

Infinite Derivative Ghost Free & Singularity Free Gravity

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Phys. Rev. Lett. (2012), JCAP (2012, 2011), JCAP (2006)

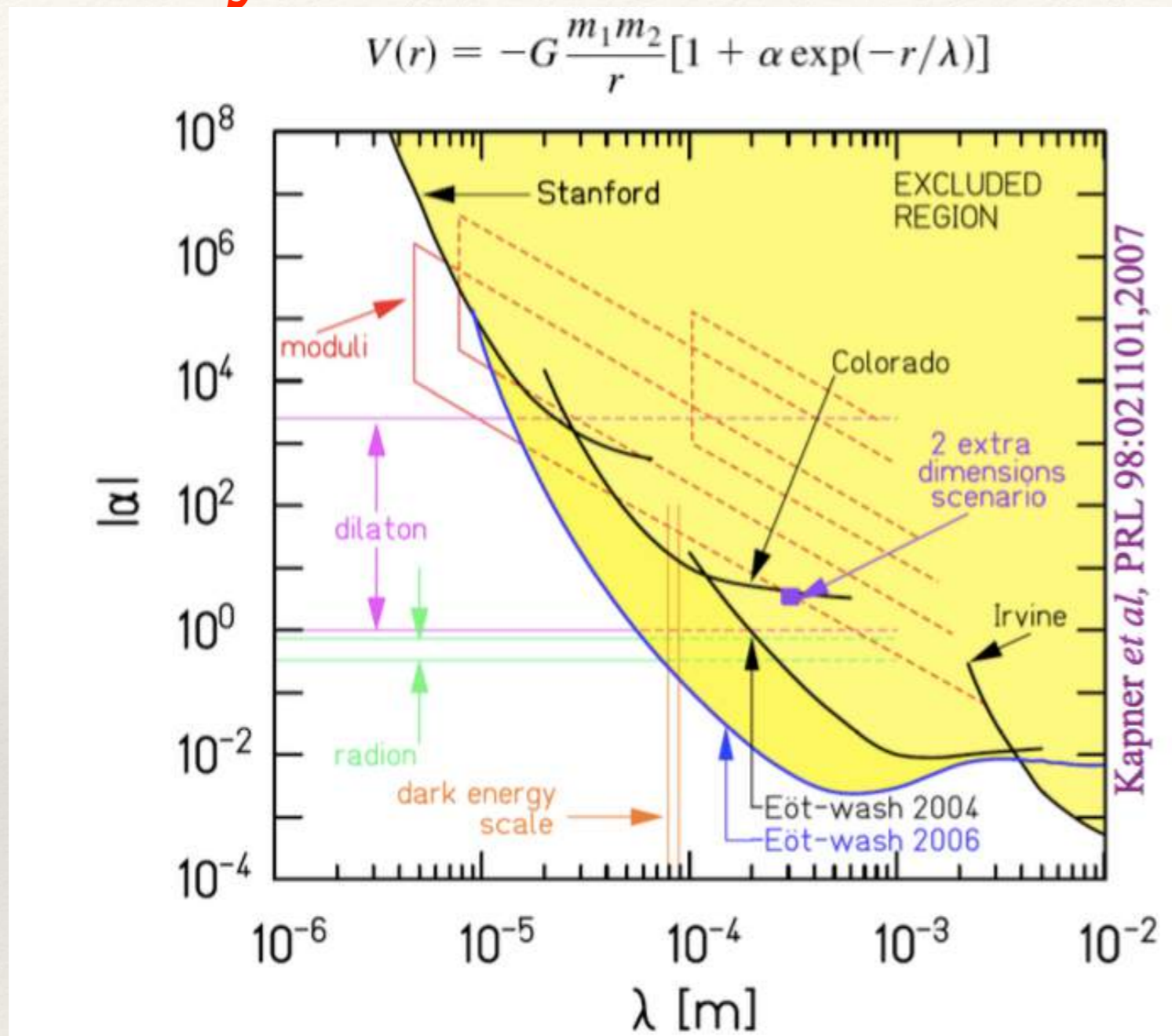
Class.Quant. Grav. (2013), Phys. Rev. D (2014), 1412.3467 (Class. Quant. Grav. 2014),

1503.05568 (Phys. Rev. Lett. 2015), 1509.01247 (Phys. Rev. D, 2015), 1602.08475,

1603.03440, 1604.01989, 1701.01009

**Einstein's GR is well behaved in IR, but UV is Pathetic;
Aim is to address the UV aspects of Gravity**

Energy Ladder : Very Little do we know about Gravity



$(10^{27} \text{ eV})^4$

$(10^{-2} \text{ eV})^4$

$(10^{-3} \text{ eV})^4$

No departure from Newtonian Gravity

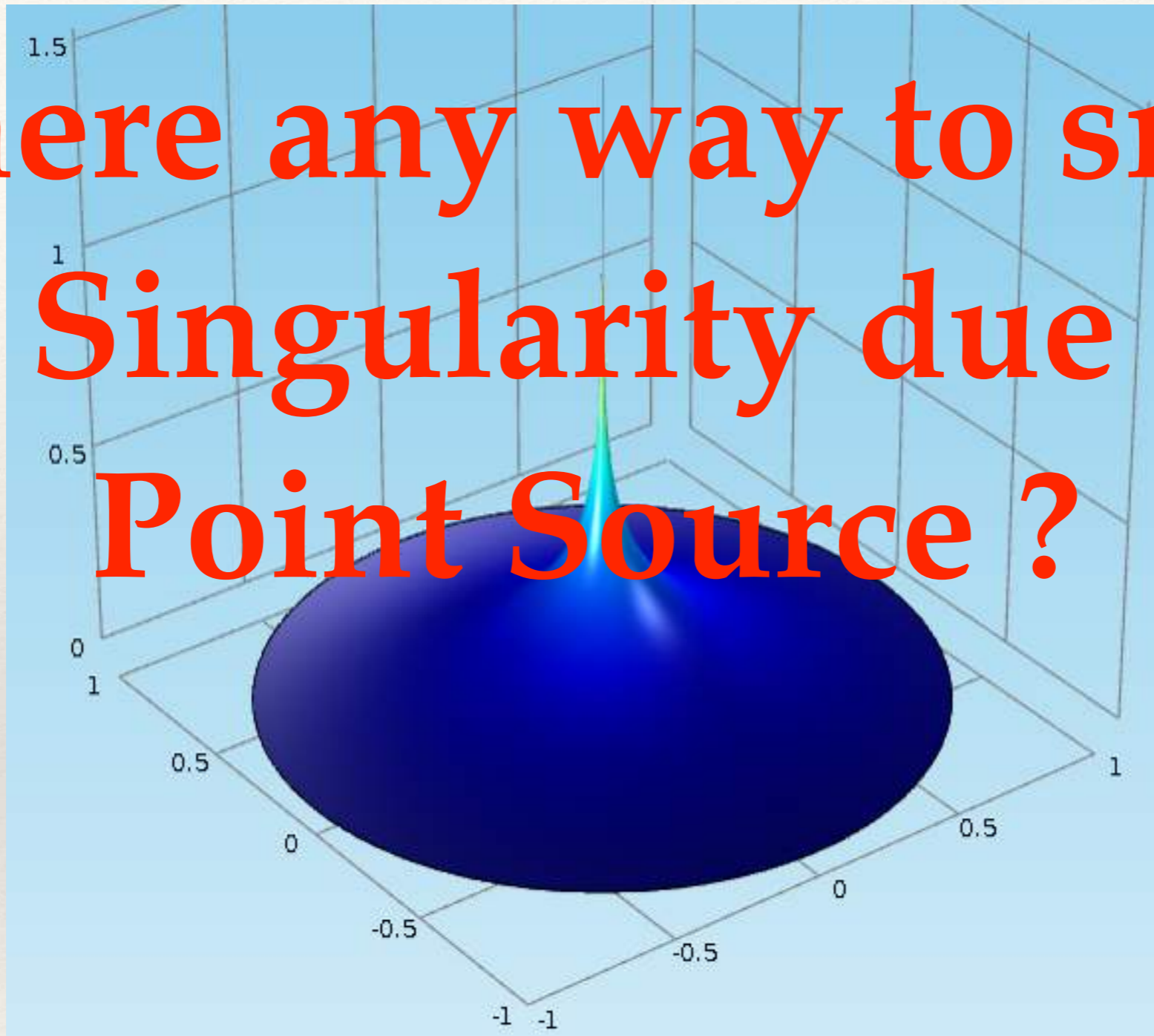
up to

$$10^{-5} \text{ m} \sim 100 \text{ (eV)}^{-1}$$

or, $M \sim 10^{-2} \text{ eV}$

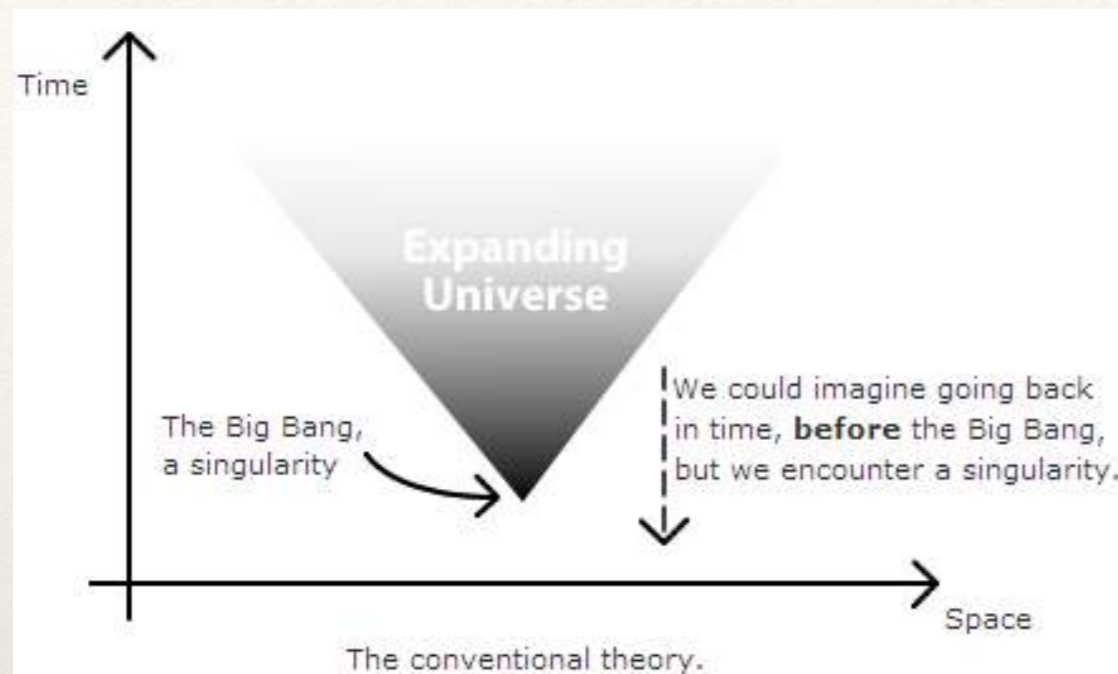
Einstein Gravity

Is there any way to smear
the Singularity due to a
Point Source ?



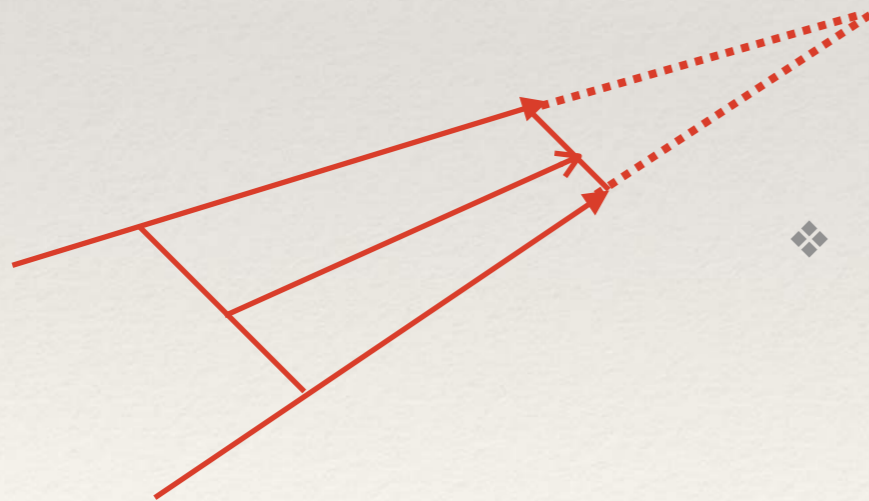
$$ds^2 = \left(1 - \frac{2Gm}{r} \right) dt^2 - \frac{dr^2}{\left(1 - \frac{2Gm}{r} \right)}$$

Cosmological Singularity



Big Bang Singularity,

Space Time have an edge

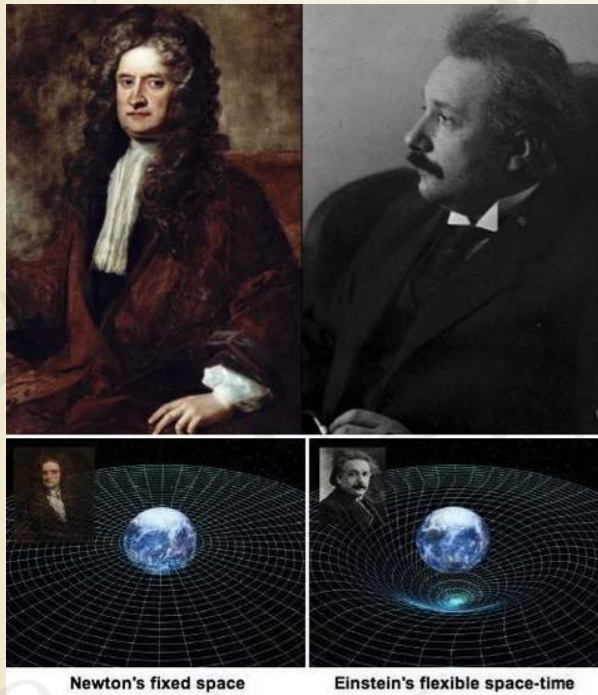


$$\rho + p \geq 0$$

- ❖ A singularity would always imply focusing of geodesics, but **focusing alone cannot imply a singularity**

