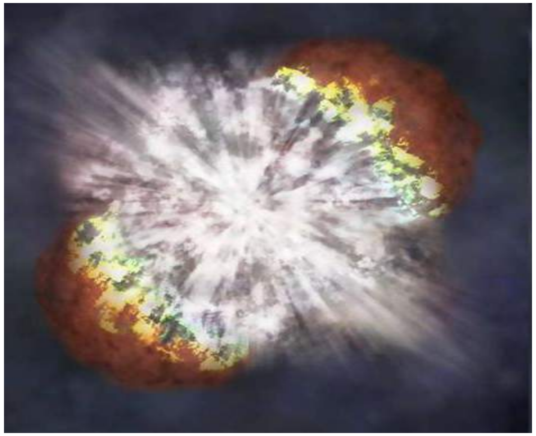
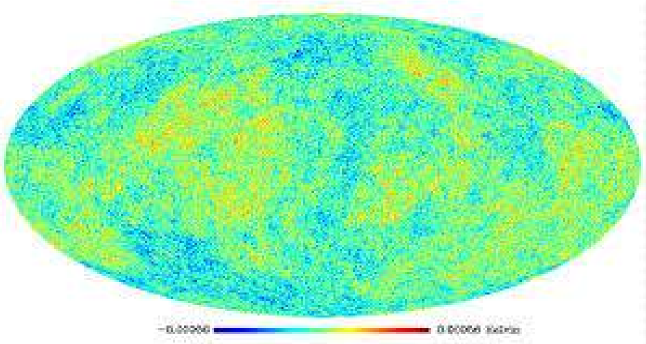
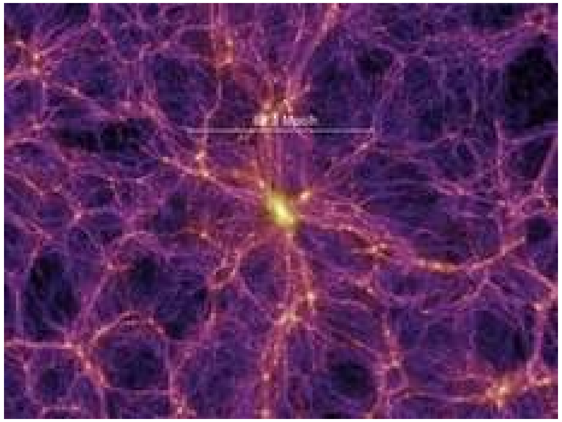
A phenomenological approach to the properties of dark matter







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M. Kunz, S.N., I. Sawicki, arXiv: 1507.01486 & 1604.05701

Main points of the talk

- Brief introduction to the standard cosmological model
- The observational data (SnIa, CMB, WL, BAO)
- Dark Matter (DM) perturbations and their sound speed cs2, a phenomenological approach
- Constraints on other DM properties (w,cvis...)
- Conclusions

The Standard Cosmological model

Einstein equations:

$$\underbrace{R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R}_{G_{\mu\nu}} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

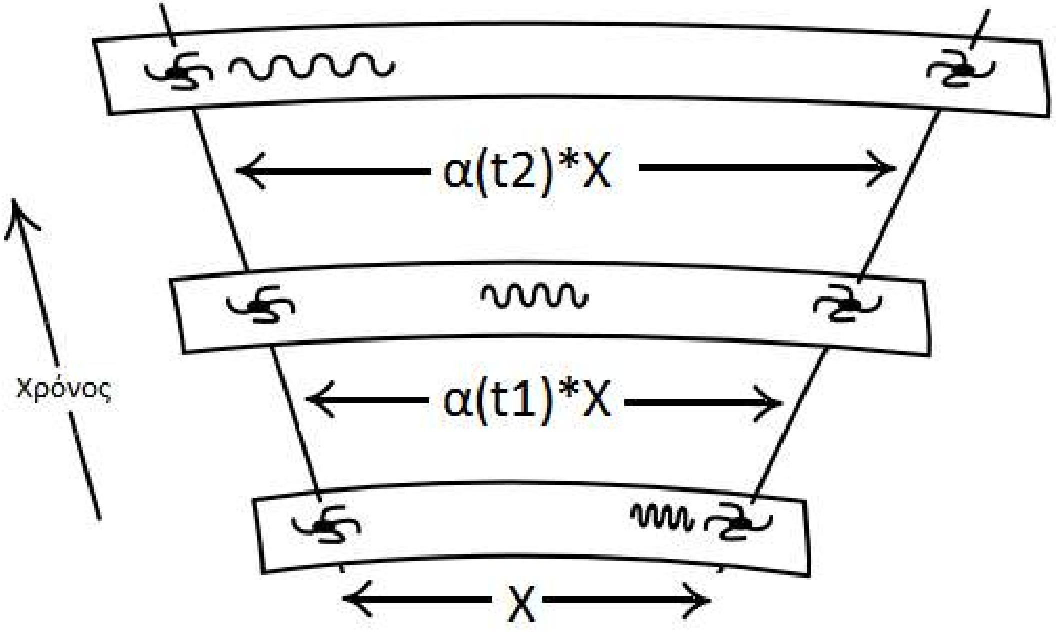
Cosmological Constant

$$T^{\mu}_{\nu} = Pg^{\mu}_{\nu} + (\rho + P)U^{\mu}U_{\nu}$$

Friedmann-Lemaitre-Robertson-Walker (FLRW) metric:

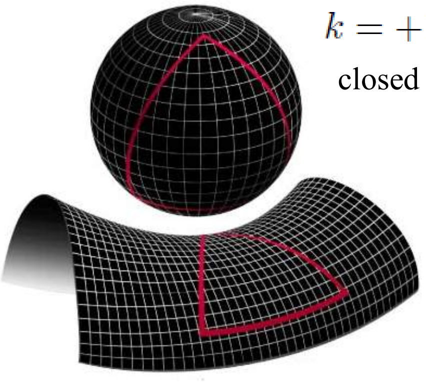
$$ds^2 = c^2dt^2 - \alpha(t)^2\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin(\theta)^2d\phi^2)\right)$$

Scale factor α(t):



The Standard Cosmological model

The curvature:



Friedmann equations (1924):

$$H^2(\alpha) = \left(\frac{\dot{\alpha}}{\alpha}\right)^2 = \frac{8\pi G}{3}\rho(\alpha) - \frac{k}{\alpha^2}$$

$$\frac{\ddot{\alpha}}{\alpha} = -\frac{4\pi G}{3}(\rho(\alpha) + P(\alpha))$$

Continuity equations:

(via Bianchi identities)

$$\nabla_\nu T^{\mu\nu} = 0 \quad \Longrightarrow \quad \dot{\rho} + 3H(\rho + P) = 0$$

The Standard Cosmological model

Hubble (1929): The Universe is expanding

Redshift of distant galaxies

Riess et al. (1998): ...and it's also accelerating!

Type Ia supernovae

2nd Friedmann equation: $\frac{\ddot{\alpha}}{\alpha} = -\frac{4\pi G}{3}\left(\rho(\alpha) + 3P(\alpha)\right)$ $\Longrightarrow$ $P < -\frac{\rho}{3}$

Equation of state

$$P = w\,\rho \left\{ \begin{array}{lll} w = 0 & \text{Non-relativistic matter} & P << \rho \\ w = \frac{1}{3} & \text{Relativistic matter (photons etc)} & P = \frac{1}{3}\rho \end{array} \right.$$

$$P < -\frac{\rho}{3} \Longrightarrow w < -\frac{1}{3}$$

The known forms of matter cannot explain the accelerated expansion of the Universe!

The Standard Cosmological model

Fractional density parameters:

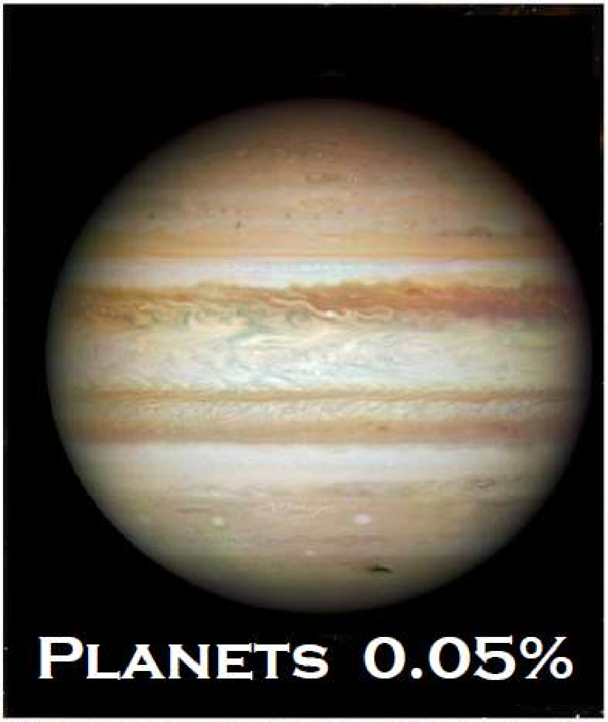
$$\rho_c(t) = \frac{3H^2}{8\pi G}$$

$$\Omega(t) \equiv \frac{\rho}{\rho_c}$$

$$\Omega_{K,0} = -\frac{k}{H_0^2 a_0^2}$$

1st Friedmann equation:

$$\boxed{H(\alpha)^2 = H_0^2 \left(\Omega_{b,0}\alpha^{-3} + \Omega_{c,0}\alpha^{-3} + \Omega_{r,0}\alpha^{-4} + \Omega_{K,0}\alpha^{-2} + \Omega_{DE,0}\alpha^{-3(1+w)}\right)}$$

