Structure formation under the spell of General Relativity

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Outline:

- Part 1: Going beyond the Newtonian approximation in N-body simulations. [Including radiation perturbations.]
- Part 2: The intrinsic matter bispectrum in second order perturbation theory, numerical solution and analytical insights.

Cosmological inference

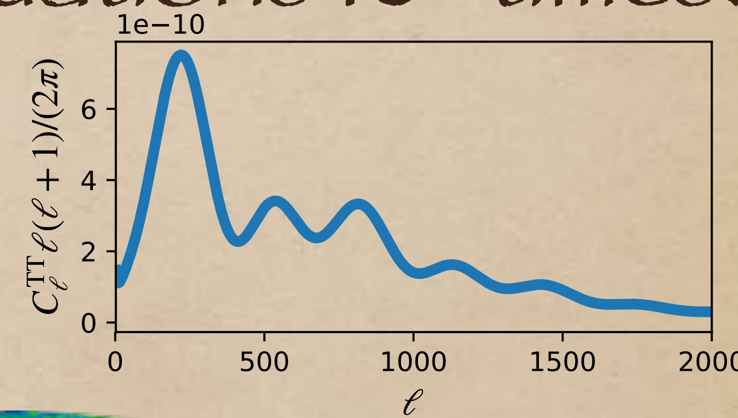
- Given some data D and some model M(...), what are the bounds on the parameters of M?
- Solve Einstein-Boltzmann equations 10^6 times.

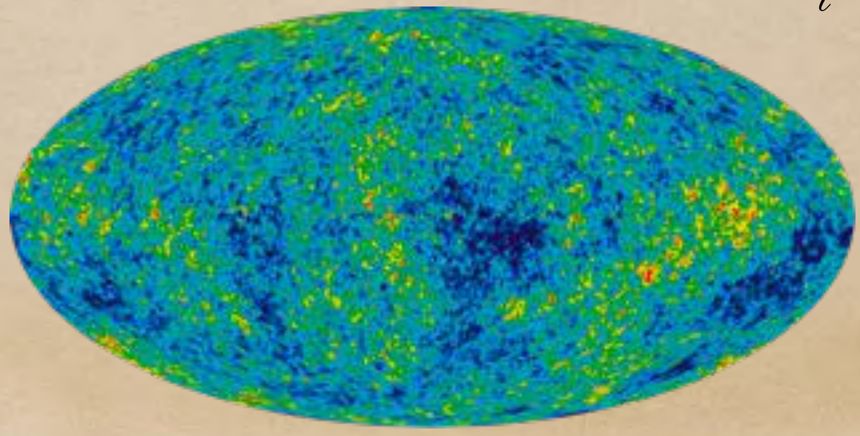
Ω_{cdm} Ω_b H_0

z_{reio} A_s n_s

(...)

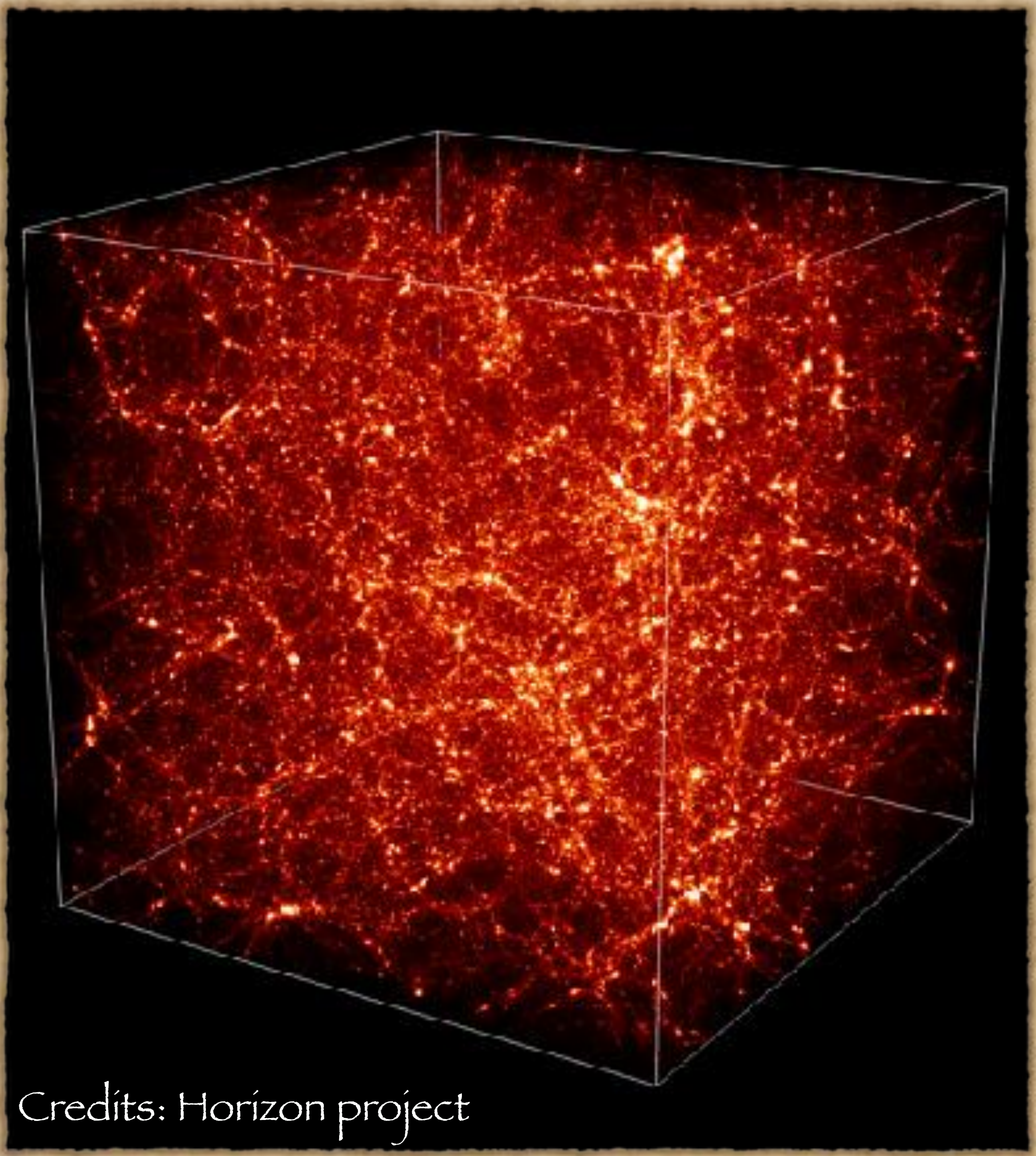
CLASS/CAMB





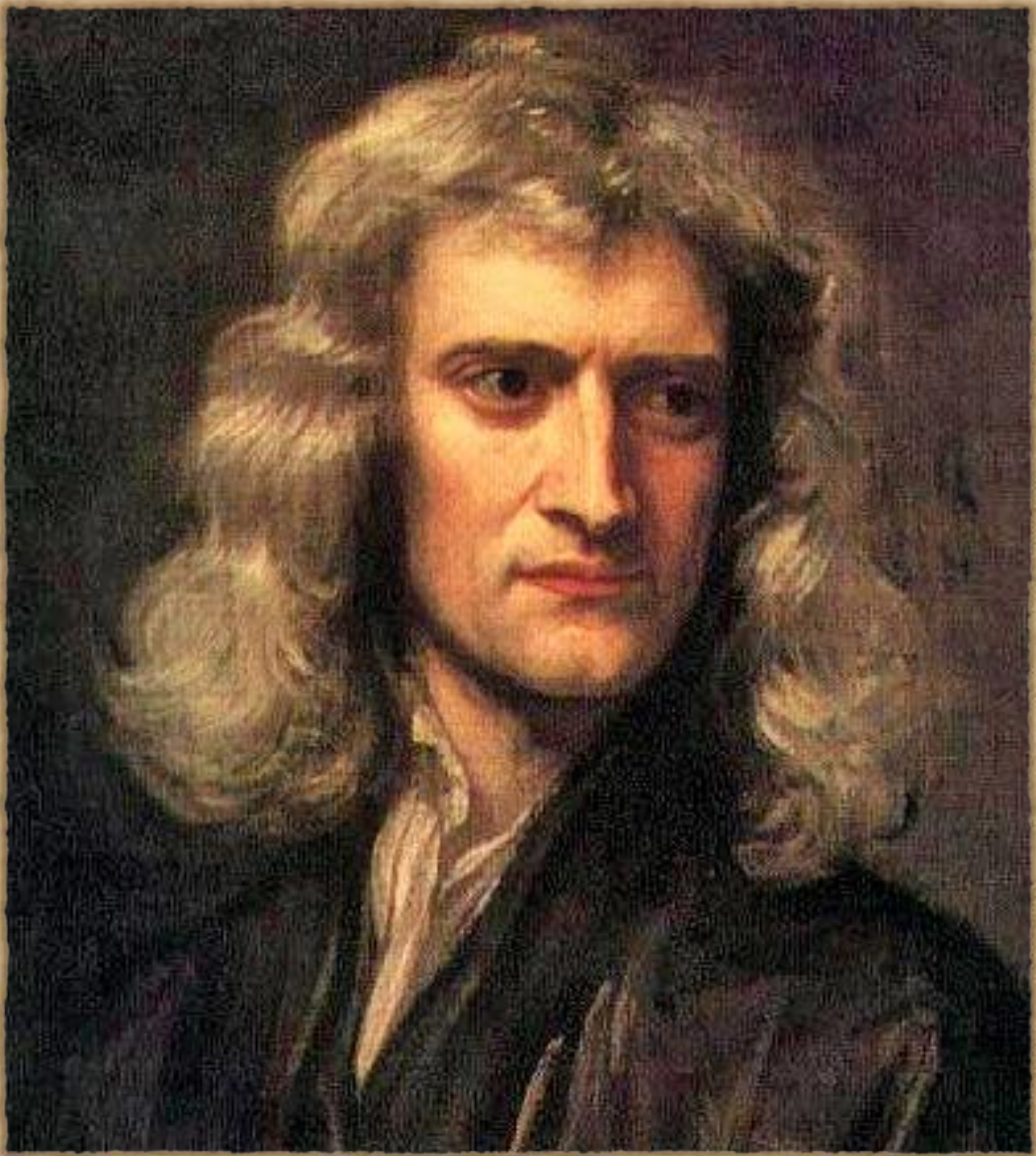
The problem

- Large Scale Structure (LSS) formation is a nonlinear process.
- Nonlinear Einstein-Boltzmann system not numerically tractable.



Newtonian approximation

- Solve Newtonian equations of motion for non-relativistic particles on a background that expands according to General Relativity.
- Can we go beyond this approximation?



