RELATIVISTIC PERTURBATION THEORY IN ΛCDM AND BEYOND: EFFECTS ON COSMOLOGICAL DYNAMICS AND OBSERVATIONS

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JCAP **1601**, (2016)

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JCAP **1804**, (2018)

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OUTLINE

1. Cosmological PT in a nutshell

2. GR effects in cosmological dynamics
   - GR cosmological dynamics beyond linear order

3. GR effects in cosmological observations
   - Effects on the parameter estimation
     - Bias in the constraints
     - Bias in the best-fit values

Our model

Real Universe = homogeneous and isotropic FRW + perturbations

Our goal

$$\Phi = \Phi_p \times \text{ Transfer function } \times \text{ Growth function}$$

Our approximations

- **Relativistic perturbation theory** is an expansion in powers of the amplitude of the perturbations around an homogeneous FRW background

  Applicability: largest scales/early times, where the fluctuations are small

  Fully GR, but restricted to regimes where the matter density perturbations are small

- **Newtonian approximation** of GR involves weak gravitational fields and slow motion of particles

  Applicability: scales λ such that Schwarzschild radius $\ll \lambda \ll$ Hubble horizon

  Well suited to deal with non-linearities, e.g. using N-body simulations, but Newtonian

Step 1: primordial

$$\Phi = \Phi_p \times \text{ Transfer function } \times \text{ Growth function}$$

Primordial perturbation set at very early times, at the end of inflation

Standard paradigm:

- (almost) Gaussian
- (almost) scale-invariant

Step 2: transfer function

$$\Phi = \Phi_p \times \text{ Transfer function } \times \text{ Growth function}$$

Describes the evolution of perturbations during radiation era, the radiation-matter transition and early in matter era

The transfer function is found from the Einstein-Boltzmann equations for the tight coupling CDM + baryons + photons

- analytical solutions for horizon crossing in radiation era and early-matter era $\Rightarrow$ match
- BBKS fitting formula for CDM + photons
- Eisenstein & Hu fitting formula for CDM + baryons + photons
- CAMB, CLASS etc..

Step 3: growth function

$$\Phi = \Phi_p \times \text{ Transfer function } \times \text{ Growth function}$$

Describe the evolution of perturbations at late times:
deep matter- and Λ-dominated eras

- late times $\Rightarrow$ CDM + Λ
- mildly non-linear $\Rightarrow$ no pressure
- usually stop at I order

The caveat: gauge issue

Relativistic cosmological PT "in practice":

- Start FRW

$$ds^2 = a^2 \left[-d\eta^2 + \delta_{ij} dq^i dq^j \right]$$

- Add perturbations

$$ds^2 = a^2 \left\{ -(1+2\psi)\, d\eta^2 + 2B_i d\eta dx^i + \left[(1-2\phi)\, \delta_{ij} + \chi_{ij} \right] dx^i dx^i \right\}$$

- Calculate the perturbations of what you need
- Solve your equations

Perturbations are gauge-dependent!!!

So..

- use gauge-invariant variables
- use smart gauges

Non-linear cosmological PT in GR

Which are the differences between GR and Newtonian cosmological dynamics beyond linear theory?

RELATIVISTIC PERTURBATIONS IN ΛCDM COSMOLOGY: EULERIAN AND LAGRANGIAN APPROACHES

Villa & Rampf JCAP **01** 030 (2016)

Aim & procedure

We want to

- study the perturbations of the space-time metric and fluid variables in GR
  - up to II order in PT
  - in the Λ + CDM era
  - include all the degrees of freedom - scalar, vector and tensor
- identify relativistic corrections to Newtonian PT
  $\Rightarrow$ choose gauges with a clean correspondence to Newtonian gravity

How:

1. solve Einstein equations in the Eulerian frame
2. perform a gauge transformation
3. get the results in the Lagrangian frame

The Lagrangian frame: the synchronous-comoving gauge

In the Lagrangian picture the dynamics is described with respect to a coordinate system attached to the matter

In GR we use the synchronous-comoving gauge

- Spatial coordinates are constant along the geodesic of the matter
- The time coordinate coincides with the proper time of the fluid

Applicability: intermediate non-linear scales, until caustic formation

The synchronous-comoving gauge

$$ds^2 = a^2 \left[-d\tau^2 + \gamma_{ij} dq^i dq^j \right]$$

In the Newtonian limit the Einstein equations take the classical Newtonian form of the equations of motion in the Lagrangian formulation

