Nonlinear GR effects in structure formation: from approximations to full numerical relativity simulations

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Helsinki 2/11/2016

Outline

- standard ΛCDM cosmology and a basic question
- non-linear Post-Friedmann ΛCDM: a new weak-field/post-Newtonian type approximation scheme for cosmology
- cosmological frame dragging from Newtonian N-body simulations
- the back-reaction problem
- full Numerical Relativity cosmological simulations

Credits: first part

- Irene Milillo, Daniele Bertacca, MB and Andrea Maselli, *The missing link: a nonlinear post-Friedmann framework for small and large scales* [arXiv: 1502.02985], Physical Review D, **92**, 023519 (2015)
- MB, Dan B. Thomas and David Wands, *Computing General Relativistic effects from Newtonian N-body simulations: Frame dragging in the post-Friedmann approach,* Physical Review D, **89**, (2014) 044010 [arXiv:1306.1562]
- Dan B. Thomas, MB, Kazuya Koyama, Baojiu Li and Gong-bo Zhao *f(R) gravity on non-linear scales: The post-Friedmann expansion and the vector potential,* JCAP, 1507 (2015) 07, 051 [arXiv:1503.07204]
- V. Salvatelli, N. Said, MB, A. Melchiorri & D. Wands, *Indications of a Late-Time Interaction in the Dark Sector*, PRL **113** (2014)181301 [arXiv: 1406.7297]
- C. Rumpf, E. Villa, D. Bertacca and M. Bruni, *Lagrangian theory for cosmic structure for- mation with vorticity: Newtonian and post-Friedmann approximations*, Phys. Rev. D **94** (2016) 083515 [arXiv:1607.05226]
- A. Maselli, B. Bruni & D. Thomas, Interacting vacuum-energy in a Post-Friedmann expanding Universe (to be submitted)

Featured in Physics | Editors' Suggestion

Departures from the Friedmann-Lemaitre-Robertston-Walker Cosmological Model in an Inhomogeneous Universe: A Numerical Examination

John T. Giblin, Jr., James B. Mertens, and Glenn D. Starkman

Phys. Rev. Lett. **116**, 251301 (2016) – Published 24 June 2016

Cosmologists have begun using fully relativistic models to understand the effects of inhomogeneous matter distribution on the evolution of the Universe.

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Credits: second part

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Effects of Nonlinear Inhomogeneity on the Cosmic Expansion with Numerical Relativity

Eloisa Bentivegna and Marco Bruni

Phys. Rev. Lett. **116**, 251302 (2016) – Published 24 June 2016

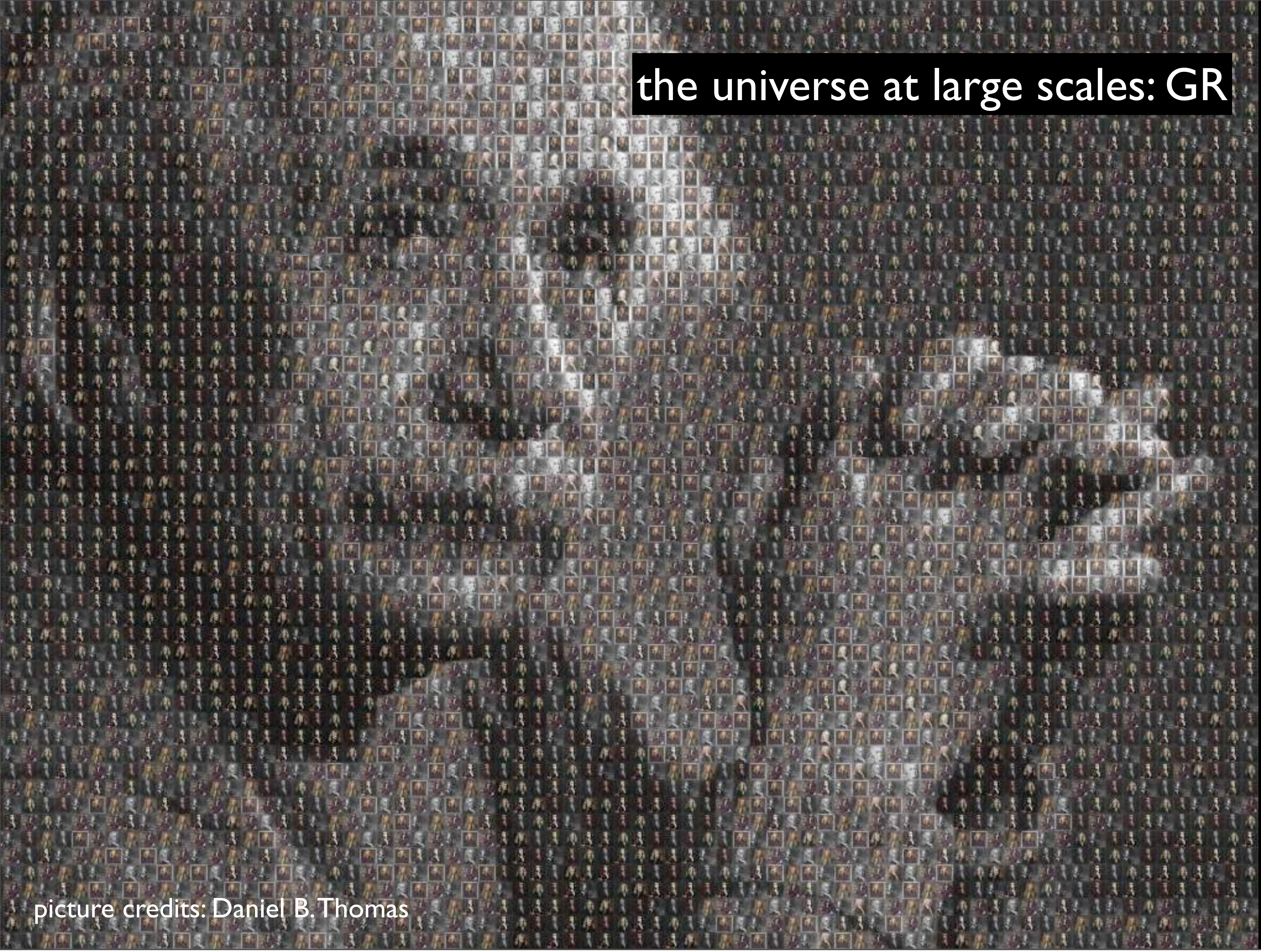
Cosmologists have begun using fully relativistic models to understand the effects of inhomogeneous matter distribution on the evolution of the Universe.

Show Abstract +

Standard ΛCDM Cosmology

- Recipe for modeling based on 3 main ingredients:
  1. Homogeneous isotropic background, FLRW models
  2. Relativistic Perturbations (e.g. CMB), good for large scales I-order, II order, gradient expansion
  3. Newtonian study of non-linear structure formation (N-body simulations or approx. techniques, e.g. 2LPT) at small scales
- on this basis, well supported by observations, the flat ΛCDM model has emerged as the Standard "Concordance" Model of cosmology.

the universe at large scales: GR



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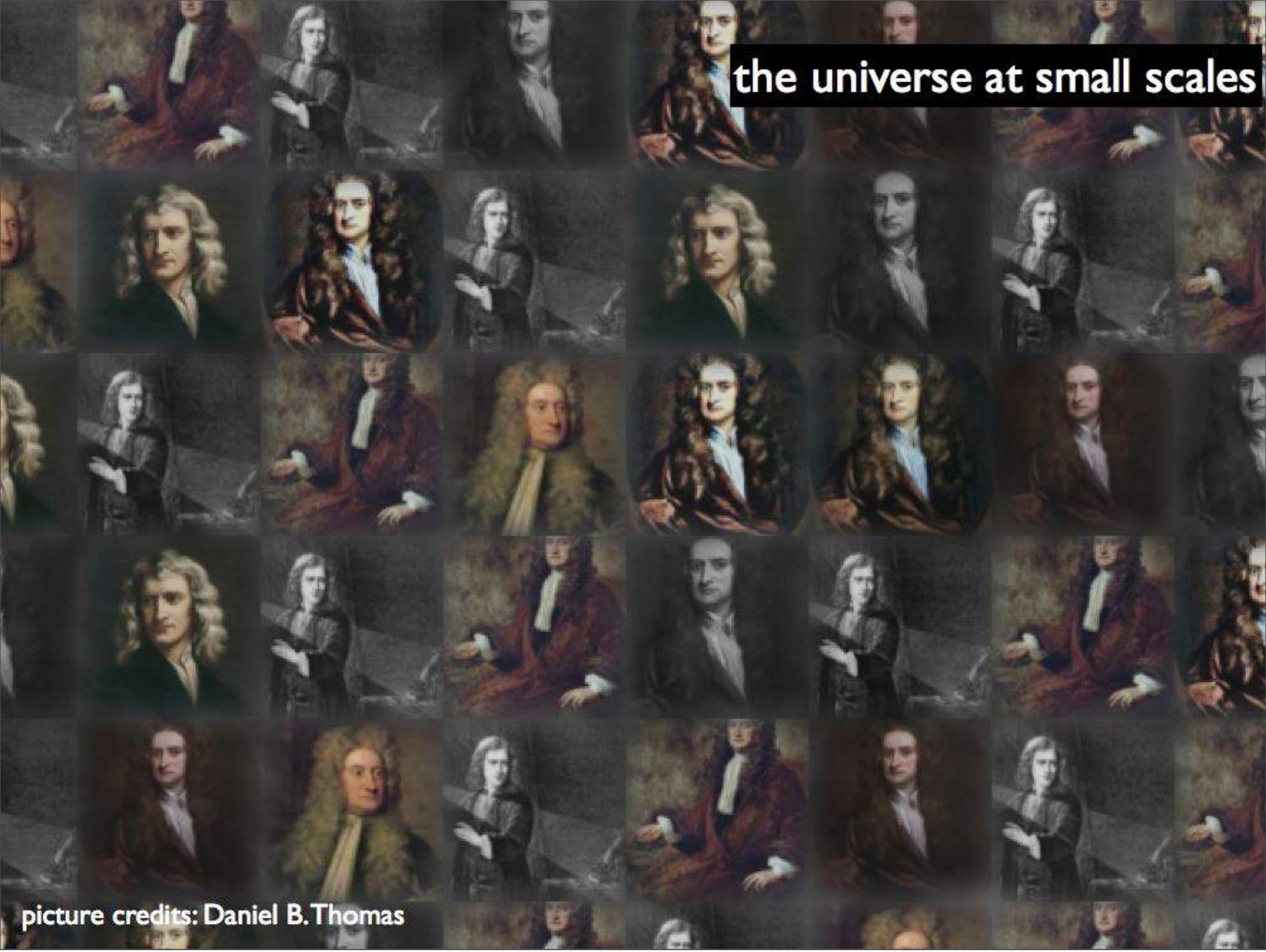

the universe at small scales
picture credits: Daniel B. Thomas

Questions on ΛCDM

- Recipe for modelling based on 3 main ingredients:
  1. Homogeneous isotropic background, FRW models
  2. Relativistic Perturbations (e.g. CMB)
  3. Newtonian study of non-linear structure formation (numerical simulations or approx. techniques)
- Is 3 enough? (more data, precision cosmology, observations and simulations covering large fraction of H^{-1}, etc...)
  - We need to bridge the gap between 2 and 3

Alternatives to ΛCDM

ΛCDM is the simplest and very successful model supporting the observations that, assuming the Cosmological Principle, are interpreted as acceleration of the Universe expansion

Going beyond ΛCDM, two main alternatives:

1. Maintain the Cosmological Principle (FLRW background), then either

   a) maintain GR + dark components (CDM+DE or UDM, or interacting CDM+vacuum)

   b) modified gravity (f(R), branes, etc...)

