Recalculating Freeze-In

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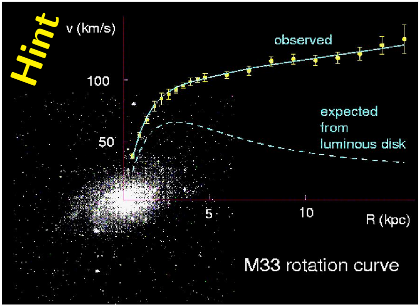
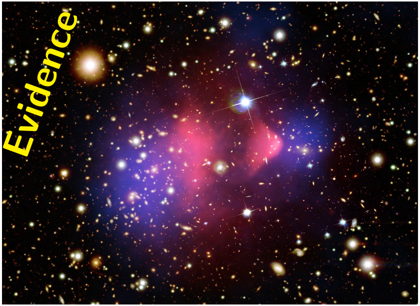
Outline

Dark Matter

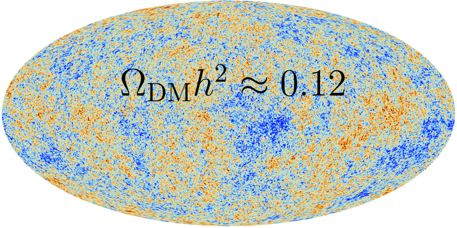
Why Particle Dark Matter



E. Corbelli and P. Salucci, *Mon. Not. Roy. Astron. Soc.* **311** *441* (2000), arXiv:astro-ph/9909252.



M. Markevitch, ESA Spec. Publ. **604** (2006) 723, astro-ph/0511345.Clowe, Bradac, *et. al.* Astrophys. J. **648**, L109 (2006), astro-ph/0608407



$$\Omega_{\rm DM} h^2 \approx 0.12$$

N. Aghanim *et al.* [Planck Collaboration], arXiv:1807.06209 [astro-ph.CO].

The Dark Matter Particle

“Ἓν οἶδα, ὅτι οὐδὲν οἶδα.”
“I know one thing, that I know nothing.”

–Socrates

- Gravitational interactions.
- Mostly electrically neutral.
- Stable or very slow decay rate.
- Non-Baryonic.
- Cold/Warm and non-relativistic today.

Freeze-in

The Freeze-in mechanism

Dark Matter production via Freeze-in: [1]

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[1]

J. R. Ellis, J. E. Kim and D. V. Nanopoulos, Phys. Lett. **145B**, 181 (1984).

L. Covi, H. B. Kim, J. E. Kim and L. Roszkowski, JHEP **0105**, 033 (2001), hep-ph/0101009

J. McDonald, Phys. Rev. Lett. **88**, 091304 (2002) , hep-ph/0106249

L. J. Hall, K. Jedamzik, J. March-Russell and S. M. West, JHEP **1003**, 080 (2010), arXiv:0911.1120 [hep-ph]

The Freeze-in mechanism

Dark Matter production via Freeze-in: [1]

- Dark Matter particle absent and out of thermal equilibrium.

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The Freeze-in mechanism

Dark Matter production via Freeze-in: [1]

- Dark Matter particle absent and out of thermal equilibrium.
- Dark Matter particle produced from decays or annihilations of plasma particles.

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[1]

J. R. Ellis, J. E. Kim and D. V. Nanopoulos, Phys. Lett. **145B**, 181 (1984).

L. Covi, H. B. Kim, J. E. Kim and L. Roszkowski, JHEP **0105**, 033 (2001), hep-ph/0101009

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The Boltzmann equation

Usually we are dealing with $\mathrm{Bath}\ \mathrm{Bath} \to \mathrm{DM}\ \mathrm{DM}$ and $\mathrm{Bath} \to \mathrm{DM}\ \mathrm{DM}$ processes. Since the DM number density **never** reaches the level of the plasma ones, $f_{DM} \ll f_\zeta(p)$. So,