“Inflation does not solve the singularity problem”

UV Modification of Gravity



UV is Pathological,

IR Part is Safe

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} + \dots \right)$$

**Gravity requires modification at small distances
and at early times**

While keeping the General Covariance

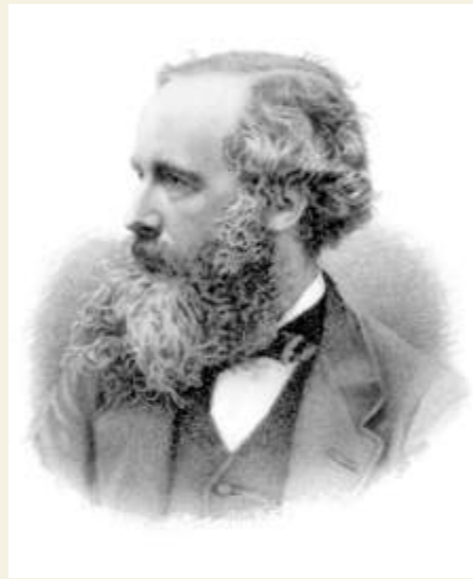


**analogous to
Born-Infeld
theory of E & M**

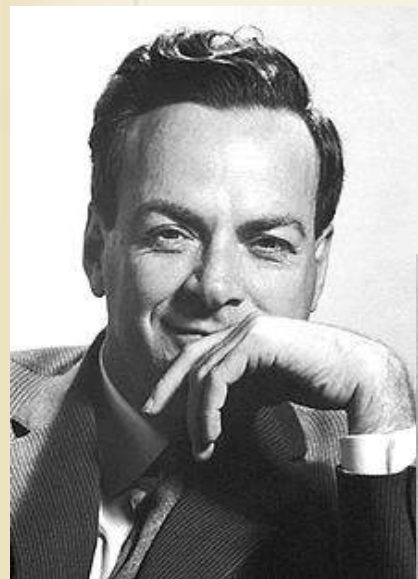
$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} \right)$$

Maxwell's Electromagnetism

Self energy of an electron is infinite in Maxwell's theory



$1/r$ -fall of Coulomb's Potential



Quantum
Electrodynamics
(QED)



Classical approach:
Born-Infeld

Born-Infeld resolves $1/r$ singularity

$$\mathcal{L}_{\text{Born-Infeld}} = b^2 \left[1 - \sqrt{1 - (\mathbf{E}^2 - \mathbf{B}^2)/b^2 - (\mathbf{E} \cdot \mathbf{B})^2/b^4} \right]$$

$$b \rightarrow \infty$$

$$\mathcal{L}_{\text{Born-Infeld}} \rightarrow \mathcal{L}_{\text{Maxwell}}$$

Maxwell

$$E_{\text{tot}} = \frac{1}{2} \int (\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H}) d^3r$$

$$D = e\hat{r}/4\pi r^2, \quad E = e\hat{r}/4\pi\epsilon r^2, \quad B = H = 0$$

$$E_{\text{tot}} = \frac{1}{32\pi^2} \int_0^\infty \frac{e^2}{r^4} 4\pi r^2 dr = \infty$$

Born-Infeld

$$\nabla \cdot \mathbf{D} = e\delta^{(3)}(\mathbf{r}) \quad \mathbf{B} = 0$$

$$\mathbf{D} = \frac{e\mathbf{r}}{4\pi|\mathbf{r}|^3} \quad \mathbf{D}^2/b^2 = \frac{q^2}{r^4}$$

$$E_{\text{tot}} = 4\pi b^2 \int_0^\infty dr r^2 \left(\sqrt{1 + q^2/r^4} - 1 \right)$$

$$= \frac{4\Gamma^2(5/4)\sqrt{e^3b}}{3\pi} = 1.2361\sqrt{e^3b}$$

Dealing Gravity similar to QED is extremely hard

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} \right)$$

One loop pure gravitational action is renormalizable

Beyond two loops it is hard to compute, number of Feynman diagrams increases rapidly

Quadratic Curvature Gravity is renormalizable, but contains “Ghosts”: Vacuum is Unstable

Constructing Singularity Free & Ghost Free version for Gravity

Consistent theory of Gravity around Constant Curvature

Backgrounds

Criteria for resolving Cosmological Singularity

Divergence structures in 1 and 2-loops in a scalar Toy model

Without SUSY and SUGRA : SUSY is broken for a generic time dependent scenarios



Consistent General Covariant Quadratic Theories of Gravity with Constant Curvature Backgrounds

“Perturbative Unitarity”

“Ghost Free”

“Tachyon Free”

**“Correct degrees of freedom in
Graviton Propagator”**

Spin-2

&

Spin-0

**components
of a
Graviton
Propagator**

4th Derivative Gravity & Power Counting renormalizability

$$I = \int d^4x \sqrt{g} \left[\lambda_0 + k R + a R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} (b + a) R^2 \right]$$

$$D \propto \frac{1}{k^4 + Ak^2} = \frac{1}{A} \left(\frac{1}{k^2} - \frac{1}{k^2 + A} \right)$$

Massive Spin-0 & Massive Spin-2 (Ghost) Stelle (1977)

Utiyama, De Witt (1961), Stelle (1977)

Modification of Einstein's GR

Modification
of Graviton
Propagator

Extra propagating
degree of freedom

Challenge: to get rid of the extra dof

Ghosts

Higher Order Derivative Theory Generically Carry Ghosts (-ve Residue) with real “m” (No-Tachyon)

$$S = \int d^4x \phi \square (\square + m^2) \phi \Rightarrow \square (\square + m^2) \phi = 0$$

$$\Delta(p^2) = \frac{1}{p^2(p^2+m^2)} \sim \frac{1}{p^2} - \frac{1}{(p^2+m^2)}$$

Propagator with first order poles

Ghosts cannot be cured order by order, finite terms in perturbative expansion will always lead to Ghosts !!

$$\square e^{-\square} \phi = 0$$

No extra states other than the original dof.

Moffat (1991), Tomboulis (1997), Tseytlin (1997), Siegel (2003), Biswas, Grisaru, Siegel (2004), Biswas, Mazumdar, Siegel (2006)

Higher order Construction of Gravity in Any Arbitrary Background

$$S = S_E + S_q$$

$$S_q = \int d^4x \sqrt{-g} R_{\mu_1 \nu_1 \lambda_1 \sigma_1} \mathcal{O}_{\mu_2 \nu_2 \lambda_2 \sigma_2}^{\mu_1 \nu_1 \lambda_1 \sigma_1} R^{\mu_2 \nu_2 \lambda_2 \sigma_2}$$

Covariant derivatives

**Unknown Infinite
Functions of Derivatives**

Well defined Minkowski Limit:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$R \sim \mathcal{O}(h) \quad S_q \sim \int d^4x \sqrt{-g} \mathcal{O}(h^2)$$

Redundancies & Form Factors

$$\begin{aligned}
 S_q = & \int d^4x \sqrt{-g} [RF_1(\square)R + RF_2(\square)\nabla_\mu\nabla_\nu R^{\mu\nu} + R_{\mu\nu}F_3(\square)R^{\mu\nu} + R_\mu^\nu F_4(\square)\nabla_\nu\nabla_\lambda R^{\mu\lambda} \\
 & + R^{\lambda\sigma}F_5(\square)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\lambda R^{\mu\nu} + RF_6(\square)\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\lambda}F_7(\square)\nabla_\nu\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
 & + R_\lambda^\rho F_8(\square)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\rho R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1}F_9(\square)\nabla_{\mu_1}\nabla_{\nu_1}\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
 & + R_{\mu\nu\lambda\sigma}F_{10}(\square)R^{\mu\nu\lambda\sigma} + R_{\mu\nu\lambda}^\rho F_{11}(\square)\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\rho_1\nu\sigma_1}F_{12}(\square)\nabla^{\rho_1}\nabla^{\sigma_1}\nabla_\rho\nabla_\sigma R^{\mu\rho\nu\sigma} \\
 & + R_\mu^{\nu_1\rho_1\sigma_1}F_{13}(\square)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1\rho_1\sigma_1}F_{14}(\square)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_{\mu_1}\nabla_\mu\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma}
 \end{aligned}$$

$$= \int d^4x \sqrt{-g} [R + R\mathcal{F}_1(\square)R + R_{\mu\nu}\mathcal{F}_2(\square)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\square)R^{\mu\nu\alpha\beta}]$$

- (1) GR
- (2) Weyl Gravity
- (3) F(R) Gravity
- (4) Gauss-Bonnet Gravity
- (5) Ghost free Gravity

**UV completion of Starobinsky Inflation
up to quadratic in curvature**

Biswas, Mazumdar, Siegel, 2006,

Chialva, Mazumdar, 2013,

Koshelev, Modesto, Rachwal, Starobinsky, 2016

Linearised Equations of Motion around Minkowski

$$= \int d^4x \sqrt{-g} \left[R + R\mathcal{F}_1(\square)R + R_{\mu\nu}\mathcal{F}_2(\square)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\square)R^{\mu\nu\alpha\beta} \right]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$S_q = - \int d^4x \left[\frac{1}{2} h_{\mu\nu} a(\square) \square h^{\mu\nu} + h_{\mu}^{\sigma} b(\square) \partial_{\sigma} \partial_{\nu} h^{\mu\nu} \quad (3) \right. \\ \left. + hc(\square) \partial_{\mu} \partial_{\nu} h^{\mu\nu} + \frac{1}{2} hd(\square) \square h + h^{\lambda\sigma} \frac{f(\square)}{\square} \partial_{\sigma} \partial_{\lambda} \partial_{\mu} \partial_{\nu} h^{\mu\nu} \right]$$

$$a(\square) = 1 - \frac{1}{2} \mathcal{F}_2(\square) \square - 2\mathcal{F}_3(\square) \square$$

$$b(\square) = -1 + \frac{1}{2} \mathcal{F}_2(\square) \square + 2\mathcal{F}_3(\square) \square$$

$$c(\square) = 1 + 2\mathcal{F}_1(\square) \square + \frac{1}{2} \mathcal{F}_2(\square) \square$$

$$d(\square) = -1 - 2\mathcal{F}_1(\square) \square - \frac{1}{2} \mathcal{F}_2(\square) \square$$

$$f(\square) = -2\mathcal{F}_1(\square) \square - \mathcal{F}_2(\square) \square - 2\mathcal{F}_3(\square) \square.$$

$$R_{\mu\nu\lambda\sigma} = \frac{1}{2} (\partial_{[\lambda} \partial_{\nu} h_{\mu\sigma]} - \partial_{[\lambda} \partial_{\mu} h_{\nu\sigma]})$$

$$R_{\mu\nu} = \frac{1}{2} (\partial_{\sigma} \partial_{(\nu} h_{\mu)}^{\sigma} - \partial_{\nu} \partial_{\mu} h - \square h_{\mu\nu})$$

$$R = \partial_{\nu} \partial_{\mu} h^{\mu\nu} - \square h$$

$$a + b = 0$$

$$c + d = 0$$

$$b + c + f = 0$$

Similar analysis has been derived for dS and AdS

Graviton Propagator around Minkowski

$$a(\square)\square h_{\mu\nu} + b(\square)\partial_\sigma\partial_{(\nu}h_{\mu)}^\sigma + c(\square)(\eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \partial_\mu\partial_\nu h) \\ + \eta_{\mu\nu}d(\square)\square h + \frac{1}{4}f(\square)\square^{-1}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\lambda\sigma} = -\kappa\tau_{\mu\nu}$$

$$-\kappa\tau\nabla_\mu\tau_\nu^\mu = 0 = (c + d)\square\partial_\nu h + (a + b)\square h_{\nu,\mu}^\mu + (b + c + f)h_{,\alpha\beta\nu}^{\alpha\beta}$$

Bianchi Identity

$$a + b = 0$$

$$c + d = 0$$

$$b + c + f = 0$$

$$\Pi_{\mu\nu}^{-1\lambda\sigma} h_{\lambda\sigma} = \kappa\tau_{\mu\nu} \quad h = h^{TT} + h^L + h^T$$

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a - 3c)k^2}$$

Spin projection operators

Let us introduce

$$\begin{aligned}
 \mathcal{P}^2 &= \frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma} + \theta_{\mu\sigma}\theta_{\nu\rho}) - \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}, \\
 \mathcal{P}^1 &= \frac{1}{2}(\theta_{\mu\rho}\omega_{\nu\sigma} + \theta_{\mu\sigma}\omega_{\nu\rho} + \theta_{\nu\rho}\omega_{\mu\sigma} + \theta_{\nu\sigma}\omega_{\mu\rho}), \\
 \mathcal{P}_s^0 &= \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}, \quad \mathcal{P}_w^0 = \omega_{\mu\nu}\omega_{\rho\sigma}, \\
 \mathcal{P}_{sw}^0 &= \frac{1}{\sqrt{3}}\theta_{\mu\nu}\omega_{\rho\sigma}, \quad \mathcal{P}_{ws}^0 = \frac{1}{\sqrt{3}}\omega_{\mu\nu}\theta_{\rho\sigma},
 \end{aligned} \tag{16}$$

where the transversal and longitudinal projectors in the momentum space are respectively

$$\theta_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, \quad \omega_{\mu\nu} = \frac{k_\mu k_\nu}{k^2}.$$

Note that the operators \mathcal{P}^i are in fact 4-rank tensors, $\mathcal{P}_{\mu\nu\rho\sigma}^i$, but we have suppressed the index notation here.

