Gravitational reheating after multi-field inflation

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Success of Inflation +ΛCDM

- **ΛCDM** is in very good **agreement with CMB observations**

- Perturbations **consistent with simple single-field inflation:**
  Planck 2015 & BICEP2/Keck Oct. 2015

  ✓ **5.6σ deviation from scale invariance:** $n_s = 0.9655 \pm 0.0062$

  ✓ perturbations close to **Gaussian**:

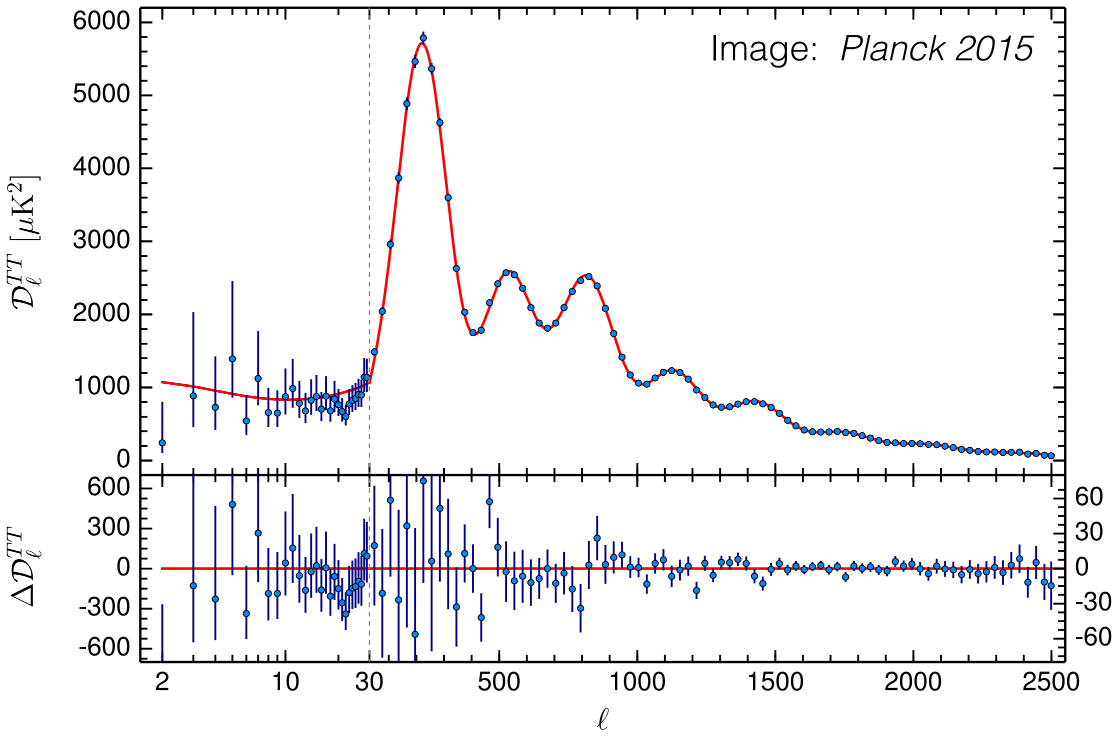
$$f_{NL}^{local} = 0.8 \pm 5.0$$

  ✓ perturbations close to **adiabatic**:

$$|\alpha_{non-ad}| < \mathcal{O}(1\%)$$

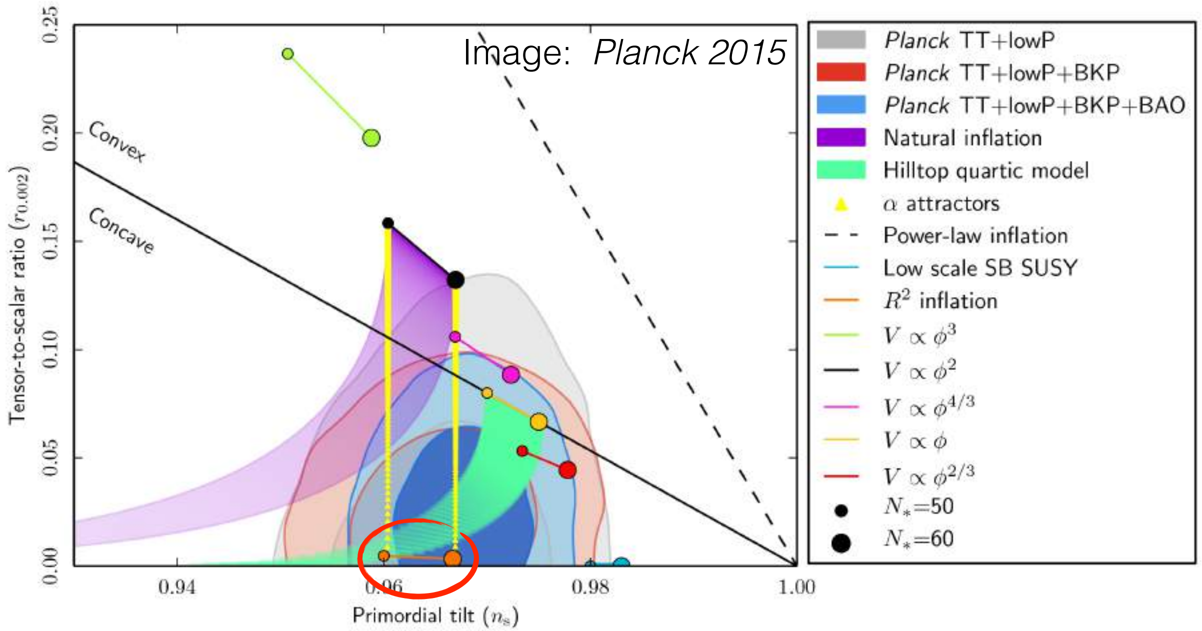
  ✓ tensor modes constrained:

$$r_{0.05} < 0.07$$



Constraining inflation models

- Current CMB data is very precise $\Rightarrow$ **we can constrain inflation models**



Two things to note:

- The predictions of a given model depend on when observable scales left the horizon - **need to know N_* when constraining models**
- Large class of **models with non-minimal gravity sectors** are in **good agreement with observations**, e.g. R^2, Higgs, α-attractors

When did observable scales leave the horizon?

- To determine N_* we **need to know how the universe evolves after inflation,** but…

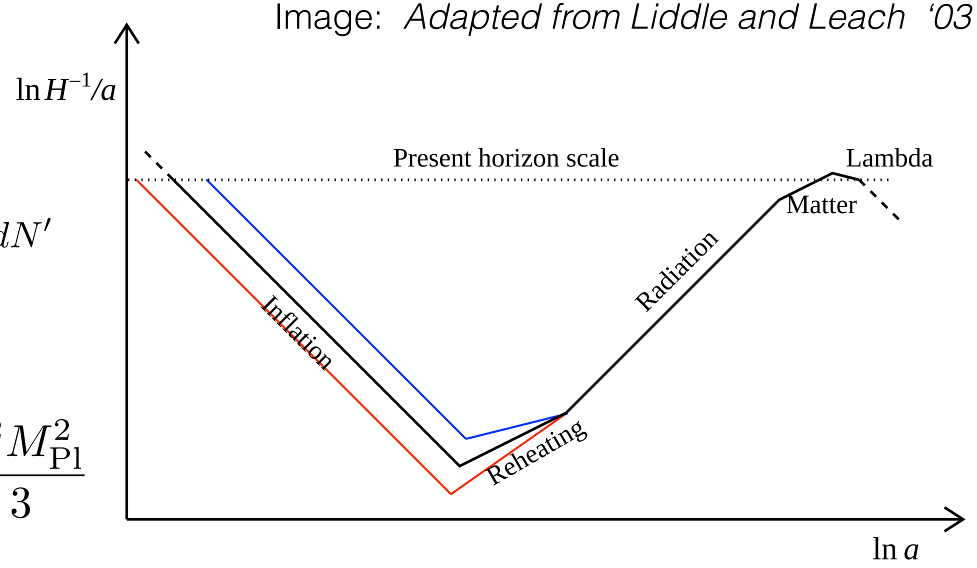
  …we still **know very little about the reheating epoch**, during which the inflaton energy is converted into the matter of our universe.

- We don't have to worry about the microphysics - **just need**:
  1. **Effective e.o.s.**
  2. **Duration**

$$N_* \approx 67 - \ln\left(\frac{k}{a_0 H_0}\right) + \frac{1}{4}\ln\left(\frac{V_*^2}{M_{\rm Pl}^4 \rho_{\rm end}}\right) + \frac{\Delta N_{rh}}{4}(3w_{rh} - 1)$$

$$\frac{1 - 3w_{\rm rh}}{12(1 + w_{\rm rh})}\ln\left(\frac{\rho_{\rm reh}}{\rho_{\rm end}}\right) \qquad w_{\rm rh} = \frac{1}{\Delta N_{\rm rh}}\int_{N_{\rm end}}^{N_{\rm rh}} \frac{p(N')}{\rho(N')} dN'$$