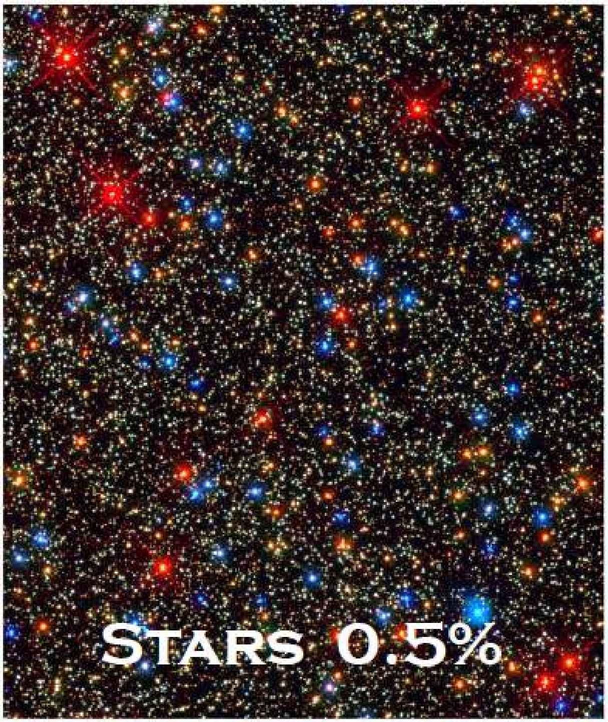


PLANETS 0.05%

PLANETS+STARS+GAS

DARK MATTER

25%



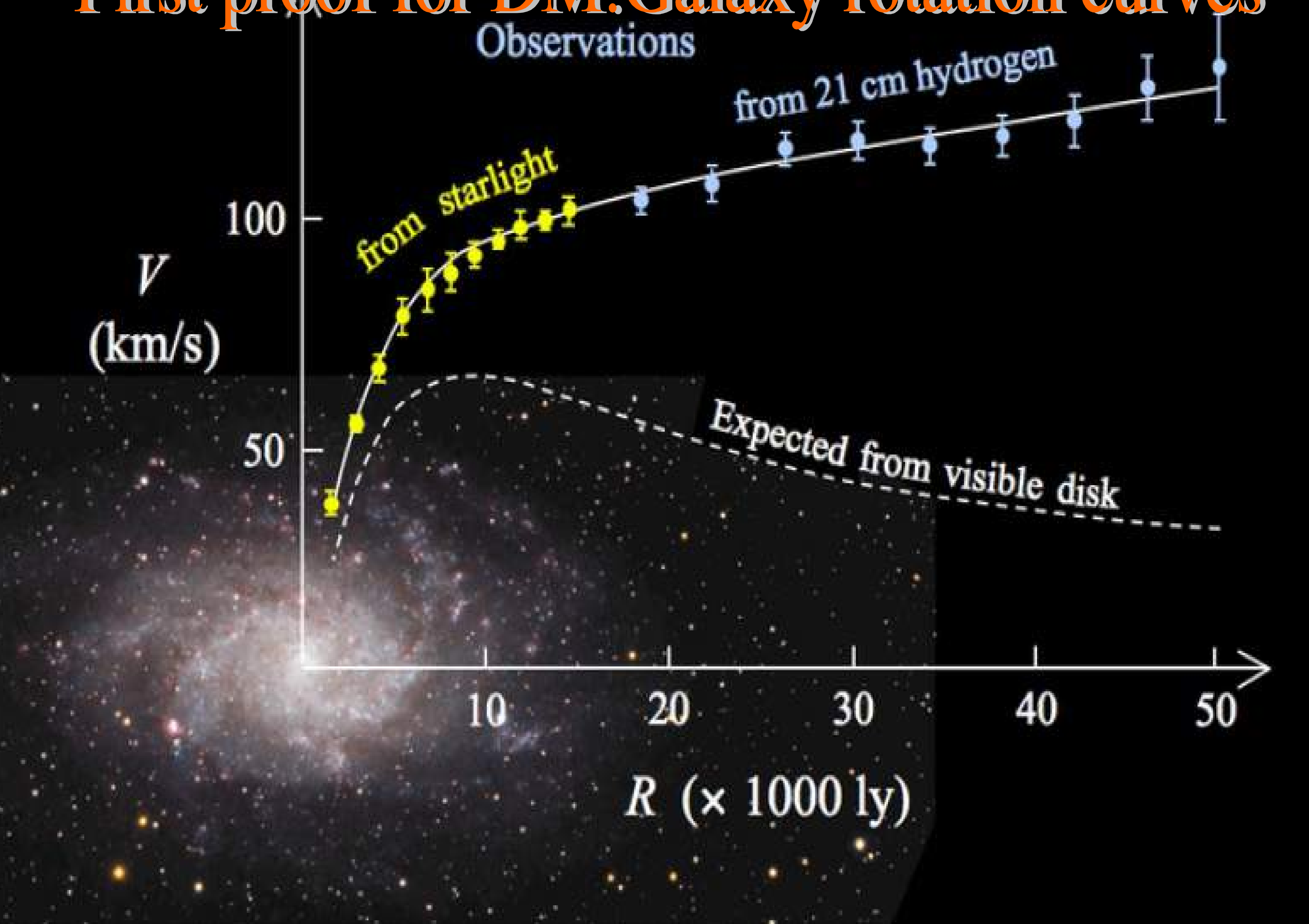
STARS 0.5%

GAS 4%

DARK ENERGY

70%

First proof for DM:Galaxy rotation curves



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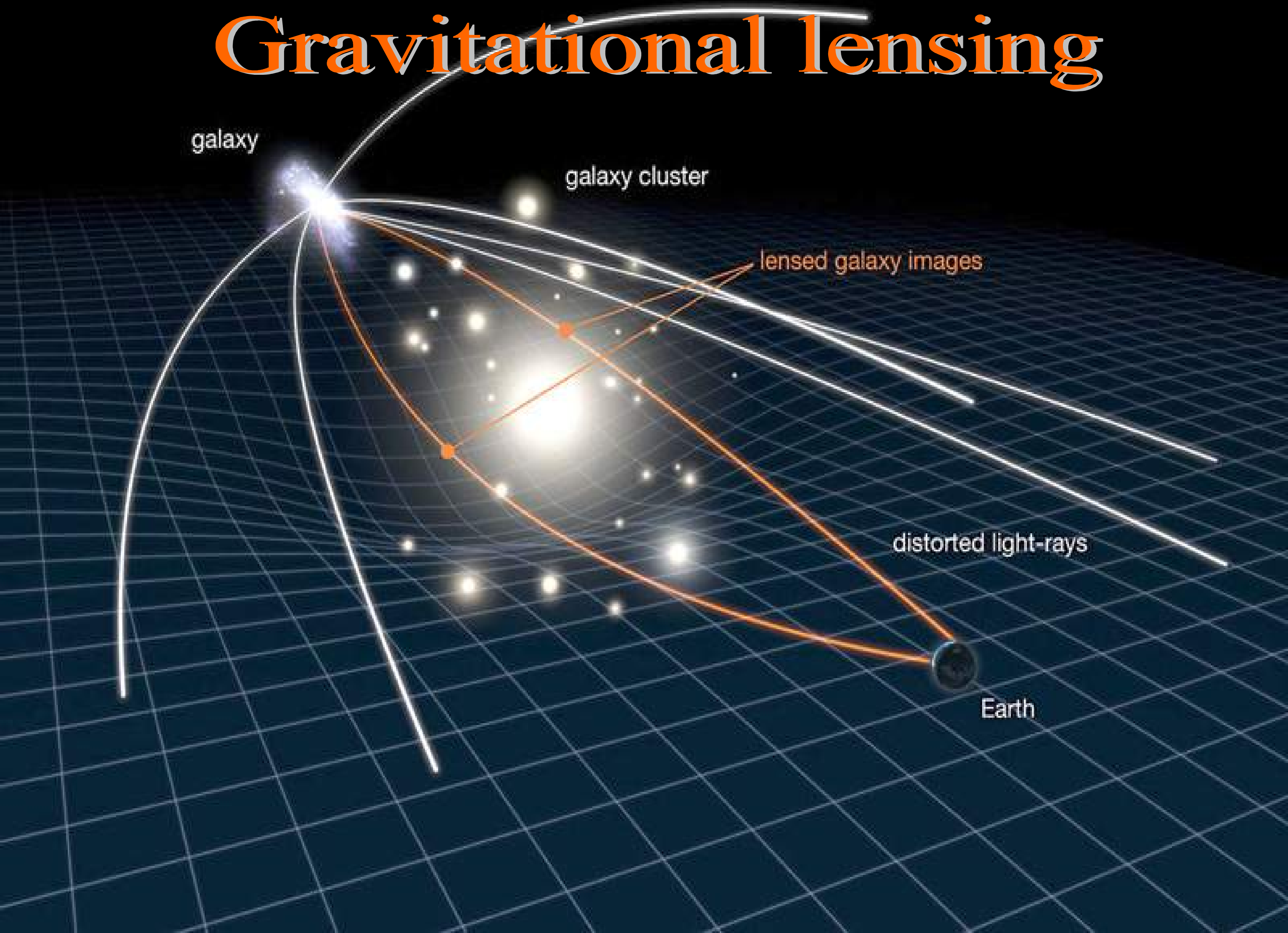
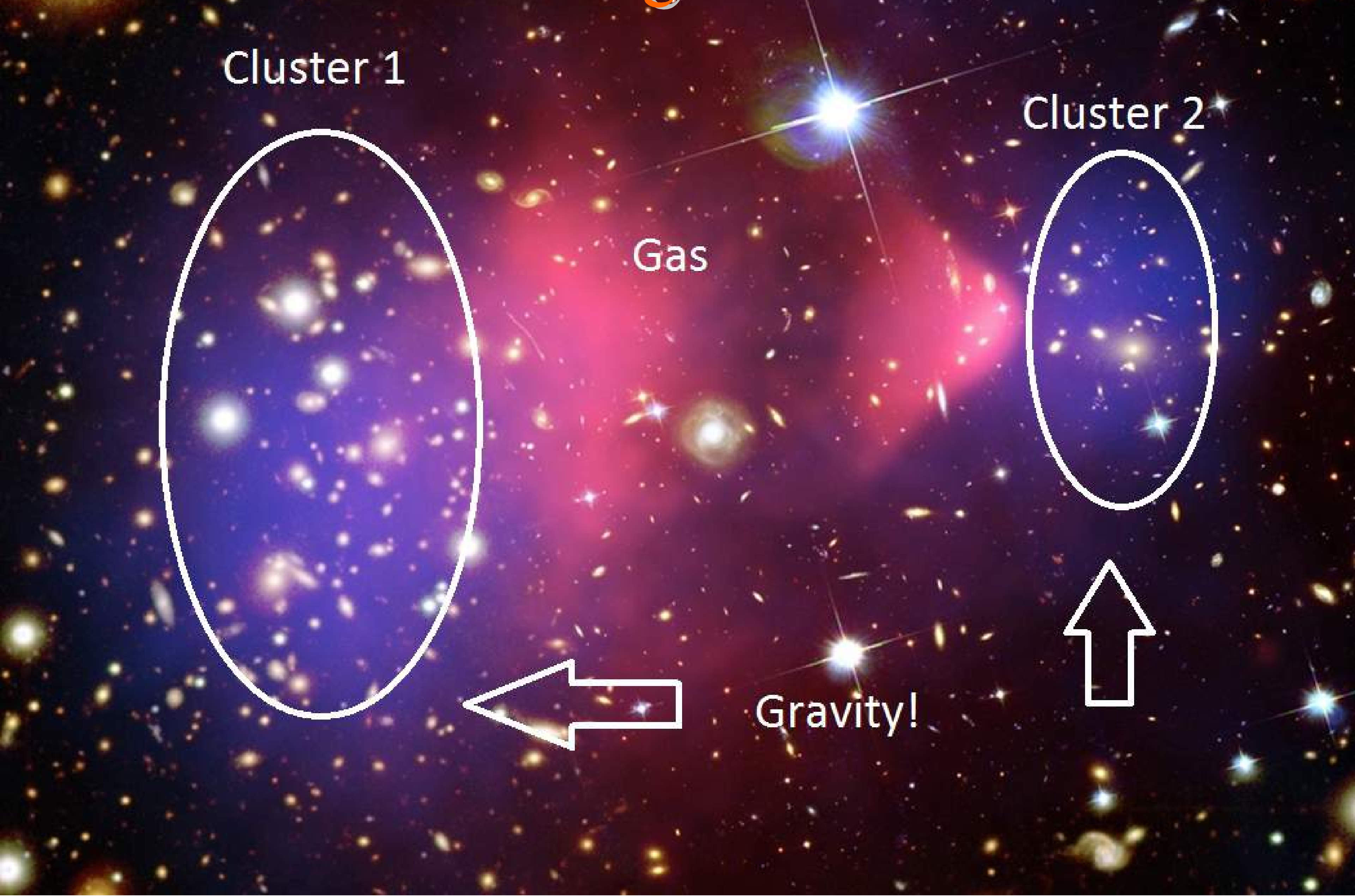
Gravitational lensing

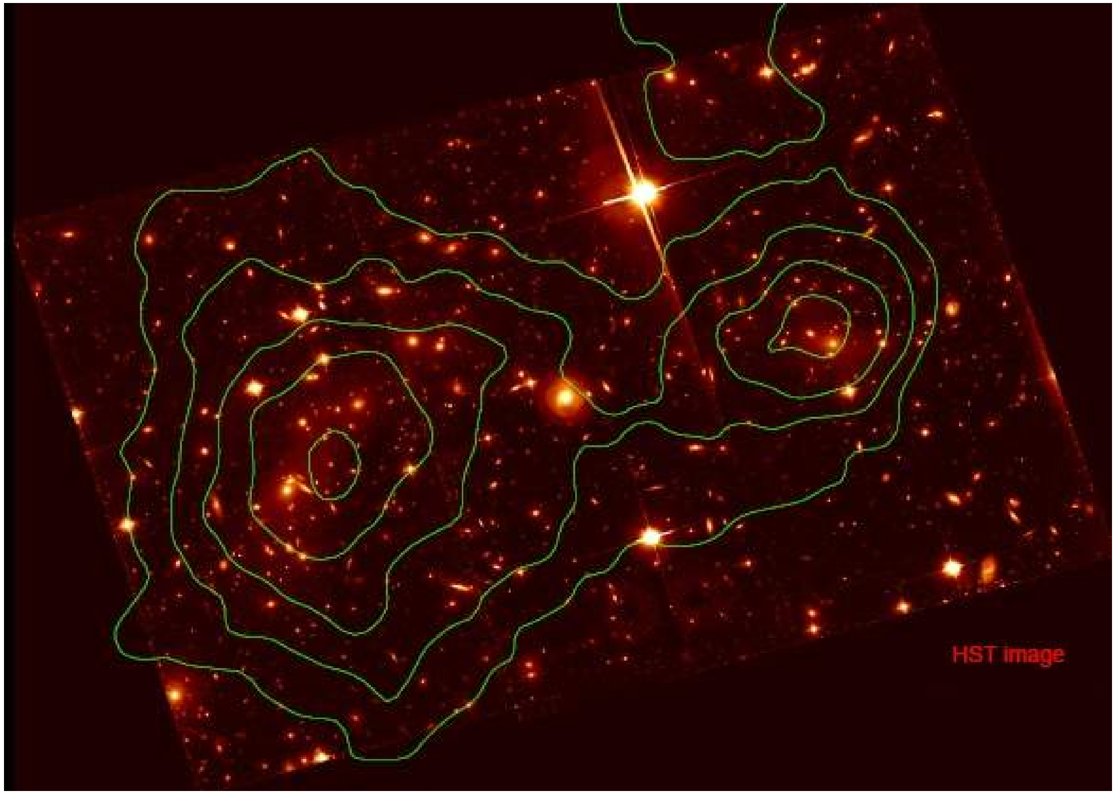


Gravitational lensing and the Bullet Cluster



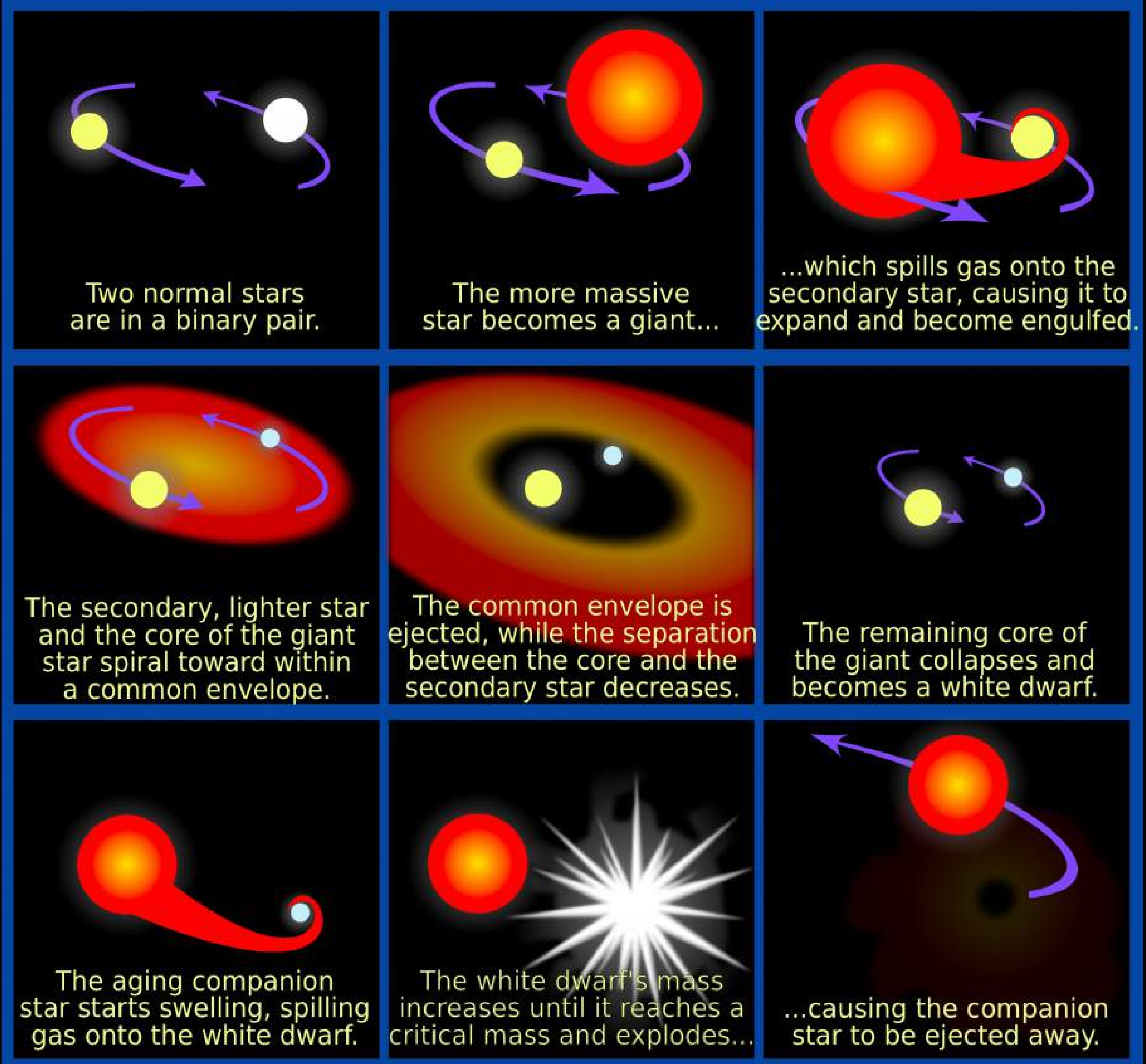
Gravitational lensing and the Bullet Cluster

Weak lensing can measure the masses of the clusters and the dark matter distribution



The Bullet Cluster with the total mass contours

The progenitor of a Type Ia supernova



Two normal stars are in a binary pair.

The more massive star becomes a giant...

...which spills gas onto the secondary star, causing it to expand and become engulfed.

The secondary, lighter star and the core of the giant star spiral toward within a common envelope.

The common envelope is ejected, while the separation between the core and the secondary star decreases.

The remaining core of the giant collapses and becomes a white dwarf.

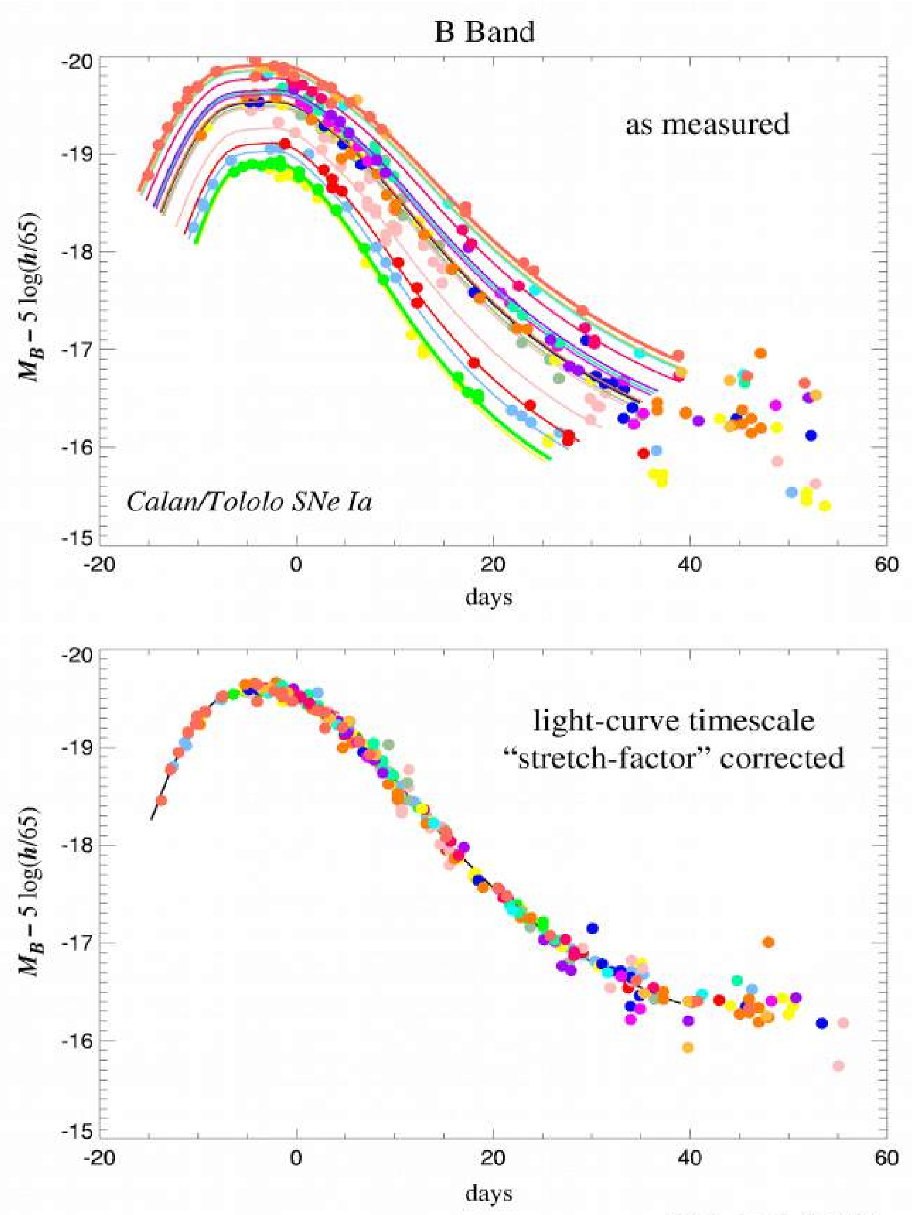
The aging companion star starts swelling, spilling gas onto the white dwarf.

The white dwarf's mass increases until it reaches a critical mass and explodes...

...causing the companion star to be ejected away.