Out of the six operators four of them, $\{\mathcal{P}^2, \mathcal{P}^1, \mathcal{P}_s^0, \mathcal{P}_w^0\}$, form a complete set of projection operators:

$$\mathcal{P}_a^i \mathcal{P}_b^j = \delta^{ij} \delta_{ab} \mathcal{P}_a^i \quad \text{and} \quad \mathcal{P}^2 + \mathcal{P}^1 + \mathcal{P}_s^0 + \mathcal{P}_w^0 = 1, \tag{17}$$

$$\mathcal{P}_{ij}^0 \mathcal{P}_k^0 = \delta_{jk} \mathcal{P}_{ij}^0, \quad \mathcal{P}_{ij}^0 \mathcal{P}_{kl}^0 = \delta_{il} \delta_{jk} \mathcal{P}_k^0, \quad \mathcal{P}_k^0 \mathcal{P}_{ij}^0 = \delta_{ik} \mathcal{P}_{ij}^0,$$

Ph.D. Thesis by K. J. Barnes, 1963

R. J. Rivers (1963)

P. Van Nieuwenhuizen,

Nucl.Phys. B60 (1973), 478.

For the above action, see:

Biswas, Koivisto, Mazumdar

1302.0532

Tree level Graviton Propagator

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a - 3c)k^2}$$

**No new propagating degree of freedom
other than the massless Graviton**

$$a(\square) = c(\square) \Rightarrow 2\mathcal{F}_1(\square) + \mathcal{F}_2(\square) + 2\mathcal{F}_3(\square) = 0$$

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + R\mathcal{F}_1(\square)R - \frac{1}{2}R^{\mu\nu}\mathcal{F}_2(\square)R_{\mu\nu} \right]$$

Without loss of generality either \mathcal{F}_1 , or \mathcal{F}_2 , or $\mathcal{F}_3 = 0$

Well known Higher Derivative limits

$$a(\square) = 1 - \frac{1}{2}\mathcal{F}_2(\square)\square - 2\mathcal{F}_3(\square)\square$$

$$b(\square) = -1 + \frac{1}{2}\mathcal{F}_2(\square)\square + 2\mathcal{F}_3(\square)\square$$

$$c(\square) = 1 + 2\mathcal{F}_1(\square)\square + \frac{1}{2}\mathcal{F}_2(\square)\square$$

$$d(\square) = -1 - 2\mathcal{F}_1(\square)\square - \frac{1}{2}\mathcal{F}_2(\square)\square$$

$$f(\square) = -2\mathcal{F}_1(\square)\square - \mathcal{F}_2(\square)\square - 2\mathcal{F}_3(\square)\square.$$

(3) GB Gravity:

$$\mathcal{L} = R + \alpha(\square)G,$$

$$a = c = -b = -d = 1$$

$$\Pi = \Pi_{GR}$$

(1) GR:

$$a(0) = c(0) = -b(0) = -d(0) = 1$$

$$\lim_{k^2 \rightarrow 0} \Pi = (\mathcal{P}^2/k^2) - (\mathcal{P}_s^0/2k^2) \equiv \Pi_{GR}$$

(2) F(R) Gravity:

$$\mathcal{L}(R) = \mathcal{L}(0) + \mathcal{L}'(0)R + \frac{1}{2}\mathcal{L}''(0)R^2 + \dots$$

$$a = -b = 1, \quad c = -d = 1 - \mathcal{L}''(0)\square$$

$$\Pi = \frac{\mathcal{P}^2}{k^2} - \frac{\mathcal{P}_s^0}{2k^2(1 + 3\mathcal{L}''(0)k^2)} \quad \Pi = \Pi_{GR} + \frac{1}{2} \frac{\mathcal{P}_s^0}{k^2 + m^2}, \quad m^2 = \frac{1}{3\mathcal{L}''(0)}$$

(4) Weyl Gravity:

$$\mathcal{L} = R - \frac{1}{m^2}C^2 \quad C^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$$

$$a = -b = 1 - (k/m)^2$$

$$c = -d = 1 - (k/m)^2/3 \text{ and } f = -2(k/m)^2/3$$

$$\Pi = \frac{\mathcal{P}^2}{k^2(1 - (k/m)^2)} - \frac{\mathcal{P}_s^0}{2k^2} = \Pi_{GR} - \frac{\mathcal{P}^2}{k^2 + m^2}$$

Complete Field Equations

Ghost-free gravity

2.3. The Complete Field Equations

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2} + R\mathcal{F}_1(\square)R + R^{\mu\nu}\mathcal{F}_2(\square)R_{\mu\nu} + C^{\mu\nu\lambda\sigma}\mathcal{F}_3(\square)C_{\mu\nu\lambda\sigma} \right)$$

Following from this we find the equation of motion for the full action S in (1) to be a combination of S_0 , S_1 , S_2 and S_3 above

$$\begin{aligned} P^{\alpha\beta} &= G^{\alpha\beta} + 4G^{\alpha\beta}\mathcal{F}_1(\square)R + g^{\alpha\beta}R\mathcal{F}_1(\square)R - 4(\nabla^\alpha\nabla^\beta - g^{\alpha\beta}\square)\mathcal{F}_1(\square)R \\ &\quad - 2\Omega_1^{\alpha\beta} + g^{\alpha\beta}(\Omega_{1\sigma}^\sigma + \bar{\Omega}_1) + 4R_\mu^\alpha\mathcal{F}_2(\square)R^{\mu\beta} \\ &\quad - g^{\alpha\beta}R_\nu^\mu\mathcal{F}_2(\square)R_\mu^\nu - 4\nabla_\mu\nabla^\beta(\mathcal{F}_2(\square)R^{\mu\alpha}) + 2\square(\mathcal{F}_2(\square)R^{\alpha\beta}) \\ &\quad + 2g^{\alpha\beta}\nabla_\mu\nabla_\nu(\mathcal{F}_2(\square)R^{\mu\nu}) - 2\Omega_2^{\alpha\beta} + g^{\alpha\beta}(\Omega_{2\sigma}^\sigma + \bar{\Omega}_2) - 4\Delta_2^{\alpha\beta} \\ &\quad - g^{\alpha\beta}C^{\mu\nu\lambda\sigma}\mathcal{F}_3(\square)C_{\mu\nu\lambda\sigma} + 4C_{\mu\nu\sigma}^\alpha\mathcal{F}_3(\square)C^{\beta\mu\nu\sigma} \\ &\quad - 4(R_{\mu\nu} + 2\nabla_\mu\nabla_\nu)(\mathcal{F}_3(\square)C^{\beta\mu\nu\alpha}) - 2\Omega_3^{\alpha\beta} + g^{\alpha\beta}(\Omega_{3\gamma}^\gamma + \bar{\Omega}_3) - 8\Delta_3^{\alpha\beta} \\ &= T^{\alpha\beta}, \end{aligned} \quad (52)$$

where $T^{\alpha\beta}$ is the stress energy tensor for the matter components in the universe and we have defined the following symmetric tensors:

$$\Omega_1^{\alpha\beta} = \sum_{n=1}^{\infty} f_{1n} \sum_{l=0}^{n-1} \nabla^\alpha R^{(l)} \nabla^\beta R^{(n-l-1)}, \quad \bar{\Omega}_1 = \sum_{n=1}^{\infty} f_{1n} \sum_{l=0}^{n-1} R^{(l)} R^{(n-l)}, \quad (53)$$

$$\Omega_2^{\alpha\beta} = \sum_{n=1}^{\infty} f_{2n} \sum_{l=0}^{n-1} R_\nu^{\mu;\alpha(l)} R_\mu^{\nu;\beta(n-l-1)}, \quad \bar{\Omega}_2 = \sum_{n=1}^{\infty} f_{2n} \sum_{l=0}^{n-1} R_\nu^{\mu(l)} R_\mu^{\nu(n-l)}, \quad (54)$$

$$\Delta_2^{\alpha\beta} = \frac{1}{2} \sum_{n=1}^{\infty} f_{2n} \sum_{l=0}^{n-1} [R_\sigma^{\nu(l)} R^{(\beta|\sigma|\alpha)(n-l-1)} - R_\sigma^{\nu;\alpha(l)} R^{\beta\sigma(n-l-1)}]_{;\nu}, \quad (55)$$

$$\Omega_3^{\alpha\beta} = \sum_{n=1}^{\infty} f_{3n} \sum_{l=0}^{n-1} C_{\nu\lambda\sigma}^{\mu;\alpha(l)} C_\mu^{\nu\lambda\sigma;\beta(n-l-1)}, \quad \bar{\Omega}_3 = \sum_{n=1}^{\infty} f_{3n} \sum_{l=0}^{n-1} C_{\nu\lambda\sigma}^{\mu(l)} C_\mu^{\nu\lambda\sigma(n-l)}, \quad (56)$$

$$\Delta_3^{\alpha\beta} = \frac{1}{2} \sum_{n=1}^{\infty} f_{3n} \sum_{l=0}^{n-1} [C_{\sigma\mu}^{\lambda\nu(l)} C_\lambda^{(\beta)\sigma\mu;\alpha(n-l-1)} - C_{\sigma\mu}^{\lambda\nu;\alpha(l)} C_\lambda^{\beta)\sigma\mu(n-l-1)}]_{;\nu}. \quad (57)$$

The trace equation is often particularly useful and below we provide it for the general action (1):

$$\begin{aligned} P &= -R + 12\square\mathcal{F}_1(\square)R + 2\square(\mathcal{F}_2(\square)R) + 4\nabla_\mu\nabla_\nu(\mathcal{F}_2(\square)R^{\mu\nu}) \\ &\quad + 2(\Omega_{1\sigma}^\sigma + 2\bar{\Omega}_1) + 2(\Omega_{2\sigma}^\sigma + 2\bar{\Omega}_2) + 2(\Omega_{3\sigma}^\sigma + 2\bar{\Omega}_3) - 4\Delta_{2\sigma}^\sigma - 8\Delta_{3\sigma}^\sigma \\ &= T \equiv g_{\alpha\beta}T^{\alpha\beta}. \end{aligned} \quad (58)$$

It is worth noting that we have checked special cases of our result against previous work in sixth order gravity given in [24] and found them to be equivalent at least to the cubic order (see Appendix C for details).

$$R^{(m)} \equiv \square^m R$$

Biswas, Conroy, Koshelev, Mazumdar
1308.2319 Class.Quant. Grav. (2014)

Cosmological
Bouncing solution is
known exactly

Biswas, Mazumdar, Siegel, 2006,

Stability of Hamiltonian

Hamiltonian Analysis for Infinite Derivative Field Theories and Gravity

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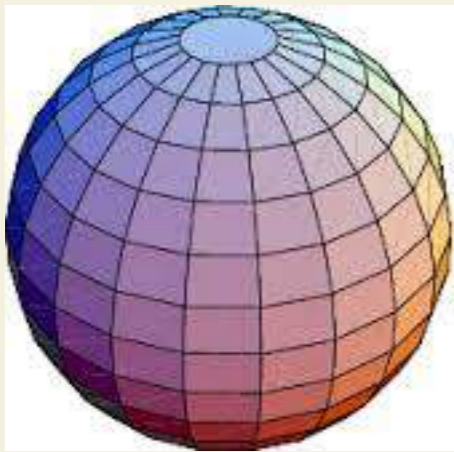
E-mail: a.mazumdar@lancaster.ac.uk, s.talaganis@lancaster.ac.uk, a.teimouri@lancaster.ac.uk

Abstract

Typically higher-derivative theories are unstable. Instabilities manifest themselves from *extra* propagating degrees of freedom, which are unphysical. In this paper, we will investigate *infinite derivative* field theories and study their *true* dynamical degrees of freedom via Hamiltonian analysis. In particular, we will show that if the infinite derivatives can be captured by a Gaussian kinetic term, i.e. *exponential of entire function*, then it is possible to prove that there are only *finite* number of dynamical degrees of freedom. This conclusion is similar to previous analyses which were performed in the context of Lagrangian analysis. We will further extend our investigation into infinite derivative theory of gravity, and in particular concentrate on *ghost free* and *singularity free* theory of gravity, which has been studied extensively in the Lagrangian approach. Here we will show from the Hamiltonian perspective that there are only finite number of degrees of freedom. For a homogeneous case, we will show that the Hamiltonian density can be bounded from below.