Alternatives to ΛCDM

Going beyond ΛCDM, two main alternatives:

2. Maintain GR, drop CP, then either

   a) try to construct an homogeneous isotropic model from averaging, possibly giving acceleration: dynamical back-reaction

   b) consider inhomogeneous models, e.g. LTB (violating the CP) or Szekeres (not necessarily violating the CP): back-reaction on observations

3. NEW: FULL GR, Numerical Relativity simulations

Questions/Motivations

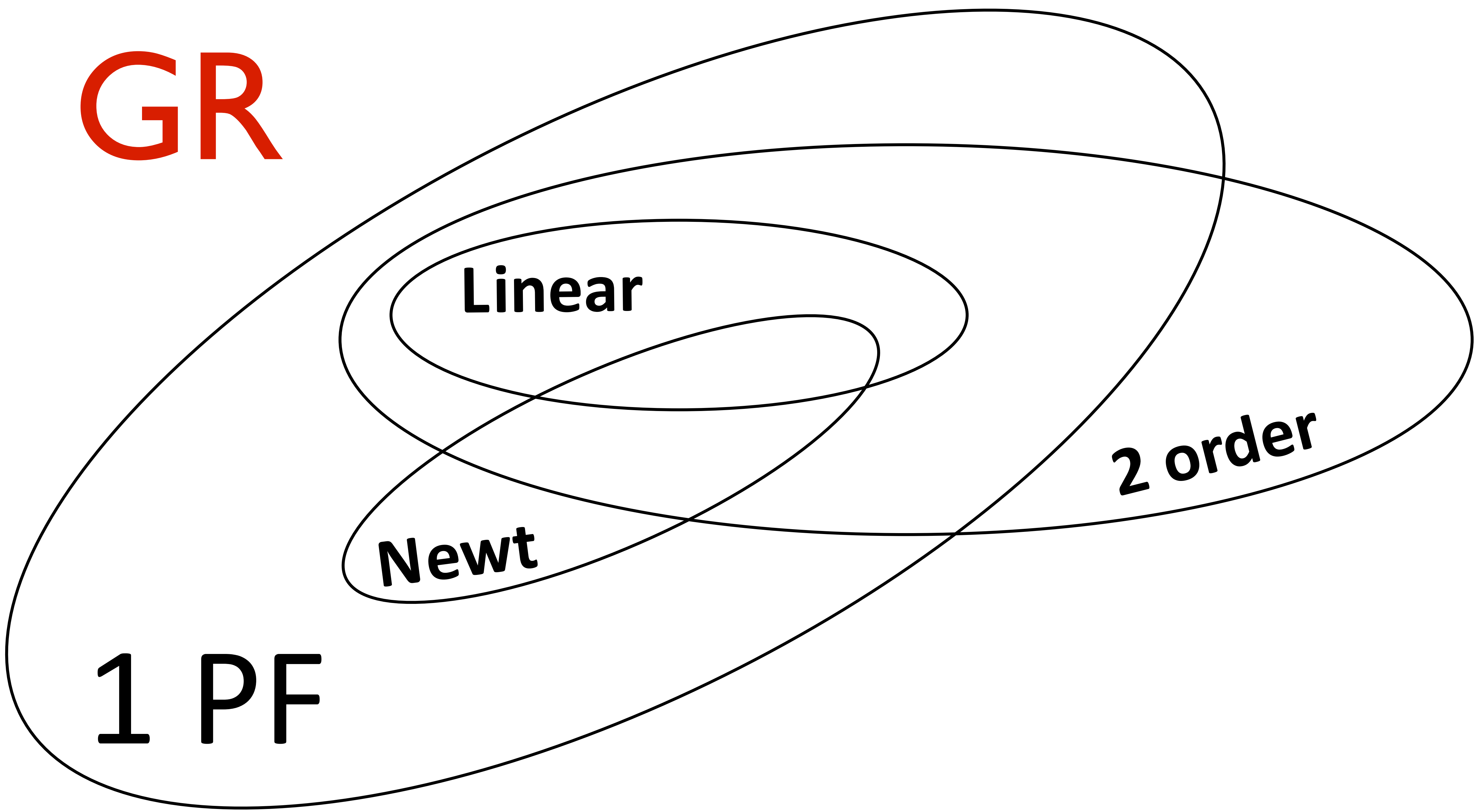
- Is the Newtonian approximation good enough to study non-linear structure formation?
  - surveys and simulations covering large fraction of H^{-1}
  - we are going to have more data: precision cosmology
  - we also need accurate cosmology: not only we want accurate observations, we also need accurate theoretical predictions (e.g.: Euclid target: N-body simulations wih 1% accuracy)
  - what if relativistic corrections are ~ few%?
    - We need to bridge the gap between small scale non-linear Newtonian approximation and large scale relativistic perturbation theory
    - We need a relativistic framework ("dictionary") to interprete N-body simulations [e.g. Chisari & Zaldarriaga (2011), Green & Wald (2012), Fidler et arXiv:1505.04756]
    - We need to go beyond the standard perturbative approach, considering non-linear density inhomogeneities within a relativistic framework

standard ΛCDM, General Relativity and non-linearity

- from now on, I assume GR and a flat ΛCDM background
- perturbation theory is only valid for small δ
- clearly, to bridge the gap between Newtonian non-linear structure formation and large scale small inhomogeneities we need to go beyond the standard perturbative approach, considering non-linear density inhomogeneities within a relativistic framework

post-Friedmann framework

- spaces of equations (not solutions!)



Post-Newtonian cosmology

- post-Newtonian: expansion in 1/c powers (more later)
- various attempts and studies:
  - Tomita Prog. Theor. Phys. 79 (1988) and 85 (1991)
  - Matarrese & Terranova, MN 283 (1996)
  - Takada & Futamase, MN 306 (1999)
  - Carbone & Matarrese, PRD 71 (2005)
  - Hwang, Noh & Puetzfeld, JCAP 03 (2008)
- even in perturbation theory it is important to distinguish post-Newtonian effects, e.g. in non-Gaussianity and initial conditions. MB, J. C. Hidalgo, N. Meures, D. Wands, ApJ 785:2 (2014) [arXiv:1307:1478], cf. Bartolo et al. CQG 27 (2010) [arXiv:1002.3759]

post-N vs. post-F

- problems of standard post-Newtonian:
  - focus on equation of motion of matter, rather than on deriving a consistent approximate solution of field equations
  - derived metric OK for motion of matter, not for photons
- post-Friedmann: something in between: start with a post-M (weak field) approach on a FLRW background, Hubble flow is not slow but peculiar velocities are small $\dot{\vec{r}} = H\vec{r} + a\vec{v}$
- post-Friedmann: we don't necessarily follow an iterative approach; aim at resummed variables in order to match standard perturbation theory in some limit

metric and matter I

starting point: the 1-PN cosmological metric (cf. Chandrasekhar 1965)

$$
\begin{aligned}
g_{00} &= -\left[1-\frac{2U_N}{c^2}+\frac{1}{c^4}(2U_N^2-4U_P)\right]+O\left(\frac{1}{c^6}\right),\\
g_{0i} &= -\frac{a}{c^3}B_i^N-\frac{a}{c^5}B_i^P+O\left(\frac{1}{c^7}\right),\\
g_{ij} &= a^2\left[\left(1+\frac{2V_N}{c^2}+\frac{1}{c^4}(2V_N^2+4V_P)\right)\delta_{ij}+\frac{1}{c^4}h_{ij}\right]+O\left(\frac{1}{c^6}\right),
\end{aligned}
$$

we assume a Newtonian-Poisson gauge: B_i is solenoidal and h_{ij} is TT, at each order 2 scalar DoF in g_{00} and g_{ij}, 2 vector DoF in frame dragging potential B_i and 2 TT DoF in h_{ij} (not GW!)

Newtonian ΛCDM, with a bonus

- insert leading order terms in E.M. conservation and Einstein equations
- subtract the background, getting usual Friedmann equations
- introduce usual density contrast by $\rho=\rho_b(1+\delta)$

from E.M. conservation: Continuity & Euler equations

$$\dot{\delta} + \frac{v^i \delta_{,i}}{a} + \frac{v^i{}_{,i}}{a}(\delta + 1) = 0\,,$$

$$\dot{v}_i + \frac{v^j v_{i,j}}{a} + \frac{\dot{a}}{a} v_i = \frac{1}{a} U_{N,i}\,.$$

Poisson

$$G^0{}_0 + \Lambda = \frac{8\pi G}{c^4} T^0{}_0 \quad \rightarrow \quad \frac{1}{c^2}\frac{1}{a^2}\nabla^2 V_N = -\frac{4\pi G}{c^2}\bar{\rho}\delta$$

Newtonian ΛCDM, with a bonus

what do we get from the ij and 0i Einstein equations?

$$\text{trace of } \; G^i{}_j + \Lambda\delta^i{}_j = \frac{8\pi G}{c^4} T^i{}_j \;\rightarrow\; \frac{1}{c^2}\frac{2}{a^2}\nabla^2(V_N - U_N) = 0\,,$$

zero "Slip"

$$\text{traceless part of } \; G^i{}_j + \Lambda\delta^i{}_j = \frac{8\pi G}{c^4} T^i{}_j \;\rightarrow\; \frac{1}{c^2}\frac{1}{a^2}\left[(V_N - U_N)_{,i}{}^{,j} - \frac{1}{3}\nabla^2(V_N - U_N)\delta^j_i\right] = 0$$

bonus

$$G^0{}_i = \frac{8\pi G}{c^4} T^0{}_i \;\rightarrow\; \frac{1}{c^3}\left[-\frac{1}{2a^2}\nabla^2 B^N_i + 2\frac{\dot a}{a^2} U_{N,i} + \frac{2}{a}\dot V_{N,i}\right] = \frac{8\pi G}{c^3}\bar\rho(1+\delta)v_i$$

- Newtonian dynamics at leading order, with a bonus: the frame dragging potential B_i is not dynamical at this order, but cannot be set to zero: doing so would forces a constraint on Newtonian dynamics
- result entirely consistent with vector relativistic perturbation theory
- in a relativistic framework, gravitomagnetic effects cannot be set to zero even in the Newtonian regime, cf. Kofman & Pogosyan (1995), ApJ 442:

magnetic Weyl tensor at leading order

$$H_{ij} = \frac{1}{2c^3}\left[B^N_{\mu,\nu(i}\varepsilon_{j)}{}^{\mu\nu} + 2v_\mu(U_N + V_N)_{,\nu(i}\varepsilon_{j)}{}^{\mu\nu}\right]$$

Post-Friedmannian ΛCDM

The 1-PF equations: vector and tensor sectors

- the frame dragging vector potential becomes dynamical at this order
- the TT metric tensor h_{ij} is not dynamical at this order, but it is instead determined by a non-linear constraint in terms of the scalar and vector potentials
- h_{ij} doesn't represent GW at this order, it is a distortion of the spatial slices in the Poisson gauge
- GW comes in at c^{-6} order, and according to Szekeres [gr-qc/9903056] the approximate set of equations should become hyperbolic at that order

so far so good...