Image: *Adapted from Liddle and Leach '03*



- Under the instant decay approximation **reheating ends when** $\Gamma \approx 3H \rightarrow \rho_{\rm rh} = \frac{\Gamma^2 M_{\rm Pl}^2}{3}$

$\Rightarrow$ we need to determine Γ

Models with non-minimal coupling

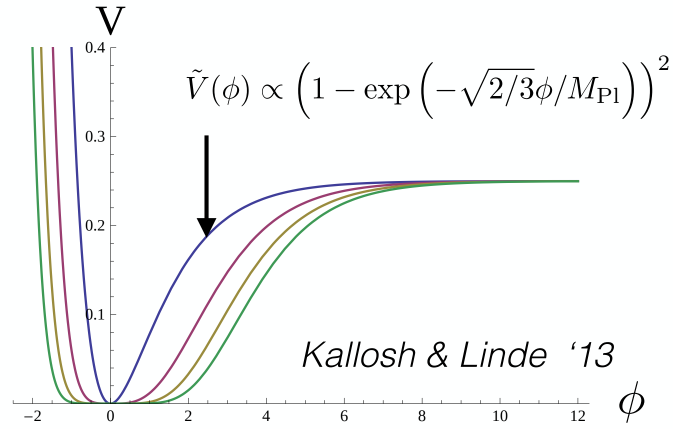
- In the context of modified gravity, field theory in curved space-time and higher-dimensional unifying particle physics theories, **non-minimal coupling between scalar fields and the Ricci scalar is common**

- e.g.

$$S_{f(R)} = \int d^4x\sqrt{-g}\,f(R) \;\Rightarrow\; \int d^4x\sqrt{-g}\left(\Phi R + \ldots\right)$$

$$(\Phi = df/dR)$$

$$S_{Higgs} = \int d^4x\sqrt{-g}\left[\frac{1}{2}\left(M_{pl}{}^2 + \xi h^2\right)R + \ldots\right]$$



- Make **conformal transformation:** $\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}$

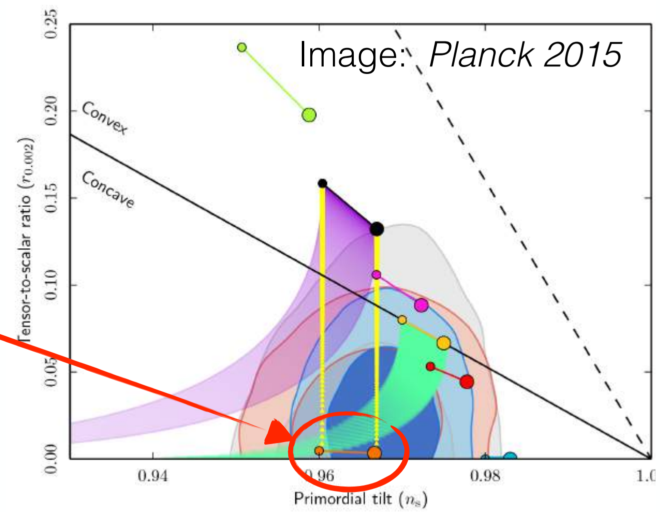
$$\Rightarrow \quad S = \int d^4x\sqrt{-\tilde{g}}\left(\frac{M_{pl}^2}{2}\tilde{R} - \frac{1}{2}s(\varphi)(\partial\varphi)^2 - \tilde{V}\right)$$

1. Flattening of potential $\quad \tilde{V} = V/\Omega^4$
2. Rescaling of field $\quad \phi = M_{\rm Pl}\sqrt{3/2}\ln\varphi$

$\Rightarrow$ **Ideal for inflation**

- All predict $\quad n_s = 1 - \dfrac{2}{N_*} \qquad r = \dfrac{12}{N_*^2}$

but N_* **depends on reheating**, which is **different for different models**



Reheating in models with non-minimal coupling

- Given that inflation models with non-minimal coupling are favoured by observations, we consider reheating in models with the following action:

$$S = \int d^4x\sqrt{-g}\left\{\frac{f(\vec{\phi})}{2}R - \frac{1}{2}h_{ab}g^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - V\right\} + S_m$$

- Have allowed for **multiple fields** with a **non-flat field space**. The presence of multiple fields is expected in the context of HEP unifying theories.

