Searching for spectator fields during inflation

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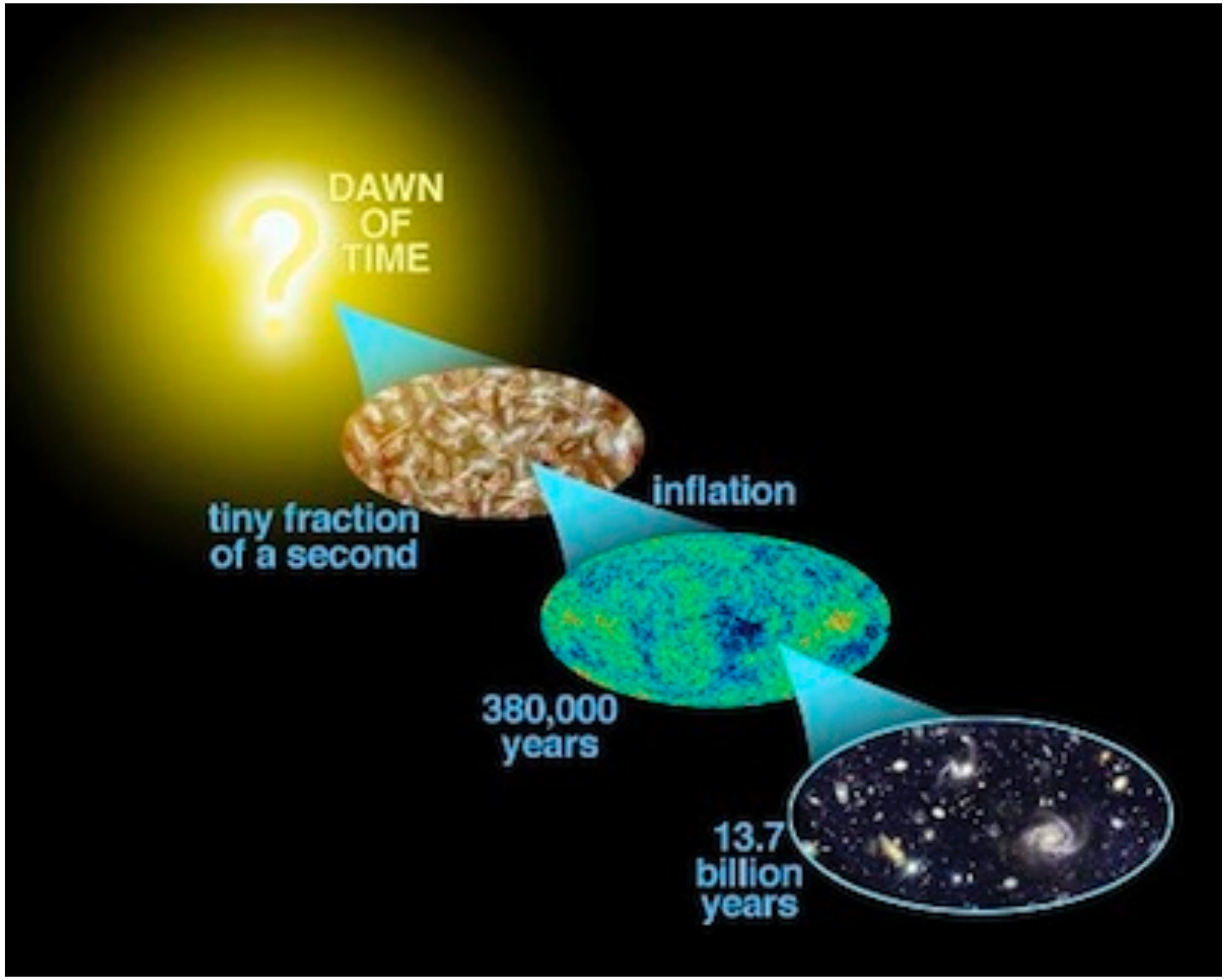
Work done with Rob Hardwick, Jesus Torrado, Vincent Vennin and David Wands
Especially ArXiV:1701.06473 + work in progress

Helsinki, 26th of April 2017

Inflation: sowing the seeds

Quantum mechanical perturbations present during inflation become temperature perturbations on the cosmic microwave background, and later galaxies thanks to the attraction of gravity

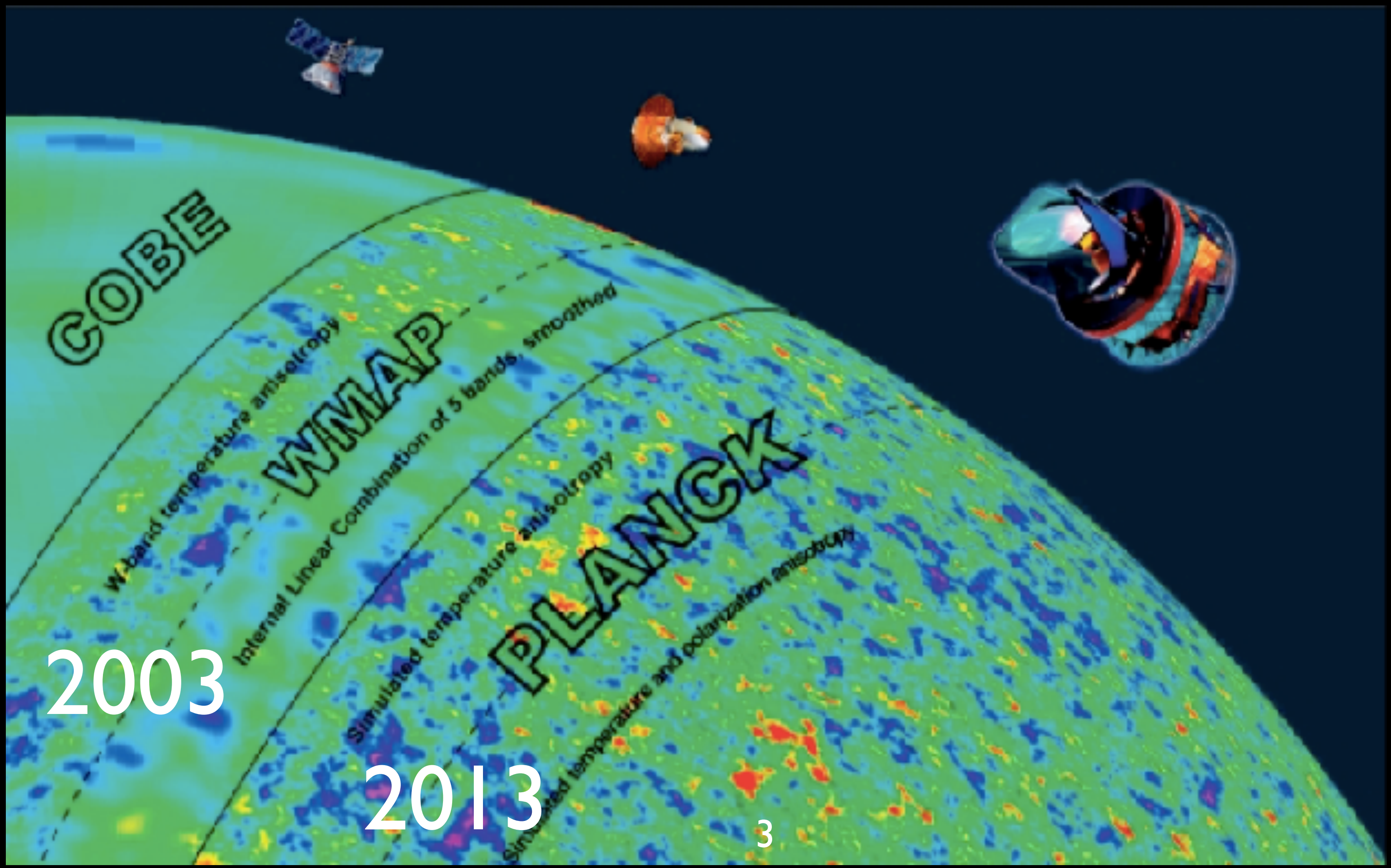
By design, the background is almost the same between inflation models. The perturbations may discriminate between models



How much have we learnt from the precision era?