Gauges in General Relativity

- The metric $g_{\mu\nu}$ is a symmetric 4 by 4 tensor having 10 d.o.f.
- SVT decomposition: 4 scalar, 4 vector and 2 tensor d.o.f.
- Diffeomorphism invariance

$$\mathbf{x}^\mu \rightarrow \mathbf{x}^\mu + \boldsymbol{\epsilon}^\mu$$

removes 2 scalar and 2 vector d.o.f.

$$g_{00} = -a^2(1+2A)\,,$$
$$g_{0i} = -a^2 B_i\,,$$
$$g_{ij} = a^2\left[\delta_{ij}\left(1+2H_{\rm L}\right) - 2H_{{\rm T}\,ij}\right]\,.$$

Newtonian gauge:

$$H_T^{(0)} = H_T^{(1)} = B^{(0)} = 0$$

Synchronous gauge:

$$A = B^{(0)} = B^{(1)} = 0$$

The N-body gauge

- There exist a unique gauge where N-body simulations are correct to first order in PT!
- ... assuming radiation perturbations can be neglected.
- 1505.04756: Fidler, Rampf, TT, Crittenden, Koyama, Wands.

Newtonian equations:

$$\dot{\delta} + \nabla \cdot \vec{v} = 0,$$
$$\left(\frac{\partial}{\partial \eta} + \frac{\dot{a}}{a} \right) \vec{v} = \nabla \Phi.$$

N-body gauge:

$$\dot{\delta} + \nabla \cdot \vec{v} = 0,$$
$$\left(\frac{\partial}{\partial \eta} + \frac{\dot{a}}{a} \right) \vec{v} = \nabla \Phi + \nabla \gamma.$$

The Poisson equation

- There exist a one-parameter family of gauges with Newtonian equations of motion.
- But N-body gauge is the only one with no "volume deformation" H_L.

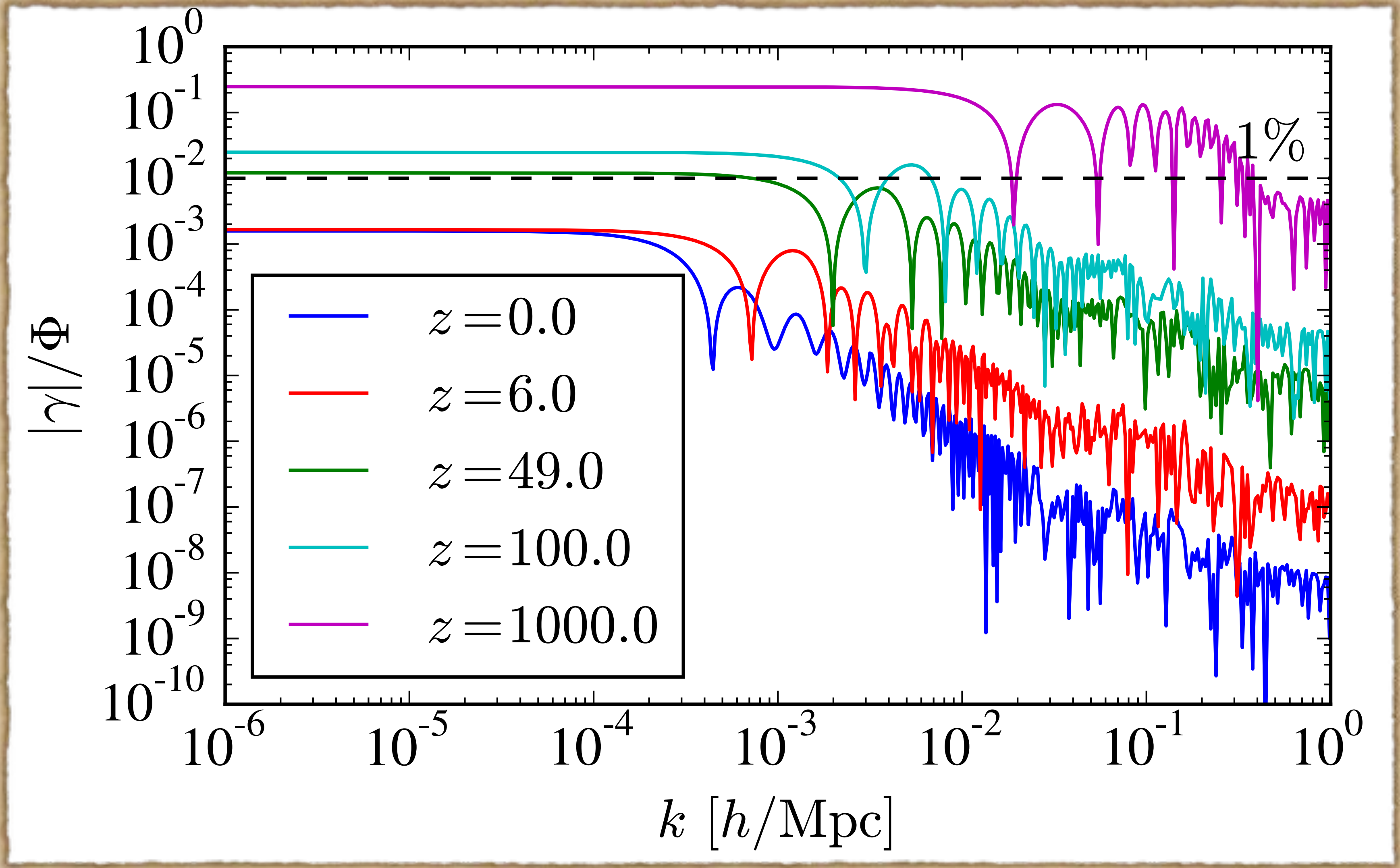
In simulation:

$$\nabla^2\Phi^N = 4\pi G a^2 \bar{\rho}\delta^N .$$

In a comoving gauge:

$$\nabla^2\Phi = 4\pi G a^2 \bar{\rho}\delta,$$

$$\delta = \delta^N + 3H_L$$

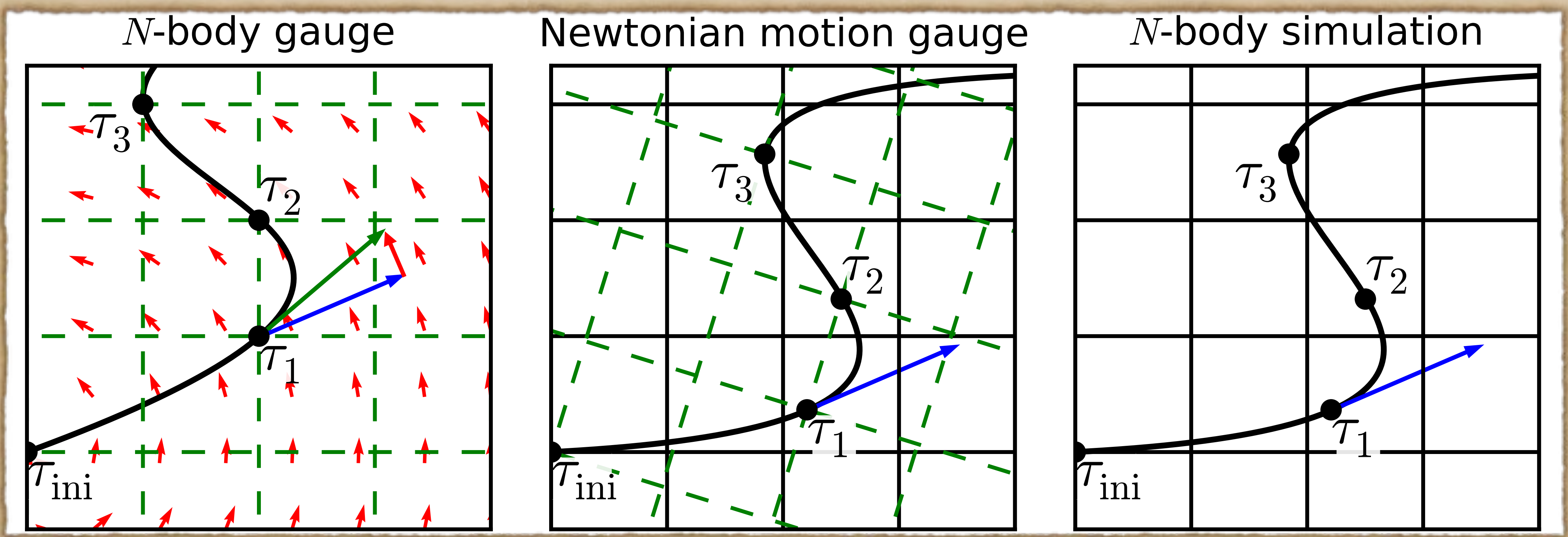


How to deal with radiation?

Just ignore? Start sufficiently late?

Add gamma to N-body simulation?

Newtonian motion



- Solve for the <u>difference</u> induced by radiation perturbations using e.g. CLASS. 1606.05588: Fidler, TT, Rampf, Crittenden, Koyama and Wands.

Newtonian motion gauges

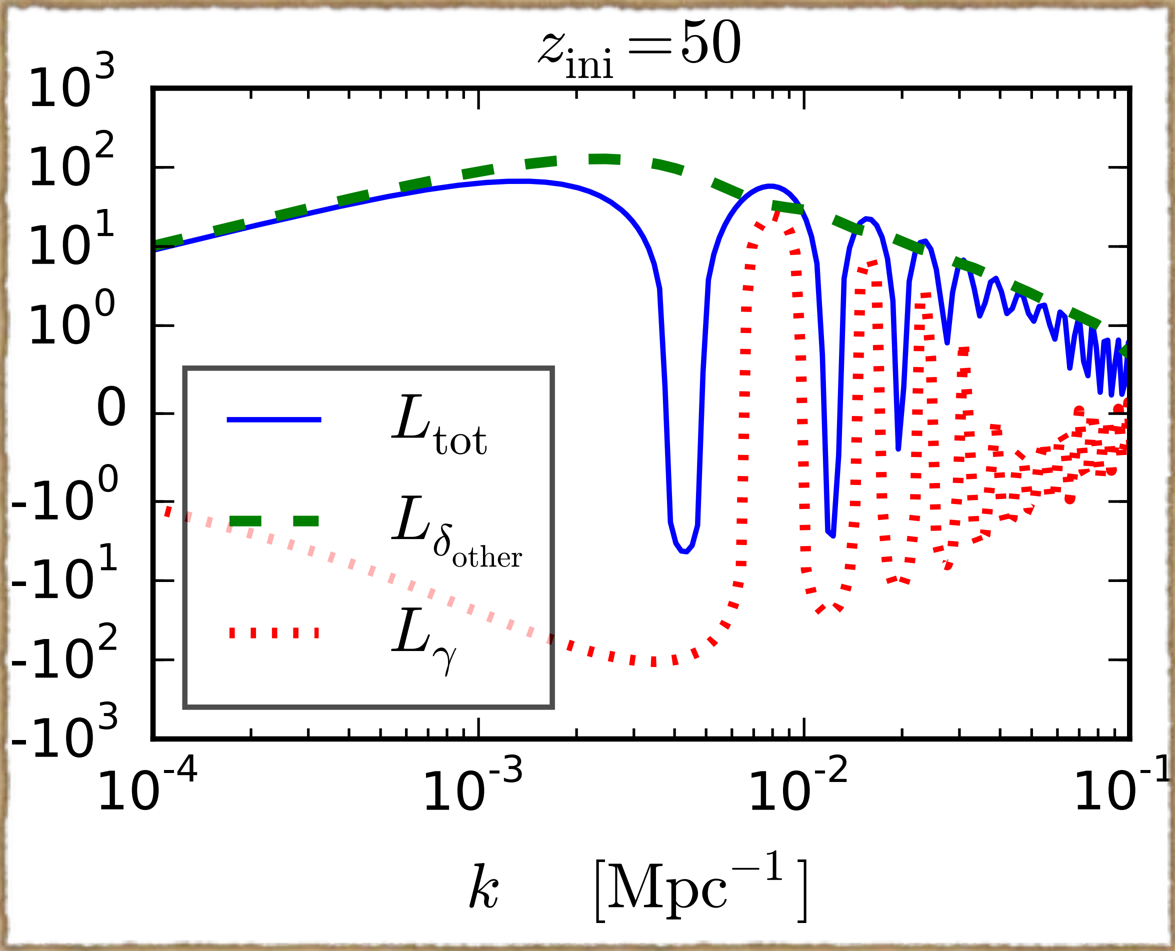
- The condition for Newtonian motion is simply

$$\vec{v} = \vec{v}^N$$

- Using the Euler equation, this condition becomes:

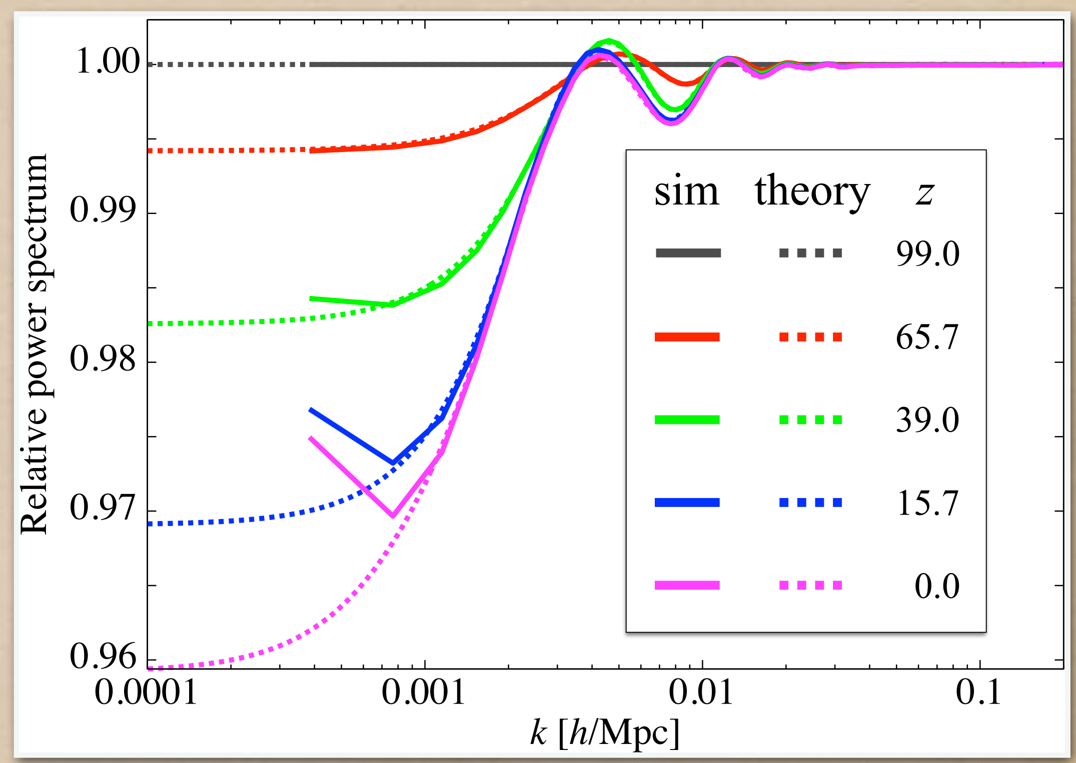
$$\gamma^{\rm Nm} = \Phi^N - \Phi$$

- Some remaining gauge freedom.



Nonlinear feedback?

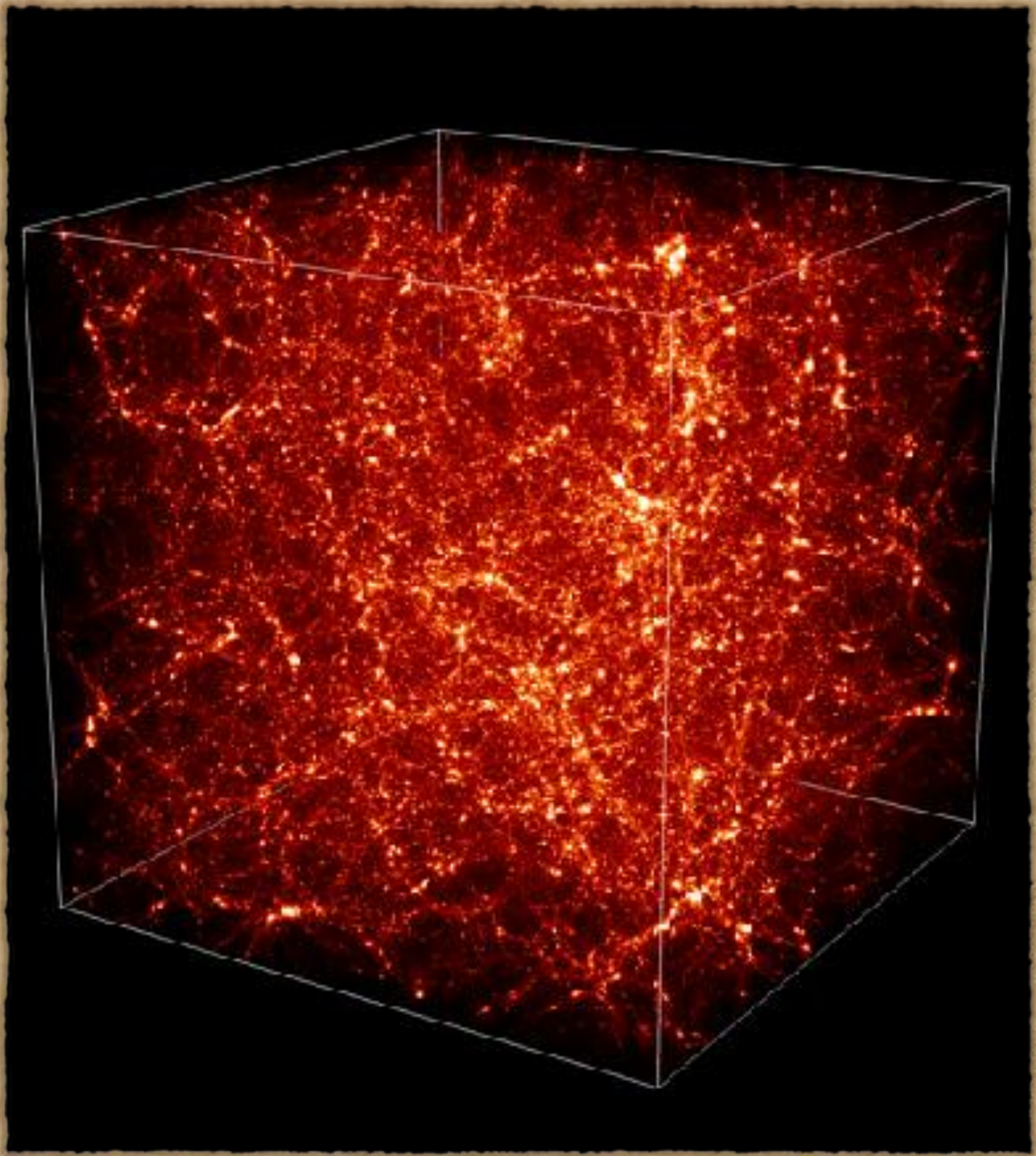
- Implement the gamma term directly in N-body
- Compare with linear Newtonian motion computation in CLASS
- 1610.04236 : Brandbyge, Rampf, TT, Leclercq, Fidler and Hannestad



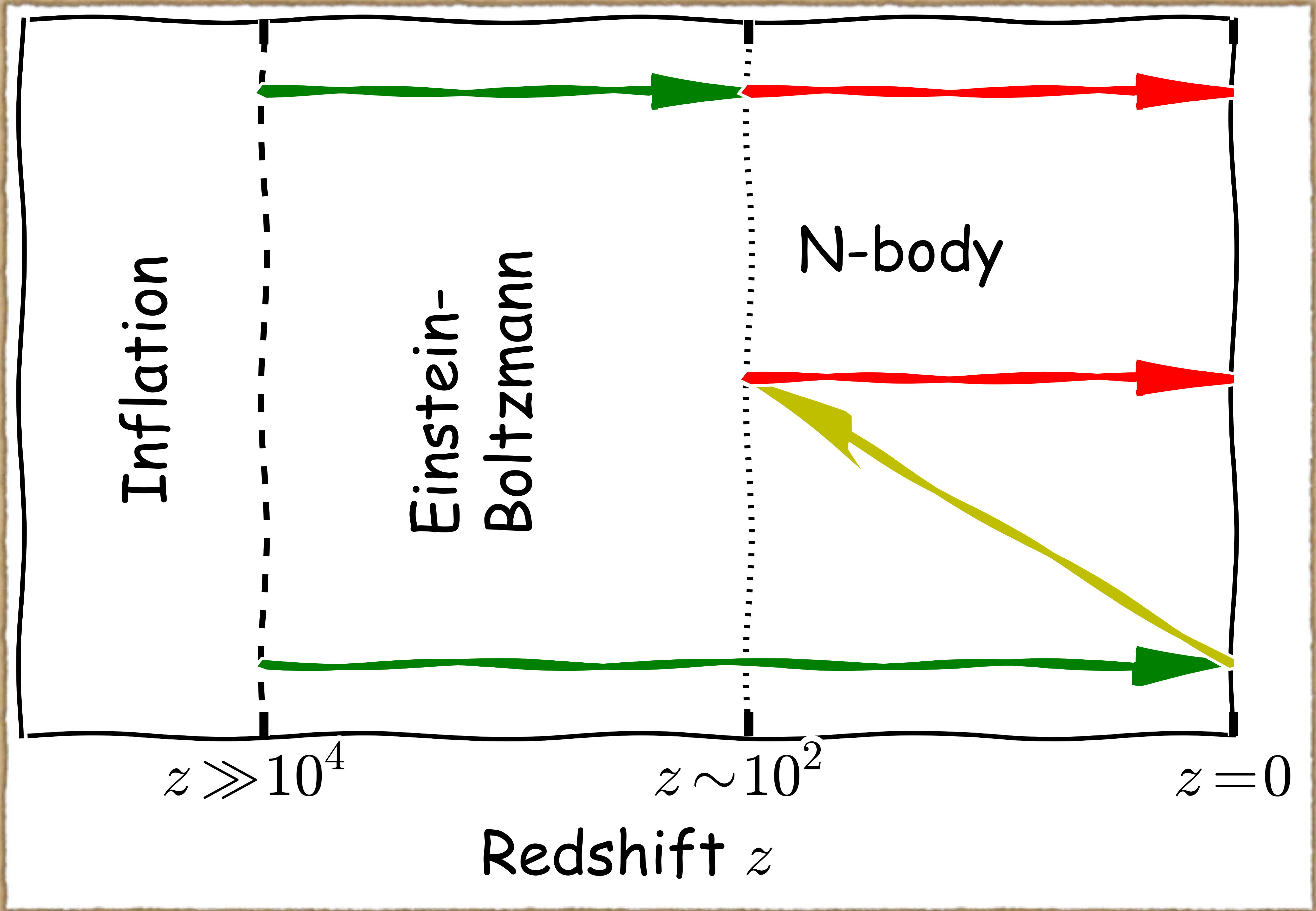
Newtonian motion gauge agrees with direct approach!