2	Hamiltonian from a Lagrangian	6
2.1	Constraints for a <i>singular</i> system	7
2.2	<i>First and second-class</i> constraints	9
2.3	Counting the degrees of freedom	10
3	Scalar field theory: a Toy model	10
3.1	Simple homogeneous scalar action	10
3.2	Finite degrees of freedom and boundedness of the Hamiltonian density	12
3.3	Scalar Lagrangian with covariant derivatives	16
3.4	Infinite derivative scalar field theory	17
3.4.1	Gaussian kinetic term and propagator	21
4	Infinite derivative gravity (IDG)	22
4.1	ADM formalism	23
4.2	ADM decomposition of an Infinite derivative gravity	25
4.3	Equivalent action and decomposition	25
5	Hamiltonian analysis	28
5.1	Hamiltonian for $f(R)$ gravity	28
5.1.1	Classification of constraints for $f(R)$ gravity	29
5.2	Number of physical degrees of freedom for $f(R)$ gravity	32
5.3	Constraints for IDG	34
5.3.1	Classifications of constraints for IDG	35
6	Physical degrees of freedom for IDG	37
6.1	Choice of $\mathcal{F}(\square)$	37
6.2	$\mathcal{F}(e^{\square})$	40
7	Conclusion	43
A	Hamiltonian density	45
B	Auxiliary fields χ_1 and η_1	46
C	$B(R - A)$ decomposition	46
D	Finding the physical degrees of freedom from propagator analysis	47

Gravitational Entropy



$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2d\Omega^2$$

$$S_W = -8\pi \oint_{r=r_H, t=\text{const}} \left(\frac{\partial \mathcal{L}}{\partial R_{rttr}} \right) q(r) d\Omega^2$$

Wald (1990, 1993), Iyer, Wald (1993)

$$S_W = \frac{Area}{4G} \left[1 + \alpha (2\mathcal{F}_1 + \mathcal{F}_2 + \underbrace{2\mathcal{F}_3}_0) R \right]$$

Holography is an IR effect

Higher order corrections yield zero entropy
“Ground State of Gravity”

Consistent theories of Gravity around dS and AdS backgrounds

$$S = \int d^4x \sqrt{-g} \left[\mathcal{P}_0 + \sum_i \mathcal{P}_i \prod_I (\hat{\mathcal{O}}_{iI} \mathcal{Q}_{iI}) \right]$$

Most generic action - “Parity Invariant” and “Torsion Free”

$$R = \bar{R} = \text{const}, \quad R_{\mu\nu} = \frac{\bar{R}}{4} \bar{g}_{\mu\nu}, \quad R^\rho_{\mu\sigma\nu} = \frac{\bar{R}}{12} (\delta^\rho_\sigma \bar{g}_{\mu\nu} - \delta^\rho_\nu \bar{g}_{\mu\sigma})$$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \Lambda + \frac{\lambda}{2} (R \mathcal{F}_1(\square) R + S_{\mu\nu} \mathcal{F}_2(\square) S^{\mu\nu} + C_{\mu\nu\lambda\sigma} \mathcal{F}_3(\square) C^{\mu\nu\lambda\sigma}) \right]$$

$$h_{\mu\nu} = h^\perp_{\mu\nu} + \bar{\nabla}_\mu A^\perp_\nu + \bar{\nabla}_\nu A^\perp_\mu + (\bar{\nabla}_\mu \bar{\nabla}_\nu - \frac{1}{4} \bar{g}_{\mu\nu} \bar{\square}) B + \frac{1}{4} \bar{g}_{\mu\nu} h$$

For pure EH action, see D'Hoker, Freedman, Mathur, Matusis, Rastelli (hep-th/9902042)

Quadratic order Action for spin-2 and spin-0 components

$$S_2 \equiv \frac{1}{2} \int dx^4 \sqrt{-\bar{g}} \widetilde{h}^{\perp\mu\nu} \left(\bar{\square} - \frac{\bar{R}}{6} \right) \left\{ 1 + \frac{2}{M_p^2} \lambda c_{1,0} \bar{R} + \frac{\lambda}{M_p^2} \left[\left(\bar{\square} - \frac{\bar{R}}{6} \right) \mathcal{F}_2(\bar{\square}) + 2 \left(\bar{\square} - \frac{\bar{R}}{3} \right) \mathcal{F}_3 \left(\bar{\square} + \frac{\bar{R}}{3} \right) \right] \right\} \widetilde{h}^{\perp}_{\mu\nu}$$

$$S_0 \equiv -\frac{1}{2} \int dx^4 \sqrt{-\bar{g}} \widetilde{\phi} \left(\bar{\square} + \frac{\bar{R}}{3} \right) \left\{ 1 + \frac{2}{M_p^2} \lambda c_{1,0} \bar{R} - \frac{\lambda}{M_p^2} \left[2(3\bar{\square} + \bar{R}) \mathcal{F}_1(\bar{\square}) + \frac{1}{2} \bar{\square} \mathcal{F}_2 \left(\bar{\square} + \frac{2}{3} \bar{R} \right) \right] \right\} \widetilde{\phi}$$

**Minkowski limit matches
with our earlier propagator**

$$\Pi_2 = \frac{i}{p^2 \left\{ 1 - \frac{2p^2}{M_p^2} [\mathcal{F}_2(-p^2) + 2\mathcal{F}_3(-p^2)] \right\}},$$

$$\Pi_0 = \frac{-i}{p^2 \left\{ 1 + \frac{2p^2}{M_p^2} [6\mathcal{F}_1(-p^2) + \frac{1}{2}\mathcal{F}_2(-p^2)] \right\}}$$

$$\widetilde{h}^{\perp}_{\mu\nu} = \frac{1}{2} M_p h^{\perp}_{\mu\nu}, \quad \widetilde{\phi} = \sqrt{\frac{3}{32}} M_p \phi$$

Biswas, Koshelev, Mazumdar

1602.08475

80th B'Day Celeb. of Carl Brans

Most generic Ghost Free Graviton Propagator in dS/AdS

$$\mathcal{T}(\bar{R}, \bar{\square}) \equiv 1 + \frac{4\bar{R}}{M_p^2} c_{1,0} + \frac{2}{M_p^2} \left[\left(\bar{\square} - \frac{\bar{R}}{6} \right) \mathcal{F}_2(\bar{\square}) + 2 \left(\bar{\square} - \frac{\bar{R}}{3} \right) \mathcal{F}_3 \left(\bar{\square} + \frac{\bar{R}}{3} \right) \right]$$

$$\mathcal{S}(\bar{R}, \bar{\square}) \equiv 1 + \frac{4\bar{R}}{M_p^2} c_{1,0} - \frac{2}{M_p^2} \left[2(3\bar{\square} + \bar{R}) \mathcal{F}_1(\bar{\square}) + \frac{1}{2} \bar{\square} \mathcal{F}_2 \left(\bar{\square} + \frac{2}{3} \bar{R} \right) \right]$$

$$\mathcal{T}(\bar{R}, \bar{\square}) \equiv e^{\tau(\bar{\square})},$$

$$\mathcal{S}(\bar{R}, \bar{\square}) \equiv \left(1 - \frac{\bar{\square}}{m^2} \right)^\epsilon e^{\sigma(\bar{\square})}$$

$\epsilon = 0$, No scalar propagating d.o.f.

Background Independent Action of Quadratic Action of Gravity

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + \alpha_0 (R, R_{\mu\nu}) + \alpha_1 (R, R_{\mu\nu}) R \mathcal{F}_1(\square) R + \alpha_2 (R, R_{\mu\nu}) R_{\mu\nu} \mathcal{F}_2(\square) R^{\mu\nu} + \alpha_3 (R, R_{\mu\nu}) C_{\mu\nu\lambda\sigma} \mathcal{F}_3 C^{\mu\nu\lambda\sigma} \right]$$

Proof is due

Newtonian Limit

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a - 3c)k^2} \quad a(\square) = c(\square) = e^{-\square/M^2}$$

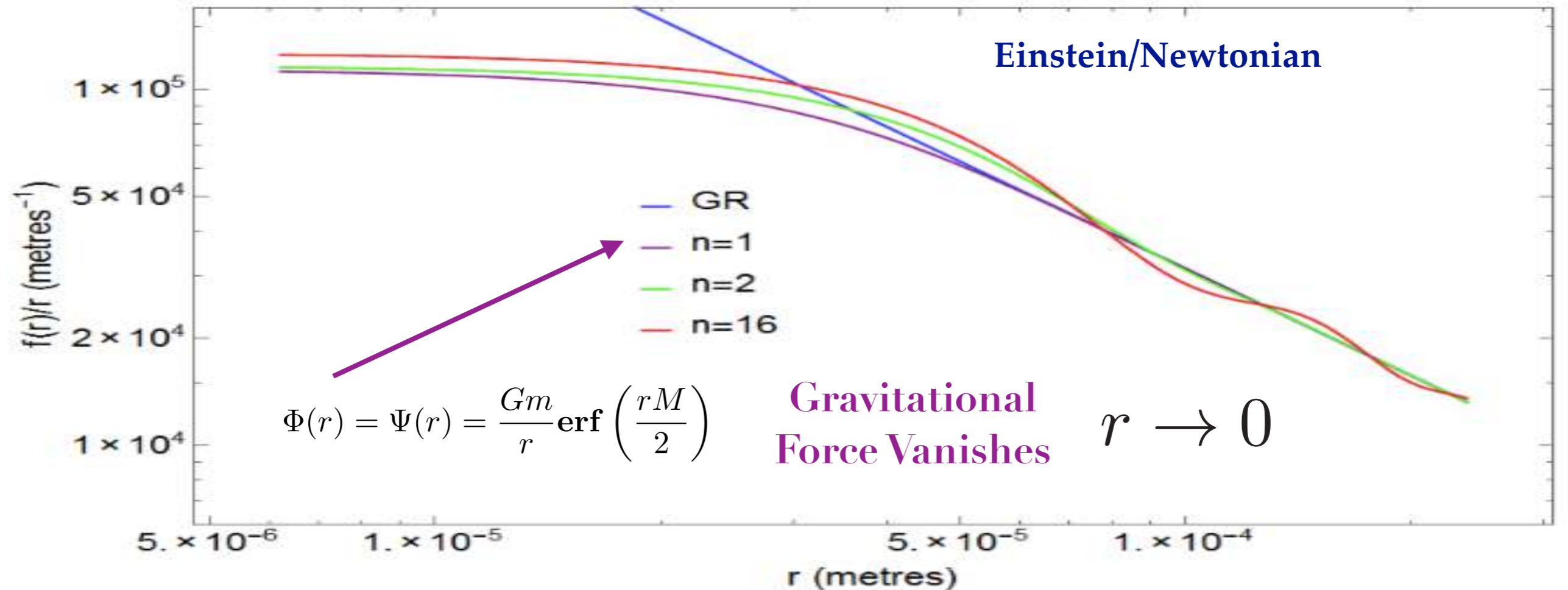
$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{-\frac{\square}{M^2}} - 1}{\square} \right] R - 2R_{\mu\nu} \left[\frac{e^{-\frac{\square}{M^2}} - 1}{\square} \right] R^{\mu\nu} \right]$$

$$ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Psi)dr^2$$

$$\Phi = \Psi = \frac{Gm}{r} \operatorname{erf} \left(\frac{rM}{2} \right)$$

Resolution of Singularity at short distances

$$a(\square) = e^{\gamma(\square)} \quad \text{Any Entire Function: } \gamma(\square) = -\frac{\square}{M^2} - \sum_N a_N \left(\frac{\square}{M^2}\right)^N$$



$$mM \ll M_p^2 \implies m \ll M_p$$

Current Bound : $M > 0.01 \text{ eV}$

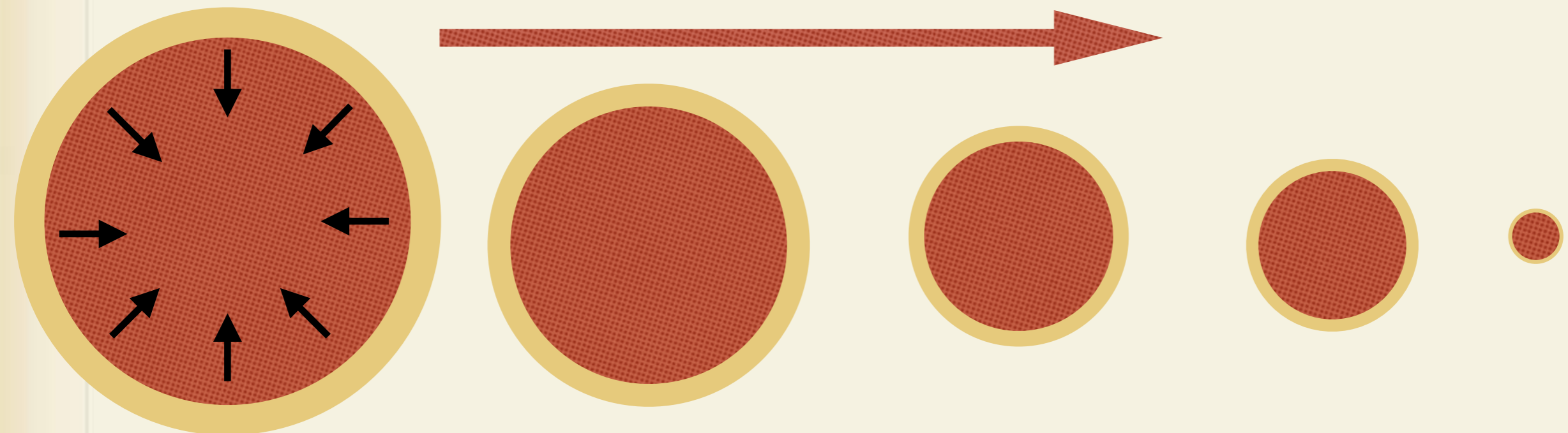
Edholm, Koshelev, Mazumdar (2016)

Frolov & Zelnikov (2015)

Dynamical Aspects

Valeri Frolov & Andrei Zelnikov
Studied various aspects in 5 papers
(2015, 2016)