$$\frac{dn_{\mathrm{DM}}}{dt} - 3Hn_{\mathrm{DM}} = \int \left(\prod_{i=1}^{2} \frac{1}{(2\pi)^3} \frac{d^3p_i}{2E_i}\ f_{\zeta_i}(p_i) \right) (2E_1\, 2E_2\, \sigma v_{\mathrm{rel}}) + \\ + 2\, m\, \Gamma^{(\mathrm{CM})} \int \frac{1}{(2\pi)^3} \frac{d^3p}{2E}\ f_\zeta(p)\ ,$$

where $f_\zeta(p) = \dfrac{1}{e^{E/T} - \zeta}$ with $\zeta = -1\,(+1)$ for fermions (bosons), and E the energy of plasma particles in the rest frame of the cosmic fluid.

Standard Freeze-in

To solve the Boltzmann, we usually make several assumptions and simplifications:

- Ignore quantum statistics, $f_\zeta(p) \approx f_0(p) = e^{-E/T}$.
- Radiation dominated expansion, $H = \sqrt{\frac{4\pi^3}{45} g_{\text{eff}}} \frac{T^2}{M_p}$.
- Entropy conservation, $\dot{s} + 3Hs = 0$.

The Boltzmann equation becomes ($Y_{\text{DM}} = n_{\text{DM}}/s$)

$$\frac{dY_{\text{DM}}}{dT} = -\frac{\delta_h}{HsT}\left[\frac{m^2 T}{2\pi^2}\,\Gamma^{(\text{CM})} K_1(m/T) + \right.$$
$$\left. \frac{T}{2(2\pi)^4}\int_{4m_{\text{DM}}^2}^{\infty} d\hat{s}\ \hat{s}\ \sqrt{\hat{s} - 4m_{\text{DM}}^2}\ \sigma v_{\text{rel}}\ K_1(\sqrt{\hat{s}}/T)\right],$$

with $\delta_h = 1 + 1/3 \frac{T}{h_{\text{eff}}} \frac{dh_{\text{eff}}}{dT}$.

Ignoring thermal masses for the moment (another usual simplification), the decay is already parametrized in a model independent fashion. What about pair annihilation?

Standard Freeze-in: Pair annihilation

For a model-independent parametrization of the $2 \to 2$ process we may assume $|\mathcal{M}|^2 = \gamma^2 \left(\frac{\hat{s}}{\Lambda^2}\right)^{d-4}$, which holds at high energy (*i.e.* ignoring all masses). The Boltzmann equation becomes

$$\frac{dY_{\rm DM}}{dT} = -\frac{45\ 4^d \gamma^2}{(4\pi)^7}\sqrt{\frac{45}{4\pi^3}}\left(\frac{\delta_h}{\sqrt{g_{\rm eff}}h_{\rm eff}}\right)\left(\frac{M_p T^{2(d-5)}}{\Lambda^{2(d-4)}}\right) I_{00}\ .$$

where $I_{00} = 2\int_0^\infty dt\, t^{2(d-4)+2}\int_1^\infty dx\,\sqrt{x^2-1}\,e^{-2\,t\,x}$.
Integrating from $T_{\rm RH}$ to the temperature where production stops (model dependent),we get the yield today

$$Y_{\rm DM,0} = \frac{45\ 4^d \gamma^2}{(4\pi)^7}\sqrt{\frac{45}{4\pi^3}}\left(\frac{\delta_h}{\sqrt{g_{\rm eff}}h_{\rm eff}}\right)\left(\frac{T_{\rm RH}^{2d-9} - T_0^{2d-9}}{(2d-9)\Lambda^{2d-9}}\right)\left(\frac{M_p}{\Lambda}\right) I_{00}.$$

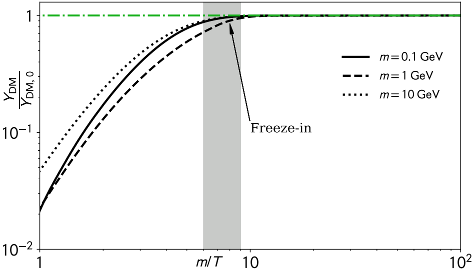
Two freeze-in cases:
IR freeze-in $(2d-9<0)$, *i.e.* production dominated at low energy.
UV freeze-in $(2d-9>0)$, *i.e.* production dominated at high energy.