Matarrese & Terranova MNRAS (1997)

Villa, Matarrese & Maino JCAP (2014)

An Eulerian frame: the Poisson gauge

In the Eulerian picture the dynamics is described with respect to a coordinate system not comoving with the matter

In GR we use the Poisson gauge

The Poisson gauge

$$ds^2 = a^2 \left\{ -\left(1+2\psi\right) d\eta^2 + 2B_i d\eta dx^i + \left[\left(1-2\phi\right)\delta_{ij} + \chi_{ij}\right] dx^i dx^i \right\}$$

$$\text{with } \partial^i B_i = 0 \text{ and } \partial^i \chi_{ij} = \chi^i_i = 0$$

Applicability: intermediate non-linear scales, accounting for vector and tensor modes generated beyond linear regime

In the Newtonian limit the Einstein equations take the classical Newtonian form of the equations of motion in the Eulerian formulation

Previous results in the literature

Poisson gauge

- formal expressions for all metric modes and fluid variables
- no explicit time dependence for all second-order quantities

Bartolo, Matarrese & Riotto JCAP (2006)

Synchronous-comoving gauge

- approximated solutions for the second-order scalars and density
- no solution for vector and tensor modes

Bartolo, Matarrese, Pantano & Riotto CQG (2010)

Solutions of Einstein equations: first-order scalar

The evolution equation for the first-order scalar is

$$\ddot{\phi}_1 + 3\mathcal{H}\dot{\phi}_1 + a^2\Lambda\phi_1 = 0$$

The solution is

$$\phi_1 = \frac{D}{a}\varphi_0$$

where D is the growing mode solution of the Newtonian evolution equation for the density perturbation δ_1^N

$$\ddot{D} + \mathcal{H}\dot{D} - \frac{3}{2}\mathcal{H}_0^2\Omega_{\mathrm{m}0}\frac{D}{a} = 0$$

The GR first-order scalar $\phi_1 = g\varphi_0$ is identical to the first-order Newtonian gravitational potential, solution of the Poisson equation in EPT

$$\nabla^2\phi_1 = \nabla^2\varphi_1^N = \frac{3}{2}\frac{\mathcal{H}_0^2\Omega_{\mathrm{m}0}}{a}\delta_1^N$$

Solutions of Einstein equations: second-order scalars

The evolution equation for the second-order scalar in g_{ij} is

$$\ddot{\phi}_2 + 3\mathcal{H}\dot{\phi}_2 + a^2\Lambda\phi_2 = S(\eta, \boldsymbol{x})$$

In fully generality, the solution can be written as

$$\phi_2(\eta, \boldsymbol{x}) = \frac{b_1}{a}(\eta)\, S_1(\boldsymbol{x}) + \frac{b_2}{a}(\eta)\, S_2(\boldsymbol{x}) + \frac{b_3}{a}(\eta)\, S_3(\boldsymbol{x}) + \frac{b_4}{a}(\eta)\, S_4(\boldsymbol{x}) + \frac{g(\eta)}{g_{\rm in}}\phi_{2_{\rm in}}(\boldsymbol{x})$$

Now:

- substitute in the evolution equation
- obtain four ODE for b_1, b_2, b_3, b_4
- solve

Solutions of Einstein equations: second-order scalars

$$\phi_2(\eta, \boldsymbol{x}) = \frac{b_1}{a}(\eta)\, S_1(\boldsymbol{x}) + \frac{b_2}{a}(\eta)\, S_2(\boldsymbol{x}) + \frac{b_3}{a}(\eta)\, S_3(\boldsymbol{x}) + \frac{b_4}{a}(\eta)\, S_4(\boldsymbol{x}) + \frac{g(\eta)}{g_{\rm in}} \phi_{2_{\rm in}}(\boldsymbol{x})$$

The growing mode solutions are

$$b_1 = -\frac{D^2}{a} + \frac{2\dot{D}^2}{3\mathcal{H}_0^2\Omega_{\rm m0}} + \frac{1}{3}\frac{DD_{\rm in}}{a_{\rm in}}$$

$$b_2 = -2D\left(\frac{D_{\rm in}}{a_{\rm in}} - \frac{D}{a}\right)$$

$$b_3 = \frac{2}{3\mathcal{H}_0^2\Omega_{\rm m0}}\left(F + D^2\right)$$

$$b_4 = -\frac{2}{3\mathcal{H}_0^2\Omega_{\rm m0}} F$$

Only two functions appear:

- D the first-order growing mode of the Newtonian density perturbation and trajectory
- F the second-order growing mode of the Newtonian trajectory

$$\boldsymbol{x} = \boldsymbol{q} + D(\eta)\boldsymbol{\Psi}_1(\boldsymbol{q}) + F(\eta)\boldsymbol{\Psi}_2(\boldsymbol{q})$$

We recognise in the GR scalar the second-order Newtonian gravitational potential

$$\nabla^2\left(\frac{b_3}{a}(\eta)\, S_3(\boldsymbol{x}) + \frac{b_4}{a}(\eta)\, S_4(\boldsymbol{x})\right) = \nabla^2\varphi_2^N = \frac{3}{2}\frac{\mathcal{H}_0^2\Omega_{\rm m0}}{a}\delta_2^N$$

Villa & Rampf JCAP (2016)

Results in the Poisson gauge

Space-time metric

$$g_{00_{\rm P}} = -a^2\left[1+2\frac{D}{a}\varphi_0 + \left(3\frac{D^2}{a^2} + \frac{5}{3}\frac{DD_{\rm in}}{aa_{\rm in}}(1-2a_{\rm nl}) + \frac{2\dot{D}^2}{3a\mathcal{H}_0^2\Omega_{\rm m0}}\right)\varphi_0^2 + 6\left(4\frac{D^2}{a^2} - \frac{10}{3}\frac{DD_{\rm in}}{aa_{\rm in}} + \frac{4}{3}\frac{\dot{D}^2}{a\mathcal{H}_0^2\Omega_{\rm m0}}\right)\Theta_0 + \frac{2D^2}{3a\mathcal{H}_0^2\Omega_{\rm m0}}\varphi_{0,l}\varphi_0^{,l} - \frac{4\left(D^2+F\right)}{3a\mathcal{H}_0^2\Omega_{\rm m0}}\Psi_0\right]$$

$$g_{0i_{\rm P}} = -\frac{8aD\dot{D}}{3\mathcal{H}_0^2\Omega_{\rm m0}}\mathcal{R}_i$$

$$g_{ij_{\rm P}} = a^2\left[\delta_{ij}\left\{1-2\frac{D}{a}\varphi_0 + \left(\frac{D^2}{a^2} - \frac{5}{3}\frac{DD_{\rm in}}{aa_{\rm in}}(1-2a_{\rm nl}) - \frac{2\dot{D}^2}{3a\mathcal{H}_0^2\Omega_{\rm m0}}\right)\varphi_0^2 - 6\left(2\frac{D^2}{a^2} - \frac{10}{3}\frac{DD_{\rm in}}{aa_{\rm in}}\right)\Theta_0 - \frac{2D^2}{3a\mathcal{H}_0^2\Omega_{\rm m0}}\varphi_{0,l}\varphi_0^{,l} + \frac{4\left(D^2+F\right)}{3a\mathcal{H}_0^2\Omega_{\rm m0}}\Psi_0\right\} + \frac{1}{2}\chi^{\rm GW}_{2ij_P}\right].$$

