DUSTY INFLATION IN BORN-INFELDIZED GRAVITY

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OUTLINE

- Born-Infeld & Born-Infeld inspired gravity.
- Generalised Born-Infeld inspired gravity. Minimal extension.
- Perfect fluid and cosmological solutions.
- Dust inflation.
- Bouncing solutions

BORN-INFELD

$$\mathcal{S}_{\text{Maxwell}} = \frac{1}{2}\int \mathrm{d}^4x\,\left(\vec{E}^2 - \vec{B}^2\right) = -\frac{1}{4}\int \mathrm{d}^4x F_{\mu\nu}F^{\mu\nu}$$

M. Born and L. Infeld. Proc.Roy.Soc.Lond. A144 .(1934)

Principle of finiteness

$$\frac{1}{2}m^2\int \mathrm{d}t\, v^2 \to mc^2\int \mathrm{d}t\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)$$

$$\mathcal{S} = -\lambda^4\int \mathrm{d}^4x\left[\sqrt{1-\lambda^{-4}(\vec{E}^2-\vec{B}^2)}-1\right]$$

$$\mathcal{S} \sim \int \mathrm{d}^4x\sqrt{\det a_{\mu\nu}}$$

$$\mathcal{S}_{\text{BIE}} = -\lambda^4\int \mathrm{d}^4x\left[\sqrt{-\det\left(\eta_{\mu\nu}+\lambda^{-2}F_{\mu\nu}\right)}-1\right]$$

BORN-INFELD

$$\mathcal{S}_{\text{BIE}} = -\lambda^4 \int d^4x \left[\sqrt{-\det\left(\eta_{\mu\nu} + \lambda^{-2} F_{\mu\nu}\right)} - 1 \right]$$

$$\mathcal{S}_{\text{BIE}} = -\lambda^4 \int d^4x \left[\sqrt{1 + \frac{1}{2\lambda^4} F_{\mu\nu}F^{\mu\nu} - \frac{1}{16\lambda^8}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right)^2} - 1 \right]$$

$$= -\lambda^4 \int d^4x \left[\sqrt{1 - \frac{\vec{E}^2 - \vec{B}^2}{\lambda^4} - \frac{(\vec{E}\cdot\vec{B})^2}{\lambda^8}} - 1 \right]$$

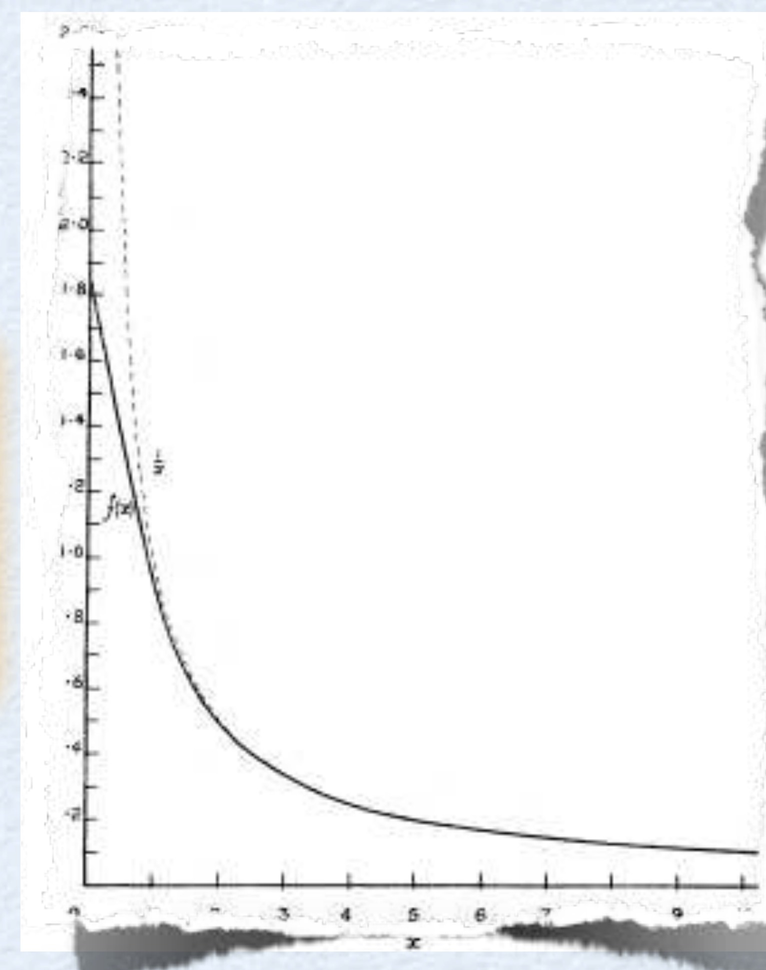
M. Born and L. Infeld. Proc.Roy.Soc.Lond. A144 .(1934)

For small electromagnetic fields it recovers Maxwell's theory:

$$\mathcal{S}_{\text{BIE}}(F_{\mu\nu} \ll \lambda^2) \simeq \mathcal{S}_{\text{Maxwell}}$$

For large electromagnetic fields it differs so that it regularizes the self-energy of point-like charged particles.

$$|\vec{E}| = \frac{1}{\sqrt{1 + \left(\frac{Q}{4\pi\lambda^2 r^2}\right)^2}} \frac{Q}{4\pi r^2}$$



BORN-INFELD

$$\mathcal{S}_{\rm BIE} = -\lambda^4 \int d^4x \left[\sqrt{-\det\left(\eta_{\mu\nu} + \lambda^{-2} F_{\mu\nu}\right)} - 1 \right]$$

M. Born and L. Infeld. Proc.Roy.Soc.Lond. A144 .(1934)

- Electric-magnetic self-duality.
- Causal propagation. Absence of shock waves and birefringence. Exceptional.
- Existence of exact finite energy soliton solutions (BIons).
- Natural low energy limit in string theory and D-branes.
- ...

For a general non-linear electrodynamics,we have the equations:

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = +\frac{\partial \vec{D}}{\partial t}, \quad \nabla \cdot \vec{D} = 0$$

$$\vec{H} = -\frac{\partial \mathcal{L}}{\partial \vec{B}} \quad \text{and} \quad \vec{D} = +\frac{\partial \mathcal{L}}{\partial \vec{E}}$$

self-duality invariance

$$\vec{D} + i\vec{B} \rightarrow e^{i\theta}(\vec{D} + i\vec{B})$$

$$\vec{E} + i\vec{H} \rightarrow e^{i\theta}(\vec{E} + i\vec{H})$$

$$\Downarrow$$

$$\vec{E} \cdot \vec{B} = \vec{D} \cdot \vec{H}$$

BORN-INFELD INSPIRED GRAVITY

$$S_{\rm DG} = \int d^4x \sqrt{-\det\left(a g_{\mu\nu} + b R_{\mu\nu} + c X_{\mu\nu}\right)}$$

S. Deser and G. Gibbons, CQG15 (1998)

Higher order curvature terms to be tuned to avoid ghosts

There is a large freedom in the choice of $X_{\mu\nu}$ and no clear immediate criterion.

BORN-INFELD INSPIRED GRAVITY

$$S_{\rm Ed} = \lambda^4 \int d^4x \sqrt{\det R_{(\mu\nu)}(\Gamma)}$$

Determinantal actions for gravity were considered by Eddington (1924) as a purely affine theory.

Couplings to matter. The metric enters as an auxiliary field that can then be integrated out.

M. Ferraris and J. Kijowski, Letters in Mathematical Physics, 5 127-135, (1981)

Levi-Civita connection

$$\Gamma^{\alpha}_{\mu\nu} = \gamma^{\alpha}_{\mu\nu} + L^{\alpha}_{\mu\nu}(Q) + K^{\alpha}_{\mu\nu}(T)$$

$$\nabla_{\mu} g_{\alpha\beta} = Q_{\mu\alpha\beta} \qquad T^{\alpha}_{\mu\nu} = \Gamma^{\alpha}_{[\mu\nu]}$$

Non-metricity — Torsion

For Einstein-Hilbert metric ⟺ Palatini

In general there are two independent traces of the Riemann tensor:

$$R^{\alpha}{}_{\alpha\mu\nu} \qquad R^{\alpha}{}_{\mu\alpha\nu}$$

also independent from $g^{\alpha\beta} R^{\mu}{}_{\alpha\beta\nu}$

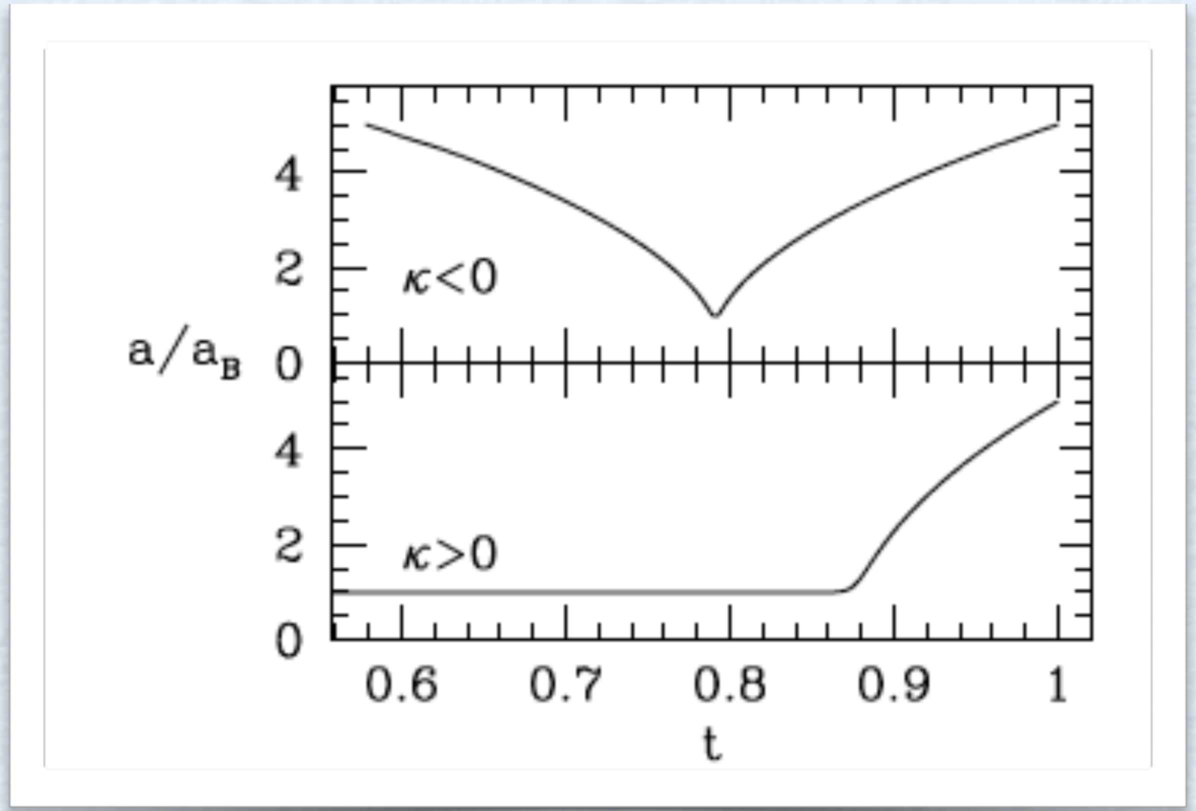
BORN-INFELD INSPIRED GRAVITY

$$S_{\rm BIP} = \lambda^4 \int d^4x \left[\sqrt{-\det\left(g_{\mu\nu} + \lambda^{-2} R_{\mu\nu}(\Gamma)\right)} - \sqrt{-\det(g_{\mu\nu})} \right]$$

D. N. Vollick, PRD 69 (2004) 064030.

In the Palatini formulation the ghost can be avoided without further corrections

Existence of bouncing solutions...