- We would like to determine:
  1. **Equation of state** during reheating
  2. The duration of reheating, i.e. Γ
  3. **Evolution of** ζ through reheating

Due to the non-minimal coupling we **get reheating even in the absence of direct couplings** between $\vec{\phi}$ and matter, i.e. $S_m = S_m\left(g_{\mu\nu}, X_m\right)$

- This is **gravitational reheating.** We will consider this minimal setup where $S_m = S_m\left(g_{\mu\nu}, X_m\right)$

Interaction terms in the Einstein frame

- Reason for gravitational reheating is most clear in **Einstein frame:**

$$g_{\mu\nu} = \frac{M_{pl}^2}{f}\tilde{g}_{\mu\nu} \quad \Rightarrow \quad S = \int d^4x\sqrt{-\tilde{g}}\left[\frac{M_{\rm Pl}^2}{2}\tilde{R} - \frac{1}{2}S_{ab}\tilde{g}^{\mu\nu}\partial_\mu\phi^a\partial_\nu\phi^b - \tilde{V}(\vec{\phi})\right] + S_m\left(\frac{M_{\rm Pl}^2}{f(\vec{\phi})}\tilde{g}_{\mu\nu}, X_m\right)$$

$$S_{ab} = \frac{M_{pl}^2}{f}\left(h_{ab} + \frac{3f_a f_b}{2f}\right) \qquad \tilde{V} = \frac{M_{pl}^4 V}{f^2}$$

Explicit interaction terms

- e.g. if in the **Jordan frame** we consider matter to consist of fermions and scalar fields:

$$S_\chi = \int d^4x\sqrt{-g}\left\{-\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - U(\chi)\right\}$$

$$S_\psi = -\int d^4x\sqrt{-g}\left\{\bar{\psi}\overleftrightarrow{\not{D}}\psi + m_\psi\bar{\psi}\psi\right\}$$

$$\not{D} = e^\mu_\alpha\gamma^\alpha\left(\partial_\mu - \Gamma_\mu - igA_\mu\right)$$

$$\Gamma_\mu = -(1/2)\Sigma^{\alpha\beta}e^\lambda_\alpha\nabla_\mu e_{\beta\lambda}$$

- Transforming to the **Einstein frame**: $\left(\Omega^2 = \frac{f}{M_{pl}^2}\right)$

$$S_{\tilde{\chi}} = \int d^4x\sqrt{-\tilde{g}}\left\{-\frac{1}{2}\tilde{g}^{\mu\nu}\mathcal{D}_\mu\tilde{\chi}\mathcal{D}_\nu\tilde{\chi} - \frac{U(\chi)}{\Omega^4}\right\}$$

$$S_{\tilde{\psi}} = -\int d^4x\sqrt{-\tilde{g}}\left\{\bar{\tilde{\psi}}\overleftrightarrow{\tilde{\not{D}}}\tilde{\psi} + \frac{m_\psi}{\Omega}\bar{\tilde{\psi}}\tilde{\psi}\right\}$$

Explicit interaction terms

$$\mathcal{D}_\mu = \partial_\mu + \tilde{\chi}\partial_\mu(\ln\Omega) \qquad \tilde{\not{D}} = \tilde{e}^\mu_\alpha\gamma^\alpha\left(\partial_\mu - \Gamma_\mu - igA_\mu\right) \qquad \tilde{\psi} = \Omega^{-3/2}\psi, \qquad \tilde{\chi} = \frac{\chi}{\Omega} \qquad \tilde{e}^\mu_\alpha = \frac{e^\mu_\alpha}{\Omega}$$

Background dynamics of oscillating inflatons

- First consider dynamics in the Einstein frame, where the inflatons are minimally coupled
- Assume ordinary matter fields are not present initially

$\Rightarrow$ **Dynamics of ϕ^a determined by the Einstein frame potential $\tilde{V}(\vec{\phi})$**

$\Rightarrow$ **Assume fields oscillate about a minimum of $\tilde{V}(\vec{\phi})$** at the end of inflation

$\Rightarrow$ Decompose $\phi^a = \phi^a_{\rm vev} + \sigma^a$ and expand the E.F. action:

$$S = \int d^4x \sqrt{-\tilde{g}} \left\{ \frac{M_{pl}^2 \tilde{R}}{2} - \frac{1}{2} S_{ab}|_{\rm vev} \tilde{g}^{\mu\nu} \partial_\mu \sigma^a \partial_\nu \sigma^b - \frac{1}{2} \tilde{V}_{ab}|_{\rm vev} \sigma^a \sigma^b \right\}$$