Matter density

$$\delta_{\rm P} = \frac{2}{3\mathcal{H}_0^2\Omega_{\rm m0}}\left[D\nabla^2\varphi_0 - 3\mathcal{H}\dot{D}\varphi_0\right] + \frac{1}{3\mathcal{H}_0^2\Omega_{\rm m0}}\left[6\frac{D^2}{a} + \frac{10}{3}\frac{DD_{\rm in}}{a_{\rm in}}(1-2a_{\rm nl}) + \frac{4}{3}\frac{\dot{D}^2}{\mathcal{H}_0^2\Omega_{\rm m0}}\right]\varphi_0\nabla^2\varphi_0$$
$$+ \frac{\mathcal{H}^2D^2}{a\mathcal{H}_0^2\Omega_{\rm m0}}\left[\frac{\dot{D}^2}{\mathcal{H}^2D^2} - 4\frac{\dot{D}}{\mathcal{H}D}\right]\varphi_0^2 + \frac{5\mathcal{H}}{3\mathcal{H}_0^2\Omega_{\rm m0}}(1+2a_{\rm nl})\dot{D}\frac{D_{\rm in}}{a_{\rm in}}\varphi_0^2 - \frac{12\mathcal{H}\dot{D}D}{a\mathcal{H}_0^2\Omega_{\rm m0}}\Theta_0$$
$$+ \frac{D}{\mathcal{H}_0^2\Omega_{\rm m0}}\left[\frac{D}{a} - \frac{20}{9}\frac{D_{\rm in}}{a_{\rm in}}a_{\rm nl}\right](\nabla\varphi_0)^2 + \frac{4\mathcal{H}\dot{F}}{3(\mathcal{H}_0^2\Omega_{\rm m0})^2}\Psi_0$$
$$+ \frac{2}{9(\mathcal{H}_0^2\Omega_{\rm m0})^2}\left[(D^2+F)(\nabla^2\varphi_0)^2 + 2D^2\varphi_0^{,l}\nabla^2\varphi_{0,l} + (D^2-F)\varphi_{0,lm}\varphi_0^{,lm}\right]$$

Villa & Rampf JCAP (2016)

Transforming

to the synchronous-comoving gauge...

Space-time metric

$$g_{00_{\rm S}} = -a^2 \qquad g_{0i_{\rm S}} = 0$$

$$g_{ij_{\rm S}} = a^2\left\{\left(1 - \frac{10}{3}\varphi_{\rm in} + \frac{50}{9}a_{\rm nl}\varphi_{\rm in}^2 + \frac{10}{9\mathcal{H}_0^2\Omega_{\rm m0}}\frac{DD_{\rm in}}{a_{\rm in}}\varphi_{0,l}\varphi_0^{,l}\right)\delta_{ij} - \frac{4}{3}\frac{D}{\mathcal{H}_0^2\Omega_{\rm m0}}\varphi_{0,ij}\right.$$

$$+\frac{40}{9\mathcal{H}_0^2\Omega_{\rm m0}}\frac{DD_{\rm in}}{a_{\rm in}}\left[\left(a_{\rm nl}-\frac{3}{2}\right)\varphi_{0,i}\varphi_{0,j} + (a_{\rm nl}-1)\varphi_0\varphi_{0,ij}\right] + \frac{1}{2}\pi_{ij_{\rm S}}$$

$$\left.+\frac{4}{9(\mathcal{H}_0^2\Omega_{\rm m0})^2}\left[D^2\varphi_{0,ik}\varphi_{0,j}^{,k} - 2F\nabla^2\Psi_0\delta_{ij} - 4F(\varphi_{0,ij}\nabla^2\varphi_0 - \varphi_{0,il}\varphi_{0,j}^{,l})\right]\right\},$$

where the tensor $\pi_{ij_{\rm S}} = \frac{16F}{9(\mathcal{H}_0^2\Omega_{\rm m0})^2}\mathcal{S}_{ij} + \left(\frac{40}{9\mathcal{H}_0^2\Omega_{\rm m0}}Dg_{\rm in} + \frac{8\dot{D}^2}{9(\mathcal{H}_0^2\Omega_{\rm m0})^2}\right)\nabla^{-2}\mathcal{S}_{ij} + \chi_{2Pij}^{\rm GW}$

is the solution of the wave equation $\ddot{\pi}_{ij_{\rm S}} + 2\mathcal{H}\dot{\pi}_{ij_{\rm S}} - \nabla^2\pi_{ij_{\rm S}} = -\frac{16F}{9(\mathcal{H}_0^2\Omega_{\rm m0})^2}\nabla^2\mathcal{S}_{ij}$.