IR and UV Freeze-in

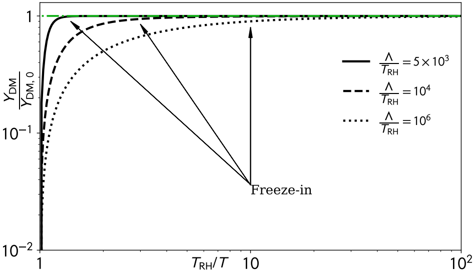
Consider χ the Dark Matter particle, and S in equilibrium with the plasma.

$$\mathcal{L}_{\rm int} = -yS\bar{\chi}\chi$$

$$\mathcal{L}_{\rm int} = -\frac{1}{\Lambda}SS\bar{\chi}\chi$$



IR



UV

Quantum statistics and Freeze-in

The Boltzmann equation for pair annihilations

Including the quantum statistical factors, the Boltzmann equation for $2 \to 2$ production channel is [2]

$$\frac{dY_{\rm DM}}{dT} = -\frac{\delta_h}{H\,s\,T} \int \frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} \frac{2E_1\,2E_2\,\sigma\,v_{\rm rel}}{\left(e^{u\cdot p_1/T}-\zeta_1\right)\left(e^{u\cdot p_2/T}-\zeta_2\right)}\ .$$

The $\zeta_{1,2}=0$ is valid for non-relativistic particles (typically low temperatures), but freeze-in can happen when particles are relativistic (especially in the UV case).

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[2] The importance of such factors for freeze-out was pointed out by G. Arcadi, O. Lebedev, S. Pokorski and T. Toma, JHEP **1908**, 050 (2019) [arXiv:1906.07659], while for freeze-in by O. Lebedev and T. Toma, Phys Let. **798B**, 134961 (2019) [arXiv:1908.05491]. Both papers introduce also a formalism that makes it relatively straightforward to apply in general. In principle micrOMEGAs 5+ (G. Bélanger, F. Boudjema, A. Goudelis, A. Pukhov and B. Zaldivar, Comput. Phys. Commun. **231** (2018) 173 [arXiv:1801.03509]) can calculate this without the inclusion of thermal masses for the moment.

Impact on the pair annihilation production

Assume a d-dimensional operator responsible for this process (since this is a high energy approximation, $m \approx \alpha T$). Then we get

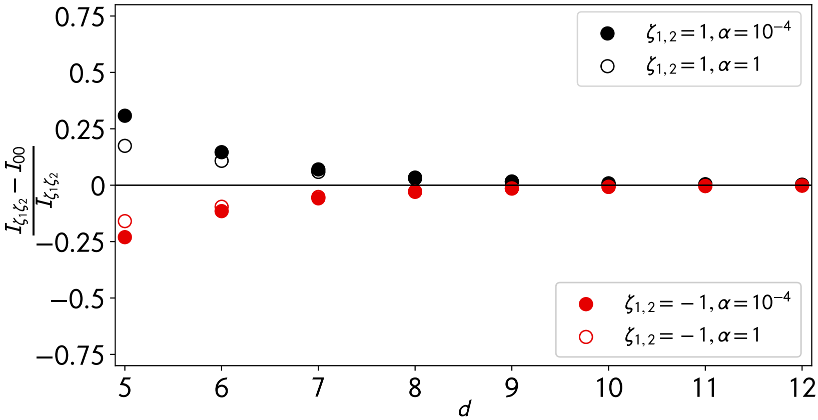
$$Y_{\rm DM,0} = \frac{45\ 4^d \gamma^2}{(4\pi)^7}\sqrt{\frac{45}{4\pi^3}}\left(\frac{\delta_h}{\sqrt{g_{\rm eff}}h_{\rm eff}}\right)\left(\frac{T_{\rm RH}^{2d-9} - T_0^{2d-9}}{(2d-9)\Lambda^{2d-9}}\right)\left(\frac{M_p}{\Lambda}\right)\ I_{\zeta_1\zeta_2}\ ,$$

