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Non-linear CMB lensing and next generation experiments

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based on:
Marozzi, F, Di Dio, Durrer, JCAP 1609 (2016) no.09, 028, arXiv:1605.08761
Marozzi, F, Di Dio, Durrer, arXiv:1612.07263
Marozzi, F, Di Dio, Durrer, PRL 118 (2017) no.21, 211301 arXiv:1612.07650

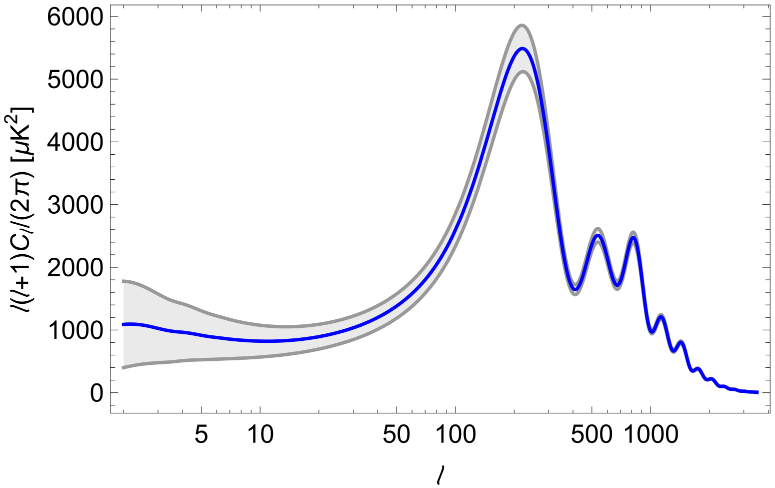
Contents

- First order lensing for $\Delta T/T$
- Beyond Born approximation and more (LSS corrections)
- From temperature to polarization
- What's new about polarization? Rotating the polarization's axes
- Are these corrections relevant?
- From analytical to numerical results

CMB: properties and features

- CMB is a well studied object in cosmology
- It has been generated at the decoupling time, during the matter dominated era
- It perfectly behaves as a black body, with a temperature $T_0 = 2.7$ K
- It has a high degree of homogeneity and isotropy
- Nevertheless, deviations from homogeneity and isotropy are present, due to local perturbations of the gravitational potential and due to scattering of baryons within the horizon

CMB temperature's fluctuations (1)



CMB temperature's fluctuations (2)

- Effects mentioned above induce the so called primary anisotropies
- Studying them allows us to gain a lot of informations about cosmological parameters
- However, these are not the only sources of anisotropies. Indeed, once a photon is emitted by the last scattering surface, it encounters inhomogeneities along its travel toward us. This generates the so called late time anisotropies
- The one we are going to talk about is weak lensing

From lensed to unlensed correlation's function (1)

- A generic scalar field transforms from lensed to unlensed coordinates as

$$\tilde{\mathcal{M}}(\tilde{x}^a) = \mathcal{M}(x^a + \delta\theta^a)$$

- At first order, this equation can be linearized as

$$\tilde{\mathcal{M}}(\tilde{x}^a) \simeq \mathcal{M}(x^a) + \theta^{(1)b}\nabla_b\mathcal{M}(x^a)$$

- In order to get corrections for C_ℓ, let's go from real space to Fourier's space at a given redshift z_s, i.e.

$$\tilde{\mathcal{M}}(z_s, \boldsymbol{\ell}) \simeq \mathcal{A}^{(0)}(\boldsymbol{\ell}) + \mathcal{A}^{(1)}(\boldsymbol{\ell})$$

From lensed to unlensed correlation's function (2)

- At this point, we can evaluate the two points correlation's function by defining

$$\delta(\boldsymbol{\ell}-\boldsymbol{\ell}')\tilde{C}_\ell = \langle \tilde{\mathcal{M}}(\boldsymbol{\ell})\bar{\tilde{\mathcal{M}}}(\boldsymbol{\ell}')\rangle$$

$$\delta(\boldsymbol{\ell}-\boldsymbol{\ell}')\tilde{C}_\ell^{(ij\ldots,i'j'\ldots)} = \langle \mathcal{A}^{(ij\ldots)}(\boldsymbol{\ell})\bar{\mathcal{A}}^{(i'j'\ldots)}(\boldsymbol{\ell}')\rangle + perm(ij\ldots,i'j'\ldots)$$

- In this way, lensed correlation's function is given by

$$\tilde{C}_\ell = C_\ell + C_\ell^{(0,11)} + C_\ell^{(1,1)}$$

From lensed to unlensed correlation's function (3)

C_ℓ	Counterpart
$C_\ell^{(0,11)}$	$\langle \theta^{(1)a}\theta^{(1)b}\rangle\langle \mathcal{M}\,\nabla_a\nabla_b\bar{\mathcal{M}}\rangle$
$C_\ell^{(1,1)}$	$\langle \theta^{(1)a}\theta^{(1)b}\rangle\langle \nabla_a\mathcal{M}\,\nabla_b\bar{\mathcal{M}}\rangle$

- First order corrections to lensing are due to the two points correlation's function of $\theta^{(1)a}$
- Thanks to Wick's theorem, the n points correlation's function of $\theta^{(1)a}$ is null, if n is odd, and is given by the two points one, for even n's

Lensing leading order corrections

- To our aim, we just need the leading part of first order deflection angles, which is

$$\theta^{(1)a} = -2\int_0^{r_s} dr' \, \frac{r_s - r'}{r_s \, r'} \nabla^a \Phi_W(r')$$

where Φ_W is the Weyl potential.