Initial conditions for N-body

- N-body simulations are initialised through a trick known as “backscaling”.
- Can we understand backscaling using the Newtonian motion gauge framework?
- 1702.03221: Fidler, TT, Rampf, Crittenden, Koyama, Wands.



What is backscaling?



Boundary condition

- The Newtonian motion gauge condition is equivalent to

$$\left(\partial_\tau + \mathcal{H}\right) \dot{H}_{\mathrm{T}} - 4\pi G a^2 \bar{\rho}_{\mathrm{cdm}} \left(H_{\mathrm{T}} - 3\zeta\right) = S$$

- In the absence of radiation, S=0 and ζ is constant so

$$H_{\mathrm{T}}(\tau) = C_+^{H_{\mathrm{T}}} D_+(\tau) + C_-^{H_{\mathrm{T}}} D_-(\tau) + 3\zeta$$

- N-body gauge has $H_{\mathrm{T}}(\tau) = 3\zeta$ so we can match the boundary condition with $C_+^{H_{\mathrm{T}}} = C_-^{H_{\mathrm{T}}} = 0$

- When S in non-zero, we add a time-dependence to the coefficients (variation of constants).

A few additional details

- Variation of constants ansatz:

$$H_{\mathrm{T}} = C_{+}^{H_{\mathrm{T}}}(\tau)D_{+}(\tau) + C_{-}^{H_{\mathrm{T}}}(\tau)D_{-}(\tau) + 3\zeta$$

- Solutions that satisfy boundary condition:

$$C_{\pm}^{H_{\mathrm{T}}}(\tau) = \pm \int_{\tau}^{\tau_{\mathrm{final}}} \tilde{S}(\tilde{\tau})D_{\mp}(\tilde{\tau})\,W(\tilde{\tau})^{-1}\mathrm{d}\tilde{\tau}$$

- W is the Wronskian $W = D_{+}\dot{D}_{-} - D_{-}\dot{D}_{+}$.

The growing mode

- Second order ODE for the linear Newtonian density contrast:

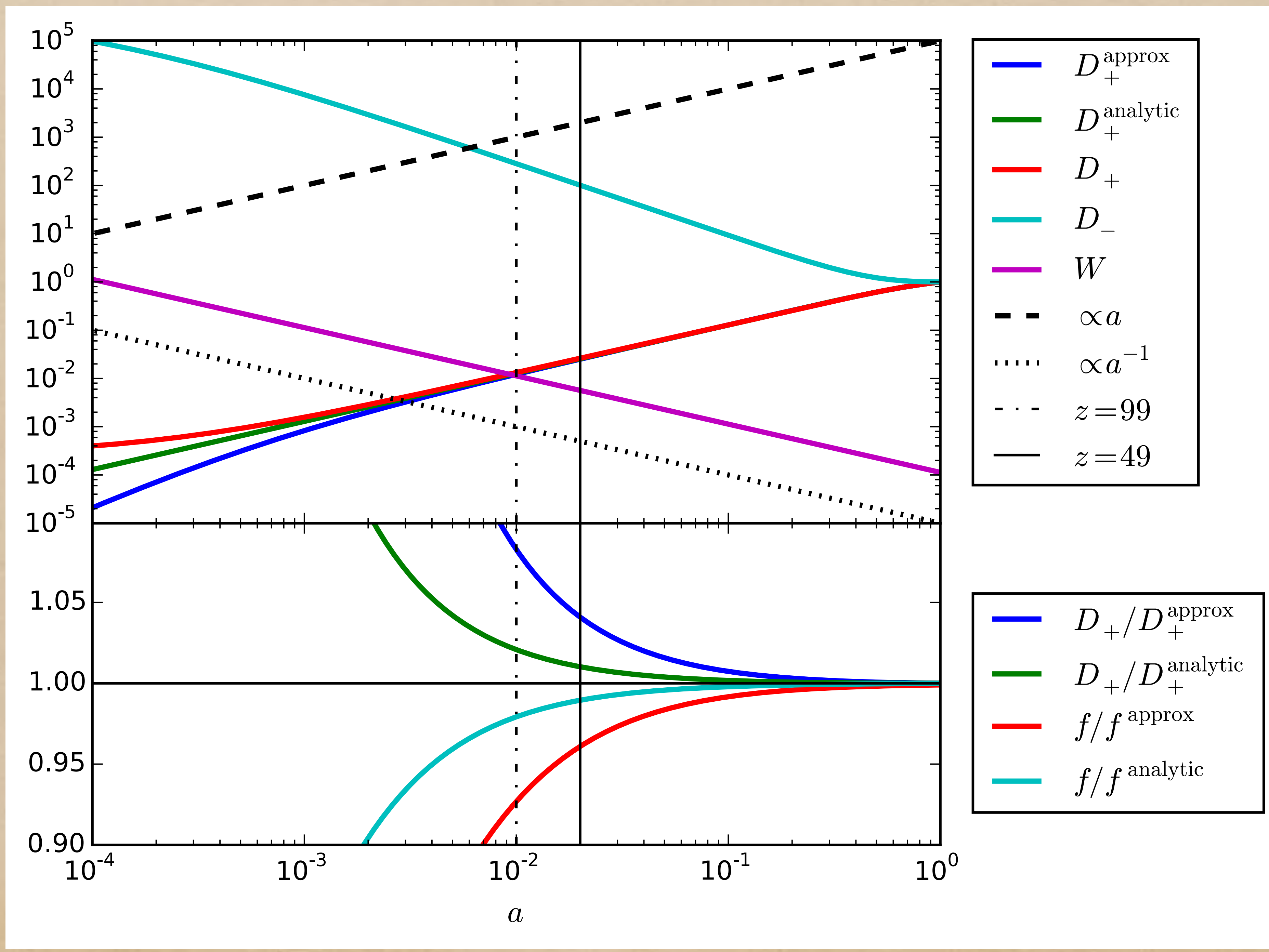
$$\ddot{D} + \mathcal{H}\dot{D} - \frac{3}{2}\frac{H_0\Omega_m}{a}D = 0$$

- If radiation can be ignored we have

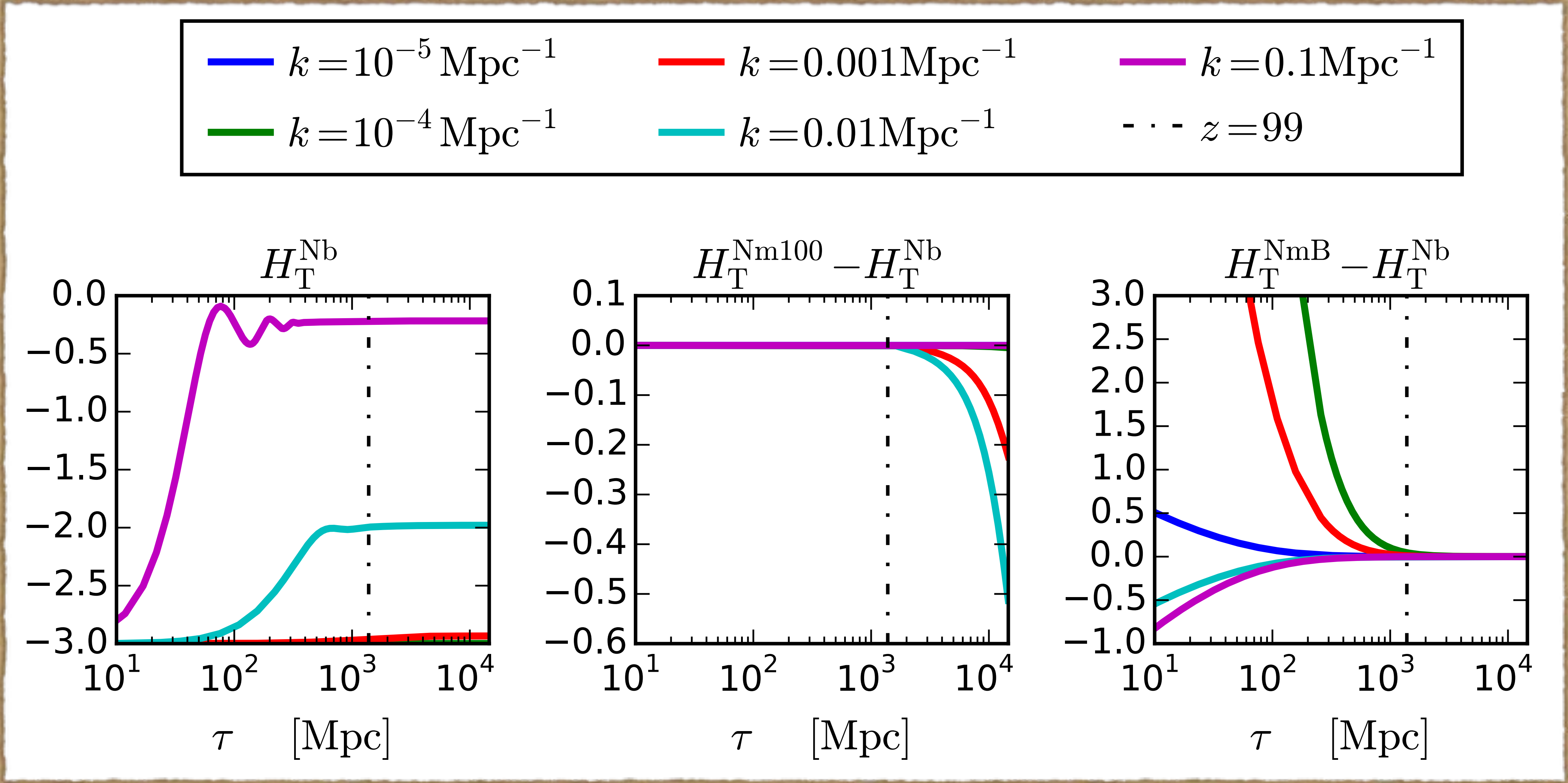
$$D_+(a) = \frac{5}{2}H_0\Omega_m\frac{\mathcal{H}}{a}\int_0^a \frac{da'}{\mathcal{H}^3(a')} = a\,{}_2F_1\left(\frac{1}{3}, 1, \frac{11}{6}, -\frac{\Omega_m}{\Omega_\Lambda}a^3\right)$$

