Testing General Relativity on Cosmological Scales

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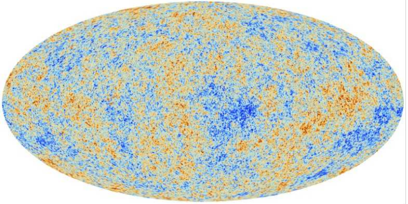
Outline

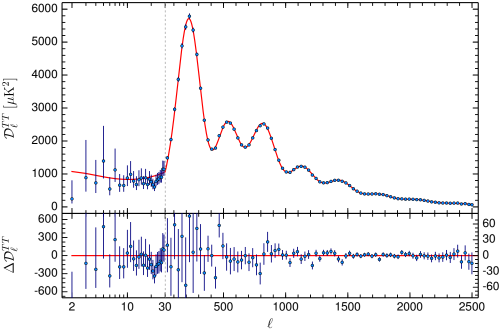
The CMB



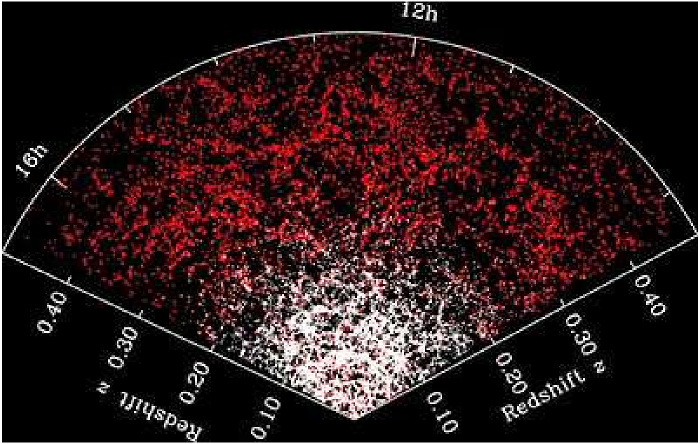
CMB sky as seen by Planck

$D_\ell = \ell(\ell+1)C_\ell/(2\pi)$

The Planck Collaboration:
Planck results 2015 XIII

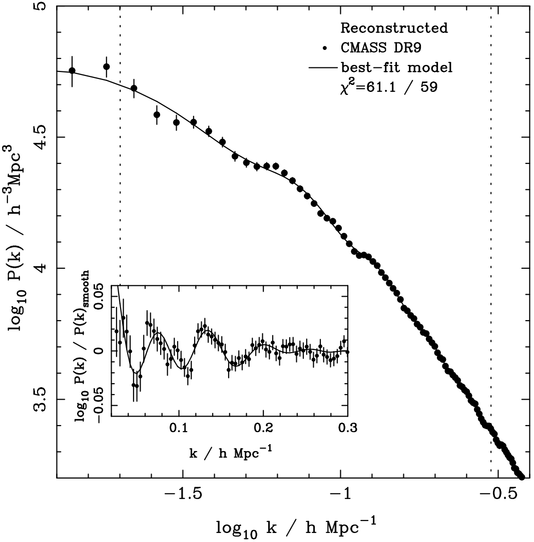


Introduction



M. Blanton and the Sloan Digital Sky Survey Team.

Galaxy power spectrum from the Sloan Digital Sky Survey (BOSS)



from Anderson et al. '12

SDSS-III (BOSS) power spectrum.

Galaxy surveys $\simeq$ matter density fluctuations, biasing and redshift space distortions.

But...

- We have to take fully into account that all observations are made on our past lightcone which is itself perturbed.
  We see density fluctuations which are further away from us, further in the past.
  We cannot observe spatial distances, we measure 2 angles and a redshift.

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- But of course much more for future surveys like DES, DESI Euclid, LSST and SKA.

Future surveys like DESI Euclid, LSST and SKA.