- Thanks to this, first order corrections are

$$C_\ell^{(0,11)} = -C_\ell(z_s)\int \frac{d^2\ell_1}{(2\pi)^2} \, (\boldsymbol{\ell}_1 \cdot \boldsymbol{\ell})^2 \; C^{\psi}_{\ell_1}(z_s, z_s)$$

$$C_\ell^{(1,1)} = \int \frac{d^2\ell_1}{(2\pi)^2} \left[(\boldsymbol{\ell} - \boldsymbol{\ell}_1)\cdot \boldsymbol{\ell}_1\right]^2 \; C^{\psi}_{|\boldsymbol{\ell}-\boldsymbol{\ell}_1|}(z_s, z_s) C_{\ell_1}(z_s)$$

where C^{ψ}_ℓ is the power spectrum of the lensing potential ψ and z_s is the redshift of the CMB

Full corrections from first order deflection angles (1)

- However, we go beyond the leading order for the first deflection angles. If we look at the correlation function, we have

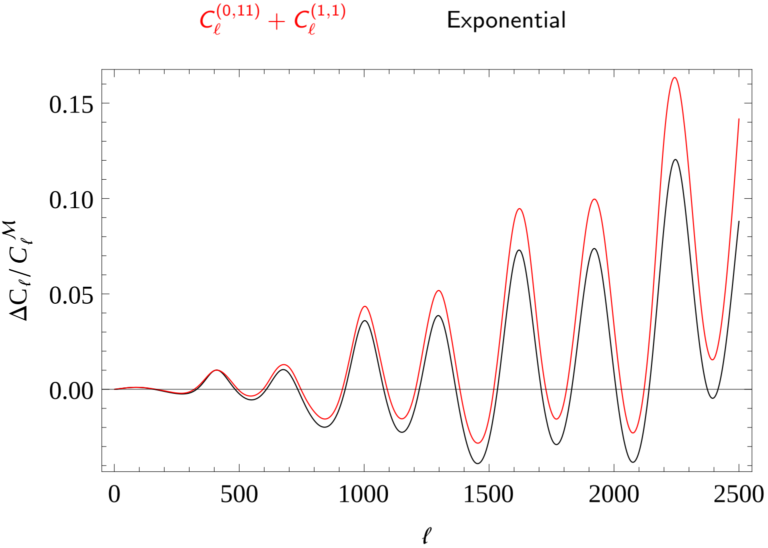
$$\tilde{\xi}(r) = \langle \tilde{\mathcal{M}}(\mathbf{x})\tilde{\mathcal{M}}(\mathbf{x}+\mathbf{r})\rangle = \langle \mathcal{M}(\mathbf{x}+\delta\boldsymbol{\theta})\mathcal{M}(\mathbf{x}+\mathbf{r}+\delta\boldsymbol{\theta}')\rangle$$
$$= \int \frac{d^2\ell}{(2\pi)^2} C_\ell \, e^{i\boldsymbol{\ell}\cdot\mathbf{r}} \langle e^{i\boldsymbol{\ell}\cdot\left(\delta\boldsymbol{\theta}-\delta\boldsymbol{\theta}'\right)}\rangle = \int \frac{d^2\ell}{(2\pi)^2} C_\ell \, e^{i\boldsymbol{\ell}\cdot\mathbf{r}} e^{-\langle[\boldsymbol{\ell}\cdot\left(\delta\boldsymbol{\theta}-\delta\boldsymbol{\theta}'\right)]^2\rangle/2}$$

- From here, we get that

$$\tilde{C}_\ell^{(1)} = \int dr \, r J_0\left(\ell r\right) \int \frac{d^2\ell'}{\left(2\pi\right)^2} C_{\ell'}^{\mathcal{M}} e^{-i\boldsymbol{\ell}'\cdot\mathbf{r}}$$
$$\times \exp\left[-\frac{\ell'^2}{2}\left(A_0\left(0\right) - A_0\left(r\right) + A_2\left(r\right)\cos\left(2\phi\right)\right)\right]$$

- This property holds because $\theta^{(1)a}$ is a gaussian stochastic field. This allows us to take into account the entire lensing correction from first order deflection angles up to all orders in perturbations theory.