D_{approx} $\qquad$ D_{analytic}

Growing and decaying

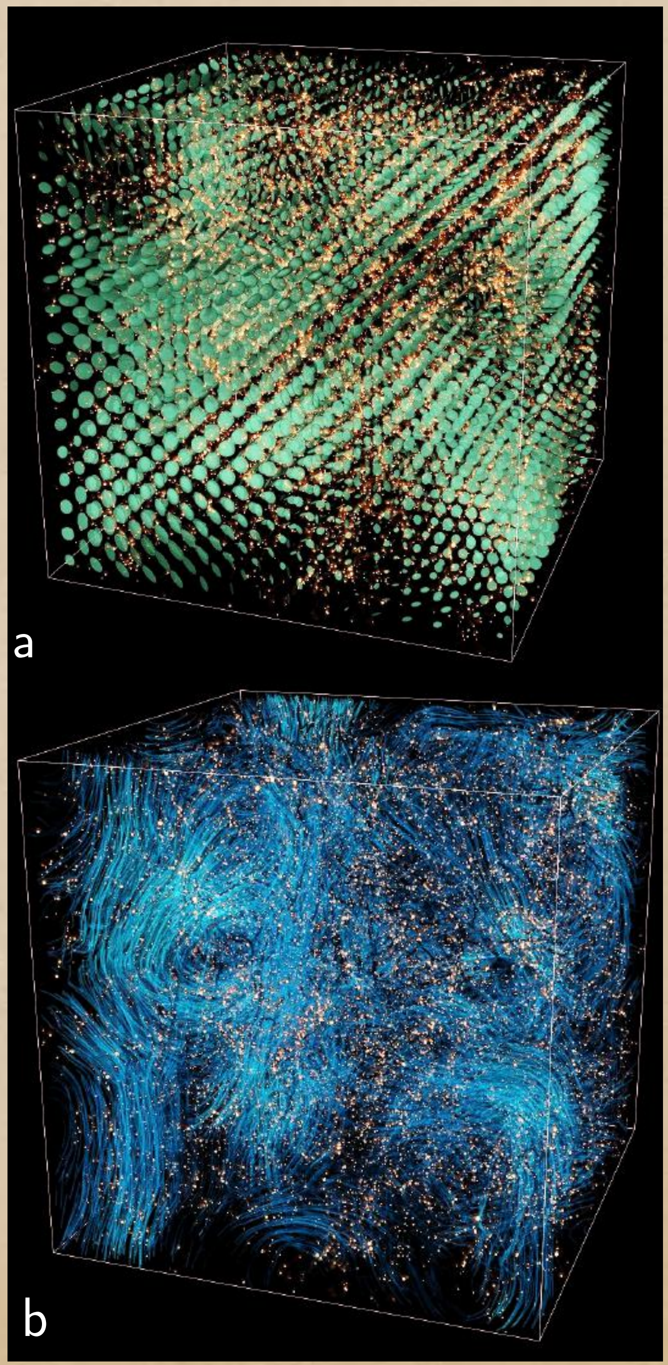


Reconstructing the metric



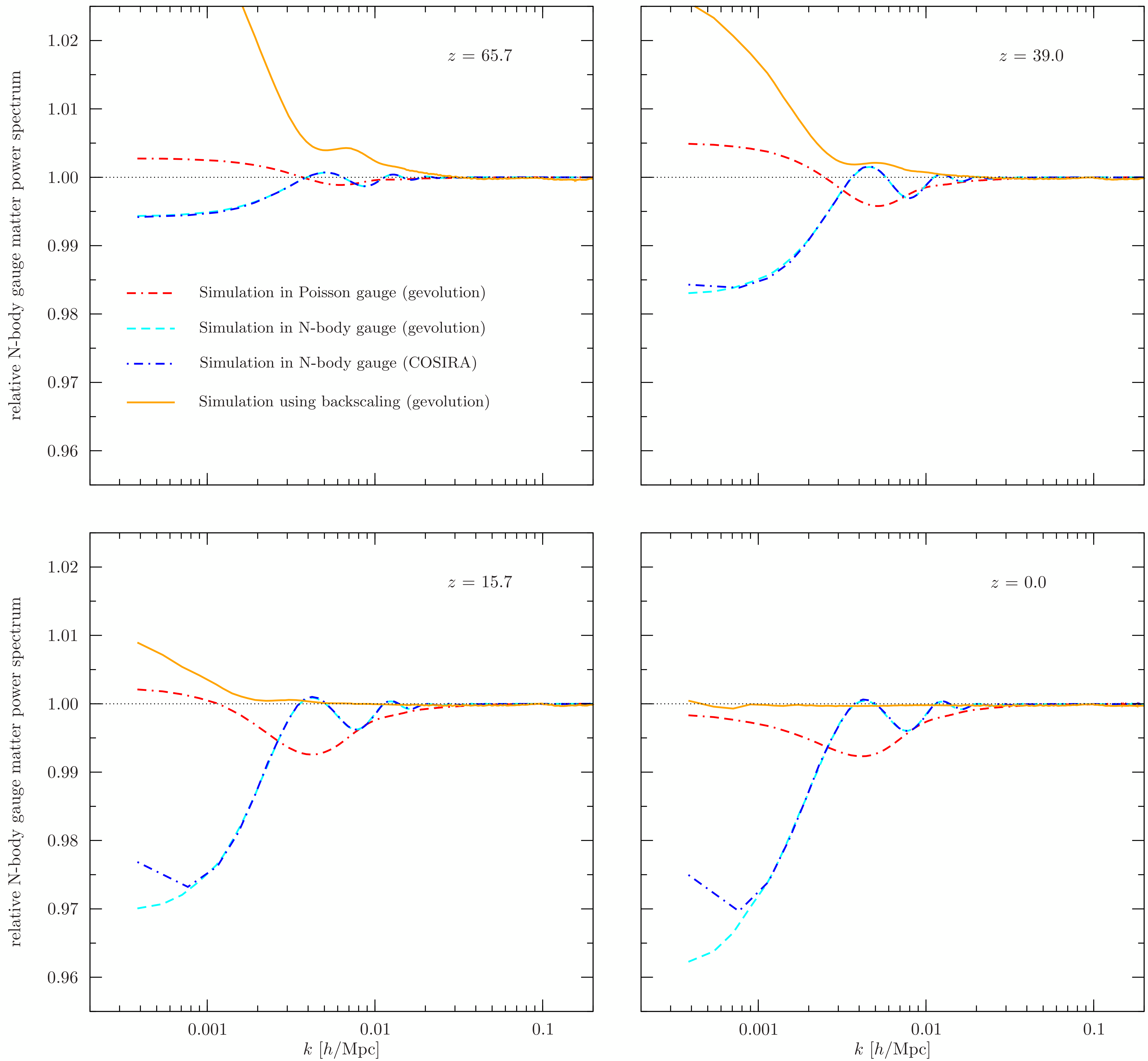
gevolution comparison

- The code gevolution by Julian Adamek et. al. is an N-body code based on a weak-field expansion of GR.
- Radiation was not included in v1.0 but has now been included in v1.1.



Including radiation

- 1702.03221: Adamek, Brandbyge, Fidler, Hannestad, Rampf, TT.
- We compare relative matter power spectra between simulations that included radiation and those that did not.



relative N-body gauge matter power spectrum
1.02
1.01
1.00
0.99
0.98
0.97
0.96
$z = 65.7$
$z = 39.0$
$z = 15.7$
$z = 0.0$
Simulation in Poisson gauge (gevolution)
Simulation in N-body gauge (gevolution)
Simulation in N-body gauge (COSIRA)
Simulation using backscaling (gevolution)
0.001
0.01
0.1
k [h/Mpc]

Part 1 conclusions

- Radiation perturbations can be included consistently in various ways.
- (Relativistic) Backscaling works very well in LCDM!

The Intrinsic Matter Bispectrum in ΛCDM

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Part 2

Based on 1602.05933

Bispectrum reminder

- Homogeneous three-point function:

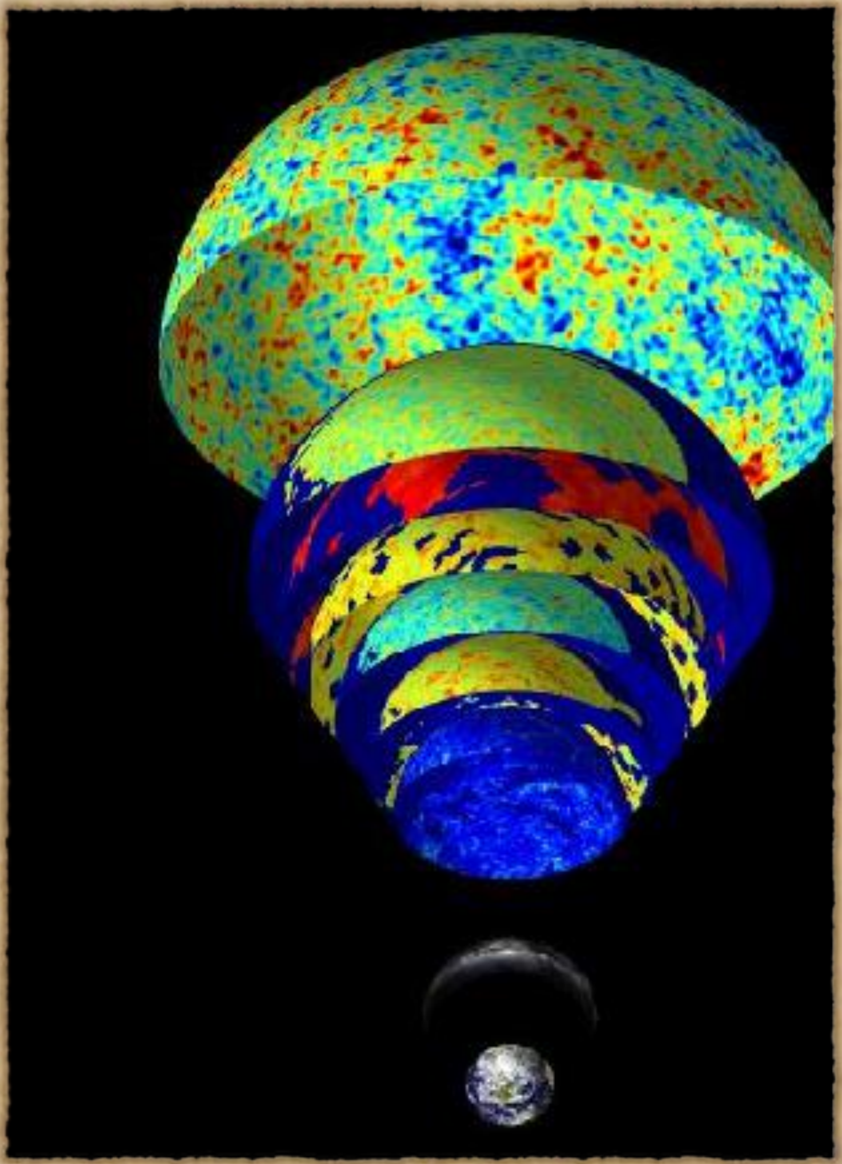
$$\xi(\mathbf{r},\mathbf{s}) = \langle R(\mathbf{x}) R(\mathbf{x}+\mathbf{r}) R(\mathbf{x}+\mathbf{s}) \rangle$$