NASA.gov

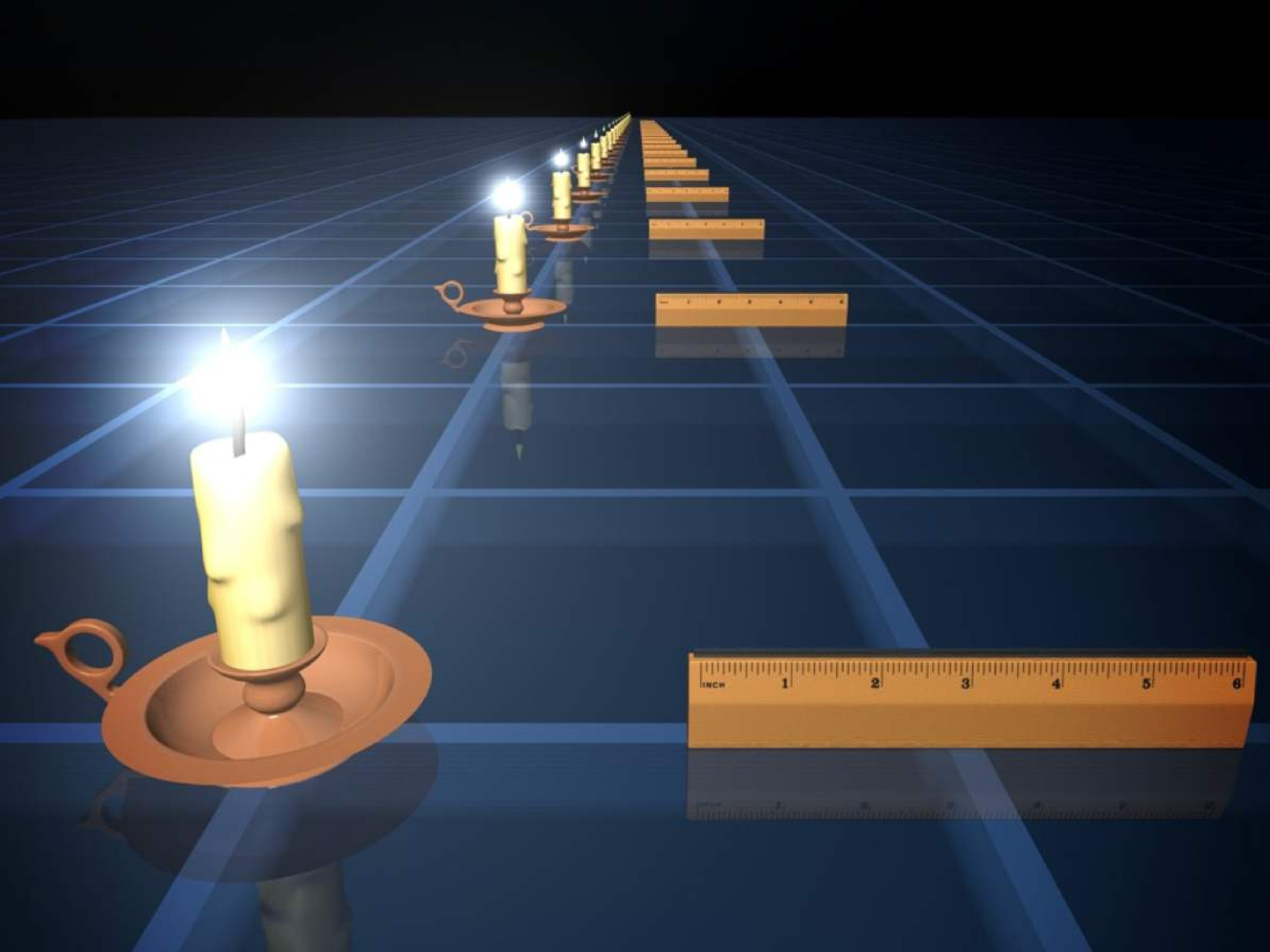
Type Ia Supernovae as standard candles



Kim, *et al.* (1997)

The lightcurves before...

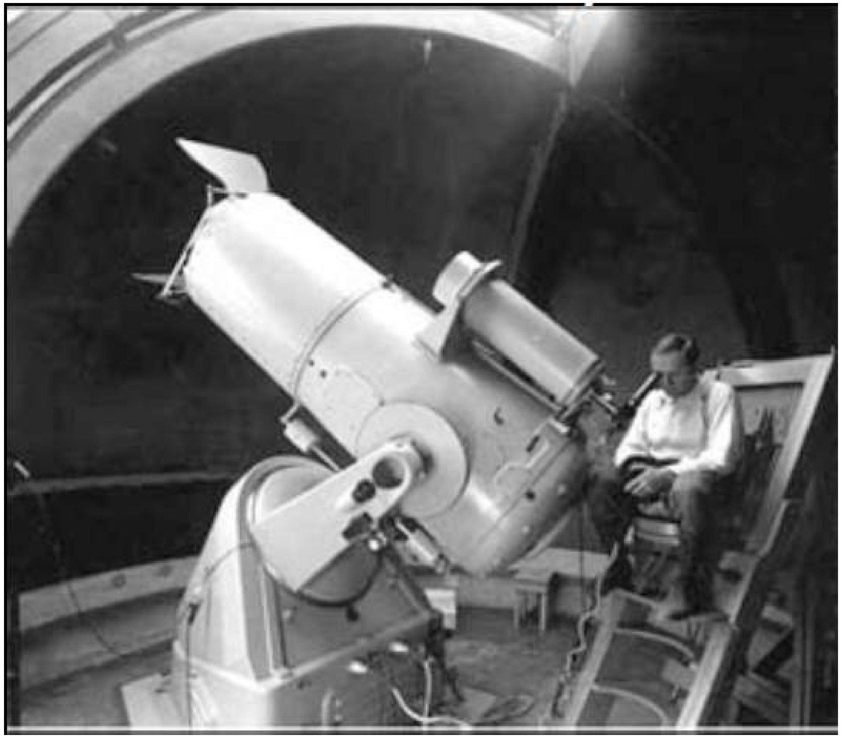
And after the corrections...


INCH
1
2
3
4
5
6

Type Ia Supernovae as standard candles

Fritz Zwicky (1934-35):

Charles Kowal (1968)



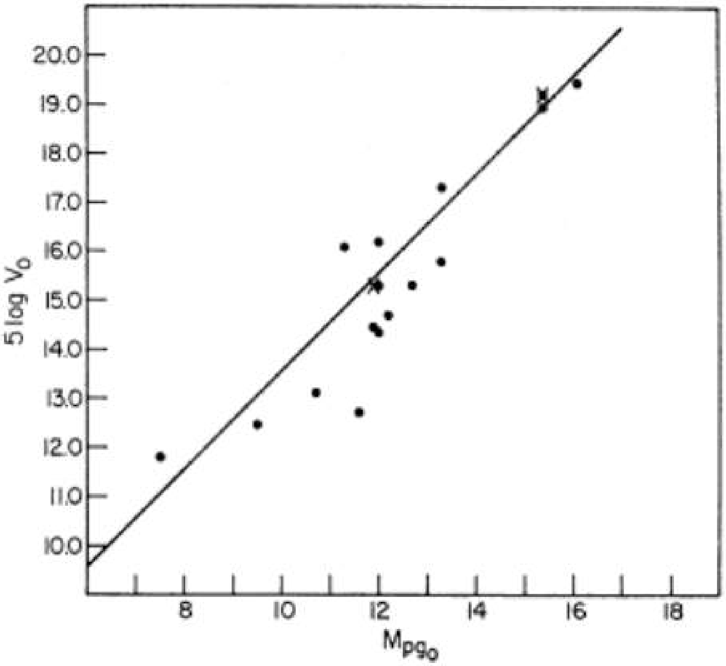


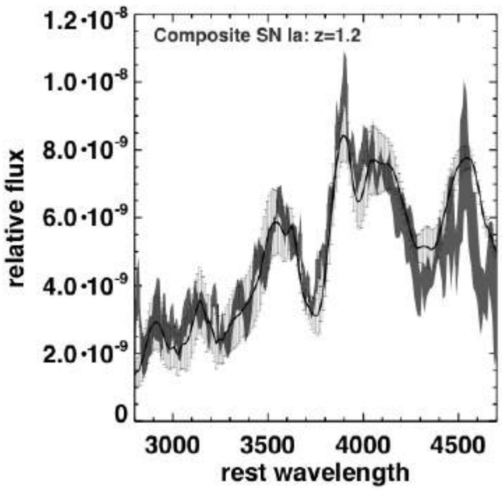
FIG. 1. The redshift–magnitude relation for supernovae of type I. The dots refer to individual supernovae, and the crosses represent averages for the Virgo and Coma clusters, as explained in the text.

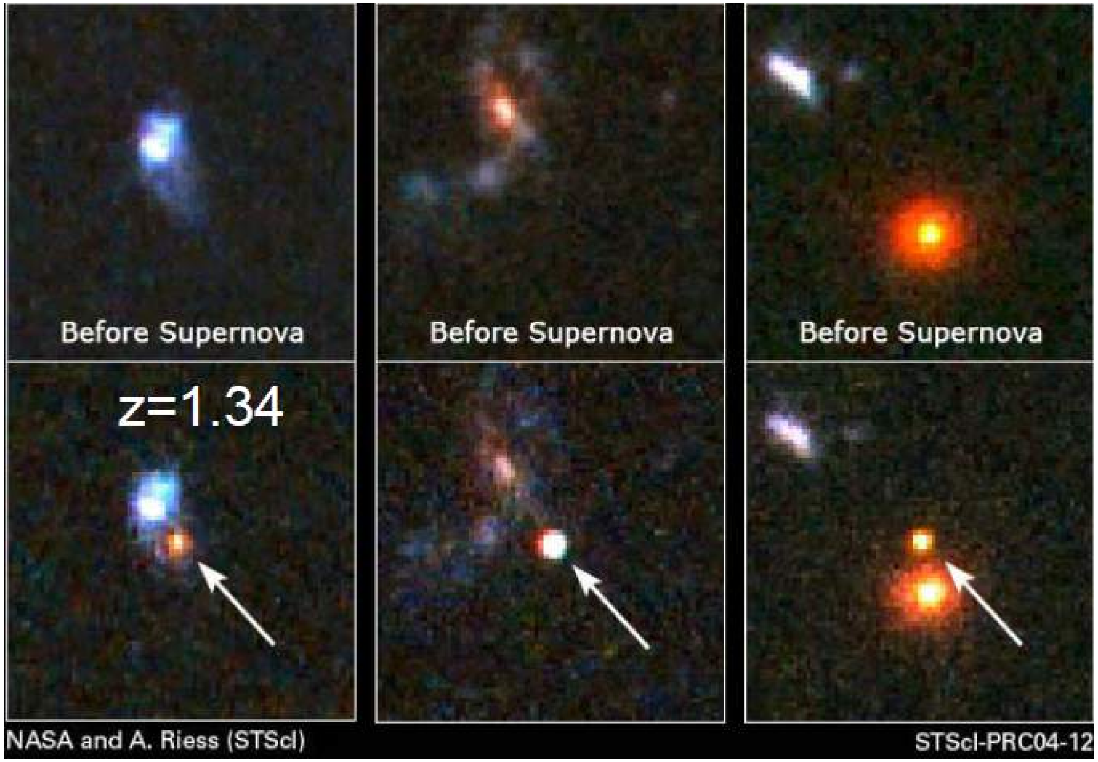
The first use of supernovae for measuring distances!

Type Ia Supernovae as standard candles

Hubble telescope:

2002-07, 23 new SnIa at z>1, eg





First measurements at z>1!

Some mathematical details about the SnIa

- The SnIa data are given in term of the dist. modulus:

$$\mu_{obs}(z_i) \equiv m_{obs}(z_i) - M$$

- Dark Energy can be described via w(z)

$$\boxed{w(z) \equiv \frac{P}{\rho}}$$

$$w(z) = -1 + \frac{1}{3}(1+z)\frac{d\ln(\delta H(z)^2)}{d\ln z}$$

$$\delta H(z)^2 = H(z)^2/H_0^2 - \Omega_{0\mathrm{m}}(1+z)^3$$

- Theoretical prediction

(flat universe)

$$D_L(z) = (1+z)\int_0^z dz' \frac{H_0}{H(z';\Omega_{0\mathrm{m}}, w_0, w_1)}$$

$$\mu_{th}(z_i) \equiv m_{th}(z_i) - M = 5log_{10}(D_L(z)) + \mu_0$$

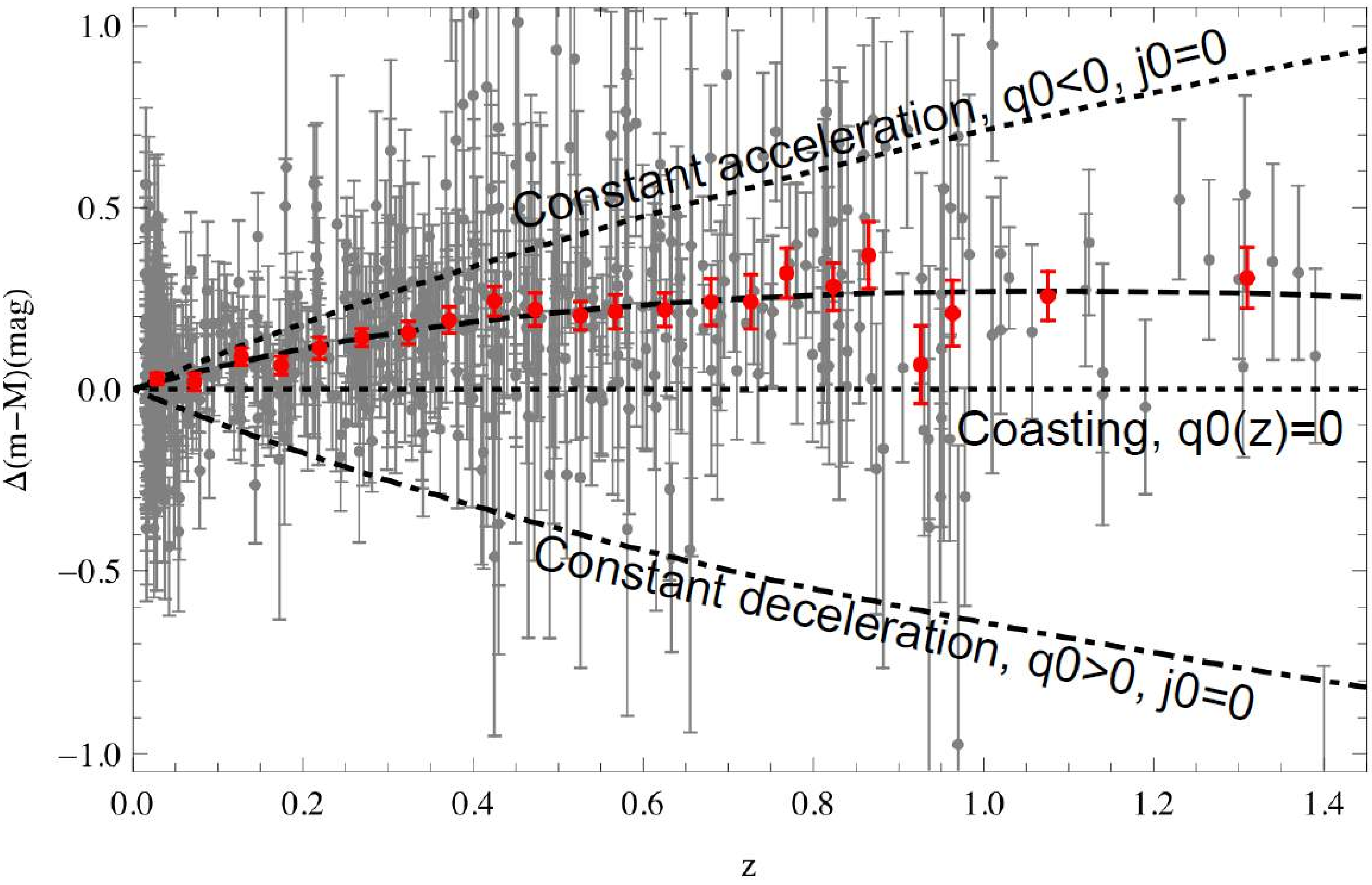
$$\mu_0 = 42.38 - 5log_{10}h$$

- Minimization:

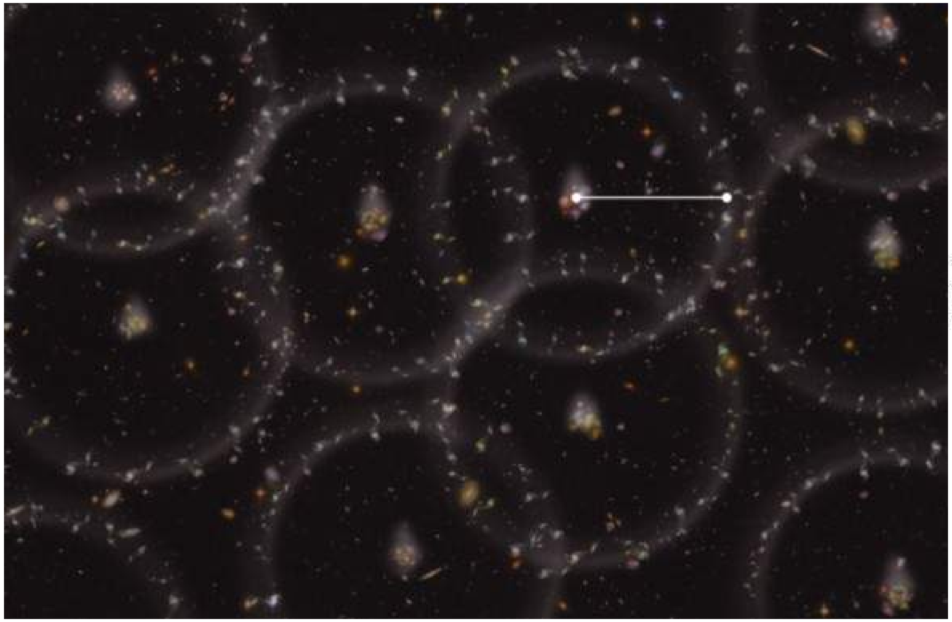
$$\chi^2_{SnIa}(\Omega_{0\mathrm{m}}, w_0, w_1) = \sum_{i=1}^{N} \frac{(\mu_{obs}(z_i) - \mu_{th}(z_i))^2}{\sigma^2_{\mu\ i}}$$

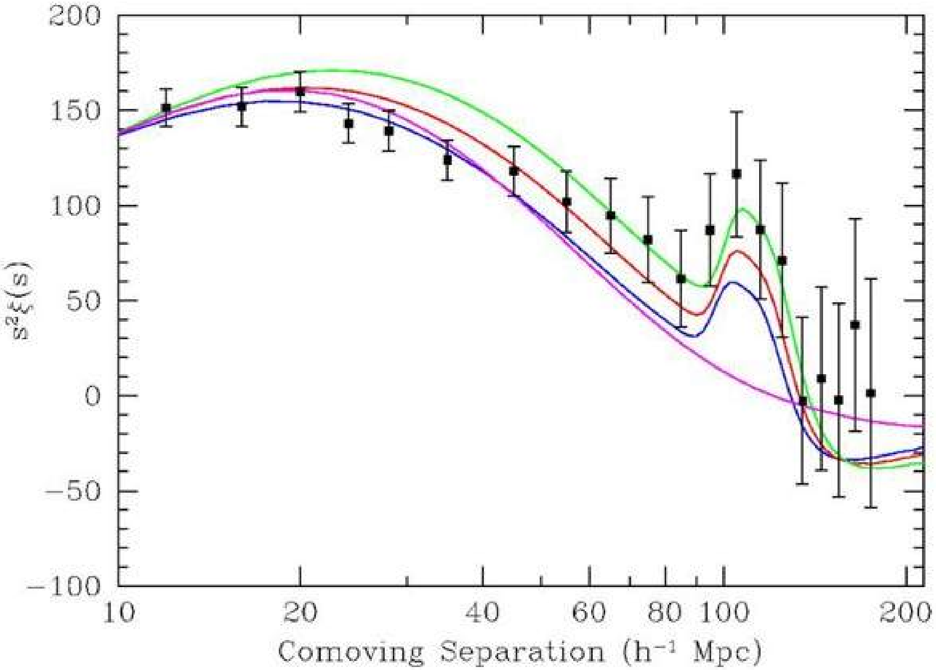
Union-2 SNe

Amanullah et al. (2010)



The Baryon Acoustic Oscillations (BAO)





1) Created by the baryons falling in and out of the potential wells (due to the photons' pressure).

2) They happen at scales where galaxies are correlated.

3) These scales are known and can be used to measure the expansion history of the Universe.

Some mathematical details about the BAO...

Probability to find two galaxies in positions 1 and 2 if they are uniformly distributed:

$$\Delta P_{\rm uniform} = \frac{\Delta V_1}{V} \times \frac{\Delta V_2}{V}$$

And if they are clustering:

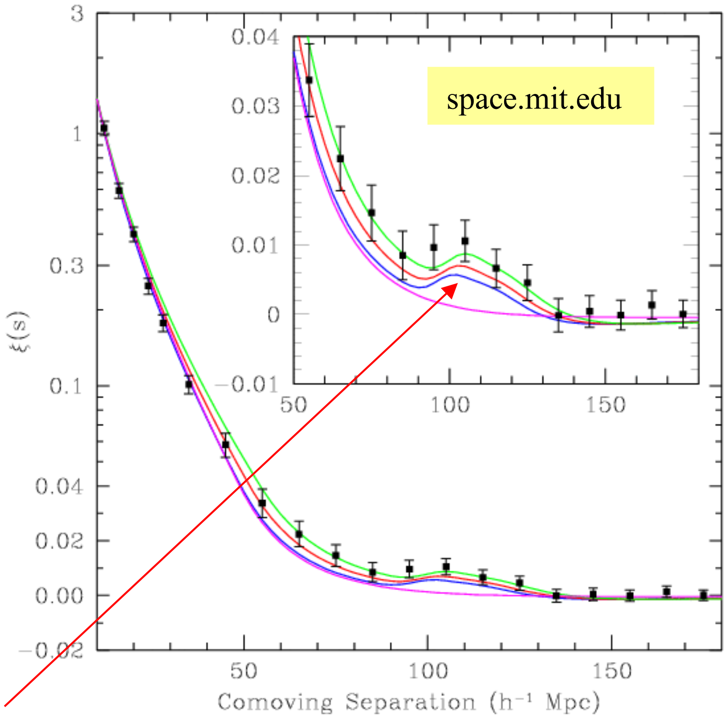
$$\Delta P = [1 + \xi_g(r)]\,\frac{\Delta V_1}{V}\frac{\Delta V_2}{V}$$

$$\xi(\vec{r}) \equiv \langle \delta(\vec{x})\delta(\vec{x}+\vec{r})\rangle \qquad \delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \langle\rho\rangle}{\langle\rho\rangle}$$

$$\Downarrow$$

$$\xi(r) = \frac{1}{(2\pi)^3}\int P(k)\frac{\sin(kr)}{kr}\,4\pi k^2 dk$$

$$P(k) \equiv \langle|\delta_k|^2\rangle$$



space.mit.edu

BAO!

Correlation function:

Related to the probability to find a galaxy at r.

Some mathematical details about the BAO...

Matter power spectrum P(k)

$$P(k) \equiv \langle |\delta_k|^2 \rangle$$

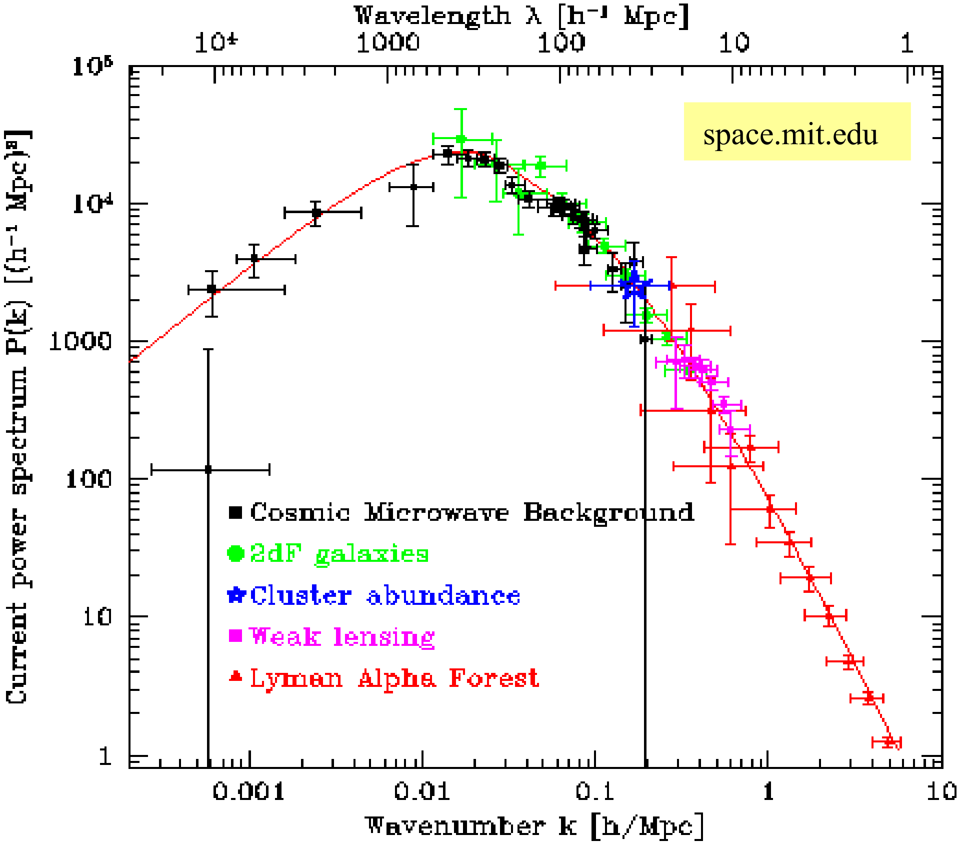
CAMB

If:

$$\xi_g(r) = \left(\frac{r}{r_0}\right)^{-(3+n)}$$

Then:

$$P_g(k) \propto k^n$$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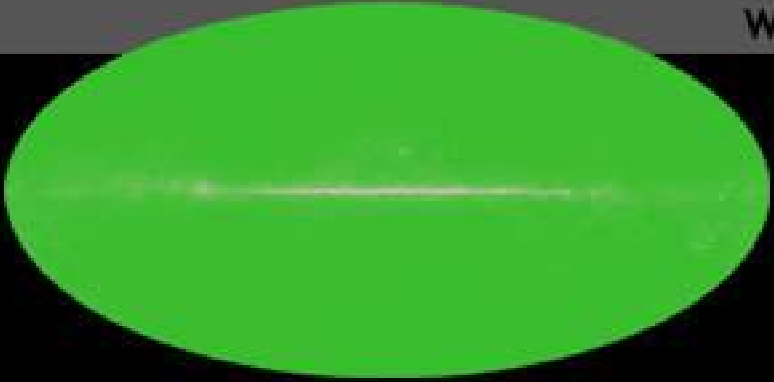
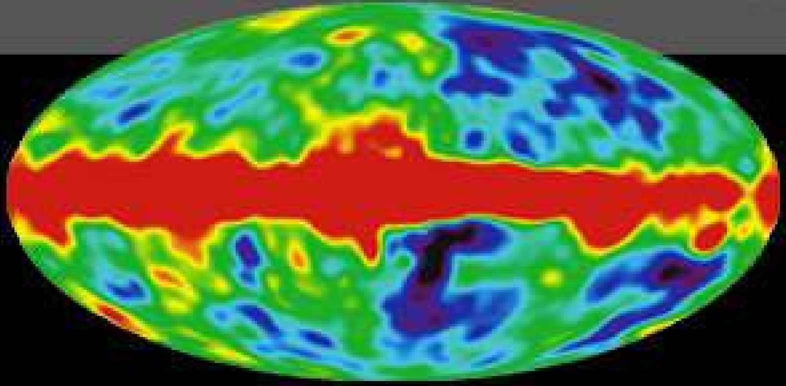
1965