Matter density

$$\delta_{\rm S} = \frac{2}{3\mathcal{H}_0^2\Omega_{\rm m0}}D\nabla^2\varphi_0 + \frac{20DD_{\rm in}}{9a_{\rm in}\mathcal{H}_0^2\Omega_{\rm m0}}\left[\left(\frac{3}{4}-a_{\rm nl}\right)(\nabla\varphi_0)^2 + (2-a_{\rm nl})\varphi_0\nabla^2\varphi_0\right]$$

$$+\frac{2}{9(\mathcal{H}_0^2\Omega_{\rm m0})^2}\left[(D^2+F)(\nabla^2\varphi_0)^2 + (D^2-F)\varphi_{0,lm}\varphi_0^{,lm}\right]$$

Villa & Rampf JCAP (2016)

Applications

Our results

- We consider the gauges which represent the Eulerian and Lagrangian approach in GR
- We provide simple solutions with a clean identification of Newtonian and GR contributions up to second-order in PT

Applications: wherever you need GR II order

- bispectrum and non-Gaussianity
- gravitaional lensing

Galaxy number counts in GR

How much wrong is to use the Newtonian approximation to model what we measure in a galaxy survey?

LENSING CONVERGENCE IN GALAXY CLUSTERING IN ΛCDM AND BEYOND

Villa, Di Dio & Lepori JCAP **04** 033 (2018)

Theoretical systematics & precision cosmology

A theoretical systematic is a "unknown known"

- **we know** that we use some assumptions
- **we usually do not know** how much impact they have

Why important now? Precision cosmology!

- current and upcoming measurements aim at 1% accuracy
- theoretical modelling has to be accurate at the same level

Lensing as a theoretical systematic

In galaxy surveys we observe the number of galaxies in a redshift bin dz and a solid angle $d\Omega$ in our past light cone

It is perturbed

- in the radial direction → redshift-space distortions
- in the transversal direction → lensing

$$\Delta(\mathbf{n}, z) = b_g \delta + \frac{1}{\mathcal{H}(z)} \partial_r (\mathbf{V} \cdot \mathbf{n}) + (5s - 2) \int_0^{r(z)} \frac{r(z) - r}{2 r(z) r} \Delta_\Omega (\Phi + \Psi) dr$$

density & RSD → $z_i \sim z_j$ and small Δz

Lensing → $z_i \neq z_j$ and integrated

$\Delta(\mathbf{n}, z)$ is linear and gauge invariant

Our aims

What we want to understand the effects of neglecting lensing in galaxy clustering:

1. bias the constraints of the cosmological parameters?
2. bias the value of the cosmological parameters that we infer from these observations?

and

3. differences of the lensing effects for different surveys?

Galaxy surveys

Photometric survey: EUCLID-like

$$0.1 < z < 2$$
$$f_{\rm sky} = 0.375$$
$$N_{\rm bins} = \text{up to } 10$$

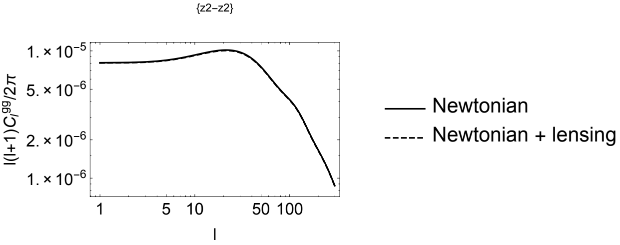
Spectroscopic survey: SKA II-like

$$0.1 < z < 2$$
$$f_{\rm sky} = 0.73$$
$$N_{\rm bins} = \sim 10^2$$

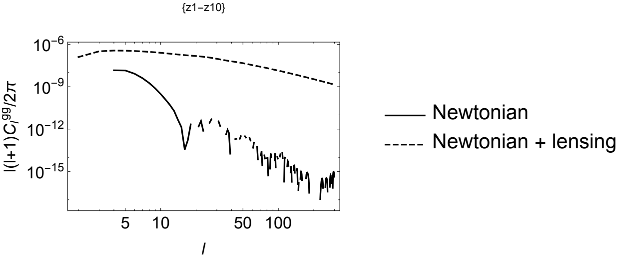
A deeper look into lensing: a spectroscopic survey

Nwt $\rightarrow z_i \sim z_j$ **and small** Δz
Lensing $\rightarrow z_i \neq z_j$ **and integrated**

Unknown known



$\Rightarrow$ the effect of lensing convergence in auto-correlations is negligible



$\Rightarrow$ far cross-bin correlations in galaxy clustering contain no information

Why (not) go beyond ΛCDM?