M. Bañados, P. G. Ferreira, PRL 105, 011101 (2010)

C. Escamilla-Rivera,M. Bañados, P. G. Ferreira, PRD85 (2012)

...however tensor instabilities at the bounce.

BORN-INFELD INSPIRED GRAVITY

Much more information on
Born-Infeld gravity soon!!!
(Hopefully in April)

Born-Infeld inspired modifications of gravity

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Abstract

General Relativity has shown an outstanding observational success in the scales where it has been directly tested. However, modifications have been intensively explored in the regimes where GR seems either incomplete or signals its own limit of validity. In particular, the existence of spacetime singularities and the breakdown of unitarity at energies near the Planck scale strongly suggest that GR needs to be modified at high energies or when the involved curvatures are high. Born-Infeld inspired gravity theories have shown an extraordinary ability to regularize the gravitational dynamics, leading to nonsingular cosmologies and regular black hole space-times in a very robust manner and without resorting to quantum gravity effects. This has boosted the interest in these theories in applications to compact objects, gravitational collapse, inflationary scenarios, early and late-time cosmological singularities, black hole and wormhole physics, among others. We review the motivations, various formulations, and main results achieved within this type of extensions beyond Einstein's gravity, including their observational viability, and provide an overview of current open problems and future research opportunities.

Keywords:

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EXTENDED BORN-INFELD GRAVITY

Our proposal to extend it is...

$$\mathcal{S} = \lambda^4 \int \mathrm{d}^4x \sqrt{-\det\left(g_{\mu\nu} + \lambda^{-2} R_{\mu\nu}\right)} = \lambda^4 \int \mathrm{d}^4x \sqrt{-g}\, \det \sqrt{\delta^\mu{}_\nu + \lambda^{-2} g^{\mu\alpha} R_{\alpha\nu}}$$

⇒

$$\mathcal{S} = \lambda^4 \int \mathrm{d}^4x \sqrt{-g}\, \det \sqrt{\hat{g}^{-1}\hat{q}}$$

$$q_{\alpha\nu} \equiv g_{\alpha\nu} + \lambda^{-2} R_{\alpha\nu}(\Gamma)$$

This reminds of the massive gravity potential:

$$\mathcal{S}_{MG} = \int \mathrm{d}^4x \sqrt{-g} \sum_{n=0}^{4} \frac{\beta_n}{n!(4-n)!} e_n\left(\sqrt{g^{-1}f}\right)$$

C. de Rham, G. Gabadadze, A.J.Tolley, PRL106 (2011)

S.F. Hassan, R.A. Rosen JHEP1107 (2011)

↓ elementary symmetric polynomials

Extended Born-Infeld gravity

...and so, a natural generalization of BI inspired gravity is

JBJ, L. Heisenberg and G.J. Olmo
JCAP 11 (2014) 004

$$\mathcal{S} = \tilde{\lambda}^4 \int \mathrm{d}^4x \sqrt{-g} \sum_{n=0}^{4} \beta_n e_n(\hat{M})$$

$$\hat{M} \equiv \sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}(\Gamma)}$$

$$\begin{aligned}
e_0(\hat{M}) &= 1, \\
e_1(\hat{M}) &= [\hat{M}], \\
e_2(\hat{M}) &= \frac{1}{2!}\Big([\hat{M}]^2 - [\hat{M}^2]\Big), \\
e_3(\hat{M}) &= \frac{1}{3!}\Big([\hat{M}]^3 - 3[\hat{M}][\hat{M}^2] + 2[\hat{M}^3]\Big), \\
e_4(\hat{M}) &= \frac{1}{4!}\Big([\hat{M}]^4 - 6[\hat{M}]^2[\hat{M}^2] + 8[\hat{M}][\hat{M}^3] + 3[\hat{M}^2]^2 - 6[\hat{M}^4]\Big).
\end{aligned}$$

with matter minimally coupled.

Low curvature limit

$$\mathcal{S} \simeq \int \mathrm{d}^4x \sqrt{-g} \left[\tilde{\lambda}^4 \left(\beta_0 + 4\beta_1 + 6\beta_2 + 4\beta_3 + \beta_4\right) + \frac{\tilde{\lambda}^4}{2\lambda^2}\left(\beta_1 + 3\beta_2 + 3\beta_3 + \beta_4\right) g^{\mu\nu} R_{\mu\nu}(\Gamma)\right]$$

Cosmological constant — Newton's constant

Accidental projective symmetry

$$\Gamma^\alpha_{\mu\nu} \to \Gamma^\alpha_{\mu\nu} + \delta^\alpha_\nu \xi_\mu$$

Extended Born-Infeld gravity

...and so, a natural generalization of BI inspired gravity is

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$$\mathcal{S} = \tilde{\lambda}^4 \int \mathrm{d}^4x \sqrt{-g} \sum_{n=0}^{4} \beta_n e_n(\hat{M})$$

$$\hat{M} \equiv \sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}(\Gamma)}$$

High curvature limit (where the different terms differ)

$$\begin{aligned}
e_0 &\simeq 1 \\
e_1 &\simeq |\lambda|^{-1}\left[\sqrt{\hat{P}}\right] \\
e_2 &\simeq \frac{\lambda^{-2}}{2!}\left(\left[\sqrt{\hat{P}}\right]^2 - [\hat{P}]\right) \\
e_3 &\simeq \frac{|\lambda|^{-3}}{3!}\left(\left[\sqrt{\hat{P}}\right]^3 - 3\left[\sqrt{\hat{P}}\right][\hat{P}] + 2\left[\hat{P}^{3/2}\right]\right) \\
e_4 &\simeq \frac{\lambda^{-4}}{4!}\left(\left[\sqrt{\hat{P}}\right]^4 - 6\left[\sqrt{\hat{P}}\right]^2[\hat{P}] + 8\left[\sqrt{\hat{P}}\right]\left[\hat{P}^{3/2}\right] + 3[\hat{P}]^2 - 6[\hat{P}^2]\right)
\end{aligned}$$

$\Longrightarrow$

$$\mathcal{S}_{\mathrm{EEH}} = \tilde{m}^2 \int \mathrm{d}^4x \sqrt{-g}\left(\left[\hat{g}^{-1}\hat{R}\right] - \left[\sqrt{\hat{g}^{-1}\hat{R}}\right]^2\right)$$

Eddington's theory

$$\mathcal{S}_{\mathrm{Eddington}} \simeq \beta_4 \frac{\tilde{\lambda}^4}{\lambda^4} \int \mathrm{d}^4x \sqrt{\det R_{\mu\nu}(\Gamma)}$$

$$P^{\mu}{}_{\nu} = g^{\mu\alpha} R_{\alpha\nu}(\Gamma)$$

EXTENDED BORN-INFELD GRAVITY

...and so, a natural generalization of BI inspired gravity is

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$$\mathcal{S} = \tilde{\lambda}^4 \int \mathrm{d}^4x \sqrt{-g} \sum_{n=0}^{4} \beta_n e_n(\hat{M})$$

$$\hat{M} \equiv \sqrt{\mathbb{1} + \lambda^{-2} \hat{g}^{-1} \hat{R}(\Gamma)}$$

Field equations

$$\frac{\tilde{\lambda}^4 \lambda^{-2}}{2} \left(R_{\alpha\lambda} W^{\lambda}{}_{\beta} + R_{\beta\lambda} W^{\lambda}{}_{\alpha} \right) - \mathcal{L}_G g_{\alpha\beta} = T_{\alpha\beta}$$

$$\nabla_\lambda \left(\sqrt{-g} W^{\beta\nu} \right) - \delta^\nu_\lambda \nabla_\rho \left(\sqrt{-g} W^{\beta\rho} \right) + 2\sqrt{-g} \left(\mathcal{T}^\kappa_{\lambda\kappa} W^{\beta\nu} - \delta^\nu_\lambda \mathcal{T}^\kappa_{\rho\kappa} W^{\beta\rho} + \mathcal{T}^\nu_{\lambda\rho} W^{\beta\rho} \right) = 0$$

$$\hat{W} = f_1 \hat{M}^{-1} + f_2 \mathbb{1} + f_3 \hat{M} + f_4 \hat{M}^2$$

$$\begin{aligned} f_1 &= \beta_1 e_0 + \beta_2 e_1 + \beta_3 e_2 + \beta_4 e_3 \\ f_2 &= -(\beta_2 e_0 + \beta_3 e_1 + \beta_4 e_2) \\ f_3 &= \beta_3 e_0 + \beta_4 e_1 \\ f_4 &= -\beta_4 e_0 \end{aligned}$$

The equations have the same structure for all the terms.

Minimal Born-Infeld extension

$$\mathcal{S}_{\rm min} = \lambda^2 M_{\rm Pl}^2 \int d^4x \sqrt{-g}\,{\rm Tr}\left[\sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}} - \mathbb{1}\right]$$

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$$\hat{M} \equiv \sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}(\Gamma)}$$

Metric field equations

$$(M^{-1})^{\alpha}{}_{(\mu}R_{\nu)\alpha} - {\rm Tr}(\hat{M} - \mathbb{1})\lambda^2 g_{\mu\nu} = \frac{1}{M_{\rm Pl}^2} T_{\mu\nu}$$

$\Downarrow$ $\hat{R} = \lambda^2 \hat{g}(\hat{M}^2 - \mathbb{1})$

$$\frac{1}{2}\left[\hat{g}(\hat{M} - \hat{M}^{-1}) + (\hat{M} - \hat{M}^{-1})^T \hat{g}\right] - {\rm Tr}(\hat{M} - \mathbb{1})\hat{g} = \frac{1}{\lambda^2 M_{\rm Pl}^2}\hat{T}$$

This equation allows to express $M^{\alpha}{}_{\beta}$ as a function of the matter content and the metric tensor.

Minimal Born-Infeld extension

$$\mathcal{S}_{\rm min} = \lambda^2 M_{\rm Pl}^2 \int {\rm d}^4x \sqrt{-g}{\rm Tr}\left[\sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}} - \mathbb{1}\right]$$

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$$\hat{M} \equiv \sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}(\Gamma)}$$

Connection field equations

$$\nabla_\lambda\left(\sqrt{-g}W^{\beta\nu}\right) - \delta^\nu_\lambda \nabla_\rho\left(\sqrt{-g}W^{\beta\rho}\right) + 2\sqrt{-g}\left(\mathcal{T}^\kappa_{\lambda\kappa}W^{\beta\nu} - \delta^\nu_\lambda \mathcal{T}^\kappa_{\rho\kappa}W^{\beta\rho} + \mathcal{T}^\nu_{\lambda\rho}W^{\beta\rho}\right) = 0$$

$$\hat{W} = \hat{M}^{-1}$$

We will consider solutions without torsion $\mathcal{T}^\alpha{}_{\mu\nu} = 0$

⇓

$$\nabla_\lambda\left(\sqrt{-g}g^{\rho(\nu}W^{\beta)}{}_\rho\right) = 0 \quad \Longrightarrow \quad \Gamma = \Gamma(\tilde{g}) \qquad \tilde{g}^{\mu\nu} = \sqrt{\det \hat{M}}g^{\alpha\mu}(\hat{M}^{-1})^\nu{}_\alpha$$

$$\nabla_\lambda\left(\sqrt{-g}g^{\rho[\nu}W^{\beta]}{}_\rho\right) = 0 \quad \Longrightarrow$$

We set torsion to zero a posteriori. This is a consistency equation for this Ansatz.