DESI (2018)

LSST (construction started, operation: 2022)

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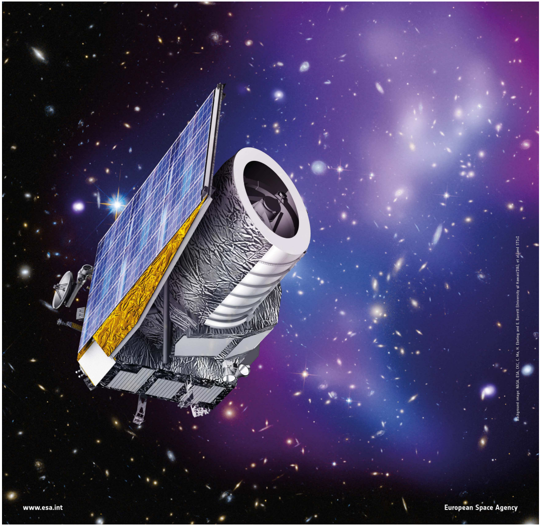
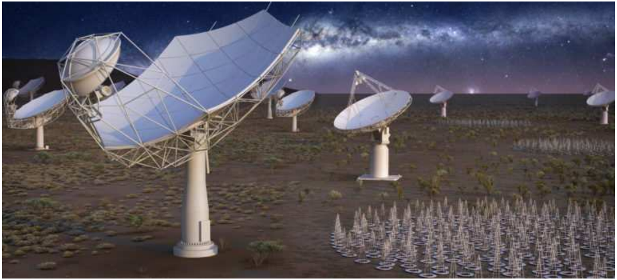


Euclid (launch 2020)



SKA (2018 $\cdots$ 202X)

Cosmological distances

In a Friedmann Universe the (comoving) radial distance is

$$r(z) = \int_0^z \frac{dz'}{H(z')} = \frac{1}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m(1+z')^3 + \Omega_K(1+z')^2 + \Omega_\Lambda}}$$

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Depending on the observational situation we measure directly $r(z)$ or

$$d_A(z) = \frac{1}{(1+z)}\chi_K(r(z)) \quad \text{the angular diameter distance}$$

$$d_L(z) = (1+z)\chi_K(r(z)) \quad \text{the luminosity distance.}$$

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- Whenever we convert a measured redshift and angle into a length scale, we make assumptions about the underlying cosmology.

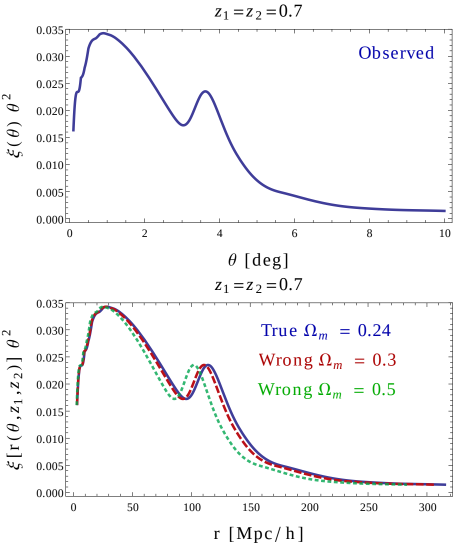
What are very large scale galaxy catalogs really measuring?

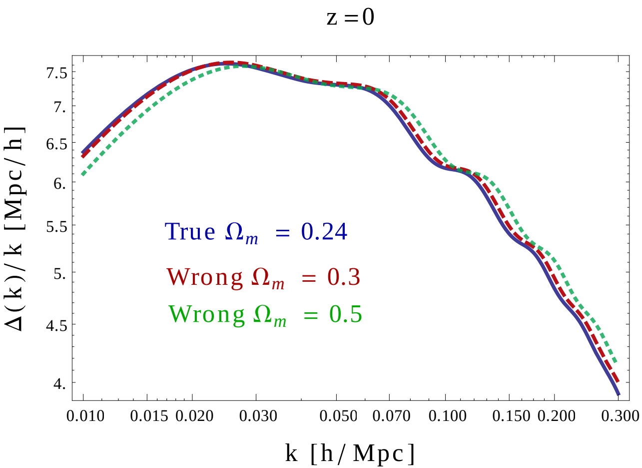
If we convert the measured $\xi(\theta, z_1, z_2)$ to a power spectrum, we have to introduce a cosmology, to convert angles and redshifts into length scales.

$$r(z_1, z_2, \theta) \stackrel{(K=0)}{=} \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta}.$$

$$r_i = r(z_i) = \int_0^{z_i} \frac{dz}{H(z)}$$

(Figure by F. Montanari)





$$\Delta(k)/k = k^2 P(k)$$

(Figure from Di Dio, Montanari, RD, Lesgourgues, [arXiv:1308.6186])

What are very large scale galaxy catalogs really measuring?