Penzias and Wilson



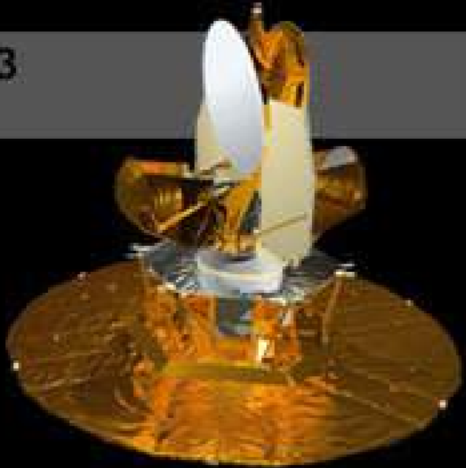
1992

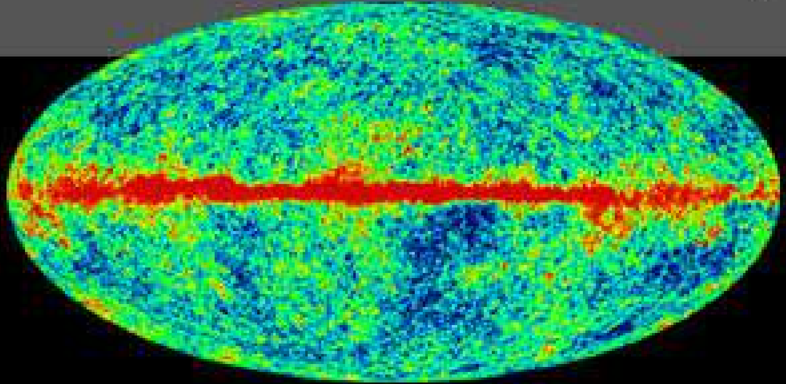
COBE

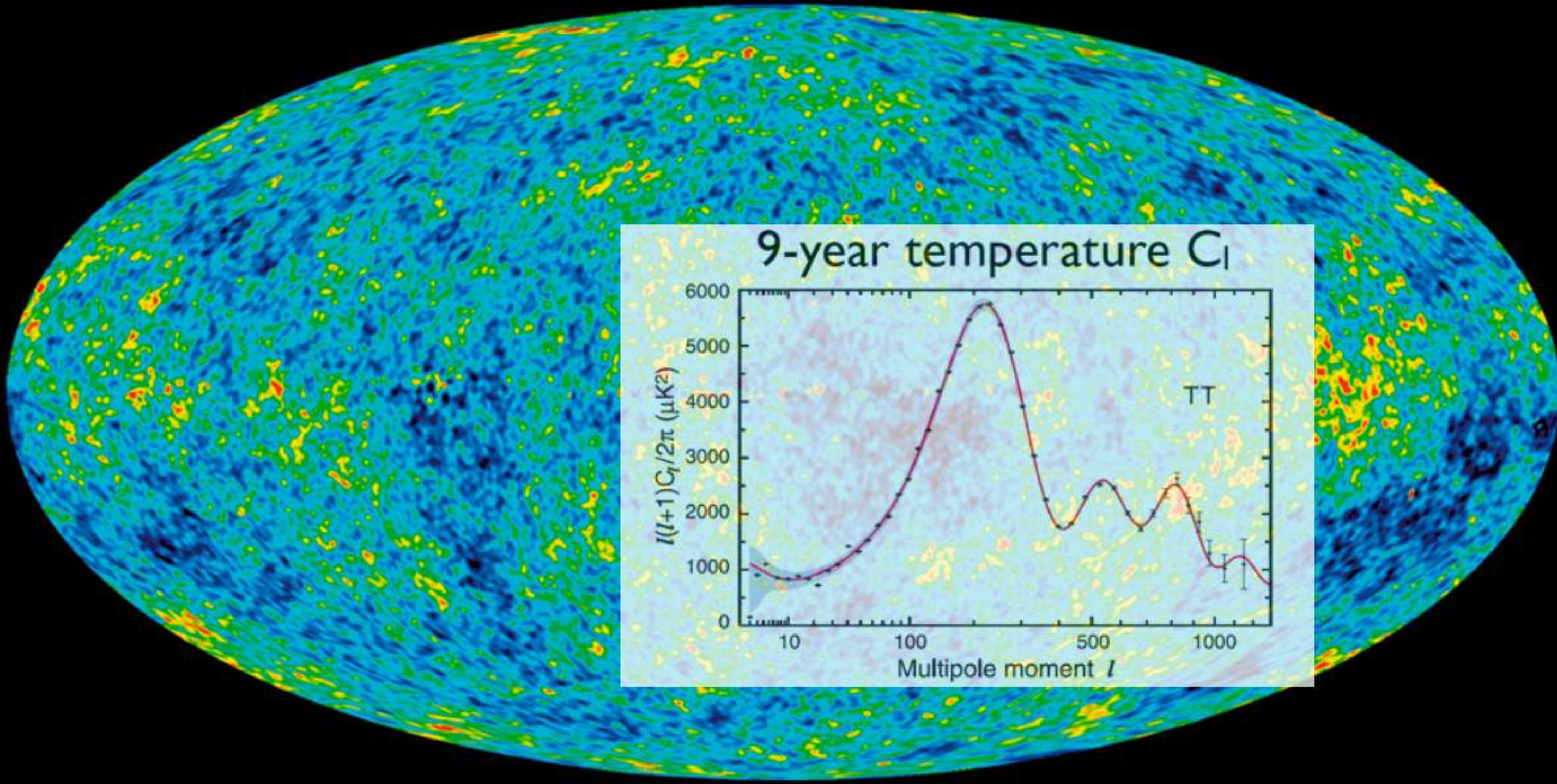


2003

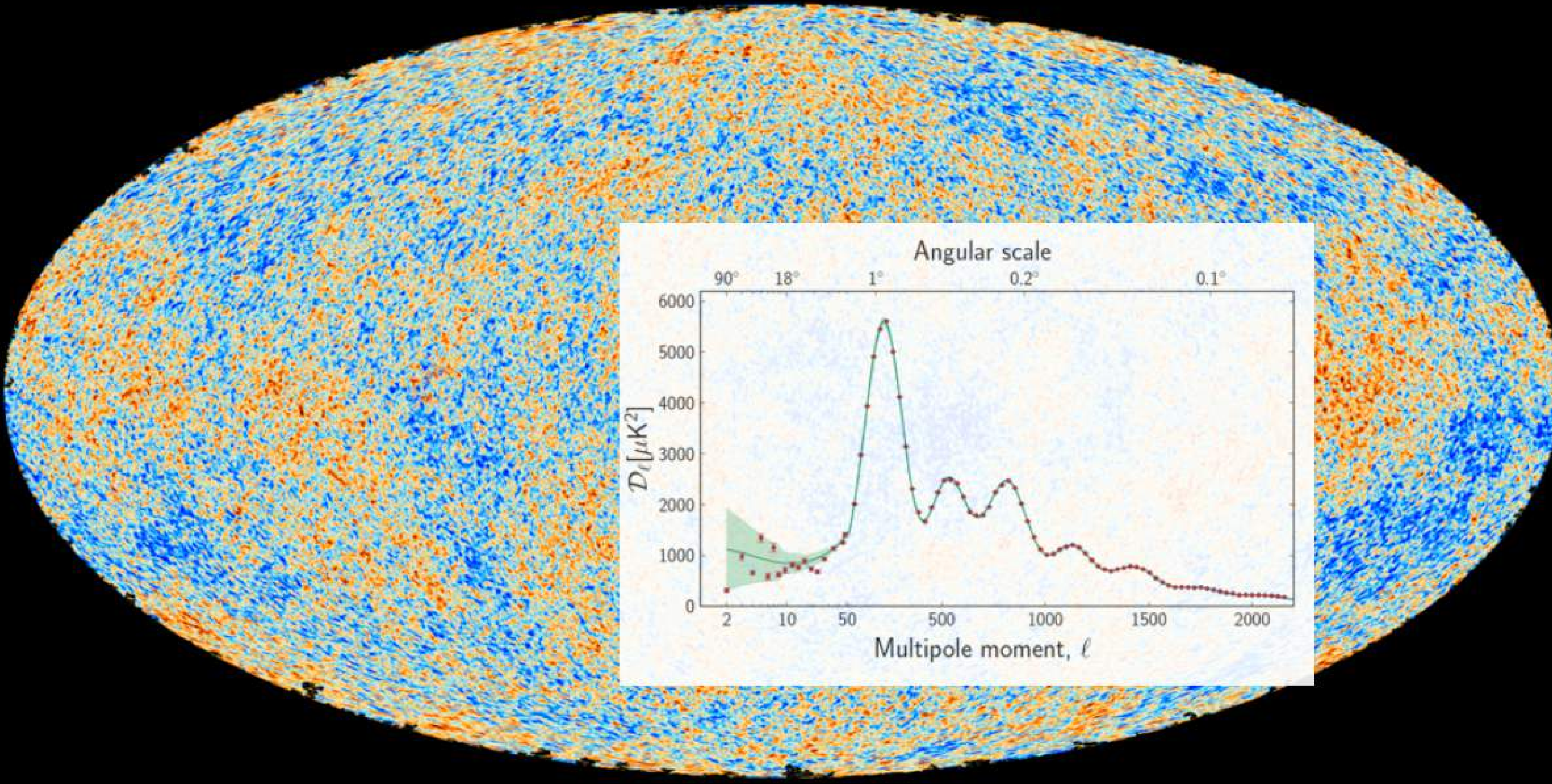
WMAP







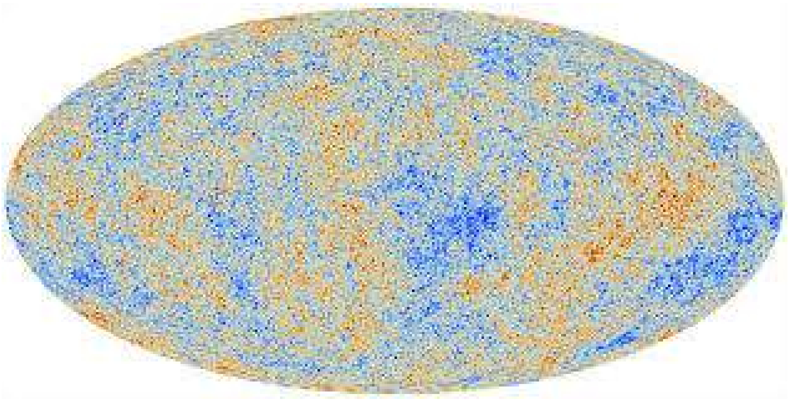
9-year temperature C_l


Angular scale
90°
18°
1°
0.2°
0.1°
6000
5000
4000
3000
2000
1000
0
$\mathcal{D}_\ell [\mu K^2]$
2
10
50
500
1000
1500
2000
Multipole moment, ℓ

Some mathematical details about the CMB...

A CMB map:

$$T(\vec{x},\hat{p},\eta) = T(\eta)\left[1+\Theta(\vec{x},\hat{p},\eta)\right]$$

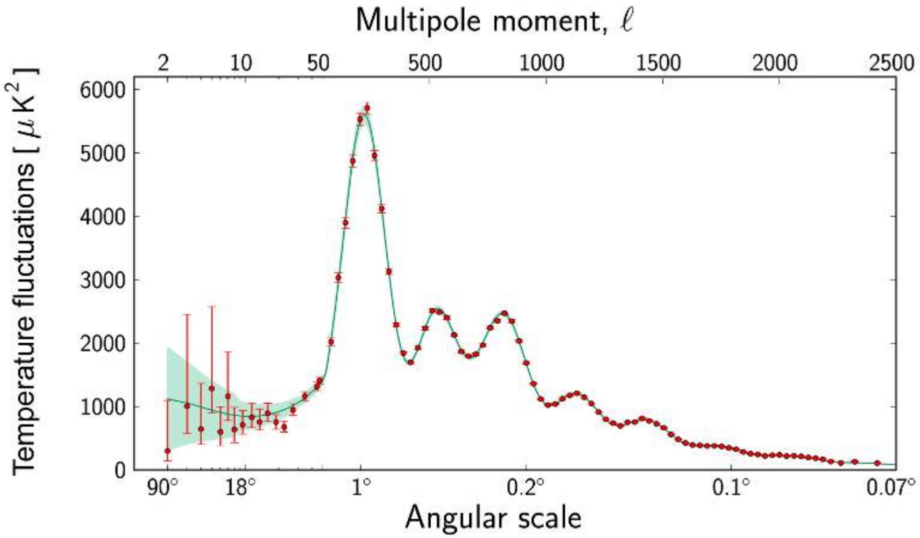


Expand in spherical harmonics:

Healpix

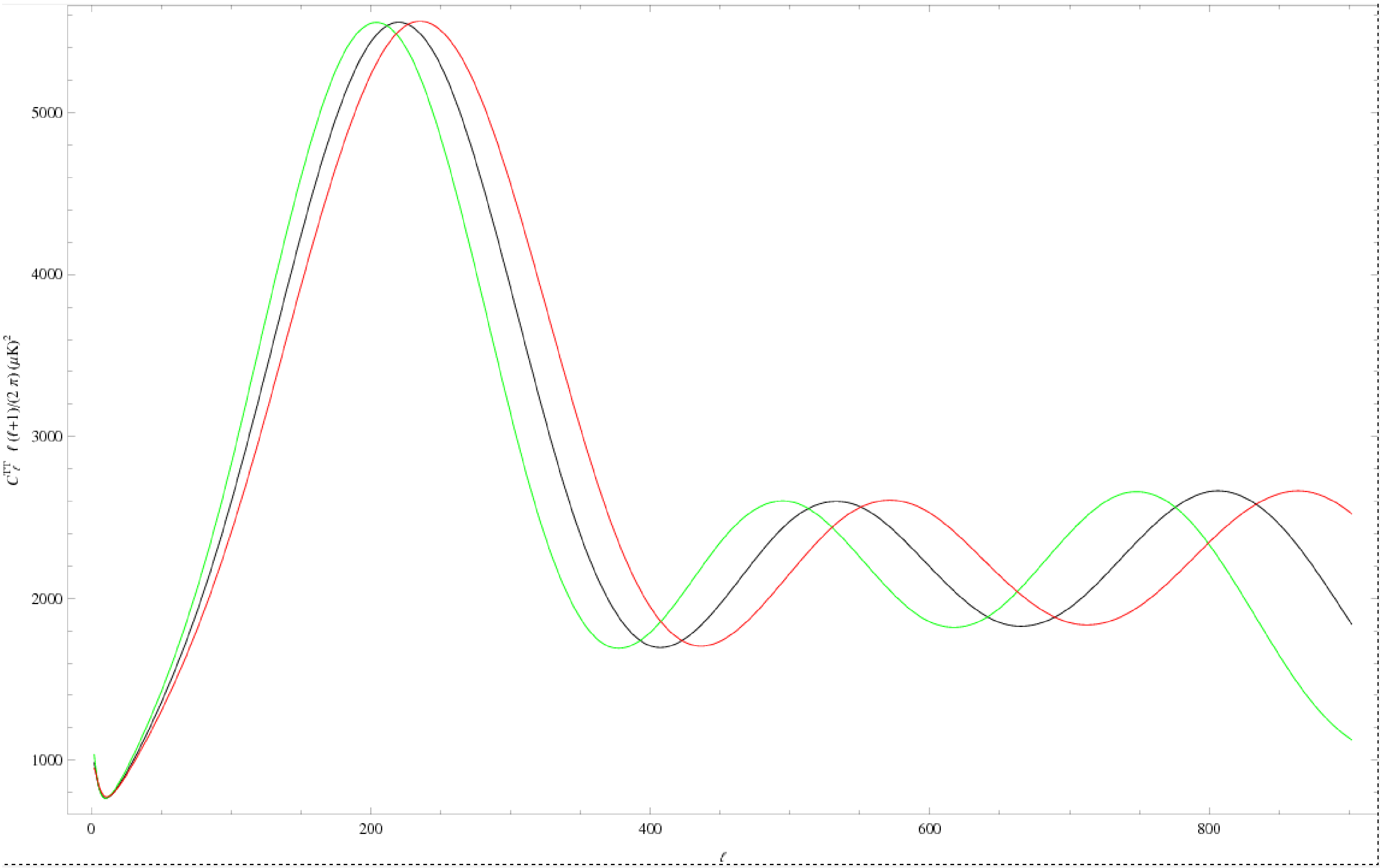
$$\Theta(\vec{x},\hat{p},\eta) = \sum_{l=1}^{\infty}\sum_{m=-l}^{l} a_{lm}(\vec{x},\eta)Y_{lm}(\hat{p})$$

$$\langle a_{lm}\rangle = 0 \qquad ; \qquad \langle a_{lm}a^{*}_{l'm'}\rangle = \delta_{ll'}\delta_{mm'}C_l$$



The physics of the CMB and the cosmological parameters

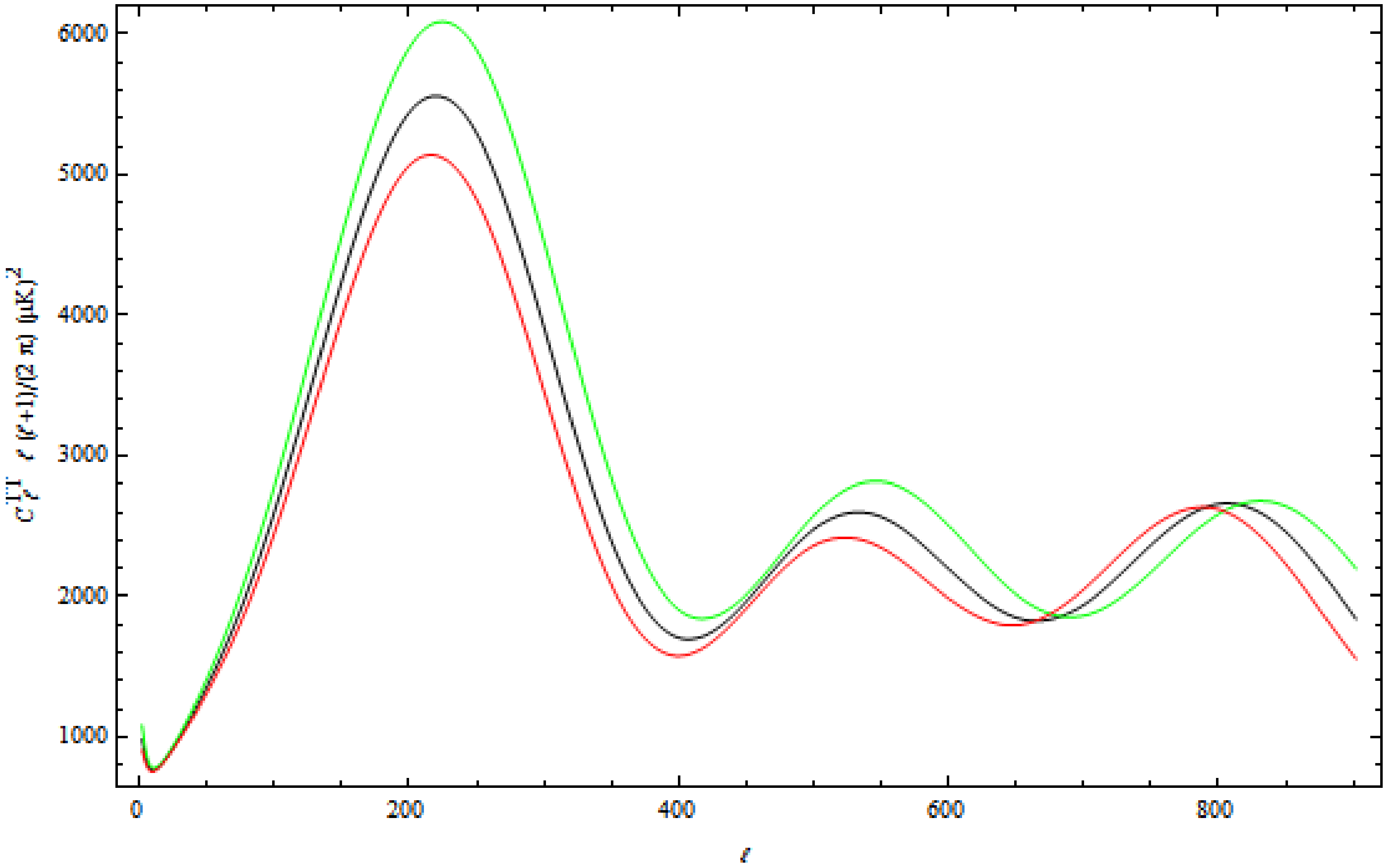
Ωk=[-0.05, 0, 0.05]



The physics of the CMB and the cosmological parameters

Ωm=[0.2038, 0.2538, 0.3038] and H0=70

Ωmh^2=[0.099862, 0.124362, 0.148862]



A phenomenological approach to Dark Matter's properties

- What is the nature of Dark Matter (DM) and what are its properties?
- What is the behaviour of the DM perturbations?
- How can we use the aforementioned data?
- How compatible is the Standard Cosmological Model with the data?

The Dark Matter perturbations and their "sound speed"

M. Kunz, S.N., I. Sawicki, arXiv: 1507.01486

Energy-momentum tensor
for perfect fluid:

$$T^\mu_\nu = P g^\mu_\nu + (\rho + P) U^\mu U_\nu$$

Equation of state w

$$P = w\,\rho$$

- $w = 0$ Non-relativistic matter
- $w = \frac{1}{3}$ Radiation

Sound of perturbations
cs^2:

$$\delta P = c_s^2 \delta\rho$$

- $c_s^2 = 1$ Quintessence
- $c_s^2 = 0$ "Usual" Dark Matter

The Dark Matter perturbations and their “sound speed”

M. Kunz, S.N., I. Sawicki, arXiv: 1507.01486

If $0 < c_s^2 \leq 1$ for DM, then this affects large structure in the Universe!