To diagonalise the action we introduce the **mass eigen-basis**: $\sigma^a = \alpha^A e^a_A$

$$\tilde{V}^a{}_b e^b_A \equiv S^{ac} \tilde{V}_{cb} e^b_A = m^2_{\hat{A}} e^a_A$$

$$\Rightarrow \qquad S = \int d^4x \sqrt{-\tilde{g}} \left\{ \frac{M_{\rm Pl}^2 \tilde{R}}{2} - \frac{1}{2} \delta_{AB} \left(\tilde{g}^{\mu\nu} \partial_\mu \alpha^A \partial_\nu \alpha^B + m^2_{\hat{A}} \alpha^A \alpha^B \right) \right\}$$

Background dynamics of oscillating inflatons

- Assuming an **FLRW** metric: $d\tilde{s}^2 = -d\tilde{t}^2 + \tilde{a}^2 \delta_{ij} dx^i dx^j$

$$\Rightarrow \qquad \frac{d^2}{d\tilde{t}^2}\left(\tilde{a}^{3/2}\alpha^A\right) + \left[m_A^2 - \left(\frac{9}{4}\tilde{H}^2 + \frac{3}{2}\frac{d\tilde{H}}{d\tilde{t}}\right)\right]\left(\tilde{a}^{3/2}\alpha^A\right) = 0$$

- Assuming $m_A^2 \gg \tilde{H}^2,\ d\tilde{H}/d\tilde{t}$ $\quad \Rightarrow \quad \alpha^A \simeq \frac{\alpha_0^A}{\tilde{a}^{3/2}} \cos[m_A \tilde{t} + d_A]$

$$\tilde{H}^2 = \frac{1}{6M_{\rm Pl}^2}\sum_A \left[\left(\frac{d\alpha^A}{d\tilde{t}}\right)^2 + m_A^2(\alpha^A)^2\right] = \sum_A \frac{(\alpha_0^A)^2 m_A^2}{6M_{\rm Pl}^2 \tilde{a}^3}\left(1 + \frac{3\tilde{H}}{2m_A}\sin(2(m_A\tilde{t} + d_A)) + \mathcal{O}(\tilde{H}^2/m_A^2)\right)$$

Universe essentially **evolves like matter-dominated universe during reheating**
i.e. $\omega_{rh} = 0$

Oscillatory component of $\tilde{H}$ is sub-leading order in $\tilde{H}/m_A$

$$\frac{d\tilde{H}}{d\tilde{t}} \simeq -\frac{3}{2}\tilde{H}^2\left(1 - \frac{1}{\tilde{H}^2}\sum_A \frac{(\alpha_0^A)^2 m_A^2}{6M_{\rm Pl}^2\tilde{a}^3}\cos(2(m_A\tilde{t} + d_A))\right)$$

$d\tilde{H}/d\tilde{t}$ has an oscillatory component even at leading order, but $d\tilde{H}/d\tilde{t} \sim \mathcal{O}(\tilde{H}^2)$

Background dynamics in the Jordan frame

- Would like to use the E.F. results to determine dynamics in Jordan frame.
- Under the conformal transformation we have: $(\Omega^2 = f(\phi)/M_{pl}^2)$

$$d\tilde{s}^2 = -d\tilde{t}^2 + \tilde{a}^2(\tilde{t})\delta_{ij}d\tilde{x}^i d\tilde{x}^j = \Omega^2 ds^2 = \Omega^2\left(-dt^2 + a^2(t)\delta_{ij}dx^i dx^j\right)$$

$$\Rightarrow \qquad \tilde{a} = \Omega a, \quad d\tilde{t} = \Omega dt, \quad d\tilde{x}^i = dx^i \quad \text{and} \quad \tilde{H} = \frac{1}{\Omega}\left(H + \frac{\dot{\Omega}}{\Omega}\right)$$

- **Assuming** $f|_{\mathrm{vev}} = M_{\mathrm{Pl}}^2$ and expanding:

$$f = M_{pl}^2\left(1 + \frac{f_A\alpha^A}{M_{Pl}^2} + \frac{1}{2}\frac{f_{AB}\alpha^A\alpha^B}{M_{Pl}^2} + \ldots\right)$$