We now consider fluctuations in the matter distribution and in the geometry first to linear order. (See C. Bonvin & RD , 2011; Challinor & Lewis, 2011, J. Yoo et al. 2009; J. Yoo 2010)

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We can count the galaxies inside a redshift bin and small solid angle, $N(\mathbf{n}, z)$ and measure the fluctuation of this count:

$$\Delta(\mathbf{n}, z) = \frac{N(\mathbf{n}, z) - \bar{N}(z)}{\bar{N}(z)}.$$

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$$\Delta(\mathbf{n}, z) = \frac{N(\mathbf{n}, z) - \bar{N}(z)}{\bar{N}(z)}.$$

$$\xi(\theta, z, z') = \langle \Delta(\mathbf{n}, z)\Delta(\mathbf{n}', z') \rangle \,, \qquad \mathbf{n} \cdot \mathbf{n}' = \cos\theta \,.$$

This quantity is directly measurable $\Rightarrow$ gauge invariant.

Putting the density and volume fluctuations together one obtains the galaxy number density fluctuations to 1st order

$$\begin{aligned}\Delta(\mathbf{n},z) &= D_s - 2\Phi + \Psi + \frac{1}{\mathcal{H}}\left[\dot\Phi + \partial_r(\mathbf{V}\cdot\mathbf{n})\right] \\ &+ \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r(z)\mathcal{H}}\right)\left(\Psi + \mathbf{V}\cdot\mathbf{n} + \int_0^{r(z)} dr(\dot\Phi + \dot\Psi)\right) \\ &+ \frac{1}{r(z)}\int_0^{r(z)} dr\left[2 - \frac{r(z)-r}{r}\Delta_\Omega\right](\Phi + \Psi).\end{aligned}$$

(C. Bonvin & RD '11)

The total galaxy density fluctuation per redshift bin, per sold angle

Putting the density and volume fluctuations together one obtains the galaxy number density fluctuations

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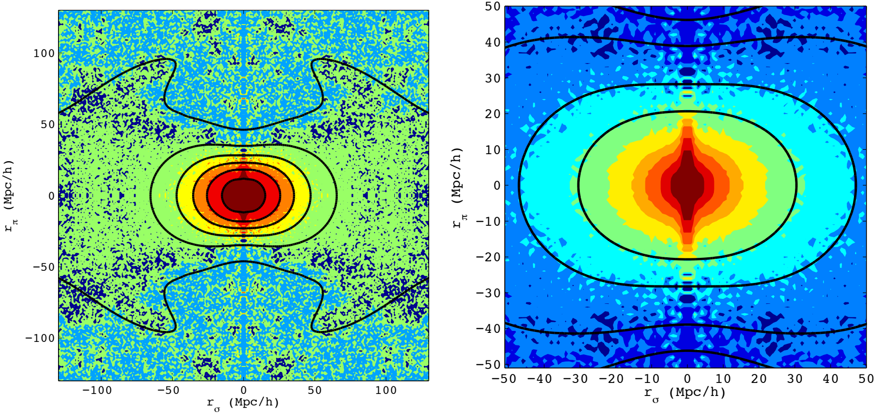
(C. Bonvin & RD '11)

(from Reid et al. '12)