- Planck completes a trilogy of CMB experiments
- Planck: 25 times better sensitivity and 3 times better resolution than WMAP, the previous best experiment

1992



We observe so much yet see so little…

- It is a highly non trivial and remarkable and disappointing statement that we can explain the statistical property of 10^7 CMB pixels with just two primordial numbers (+ background parameters)
- We have only measured the amplitude and spectral index of the power spectrum
- Is this evidence that inflation was simple?

Questions

- Is inflation the correct paradigm?
- If yes, how likely is the existence of more than one light scalar field?
- If there was more than one light scalar field, how likely are we to have already observed this?
- Given that we haven't, how likely are we to detect it in the future?
- How likely is it undetectable, even in principle?

The key Planck plot for inflation

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_s} \qquad \mathcal{P}_s = A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$$

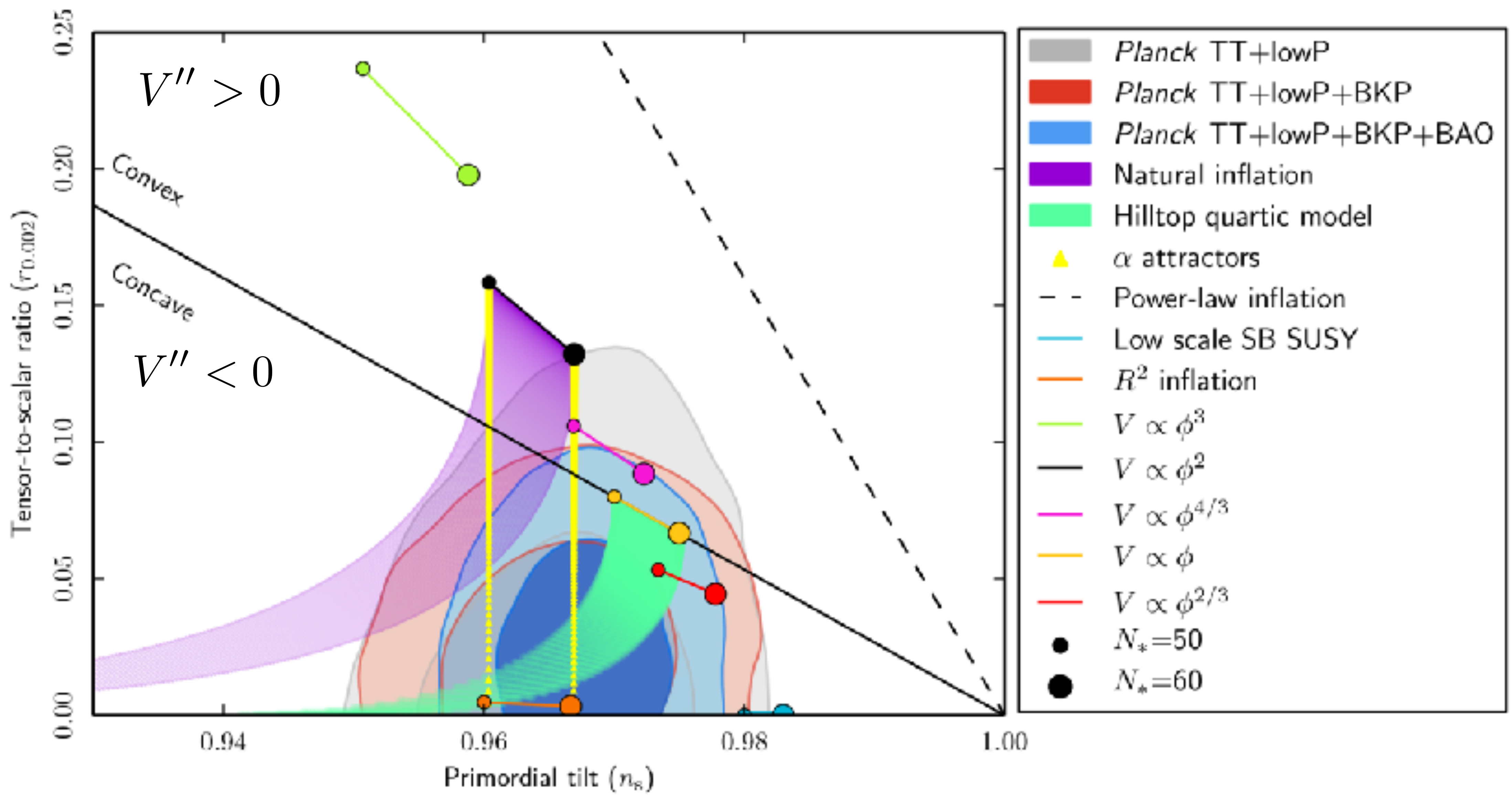
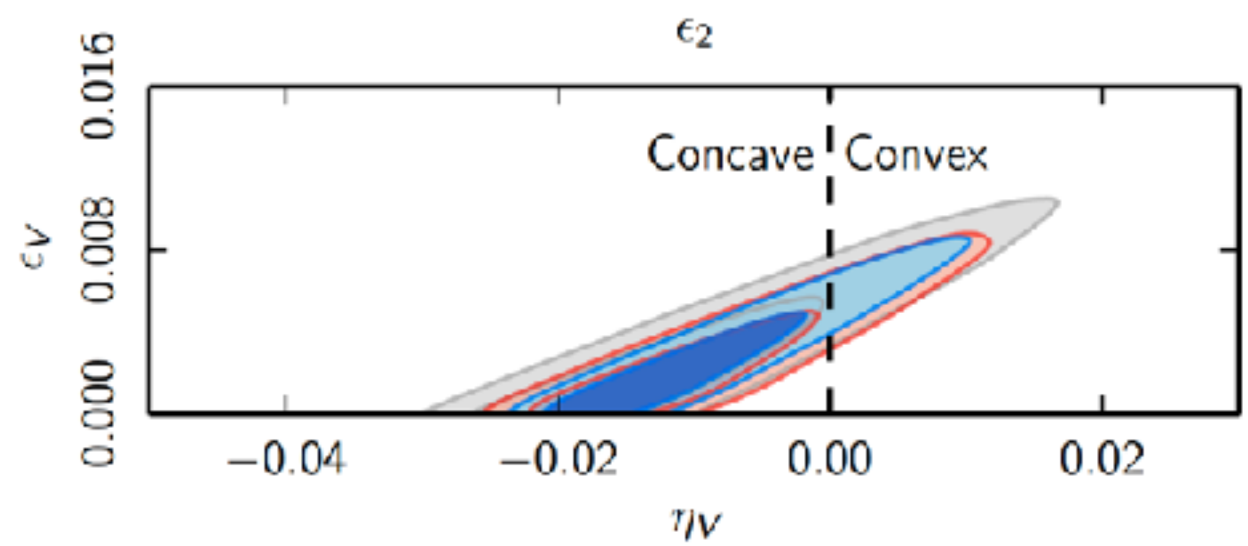


Fig. 55. Marginalized joint 68% and 95% CL regions for n_s and r at $k = 0.002\,\mathrm{Mpc}^{-1}$ from *Planck* alone and in combination with its cross-correlation with BICEP2/Keck Array and/or BAO data compared with the theoretical predictions of selected inflationary models. Note that the marginalized joint 68% and 95% CL regions have been obtained by assuming $\mathrm{d}n_s/\mathrm{d}\ln k = 0$.