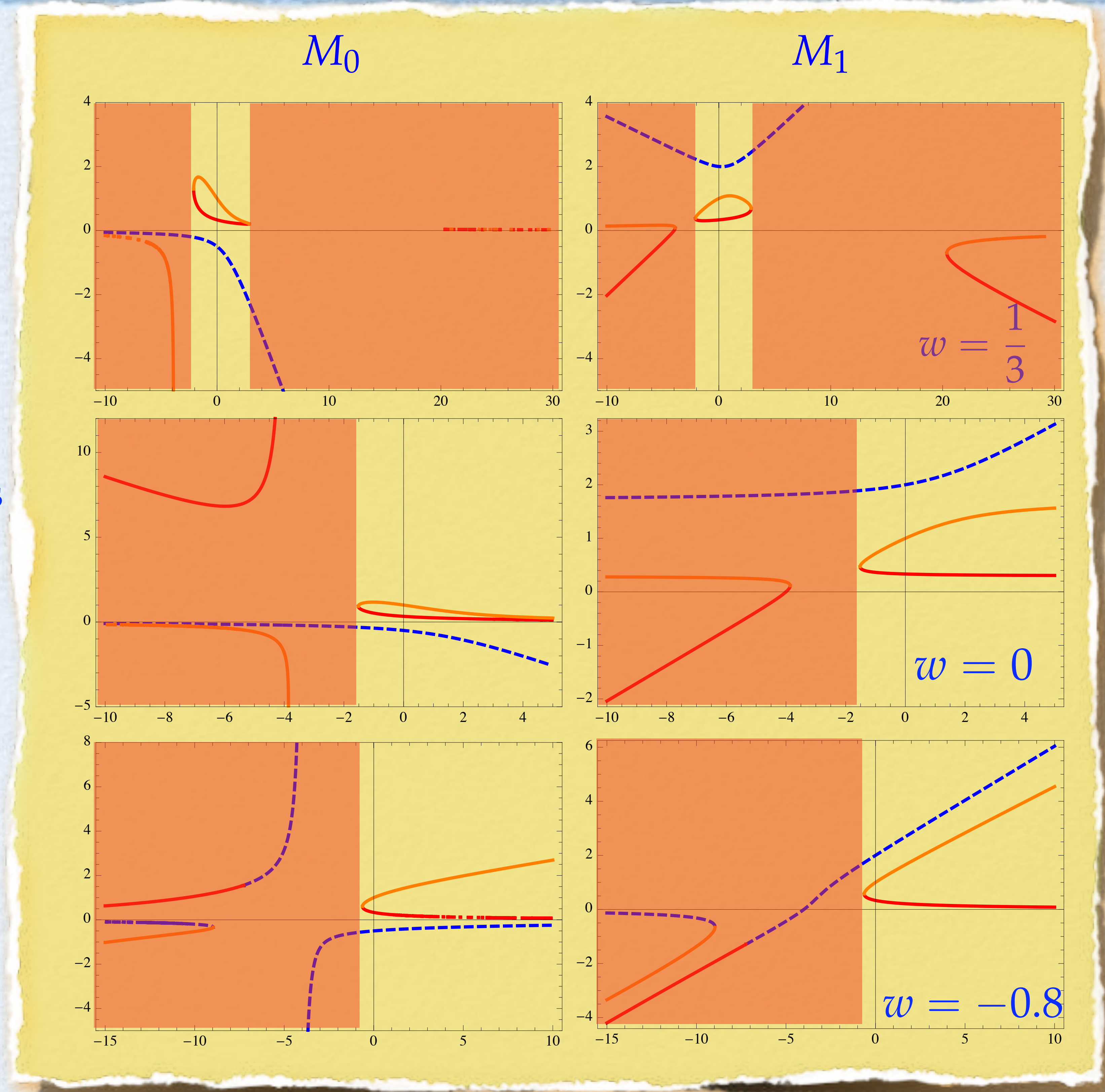
Perfect fluid solutions

$$T^{\mu}{}_{\nu} = \begin{pmatrix} -\rho & \mathbf{0} \\ \mathbf{0} & p\,\mathbb{1}_{3\times 3} \end{pmatrix}$$

$$M^{\mu}{}_{\nu} = \begin{pmatrix} M_0 & \mathbf{0} \\ \mathbf{0} & M_1\,\mathbb{1}_{3\times 3} \end{pmatrix}$$

$$\begin{aligned} \frac{1}{M_0} + 3M_1 &= 4 + \tilde{\rho} \\ M_0 + 2M_1 + \frac{1}{M_1} &= 4 - \tilde{p} \end{aligned}$$

Metric field equations



In general we find 3 branches of solutions, but only two of them are physical. Out of those two, only one is continuously connected with GR.

Perfect fluid solutions

$$T^{\mu}{}_{\nu} = \begin{pmatrix} -\rho & \mathbf{0} \\ \mathbf{0} & p\mathbb{1}_{3\times3} \end{pmatrix}$$

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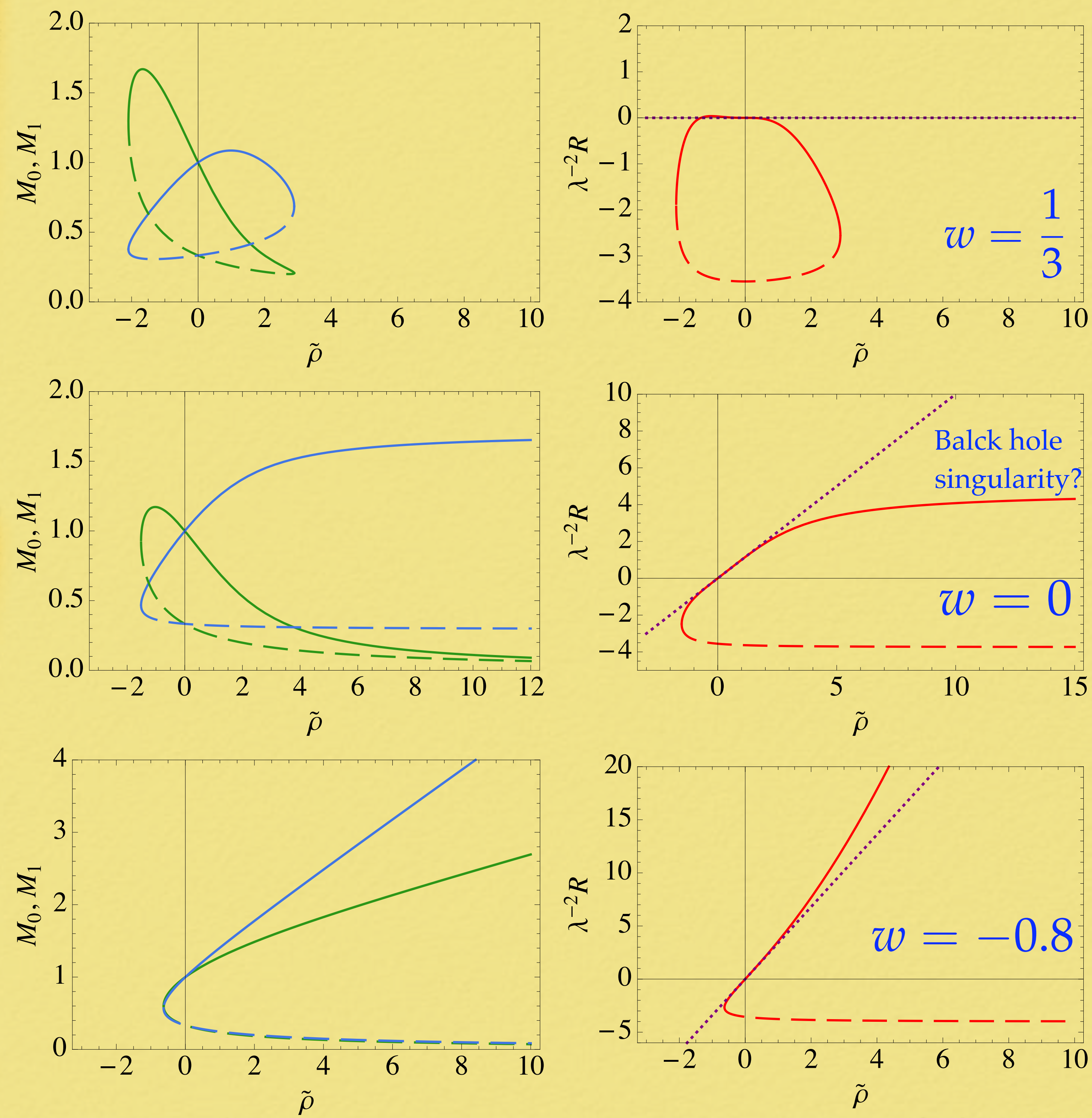
$$\frac{1}{M_0} + 3M_1 = 4 + \tilde{\rho}$$
$$M_0 + 2M_1 + \frac{1}{M_1} = 4 - \tilde{\rho}$$

Metric field equations



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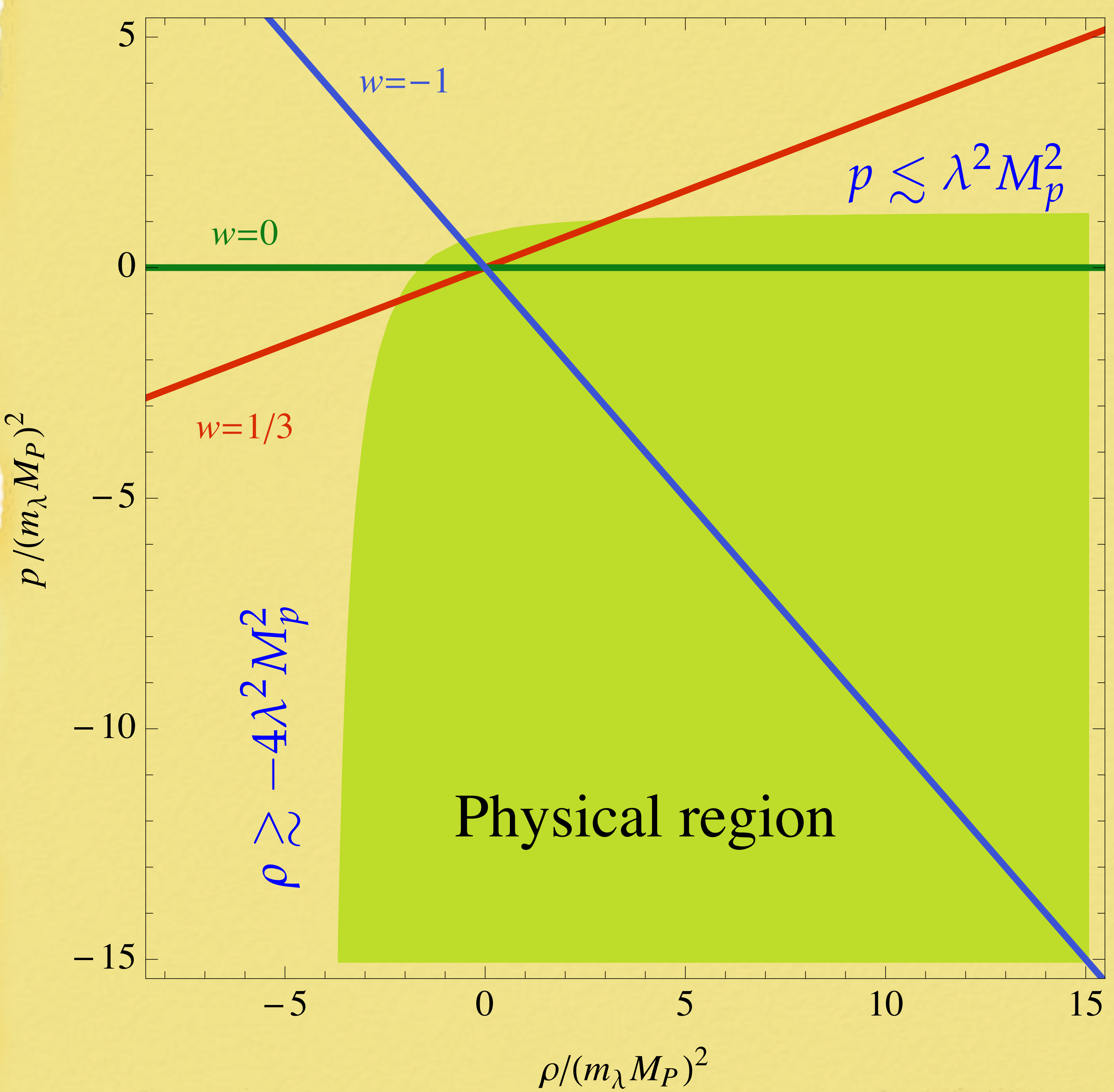
Metric field equations



In general we find 3 branches of solutions, but only two of them are physical. Out of those two, only one is continuously connected with GR.