The angular power spectrum of galaxy density fluctuations

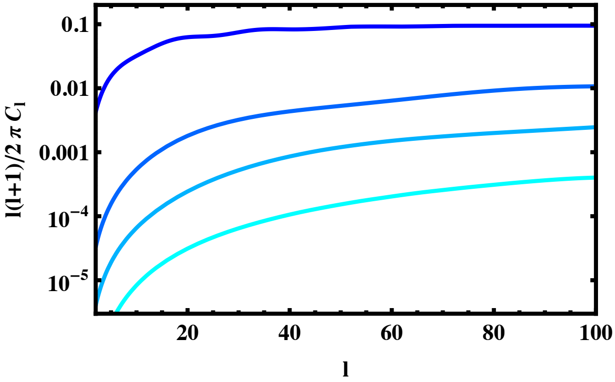
For fixed z, we can expand $\Delta(\mathbf{n}, z)$ in spherical harmonics,

$$\Delta(\mathbf{n}, z) = \sum_{\ell m} a_{\ell m}(z) Y_{\ell m}(\mathbf{n}), \qquad C_\ell(z, z') = \langle a_{\ell m}(z) a^*_{\ell m}(z') \rangle .$$

$$\xi(\theta, z, z') = \langle \Delta(\mathbf{n}, z) \Delta(\mathbf{n}', z') \rangle = \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell(z, z') P_\ell(\cos\theta)$$

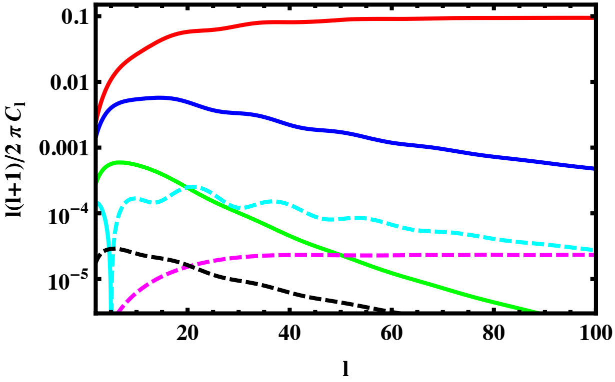
$$\cos\theta = \mathbf{n} \cdot \mathbf{n}'$$

The transverse power spectrum, $z' = z$ (from Bonvin & RD '11)



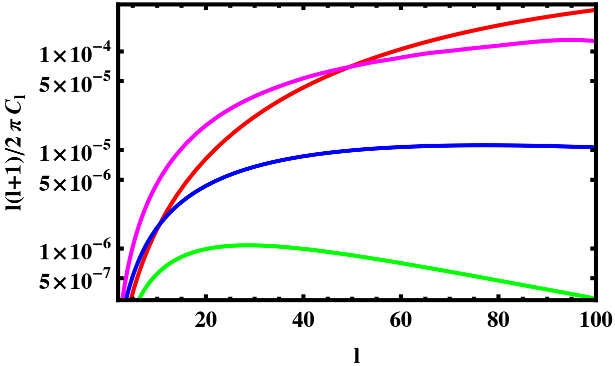
$z = 0.1$, $z = 0.5$, $z = 1$ and $z = 3$.

Contributions to the transverse power spectrum at redshift $z = 0.1,\ \Delta z = 0.01$ (from Bonvin & RD '11)



C_ℓ^{DD} (red), C_ℓ^{zz} (green), $2C_\ell^{Dz}$ (blue), $C_\ell^{Doppler}$ (cyan), $C_\ell^{lensing}$ (magenta) C_ℓ^{grav} (black).

Contributions to the transverse power spectrum at redshift $z = 3$, $\Delta z = 0.3$ (from Bonvin & RD '11)



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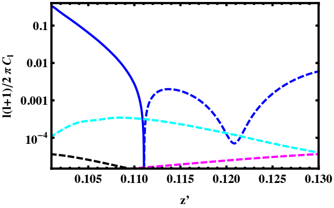
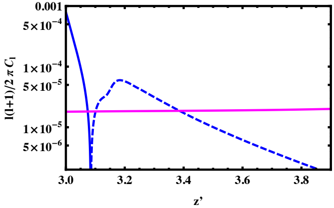
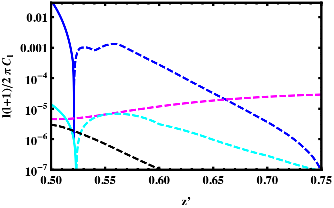
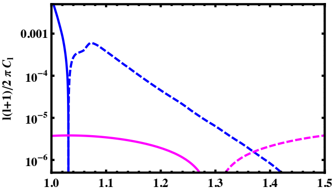
The radial power spectrum