Time

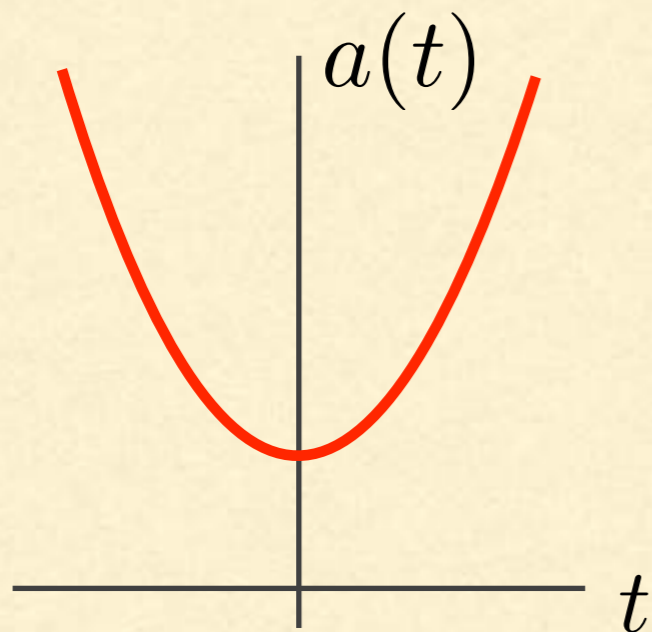


Conclusion: A lump of matter without Horizon
and without Singularity in a Linear regime

Cosmological Singularity can be resolved in a Full Non-linear Regime

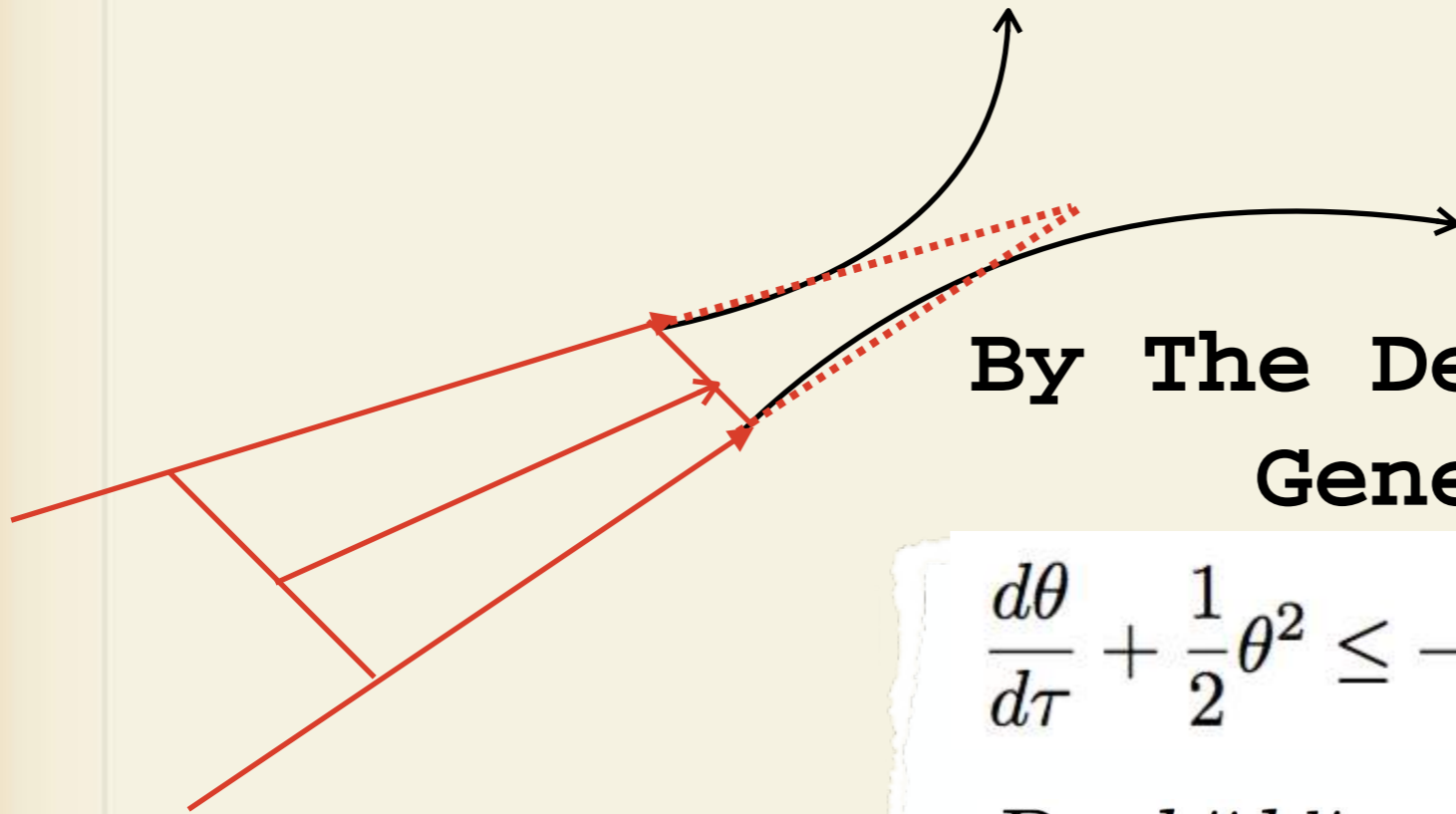
$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\square}{M^2} - 1}}{\square} \right] R + \Lambda \right]$$

$$\square R = c_1 R + c_2$$



$$a(t) = a_0 \cosh \left(\sqrt{\frac{\Lambda}{6M_{pl}^2}} t \right)$$

Cosmological Singularity can be resolved without an Ansatz



By The Defocusing Theorem of General Relativity

$$\frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq -R_{\mu\nu}k^\mu k^\nu$$

$$R_{\mu\nu}k^\mu k^\nu = \kappa T_{\mu\nu}k^\mu k^\nu$$

$$R_{\mu\nu}k^\mu k^\nu \geq 0, \quad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq 0$$

Einstein-Hilbert:

Infinite derivative Gravity:

$$R_{\mu\nu}k^\mu k^\nu \leq 0, \quad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \geq 0$$

3 Criteria for Defocusing Null Congruences without Ghosts & Tachyons

$$\frac{f(\bar{\square})\square}{a(\bar{\square})}R^{(L)} > 0 \Rightarrow \frac{a(\bar{\square}) - c(\bar{\square})}{a(\bar{\square})}R^{(L)} > 0$$

$$c(\bar{\square}) = \frac{a(\bar{\square})}{3} [1 + 2(1 - \alpha M_P^{-2}\square)\tilde{a}(\bar{\square})]$$

$$S = \frac{1}{2} \int d^4x \sqrt{-g} [M_P^2 R + R\mathcal{F}_1(\bar{\square})R]$$

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2}$$

**Massless Graviton
for : a=c**

(1) Infinite Derivatives

Locality leads to Starobinsky Model, which requires Tachyonic massive Spin-0 states to resolve singularity, but it cannot give Inflation !

(2) Massless Spin-2,

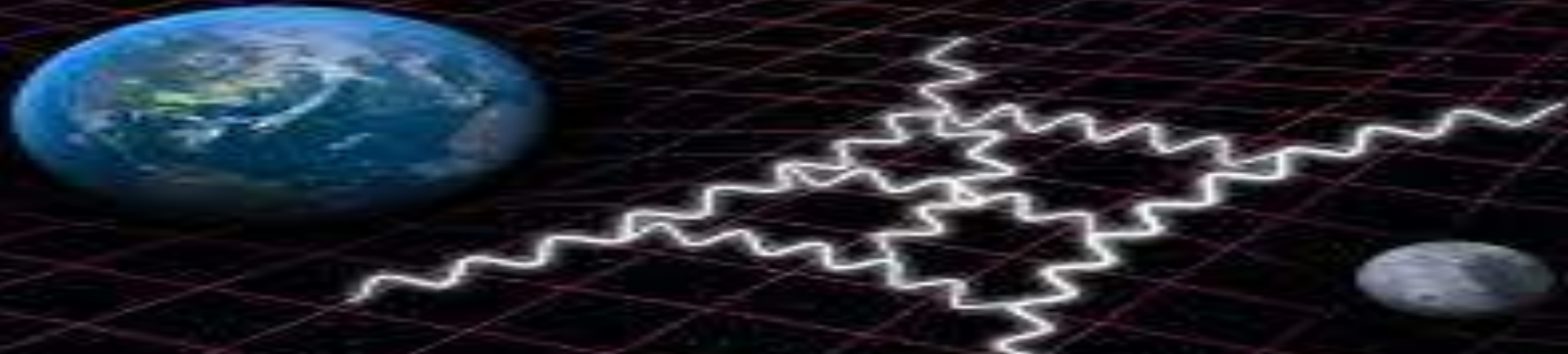
(3) Non-Tachyonic Massive Spin-0

$$\Pi(-k^2) = \frac{1}{a(-k^2)} \left[\frac{\mathcal{P}^2}{k^2} - \frac{1}{2\tilde{a}(-k^2)} \left(\frac{\mathcal{P}_s^0}{k^2} - \frac{\mathcal{P}_s^0}{k^2 + m^2} \right) \right]$$

$$S = \frac{1}{2} \int d^4x \sqrt{-g} [M_p^2 R + cR^2]$$

$$\Pi_{R^2} = \Pi_{GR} + \frac{1}{2} \frac{\mathcal{P}_s^0}{k^2 + m^2}$$

Quantum Aspects



How to make Gravity UV Finite ?

Could we make Gravity weak in UV?

Some interesting progress have been made:

Gravitational entropy, Boundary action, Hamiltonian, Quantum loop corrections, Ultra high energy scatterings, etc.

Toy model based on Symmetries

GR e.o.m : $g_{\mu\nu} \rightarrow \Omega g_{\mu\nu}$

Around Minkowski space the e.o.m are invariant under:

$$h_{\mu\nu} \rightarrow (1 + \epsilon)h_{\mu\nu} + \epsilon\eta_{\mu\nu}$$

Construct a scalar field theory with infinite derivatives whose e.o.m are invariant under

$$\phi \rightarrow (1 + \epsilon)\phi + \epsilon$$

$$S_{free} = \frac{1}{2} \int d^4x (\phi \square a(\square) \phi) \quad a(\square) = e^{-\square/M^2}$$

$$S_{int} = \frac{1}{M_p} \int d^4x \left(\frac{1}{4} \phi \partial_\mu \phi \partial^\mu \phi + \frac{1}{4} \phi \square \phi a(\square) \phi - \frac{1}{4} \phi \partial_\mu \phi a(\square) \partial^\mu \phi \right)$$

$$\Pi(k^2) = -\frac{i}{k^2 e^{\bar{k}^2}}$$

Quantum aspects

- **Superficial degree of divergence goes as**

$E = V - I$. Use Topological relation : $L = 1 + I - V$

$$E = 1 - L \quad E < 0, \text{ for } L > 1$$

- **At 1-loop, the theory requires counter term, the 1-loop, 2 point function yields Λ^4 divergence**
- **At 2-loops, the theory does not give rise to additional divergences, the UV behaviour becomes finite, at large external momentum, where dressed propagators gives rise to more suppression than the vertex factors**

Towards understanding the ultraviolet behavior of quantum loops in infinite-derivative theories of gravity

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Abstract

In this paper we will consider quantum aspects of a non-local, infinite derivative scalar field theory - a *toy model* depiction of a covariant infinite derivative, non-local extension of Einstein's general relativity which has previously been shown to be free from ghosts around the Minkowski background. The graviton propagator in this theory gets an exponential suppression making it *asymptotically free*, thus providing strong prospects of resolving various classical and quantum divergences. In particular, we will find that at 1-loop, the 2-point function is still divergent, but once this amplitude is renormalized by adding appropriate counter terms, the ultraviolet (UV) behavior of all other 1-loop diagrams as well as the 2-loop, 2-point function remains well under control. We will go on to discuss how one may be able to generalize our computations and arguments to arbitrary loops.