- at leading order, we have obtained Newtonian cosmology equations
- the corresponding metric is a consistent approximate solution of EFE in the Newtonian regime, valid for scales <<H^{-1}
- how about large linear scales?

linearized equations

linearized equations for the resummed variables: standard scalar and vector perturbation equations in the Poisson gauge

$$\nabla^2\psi_P - \frac{3}{c^2}a^2\left[\frac{\dot{a}}{a}\dot{\psi}_P + \left(\frac{\dot{a}}{a}\right)^2\phi_P\right] = 4\pi G\bar{\rho}a^2\delta\ ,$$

$$-\nabla^2(\psi_P - \phi_P) + \frac{3}{c^2}a^2\left[\frac{\dot{a}}{a}(\dot{\phi}_P + 3\dot{\psi}_P) + 2\frac{\ddot{a}}{a}\phi_P + \left(\frac{\dot{a}}{a}\right)^2\phi_P + \ddot{\psi}_P\right] = 0$$

$$\nabla^2\left(\frac{\dot{a}}{a}\phi_P + \dot{\psi}_P\right) = -4\pi G a\bar{\rho}\theta\ ,$$

$$\frac{1}{c^2}\nabla^2\nabla^2(\phi_P - \psi_P) = 0\ ,$$

$$\dot{\delta} + \frac{\theta}{a} - \frac{3}{c^2}\dot{\psi}_P = 0\ ,$$

$$\dot{\theta} + \frac{\dot{a}}{a}\theta + \frac{1}{a}\nabla^2\phi_P = 0\ .$$

cf. Ma & Bertschinger, ApJ (1994)

nonlinear post-Friedmann framework: applications

frame-dragging potential from N-body simulations

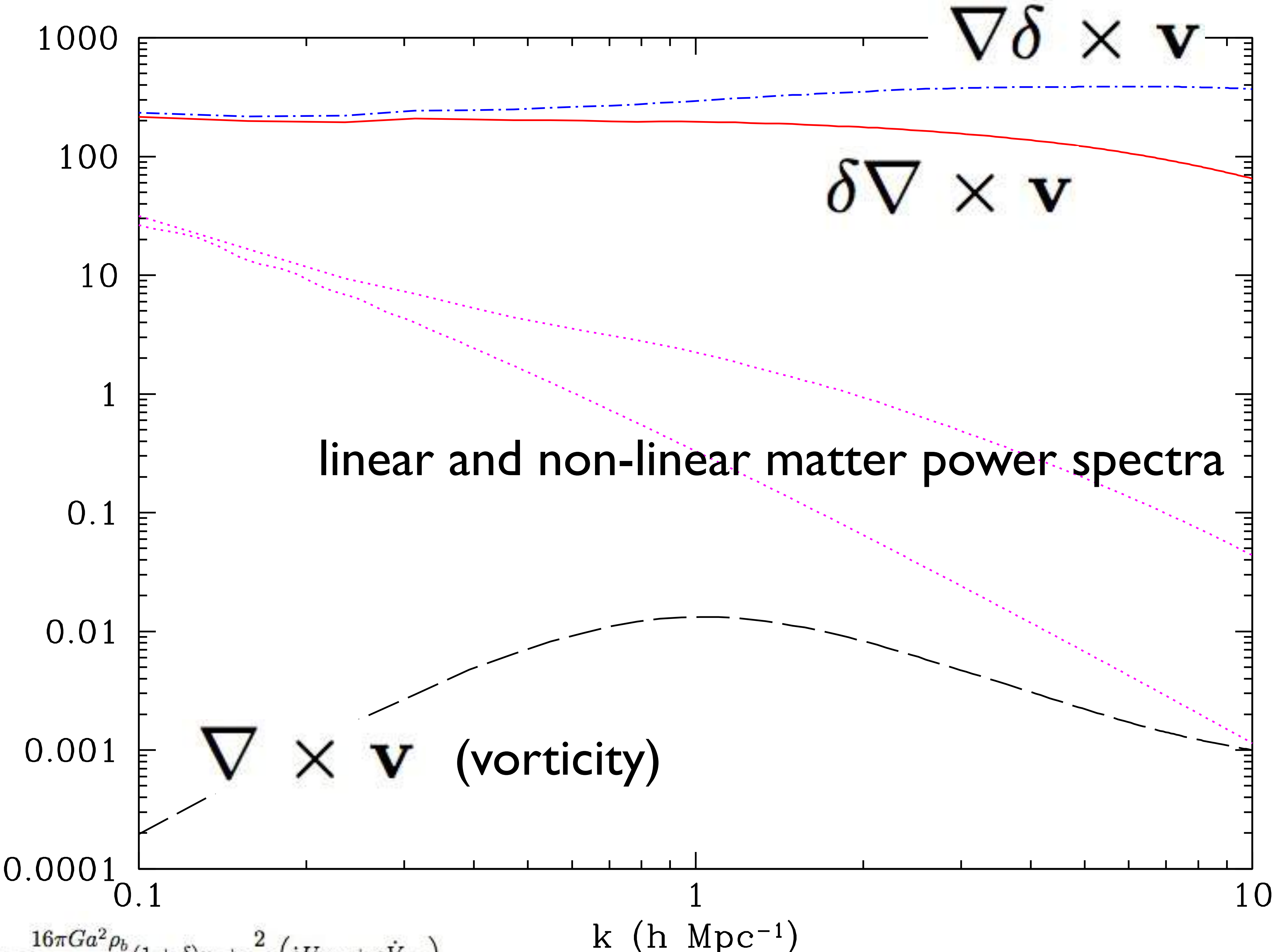
- Simulations at leading order in the post-Friedmann expansion
- dynamics is Newtonian, but a frame-dragging vector potential is sourced by the vector part of the Newtonian energy current

$$\nabla \times \nabla^2 \mathbf{B}^N = -(16\pi G\bar{\rho} a^2)\nabla \times [(1+\delta)\mathbf{v}].$$

frame-dragging potential from N-body simulations

- first calculation of an intrinsically relativistic quantity in fully non-linear cosmology
- three runs of N-body simulations with 1024^3 particles and 160 h^{-1} Mpc (Gadget-2)
- publicly available Delauney Tessellation Field Estimator (DTFE) used to extract the velocity field. cf. Pueblas & Scoccimarro (2009)
- MB, D. B. Thomas and D. Wands, Physical Review (2014), 89, 044010 [arXiv:1306.1562] - Dan B. Thomas, MB and David Wands (2015) [arXiv:1501.00799]

power spectra: sources

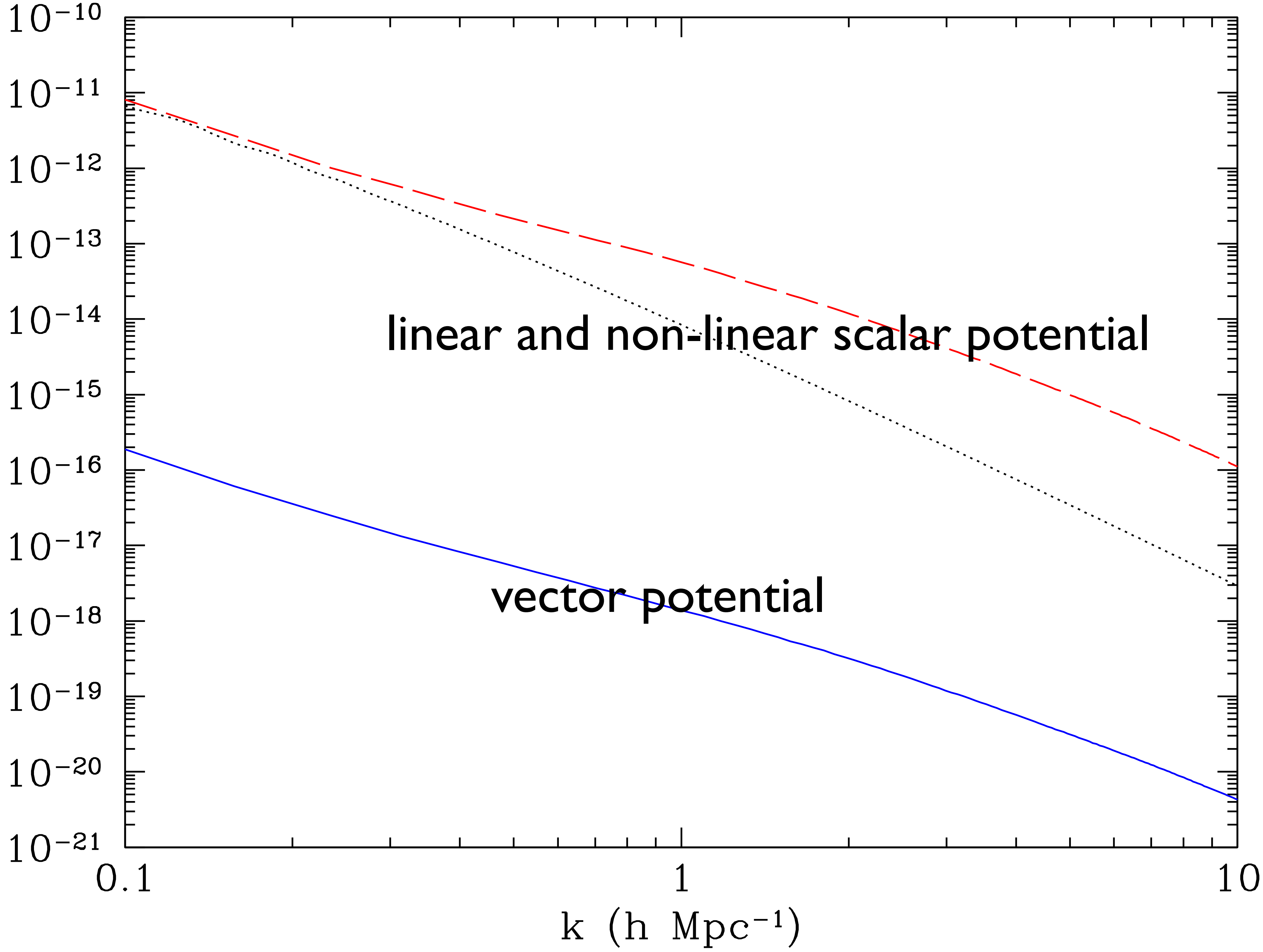


$$\frac{1}{c^3}\nabla^2 P_i^N = -\frac{16\pi G a^2 \rho_b}{c^3}(1+\delta)v_i + \frac{2}{c^3}\left(\dot{a}U_{N,i} + a\dot{V}_{N,i}\right)$$