COSMOLOGICAL SOLUTIONS

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

$$d\tilde{s}^2 = -\tilde{N}^2(t) dt^2 + \tilde{a}^2(t) \delta_{ij} dx^i dx^j$$

$$\tilde{N}^2(t) = N^2(t)\sqrt{M_0 M_1^{-3}} \qquad \tilde{a}^2(t) = \frac{a^2(t)}{\sqrt{M_0 M_1}}$$

$$\hat{M} \equiv \sqrt{\mathbb{1} + \lambda^{-2}\hat{g}^{-1}\hat{R}(\Gamma)}$$

$$\hat{G}(\tilde{g}) \equiv \hat{R}(\tilde{g}) - \frac{1}{2}\hat{\tilde{g}}\,\mathrm{Tr}(\hat{\tilde{g}}^{-1}\hat{R}) = \lambda^2 \hat{g}\left[\left(\hat{M}^2 - \mathbb{1}\right) - \frac{1}{2}\hat{M}\mathrm{Tr}\left(\hat{M} - \hat{M}^{-1}\right)\right]$$

$$\mathcal{A}^2 \equiv \tilde{a}^2 / a^2$$

$$G_{00}(\tilde{g}) = 3\left(\frac{\dot{\tilde{a}}}{\tilde{a}}\right)^2 = 3\left(H + \frac{\dot{\mathcal{A}}}{\mathcal{A}}\right)^2 = 3H^2\left[1 - 3(\rho + p)\left(\partial_\rho \ln \mathcal{A} + c_s^2 \partial_p \ln \mathcal{A}\right)\right]^2$$

$$\dot{\rho} = -3H(\rho + p)$$

$$\frac{H^2}{\lambda^2 N^2} = -\frac{(M_0^2 - 1)W_0 - 3(M_1^2 - 1)W_1}{6W_0\left[1 - 3(\rho + p)\left(\partial_\rho \ln \mathcal{A} + c_s^2 \partial_p \ln \mathcal{A}\right)\right]^2}$$

Modified Friedmann equation

COSMOLOGICAL SOLUTIONS

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

$$d\tilde{s}^2 = -N^2 (M_0 M_1^{-3})^{1/2} dt^2 + \frac{a(t)^2}{\sqrt{M_0 M_1}} \delta_{ij} dx^i dx^j$$

$$\lambda^{-2} H^2 = \frac{1 - M_0^2 + 3M_0 M_1 - \frac{3M_0}{M_1}}{6\left[1 - 3(\rho + p)\partial_\rho \ln[(M_0 M_1)^{-1/4}]\right]^2}$$

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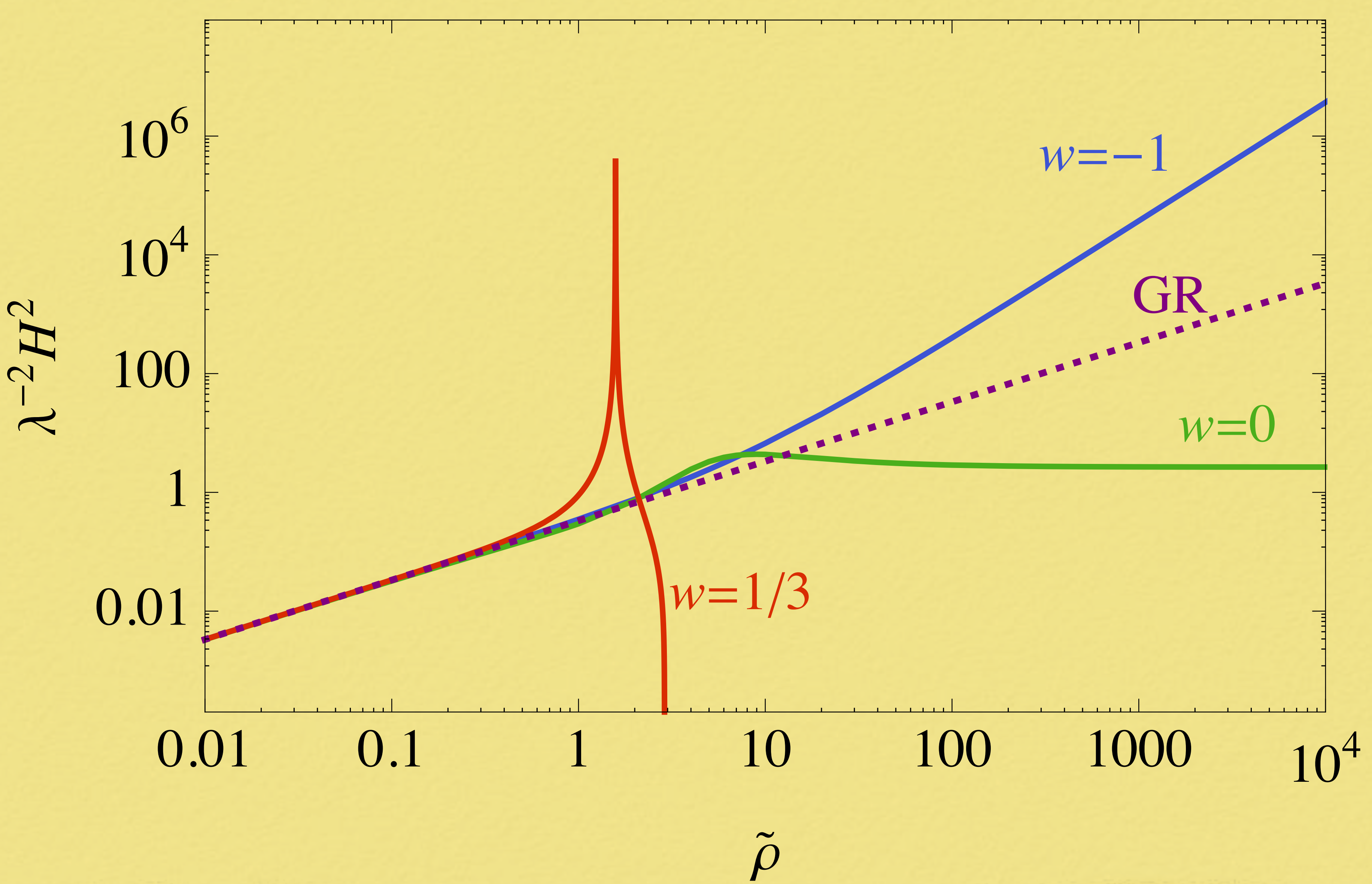
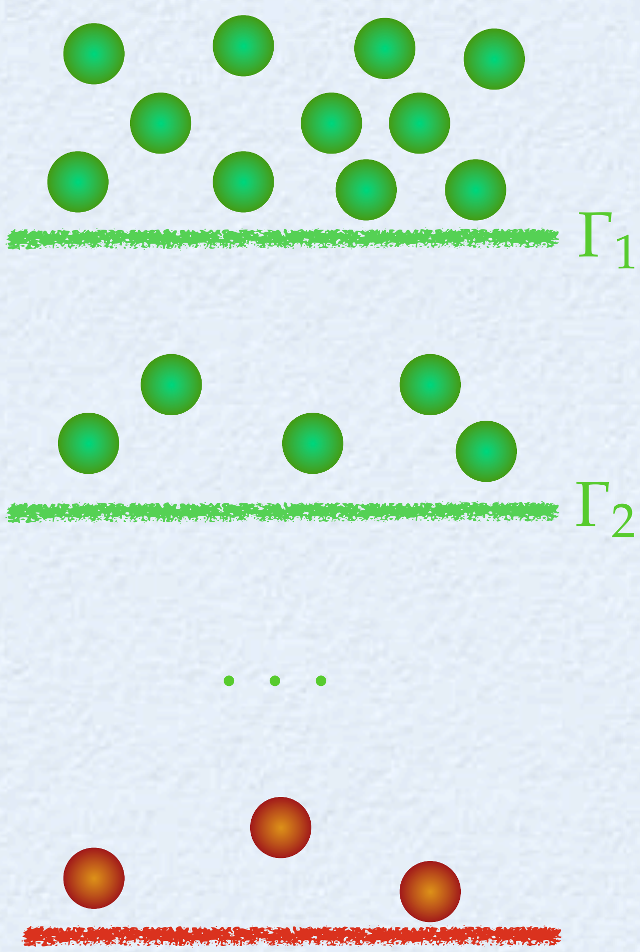
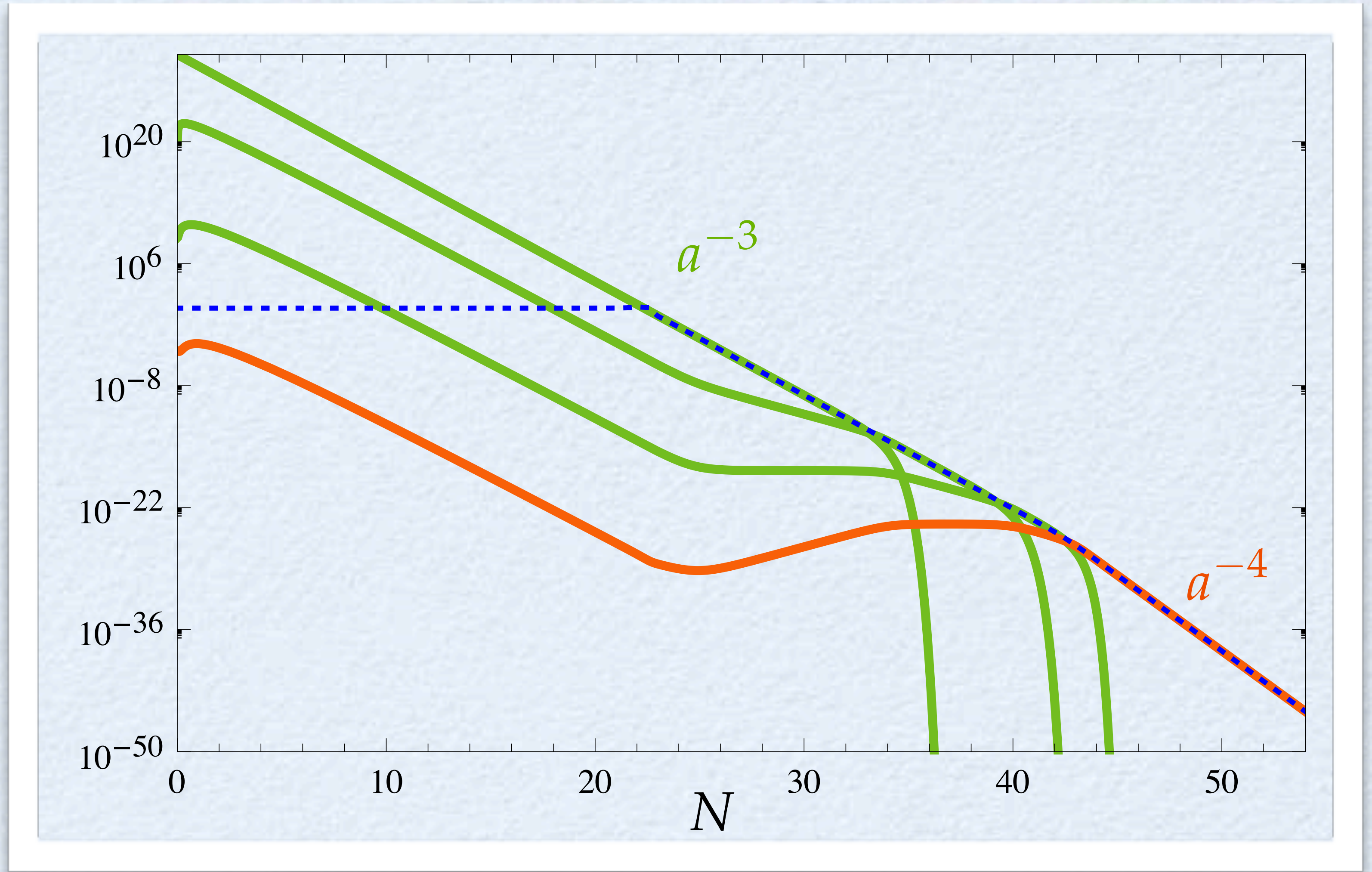