Ultra High Energy Scatterings of Scalar Gravitons

High-Energy Scatterings in Infinite-Derivative Field Theory and Ghost-Free Gravity

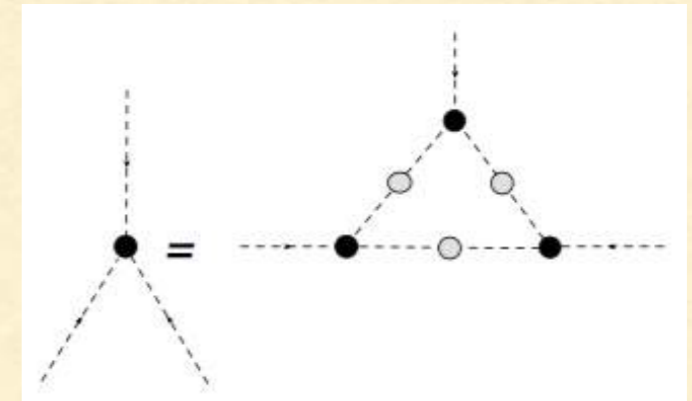
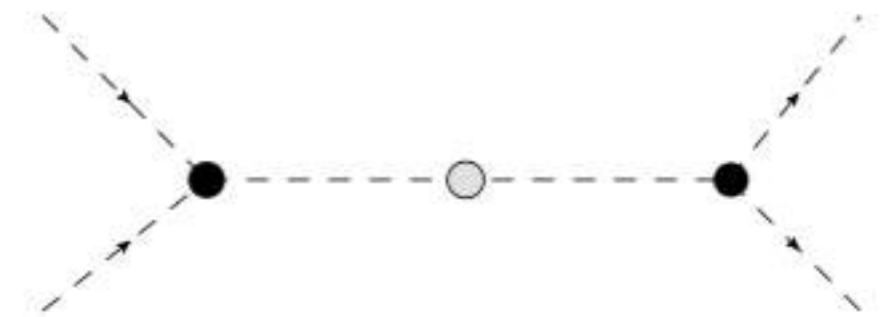
Spyridon Talaganis and Anupam Mazumdar

Consortium for Fundamental Physics, Lancaster University, LA1 4YB

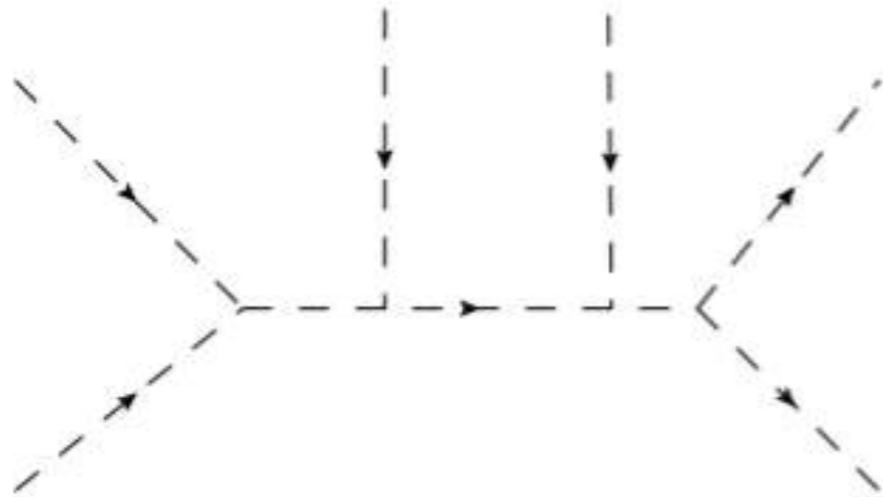
March 14, 2016

Abstract

In this paper, we will consider scattering diagrams in the context of infinite-derivative theories. First, we examine a finite-order higher-derivative scalar field theory and find that we cannot eliminate the external momentum divergences of scattering diagrams in the regime of large external momenta. Then, we employ an infinite-derivative scalar toy model and obtain that the external momentum dependence of scattering diagrams is convergent as the external momenta become very large. In order to eliminate the external momentum divergences, one has to dress the bare vertices of the scattering diagrams by considering renormalised propagator and vertex loop corrections to the bare vertices. Finally, we investigate scattering diagrams in the context of a scalar toy model which is inspired by a *ghost-free* and *singularity-free* infinite-derivative theory of gravity, where we conclude that infinite derivatives can eliminate the external momentum divergences of scattering diagrams and make the scattering diagrams convergent in the ultraviolet.

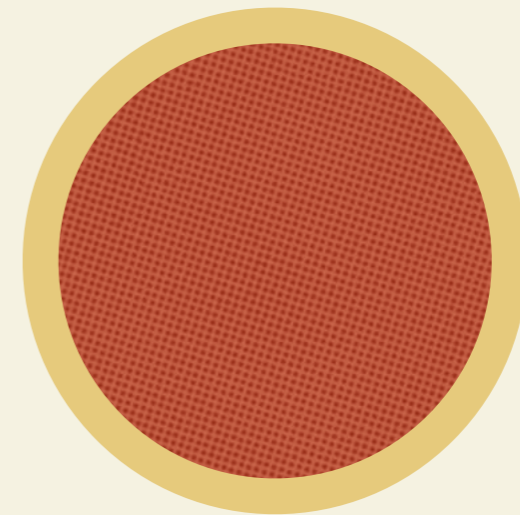


Ultra High Energy Scatterings of many Scalar Gravitons : **preliminary computations**



$$\mathcal{M}' \sim e^{-\left(\frac{p}{M_{eff}}\right)^2},$$
$$M_{eff} = \left(\frac{54}{125n - 206}\right)^{1/2} M$$

**Non-Locality can be spread out on
Blackhole Horizon scale**



$$r \sim r_* \sim (2GM_{star}) \gg M$$

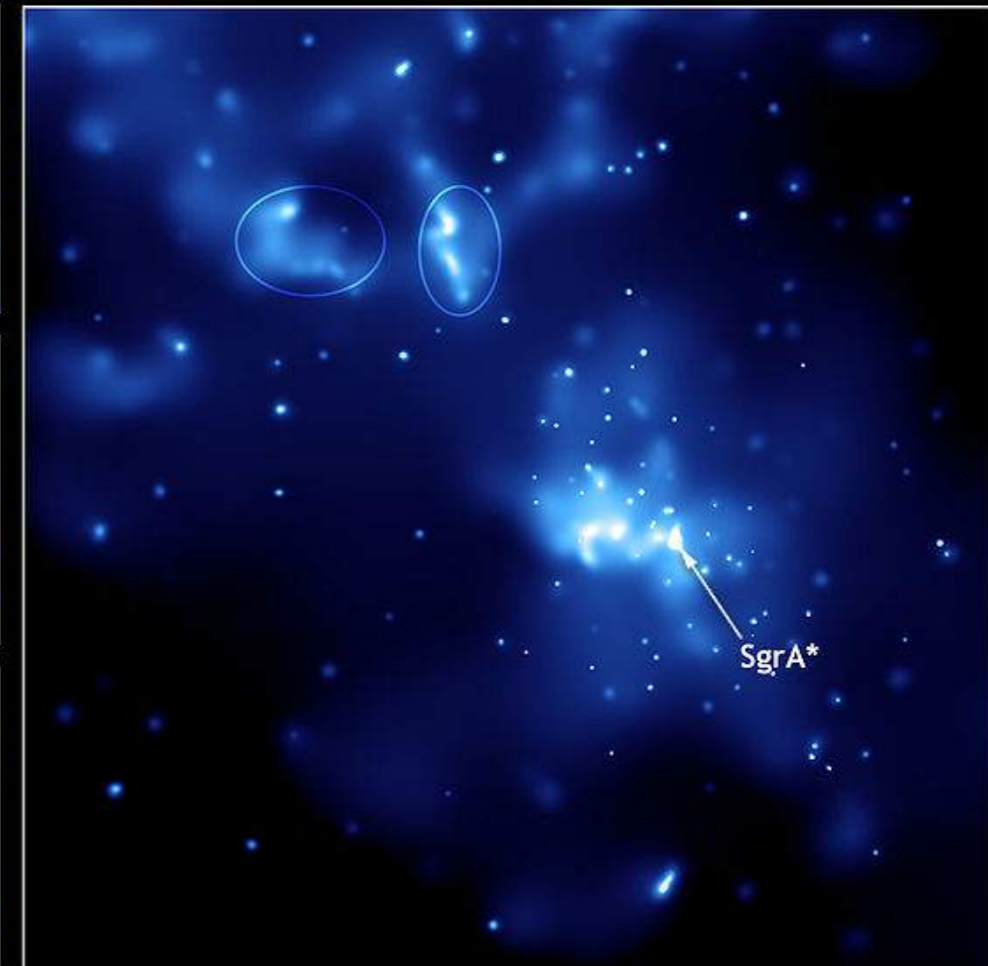
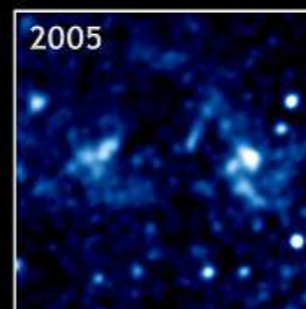
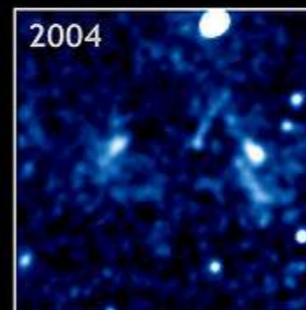
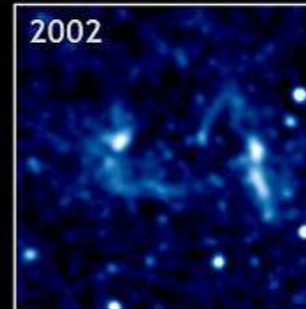
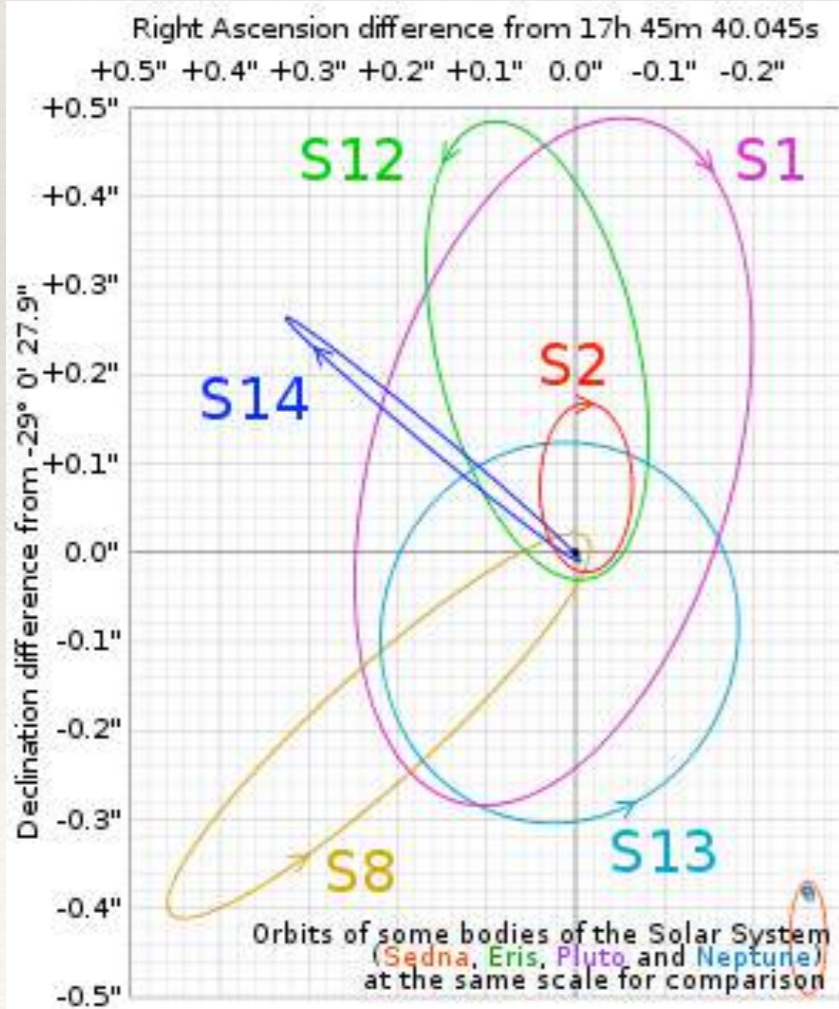
Conjecture : Gravity can be made weak not to form a trapped surface

Horizon or No Horizon!



Remnant of Non-locality

Event Horizon Telescope



Conclusions

- We have constructed a Ghost Free & Singularity Free Theory of Gravity.
- Studying singularity theorems, Hawking radiation, Non-Singular Bouncing Cosmology , , many interesting problems has been studied in this framework.
- Holography is not a property of UV, becomes part of an IR effect.
- Quantum computations also show that Infinite Derivative Gravity can ameliorate UV behaviour.
- Ultra-High energy graviton scatterings do not blow up.

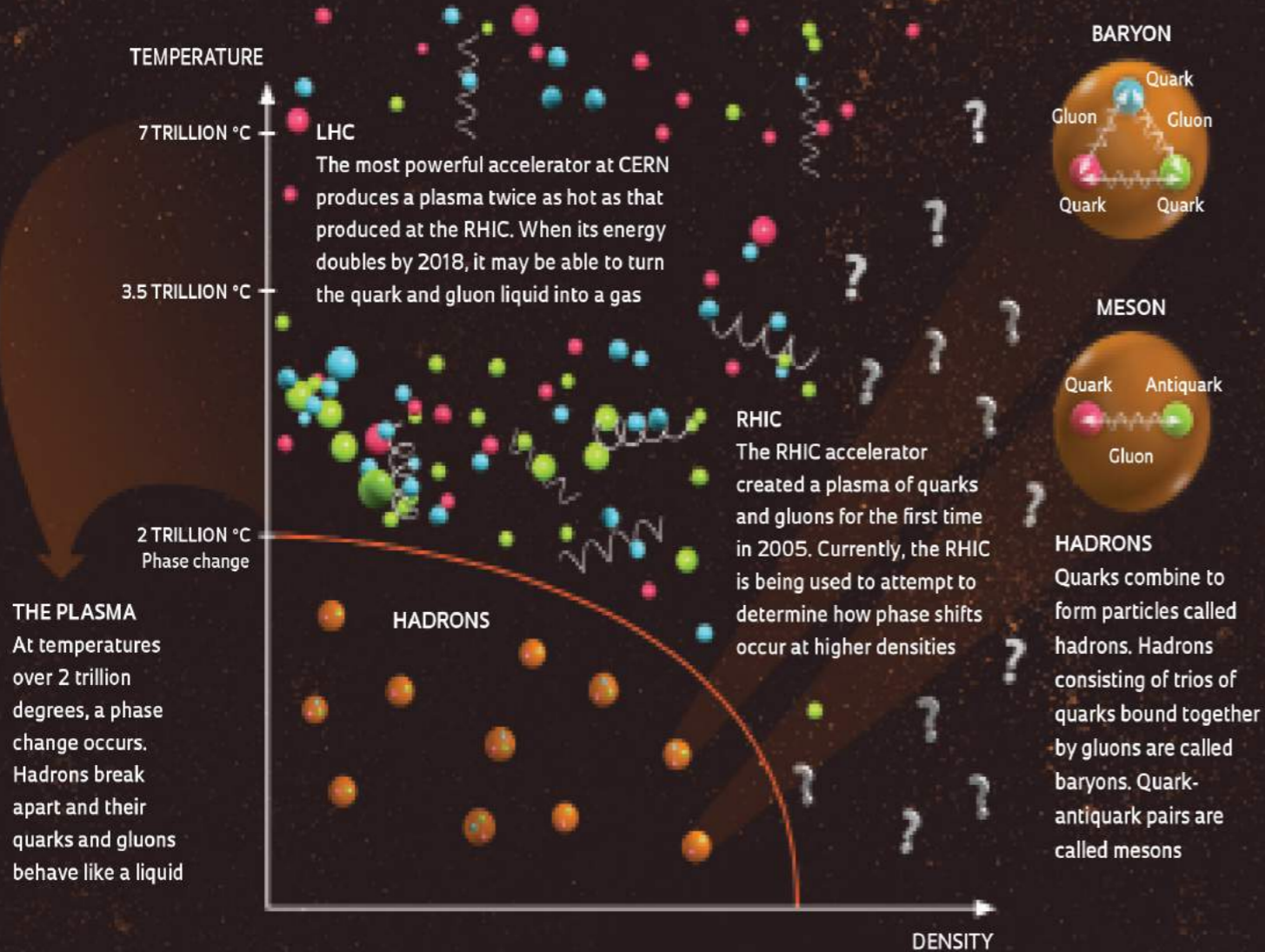
**All these consequences have ramifications for
Blackhole, Inflation & Quantum aspects of Gravity:
Both are Time Dependent Problems**