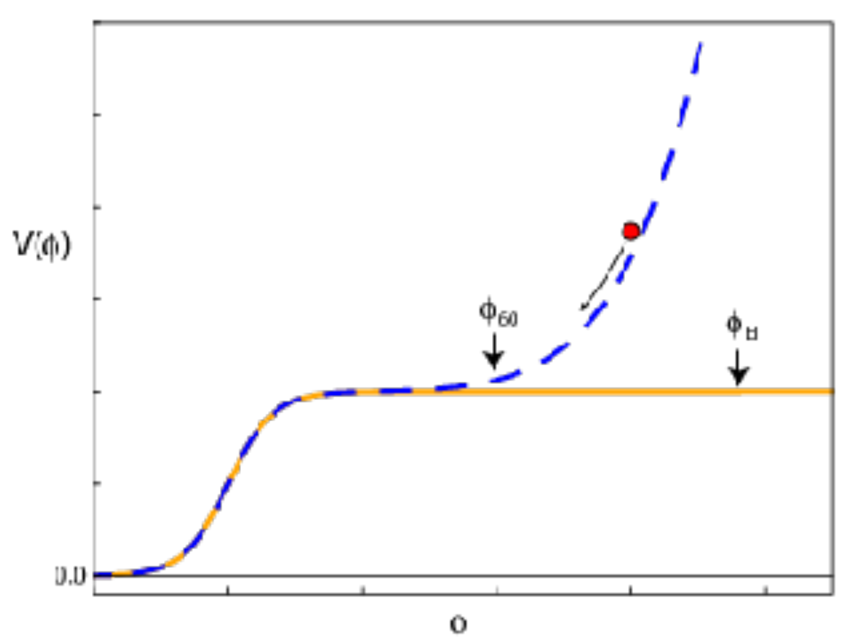
Again for single-field inflation

$$\epsilon_V = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta_V = M_p^2 \frac{V''}{V}$$

$$r = 16\epsilon_V, \qquad n_s - 1 = -6\epsilon_V + 2\eta_V$$



Planck 2015 data, excluding Bicep and Keck



A plateau model of inflation, V`` changes sign between when observable modes cross the horizon, and the end of inflation

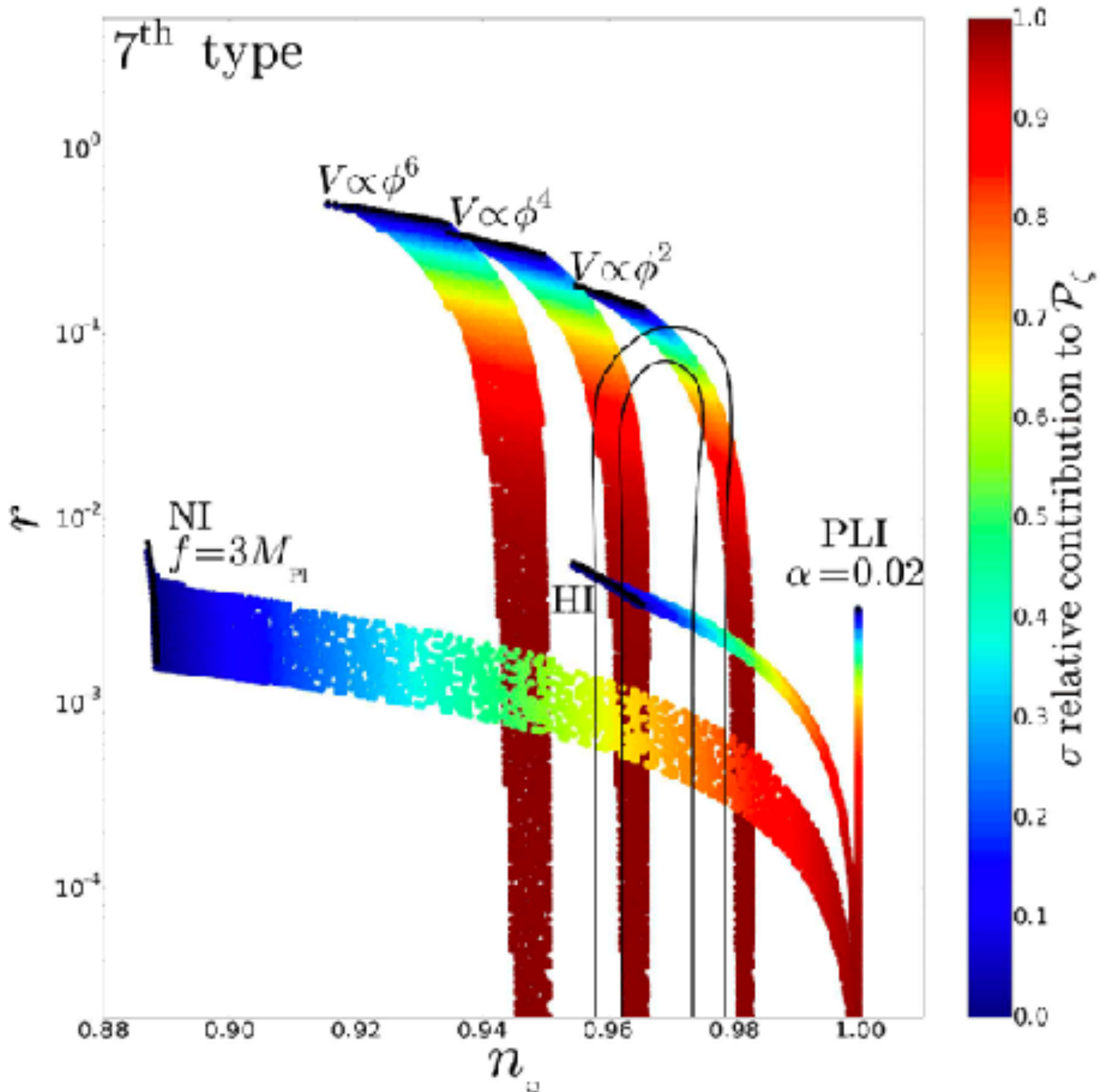
$$\epsilon_V \sim \eta_V^2$$

$$r \sim (n_s - 1)^2 \sim \text{few} \times 10^{-3}$$

General multifield inflation

If the perturbations from a field other than the inflation dominate

$$r \ll 16\epsilon_V\,, \qquad n_s - 1 = -2\epsilon_V + 2\eta_\sigma \qquad\qquad \eta_\sigma = \frac{m_\sigma^2}{3H^2}$$



Two classes of preferred models:

1. Single-field inflation with a plateau potential

2. Two field inflation, inflation field with quartic potential, very light second field

3. No inflaton potential is generically a good fit in both the single field and spectator field limits

Encyclopedia curvatonis; Vennin et al

What about non-Gaussianity?

- Sometimes presented as a “definitive” test of multifield inflation before Planck
- How likely was $10<f_{NL}<30$ really? Compared to $f_{NL}>30$?
- What can we learn from a non-detection?
- With r we have already learnt a lot and got a physically motivated future target

A worked model comparison: The curvaton scenario

- An alternative model to single-field inflation for the origin of structures. The inflaton drives inflation while the curvaton generates curvature perturbations (hence the name)
- The curvaton is a light field which

1. has a subdominant energy density during inflation
2. Is long lived (compared to the inflaton)
3. (Potentially) generates the primordial curvature perturbation

Enqvist and Sloth, Lyth and Wands, Moroi and Takahashi '01 + many later works with a strong Finnish contingent