Dust inflation

JBJ, L. Heisenberg , G.J. Olmo & C. Ringeval
JCAP 1511 (2015) 046

$$\dot{\rho}_1 + 3H\rho_1 = -\Gamma_1\rho_1$$
$$\dot{\rho}_2 + 3H\rho_2 = \Gamma_1\rho_1 - \Gamma_2\rho_2$$
$$\dots$$
$$\dot{\rho}_r + 4H\rho_r = \Gamma_n\rho_n$$





Dust inflation

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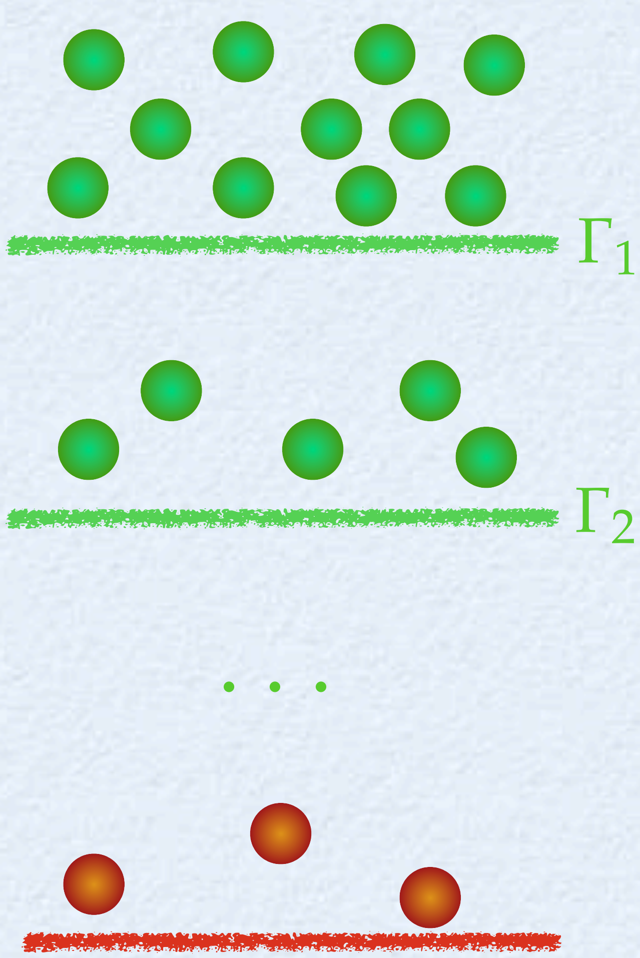
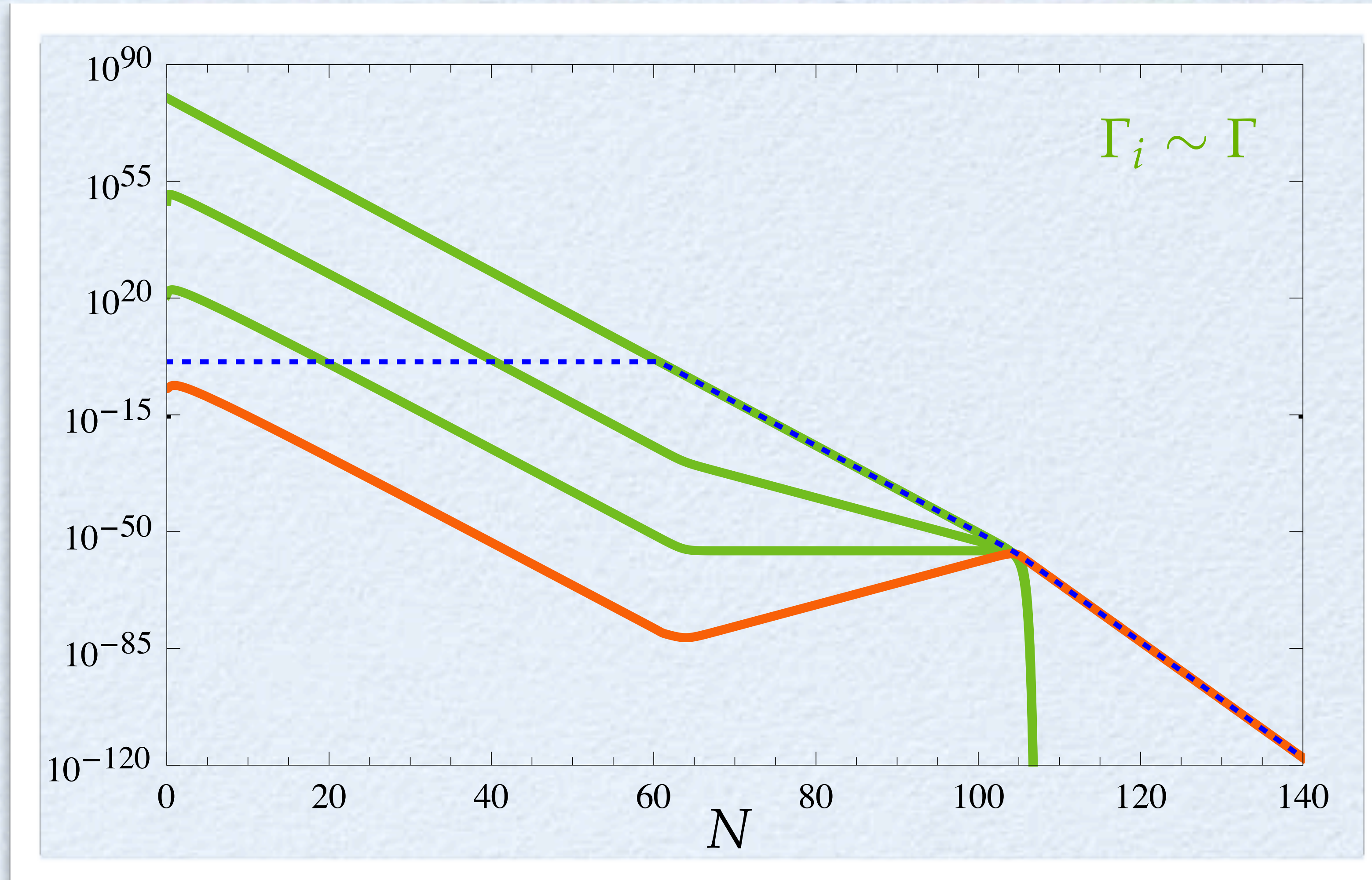
JCAP 1511 (2015) 046

$$\dot\rho_1 + 3H\rho_1 = -\Gamma_1\rho_1$$
$$\dot\rho_2 + 3H\rho_2 = \Gamma_1\rho_1 - \Gamma_2\rho_2$$
$$\ldots$$
$$\dot\rho_r + 4H\rho_r = \Gamma_n\rho_n$$





Dust inflation

JBJ, L. Heisenberg , G.J. Olmo & C. Ringeval
JCAP 1511 (2015) 046

$$\dot{\rho}_1 + 3H\rho_1 = -\Gamma_1\rho_1$$
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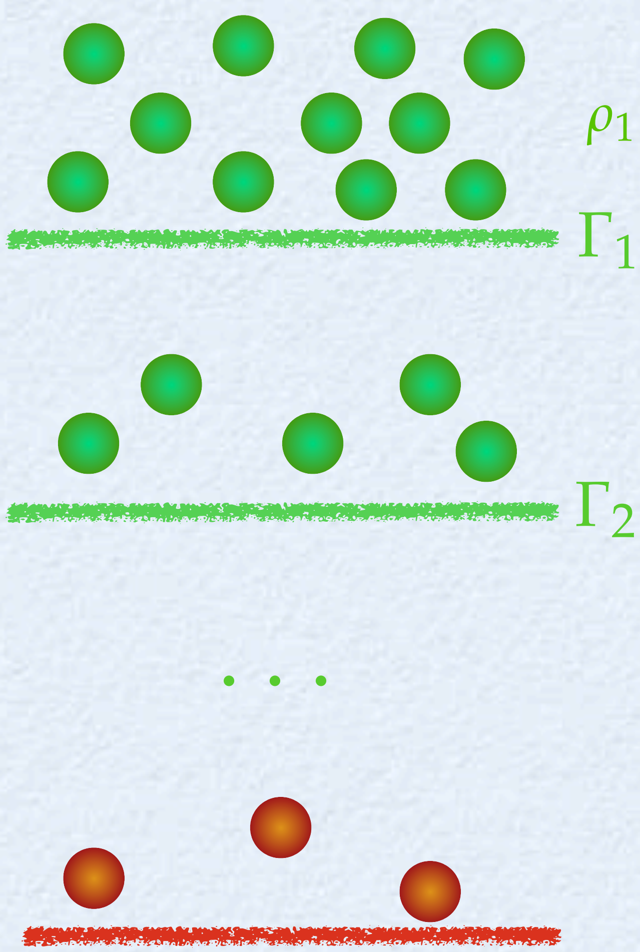