Extra Slides

Asymptotically Free QCD

Attempts to melt hadrons

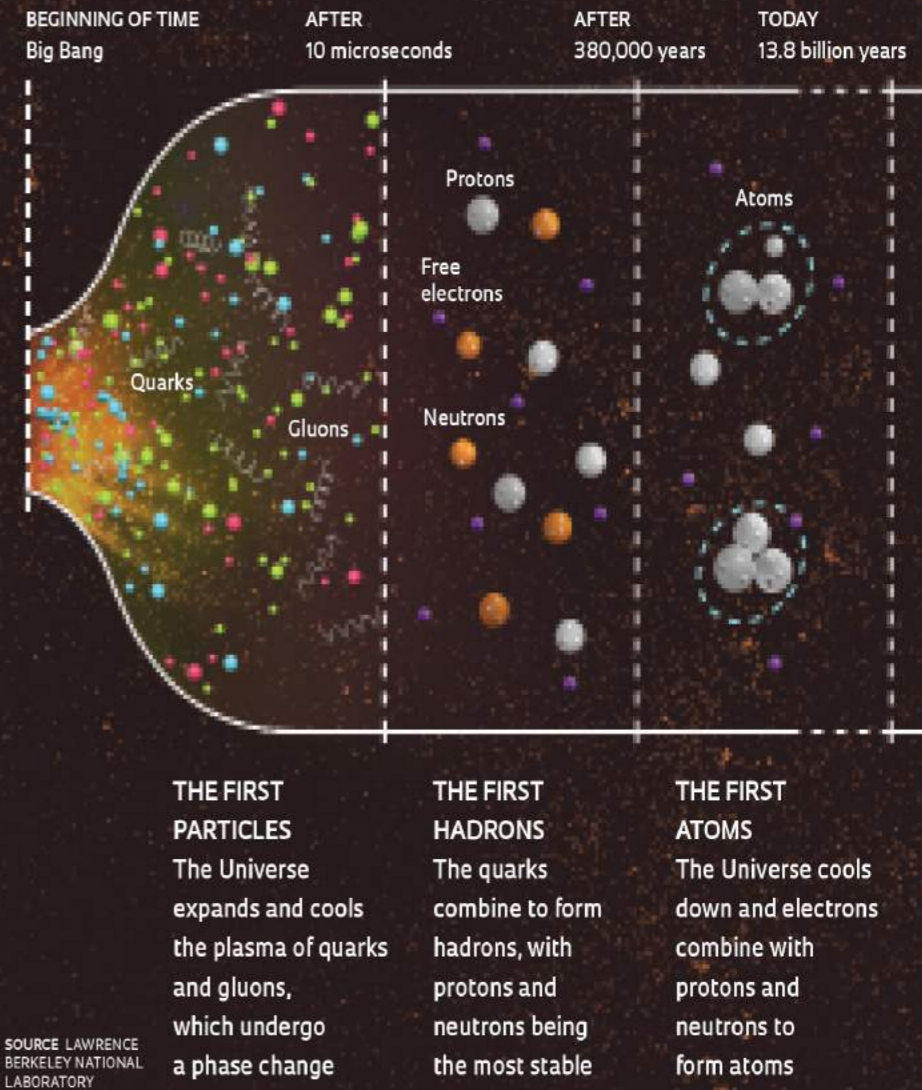
Experiments begin to map the behavior of quarks and gluons at different temperatures and densities



SOURCE BROOKHAVEN NATIONAL LABORATORY

THE HISTORY OF THE UNIVERSE

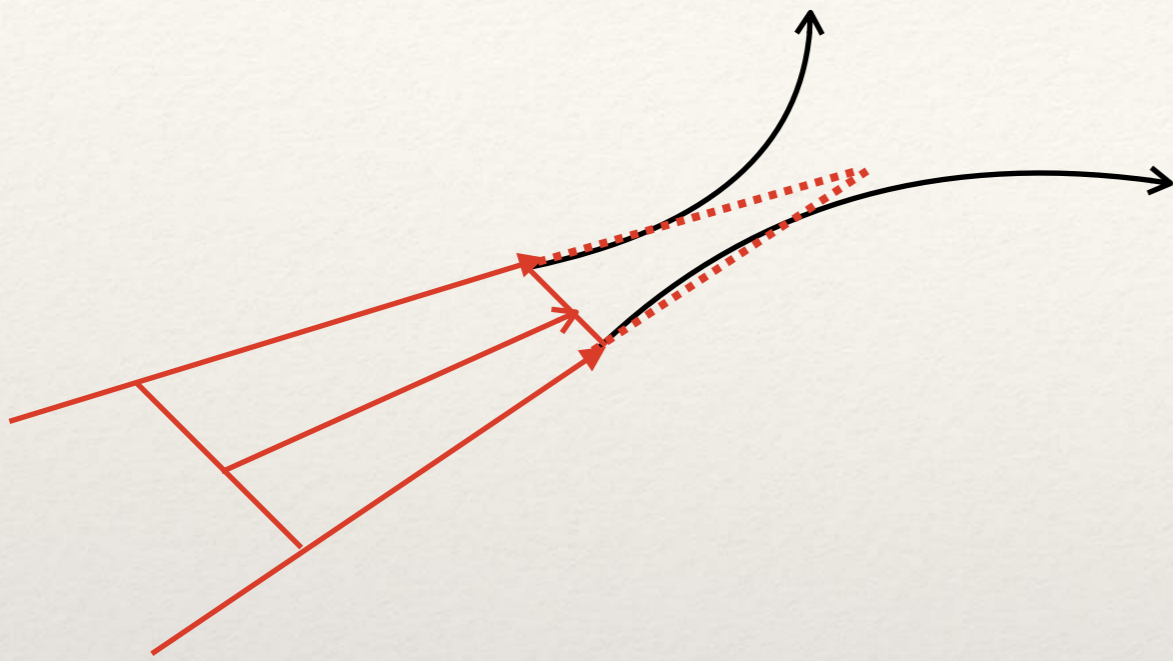
13.8 billion years ago, the universe was filled by a plasma of quarks and gluons, which generated protons and neutrons and then atoms



Can we realise Asymptotically Free Gravity?

Can we weaken Gravity at short distances and small time scales?

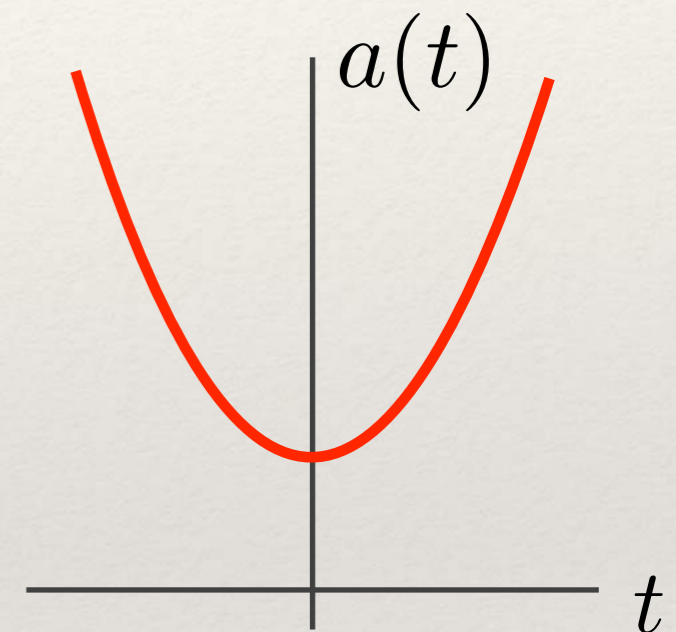
Non-Singular Bouncing Solutions: UV completion of Starobinsky Inflation



Linear Solution

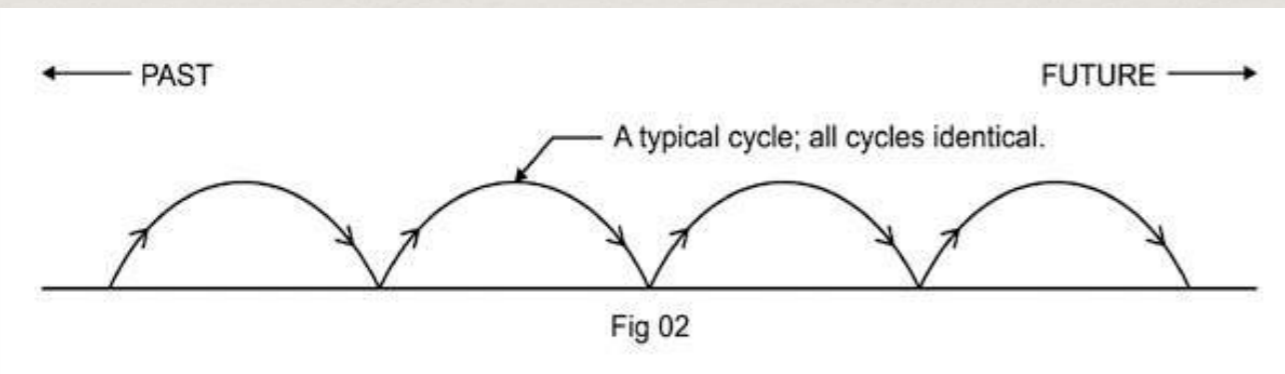
$h \sim \text{diag}(0, A \sin \lambda t, A \sin \lambda t, A \sin \lambda t)$ with $A \ll 1$

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\square}{M^2}} - 1}{\square} \right] R + \Lambda \right]$$



Non-Linear Solution

$$a(t) = \cosh \left(\sqrt{\frac{r_1}{2}} t \right)$$



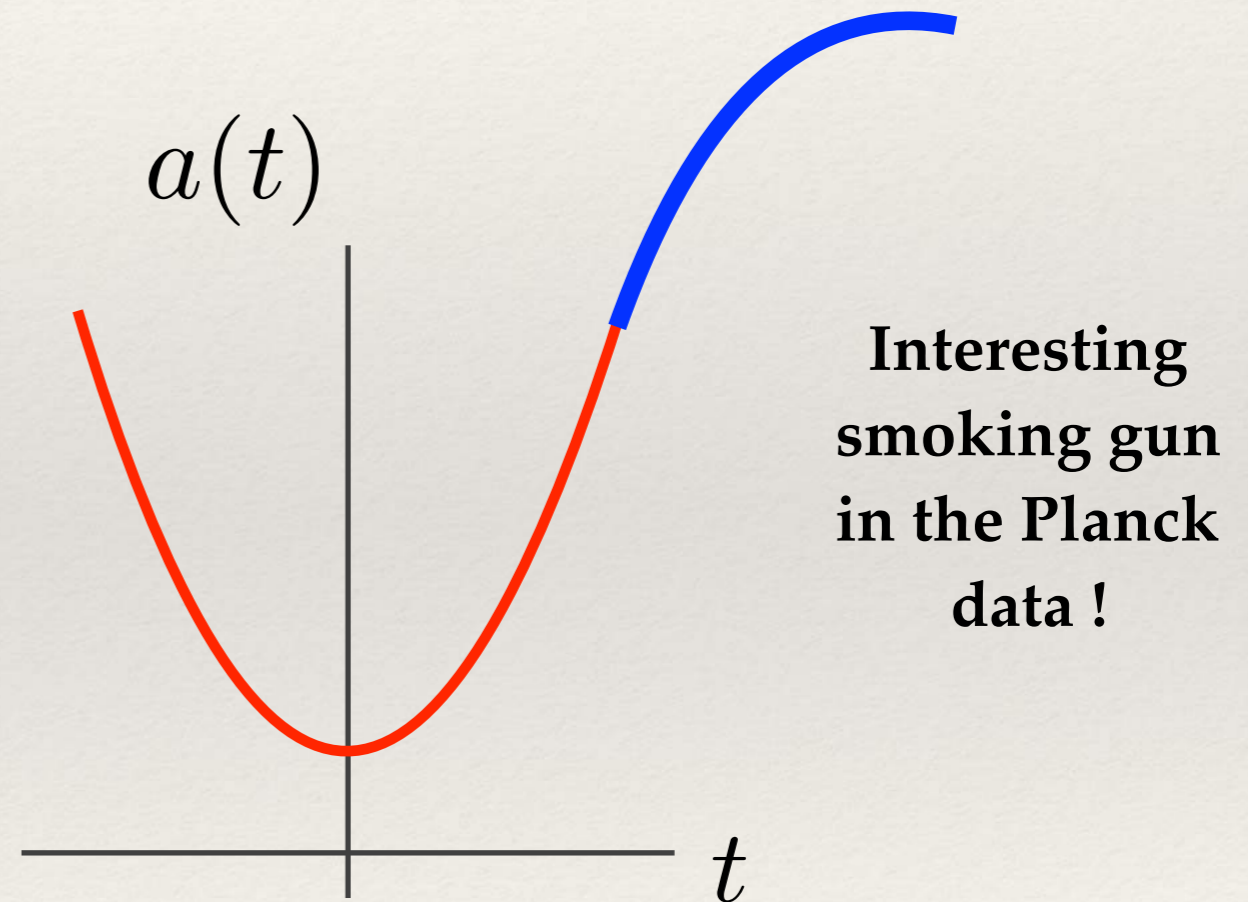
Nonlocal Gravity & Cosmological Singularity