Full corrections from first order deflection angles (2)



Summary

- Lensing corrections due to $\theta^{(1)a}$ smooth the peak of the $\Delta T/T$ spectrum
- Because of the gaussianity of $\theta^{(1)a}$, these corrections can be re-summed in order to give the full change due to first order defelction's angles
- This modifies the spectrum of $\sim 10\%$ for $\ell \leq 2500$
- Exponentiation reduces the amount of the correction about 20% so, it's crucial to take into account this effect for CMB precision cosmology
- Nevertheless, we can infer that the order of magnitude of the correction is properly taken into account even if we don't consider the exponentiation

Non-linear effects

At the next-to-leading order, several effects have to be taken into account

- Relaxing the Born Approximation
- Higher order terms for the gravitational potential
- Tensorial nature of light polarization

Beyond the Born approximation: next-to leading order corrections (1)

- So far, we have evaluate the lensing leading corrections to C_ℓ's due to first order deflection's angles
- Because we have been interested in $\theta^{(1)a}$, which are already first order, we have evaluated them along the unperturbed photons' geodesics: this is the so called Born approximation
- Going beyond this approximation is fundamental as long as we want to evaluate higher orders deflection's angles

Beyond the Born approximation: next-to leading order corrections (2)

- More specifically, $C_\ell^{(0,11)}$ and $C_\ell^{(1,1)}$ are of order ψ^2 whereas $\theta^{(1)a} \sim \psi$
- At the next to leading order, corrections to spectrum will be $\sim \psi^4$ and $\theta^{(n)a} \sim \psi^n$
- From here, it follows that we need to evaluate deflection's angles up to third order
- $\theta^{(4)a}$ does not contribute because its stochastic average is null. The same happens for $\theta^{(2)a}$ at the linear order

Beyond the Born approximation: next-to leading order corrections (3)

- Computation of higher order corrections can be done via deflection angles $\theta^{(n)a}$ or amplification matrix

$$(\Psi^a_b)^{(n)} = -\frac{\partial \theta^{(n)a}}{\partial \theta^b_o}, \qquad \text{for } n \geq 1$$

- These approaches are equivalent for the lensing leading terms[1]. Indeed, lensing leading terms for the amplification matrix are consistent with the iterative solution of the so called lens equation

$$\Psi^a_b = \frac{2}{\eta_o - \eta_s} \int_{\eta_s}^{\eta_o} d\eta' \frac{\eta' - \eta_s}{\eta_o - \eta'} \hat{\gamma}_0^{ac} \partial_c \partial_d \psi(\eta', \eta_o - \eta', \theta^a) \left[\delta^d_b - \Psi^d_b\right]$$

[1] F, Gasperini, Marozzi, Veneziano, JCAP 1508 (2015) no.08, 020

Amplification matrix and deflection's angles

- A direct evaluation of these angles via the GLC gauge gives

$$\theta^{(2)a} = -2\int_0^{r_s} dr' \, \frac{r_s - r'}{r_s\, r'} \nabla_b \nabla^a \Phi_W(r')\theta^{(1)b}(r')$$

$$\theta^{(3)a} = -2\int_0^{r_s} dr' \, \frac{r_s - r'}{r_s\, r'} \left[\nabla_b \nabla^a \Phi_W(r')\theta^{(2)b}(r') + \frac{1}{2}\nabla_b \nabla_c \nabla^a \Phi_W(r')\theta^{(1)b}(r')\theta^{(1)c}(r')\right]$$

- These expressions are perfectly consistent with the iterative solutions of the lens equation, beyond the Born approximation[2]

$$(\Psi^a_b)^{(2)} = 2\int_0^{r_s} dr' \, \frac{r_s - r'}{r_s\, r'} \, \gamma^{ac}\left[\partial_c\partial_b\partial_d\psi(r')\theta^{(1)d} - \partial_c\partial_d\psi(r')\Psi^{d(1)}_b\right] ,$$

$$(\Psi^a_b)^{(3)} = 2\int_0^{r_s} dr' \, \frac{r_s - r'}{r_s\, r'} \, \gamma^{ac}\left[\partial_c\partial_b\partial_d\psi(r')\theta^{(2)d} + \frac{1}{2}\partial_c\partial_b\partial_d\partial_e\psi(r')\theta^{(1)d}\theta^{(1)e} - \partial_c\partial_d\partial_e\psi(r')\theta^{(1)e}\Psi^{d(1)}_b - \partial_c\partial_d\psi(r')\Psi^{d(2)}_b\right]$$

[2] differently from Hagstotz, Schafer, Merkel, Mon.Not.Roy.Astron.Soc. 454 (2015) no.1, 831-838

Next to leading order corrections to $\Delta T/T$ (1)

- Just as already done for the leading order, our anisotropies' temperature field will be corrected by higher order deflection's angles as

$$\tilde{\mathcal{M}}(x^a) = \mathcal{M}\left(x^a + \delta\theta^a\right) \simeq \mathcal{M}(x^a) + \sum_{i=1}^{4} \theta^{(i)b}\nabla_b\mathcal{M}(x^a) + \frac{1}{2}\sum_{i+j\leq 4} \theta^{(i)b}\theta^{(j)c}\nabla_b\nabla_c\mathcal{M}(x^a)$$
$$+ \frac{1}{6}\sum_{i+j+k\leq 4}\theta^{(i)b}\theta^{(j)c}\theta^{(k)d}\nabla_b\nabla_c\nabla_d\mathcal{M}(x^a) + \frac{1}{24}\theta^{(1)b}\theta^{(1)c}\theta^{(1)d}\theta^{(1)e}\nabla_b\nabla_c\nabla_d\nabla_e\mathcal{M}(x^a)$$