- In Fourier space:

$$\begin{aligned}
\langle R(\mathbf{k_1}) R(\mathbf{k_2}) R(\mathbf{k_3}) \rangle &= \iiint d\mathbf{x} d\mathbf{y} d\mathbf{z} e^{-i(\mathbf{k_1}\cdot\mathbf{x}+\mathbf{k_2}\cdot\mathbf{y}+\mathbf{k_3}\cdot\mathbf{z})} \langle R(\mathbf{x}) R(\mathbf{y}) R(\mathbf{z}) \rangle \\
&= (2\pi)^3 \delta^D(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}) \iint d\mathbf{r} d\mathbf{s} e^{-i(\mathbf{k_2}\cdot\mathbf{r}+\mathbf{k_3}\cdot\mathbf{s})} \xi(\mathbf{r},\mathbf{s}) \\
&= (2\pi)^3 \delta^D(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}) B(\mathbf{k_1},\mathbf{k_2})
\end{aligned}$$

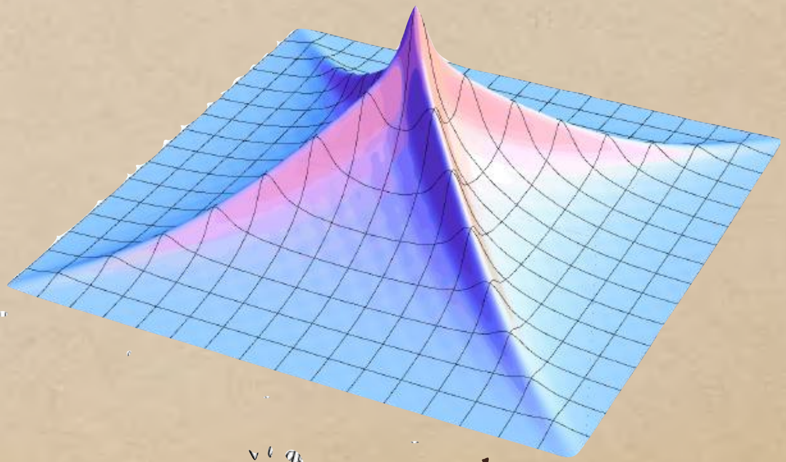
Observing primordial ICs

- The intrinsic bispectrum acts as a foreground for the primordial bispectrum.
- The intrinsic bispectrum should be computed and subtracted.



Why is this interesting?

1. No prim. bisp. for Gaussian ICs.
2. Simplest inf. predicts almost Gaussian ICs.
3. Departures produced by more complicated setups.
4. Non-Gaussianity may distinguish classical versus quantum generation of ICs.



Martin and Vennin [1510.04038]

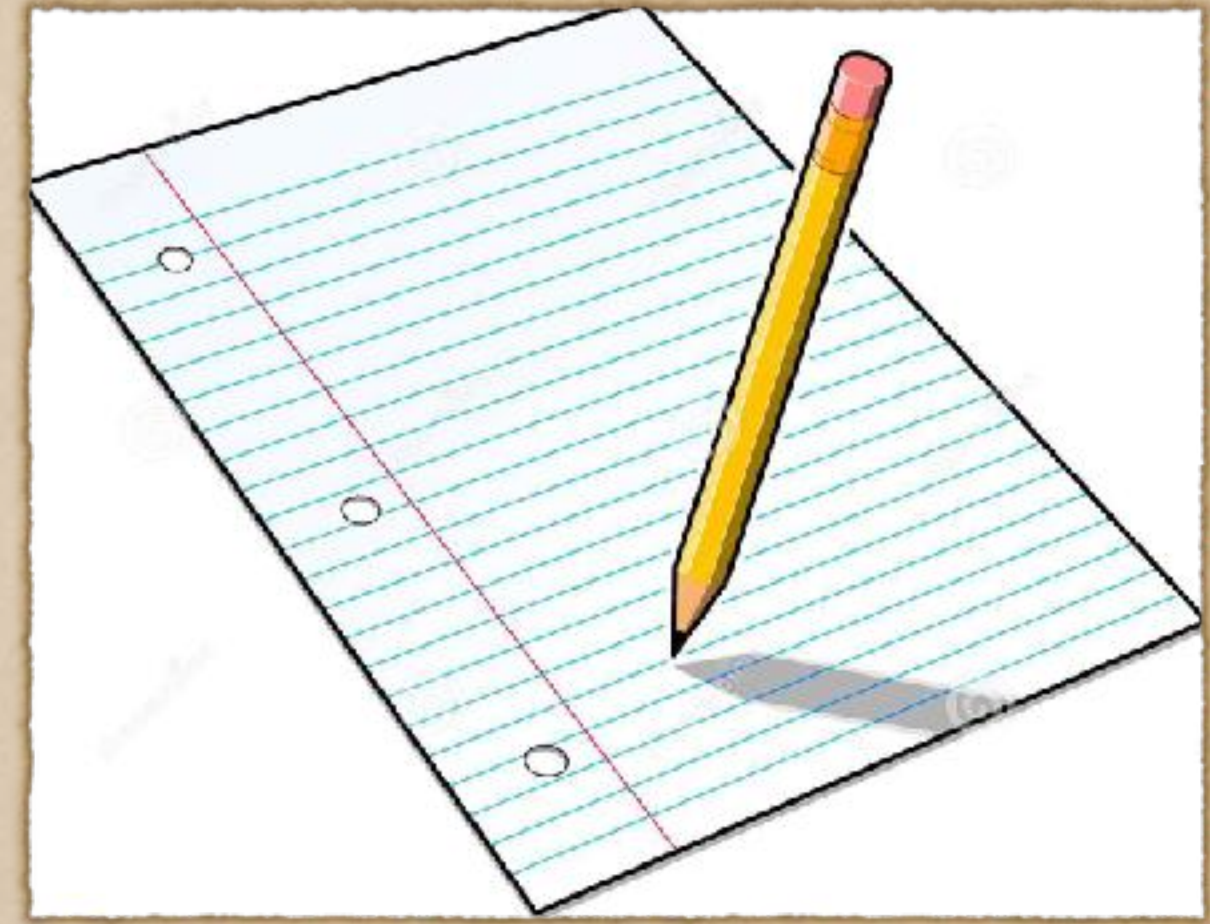
Maldacena [1508.01082]

Perturbation theory

- The intrinsic bispectrum is generated by non-linear evolution.

$$\delta(\tau, \mathbf{x}) = \delta^{(1)} + \frac{1}{2}\delta^{(2)} + \cdots$$

$$\theta(\tau, \mathbf{x}) = \theta^{(1)} + \frac{1}{2}\theta^{(2)} + \cdots$$



- Fluid equations:

$$\dot{\delta} = -\partial_j \left[(1+\delta)\partial_j \nabla^{-2}\theta\right]$$

$$\dot{\theta} = -\mathcal{H}\theta - \partial_i\partial_j\nabla^{-2}\theta\partial_j\partial_i\nabla^{-2}\theta - \partial_j\nabla^{-2}\theta\partial_j\theta - \frac{3}{2}\frac{H_0^2\Omega_m}{a}\delta,$$

A few details...

- Perturbative solution:

$$\dot{\delta} = -\partial_j \left[(1+\delta)\partial_j \nabla^{-2}\theta\right]$$

$$\dot{\theta} = -\mathcal{H}\theta - \partial_i\partial_j\nabla^{-2}\theta\partial_j\partial_i\nabla^{-2}\theta - \partial_j\nabla^{-2}\theta\partial_j\theta - \frac{3}{2}\frac{H_0^2\Omega_m}{a}\delta,$$

- 2nd order PDE for $\delta^{(1)}(\tau, \mathbf{x})$:

$$\ddot{\delta}^{(1)} + \mathcal{H}\dot{\delta}^{(1)} = \frac{3}{2}\frac{H_0^2\Omega_m}{a}\delta^{(1)}$$

- Separation of variables:

$$\delta^{(1)}(\tau, \mathbf{x}) \equiv D(\tau)\tilde{\delta}(\mathbf{x})$$

- 2nd order PDE for $\delta^{(2)}(\tau, \mathbf{x})$:

$$\ddot{\delta}^{(2)} = -\mathcal{H}\dot{\delta}^{(2)} + 2\left(\mathcal{H}\dot{D}D + \ddot{D}D + \dot{D}^2\right)\left[\partial_j\nabla^{-2}\tilde{\delta}\partial_j\tilde{\delta} + \tilde{\delta}^2\right] +$$

$$+\frac{3}{2}\frac{H_0^2\Omega_m}{a}\delta^{(2)} + 2\dot{D}^2\left[\partial_i\partial_j\nabla^{-2}\tilde{\delta}\partial_i\partial_j\nabla^{-2}\tilde{\delta} + \partial_j\nabla^{-2}\tilde{\delta}\partial_j\tilde{\delta}\right]$$

Computing the bispectrum

- Second order density in Fourier space:

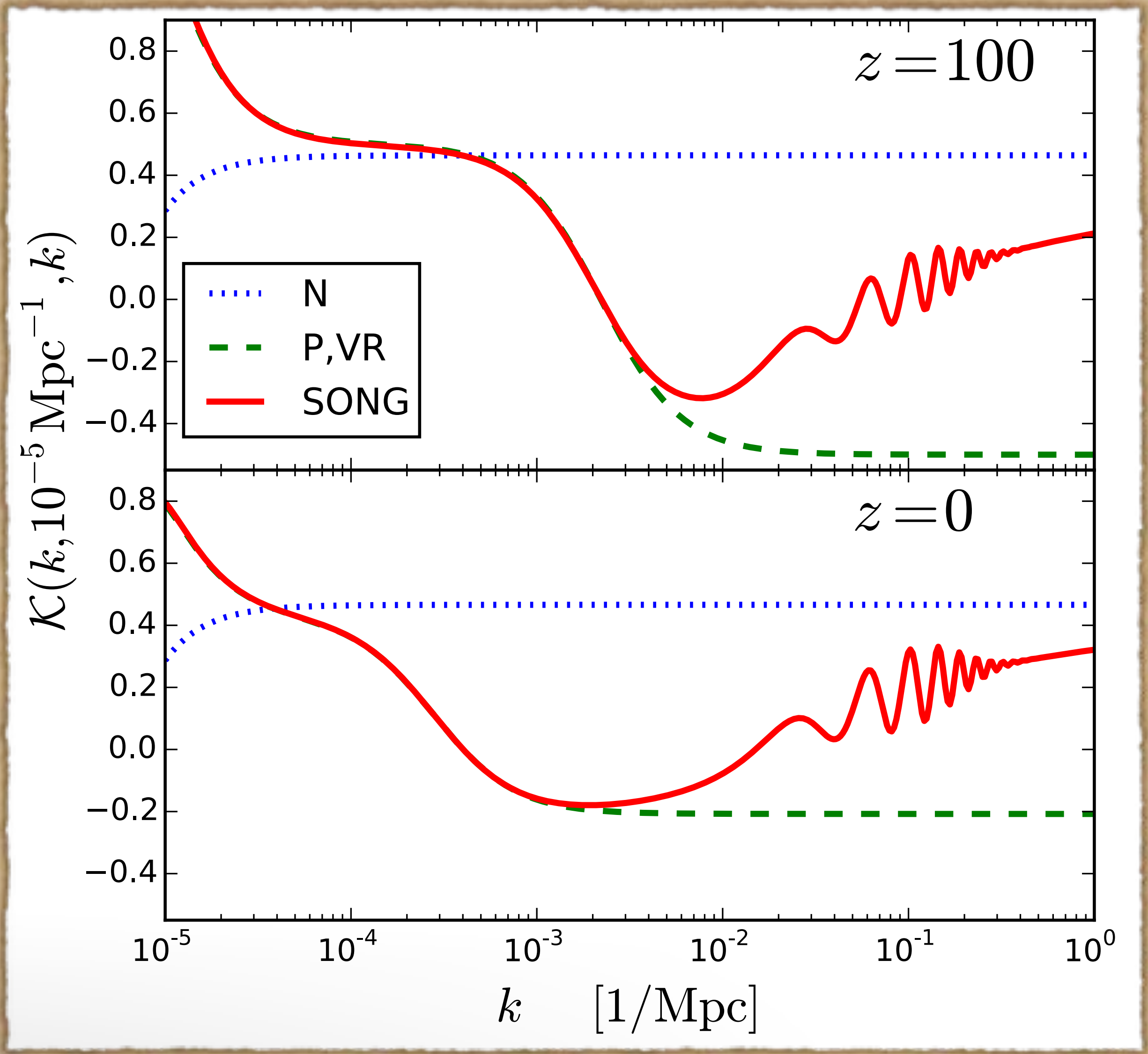
$$\frac{1}{2}\delta^{(2)}(\tau,\mathbf{k}) = \int \frac{d\mathbf{k_1} d\mathbf{k_2}}{(2\pi)^3} \delta^D(\mathbf{k}-\mathbf{k_1}-\mathbf{k_2}) \mathcal{K}(k_1,k_2,k) \delta^{(1)}(\mathbf{k_1}) \delta^{(1)}(\mathbf{k_2})$$

- Compute $\mathcal{K}(k_1,k_2,k)$ analytically [Villa&Rampf:1505.04782]

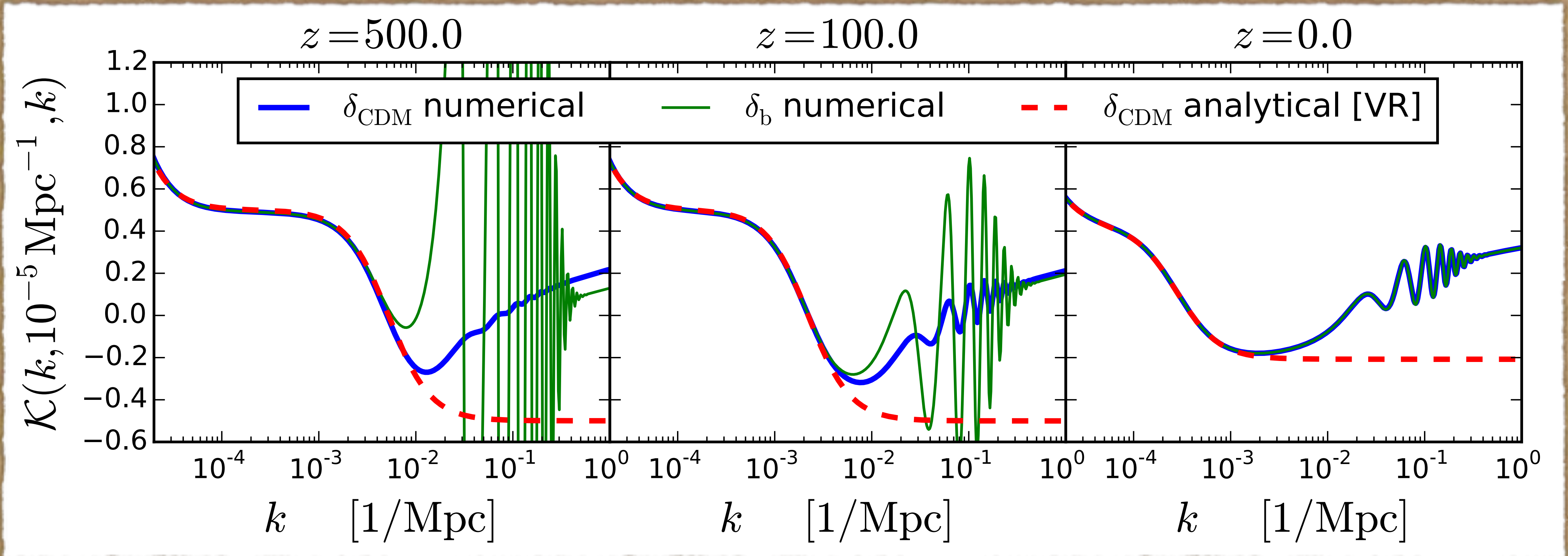
- Compute $\mathcal{K}(k_1,k_2,k)$ numerically using SONG: https://github.com/coccoinomane/song

Squeezed problem

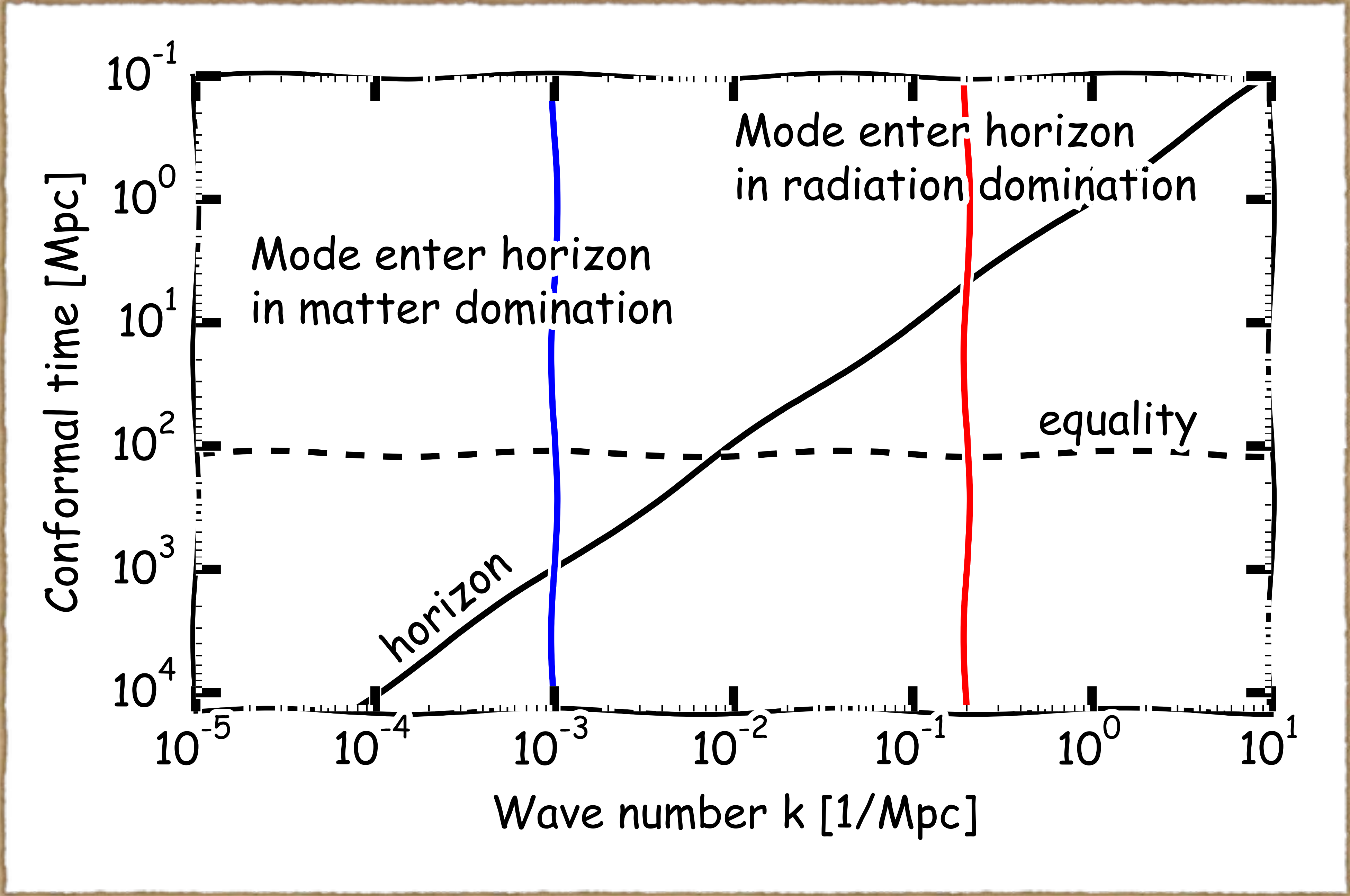
- Two large modes: OK!
- Two small modes: OK!
- Small + large: problem!