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\square}{M^2}} - 1}{\square} \right] R + \Lambda \right]$$

$$a(t) = a_0 \cosh \left(\sqrt{\frac{\Lambda}{6M_{pl}^2}} t \right)$$

**Cosmological
Constant at Bounce**

$$M \sim \Lambda^{1/4}$$



Biswas, AM, PRD (2014)

“Einstein Gravity Does Not Permit Such Solution”

Hawking-Penrose Singularity Theorems & Raychaudhuri Equation

$$\frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq -R_{\mu\nu}k^\mu k^\nu \quad \theta = \nabla_\mu k^\mu$$

$$R_{\mu\nu}k^\mu k^\nu = \kappa T_{\mu\nu}k^\mu k^\nu \quad \text{General Relativity}$$

$$R_{\mu\nu}k^\mu k^\nu \geq 0, \quad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq 0$$

$$= \int d^4x \sqrt{-g} \left[R + R\mathcal{F}_1(\square)R + R_{\mu\nu}\mathcal{F}_2(\square)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\square)R^{\mu\nu\alpha\beta} \right]$$

$$R_{\mu\nu}^{(L)}k^\mu k^\nu = \frac{1}{a(\bar{\square})} \left[\kappa T_{\mu\nu}k^\mu k^\nu - \frac{(k^0)^2}{2} f(\bar{\square})\square R^{(L)} \right]$$

Defocusing: $R_{\mu\nu}^L k^\mu k^\nu \leq 0$

Does Higgs Play a Role During Inflation with Einstein Gravity ?

$$S \sim \int \sqrt{g} d^4 x \left[R + \xi R H^2 + \dots \right]$$

$$\xi \sim \mathcal{O}(10^3 - 10^4)$$

$$S \sim \int \sqrt{g} d^4 x \left[R + \alpha_1 R^2 + \alpha_2 R^{\mu\nu} R_{\mu\nu} + \alpha_3 R^{\mu\nu\lambda\sigma} R_{\mu\nu\lambda\sigma} \right]$$

Higgs is Lost in the Myriad of Gravitational Terms !!!!

SM Higgs or 750 GeV Scalar at best plays a role of a Curvaton, but not as an Inflaton

Big Bounce & Cosmological Constant

$$\Lambda \sim M^4 \sim 10^{96} (\text{eV})^4$$

PRE-EXISTING UNIVERSE
Collapse due to gravity

SPACE-TIME IS CLASSICAL

SPACE-TIME
IS CLASSICAL

No Singularity due
to weakening of
Gravity

Gravitational Entropy for (A) dS

$$S = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left[R - 2\Lambda + \alpha \left(R\mathcal{F}_1 R + R_{\mu\nu}\mathcal{F}_2 R^{\mu\nu} + R_{\mu\nu\lambda\sigma}\mathcal{F}_3 R^{\mu\nu\lambda\sigma} \right) \right]$$

$$\Lambda = \pm \frac{(D-1)(D-2)}{2\ell^2}$$

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$

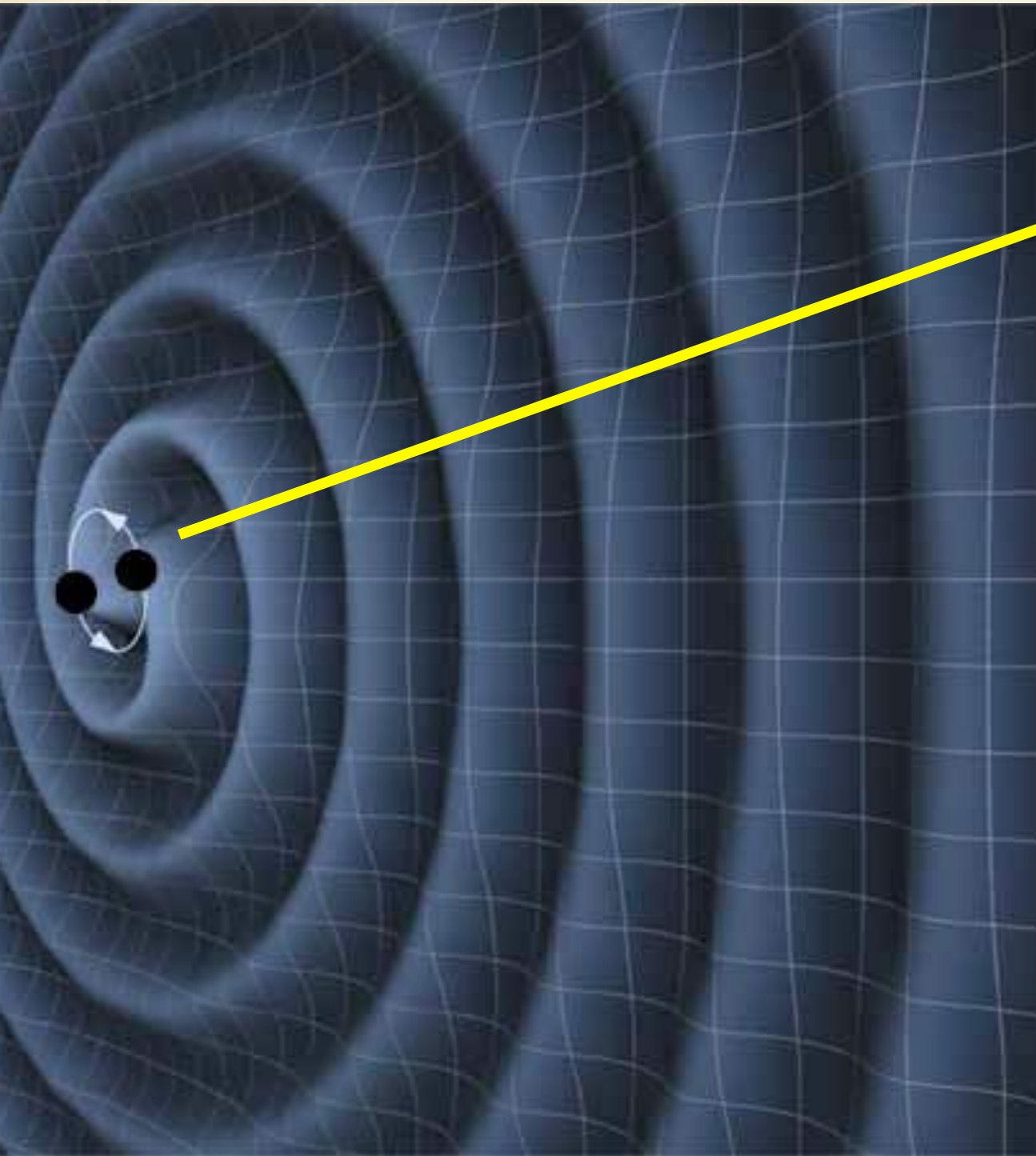
$$f(r) = \left(1 \mp \frac{r}{\ell^2} \right)$$

$$S_W^{(A)dS} = \frac{A_H^{(A)dS}}{4G_D} \left(1 \pm \frac{2\alpha}{\ell^2} (f_{1_0} D(D-1) + f_{2_0} (D-1) + 2f_{3_0}) \right)$$

For $+$ α , dS entropy can be 0

This has important consequences for a non-singular cosmology

Gravitational Waves



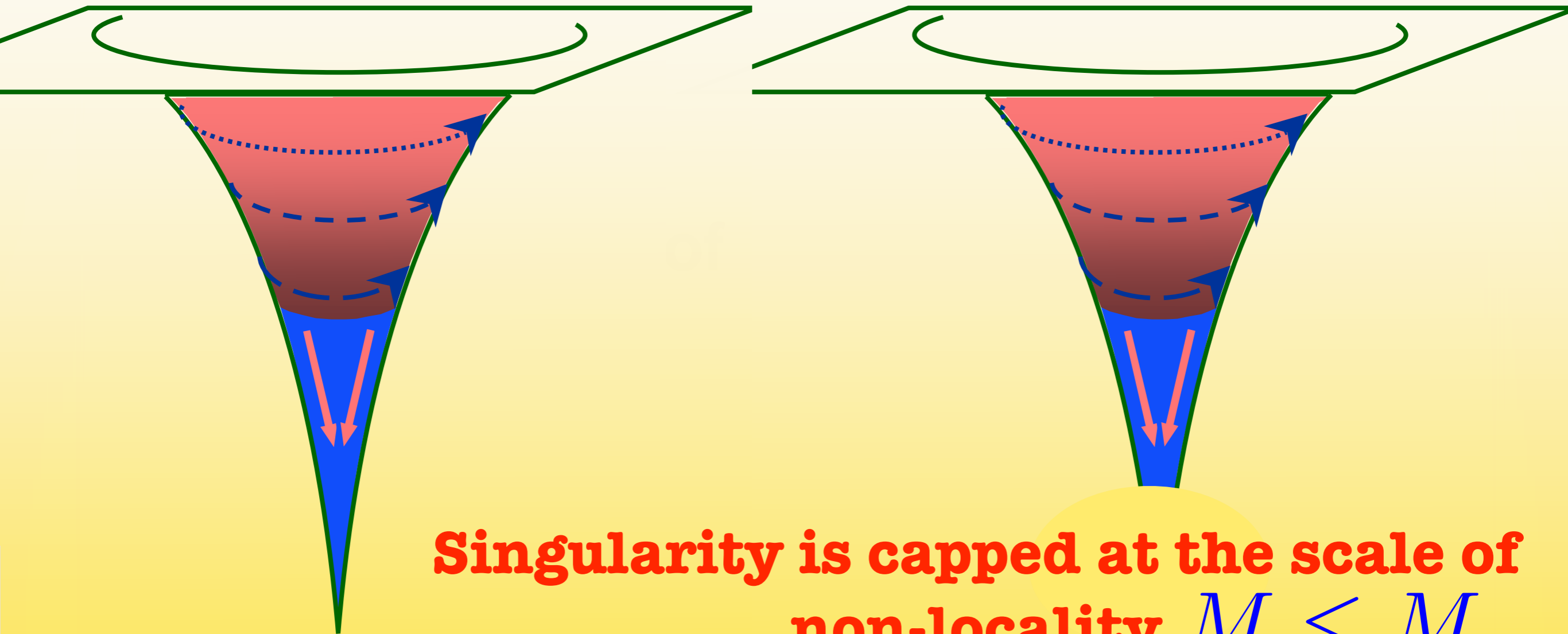
$$\bar{h}_{jk} \approx G \frac{\omega^2 (ML^2)}{r}$$

Large r
limit

$$\bar{h}_{jk} \approx G \frac{\omega^2 (ML^2)}{r} \operatorname{erf} \left(\frac{r M_P}{2} \right)$$

$r \Rightarrow 0$, **No Singularity**

Where would you expect the modifications?



Singularity is capped at the scale of non-locality $M \leq M_p$

Remnants of stringy Gravity

M_p

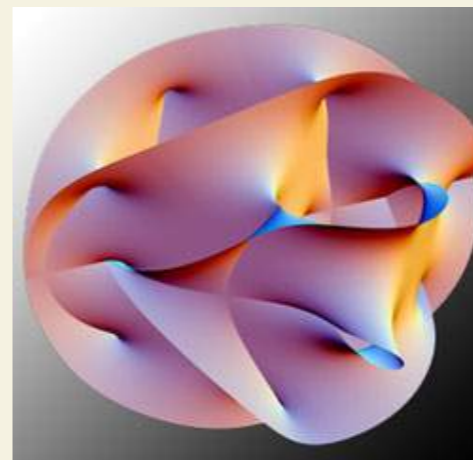
$$\mathcal{L}^{10d} \sim R + R^4 + \dots \quad \kappa^2 = g_s^2 (\alpha')^4$$

Perturbative string theory has α' & g_s corrections

For all orders : String field theory

m_W

m_s



m_{KK}

$$\mathcal{L}^{4d} \sim R + \sum_i c_i R \left(\frac{\square}{m_{kk}} \right)^i R + \dots$$

1 – loop in g_s all orders in α'

**Loop quantum gravity
or
Dynamical Triangulation approach**



Wilson loops



Non-local objects

It would be interesting to establish the connection