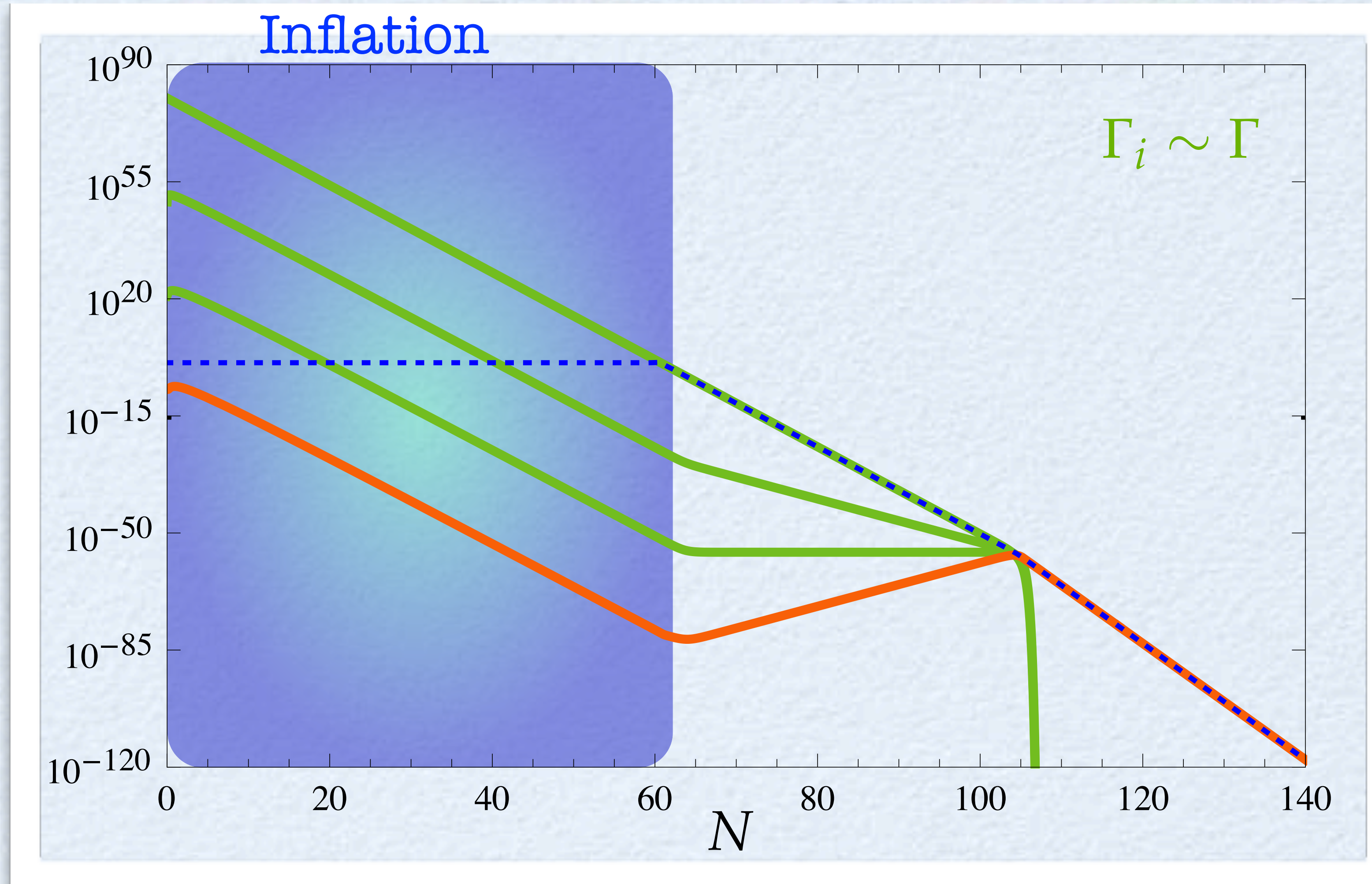

quasi de Sitter phase $\quad H^2 \simeq \dfrac{8\lambda^2}{3}$

super-inflation $\quad \epsilon_1 \equiv -\dfrac{\mathrm{d}\log H}{\mathrm{d}N} < 0$ ⇒ Blue spectrum

DUST INFLATION

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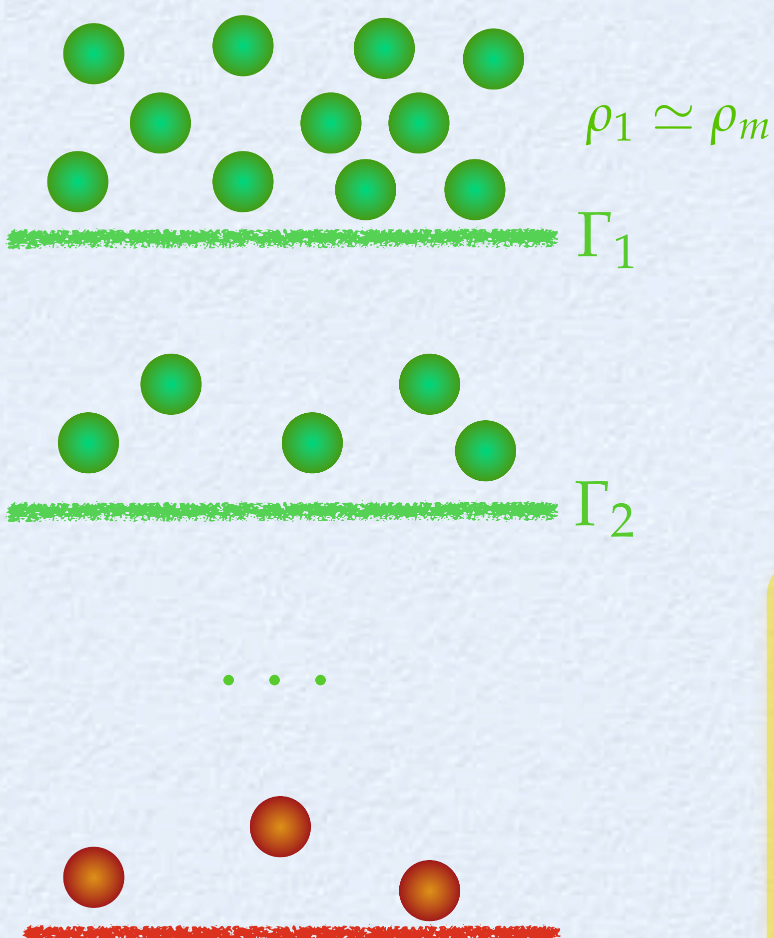
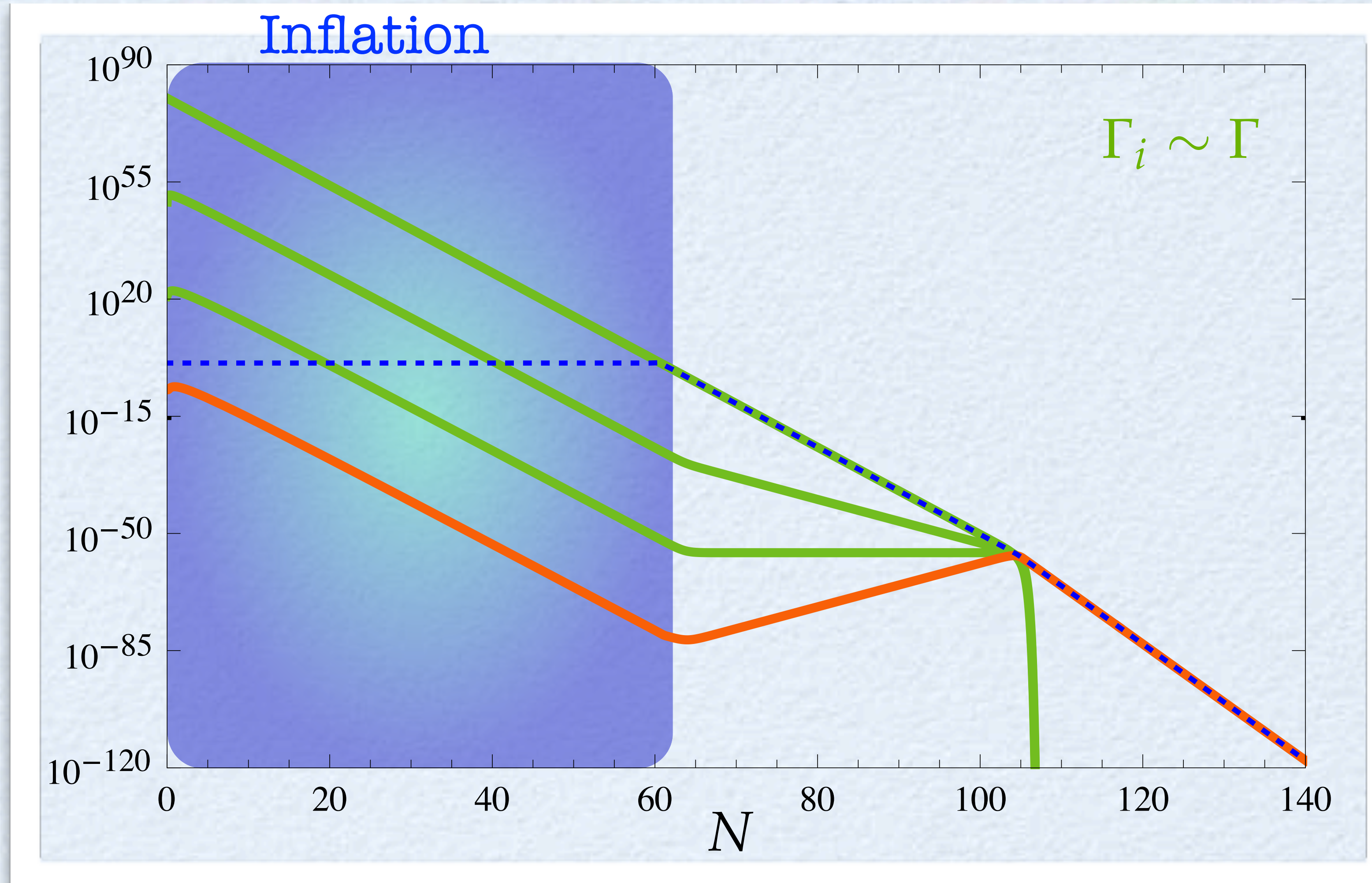


$$\rho_r \simeq \left(\prod_{i=1}^{n} \frac{\Gamma_i}{H_I}\right)\rho_1 \equiv \left(\frac{\Gamma}{H_I}\right)^n \rho_1$$



Duration of inflation

$$\rho_m \simeq \lambda^2 M_p^2 (\simeq H_I^2) \Rightarrow \Delta N_{inf} \simeq \frac{1}{3}\log\left(\frac{\rho_{m,\text{ini}}}{H_I^2 M_p^2}\right) \simeq \frac{1}{3}\log\left(\frac{\rho_{m,\text{ini}}}{\lambda^2 M_p^2}\right)$$

There is an upper bound for the duration of inflation $\bar{\rho}_{\max} \simeq \frac{24}{\sqrt{3}\bar{\Gamma}_n X_n}$

$$\Delta N_{\text{inf}} \simeq \frac{1}{3}\ln\left(\frac{\sqrt{3}}{\prod_{i=1}^{n}\bar{\Gamma}_i}\right) + \frac{n-1}{6}\ln\left[\frac{24}{(n-1)^2}\right] + \frac{1}{3}\ln[(n-1)!]$$

Dust inflation

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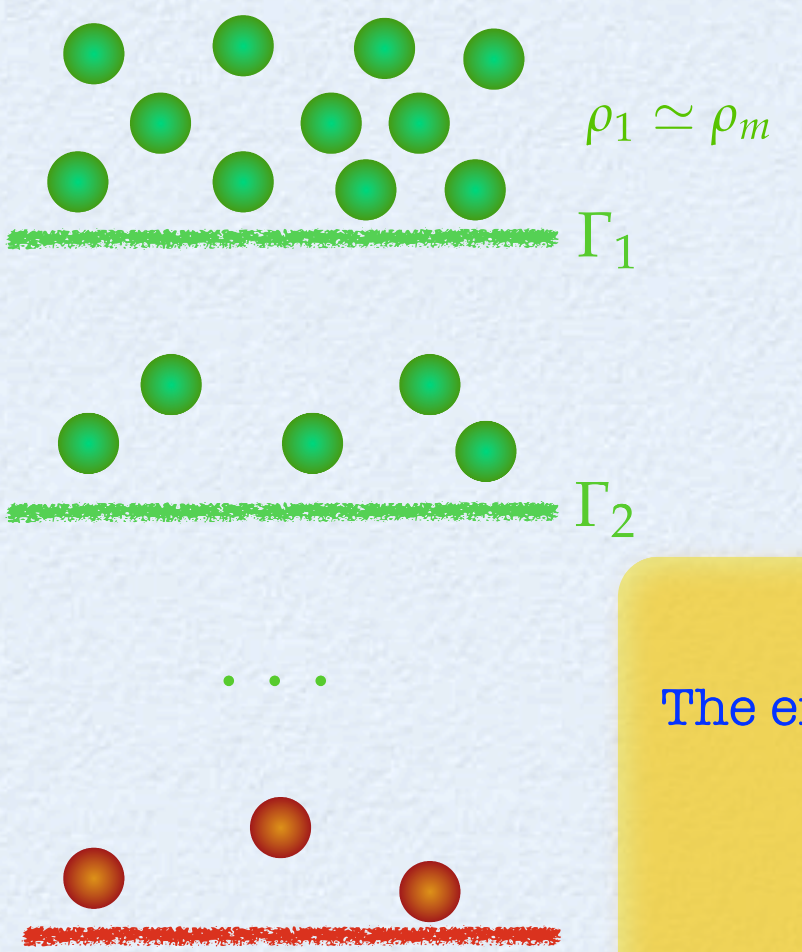