The radial power spectrum $C_\ell(z, z')$
for $\ell = 20$
Left, top to bottom: $z = 0.1,\ 0.5,\ 1,$
top right: $z = 3$

Standard terms (blue), $C_\ell^{lensing}$ (magenta),
$C_\ell^{Doppler}$ (cyan), C_ℓ^{grav} (black),
(from Bonvin & RD '11)

Measuring the lensing potential

At $z = z'$ density and rsd dominate the signal. In both, the transversal and the radial power spectrum, the potential terms are relevant only at very low ℓ .

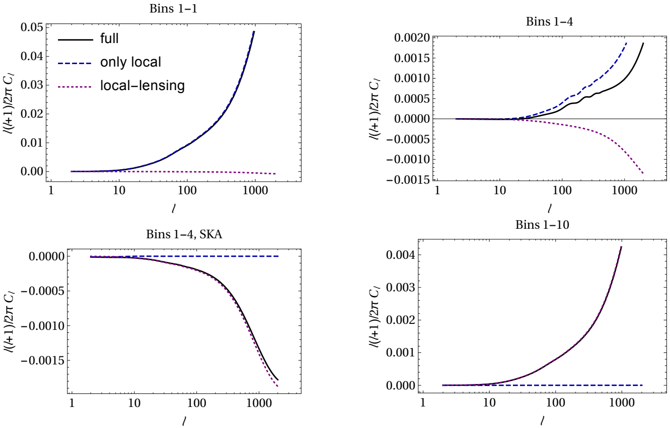
At $z < z'$ we truly measure $\langle D(z)\kappa(z')\rangle$.

$$\kappa(\mathbf{n}, z) = -\int_0^{r(z)} \frac{dr(r(z)-r)}{r(z)r}\Delta_2\Psi(r\mathbf{n}, z)$$

Well separated redshift bins measure mainly the lensing-density correlation:

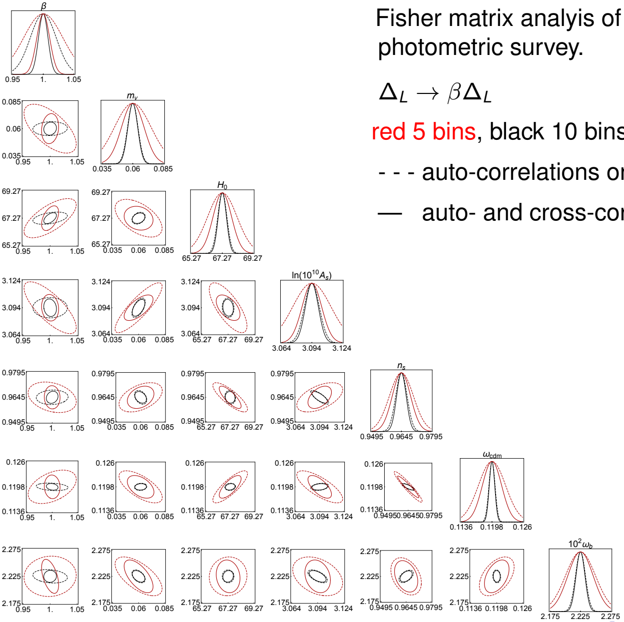
$$\langle \Delta(\mathbf{n},z)\Delta(\mathbf{n}',z')\rangle \simeq \langle \Delta^L(\mathbf{n},z)\delta(\mathbf{n}',z')\rangle \quad z > z'$$

$$\Delta^L(\mathbf{n},z) = (2-5s(z))\kappa(\mathbf{n},z)$$



(Montanari & RD [1506.01369])

Testing GR with the lensing potential



Fisher matrix analyis of an Euclid-like photometric survey.