The only difference between this the $\zeta_{1,2} = 0$ approach comes from

$$I_{\zeta_1,\zeta_2} = \int_\alpha^\infty dt\, t^{2d-6}\int_1^\infty dx\frac{\log\left[\frac{\left[e^{k_+} - \zeta_1\right]\left[e^{k_+} - \zeta_2\right]}{\left[e^{k_-} - \zeta_1\right]\left[e^{k_-} - \zeta_2\right]}\right]}{\left(e^{2t\,x} - \zeta_1\zeta_2\right)}\ ,$$

with $k_\pm = t\,x \pm \sqrt{t^2 - \alpha^2}\,\sqrt{x^2 - 1}$.

Impact on the pair annihilation production



Production due to boson (fermion) annihilation *underestimates* (*overestimates*) the relic abundance of DM.

A model

Consider the case of neutrino frozen-in DM with the neutrino masses being produced from the VEV of a Majoron-like particle, S. [3] Lightest sterile neutrino, ν, is a DM candidate with

$$-\Delta\mathcal{L} = \frac{\lambda}{2} s\,\nu\nu\ .$$

Focusing on the non-thermal S, with $S \to s + v_s$ (S is at zero temperature). That is, we assume absence of s in the early Universe, produced via:

- Pair annihilation $hh \to ss$ (always active).
- Fusion $hh \to s$ (active for $2m_h < m_s$).
- Decay $h \to ss$ (active for $m_h > 2m_s$).

---

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$$Y_{\rm DM} \approx 2BR_{s\to\nu\nu} Y_s\ .$$

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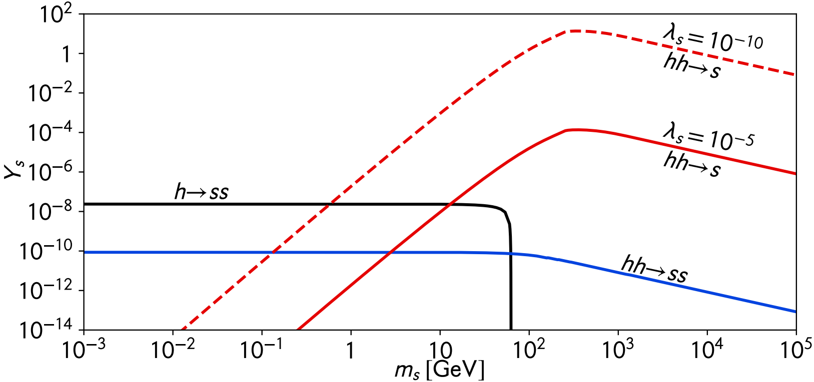
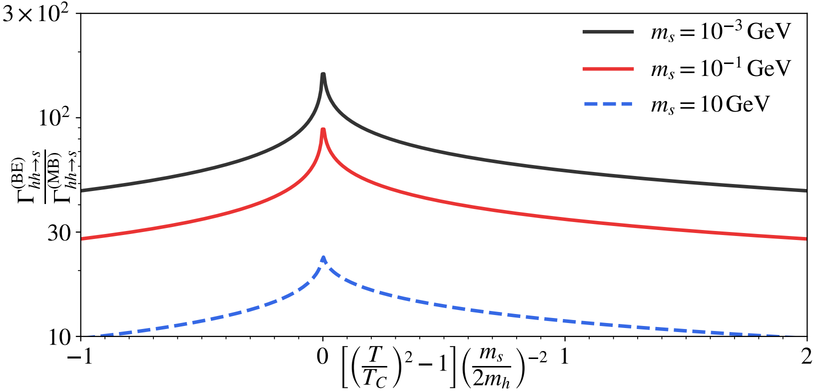
Majoron production



Production via $hh \to s$ can easily dominate the production!