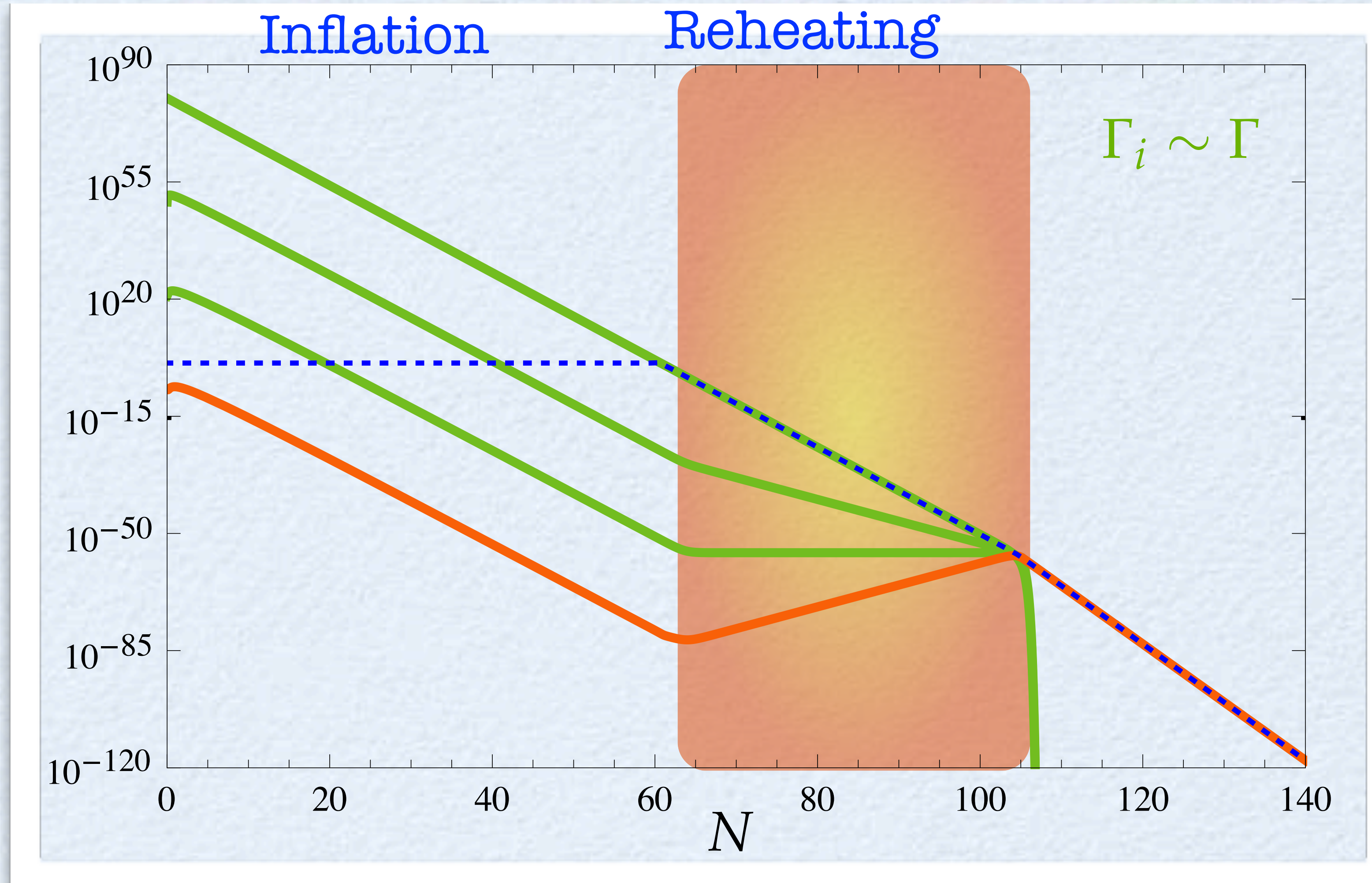

Duration of reheating

The end of reheating is set by the smallest decay rate: $\min(\Gamma_i) \simeq 3H$

$$H^2 = e^{-3(N-N_{end})}H_I^2 \quad \Rightarrow \quad \Delta N_{reh} \simeq \frac{2}{3}\log\left(\frac{H_I}{\Gamma}\right) \simeq \frac{2}{3}\log\left(\frac{\lambda}{\Gamma}\right)$$

Imposing that reheating ends before BBN gives $\min(\Gamma_i) \geq \frac{\sqrt{3\rho_{\rm nuc}}}{2M_{\rm Pl}} \simeq 10^{-41}M_{\rm Pl}$

DUST INFLATION

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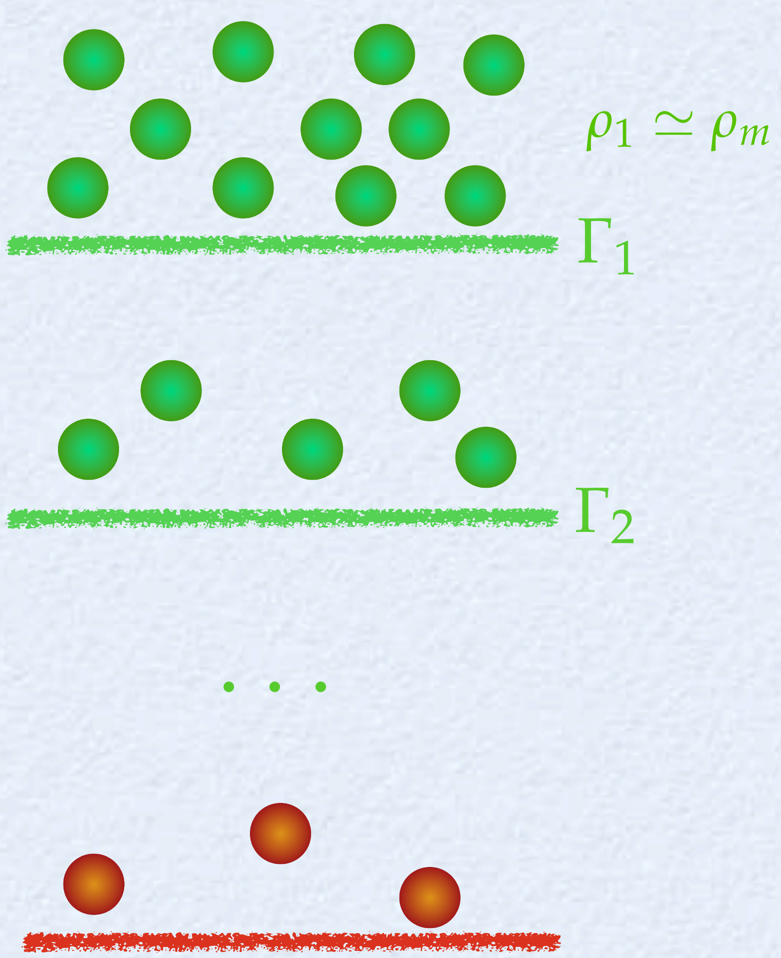
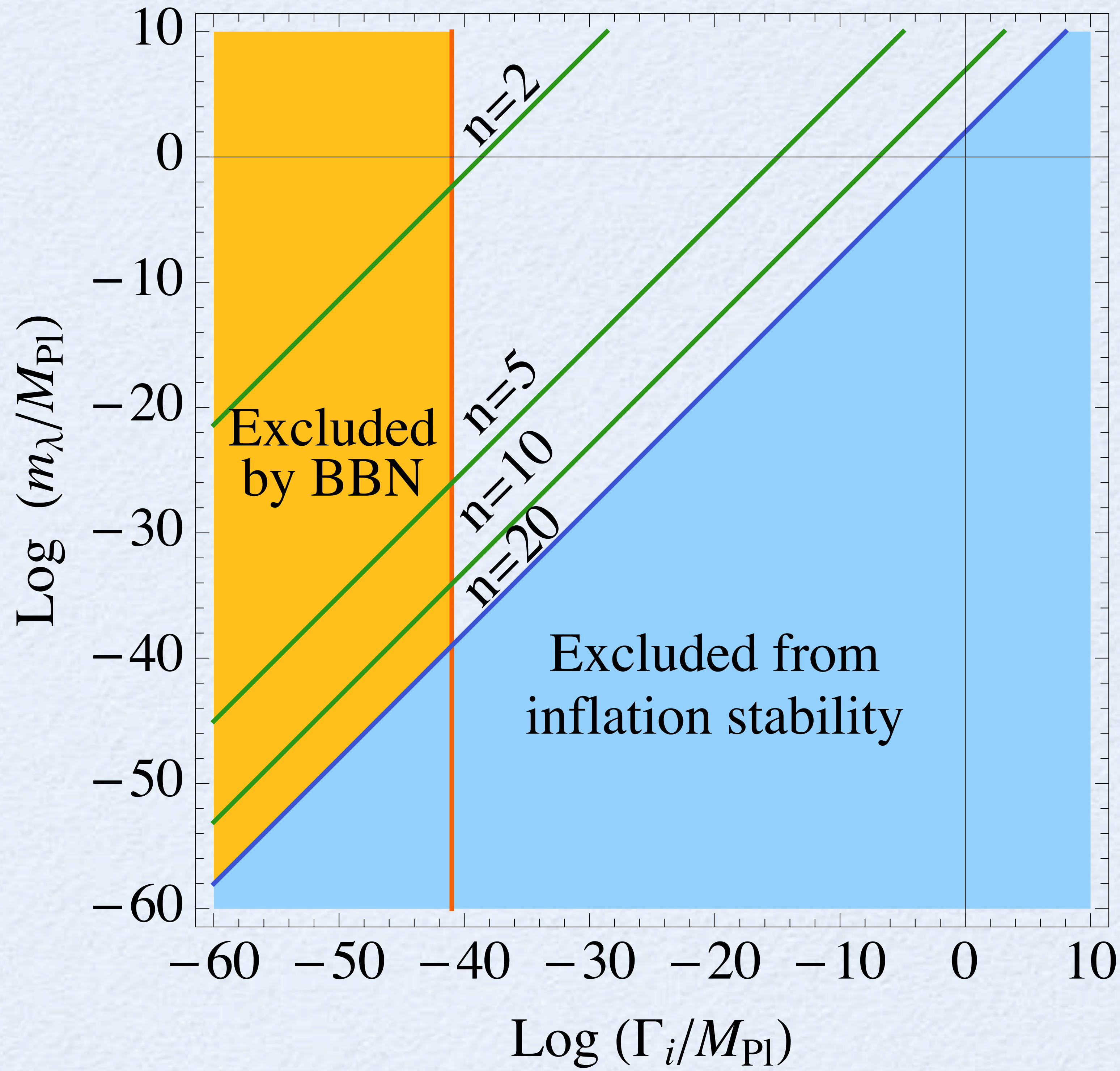




Tensor perturbations

$$\delta g_{\mu\nu} = \begin{pmatrix} 0 & \vec{0} \\ \vec{0} & a^2 h_{ij} \end{pmatrix} \qquad \delta T^{\mu}{}_{\nu} = \begin{pmatrix} 0 & \vec{0} \\ \vec{0} & \Pi^i{}_j \end{pmatrix}$$

All matrices commute at first order in tensor perturbations.