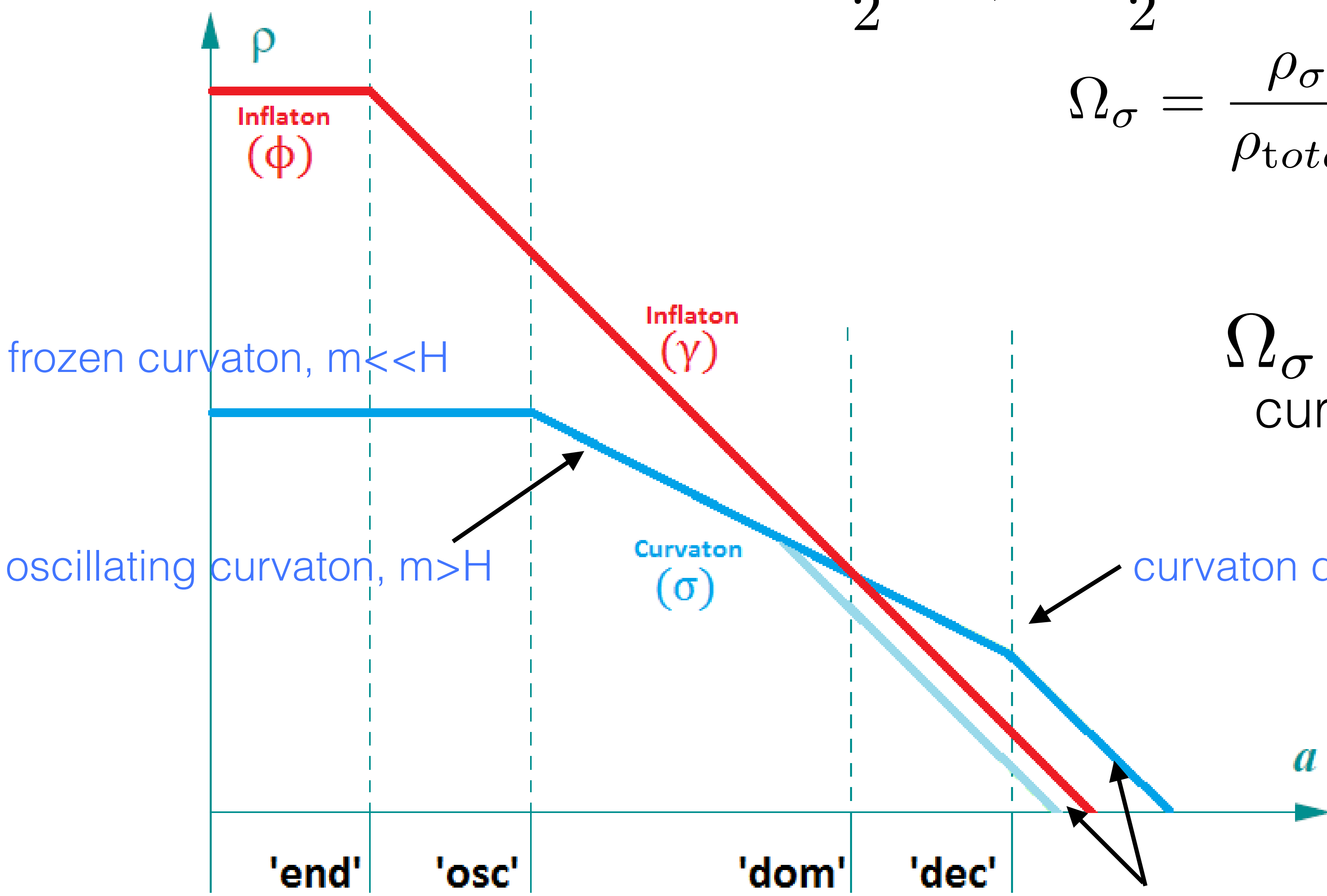
Curvaton (σ) background evolution:

Log of scale factor versus log of energy density

$$V = \frac{1}{2}M^2\phi^2 + \frac{1}{2}m^2\sigma^2$$

$$\Omega_\sigma = \frac{\rho_\sigma}{\rho_{total}}$$ measured at the curvaton decay time

$\Omega_\sigma = 1$ is an attractor if the curvaton decays late enough



The longer the curvaton lives, the larger its relative energy density becomes

What non-Gaussianity does the (quadratic) curvaton predict?

Local non-Gaussianity $\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{\rm NL} (\zeta_G^2(\mathbf{x}) - \langle \zeta_G^2(\mathbf{x}) \rangle)$

- The curvature perturbation is approximately $\zeta \simeq \Omega_\sigma \frac{\delta\rho_\sigma}{\rho_\sigma} \simeq \Omega_\sigma \left(\frac{\delta\sigma}{\sigma} + \left(\frac{\delta\sigma}{\sigma} \right)^2 \right)$
- Local non-Gaussianity is generated: $f_{NL} \sim 1/\Omega_\sigma$
- The Planck constraint $f_{NL}<10$, tells us $\Omega_\sigma>0.1$. A priori, $\Omega_\sigma \sim 10^{-5}$ (and $f_{NL} \sim 10^5$) was possible.
- If the curvaton dominates before decay, $\Omega_\sigma=1$ and $f_{NL}=-5/4$
- In terms of a linear scale on $-5/4<f_{NL}<10^5$ - 99.99% has already been ruled out
- In terms of a linear scale on $10^{-5}<\Omega_\sigma<1$ - 10% has been ruled out
- A highly subdominant curvaton is totally ruled out, so the dominant curvaton case becomes our "prediction". Detecting $f_{NL}=3$ or 7 seems unlikely, although it is compatible with the model

The simplest curvaton scenario

$$V = \frac{1}{2}M^2\phi^2 + \frac{1}{2}m^2\sigma^2$$

- Parameter constraints were originally made by Bartolo and Liddle (2002), the data allowed so much freedom they restricted the model to i) the Gaussian case ii) negligible inflaton perturbations
- CB, Cortês and Liddle (2014) revisited the model. Observations now constrain all realisations of the model.
- Rob Hardwick & CB (2015) performed the first Bayesian analysis of the curvaton scenario, a model comparison technique which penalises unwanted free parameters (Occam's razor)
- The additional three curvaton parameters are its mass m, its field value at horizon crossing sigma$*$ and its decay rate Gamma

The models

- Baseline LCDM model
- Quadratic single-field inflation (1 free parameter)
- 3 variations on the curvaton scenario (all with 4 free parameters)

1. Mixed inflaton-curvaton scenario (most general case)
2. Pure curvaton scenario (negligible inflaton perturbations)
3. Dominant curvaton scenario (negligible inflaton perturbations and the curvaton dominates the background density before it decays)

Each scenario is a subset of the one above

Cases 2 and 3 predict negligible tensor perturbations. Case 3 also f_{NL}=-5/4

The priors - these need to be specified

Model Scenarios	Prior ranges and constraints ($M_P = 1$)
Single field inflaton	$-4 > \log_{10}(M) > -6.75$
Mixed inflaton-curvaton	$-4 > \log_{10}(M) > -15.3$
	$\log_{10}(M/2) > \log_{10}(m) > -39$
	$\lvert\sigma_*\rvert < 0.01$
	$\log_{10}(m) > \log_{10}(\Gamma_\sigma) > -39$=BBN energy scale
Pure curvaton	Mixed inflaton-curvaton subset with extra constraint:
	negligible inflaton perturbations $\mathcal{P}_\zeta^\phi \lesssim 0.01\mathcal{P}_\zeta^{total}$
Dominant curvaton decay	Pure curvaton subset with extra constraint:
	The curvaton dominates the background energy density before decay
ΛCDM concordance	$1.02 > n_s > 0.9$
	$3.2 > \ln(10^{10}\mathcal{P}_\zeta) > 3.0$
	$r = f_{NL} = 0$

A log prior is standard for parameters where the order of magnitude is unknown
The curvaton vev has to be small in order for the curvaton perturbations to dominate $\sigma_*^2 < \epsilon$
but we put this in by hand and use a linear prior (more later)