- Equivalently, in Fourier space

$$\tilde{\mathcal{M}}(x^a) \simeq \mathcal{A}^{(0)}(x^a) + \sum_{i=1}^{4}\mathcal{A}^{(i)}(x^a) + \sum_{i+j\leq 4,\ 1\leq i\leq j}\mathcal{A}^{(ij)}(x^a) + \sum_{i+j+k\leq 4,\ 1\leq i\leq j\leq k}\mathcal{A}^{(ijk)}(x^a) + \mathcal{A}^{(1111)}(x^a)$$

Next to leading order corrections to $\Delta T/T$ (2)

- Following the same formalism as before, now we have that

$$\begin{aligned}\tilde{C}_\ell = &\ C_\ell \\ &+ C_\ell^{(0,11)} + C_\ell^{(1,1)} \\ &+ C_\ell^{(0,1111)} + C_\ell^{(1,111)} + C_\ell^{(11,11)} \\ &+ C_\ell^{(0,22)} + C_\ell^{(0,13)} \\ &+ C_\ell^{(1,3)} + C_\ell^{(2,2)} \\ &+ C_\ell^{(1,12)} + C_\ell^{(2,11)}\end{aligned}$$

- First line is the already evaluated linear order and refers to terms $\langle \theta^{(1)a}\theta^{(1)b}\rangle\langle \mathcal{M}\,\nabla_a\nabla_b\bar{\mathcal{M}}\rangle$ and $\langle \theta^{(1)a}\theta^{(1)b}\rangle\langle \nabla_a\mathcal{M}\,\nabla_b\bar{\mathcal{M}}\rangle$

Next to leading order from $\theta^{(1)a}$ - First group

C_ℓ	Counterpart
$C_\ell^{(0,1111)}$	$\langle\theta^{(1)a}\theta^{(1)b}\theta^{(1)c}\theta^{(1)d}\rangle\langle\mathcal{M}\,\nabla_a\nabla_b\nabla_c\nabla_d\bar{\mathcal{M}}\rangle$
$C_\ell^{(1,111)}$	$\langle\theta^{(1)a}\theta^{(1)b}\theta^{(1)c}\theta^{(1)d}\rangle\langle\nabla_a\mathcal{M}\,\nabla_b\nabla_c\nabla_d\bar{\mathcal{M}}\rangle$
$C_\ell^{(11,11)}$	$\langle\theta^{(1)a}\theta^{(1)b}\theta^{(1)c}\theta^{(1)d}\rangle\langle\nabla_a\nabla_b\mathcal{M}\,\nabla_c\nabla_d\bar{\mathcal{M}}\rangle$

These terms take into account all the next to leading order corrections due to $\theta^{(1)a}$. Their effect is consistently included in the exponentiation, previously shown

Limber approximation and null terms

C_ℓ	Counterpart
$C_\ell^{(0,13)}$	$\langle\theta^{(1)a}\theta^{(3)b}\rangle\langle\mathcal{M}\,\nabla_a\nabla_b\bar{\mathcal{M}}\rangle$
$C_\ell^{(0,22)}$	$\langle\theta^{(2)a}\theta^{(2)b}\rangle\langle\mathcal{M}\,\nabla_a\nabla_b\bar{\mathcal{M}}\rangle$

- Limber approximation applies when the argument of integration in k-space does not oscillate too much and rapidly decreases for $k \to \infty$
- This regime is valid for our terms
- By applying them, we get that $C_\ell^{(0,13)} = -C_\ell^{(0,22)}$

Next to leading order gaussian terms - Second group

C_ℓ	Counterpart
$C_\ell^{(1,3)}$	$\langle\theta^{(1)a}\theta^{(3)b}\rangle\langle\nabla_a\mathcal{M}\,\nabla_b\bar{\mathcal{M}}\rangle$
$C_\ell^{(2,2)}$	$\langle\theta^{(2)a}\theta^{(2)b}\rangle\langle\nabla_a\mathcal{M}\,\nabla_b\bar{\mathcal{M}}\rangle$

- These terms come from the two points correlation function of deflection's angles up to third order
- Following the exponentiation for $\theta^{(2)a}$ and $\theta^{(3)a}$ would take into account also these terms, just as done for the leading order

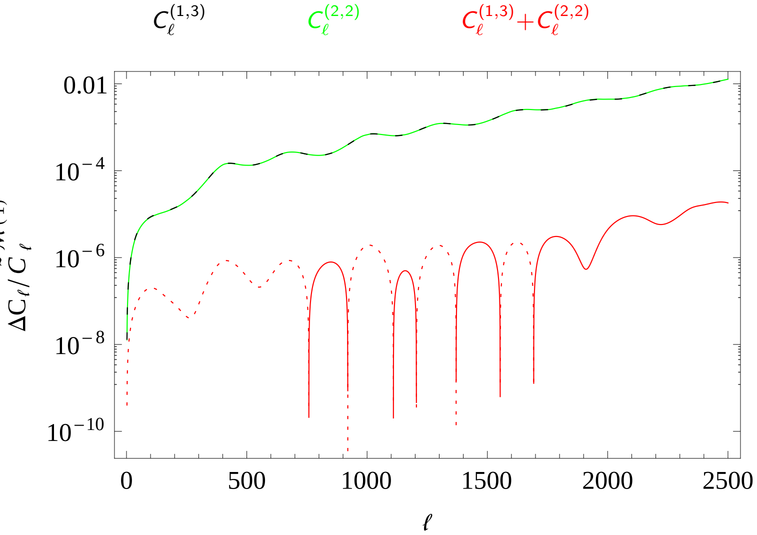
Next to leading order non gaussian terms - Third group