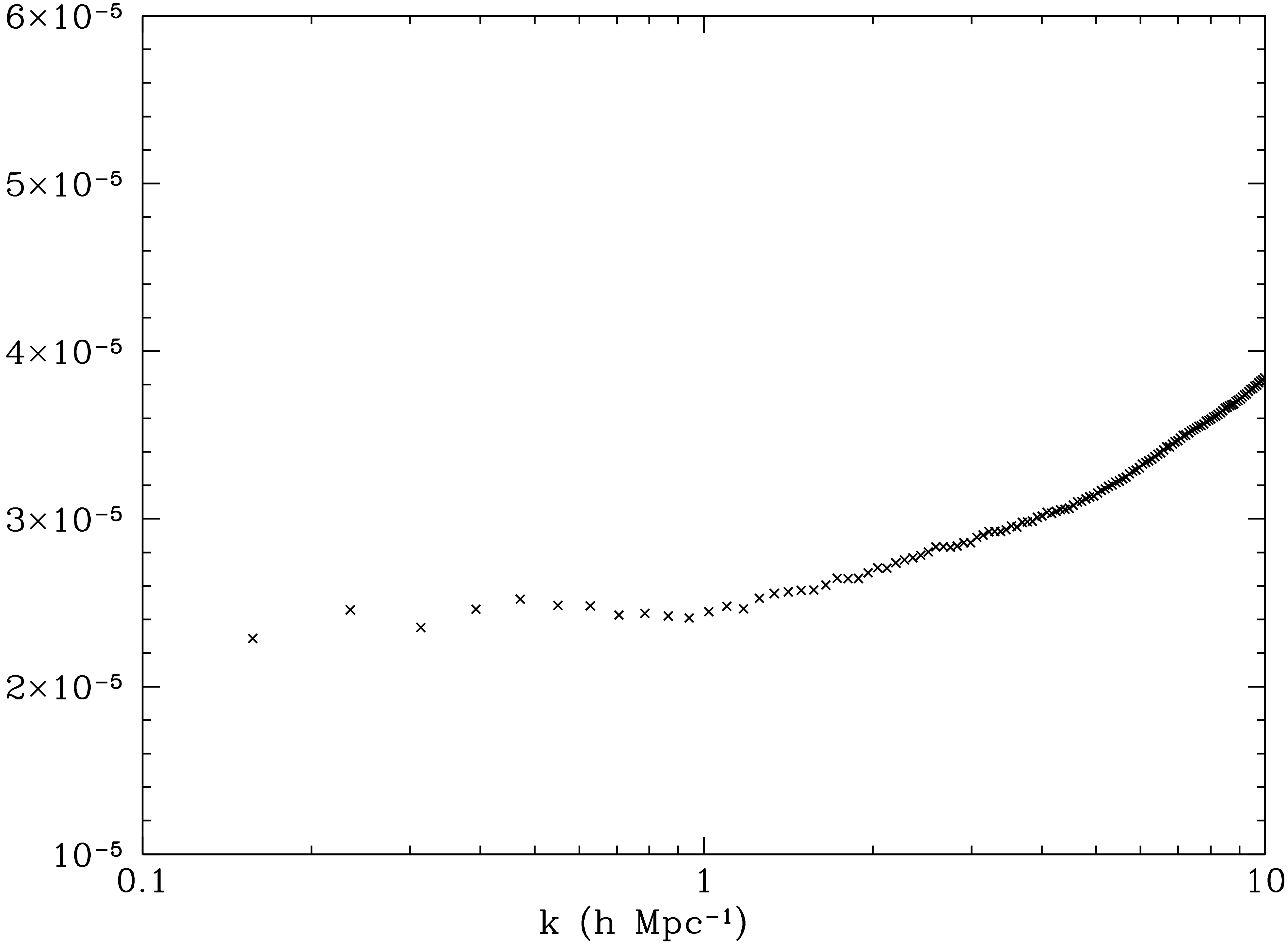
$$\nabla \times \nabla^2 \vec{P}^N = -\left(16\pi G \rho_b a^2\right) \nabla \times [(1+\delta)\vec{v}]$$

$$\nabla \times [(1+\delta)\vec{v}] = (\nabla\delta) \times \vec{v} + (1+\delta)\,\nabla \times \vec{v}$$

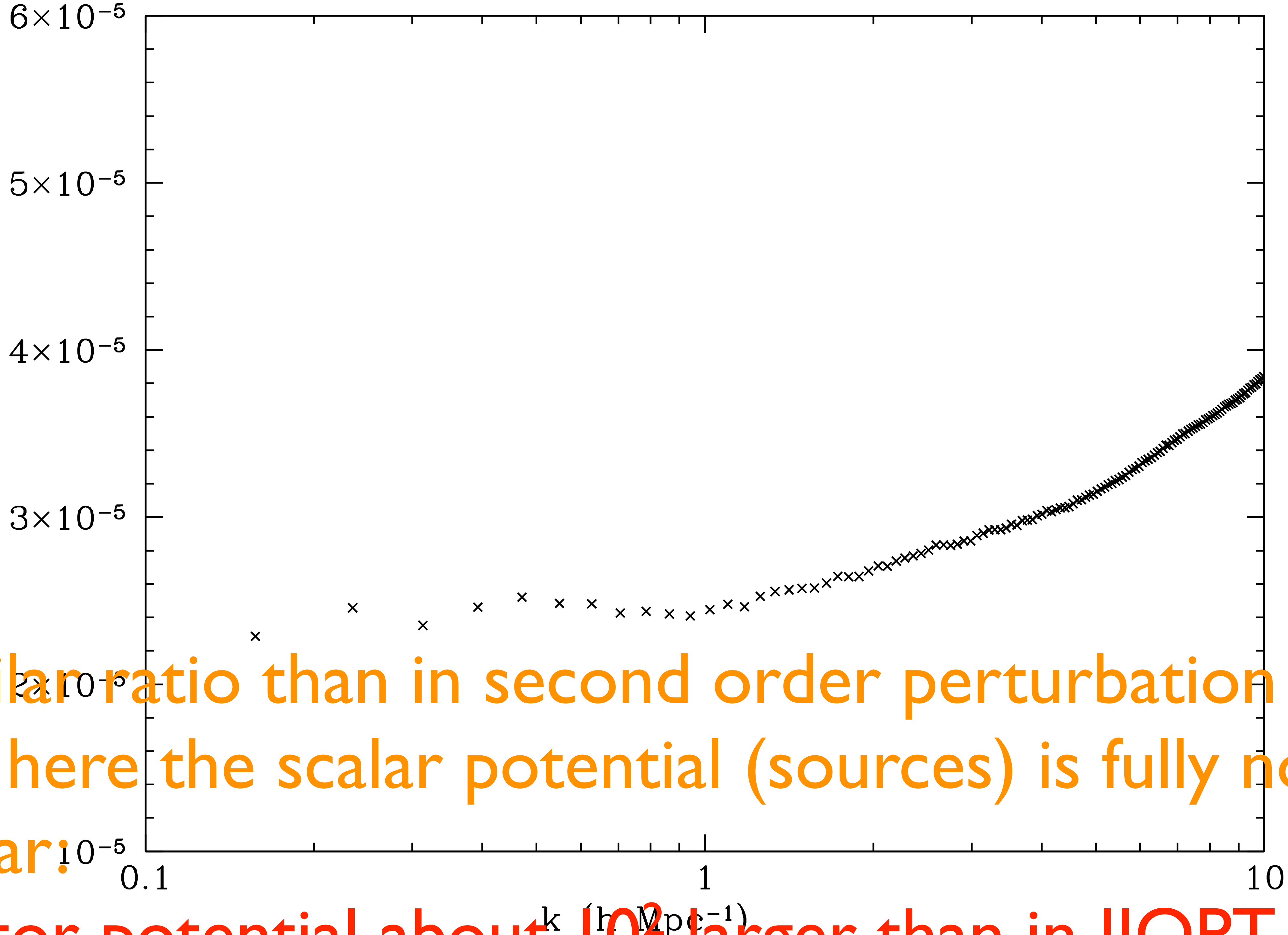
scalar and vector potentials



ratio of the potentials



ratio of the potentials



similar ratio than in second order perturbation theory but here the scalar potential (sources) is fully non-linear:

vector potential about 10^2 larger than in IIOPT

cf. Lu, Ananda, Clarkson & Maartens (2009)

post-F: other work

- weak lensing: D. B. Thomas, M. Bruni and D. Wands, [arXiv:1403.4947]
  - lensing computed up to c^{-4} valid on fully non-linear scales; effects on convergence/weak lensing E-modes negligible, currently probably not detectable; B-modes estimate says it is very small.
  - need thinking about other possible detectable effects
- extended paper with more details on the simulations and the vector potential; Thomas, Bruni & Wands [arXiv:1501.00799]
- post-F f(R) expansion and vector potential; D.B. Thomas, MB, K. Koyama, Baojiu Li and Gong-bo Zhao [arXiv:1503.07204]
- post-F "Lagrangian version": sync-comoving gauge formulation; Rampf, Villa, Bertacca & MB, [arXiv:1607.05226]

post-F vector potential in f(R)