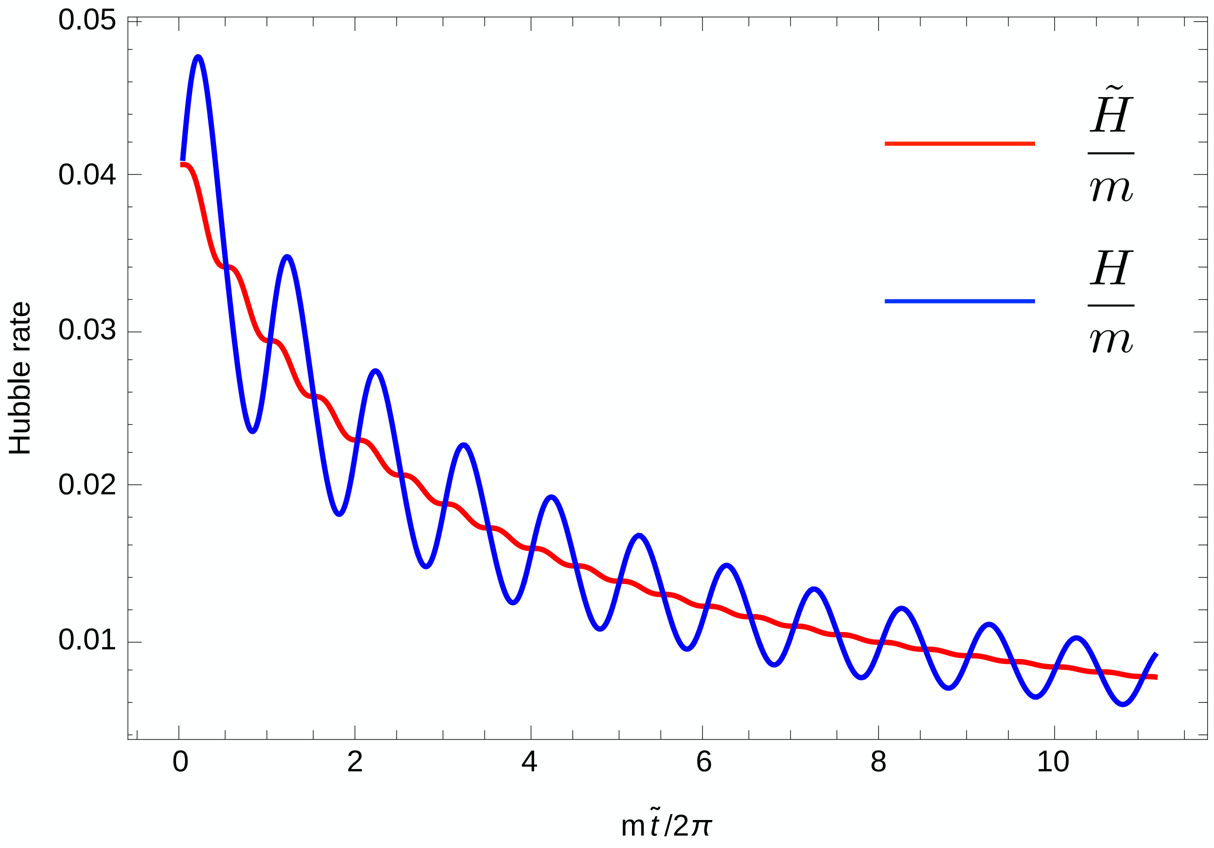
$$\Rightarrow \qquad H \simeq \tilde{H}\left(1 + \frac{1}{\tilde{H}}\sum_A \frac{f_A}{2M_{\mathrm{Pl}}^2}\frac{\alpha_0^A m_A}{\tilde{a}^{3/2}}\sin\left(m_A\tilde{t} + d_A\right)\right)$$

On average, evolution of H in the Jordan frame is like that of matter-dominated universe, but there is an **oscillatory component** that is **not suppressed**

$\Rightarrow \quad \dot{H} \sim \mathcal{O}(m_A\tilde{H})$ compared to $d\tilde{H}/d\tilde{t} \sim \mathcal{O}(\tilde{H}^2)$ in Einstein frame

Background dynamics comparison

- A single-field example with $\alpha_0 = 0.1 M_{\rm Pl}$ and $\dfrac{f_\alpha}{2M_{\rm Pl}} = 0.1$



Bogoliubov approach: scalar case

- In the flat-space QFT approach to determining decay rates we view the oscillating scalar fields as a **collection of massive zero-momentum particles that decay** into matter
  - Presents itself naturally in the Einstein frame
  - Is limited to the perturbative regime

- An alternative approach is based on **QFT in a time-varying classical background**
  Let us begin by considering the χ field in the Jordan frame:

$$S_\chi = \int dt d^3x a^3 \frac{1}{2}\left[\dot{\chi}^2 - \frac{1}{a^2}(\nabla\chi)^2 - m_\chi^2\chi^2\right]$$