Reason: There's a sound horizon at scales:

$$k_J(z) \equiv \frac{H(z)}{(1+z)c_s}$$

Two cases:

- Outside the horizon => cs^2 is irrelevant.

  $k \ll k_J$

- Inside the horizon => cs^2 “behave like pressure” and try to erase the structures.

  $k \gg k_J$

The Dark Matter perturbations and their "sound speed"

M. Kunz, S.N., I. Sawicki, arXiv: 1507.01486

Methodology:

Split DM in two parts

$c_s^2 = 0$ Usual DM

$0 < c_s^2 \leq 1$ DM with a cs^2

But keep the background fixed to ΛCDM

(dark degeneracy)

$$\Omega_X(a) + \Omega_c(a) = \Omega_\Lambda(a) + \Omega_c^{\text{Planck}}(a)$$

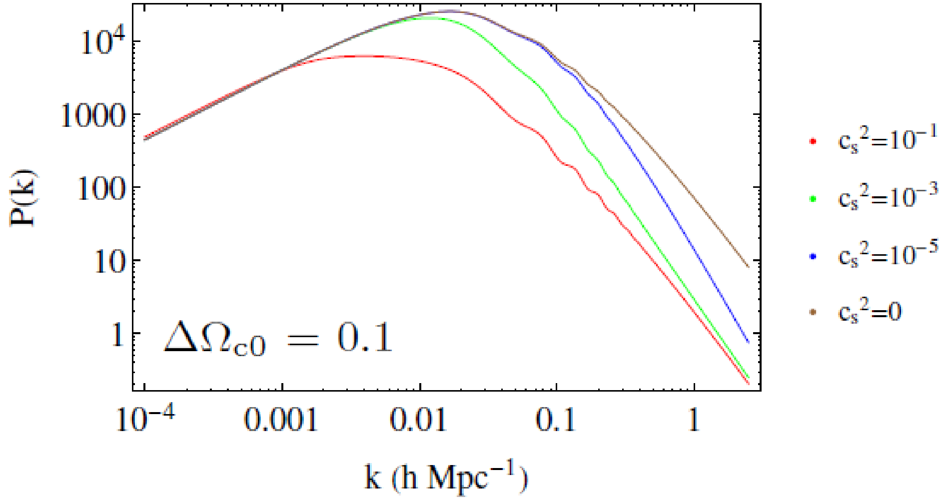
$$1 + w_X(a) = \frac{\Delta\Omega_{c0}}{\Delta\Omega_{c0} + \Omega_{\Lambda 0} a^3}$$

$$\Delta\Omega_c \equiv \Omega_c^{\text{Planck}} - \Omega_c$$

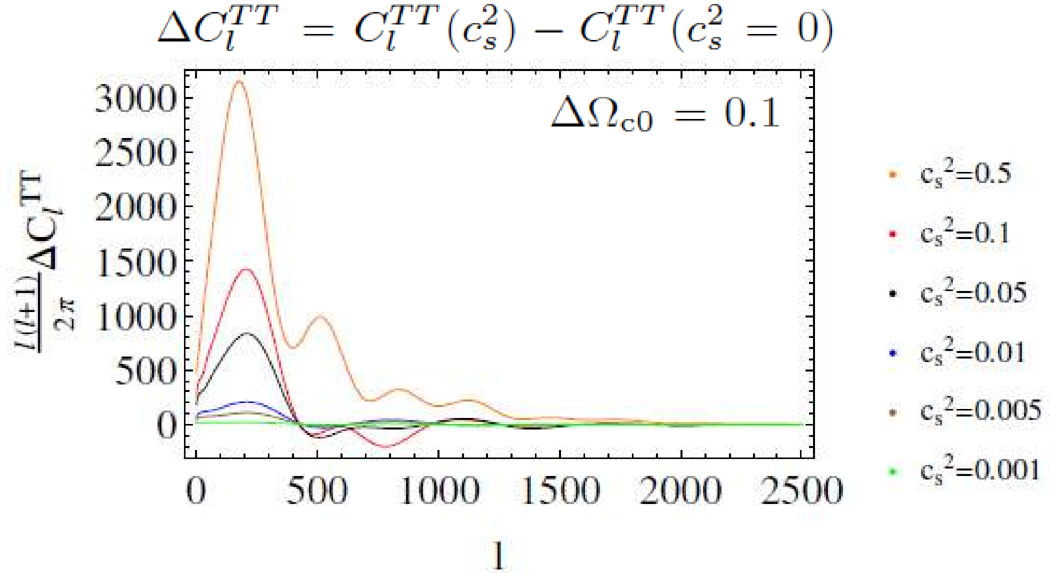
The Dark Matter perturbations and their “sound speed”

M. Kunz, S.N., I. Sawicki, arXiv: 1507.01486

It affects the power spectrum P(k):

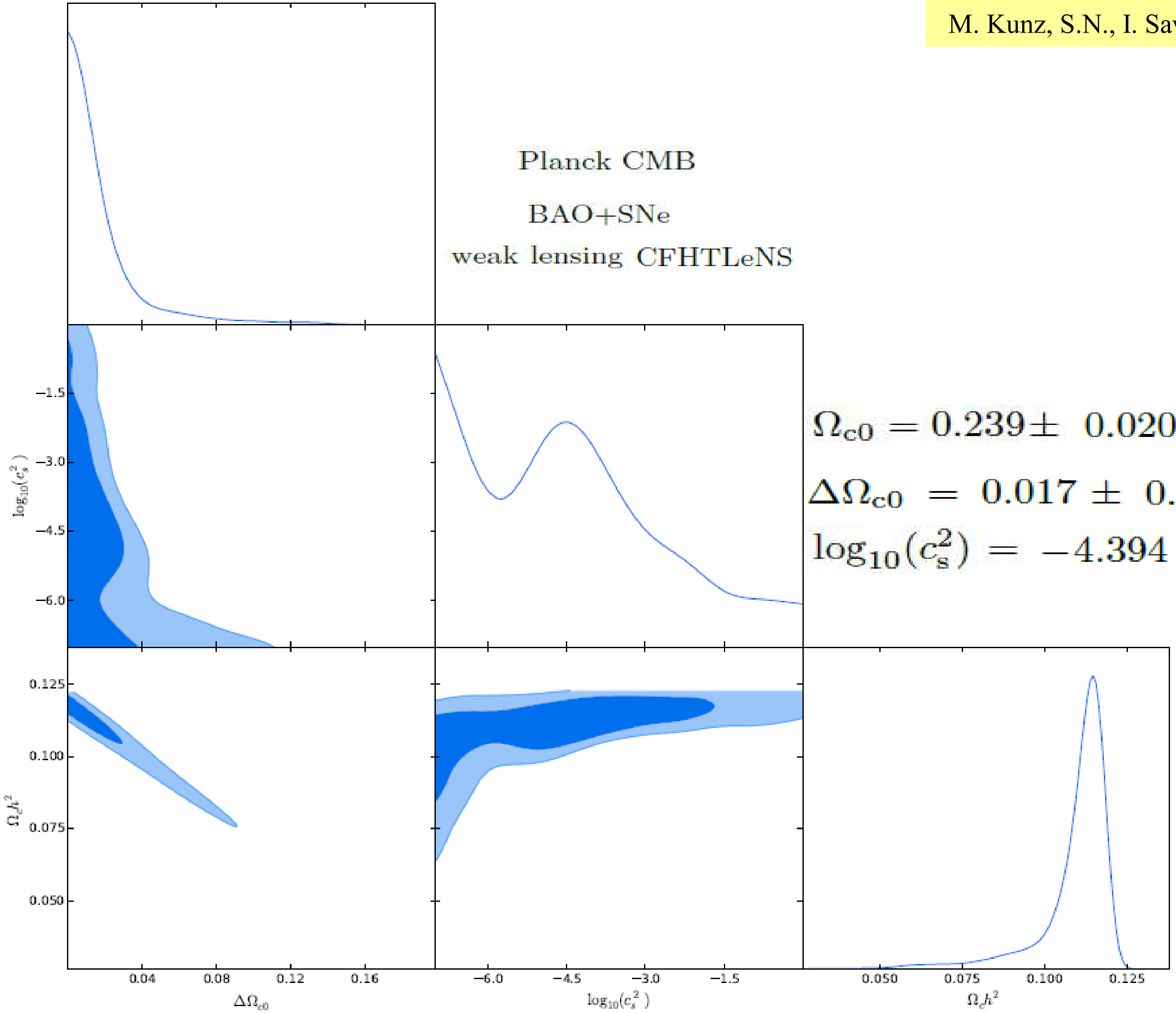


It also affects the CMB:


$$\Delta C_l^{TT} = C_l^{TT}(c_s^2) - C_l^{TT}(c_s^2 = 0)$$

The Dark Matter perturbations and their "sound speed"

M. Kunz, S.N., I. Sawicki, arXiv: 1507.01486



$$\Omega_{c0} = 0.239 \pm\ 0.020$$

$$\Delta\Omega_{c0} = 0.017 \pm 0.019$$

$$\log_{10}(c_s^2) = -4.394 \pm 1.779$$

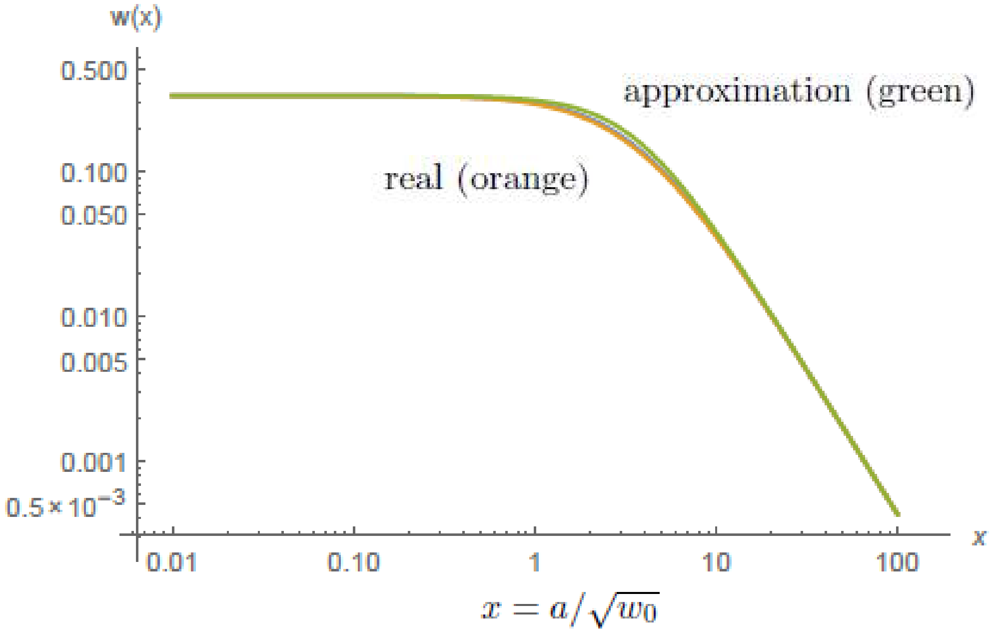
The Dark Matter equation of state w

M. Kunz, S.N., I. Sawicki, arXiv: 1604.05701

Now, replace the usual DM with the new one, which now also has an EoS w

- w = constant — Can be small and constant (CDM)
- $w(a) = \frac{1}{3+a^2/w_0^2}$ — DM can be relativistic (w~1/3) at early times (a<<1, z>>1)

For x~m/T we have:
When x<<1 -> w = 1/3,
while for x>>1 we have
w~1/a^2.



The Dark Matter equation of state w

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The Boltzmann Hierarchy
(see Ma & Bertschinger)
(+f)...(-b)

$$f(\boldsymbol{k}, \hat{\boldsymbol{n}}, q, t) = f_0(q,t)\left(1 + \Psi(\boldsymbol{k}, \hat{\boldsymbol{n}}, q, t)\right)$$

$$f_0(p,t) = \frac{g}{(2\pi)^3}\left[e^{\varepsilon(p,t)/aT} \pm 1\right]^{-1}$$

$$\partial_t \Psi + i\frac{q}{\varepsilon(q,t)}(k\hat{n})\Psi + \frac{d\ln f_0(q)}{d\ln q}\left[\dot{\phi} - i\frac{q}{\varepsilon(q,t)}(k\hat{n})\psi\right] = 0$$

Background quantities:

$$\bar{n}(a)a^3 \propto \int dq q^2 f_0(q,a)\,,$$

$$\bar{\rho}(a)a^4 \propto \int dq q^2 \varepsilon(q,a) f_0(q,a)\,,$$

$$\bar{p}(a)a^4 \propto \int dq q^2 \frac{q^2}{3\varepsilon(q,a)} f_0(q,a)\,,$$

The Dark Matter equation of state w

M. Kunz, S.N., I. Sawicki, arXiv: 1604.05701

If relativistic $q \approx \varepsilon$ then $\bar{p} = \bar{\rho}/3$

If non-relativistic: $\varepsilon \approx am$

$$\bar{\rho}(a)a^3 \propto m \int dq q^2 f_0(q) = m\bar{n}(a)\,,$$

$$\bar{p}(a)a^5 \propto \int dq q^4 f_0(q)\,.$$

Equation of state:

$$w(a) = \frac{\bar{p}}{\bar{\rho}} \propto \frac{1}{a^2}$$

The Dark Matter equation of state w

M. Kunz, S.N., I. Sawicki, arXiv: 1604.05701

Pressure and density

Perturbations:

$$\delta\rho(k,a) = \frac{4\pi}{a^4}\int dq q^2 \varepsilon(q,a) f_0(q,a)\Psi_0(k,q,a)\,,$$

$$\delta p(k,a) = \frac{4\pi}{a^4}\int dq q^2 \frac{q^2}{3\varepsilon(q,a)} f_0(q,a)\Psi_0(k,q,a)$$

$\Psi_0(k,q,a)$ is weakly varying, see Komatsu arXiv:1003.0942

So, if non-relativistic: $\varepsilon \approx am$

$$c_s^2(a) \approx w \propto 1/a^2$$

Initial conditions for constant w for DM

M. Kunz, S.N., I. Sawicki, arXiv: 1604.05701

Some remain the same:

$$h = C(k\tau)^2\,,$$

$$\eta = 2C - \frac{5+4\Omega_\nu}{6(15+4\Omega_\nu)}C(k\tau)^2$$

$$\delta_\gamma = \delta_\nu = -\frac{2}{3}C(k\tau)^2 \qquad \sigma_\gamma = 0\,, \qquad \sigma_\nu = \frac{4}{3}\frac{C(k\tau)^2}{15+4\Omega_\nu}$$

$$\theta_\gamma = -\frac{C}{18}k^4\tau^3\,, \qquad \theta_\nu = -\frac{C}{18}\frac{23+4\Omega_\nu}{15+4\Omega_\nu}k^4\tau^3$$

Other change:

$$\delta_c = -\frac{(1+w)C(k\tau)^2}{2(4+3c_s^2-6w)}\times \\ \times\left((4-3c_s^2) - \frac{48}{15+4\Omega_\nu}\frac{c_{\rm vis}^2}{1+w}(c_s^2-w)\right),$$

$$\theta_c = -\frac{Ck^4\tau^3}{2(4+3c_s^2-6w)}\times \\ \times\left(c_s^2 + \frac{16}{3(15+4\Omega_\nu)}\frac{c_{\rm vis}^2}{1+w}(2+3c_s^2-3w)\right),$$

$$\sigma_c = \frac{16C(k\tau)^2}{3(15+4\Omega_\nu)}\frac{c_{\rm vis}^2}{1+w}\,,$$

Initial conditions for relativistic DM

M. Kunz, S.N., I. Sawicki, arXiv: 1604.05701

DM will behave like neutrinos: $\Omega_\nu \to \Omega_\nu + \Omega_c$

$$h = C(k\tau)^2\,,$$

$$\eta = 2C - \frac{5 + 4(\Omega_\nu + \Omega_c)}{6(15 + 4(\Omega_\nu + \Omega_c))}C(k\tau)^2$$