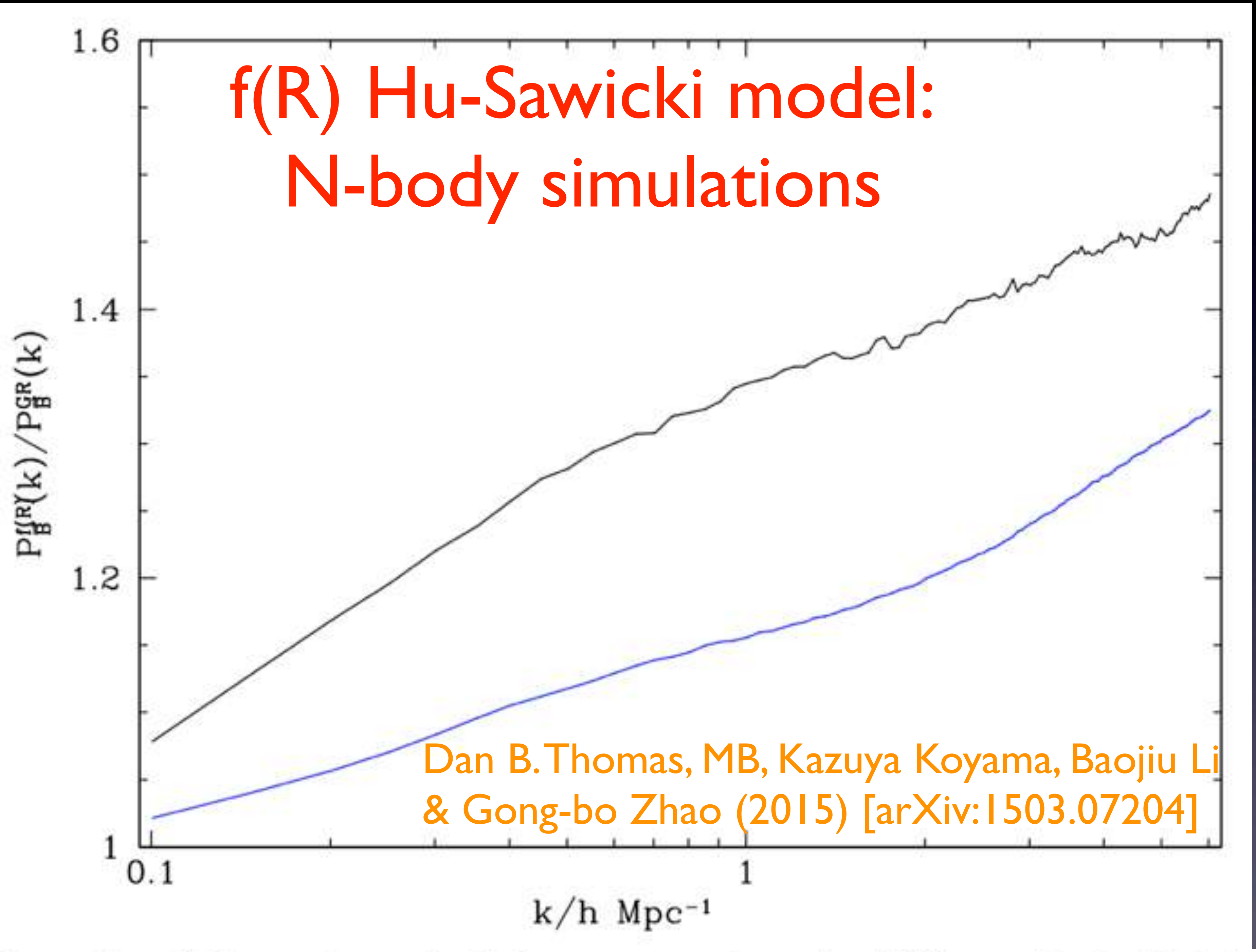
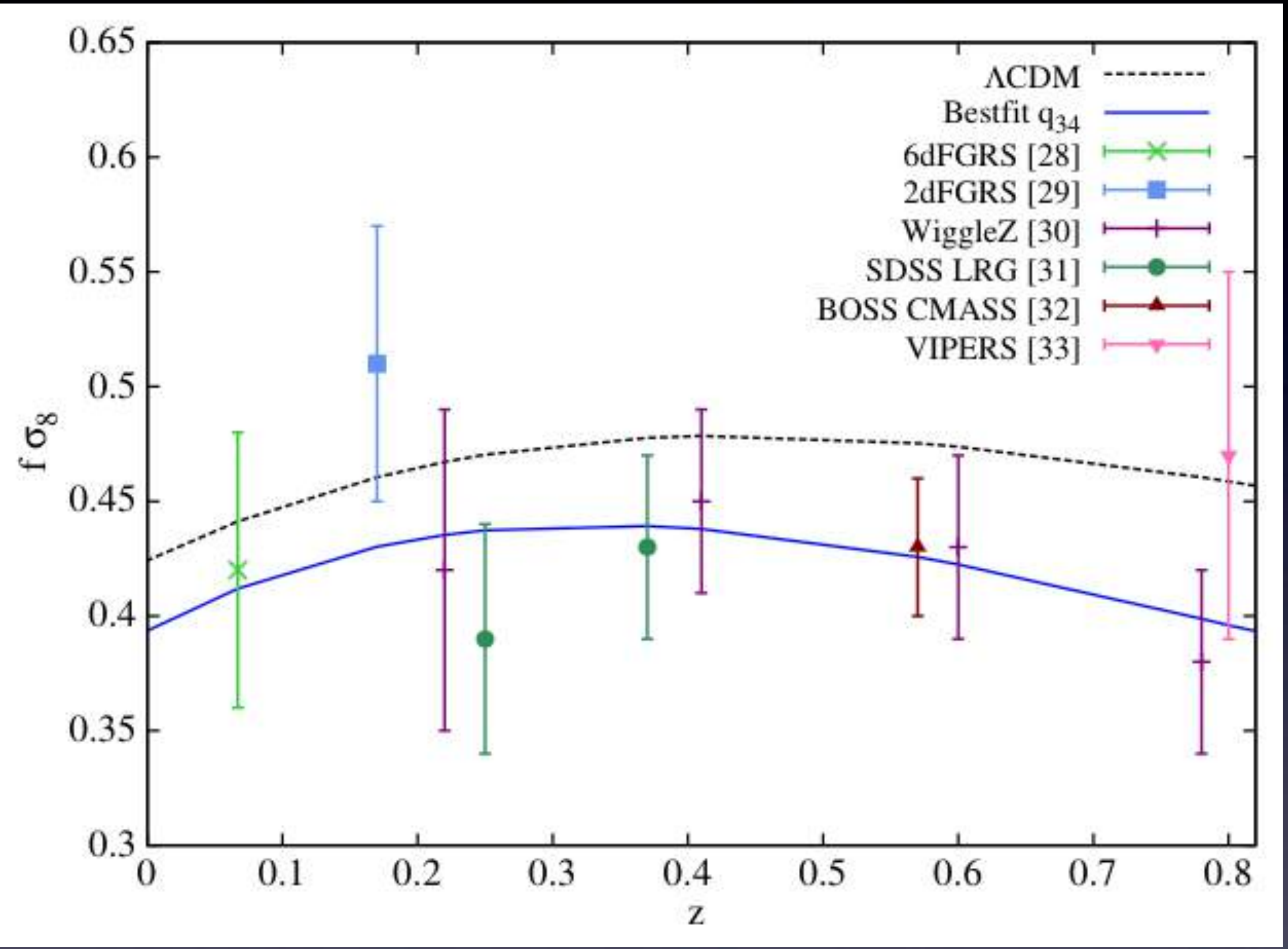


FIG. 3: The ratio of the vector potential power spectrum in $f(R)$ gravity to that in GR, for $|f_{R_0}| = 10^{-5}$. The blue curve shows the ratio at redshift one, and the black curve shows the ratio at redshift zero.

iVCDM



- iVCDM (Salvatelli, Said, MB & Wands, PRL 113, 181301, 2014): in view of simulations, compute leading order post-F for iVCDM from Einstein field equations, Maselli et al, in progress

back to basic...

Newtonian Cosmology

1. Newtonian self-gravitating fluid: described by the continuity, Euler and Poisson equations

2. rescale physical coordinates to comoving coordinates $\dot{\vec{r}} = H\vec{r} + a\vec{v}$

dust: p=0

$$\frac{d\delta}{dt} + \frac{\vec{\nabla}\cdot\vec{v}}{a}(1+\delta)$$

$$\frac{d\vec{v}}{dt} + \frac{\dot{a}}{a}\vec{v} = -\vec{\nabla}\phi$$

$$\nabla^2\phi = 4\pi G\rho_b\delta$$

note: convective time derivative

Linear perturbations

- for dust, linearise, combine continuity and Euler, substitute from Poisson, to get

$$\delta'' + \frac{3}{2a}\delta' - \frac{3}{2a^2}\delta = 0\,,$$

- In GR, for a w=constant fluid, use energy and momentum conservation equations, and the Energy constraint, to get (Δ gauge-invariant)

$$\Delta'' + \frac{3}{2S}(1-3w)\Delta' + \frac{3}{2S^2}(3w^2-2w-1)\Delta - \frac{wD^2\Delta}{H_0^2\Omega_0}S^{1+3w} = 0$$

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$$\Delta'' + \frac{3}{2S}(1-3w)\Delta' + \frac{3}{2S^2}(3w^2-2w-1)\Delta - \frac{wD^2\Delta}{H_0^2\Omega_0}S^{1+3w} = 0$$

Solution in EdS and top-hat

$$a(t) = a_i \left(\frac{t}{t_i}\right)^{2/3} ,$$

$$\delta(t) = \delta_+ a(t) + \delta_- a(t)^{-3/2}$$

- top-hat turnaround and collapse time: characterized by the value of δ at these events:

$$\delta_T = 1.06 \quad \delta_c = 1.696$$

the Averaging, BR & Fitting program

- Strictly speaking, Einstein Field Equations (EFE) describe the fundamental interaction, gravity.
- Only the truly inhomogeneous universe obeys EFE, precisely in the same way that in the Newtonian N-body problem each particle interact will all others
- Thus, in principle we should simulate inhomogeneous models and extract an average expansion a-posteriori
- Instead, we first ***assume*** the existence of a ***fitting*** homogeneous isotropic metric, ***then*** solve EFE for this (FLRW models).
- We should instead average EFE, obtaining an effective homogeneous limit that satisfies EFE with effective back-reaction terms.

Buchert's approach to the averaging problem(*)

- consider an irrotational dust spacetime [(-,+,+,+) and c=1] and adopt synchronous comoving coordinates, so that the line element reads

$$ds^2 = -dt^2 + h_{ab}(\vec{x}, t)dx^a dx^b,$$

where h_{ab} is the spatial metric of the constant t hypersurfaces, with determinant h.

then the average of a scalar Ψ on a compact coordinate domain $\mathcal{D}$ and the proper volume $V_{\mathcal{D}}$ is defined as

$$\langle \Psi \rangle_{\mathcal{D}} = \frac{1}{V_{\mathcal{D}}} \int_{\mathcal{D}} d^3x \sqrt{h} \Psi.$$

$$V_{\mathcal{D}} := \int_{\mathcal{D}} d^3x \sqrt{h}$$

(*) see e.g.: Buchert (2008), GRG 40(2), pp.467-527
Buchert (2011) CQG 28(1), p.4007.