$\Delta_L \to \beta\Delta_L$

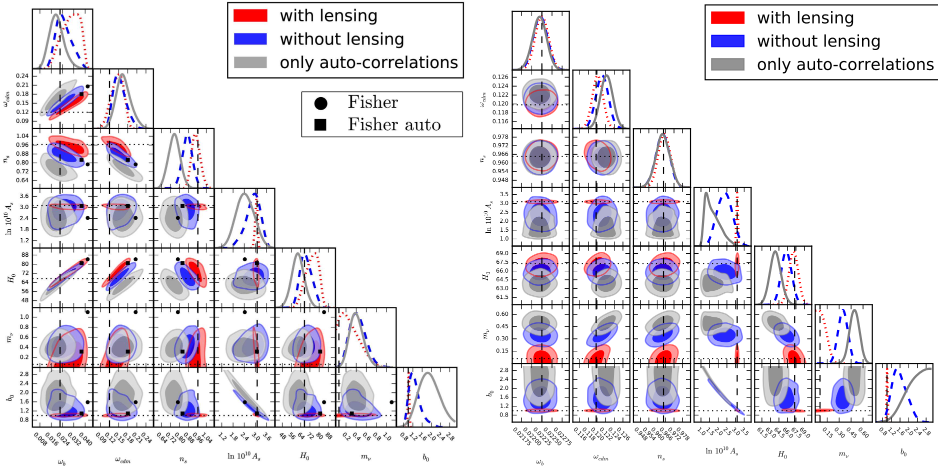
red 5 bins, black 10 bins

- - - auto-correlations only

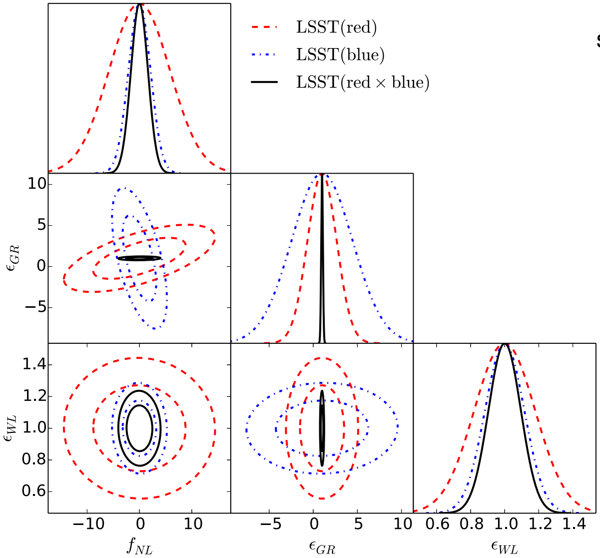
— auto- and cross-correlations

(Montanari & RD [1506.01369])

Neglecting the lensing potential biases cosmological parameters

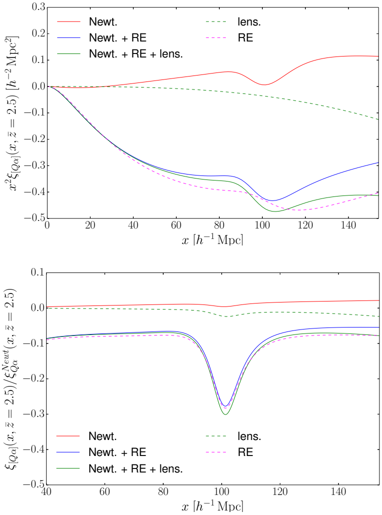


W. Cardona, RD, F. Montanari & M. Kunz, [1603.06481]



standard parameters fixed

Alonso & Ferreira [1507.03550]



The antisymmetric part of the quasar–Ly-α cross correlation function. Contrary to the quasars, the Ly-α signal has no lensing term.
The relativistic term is dominated by the Doppler contribution.

V. Iršič, E. Di Dio & M. Viel [1510.03436]

2nd order number counts

In LSS, on intermediate scales, weakly non-linear effects become important. We can calculate them by going to 2nd order.