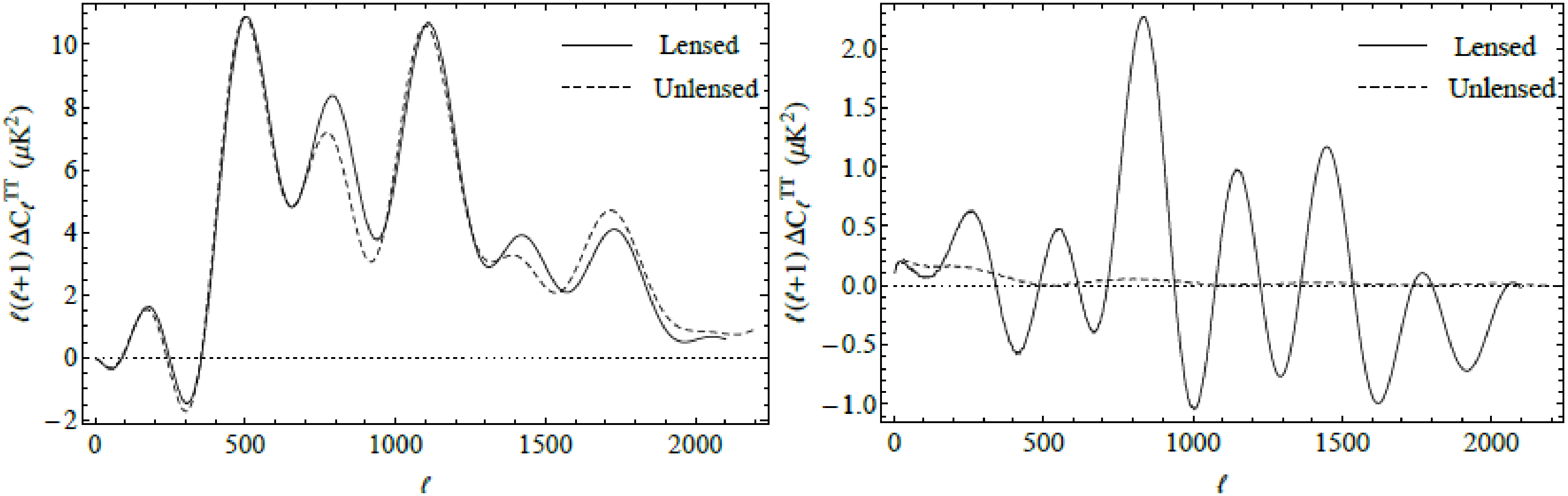
$$\delta_c = \delta_\gamma = \delta_\nu = -\frac{2}{3}C(k\tau)^2$$

$$\theta_c = \theta_\nu = -\frac{C}{18}\frac{23 + 4(\Omega_\nu + \Omega_c)}{15 + 4(\Omega_\nu + \Omega_c)}k^4\tau^3$$

$$\sigma_c = \sigma_\nu = \frac{4}{3}\frac{C(k\tau)^2}{15 + 4(\Omega_\nu + \Omega_c)}\,.$$

Difference between constant and evolving w for DM

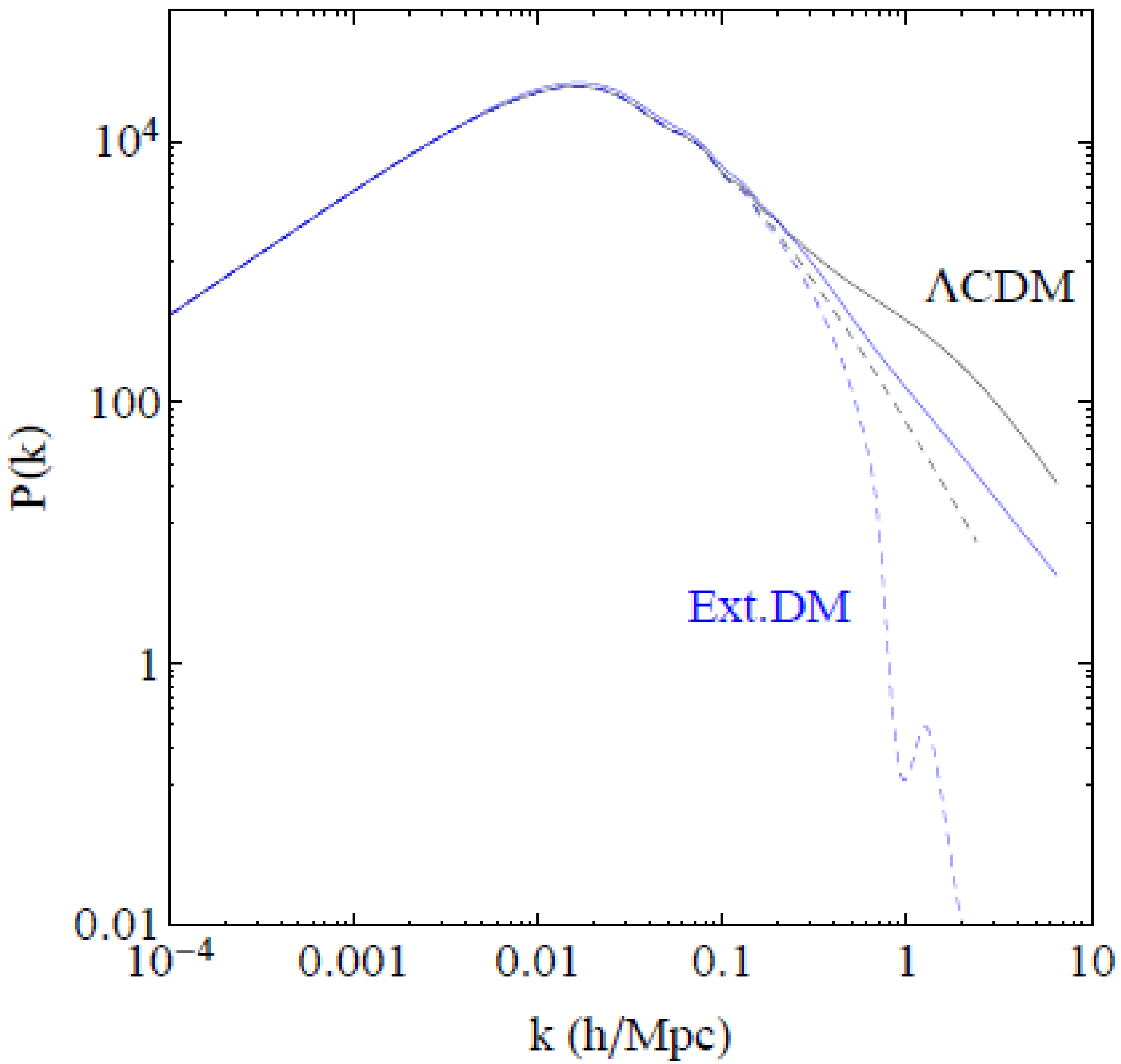
M. Kunz, S.N., I. Sawicki, arXiv: 1604.05701



initially relativistic (left) and constant (right)

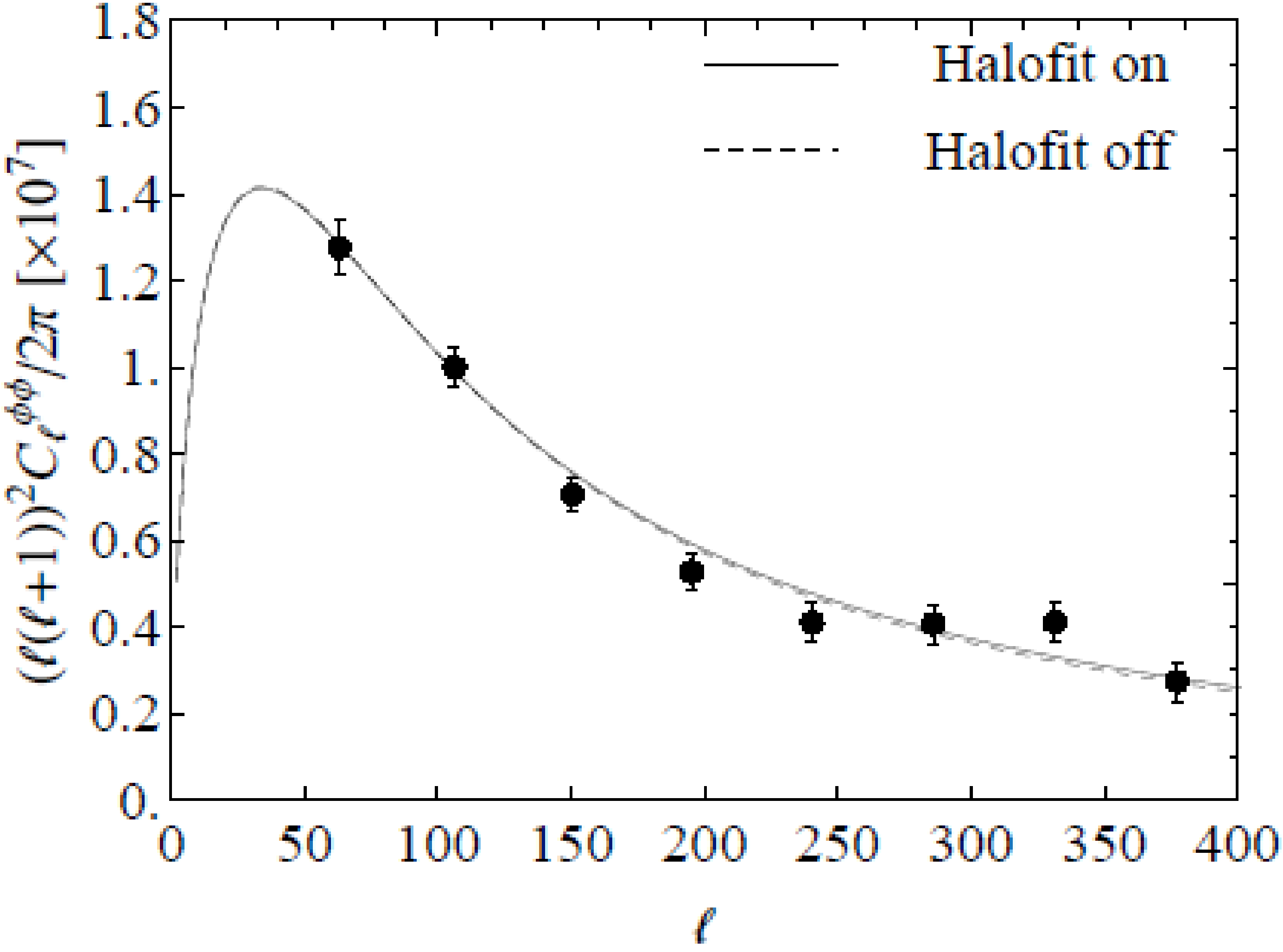
Some problems with Halofit

M. Kunz, S.N., I. Sawicki, arXiv: 1604.05701



Some problems with Halofit

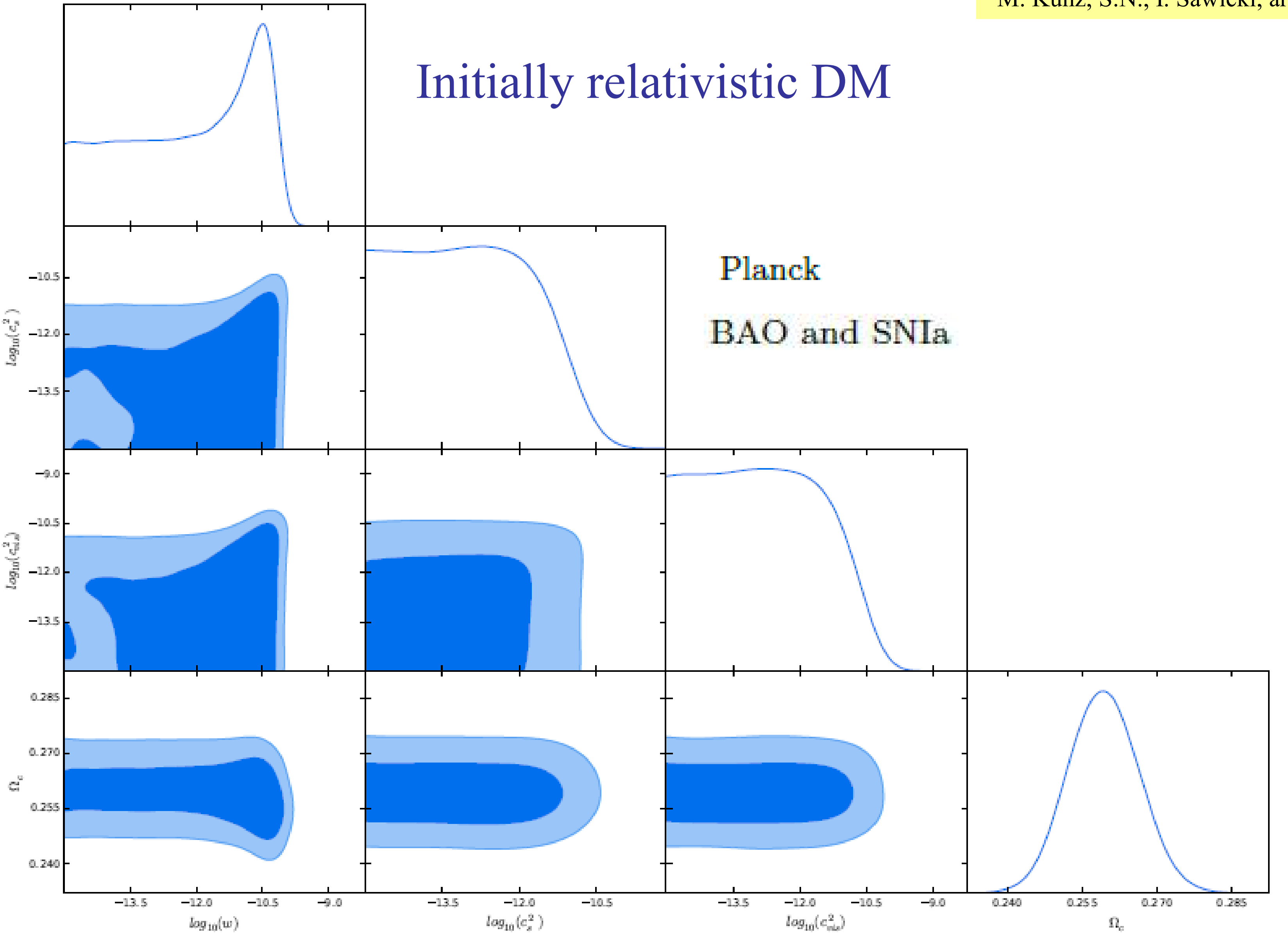
M. Kunz, S.N., I. Sawicki, arXiv: 1604.05701