C_ℓ	Counterpart
$C_\ell^{(1,12)}$	$\langle\theta^{(1)a}\theta^{(1)b}\theta^{(2)c}\rangle\langle\nabla_a\mathcal{M}\,\nabla_b\nabla_c\bar{\mathcal{M}}\rangle$
$C_\ell^{(2,11)}$	$\langle\theta^{(2)a}\theta^{(1)b}\theta^{(1)c}\rangle\langle\nabla_a\mathcal{M}\,\nabla_b\nabla_c\bar{\mathcal{M}}\rangle$

- These terms come from the three points correlation function of deflection's angles up to third order
- Because of that, they cannot be taken into account by the exponentiation method[3]

[3] This method has been used in Pratten, Lewis, JCAP 1608 (2016) no.08, 047

Second group - Numerical results (1)



Second group - Numerical results (2)

- Each term of second group gives a huge modification for the spectrum ($\sim 1\%$)
- However, the total contribution shows a significant cancellation between these terms (3 orders of magnitude)
- This cancellation can be understood by looking at the analytical expressions of these terms

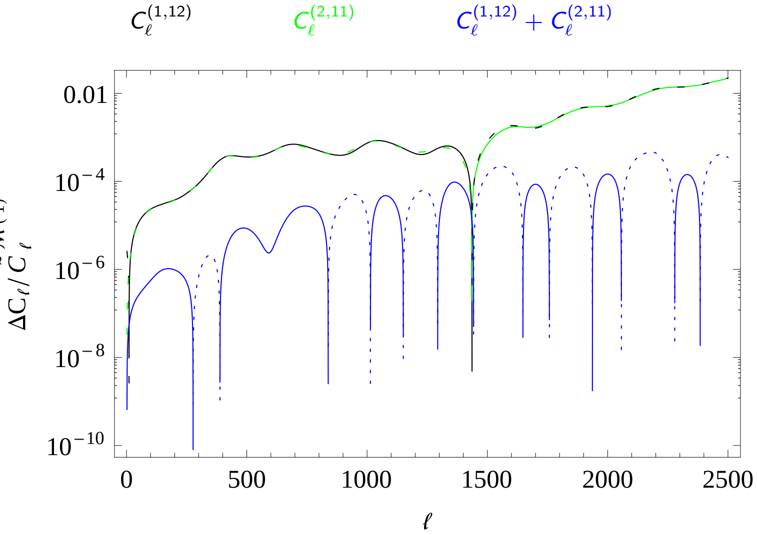
$$C_\ell^{(1,3)} = -\int \frac{d^2\ell_1}{(2\pi)^2} \int \frac{d^2\ell_2}{(2\pi)^2} \left[(\boldsymbol{\ell} - \boldsymbol{\ell}_1)\cdot\boldsymbol{\ell}_1\right]^2 \left[(\boldsymbol{\ell} - \boldsymbol{\ell}_1)\cdot\boldsymbol{\ell}_2\right]^2 C_{\ell_1}(z_s)$$
$$\times \int_0^{r_s} dr' \frac{(r_s - r')^2}{r_s^2\, r'^4} C^{\psi}_{\ell_2}(z', z') P_R\left(\frac{|\boldsymbol{\ell} - \boldsymbol{\ell}_1| + 1/2}{r'}\right) \left[T_{\Psi+\Phi}\left(\frac{|\boldsymbol{\ell} - \boldsymbol{\ell}_1| + 1/2}{r'}, z'\right)\right]^2$$

$$C_\ell^{(2,2)} = \int \frac{d^2\ell_1}{(2\pi)^2} \int \frac{d^2\ell_2}{(2\pi)^2} \left[(\boldsymbol{\ell} - \boldsymbol{\ell}_1 + \boldsymbol{\ell}_2)\cdot\boldsymbol{\ell}_1\right]^2 \left[(\boldsymbol{\ell} - \boldsymbol{\ell}_1 + \boldsymbol{\ell}_2)\cdot\boldsymbol{\ell}_2\right]^2 C_{\ell_1}(z_s)$$
$$\times \int_0^{r_S} dr' \frac{(r_s - r')^2}{r_s^2\, r'^4} C^{\psi}_{\ell_2}(z', z') P_R\left(\frac{|\boldsymbol{\ell} - \boldsymbol{\ell}_1 + \boldsymbol{\ell}_2| + 1/2}{r'}\right) \left[T_{\Psi+\Phi}\left(\frac{|\boldsymbol{\ell} - \boldsymbol{\ell}_1 + \boldsymbol{\ell}_2| + 1/2}{r'}, z'\right)\right]^2$$