In quantising the field we use conformal time $ad\eta = dt$ and define the canonically normalised field $u = a\chi$

$$S_u = \int d\eta d^3x \frac{1}{2}\left[u'^2 - (\nabla u)^2 - \left(a^2 m_\chi^2 - \frac{a''}{a}\right)u^2\right]$$

$$\Rightarrow \quad u_k'' + w_k^2 u_k = 0 \quad \text{with} \quad w_k^2 = k^2 + a^2 m_\chi^2 - \frac{a''}{a} \longleftarrow \frac{a''}{a} = a^2(\dot{H} + 2H^2)$$

u has a **time-varying mass due to time-dependence of gravitational background**

Bogoliubov approach: scalar case

- On **choosing appropriate mode-functions**:

$$u_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2w(\eta)}} \exp\left[-i\int_{-\infty}^{\eta} d\eta' w_k(\eta')\right] + \frac{\beta_k(\eta)}{\sqrt{2w(\eta)}} \exp\left[i\int_{-\infty}^{\eta} d\eta' w_k(\eta')\right]$$

$\Rightarrow$ E.o.m. are satisfied if

$$\alpha_k'(\eta) = \frac{w_k'}{2w_k} \exp\left[2i\int_{-\infty}^{\eta} d\eta' w_k(\eta')\right] \beta_k(\eta),$$

$$\beta_k'(\eta) = \frac{w_k'}{2w_k} \exp\left[-2i\int_{-\infty}^{\eta} d\eta' w_k(\eta')\right] \alpha_k(\eta)$$

$\Rightarrow$ Commutation relations satisfied if $|\alpha_k|^2 - |\beta_k|^2 = 1$

$\Rightarrow$ **Hamiltonian diagonalised**, with

$$E_k = w_k(1/2 + |\beta_k(\eta)|^2)$$

$\Rightarrow$ We **interpret** $|\beta_k(\eta)|^2$ **as the number density of particles**

Even if $\beta_k = 0$ initially, $\beta_k(\eta)$ **evolves away from zero if** $w_k' \neq 0$

$\Rightarrow$ **corresponds to particle production**

Bogoliubov approach: scalar case

- The Bogoliubov approach is more widely applicable, but we can still consider the **perturbative regime** where $f_A\alpha^A/(2M_{\rm Pl}^2) \ll 1\,,\;\; \beta_k(\eta) \ll 1$ and $\alpha_k(\eta) - 1 \ll 1$

$$\beta_k(\eta) \simeq \int_{\eta_0}^{\eta} d\eta' \frac{w_k'}{2w_k} \exp\left[-2i\int_{-\infty}^{\eta'} d\eta'' w_k(\eta'')\right]$$

- Evaluate $\beta_k(\eta)$ using the **stationary phase approx.**

Stationary when $w_k = \dfrac{\tilde{a} m_A}{2} \quad \Rightarrow \quad \dfrac{k^2}{\tilde{a}^2(\eta_k^A)} = \dfrac{m_A^2}{4}\left(1 - \dfrac{4m_\chi^2}{m_A^2}\right)$

$k/\tilde{a}$ corresponds to momentum of produced particle, so this **agrees with what we expect from kinematics**

- Determine the decay rate into $\tilde{\chi}$ using the continuity equation in the E.F.

$$\tilde{\nabla}^\mu \tilde{T}_{\mu\nu}^{(\tilde{\chi})} = -\frac{\Omega_\nu}{\Omega}\tilde{T}^{(\tilde{\chi})} \quad \Rightarrow \quad \text{we } \textbf{recover} \qquad \tilde{\Gamma}_{\alpha^A\to\chi\chi} = \frac{\left[f_A(2m_\chi^2 + m_A^2)\right]^2}{128\pi M_{pl}^4 m_A}\left(1 - \frac{4m_\chi^2}{m_A^2}\right)^{1/2}$$

Bogoliubov approach: scalar case

- Almost trivial to see that the **calculation is the same if we start in the Einstein frame**:

$$S_{\tilde\chi} = \int d\tilde t d^3x \tilde a^3 \frac{1}{2}\left[\left(\frac{d\tilde\chi}{d\tilde t}\right)^2 - \frac{1}{\tilde a^2}(\nabla\tilde\chi)^2 - \left(m_\chi^2 - \frac{f_A\alpha^A}{2M_{\rm Pl}^2}(m_A^2 + 2m_\chi^2)\right)\tilde\chi^2\right]$$

- Use conformal time, $\tilde a d\eta = d\tilde t$, and define **canonically normalised field** $\tilde u = \tilde a\tilde\chi$

$$S_{\tilde u} = \int d\eta d^3x \frac{1}{2}\left[\tilde u'^2 - (\nabla\tilde u)^2 - \left(\tilde a^2\left(m_\chi^2 - \frac{f_A\alpha^A}{2M_{\rm Pl}^2}(m_A^2 + m_\chi^2)\right) - \frac{\tilde a''}{\tilde a}\right)\tilde u^2\right]$$