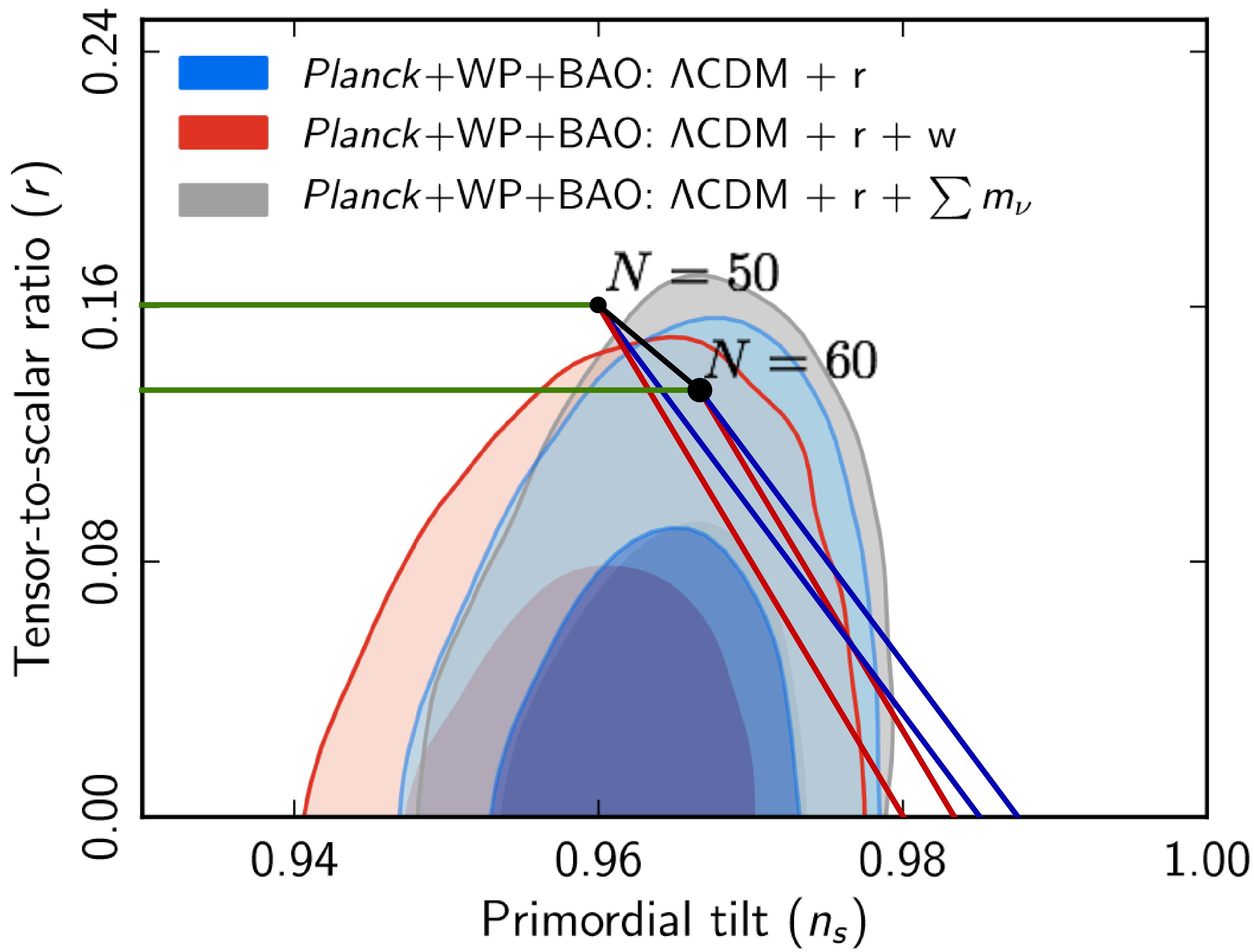
Curvaton constraints

$$V = \frac{1}{2}M^2\phi^2 + \frac{1}{2}m^2\sigma^2$$

inflaton dominates

$$\frac{P_\zeta^\phi}{P_\zeta^\sigma}$$

curvaton dominates



Red lines have $\eta_\sigma \ll \epsilon$

Blue lines have $\eta_\sigma = \eta_\phi/4$

Green lines are the inflating curvaton regime, where it drives a second period of inflation.

Planck/Bicep/Keck 2015 Results

Model Scenarios	$\Delta\chi^2_{eff}$	$\ln(\mathcal{E}/\mathcal{E}_{ref})$
Single field inflaton	10.4	-7.0
Mixed inflaton-curvaton	2.7	-5.4
Pure curvaton	4.8	-4.7
Dominant curvaton decay	4.8	-4.7

The effective chi squared values give the best fit (smaller is better)
The right hand column gives the Bayesian evidence ratios.
The curvaton is not disfavoured, despite having 3 extra parameters
The curvaton is weakly/moderately favoured over quadratic inflation

Hardwick & CB 2015

The Jeffrey's scale

$\lvert\ln(\mathcal{E}/\mathcal{E}_{ref})\rvert$	**Interpretation**
< 1	*Inconclusive*
$1 - 2.5$	*Weak evidence*
$2.5 - 5$	*Moderate evidence*
> 5	*Strong evidence*

Why does the curvaton do well?

- The quadratic single-field model is not a good fit
- The vast majority of the parameter space matches the dominant pure curvaton scenario - f_{NL}=-5/4
- Therefore the evidence ratios are similar for all three curvaton cases
- The “tight” f_{NL} constraint does not change our results much, it needs to decrease by an order of magnitude (unless f_{NL} is detected)
- Our results are not very sensitive to the choice of most priors
- However, we do force the curvaton VEV to be very small compared to the inflatons, we were not testing whether the curvaton scenario is likely
- See Encyclopedia curvatonis by Vennin et al for in depth results and different prior choices. They find that the evidence for Starobinsky inflation is robust to the addition of a second field, and that quartic inflation with a curvaton is competitive using a log prior on sigma_*

Constructing a prior for the field value

- We use the Langevin equation

$$\frac{\mathrm{d}\sigma}{\mathrm{d}N} = -\frac{V_{,\sigma}(\sigma)}{3H^2} + \frac{H}{2\pi}\xi$$

- This described the “drift” of the spectator field sigma, in units of efolding time N

- The slow-roll approximation is appropriate whenever the stochastic noise term xi is not negligible. It is Gaussian with unit variance.

- Related to the Fokker-Planck equation for a probability distribution P

$$\frac{\partial P(\sigma,N)}{\partial N} = \frac{\partial}{\partial\sigma}\left[\frac{V_{,\sigma}(\sigma)}{3H^2}P(\sigma,N)\right] + \frac{H^2}{8\pi^2}\frac{\partial^2}{\partial\sigma^2}\left[P(\sigma,N)\right]$$

Stationary solution

- In general, no analytic solution exists, however there is a well known stationary solution valid in de-Sitter space (early works by Starobinsky)

$$P_{\rm stat}(\sigma) \propto \exp\left[-\frac{8\pi^2 V(\sigma)}{3H^4}\right]$$

- Found by rewriting the Fokker-Planck equation in terms of a probability current

$$\partial P/\partial N = -J_{,\sigma} \qquad J \equiv -V_{,\sigma}P/(3H^2) - H^2 P_{,\sigma}/(8\pi^2)$$

- For a quadratic test field, this gives a Gaussian distribution satisfying

$$\sqrt{\langle\sigma^2\rangle} \sim H^2/m$$

Is the stationary solution enough?