Baryon Acoustic Oscillations

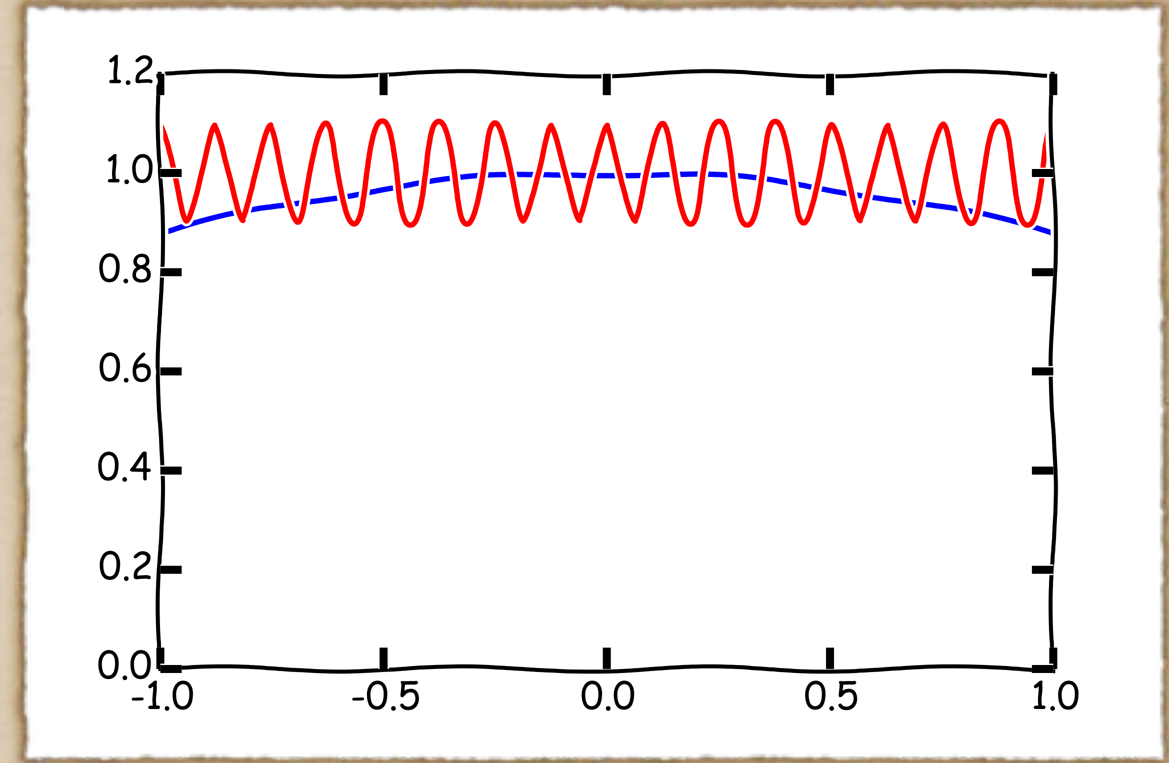


What went wrong?

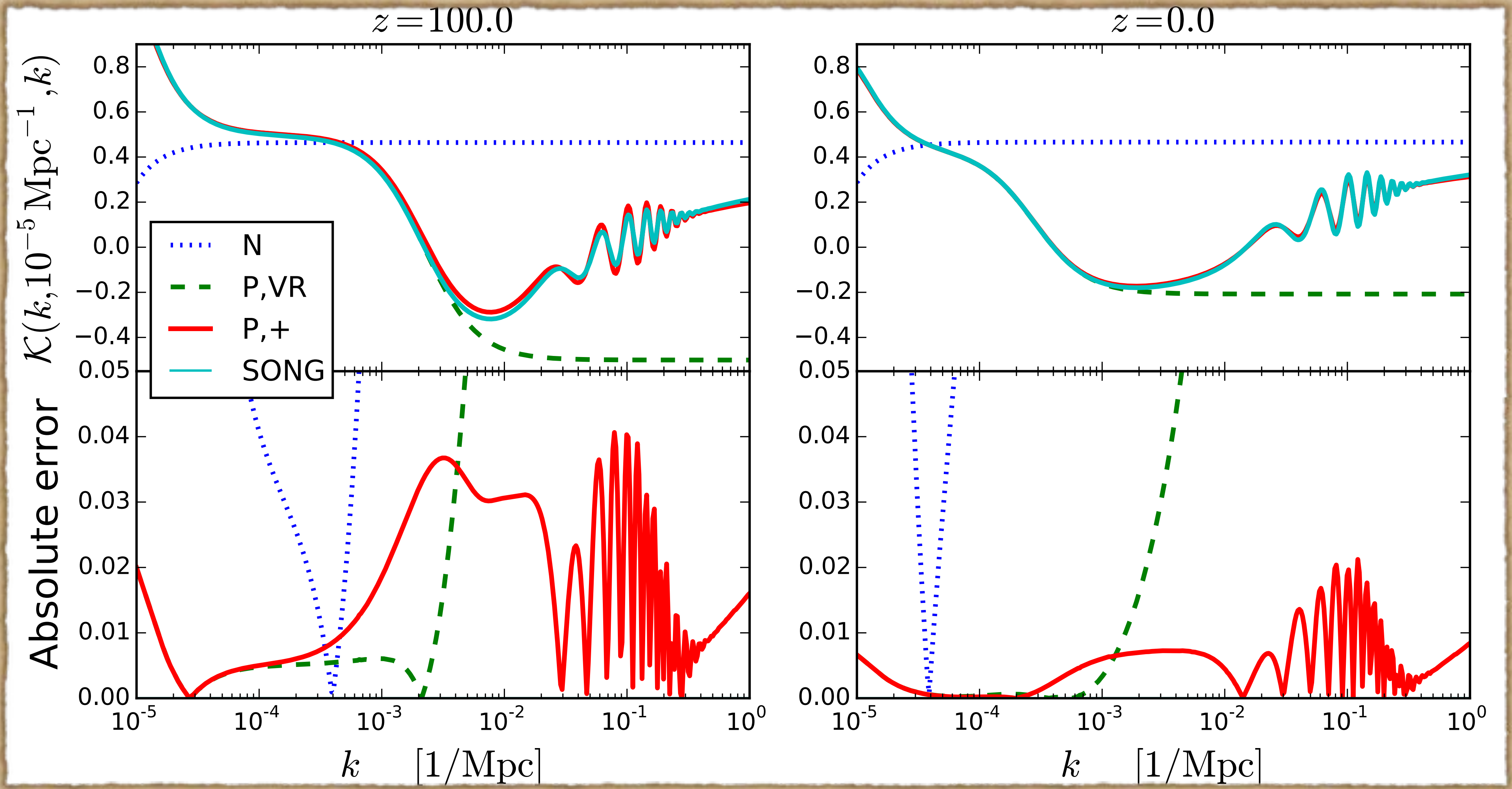


Separate Universes

- Assume that the long wavelength mode is constant for all scales of interest.
- Absorb long wavelength in background by local change of coordinates.

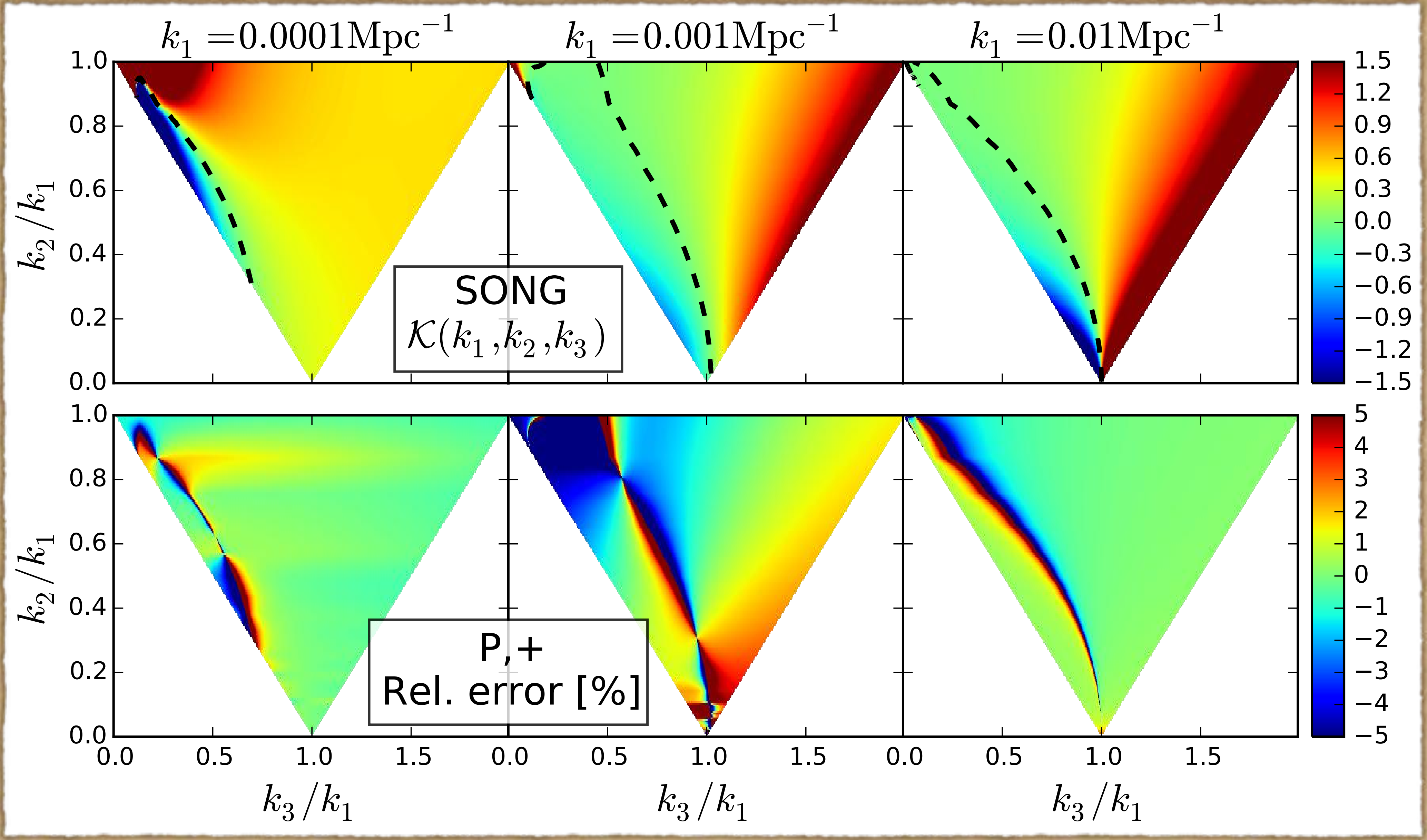


$$\mathcal{K}_{P,+} = \mathcal{K}_{P,\mathrm{VR}} - \frac{1}{2}\left(f + \frac{3u}{2}\right)\left[\frac{\mathcal{H}^2}{k^2} + 3f\frac{\mathcal{H}^4}{k^4}\right]\frac{d\log T}{d\log k}\left(\frac{k_2}{k_1} - \frac{k_1}{k_2}\right)^2$$



Squeezed configuration

$$\mathcal{K}\left(k, 10^{-5}\mathrm{Mpc}^{-1}, k\right)$$



Full kernel at redshift z=0

Conclusions for part 2

- 1% accurate analytic formula for bispectrum.
- Fast and accurate numerical code SONG.
- Future: Observables and initial conditions for N-body simulations.