading at high n to the ”cut-off”

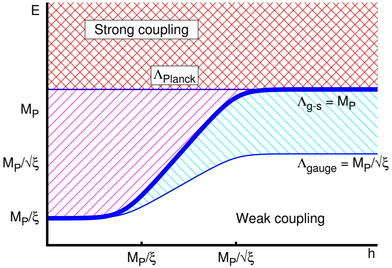
$$\Lambda \sim M_P$$

Cut-off grows with the field background

Jordan frame



Einstein frame



Relation between cut-offs in different frames:

$$\Lambda_{\text{Jordan}} = \Lambda_{\text{Einstein}}\Omega$$

Relevant scales

Hubble scale $H \sim \lambda^{1/2}\frac{M_P}{\xi}$

Energy density at inflation

$$V^{1/4} \sim \lambda^{1/4}\frac{M_P}{\sqrt{\xi}}$$

Reheating temperature $M_P/\xi < T_{\text{reheating}} < M_P/\sqrt{\xi}$

Bezrukov:2011jz

Shift symmetric UV completion allows to have effective theory during inflation

$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - U_0 \left(1 + \sum u_n e^{-n\cdot\chi/M}\right)$$
$$= \frac{(\partial_\mu \chi)^2}{2} - U_0 \left(1 + \sum \frac{1}{k!} \left[\frac{\delta\chi}{M}\right]^k \sum n^k u_n e^{-n\cdot\bar{\chi}/M}\right)$$

Effective action (from quantum corrections of loops of $\delta\chi$)

$$\mathcal{L}_{\text{eff}} = f^{(1)}(\chi)\frac{(\partial_\mu \chi)^2}{2} - U(\chi) + f^{(2)}(\chi)\frac{(\partial^2 \chi)^2}{M^2} + f^{(3)}(\chi)\frac{(\partial \chi)^4}{M^4} + \cdots$$

All the divergences are absorbed in u_n and in $f^{(n)} \sim \sum f_l e^{-n\chi/M}$

UV completion requirement

Shift symmetry (or scale symmetry in the Jordan frame) is respected

$$\chi \mapsto \chi + \text{const}$$

Connection of inflationary and low energy physics requires more assumptions on the UV theory

$$\lambda U(\bar{\chi}+\delta\chi) = \lambda\left(U(\bar{\chi}) + \frac{1}{2}U''(\bar{\chi})(\delta\chi)^2 + \frac{1}{3!}U'''(\bar{\chi})(\delta\chi)^3 + \cdots\right)$$

in one loop: $\lambda U''(\bar{\chi})\bar{\Lambda}^2,\ \lambda^2(U''(\bar{\chi}))^2\log\bar{\Lambda}$,

in two loops: $\lambda U^{(IV)}(\bar{\chi})\bar{\Lambda}^4,\ \lambda^2(U''')^2\bar{\Lambda}^2,\ \lambda^3 U^{(IV)}(U'')^2(\log\bar{\Lambda})^2$,

If no power law divergences are generated

then the loop corrections are arranged in a series in λ

$$U(\chi) = \lambda U_1(\chi) + \lambda^2 U_2(\chi) + \lambda^3 U_3(\chi) + \cdots$$

A rule to fix the finite parts of the counterterm functions $U_i(\chi)$

Example – dimensional regularisation + $\overline{\text{MS}}$

RG improved potential for Higgs inflation

$$U_{\text{RG improved}}(\chi) = \frac{\lambda(\mu)}{4}\frac{M_P^4}{\xi^2}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)^2$$

with

$$\mu^2 = \alpha^2 m_t^2(\chi) = \alpha^2 \frac{y_t^2(\mu)}{2}\frac{M_P^2}{\xi}\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)$$

- Large λ – slow (logarithmic) running, no noticeable change compared to tree level potential
- Small λ – $\delta\lambda$ significant, may give interesting “features” in the potential (“critical inflation”, large r)
- Most complicated – how really λ behave in HI?

Note on the choice of μ

- μ is the scale appearing in (dimensional) regularization
- No questions asked in the "usual" case of renormalizable theories – only space/field independent choice gives regularization that is not-breaking renormalizability.
- HI is not renormalizable – multiple choices possible

The choice for this talk:

In Jordan frame: $\mu^2 \propto M_P^2 + \xi h^2$

In Einstein frame: $\mu^2 \propto \text{const}$

Adding required counterterms to the action

- In principle – HI is not renormalizable, all counterterms appear at some loop order
- Let us try to add only the *required* counterterms at each order in loop expansion

$$\mathcal{L} = \frac{(\partial\chi)^2}{2} - \frac{\lambda}{4}F^4(\chi) + i\bar{\psi}_t\partial\!\!\!/\psi_t + \frac{y_t}{\sqrt{2}}F(\chi)\bar{\psi}_t\psi_t$$

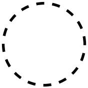
$$F(\chi) \equiv \frac{h(\chi)}{\Omega(\chi)} \approx \left\{ \begin{array}{ll} \chi & , \chi < \frac{M_P}{\xi} \\ \frac{M_P}{\sqrt{\xi}}\left(1 - e^{-\sqrt{2/3}\chi/M_P}\right)^{1/2} & , \chi > \frac{M_P}{\xi} \end{array} \right\}$$