- Most papers use this result, it is 1) easy and 2) slow-roll inflation is close to de-Sitter space

- The stationary solution is an attractor, but even in exact de-Sitter space the relaxation time to reach the attractor is

$$N_{\rm relax} \sim \frac{H^2}{m^2} \sim \eta_\sigma^{-1}$$

- This is typically well over 100 efoldings, potentially much more

- The Hubble parameter changes on a time scale

$$N_H = \frac{1}{\epsilon_1}$$

The stationary solution

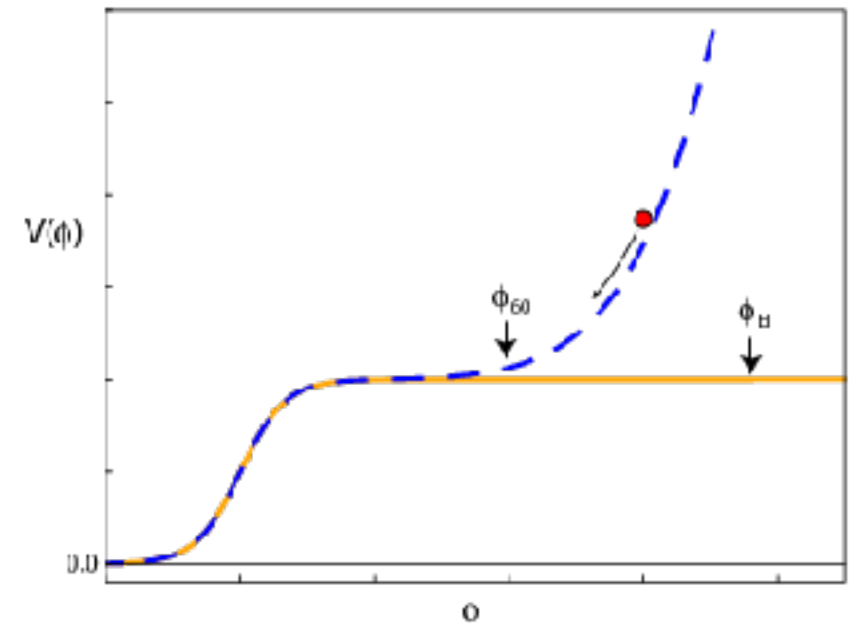
$$P_{\rm stat}(\sigma) \propto \exp\left[-\frac{8\pi^2 V(\sigma)}{3H^4}\right]$$

- Is valid provided that:
  1. Inflation has lasted long enough
  2. Inflation is of the plateau of hilltop variety, with H varying very slowly

$$N_{\rm relax} \sim \frac{H^2}{m^2} \sim \eta_\sigma^{-1} \qquad N_H = \frac{1}{\epsilon_1}$$

- Even with the currently preferred models such as Starobinsky inflation, one needs to extrapolate the model well past 60 efoldings in order to use the stationary solution

- In models where the spectator field generates the perturbations, the spectral index requires a large epsilon (large-field inflation) and the stationary solution is never valid



Going beyond the stationary solution

- For a massless field in de-Sitter, the variance of sigma grows like N
- In realistic models, H decreases so the de-Sitter limit is a lower bound on the variance
- In quartic inflation, the Hubble rate grows ~ N and the variance of sigma grows much faster, as N^3

Power law inflation

- Inflation driven by potential ϕ^p with a quadratic spectator field

- High up the potential, there is eternal inflation and H~constant in that regime. Assuming our Hubble patch came out of eternal inflation

$$\langle \sigma^2_{\rm end} \rangle \simeq \langle \sigma^2_{\rm eternal} \rangle + \frac{p}{p+2} M^2_{\rm Pl}$$

- The first term will typically dominate if the equilibrium distribution was reached during eternal inflation

- Even if the first term is zero, the typical field value is around the Planck scale

Spectator field driven inflation

- Even if the perturbations of the second field are negligible, it can change the predictions of single-field inflation

- Because the “pivot point” at which observable scales exit the horizon during inflation depend on the expansion history of the Universe after the first phase of inflation ends

- If there are N_2 efolds of spectator-field inflation, then the will be N_2 efolds less inflation by the first field

Probability of secondary inflation

Expected efolds of secondary inflation

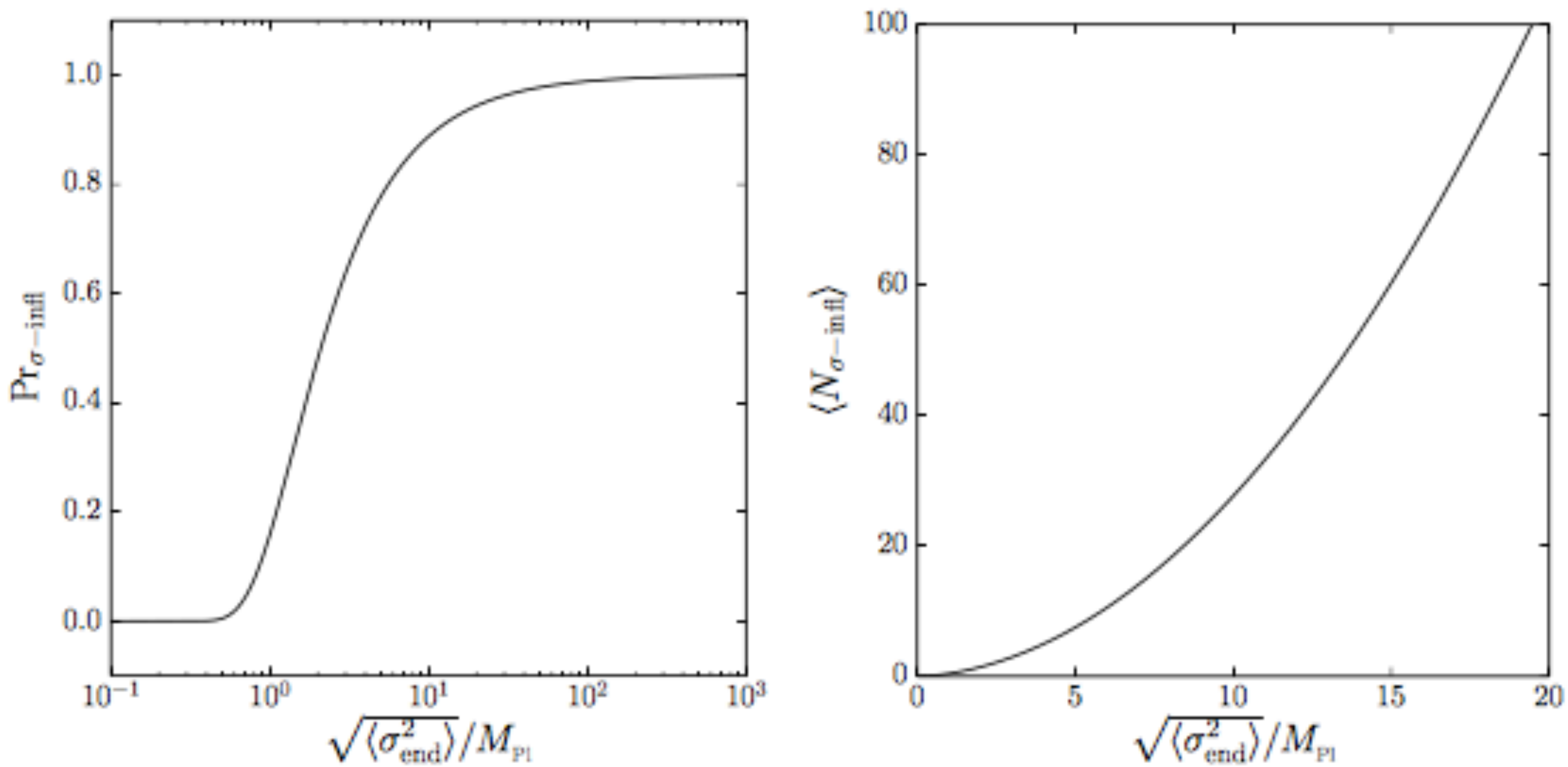


Figure 2. A quadratic spectator field σ can trigger a second phase of inflation if $|\sigma_{\mathrm{end}}| > \sqrt{2}M_{\mathrm{Pl}}$. Assuming a centred Gaussian distribution with variance $\langle\sigma^2_{\mathrm{end}}\rangle$, the left panel displays the probability for such a condition to be satisfied, while the mean number of e-folds realised in the second phase of inflation is given in the right panel.