- Moreover, it's consistent with literature[4]

[4] Pratten, Lewis, JCAP 1608 (2016) no.08, 047
Krause, Hirata, Astron.Astrophys. 523 (2010) A28

Third group - Numerical results (1)



Third group - Numerical results (2)

- Even these terms would give a huge contributions separately ($\sim 1\%$)
- Third group shows a cancellation too. However, this turns out to be smaller than what happens for the second group (2 orders of magnitude)
- Once again, this cancellation can be understood by looking at the analytical expressions

$$C_\ell^{(1,12)} = -2 \int \frac{d^2\ell_1}{(2\pi)^2} \int \frac{d^2\ell_2}{(2\pi)^2} (\boldsymbol{\ell}_1 \cdot \boldsymbol{\ell}_2) \left[(\boldsymbol{\ell} - \boldsymbol{\ell}_1) \cdot \boldsymbol{\ell}_2\right] \left[(\boldsymbol{\ell} - \boldsymbol{\ell}_1) \cdot \boldsymbol{\ell}_1\right]^2 C_{\ell_1}(z_s) C_{\ell_2}^{\psi}\left(z_s, z'\right)$$
$$\times \int_0^{r_s} dr' \frac{(r_s - r')^2}{r_s^2\, r'^4} P_R\left(\frac{|\boldsymbol{\ell} - \boldsymbol{\ell}_1| + 1/2}{r'}\right) \left[T_{\Psi+\Phi}\left(\frac{|\boldsymbol{\ell} - \boldsymbol{\ell}_1| + 1/2}{r'}, z'\right)\right]^2$$

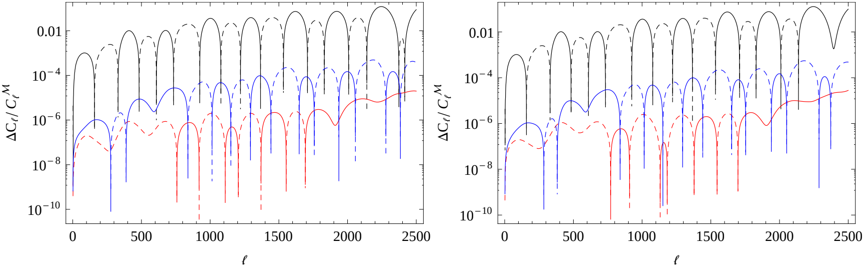
$$C_\ell^{(2,11)} = 2 \int \frac{d^2\ell_1}{(2\pi)^2} \int \frac{d^2\ell_2}{(2\pi)^2} (\boldsymbol{\ell}_1 \cdot \boldsymbol{\ell}_2) \left[(\boldsymbol{\ell} - \boldsymbol{\ell}_1 + \boldsymbol{\ell}_2) \cdot \boldsymbol{\ell}_2\right] \left[(\boldsymbol{\ell} - \boldsymbol{\ell}_1 + \boldsymbol{\ell}_2) \cdot \boldsymbol{\ell}_1\right]^2 C_{\ell_1}(z_s) C_{\ell_2}^{\psi}\left(z_s, z'\right)$$
$$\times \int_0^{r_s} dr' \frac{(r_s - r')^2}{r_s^2\, r'^4} P_R\left(\frac{|\boldsymbol{\ell} - \boldsymbol{\ell}_1 + \boldsymbol{\ell}_2| + 1/2}{r'}\right) \left[T_{\Psi+\Phi}\left(\frac{|\boldsymbol{\ell} - \boldsymbol{\ell}_1 + \boldsymbol{\ell}_2| + 1/2}{r'}, z'\right)\right]^2$$

- These contributions are not considered in literature[5] and turns out to be the dominant ones

[5] Pratten, Lewis, JCAP 1608 (2016) no.08, 047

Linear power spectrum vs HaloFit

Exponentiation Second group Third group



Linear power spectrum HaloFit

Other next-to-leading order corrections - LSS (1)

- Post-Born corrections are not the only ones which appear at next-to-leading order
- First of all, we need to consider terms from higher order Weyl potential: in particular $\Phi_W \approx \Phi_W^{(1)} + \Phi_W^{(2)} + \Phi_W^{(3)}$
- These corrections can be classified as well as we did for the post-Born ones, involving

$$\langle \Phi_W^{(2)}(z,\ell)\bar{\Phi}_W^{(2)}(z',\ell')\rangle \qquad \text{and} \qquad \langle \Phi_W^{(1)}(z_1,\ell_1)\bar{\Phi}_W^{(3)}(z_2,\ell_2)\rangle$$

$$\langle \Phi_W^{(1)}(z_1,\ell_1)\Phi_W^{(1)}(z_2,\ell_2)\Phi_W^{(2)}(z_3,\ell_3)\rangle \sim b_{\ell_1\ell_2\ell_3}^{\Phi\Phi\Phi^{(2)}}(z_1,z_2,z_3)$$

where $b_{\ell_1\ell_2\ell_3}^{\Phi\Phi\Phi^{(2)}}(z_1,z_2,z_3)$[6] is the reduced bispectrum