Bucherts averaging

- From V, we can then define the average scale factor

$$V_{\mathcal{D}} := \int_{\mathcal{D}} d^3x\sqrt{h}$$

$$a_{\mathcal{D}} \equiv (V_{\mathcal{D}}/V_{\mathcal{D}ini})^{1/3}$$

- then, the key to getting BR through averaging is the non-commutativity of the time derivative and the spatial averaging

$$\partial_t\langle\Psi\rangle_{\mathcal{D}} - \langle\partial_t\Psi\rangle_{\mathcal{D}} = \langle\Theta\Psi\rangle_{\mathcal{D}} - \langle\Theta\rangle_{\mathcal{D}}\langle\Psi\rangle_{\mathcal{D}}$$

- then, averaging the continuity equation, Hamiltonian constraints and the Raychaudhuri equation gives effective Friedmann equations

$$\dot{\langle\rho\rangle}_{\mathcal{D}} = -3\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\langle\rho\rangle_{\mathcal{D}}$$

$$\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 = \frac{8\pi G}{3}\langle\rho\rangle_{\mathcal{D}} - \frac{1}{6}(\mathcal{Q}_{\mathcal{D}} + \langle\mathcal{R}\rangle_{\mathcal{D}})$$

$$\left(\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right) = -\frac{4\pi G}{3}\langle\rho\rangle_{\mathcal{D}} + \frac{1}{3}\mathcal{Q}_{\mathcal{D}},$$

Bucherť's averaging

- in the effective Friedmann equations

$$\left(\frac{\dot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right)^2 = \frac{8\pi G}{3}\langle\rho\rangle_{\mathcal{D}} - \frac{1}{6}(\mathcal{Q}_{\mathcal{D}} + \langle\mathcal{R}\rangle_{\mathcal{D}})$$
$$\left(\frac{\ddot{a}_{\mathcal{D}}}{a_{\mathcal{D}}}\right) = -\frac{4\pi G}{3}\langle\rho\rangle_{\mathcal{D}} + \frac{1}{3}\mathcal{Q}_{\mathcal{D}},$$

- the term $\langle\mathcal{R}\rangle_{\mathcal{D}}$ represents the average of the spatial Ricci scalar, while

$$\mathcal{Q}_{\mathcal{D}} \equiv \frac{2}{3}\left(\langle\Theta^2\rangle_{\mathcal{D}} - \langle\Theta\rangle^2_{\mathcal{D}}\right) - 2\langle\sigma^2\rangle_{\mathcal{D}}.$$

- is the back-reaction term, which can be positive. If this term satisfies $\mathcal{Q}_{\mathcal{D}} > 4\pi G\langle\rho\rangle_{\mathcal{D}}$ then clearly it can act as Dark Energy

Bucherts averaging

$$\mathcal{Q}_{\mathcal{D}} \equiv \frac{2}{3}\left(\langle\Theta^2\rangle_{\mathcal{D}} - \langle\Theta\rangle_{\mathcal{D}}^2\right) - 2\langle\sigma^2\rangle_{\mathcal{D}}.$$

- So, we can get an accelerated expansion of the averaged volume if $\mathcal{Q}_{\mathcal{D}} > 4\pi G\langle\rho\rangle_{\mathcal{D}}$, i.e. if the non-local variance of the local expansion dominates.

- Even if the local expansion rate is slowing down, this non-local effects may cause acceleration.

- This non local effect is in essence the main argument of those supporting the idea that back-reaction can be important against the argument - used by detractors - that local perturbations are always very small.

- Big bonus: there is no coincidence problem. Not only because there isnt a real additional DE, but really because the effective BR DE, the variance of Θ, grows naturally as structure grows.

Full GR
Numerical Relativity
Simulations

Eloisa Bentivegna & MB, PRL 116, 251302 (2016)

cf. J.T. Giblin Jr., J.B. Mertens & G.D. Starkman, PRL 2016, 251301 (2016)

Assumptions and procedure

- Initial conditions: a small δ 10^{-2}-10^{-6} on EdS background

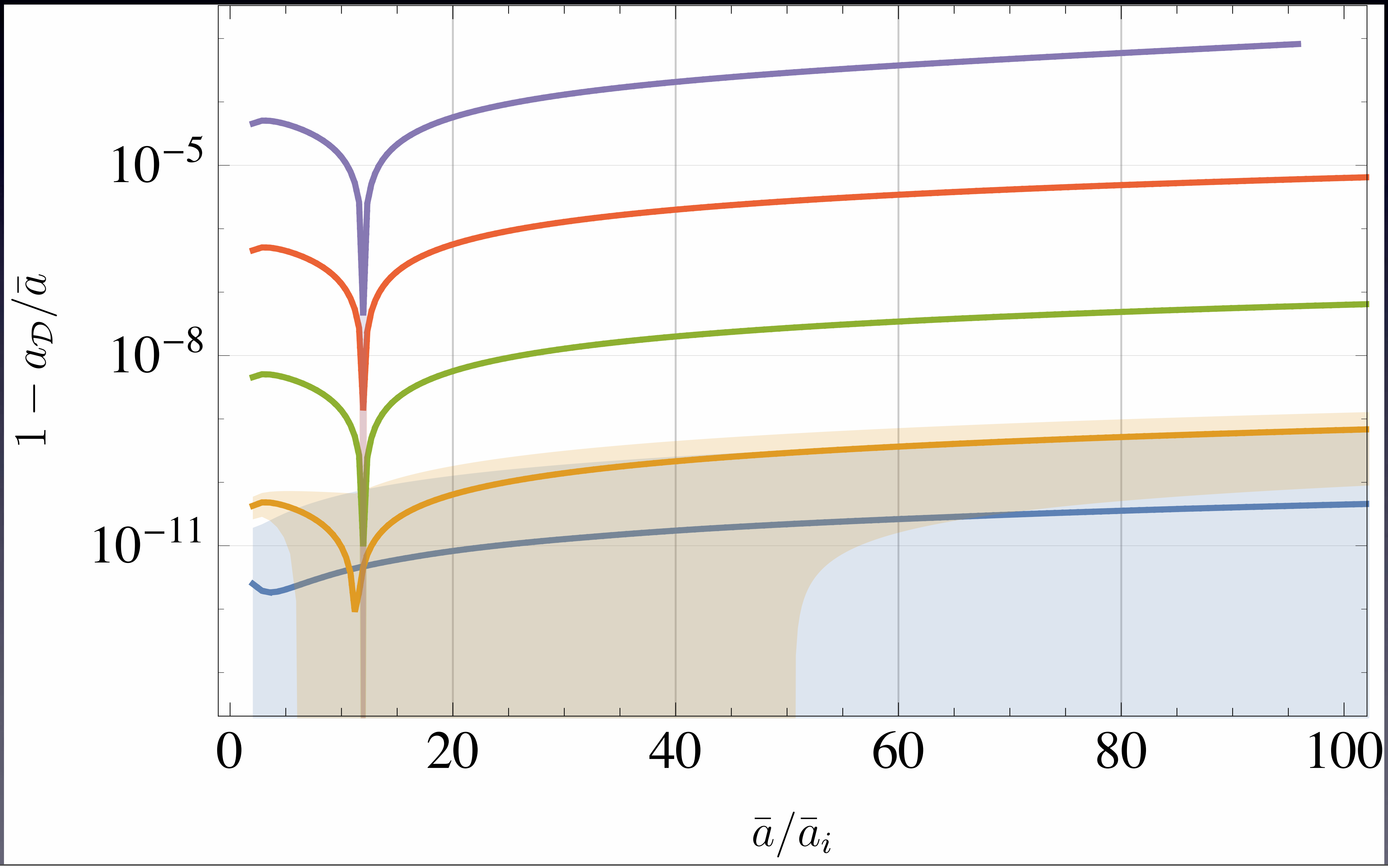
$$\rho_i = \bar{\rho}_i \left(1 + \delta_i \sum_{j=1}^{3} \sin \frac{2\pi x^j}{L}\right)$$

- synchronous-comoving gauge, irrotational fluid (Lagrangian approach)
- Integrate EFE using the Einstein Toolkit, freey available open source infrastructure for Numerical Relativity
- use a variant of BSSN formulation of EFE

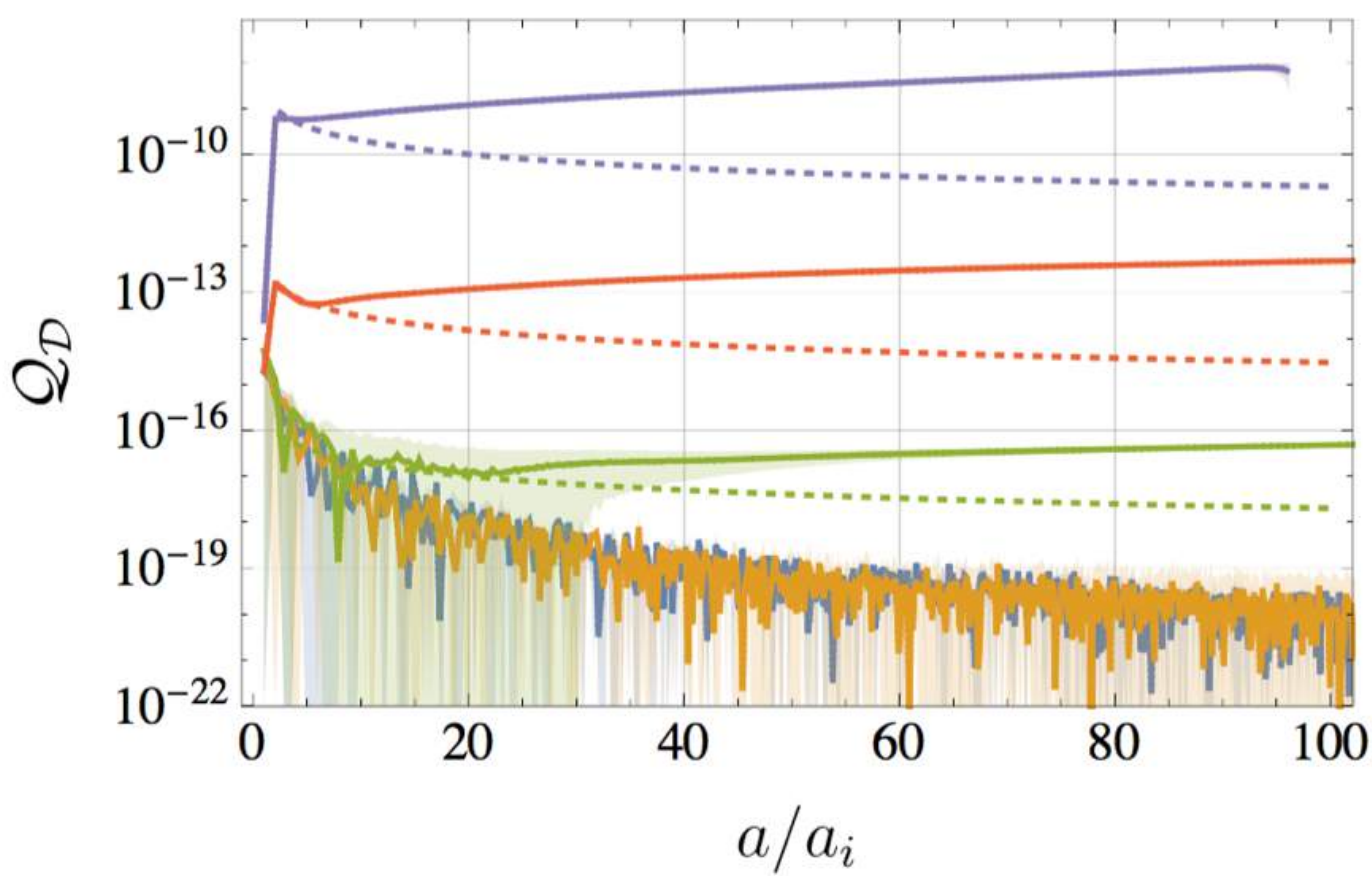
Assumptions and procedure

- solve initial constraint
- evolve EFE with periodic boundary conditions on comoving box of size L
- initial conditions: perturbations of EdS with $H_i^{-1}=L/4$
- domain discretised with 160^3 points
- compare average quantities and EdS evolution
- measure local quantities (expansion and density)