…but $\tilde a = \Omega a$ and $\tilde\chi = \chi/\Omega$ $\quad\Rightarrow\quad$ $\tilde u = u$

$\Rightarrow$ Whichever frame we start in ultimately we need to solve $u_k'' + w_k^2 u_k = 0$
However, the **interpretation is different**

J.F. $\quad w_k^2 = k^2 + a^2 m_\chi^2 - \frac{a''}{a} \quad\Rightarrow$ time-dependence of w_k due to oscillatory a
- **gravitational particle production interpretation**

E.F. $\quad w_k^2 = k^2 + \tilde a^2\left(m_\chi^2 - \frac{f_A\alpha^A}{2M_{\rm Pl}^2}(m_A^2 + 2m_\chi^2)\right) - \frac{\tilde a''}{\tilde a} \quad\Rightarrow$ time-dependence of w_k due to **explicit interaction terms**

Starobinsky's inflation

- As a simple example, let us **consider Starobinsky's R² inflation model**:

$$S_S = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left(R + \frac{1}{6M^2}R^2\right)$$

- It **can be re-expressed as a scalar-tensor theory**:

$$S_S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}\left(1+\frac{\xi\phi}{M_{\rm Pl}^2}\right)R - \lambda^2\phi^2\right]$$

To see equivalence: Eom gives $\phi = \dfrac{\xi R}{4\lambda^2}$. Substitute back into action and take $M^2 = \dfrac{4\lambda^2}{3\xi^2}M_{\rm Pl}^2$

Relevant quantities are:

$$f = M_{\rm Pl}^2\left(1+\frac{\xi\phi}{M_{\rm Pl}^2}\right) \qquad \tilde{V} = \frac{\lambda^2\phi^2}{\left(1+\frac{\xi\phi}{M_{\rm Pl}^2}\right)^2} \qquad S_{\phi\phi} = \frac{3\xi^2}{2M_{\rm Pl}^2\left(1+\frac{\xi\phi}{M_{\rm Pl}^2}\right)^2}$$

$$\phi_{\rm vev} = 0 \qquad S^{\phi\phi}\tilde{V}_{\phi\phi}|_{\rm vev} = M^2 \qquad \alpha = \sqrt{\frac{3}{2}}\frac{\xi}{M_{\rm Pl}}\phi \qquad f_\alpha = \sqrt{\frac{2}{3}}M_{\rm Pl}$$

Starobinsky's inflation

- **Assuming daughter particles to be light** $\Rightarrow$ dominant decay channel is into scalars:

$$\tilde{\Gamma} \simeq \frac{N_s f_\alpha^2 M^3}{128\pi M_{\rm Pl}^4}$$

- Assuming instant thermalisation, **use this to determine the reheating temperature:**

- Define end of reheating when $\tilde{\Gamma} = 3\tilde{H}$ $\Rightarrow$ $\tilde{\rho}_{\rm rh} = \frac{1}{3}\tilde{\Gamma}^2 M_{\rm Pl}^2 = \frac{\pi^2}{30} g_*(T_{\rm rh}) T_{\rm rh}^4$

$$\Rightarrow \quad \boxed{T_{\rm rh} \simeq 3.7 \times 10^9 \,{\rm GeV}}$$

- Compare this with **Higgs inflation**, where $T_{\rm rh} \sim 10^{13}$ GeV due to **non-gravitational coupling** Bezrukov + Gorbunov '12

- This affects how many e-folds before end of inflation observable scales left the Horizon:

$$\Delta N_* \simeq \frac{1}{3} \ln \left(\frac{T_{\rm rh}({\rm R}^2)}{T_{\rm rh}({\rm h})} \right) \simeq -3 \quad \Rightarrow \quad \begin{aligned} \text{Higgs-inflation: } n_s &= 0.967, \quad r = 0.0032, \\ R^2\text{-inflation: } n_s &= 0.965, \quad r = 0.0036. \end{aligned}$$

Summary

- CMB data is now so precise that in order to constrain inflationary models we need to correctly determine how long before the end of inflation observable scales left the horizon.

- This in turn requires us to know about the post-inflation evolution of the universe, including reheating.

- Inflation models with non-minimal coupling are well motivated and observationally favoured, so it is important to study reheating in this class of models, and to determine observable consequences of having multiple fields.

- In this class of models, even in the absence of direct coupling between the inflaton sector and matter, reheating can take place gravitationally.

- We have developed a formulation of multi-field gravitational particle production using the Bogoliubov approach, which can be applied to both perturbative reheating and preheating.

Thank you!