Doing quantum calculations we should add

$$\mathcal{L} + \mathcal{L}_{\text{1-loop}} + \delta\mathcal{L}_{\text{1-loop c.t.}} + \cdots$$

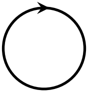
Counterterms: λ modification

Calculating vacuum energy



$$= \frac{1}{2}\,\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right]$$

$$= \frac{9\lambda^2}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} - \ln\frac{\lambda(F^4)''}{4\mu^2} + \frac{3}{2}\right)\left(F'^2 + \frac{1}{3}F''F\right)^2 F^4,$$

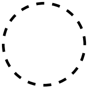


$$= -\,\mathrm{Tr}\ln\left[i\partial\!\!\!/ + y_t F\right]$$

$$= -\frac{y_t^4}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} - \ln\frac{y_t^2F^2}{2\mu^2} + \frac{3}{2}\right)F^4$$

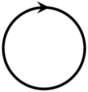
Counterterms: λ modification

Calculating vacuum energy



$$= \frac{1}{2}\,\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right]$$

$$\delta\mathcal{L}_{\rm ct} = \frac{9\lambda^2}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} + \color{red}{\delta\lambda_{1a}}\right)\left(F'^2 + \frac{1}{3}F''F\right)^2 F^4,$$



$$= -\,\mathrm{Tr}\ln\left[i\partial\!\!\!/ + y_t F\right]$$

$$\delta\mathcal{L}_{\rm ct} = -\frac{y_t^4}{64\pi^2}\left(\frac{2}{\bar{\varepsilon}} + \delta\lambda_{1b}\right) F^4$$

Small χ : $F'^4 F^4 \sim \chi^4 \sim F^4$

Large χ : $F'^4 F^4 \sim e^{-4\chi/\sqrt{6}M_P}$, and $F^4 \sim M_P^4/\xi^2$

$\delta\lambda_{1b}$ – just λ redefinition, while $\color{red}{\delta\lambda_{1a}}$ is not!

Modified "evolution" of $\lambda(\mu)$

For RG we should in principle write infinite series

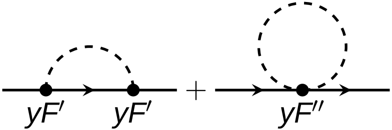
$$\frac{d\lambda}{d\ln\mu} = \beta_\lambda(\lambda, \lambda_1, a \dots)$$
$$\frac{d\lambda_1}{d\ln\mu} = \beta_{\lambda_1}(\lambda, \lambda_1, \dots)$$
$$\dots$$

- Assuming δ_i are small and have the same hierarchy, as the loop expansion, we truncate this to just first equation.
- Neglect change of $\delta\lambda_1$ between $\mu \sim M_P/\xi$ and $M_P/\sqrt{\xi}$

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda \left[\left(F'^2 + \frac{1}{3} F'' F \right)^2 - 1 \right],$$

Counterterms: Top Yukawa coupling

Calculating propagation of the top quark in the background χ



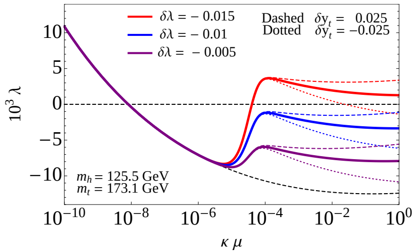
$$\delta\mathcal{L}_{\rm ct} \sim \left(\#\frac{y_t^3}{\bar{\varepsilon}} + \delta y_{t1}\right) F'^2 F \bar{\psi}\psi + \left(\#\frac{y_t\lambda}{\bar{\varepsilon}} + \delta y_{t2}\right) F''(F^4)'' \bar{\psi}\psi$$

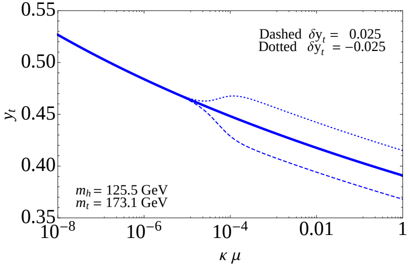
$$y_t(\mu) \to y_t(\mu) + \delta y_t \left[F'^2 - 1\right]$$

Threshold effects at M_P/ξ summarized by two new arbitrary constants $\delta\lambda$, δy_t

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda \left[\left(F'^2 + \tfrac{1}{3} F'' F \right)^2 - 1 \right]$$

$$y_t(\mu) \to y_t(\mu) + \delta y_t \left[F'^2 - 1 \right]$$



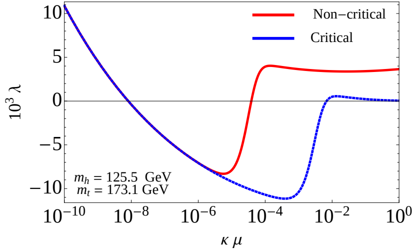


Modified λ evolution can make the potential positive again

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda \left[\left(F'^2 + \tfrac{1}{3} F'' F \right)^2 - 1 \right]$$

$$y_t(\mu) \to y_t(\mu) + \delta y_t \left[F'^2 - 1 \right]$$

Effect on the potential



(Red curve: $\xi = 1500$, $\delta y_t = 0.025$, $\delta\lambda = -0.015$)