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D. Bertacca, R. Maartens, and C. Clarkson [1405.4403,1406.0319]

J. Yoo and M. Zaldarriaga [1406.4140]

E. Di Dio, G. Marozzi, F. Montanari & RD [1407.0376]

2nd order number counts

The dominant terms are ($\propto (k/\mathcal{H})^4\Psi^2$)
(Di Dio, Marozzi, Montanari & RD, [1510.04202], Nielsen & RD [1606.02113])

$$\begin{aligned}\Delta^{(2)Leading}(\mathbf{n},z) \simeq\ & \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} - 2\kappa^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v\partial_r^3 v \\ & +\mathcal{H}^{-1}\left(\partial_r v\partial_r\delta + \partial_r^2 v\,\delta\right) - 2\delta\kappa + \nabla_a\delta\nabla^a\psi \\ & +\mathcal{H}^{-1}\left(-2\partial_r^2 v\,\kappa + \nabla_a\partial_r^2 v\nabla^a\psi\right) + 2\left(\kappa\right)^2 - 2\nabla_b\kappa\nabla^b\psi \\ & -\frac{2}{r(z)}\int_0^{r(z)} dr\frac{r(z)-r}{r}\Delta_2\left(\nabla^b\Psi_1\nabla_b\Psi_1\right) - 4\int_0^{r(z)}\frac{dr}{r}\nabla^a\Psi_1\nabla_a\kappa\,.\end{aligned}$$

$$\Delta^{(1)Leading} = \delta_\rho^{(1)} + \frac{1}{\mathcal{H}_s}\partial_r^2 v^{(1)} - 2\kappa^{(1)}$$

$$\psi = -2\int_0^{r(z)} dr\frac{r-r(z)}{r(z)r}\Psi\,, \quad \kappa = -\Delta_2\psi$$

$$\Psi_1 = \frac{1}{r(z)}\int_0^{r(z)} dr\Psi$$

The bispectrum

$$B\left(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, z_1, z_2, z_3\right) = \left\langle \Delta\left(\mathbf{n}_1, z_1\right) \Delta\left(\mathbf{n}_2, z_2\right) \Delta\left(\mathbf{n}_3, z_3\right) \right\rangle$$

Expanding in spherical harmonics gives

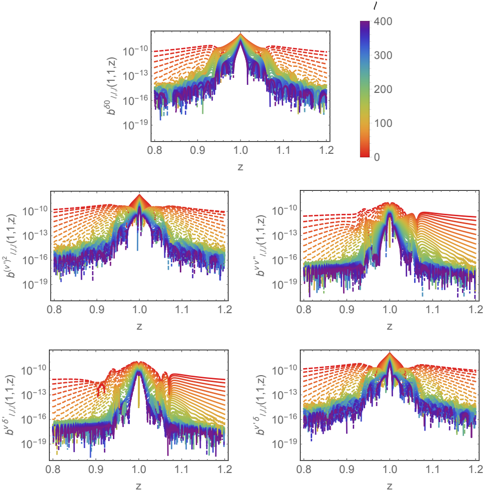
$$B\left(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, z_1, z_2, z_3\right) = \sum B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3}(z_1, z_2, z_3) Y_{\ell_1 m_1}(\mathbf{n}_1) Y_{\ell_2 m_2}(\mathbf{n}_2) Y_{\ell_3 m_3}(\mathbf{n}_3) \,.$$

Statistical isotropy fully determines the *m*-dependence of these coefficients,

$$B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3}(z_1, z_2, z_3) = \mathcal{G}^{m_1, m_2, m_3}_{\ell_1, \ell_2, \ell_3} b_{\ell_1, \ell_2, \ell_3}(z_1, z_2, z_3) \,,$$

where $\mathcal{G}^{m_1, m_2, m_3}_{\ell_1, \ell_2, \ell_3}$ is the Gaunt integral.