The Dark Matter equation of state w

M. Kunz, S.N., I. Sawicki, arXiv: 1604.05701

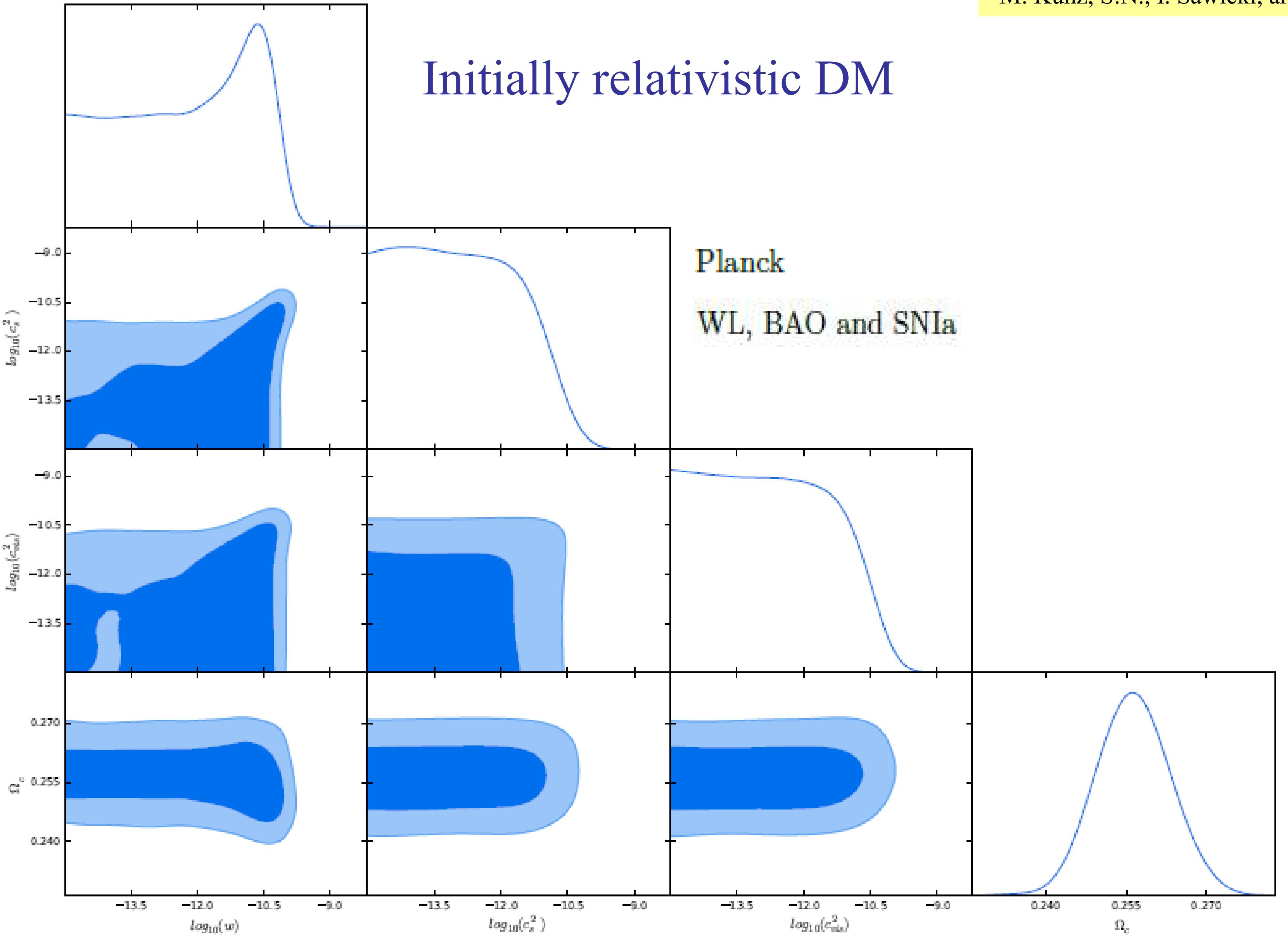
Initially relativistic DM



The Dark Matter equation of state w

M. Kunz, S.N., I. Sawicki, arXiv: 1604.05701

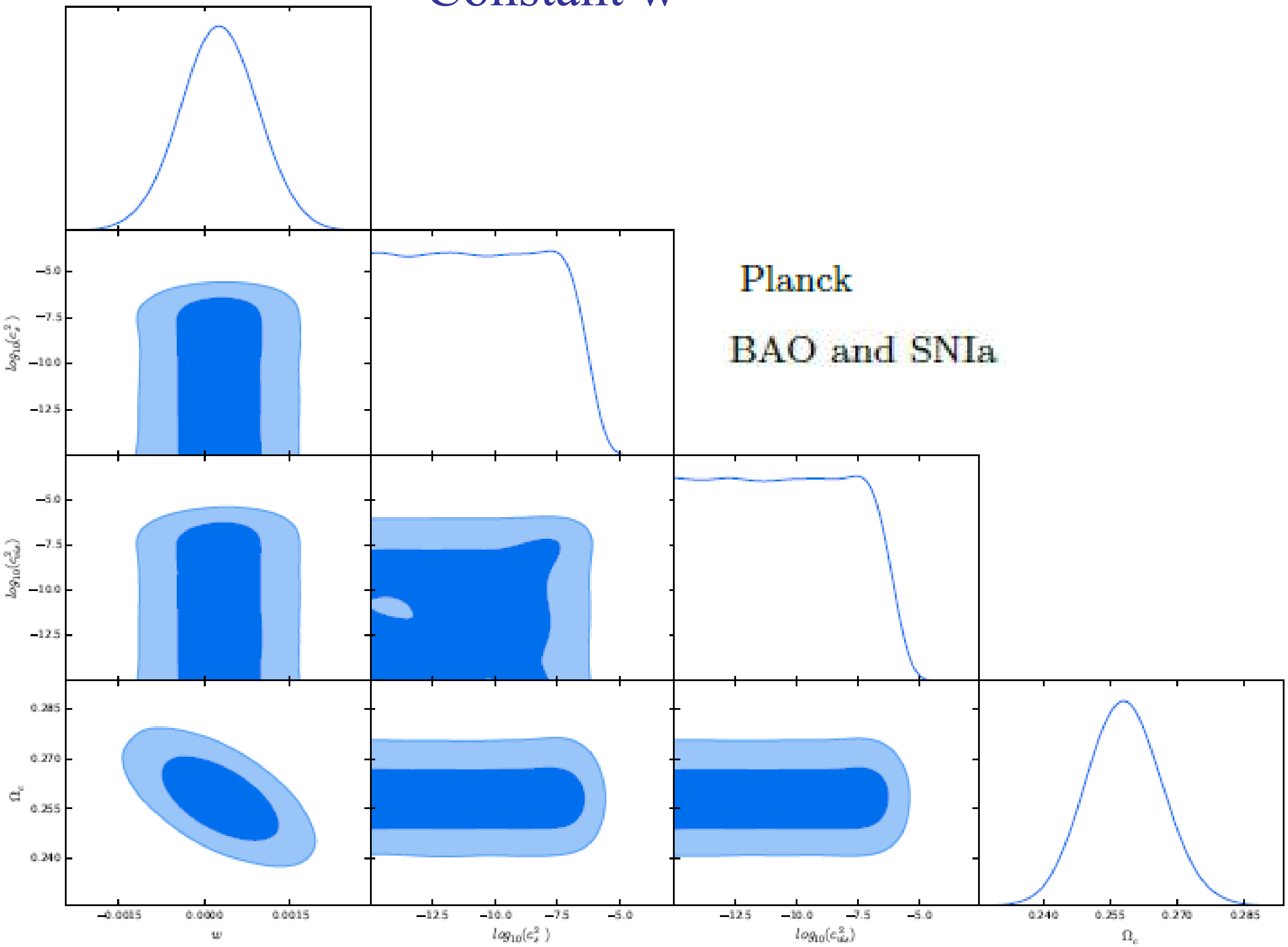
Initially relativistic DM



The Dark Matter equation of state w

M. Kunz, S.N., I. Sawicki, arXiv: 1604.05701

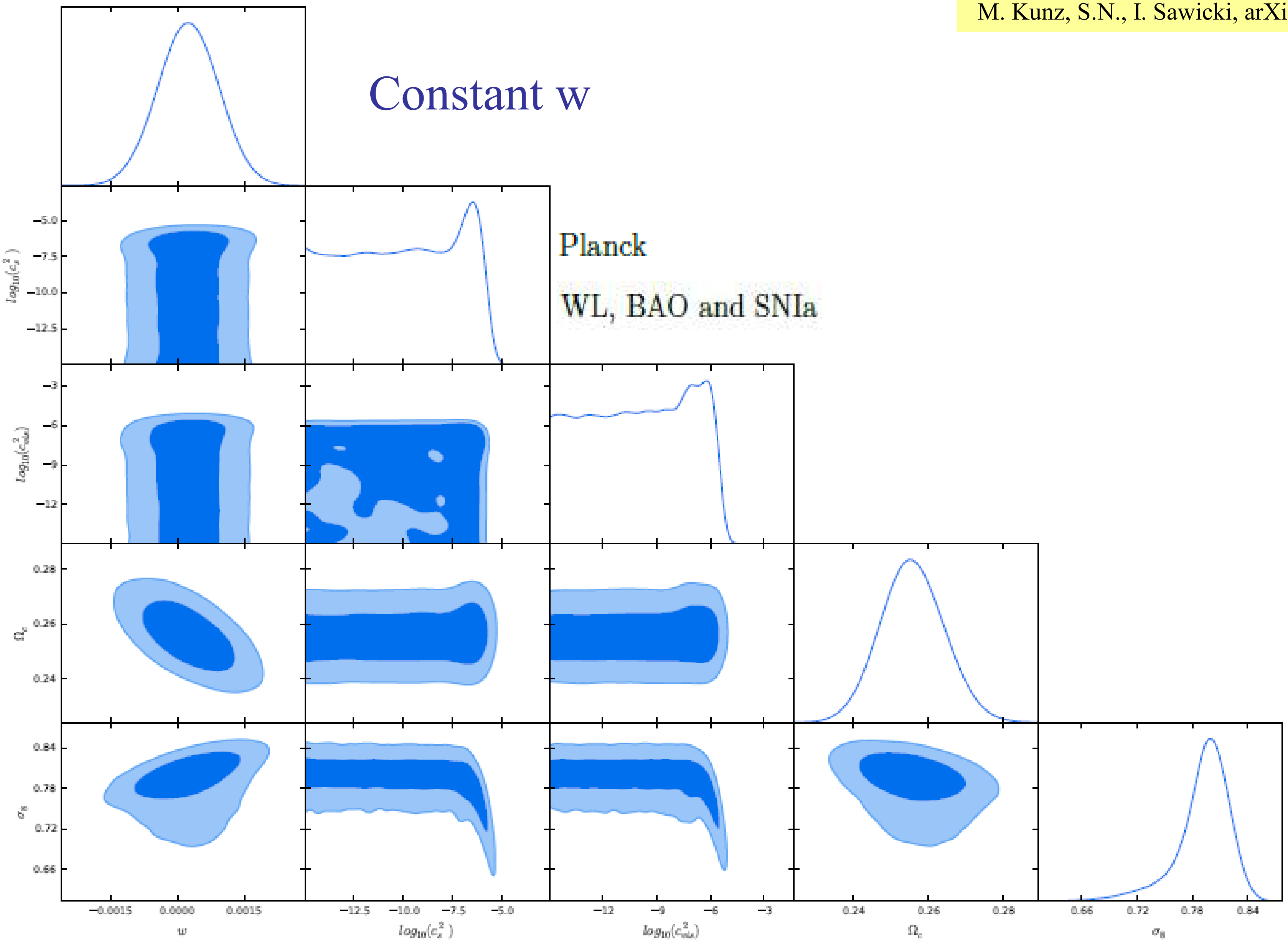
Constant w



The Dark Matter equation of state w

M. Kunz, S.N., I. Sawicki, arXiv: 1604.05701

Constant w



Conclusions

- Brief intro and discussion of the data (SnIa, CMB, WL, BAO).

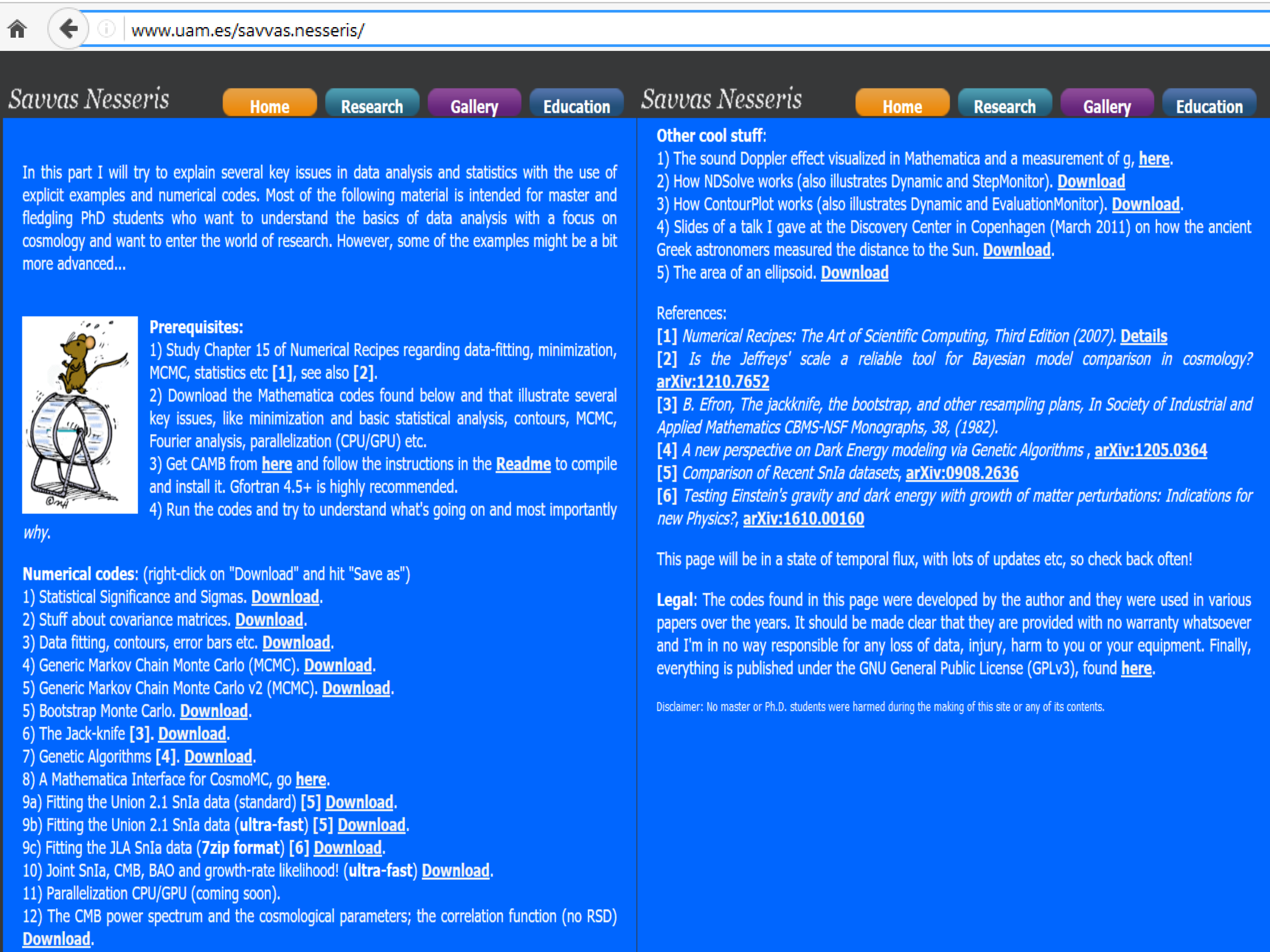
- Discussion of DM perturbations and their effects on large scale structure.

- Constraints on the sound speed of DM: $\Delta\Omega c \sim 0.017$ (6.6% of total DM) with cs^2~10^-4.4

- Constraints on DM EoS w(z):

  Constant w: w~0, cs^2<10^-6

  Initially relativistic (WDM): m>100eV, cs^2<10^-7



In this part I will try to explain several key issues in data analysis and statistics with the use of explicit examples and numerical codes. Most of the following material is intended for master and fledgling PhD students who want to understand the basics of data analysis with a focus on cosmology and want to enter the world of research. However, some of the examples might be a bit more advanced...

Prerequisites:

1) Study Chapter 15 of Numerical Recipes regarding data-fitting, minimization, MCMC, statistics etc **[1]**, see also **[2]**.
2) Download the Mathematica codes found below and that illustrate several key issues, like minimization and basic statistical analysis, contours, MCMC, Fourier analysis, parallelization (CPU/GPU) etc.
3) Get CAMB from **here** and follow the instructions in the **Readme** to compile and install it. Gfortran 4.5+ is highly recommended.
4) Run the codes and try to understand what's going on and most importantly *why*.

Numerical codes: (right-click on "Download" and hit "Save as")

1) Statistical Significance and Sigmas. **Download**.
2) Stuff about covariance matrices. **Download**.
3) Data fitting, contours, error bars etc. **Download**.
4) Generic Markov Chain Monte Carlo (MCMC). **Download**.
5) Generic Markov Chain Monte Carlo v2 (MCMC). **Download**.
5) Bootstrap Monte Carlo. **Download**.
6) The Jack-knife **[3]**. **Download**.
7) Genetic Algorithms **[4]**. **Download**.
8) A Mathematica Interface for CosmoMC, go **here**.
9a) Fitting the Union 2.1 SnIa data (standard) **[5]** **Download**.
9b) Fitting the Union 2.1 SnIa data (**ultra-fast**) **[5]** **Download**.
9c) Fitting the JLA SnIa data (**7zip format**) **[6]** **Download**.
10) Joint SnIa, CMB, BAO and growth-rate likelihood! (**ultra-fast**) **Download**.
11) Parallelization CPU/GPU (coming soon).
12) The CMB power spectrum and the cosmological parameters; the correlation function (no RSD) **Download**.

Other cool stuff:

1) The sound Doppler effect visualized in Mathematica and a measurement of g, **here**.
2) How NDSolve works (also illustrates Dynamic and StepMonitor). **Download**
3) How ContourPlot works (also illustrates Dynamic and EvaluationMonitor). **Download**.
4) Slides of a talk I gave at the Discovery Center in Copenhagen (March 2011) on how the ancient Greek astronomers measured the distance to the Sun. **Download**.
5) The area of an ellipsoid. **Download**

References:

[1] *Numerical Recipes: The Art of Scientific Computing, Third Edition (2007).* **Details**
[2] *Is the Jeffreys' scale a reliable tool for Bayesian model comparison in cosmology?* **arXiv:1210.7652**
[3] *B. Efron, The jackknife, the bootstrap, and other resampling plans, In Society of Industrial and Applied Mathematics CBMS-NSF Monographs, 38, (1982).*
[4] *A new perspective on Dark Energy modeling via Genetic Algorithms* , **arXiv:1205.0364**
[5] *Comparison of Recent SnIa datasets,* **arXiv:0908.2636**
[6] *Testing Einstein's gravity and dark energy with growth of matter perturbations: Indications for new Physics?,* **arXiv:1610.00160**

This page will be in a state of temporal flux, with lots of updates etc, so check back often!

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