Metric field equations

$$\frac{1}{2}\left[(\hat{M} - \hat{M}^{-1}) + \hat{g}^{-1}(\hat{M} - \hat{M}^{-1})^T \hat{g}\right] - \mathrm{Tr}(\hat{M} - \mathbb{1})\mathbb{1} = \frac{1}{\lambda^2 M_p^2}\hat{T} \Longrightarrow \delta M^i{}_j = \frac{1}{\lambda^2 M_p^2}\frac{1}{1 + M_1^{-2}}\Pi^i{}_j$$

Auxiliary metric

$$\tilde{g}^{\mu\nu} = \sqrt{\det \hat{M}} g^{\alpha\mu}(\hat{M}^{-1})^{\nu}{}_{\alpha} \Longrightarrow h^i{}_j = \tilde{h}^i{}_j - \frac{1}{\lambda^2 M_p^2}\frac{1}{M_1 + M_1^{-1}}\Pi^i{}_j$$

$\delta M^i{}_j$ vanishes and both metric perturbations coincide in the absence of anisotropic stresses.

An analogous result was found for the original Born-Infeld gravity theory in C. Escamilla-Rivera, M. Banados, P. G. Ferreira, PRD85 (2012).

It is actually true for any theory of the form

$$\mathcal{S} \sim \int \mathrm{d}^4x \sqrt{-g} F(\hat{g}^{-1}, \hat{R})$$

JBJ, L. Heisenberg and G.J. Olmo JCAP 1506 (2015).

Tensor perturbations

$$\delta g_{\mu\nu} = \begin{pmatrix} 0 & \vec{0} \\ \vec{0} & a^2 h_{ij} \end{pmatrix} \qquad \delta T^{\mu}{}_{\nu} = \begin{pmatrix} 0 & \vec{0} \\ \vec{0} & \Pi^{i}{}_{j} \end{pmatrix}$$

All matrices commute at first order in tensor perturbations.

$$(M^{-1})^{\alpha}{}_{(\mu} R_{\nu)\alpha} - \mathrm{Tr}(\hat{M} - \mathbb{1})\lambda^2 g_{\mu\nu} = \frac{1}{M_{\rm Pl}^2} T_{\mu\nu} \Rightarrow \delta R^{i}{}_{j}(\tilde{g}) = \frac{\sqrt{M_0 M_1^3}}{M_p^2} \Pi^{i}{}_{j}$$

Same equation as in GR with a modified Newton's constant.

$$\ddot{\tilde{h}}_{ij} + \left(3\tilde{H}(t) - \frac{\dot{\tilde{n}}(t)}{\tilde{n}(t)}\right)\dot{\tilde{h}}_{ij} - \frac{\tilde{n}(t)^2}{\tilde{a}(t)^2}\nabla^2 \tilde{h}_{ij} = 2\frac{\sqrt{M_0 M_1^3}}{M_P^2}\Pi_{ij} \Rightarrow \boxed{\tilde{\mathrm{h}}''_{ij} - \left(\nabla^2 + \frac{\tilde{a}''}{\tilde{a}}\right)\tilde{\mathrm{h}}_{ij} = 0}$$

$\tilde{n} = 1$

$\Pi^{i}{}_{j} = 0$

$\tilde{\mathrm{h}}_{ij} = \tilde{a}\tilde{h}_{ij}$

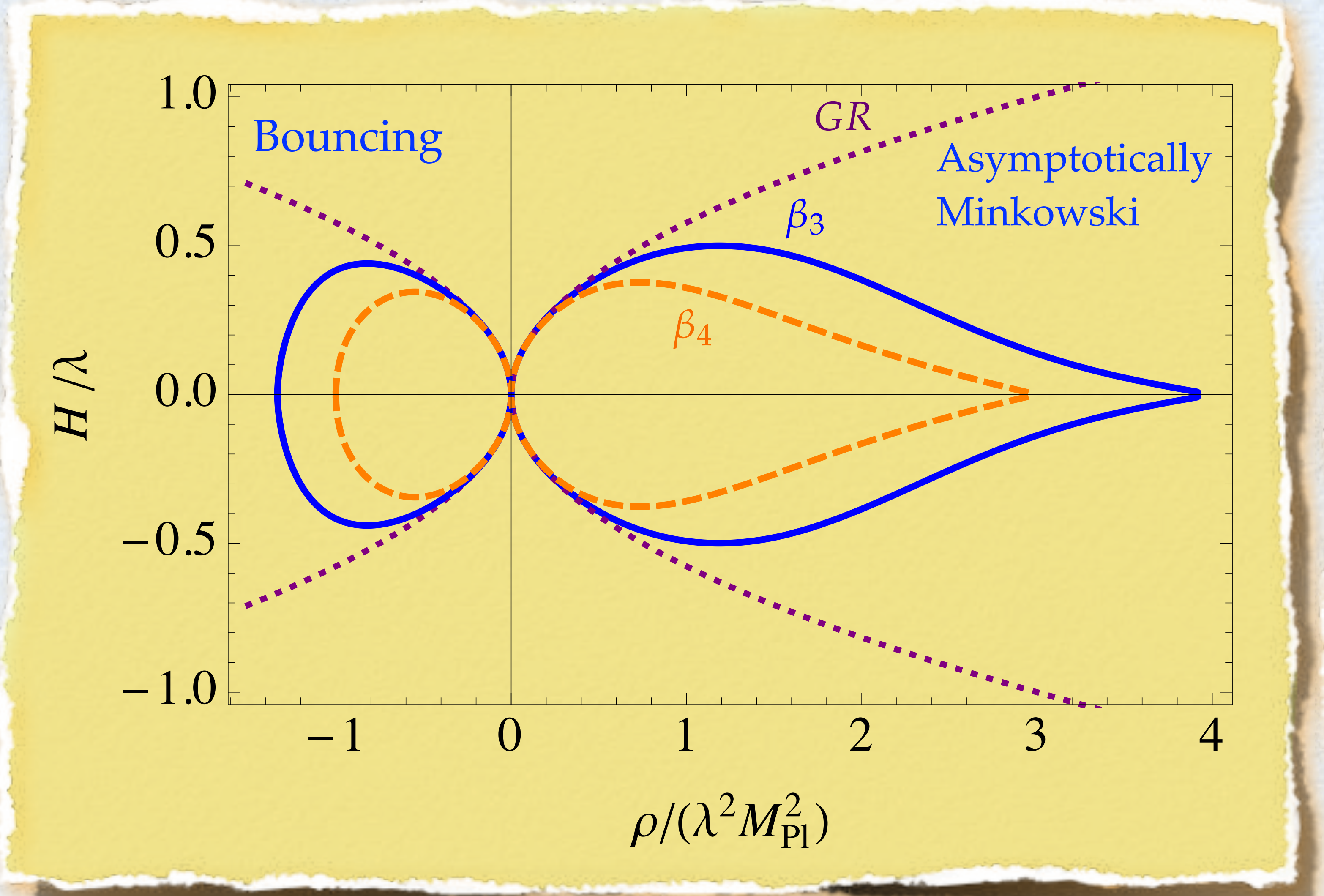
The tensor perturbations see the auxiliary metric. In the quasi de Sitter regime, we have

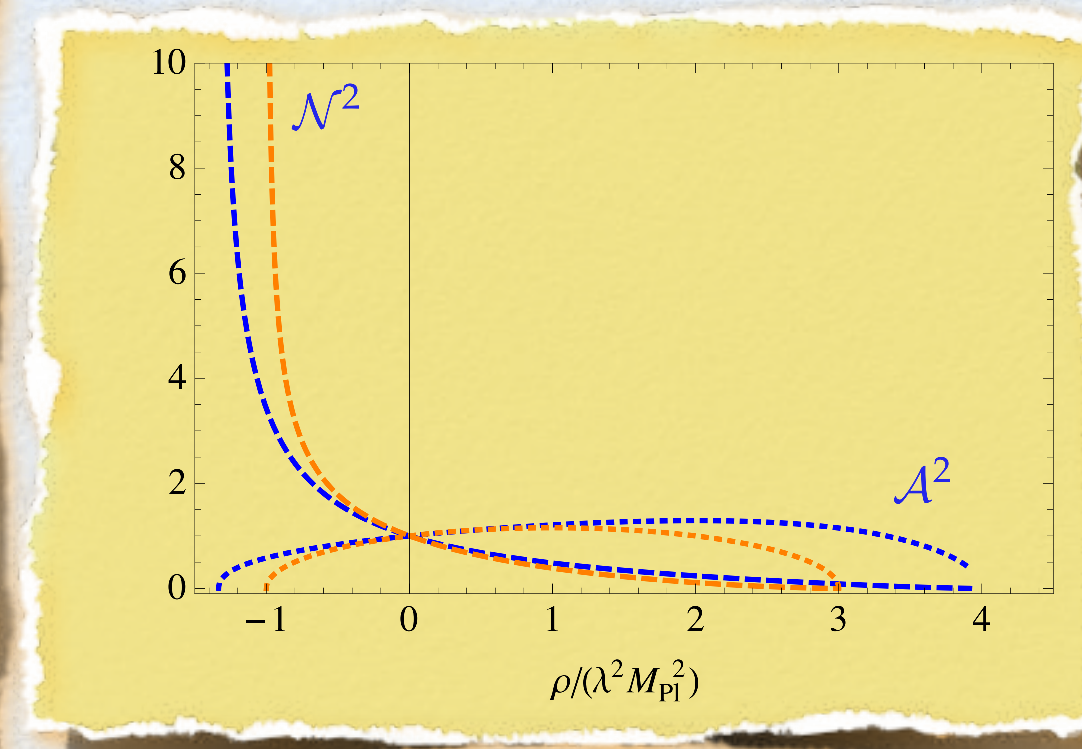
$$\tilde{H}^2 = \frac{1}{16}H^2 \simeq \frac{1}{16}H_I^2 n^2(t) \simeq \frac{1}{16}H_I^2 \tilde{n}^2(t)\sqrt{\frac{\rho_{\rm m,ini}}{(20-14\sqrt{2})\lambda^2 M_p^2}}\left(\frac{\tilde{a}}{\tilde{a}_{ini}}\right)^{-6} \longrightarrow \boxed{w_{\rm eff} = 1}$$

No generation of tensor perturbations!

$$a = a_{ini}\left(\frac{\tilde{a}}{\tilde{a}_{ini}}\right)^4 \qquad \frac{\tilde{n}^2}{n^2} \simeq \sqrt{\frac{(20-14\sqrt{2})\lambda^2 M_p^2}{\rho}} \qquad \frac{\tilde{a}^2}{a^2} \simeq \sqrt{\frac{(2-\sqrt{2})\rho}{\lambda^2 M_p^2}}$$

Bouncing solutions





Tensor instabilities observed in the original Born-Infeld gravity

C. Escamilla-Rivera, M. Bañados, P. G. Ferreira, PRD85 (2012)

Tensor instabilities could be avoided in the β_3 case.

$$\ddot{h}_{ij} + \left(3\tilde{H}(t) - \frac{\dot{\tilde{n}}(t)}{\tilde{n}(t)}\right)\dot{h}_{ij} - \frac{\tilde{n}(t)^2}{\tilde{a}(t)^2}\nabla^2 h_{ij} = 0$$

work in progress with L. Heisenberg, Diego Rubiera and G.J. Olmo

Prospects

- Possibility of stabilizing bouncing solutions with non-trivial sound speeds.
- Gravitational collapse. Singularity free black hole solutions.
- Scalar perturbations in dust inflation. Presence of instabilities.
- Role of torsion. Further explore the general action and possible extensions.
- ...

COSMOLOGICAL SOLUTIONS

$$ds^2 = -N(t)^2 dt^2 + a(t)^2 \delta_{ij} dx^i dx^j$$

$$d\tilde{s}^2 = -\tilde{N}^2(t) dt^2 + \tilde{a}^2(t) \delta_{ij} dx^i dx^j$$

$$\tilde{g}^{\mu\nu} = \sqrt{\det \hat{M}} g^{\alpha\mu} (\hat{M}^{-1})^{\nu}{}_{\alpha}$$

$$\tilde{a}^2(t) = \frac{a^2(t)}{\sqrt{M_0 M_1}}$$

$$\tilde{N}^2(t) = N^2(t) \sqrt{M_0 M_1^{-3}}$$

The signature
is preserved