Example: relativistic effects on fusion

For $m_s \ll 2m_{h,0}$, fusion happens at $T \approx T_C$ where the Higgs is almost massless.



Forbidden Freeze-in

Production via Forbidden Freeze-in

Forbidden freeze-in: Plasma particles produce Dark Matter via kinematically forbidden decays. [4]

Assuming $\Gamma = \frac{\gamma^2}{8\pi} m \left(\frac{m}{\Lambda}\right)^{2(d-4)}$ and $m = \alpha T$, the yield of DM becomes

$$Y_{\mathrm{DM},0} = \frac{45\left(\gamma\alpha^{d-2}\right)^2}{(2\pi)^6}\sqrt{\frac{45}{4\pi}}\left(\frac{\delta_h}{\sqrt{g_{\mathrm{eff}}}h_{\mathrm{eff}}}\right)\left(\frac{T_{\mathrm{RH}}^{2d-9}-T_0^{2d-9}}{\Lambda^{2d-9}(2d-9)}\right)\left(\frac{M_p}{\Lambda}\right)I_\zeta\ ,$$

with $T_0 = \dfrac{2m_{\mathrm{DM}}}{\alpha}$, and $I_\zeta = \int_0^\infty dx \dfrac{\left(x^2-1\right)^{1/2}}{e^{\alpha x}-\zeta}$. Similar behavior as in the $2 \to 2$ case.

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[4] Behavior was noted in:

V. S. Rychkov and A. Strumia, Phys. Rev. D **75**, 075011 (2007), hep-ph/0701104

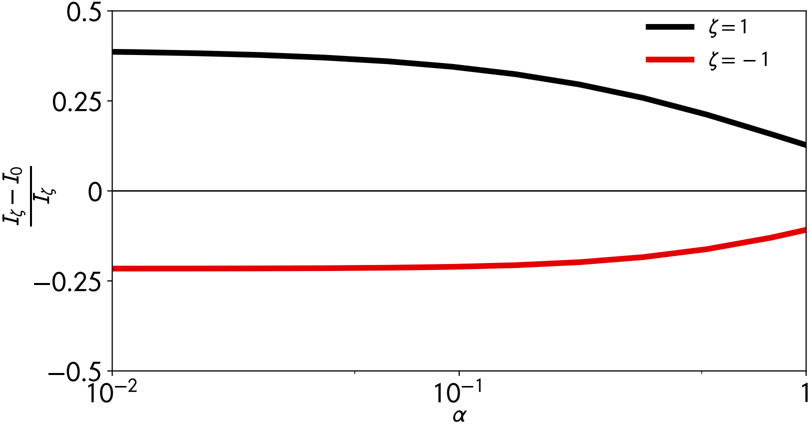
A. Strumia, JHEP **1006**, 036 (2010), arXiv:1003.5847 [hep-ph].

M. J. Baker, M. Breitbach, J. Kopp and L. Mittnacht, JHEP **1803**, 114 (2018), arXiv:1712.03962 [hep-ph].

L. Bian and Y. L. Tang, JHEP **1812**, 006 (2018), arXiv:1810.03172 [hep-ph].

Forbidden freeze-in general treatment: L. Darmé, A. Hryczuk, DK, L. Roszkowski, arXiv:1908.05685

Impact of quantum statistics



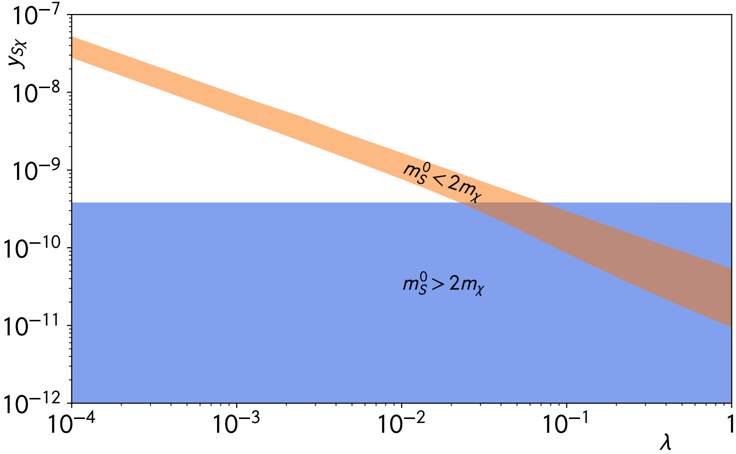
The situation is a bit better than the pair annihilation case. However, production due to boson (fermion) annihilation *underestimates* (*overestimates*) the relic abundance of DM.