Three main reasons

1. GR works well, we know it! **But..**
   - only tested in the near Universe (solar system, BH and NS mergers)
   - cosmological observations probe a entirely new range of length and time scales
2. **so** are we allowed to assume GR on cosmological scales?
3. **and** what if we do not need Dark Energy but we use the wrong theory for gravity?

$$G^{MG}_{\mu\nu} = T_{\mu\nu} \qquad \text{instead of} \qquad G_{\mu\nu} = T_{\mu\nu} + T^{DE}_{\mu\nu} \quad ?$$

Result: much fun for theorists: 30+ popular theories

The easy way: parametrizing MG

- stay linear
- scalar perturbations

$$ds^2 = a^2(\tau)\left[-(1+2\Psi)\,d\tau^2 + (1-2\Phi)\,\delta_{ij}dx^i dx^j\right]$$

- modify

the relation between metric perturbations $\frac{\Phi}{\Psi} = \gamma(k, a)$

how they are sourced by the density $\nabla^2\Psi = \frac{3}{2}\frac{\mathcal{H}_0^2\Omega_{\mathrm{m0}}}{a}\mu(k, a)\Delta$

Pros:

- simple: just two functions to compare with data
- some MG theories can be parametrized this way

We will assume no scale and time dependence and

- $\mu \Rightarrow$ parametrize Ψ
- $\Sigma \equiv \frac{\mu(1+\gamma)}{2} \Rightarrow$ parametrize $\Phi + \Psi$

Spoiler: the correlation with the lensing parameter

How much sensitive a parameter is to lensing?

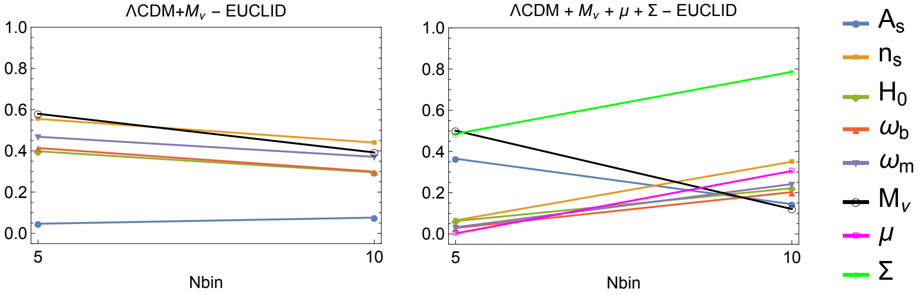
Write

$$\Delta = \Delta_g + \Delta_{rsd} + \epsilon_L \Delta_{lens}$$

Calculate the correlation between the cosmological parameters θ_α and ϵ_L

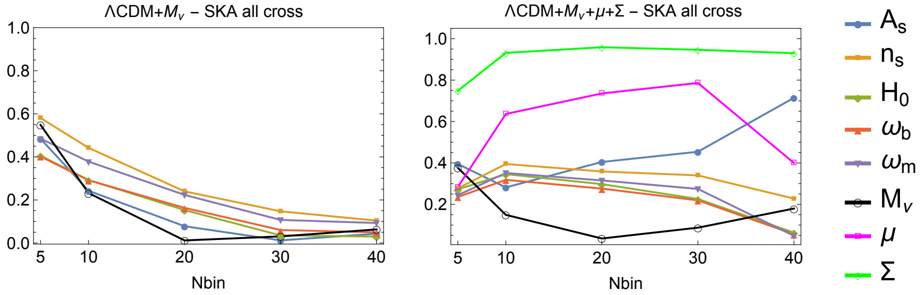
$$\rho_{\epsilon_L} = \frac{\left[\left(F^{\Phi\Phi}\right)^{-1}\right]_{\alpha\epsilon_L}}{\sqrt{\left[\left(F^{\Phi\Phi}\right)^{-1}\right]_{\alpha\alpha}\left[\left(F^{\Phi\Phi}\right)^{-1}\right]_{\epsilon_L\epsilon_L}}}$$

Lensing correlation: Euclid



Lensing correlation: SKA

All cross-bin correlations

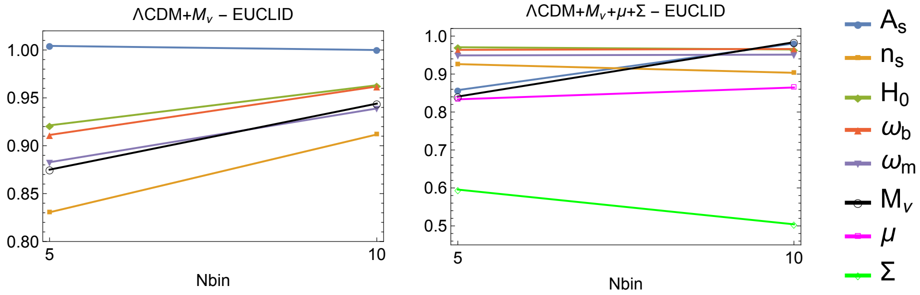


Bias in the constraints: aim and method

How much information is contained in the lensing convergence?