Quartic spectator field

$$V(\sigma) = \lambda\sigma^4$$

- Will the large spectator field values be reduced by including a self-coupling, which makes the effective mass larger with larger field values?

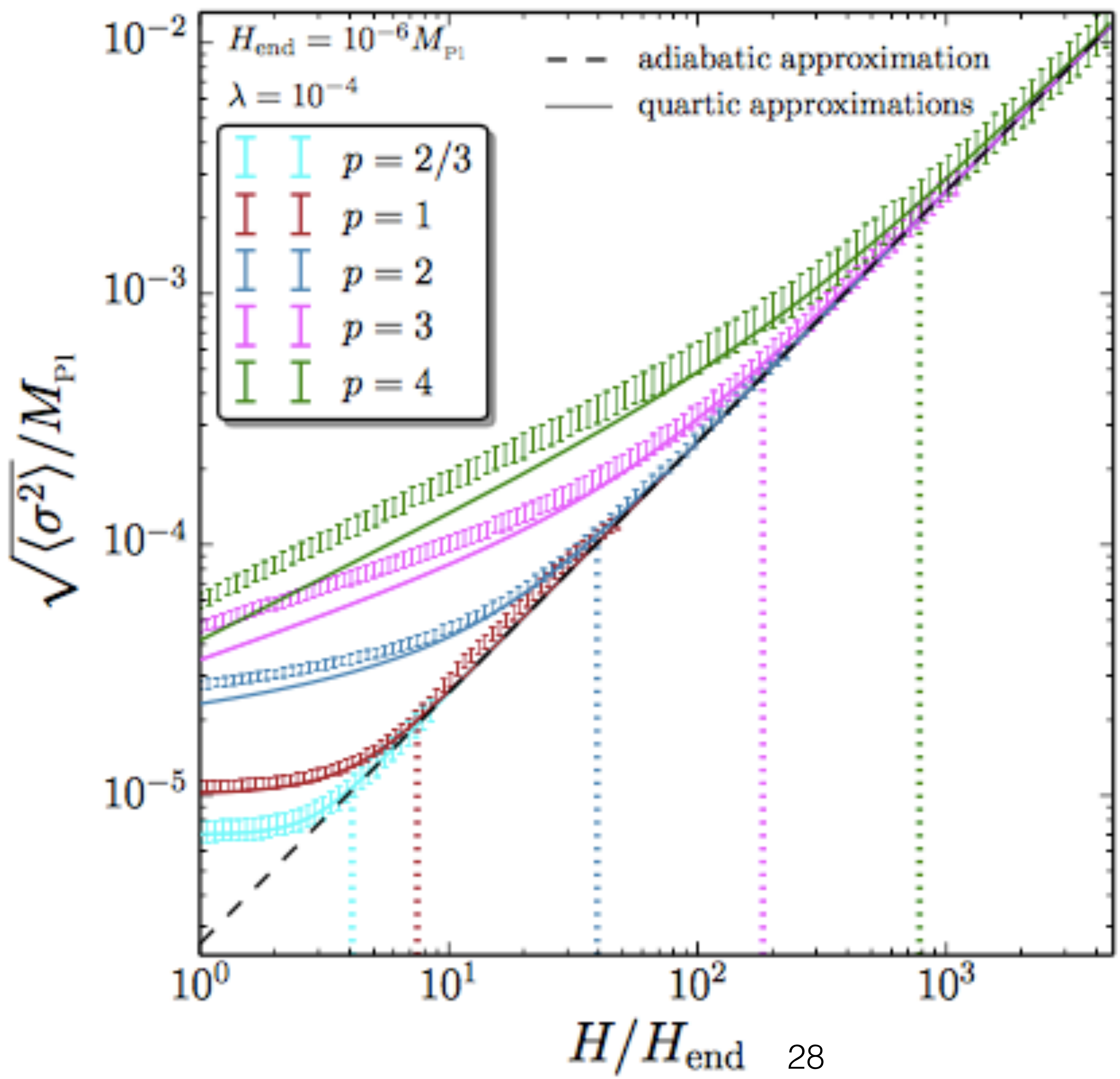
- The relaxation time becomes independent of H

$$N_{\rm relax} = \frac{1}{\sqrt{\lambda}}$$

- This means at sufficiently high H, there will always be a time when the attractor can be reached (but maybe into eternal inflation)

- The Higgs is an interesting candidate field, our results may have implications for the stability of the Universe during inflation

For large enough lambda, such that the field is initially in the stationary solution



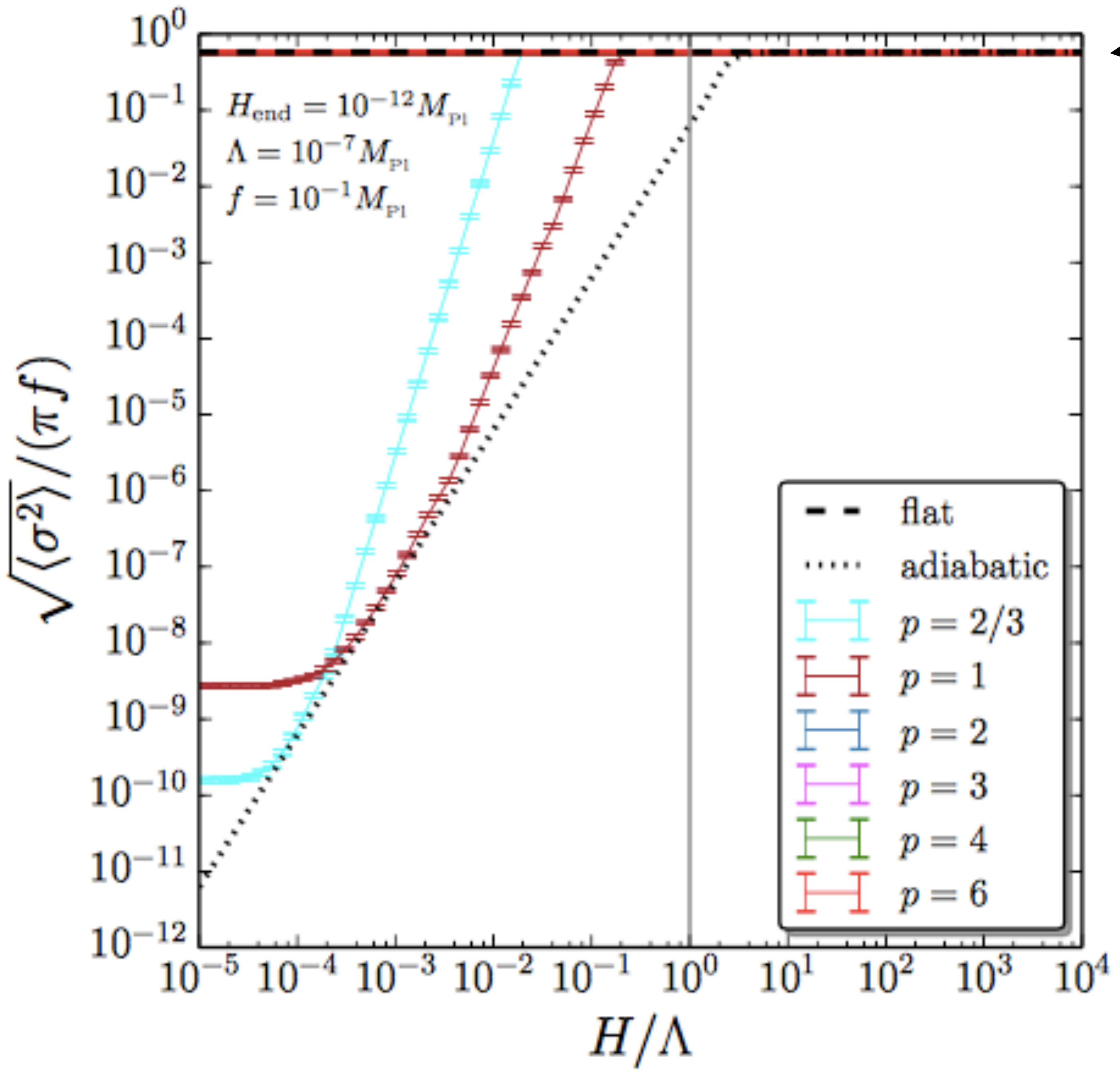
$$\lambda > \left(\frac{H_{end}}{M_{Pl}}\right)^{\frac{8}{p+2}}$$