Standard vs Forbidden Freeze-in

Comparing Freeze-in regimes (with $\zeta = 0$).

$$\mathcal{L} \supset -y_{S\chi}\; S\,\bar{\chi}\chi\; - \frac{\lambda}{4!}S^4 - \frac{1}{2}m_S^{0\,2}S^2 - m_\chi\bar{\chi}\chi\; + (\mathrm{SH-terms}),$$

$$m_S^2(T) \approx m_S^{0\,2} + \frac{\lambda}{24}T^2\ .$$



Non-standard cosmological history

Boltzmann equations

Assume there a fluid (Φ) that at some point dominates the energy density of the Universe, and later decay to plasma. For simplicity, assume that a plasma particle (S) decays to DM (IR freeze-in fo the moment). The Boltzmann equations we have to solve are

$$\begin{aligned} \frac{d\rho_\Phi}{dt} &= -3(1+w)H\rho_\Phi - \Gamma\rho_\Phi \\ \frac{ds}{dt} &= -3Hs + \frac{\Gamma}{T}\rho_\Phi - \frac{\Gamma_\chi}{T}\rho_S \\ \frac{dn_{\rm DM}}{dt} &= -3Hn_{\rm DM} + \ m_S^3 \ \frac{\Gamma_\chi}{2\pi^2} \ \int_1^\infty dt \frac{\left(t^2-1\right)^{1/2}}{e^{m_S/T \ t} \pm 1} \ . \end{aligned}$$

To avoid BBN constrains the fluid must decay away at some $T_{\rm END} \gtrsim 1 \ {\rm MeV}$ (wcich puts a bound on Γ). Since DM is not thermalized, $\Gamma_\chi \ll \Gamma, H$.

How DM freeze-in is affected? [5]

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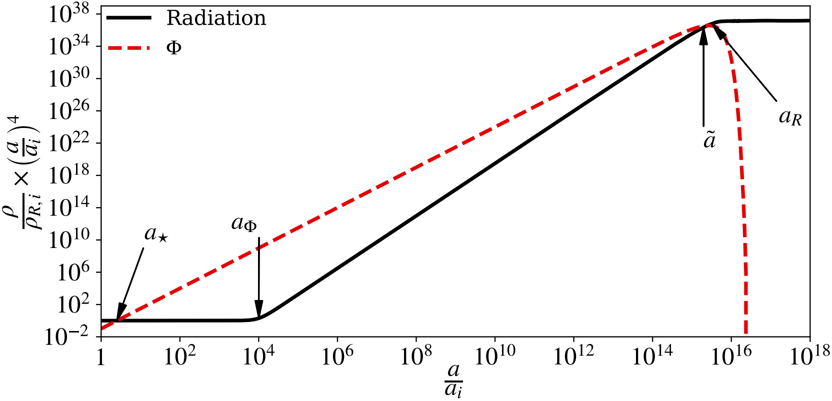
[5] There is an extensive discussion on freeze-in production of DM during such cases, such as . J. H. Chung, E. W. Kolb and A. Riotto, Phys. Rev. D **60** (1999) 063504[hep-ph/9809453], M. Drees and F. Hajkarim, JCAP **1802**, 057 (2018) [arXiv:1711.05007]., F. D'Eramo, N. Fernandez and S. Profumo, JCAP **1802**, 046 (2018) [arXiv:1712.07453], N. Bernal, F. Elahi, C. Maldonado and J. Unwin, [arXiv:1909.07992], and many others. P. Arias, DK, L. Roszkowski [work in progress...].