- compute the errors from $\Delta = \Delta_g + \Delta_{RSD}$
- compute the errors from $\Delta = \Delta_g + \Delta_{RSD} + \Delta_{lens}$
- compare

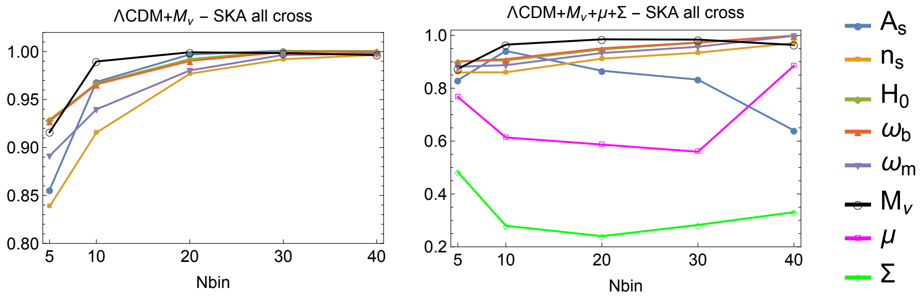
Results for EUCLID: bias in the constraints



σ **with lensing /** σ **without lensing**

Results for SKA: bias in the constraints

All cross-bin correlations



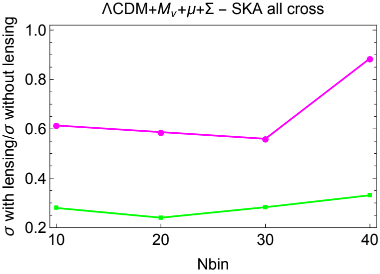
σ **with lensing /** σ **without lensing**

MG parameters: lensing vs RSD for SKA

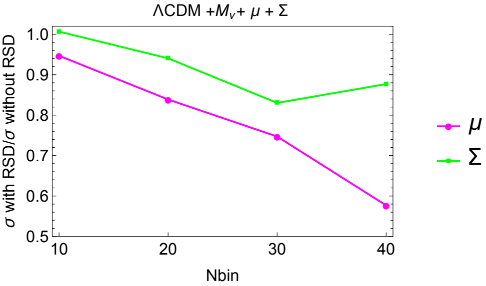
$$\Delta = \Delta_g + \Delta_{rsd} \text{ vs } \Delta = \Delta_g + \Delta_{rsd} + \Delta_{lens}$$

for Modified Gravity

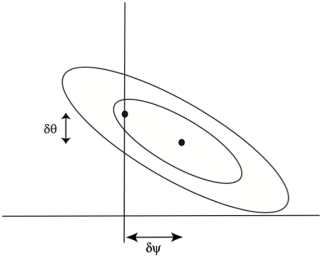
constraining power of lensing



constraining power of RSD



Lensing as a model selection problem: method



$$\{\theta_\alpha, \theta_\beta, ..., \psi = 1\}$$

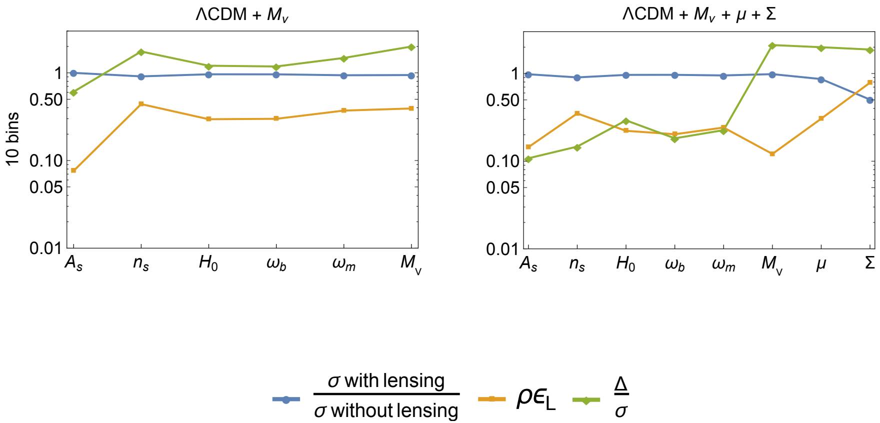
$$\{\theta_\alpha, \theta_\beta, ..., \psi = 0\}$$