[6] Di Dio, Durrer, Marozzi, Montanari, JCAP 1601 (2016) 016, arXiv:1510.04202

Other next-to-leading order corrections - LSS (2)

- Terms generated by this expansion can be classified as we did for pure post-Born corrections, by taking into account the proper correction to $\theta^{(2)}$ and $\theta^{(3)}$ due to $\Phi_W^{(2)}$ and $\Phi_W^{(3)}$
- However, using HaloFit model implies that corrections from two-points correlation functions of Bardeen potential are already included in previous results
- What is missing are terms due to three-points correlation function, involving $b^{\Phi\Phi\Phi^{(2)}}$
- Even if these corrections appear for both Second and Third group, it turns out that they are non null only in the last case, within the Limber approximation

From Temperature to Polarization's spectra (1)

- So far, we are considered just a scalar field, i.e. $\Delta T/T$
- Now, we want to find the same corrections even for other CMB spectra, in particular the polarizations' ones
- It means that we have to deal with a tensorial object P_{ab} instead of $\Delta T/T$
- More precisely, we consider the component of P_{ab}, $\mathcal{E}$ and $\mathcal{B}$ once projected on a flat 2-dimensional subspace, via a basis s_A^a which is parallel transported along the photon path

From Temperature to Polarization's spectra (2)

- Having this in mind, we get that, for the polarization' spectra, previous formula can be easily generalized by the substitution

$$C_\ell^{\mathcal{M}} \to \hat{C}_\ell^X$$

where

$$\begin{aligned}
X = \mathcal{M} \quad &\Rightarrow \quad \hat{C}_\ell^X = C_\ell^{\mathcal{M}} \\
X = \mathcal{E} \quad &\Rightarrow \quad \hat{C}_\ell^X = C_\ell^{\mathcal{E}} \cos^2 2\varphi_\ell + C_\ell^{\mathcal{B}} \sin^2 2\varphi_\ell \\
X = \mathcal{B} \quad &\Rightarrow \quad \hat{C}_\ell^X = C_\ell^{\mathcal{E}} \sin^2 2\varphi_\ell + C_\ell^{\mathcal{B}} \cos^2 2\varphi_\ell \\
X = \mathcal{E}\mathcal{M} \quad &\Rightarrow \quad \hat{C}_\ell^X = C_\ell^{\mathcal{E}\mathcal{M}} \cos 2\varphi_\ell
\end{aligned}$$

Other next-to-leading order corrections - Rotation (1)

- Moreover, polarization tensor involves also more corrections, due to the rotation of the photon's polarization from the last scattering surface to the observer
$$\tilde{P} = e^{-2i\beta} P$$
- This rotation comes from the fact that the polarization tensor is projected on the Sachs basis and this one is parallely transported along the photon's geodetic
- An exact expression for the Sachs basis has been provided in the so-called GLC gauge[7] via the conditions

$$\gamma_{ab} s^a_A s^b_B = \delta_{AB} \qquad , \qquad \nabla_\tau s^a_A = 0$$

[7] F, Marozzi, Gasperini, Veneziano, JCAP 1311 (2013) 019
F, Nugier, JCAP 1502 (2015) no.02, 002

Other next-to-leading order corrections - Rotation (2)

- In a perturbed Universe, it can be proved that this Sachs basis can be solved as

$$s_A^a = \chi_{ab}\, \bar{s}_B^b\, R_A^B$$

where

$$\chi_{ab} = \chi_{ab}^{(0)} + \chi_{ab}^{(1)} + \chi_{ab}^{(2)} + \ldots \quad \text{is a symmetric tensor}$$

$$R_A^B = \cos\beta\, \delta_A^B + \sin\beta\, \epsilon_A^B \quad \text{is a 2-D rotation matrix}$$

$$\beta = \beta^{(0)} + \beta^{(1)} + \beta^{(2)} + \ldots \quad \text{is the rotation angle}$$

$$\bar{s}_B^b = (ar)^{-2}\, \mathrm{diag}\left(1, \sin^{-1}\theta\right) \quad \text{is a particular background solution}$$

- In this way, the Sachs basis can be found up to each desired order in perturbation theory

Other next-to-leading order corrections - Rotation (3)