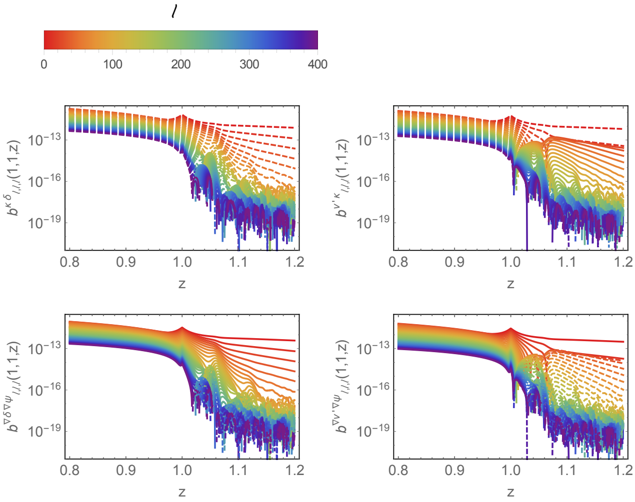
The bispectrum: Newtonian terms



The well known Newtonian terms

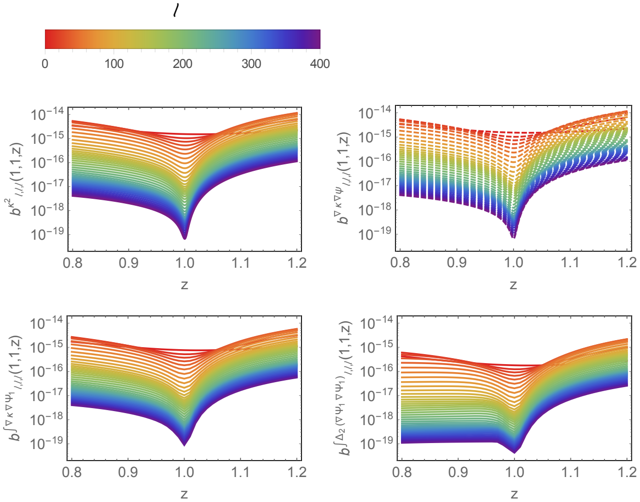
(Di Dio, RD, Marozzi & Montanari, [1510.04202])

The bispectrum: Newtonian-Lensing terms



(Di Dio, RD, Marozzi & Montanari, [1510.04202])
Terms containing Newtonian and lensing contributions.

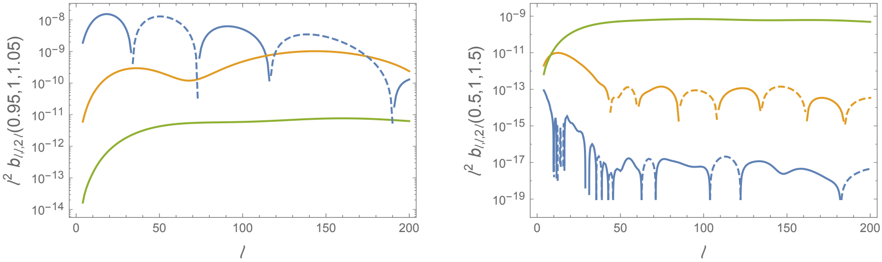
The bispectrum: Lensing terms



(Di Dio, RD, Marozzi & Montanari, [1510.04202])

Pure lensing contributions.

The bispectrum



(Di Dio, RD, Marozzi & Montanari, [1510.04202])

(density-density , density-lensing , lensing-lensing)

Conclusions

- So far cosmological LSS data mainly determined $\xi(r)$, or equivalently $P(k)$ or $B(k_1, k_2, k_3)$. These are easier to measure (less noisy) but:
  • they require an fiducial input cosmology converting redshift and angles to length scales,
  $$r = \sqrt{r(z)^2 + r(z')^2 - 2r(z)r(z')\cos\theta}\ .$$
  This complicates especially the determination of error bars in parameter estimation
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- These 3d quantities will of course be more noisy, but they also contain more information.
- These spectra are not only sensitive to the matter distribution (density) but also to the velocity (redshift space distortions) and to the perturbations of spacetime geometry (lensing) .

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- The spectra $C_\ell(z, z')$ and $b_{\ell_1,\ell_2,\ell_2}(z_1, z_2, z_3)$ depend sensitively and in several different ways on dark energy (growth factor, distance redshift relation), on the matter and baryon densities, bias etc.
- Extracting the density, velocity and the gravitational potentials by clever combinations of measurements provides a new route to not only estimate cosmological parameters but to test general relativity on cosmological scales.