- wrong model $\Delta = \Delta_g + \Delta_{rsd}$
- correct model $\Delta = \Delta_g + \Delta_{rsd} + \Delta_{lens}$

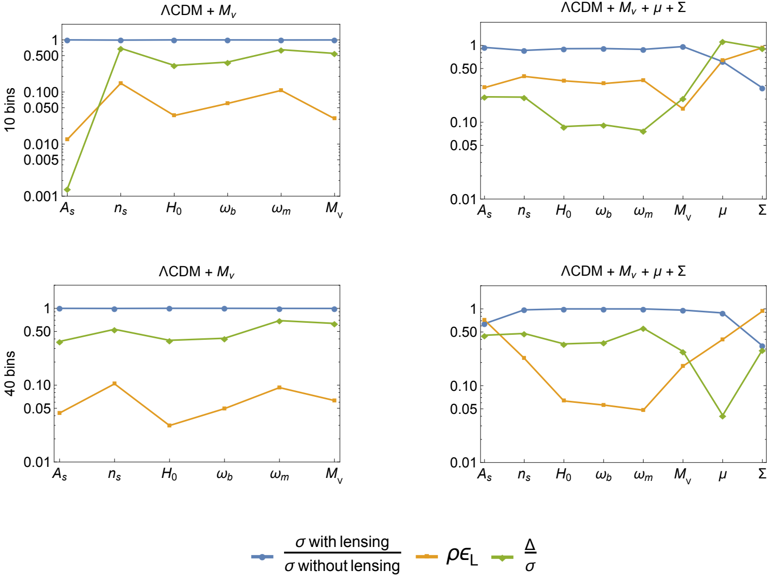
The shift in the best-fit value is

$$\delta\theta_\alpha = \sum_\beta \left(F^{\theta\theta}\right)^{-1}_{\alpha\beta} F^{\theta\epsilon_L}_\beta$$

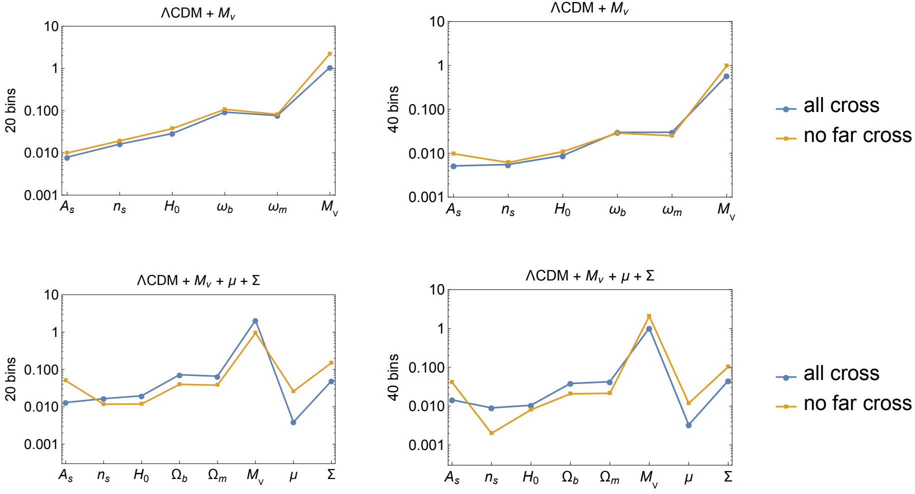
Results for EUCLID: shift



Results for SKA: shift vs N_{bins} - all cross-bin correlations



Results for SKA: all cross vs no far cross



Take-home messages

- it is crucial to include lensing convergence in galaxy clustering analyses if one wants to test GR and modifications
- for photometric surveys the estimation of cosmological parameters is biased, in particular for the modified gravity parameters
- the information contained in far-bin cross-correlations is sub-dominant