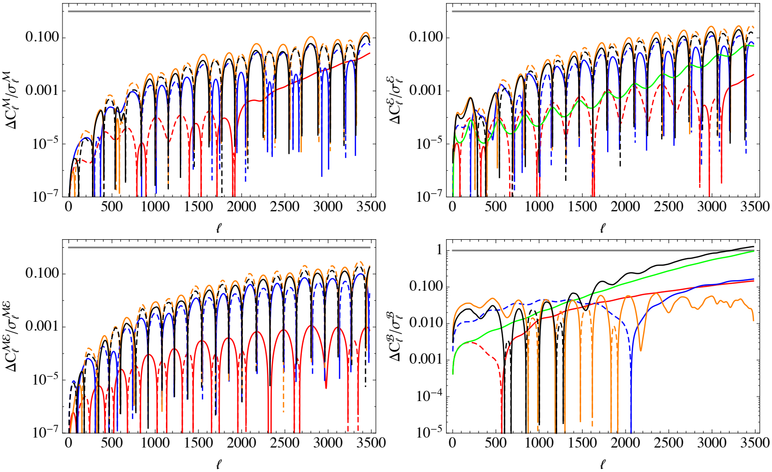
- This rotation angle β is related to the vorticity ω in the Amplification matrix
$$\mathcal{A}_{AB} = \begin{pmatrix} 1-\kappa+\gamma_1 & \gamma_2+\omega \\ \gamma_2-\omega & 1-\kappa-\gamma_1 \end{pmatrix}$$
More strictly, it can be shown that $\omega = \beta$ up to second order
- Because of this, the first non-null corrections from this angle is just $\beta^{(2)}$. This is why these corrections don't affect first order spectra
- Moreover, this implies that the unique corrections from rotation angle to polarization's spectra are related to the two-points correlation function $\langle \beta^{(2)} \beta^{(2)} \rangle$

Other next-to-leading order corrections - Rotation (4)

Term	Counterpart	TT	TE	EE	BB
$C_\ell^{\beta(2,2)}$	$\langle\beta^{(2)}(x)\beta^{(2)}(y)\rangle\langle X(x)Y(y)\rangle$	NO	NO	YES	YES
$C_\ell^{\beta(22,0)}$	$\langle\beta^{(2)}(x)\beta^{(2)}(x)\rangle\langle X(x)Y(y)\rangle$	NO	YES	YES	NO

- Even if those terms both converge, their behaviors are different. In particular, (22,0) strongly depends on smallest scale of power spectrum
- However, their contribution to the shift of cosmological parameters is not statistically relevant. So we don't take care anymore about these terms and just consider $(2,2)$ contributions

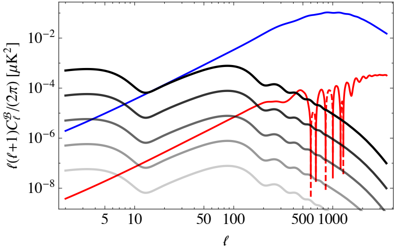
Other next-to-leading order corrections - Numerical results

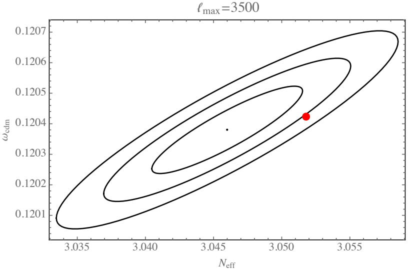


Second group[4] + Third group + LSS + Rotation[8] = Sum

[8] Not taken into account in
Lewis, Pratten, JCAP 1612 (2016) no.12, 003, arXiv:1608.01263

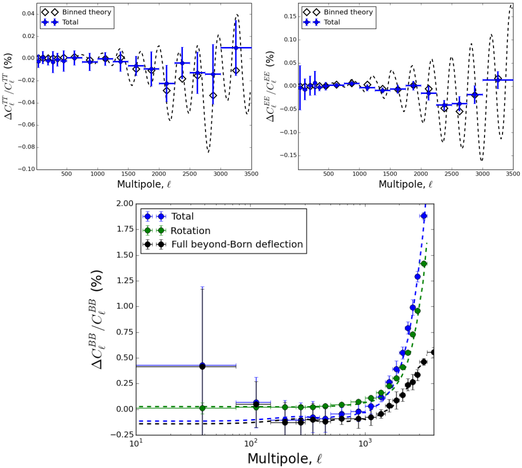
Consequences on physical observables





- Lensing-induced B-modes turn out to be of the same order as a primordial tensor-to-scalar ratio $r = 10^{-3}$
- These corrections can shift cosmological parameters evaluation of almost $2\,\sigma$

Comparing analytical and numerical results



- Our results are in very good agreement with recent numerical ones[9] on small scales

[9]Plots are courtesy of Giulio Fabbian
Fabbian, Calabrese, Carbone, arXiv:1702.03317

Summary

- Lensing corrections due to $\theta^{(1)a}$ can be taken into account non pertubatively because of its statistical properties
- At the next to leading order, also corrections due to $\theta^{(2)a}$ and $\theta^{(3)a}$ must to be considered
- Three points correlations function for these angles are no longer null. This means that method based on exponentiation breaks down at the second order
- Moreover, these terms not only are non zero but are the dominant ones
- Nevertheless, they are not the only ones involved. Also LSS corrections appear with same order of magnitude, but with opposite phase. This leads to a partial cancellation
- For polarization's spectra, also the rotation of the polarization's axis contributes. This leads to a new effects, not considered before

Conclusions

- The whole correction of these effect must be taken into account in order to interpret correctly the future measurement about primordial gravitational waves
- Moreover, these corrections can lead also to a significant shift in the estimation of cosmological parameters. A reason why, case by case, their role have to be considered in the measurement's process
- The very good (perhaps impressive!!!) agreement between our analytical method and other results obtained via N-Body simulation techniques is a strong support to these conclusions