Axionic spectator

$$V(\sigma) = \Lambda^4 \left[1 - \cos\left(\frac{\sigma}{f}\right)\right]$$

- A periodic potential provides a guaranteed way to keep the field displacement small
- When the field is effectively massless, a uniform distribution will be reached
- When the effective mass becomes large, the field should be driven to a minimum where it’s potential is effectively quadratic

Axionic spectator results



Uniform distribution

For quartic inflation (p=4) the distribution always remains flat provided that the spectator field remains light, independently of the model parameters

Playing with the potential

- Full analytic results for the curvaton scenario only exist if it has a quadratic potential
- In that case the field value is typically (super) Planckian and the curvaton perturbations are suppressed. However if the spectator field later inflates, the predictions of inflation still change because we are observing a different period of inflation
- The typical spectator field value can be suppressed by:
  1. Having at most ~1000 efolds of inflation and the field starting at zero
  2. Adding self-interactions or creating a periodic potential
- In the latter case, the field will typically not be near the quadratic minimum of the potential at the end of inflation - no analytic results exist
- The spectral index is still driven by epsilon and f_{NL}~1, so qualitative behaviour can be guessed

Isocurvature perturbations

- Adiabatic perturbations mean that locally all parts of the universe look the same, so e.g. the ratio of photons to baryons to CDM is the same everywhere
- Only multi-field inflationary models can generate (large scale) isocurvature perturbations
- Since isocurvature perturbations decay on small scales, we are close to the ultimate limits, which are at the percent level
- Are these tight constraints? Should we expect them to decay during reheating/thermalisation?
- Reheating and isocurvature perturbation survival deserves more attention

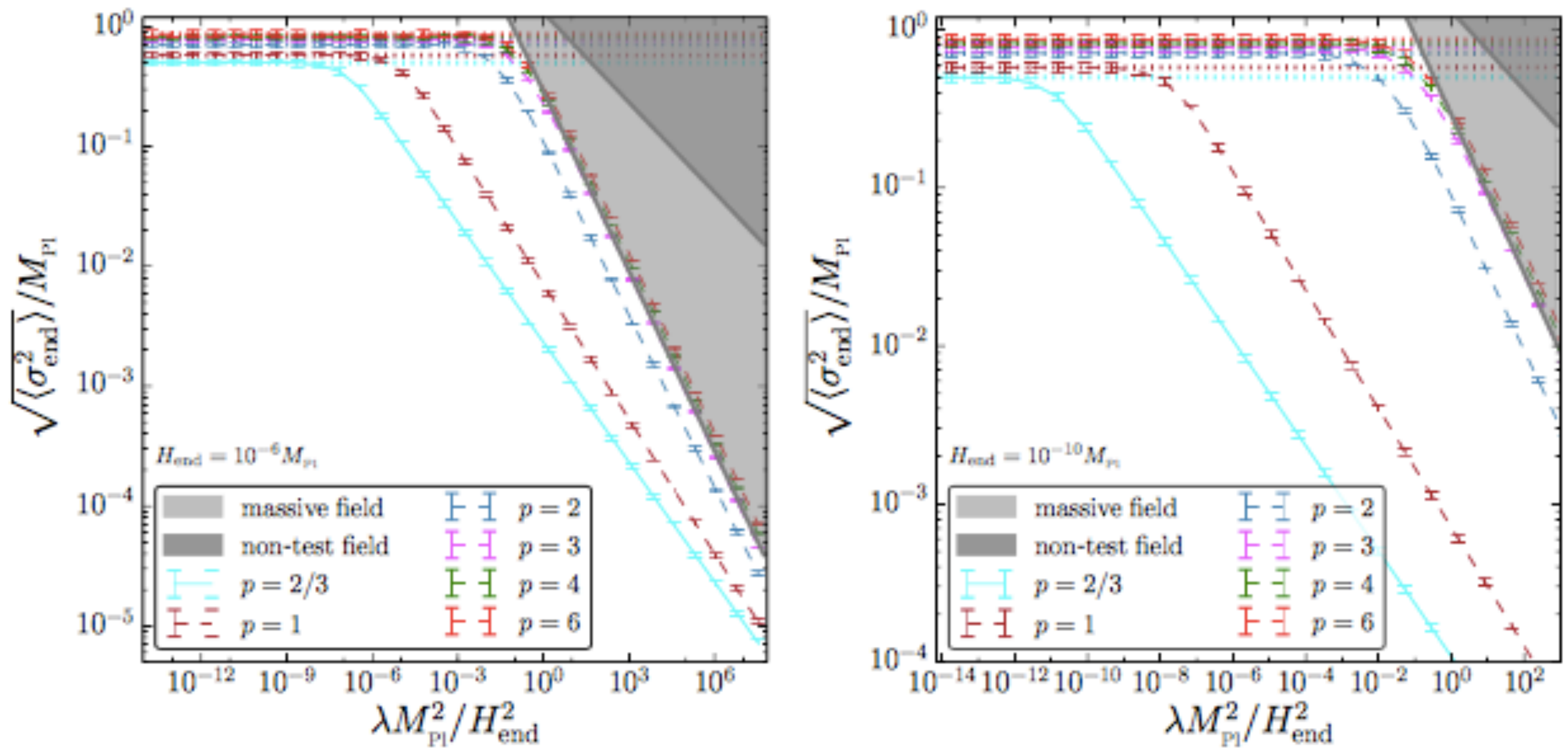
Many theories lack observables

- Bayesian reasoning provides a framework to compare models
- Results will depend on your priors, especially when the data allows a lot of freedom
- It is worth asking what we learn from “non detections”, e.g. can we disfavour complex models?
- E.g. for dark energy/modified gravity: Is there any achievable error bar which would convince “everyone” that Lambda is correct?

Conclusions

- Purely based on a detection of the spectral index and non-observation of tensor perturbations we find two "preferred" classes of inflationary models
  1. Single-field plateau inflation
  2. Spectator field generated perturbations embedded in quartic inflation
  3. Any others?
- A theoretical understanding of the priors is crucial to making a Bayesian model comparison
- The spectator field value probability distribution is especially important
- Discriminating between f_{NL}~1 and f_{NL}=0 may observationally discriminate between these two cases
- Progress on understanding the early universe remains possible after Planck. The two scenarios are extremely different at early times. Progress on reheating and isocurvature perturbation evolution/decay would help.

Setting the spectator field to be zero at the end of eternal inflation gives a “lower bound”



Light grey area excluded because the field is not light
Dark grey area means the field’s energy density dominates (not a spectator field)
For “large” self couplings the variance can become very small, but this can also make the field heavy (with a red spectral index)

How non-Gaussianity could favour the curvaton

- If non-Gaussianity was detected in the future, how quickly could we favour the curvaton over the base LCDM scenario?
- We assume all cosmological data remains the same except f_{NL}
- If f_{NL}=-5/4: the curvaton "attractor" value we need an error bar of 0.4. This would correspond to a 3-sigma detection
- If f_{NL}=10.8, the current 2-sigma upper bound from Planck then we need an error bar of about 2.6 (and the dominant curvaton scenario is ruled out). This would correspond to a 4-sigma detection
- The latter case should be "easily" achieved with Euclid, the former case maybe achievable in ~ 2 decades with Euclid, DESI, SKA…