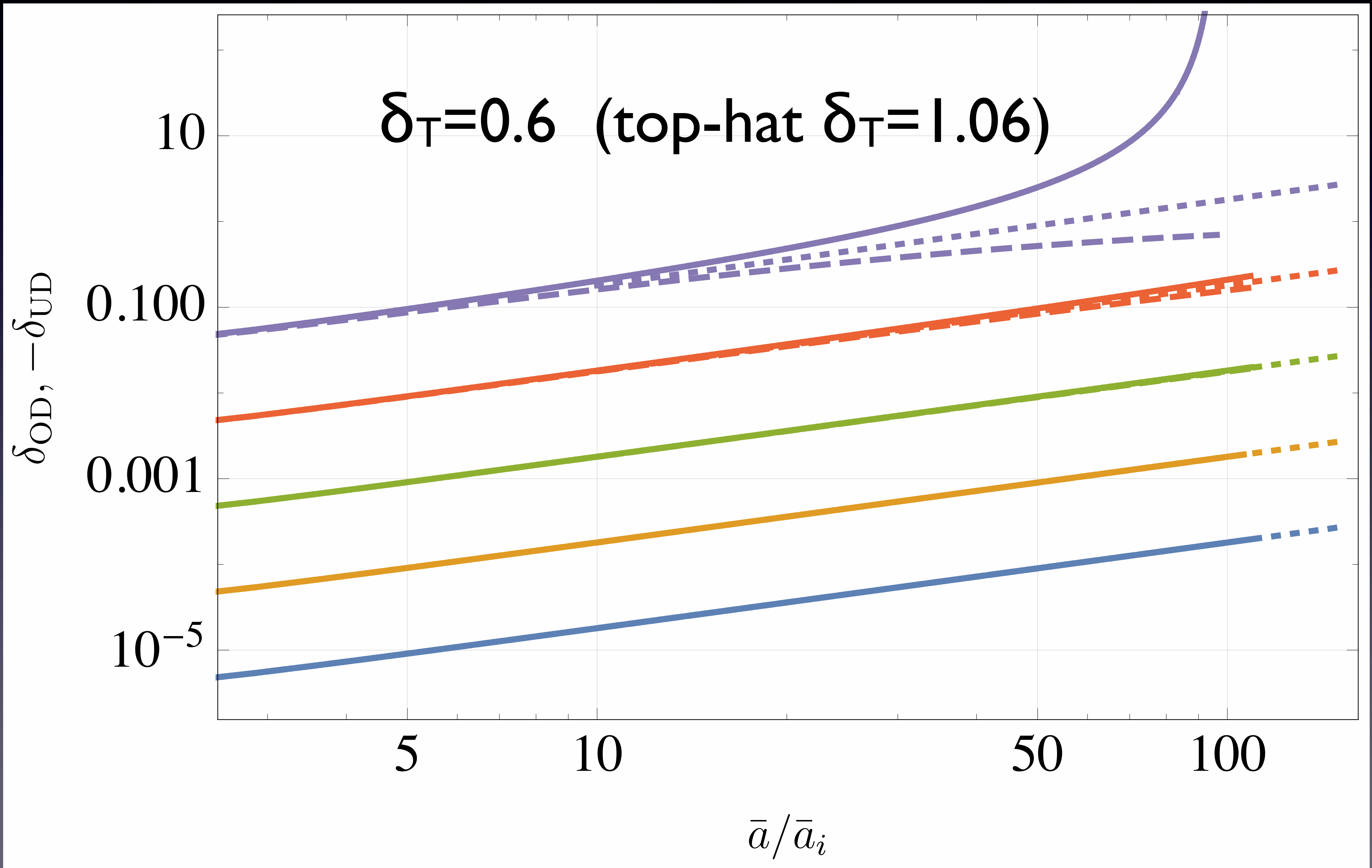
average expansion



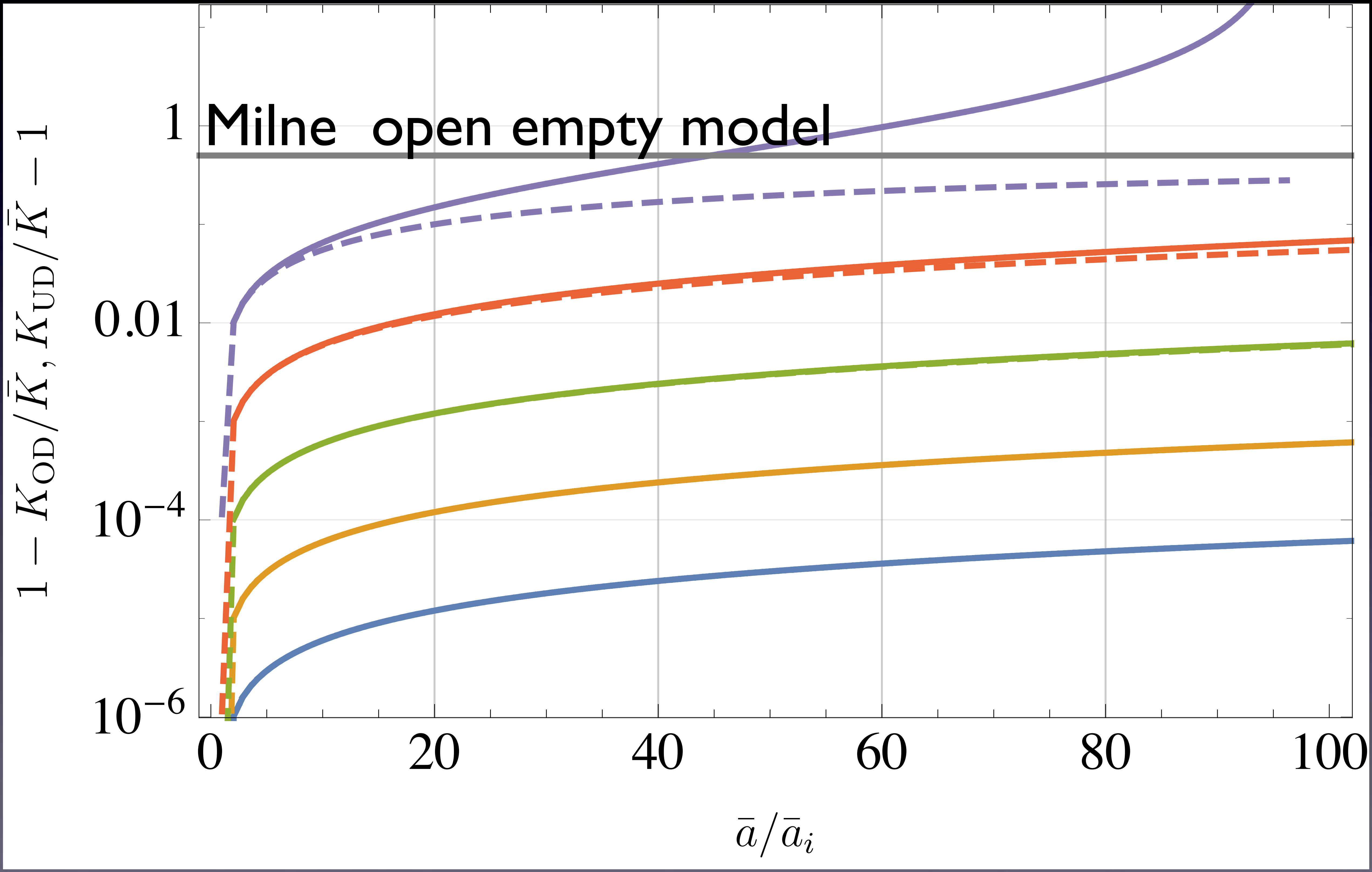
backreaction



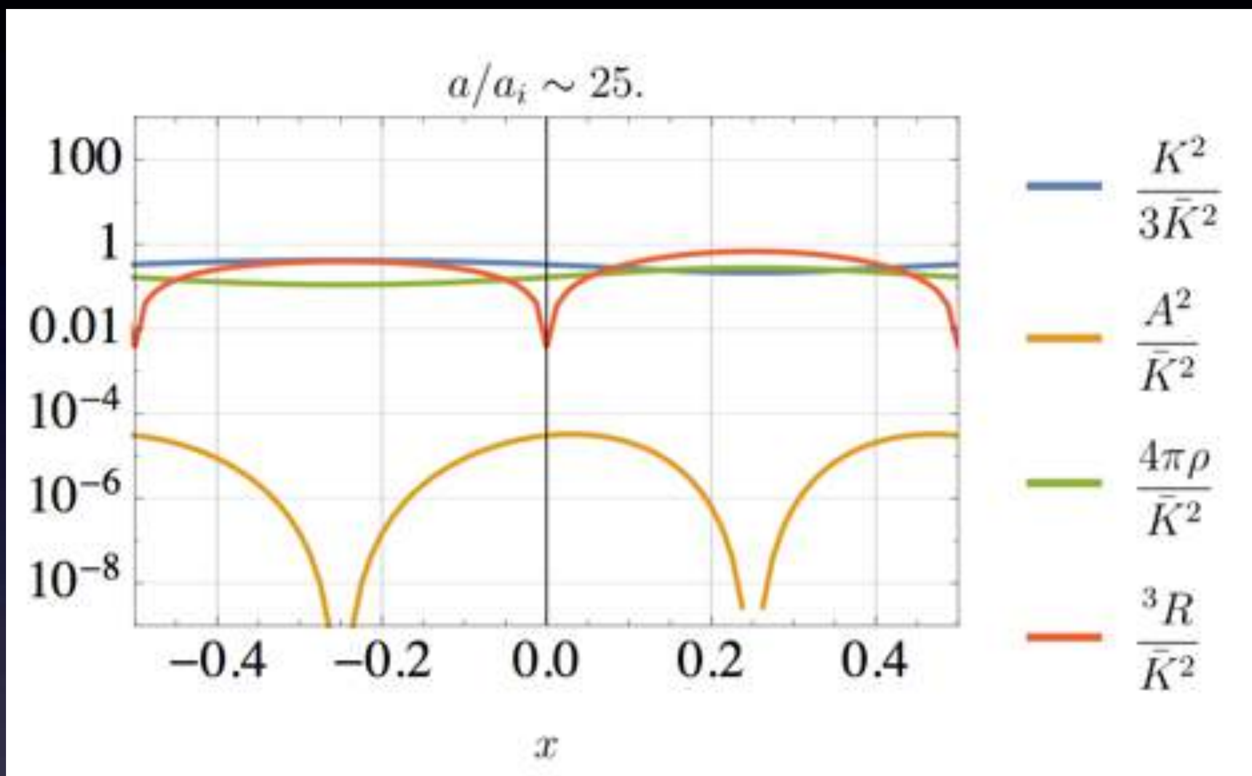
over and under densities

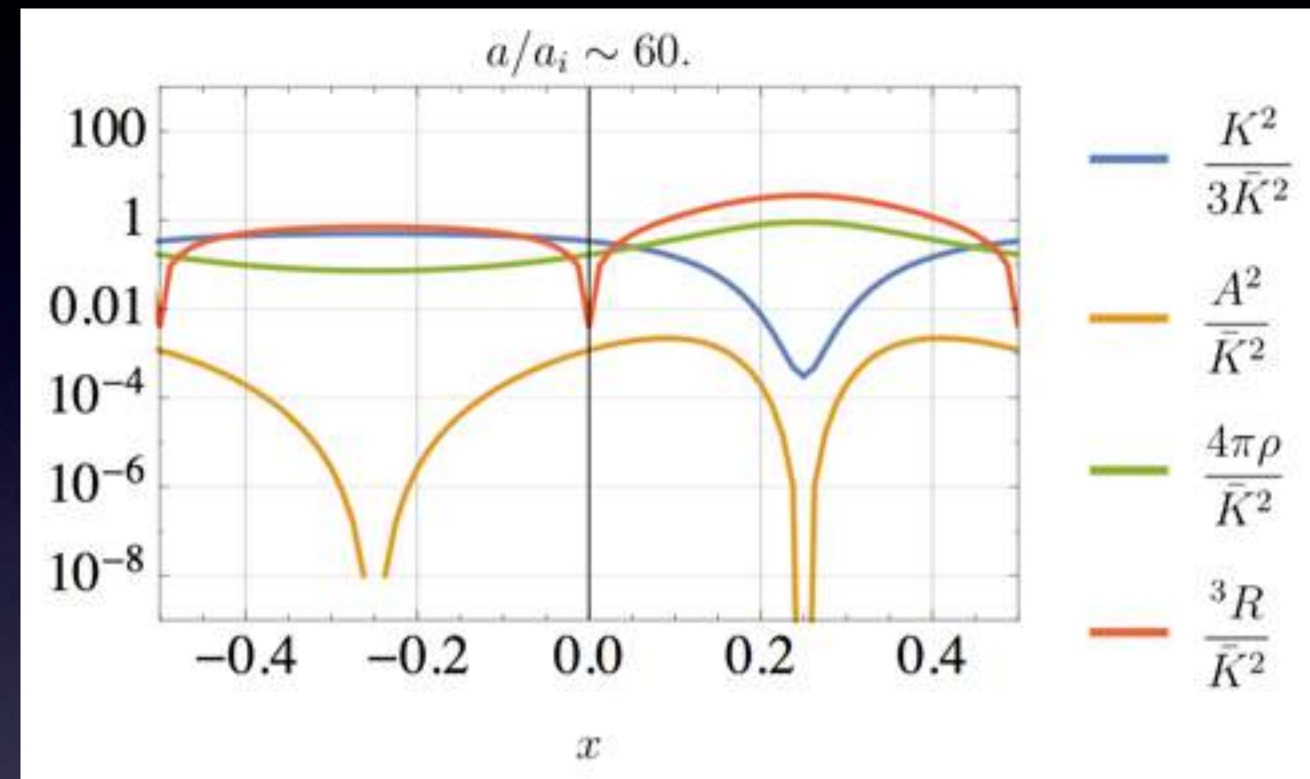


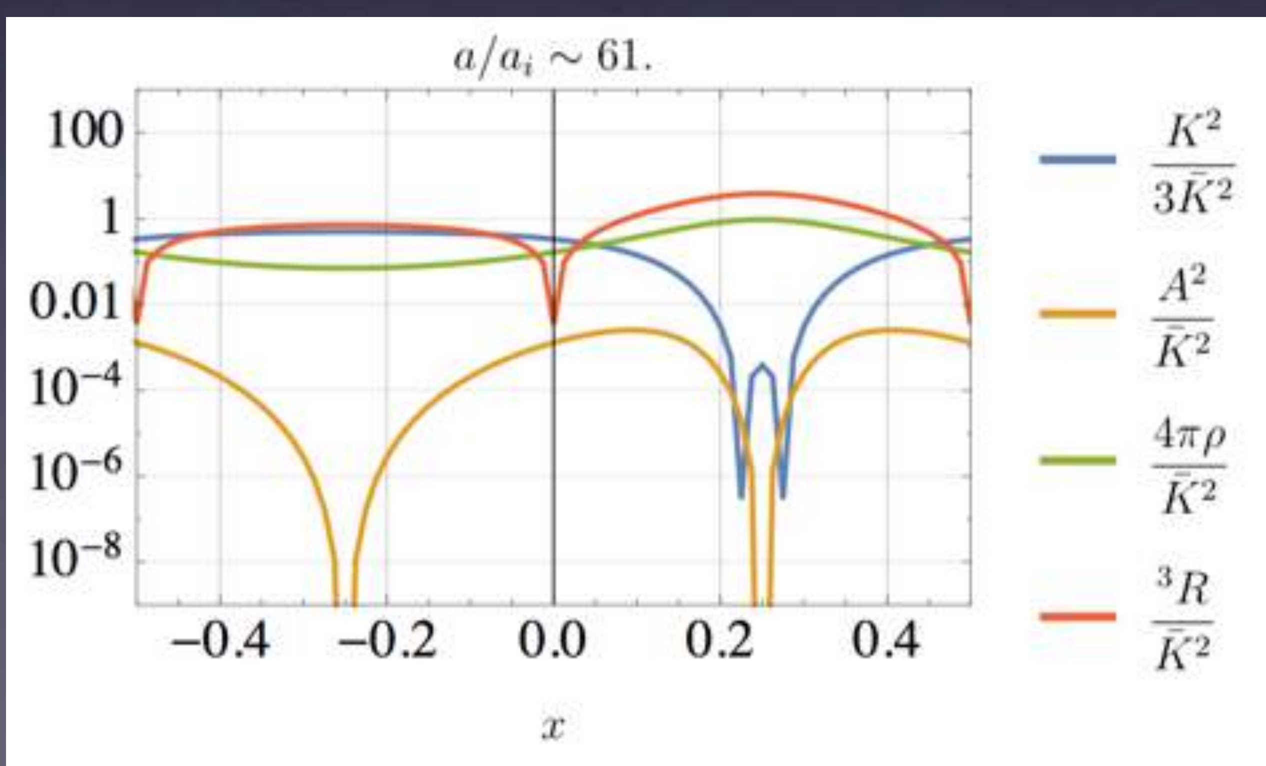
local expansion of peaks and voids

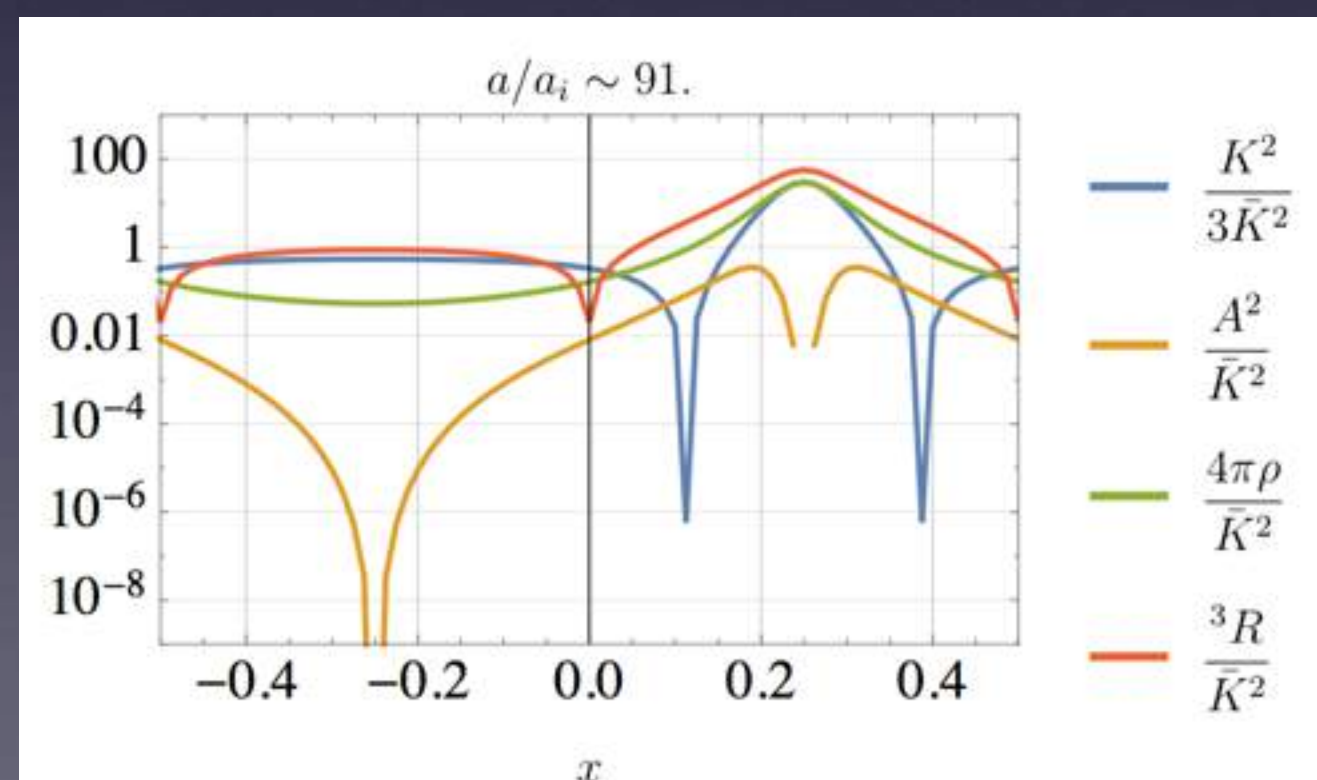


local contribution to Raychaudhuri equation









Conclusions

- post-F: framework including Newtonian and 1 GR order
  - Frame dragging small, but further work needed, e.g. lensing
  - Adamek et al.: consistent results, plus $\Phi=\Psi$ at leading order
- Full GR Numerical Relativity simulations:
  - within the fluid assumption (stop before shall crossing), backreaction is small and the box expands like EdS
  - peaks collapse much faster than standard Top-Hat
  - voids expand up to 28% faster than average
  - Gibling, Mertens & Starkman fully consistent with us

Outlook

- Bentivegna, *An automatically generated code for relativistic inhomogeneous cosmologies,* [arXiv:1610.05198]
- Giblin, Mertens, & Starkman, *Observable Deviations from Homogeneity in an Inhomogeneous Universe* [arXiv:1608.04403]
- work in progress to compare results from different codes
- work in progress to analyse in a different gauge and to extract observable quantities
- Much further work needed to obtain realistic simulations and compare with Newtonian N-body simulations