The radiation-fluid system

Assuming $3(1+w) < 4$ (possible early radiation domination), we may solve just the radiation-Φ-DM system. [6]



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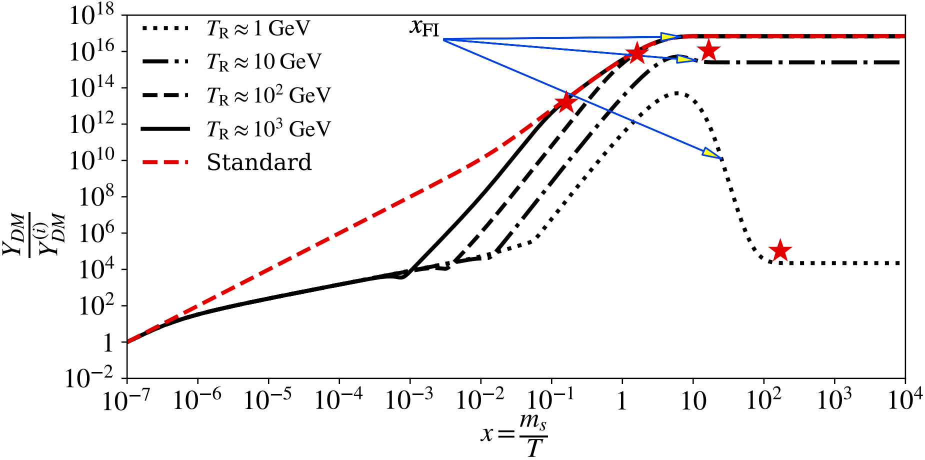
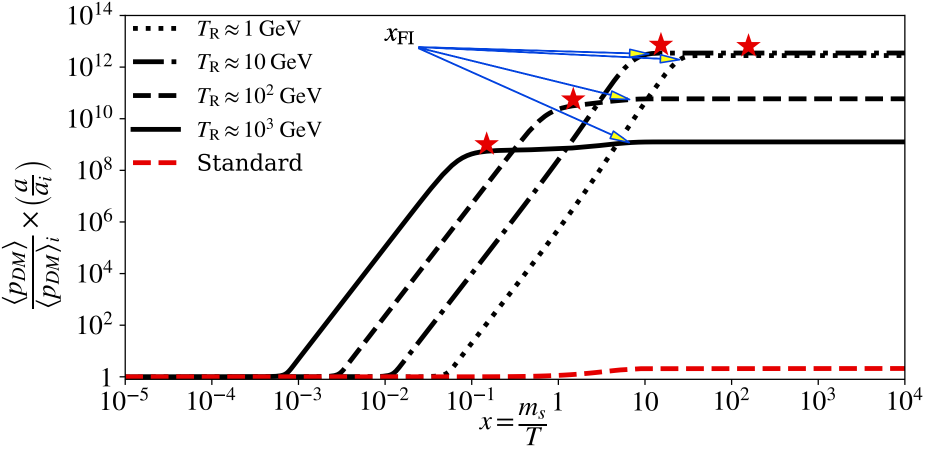
[6] To solve the system of BEs, we employ the ODE solver NaBBODES build by DK.

The freeze-in



The DM momentum



Summary

Summary

- Quantum statistics
  - Relativistic freeze-in can happen in (almost) any model.
  - Standard approach typically underestimates (overestimates) the relic for production from bosons (fermions).
- Forbidden channels
  - It should be studied, since plasma particles develop thermal masses.
  - Opens-up distinct parameter space (typically larger couplings than the standard freeze-in).
  - Re-examine models?
- Non-standard expansion
  - Freeze-in happens typically at high temperatures (unknown physics may exist).
  - The result is a different scaling of DM production.
  - Possibility for larger couplings without over-closing the Universe.
  - What's up with the momentum?

Thank you!