T70 Group - Theoretical Physics of the Early Universe
Physics Department
Technische Universität München

Gradient effects on false vacuum decay in gauge theory

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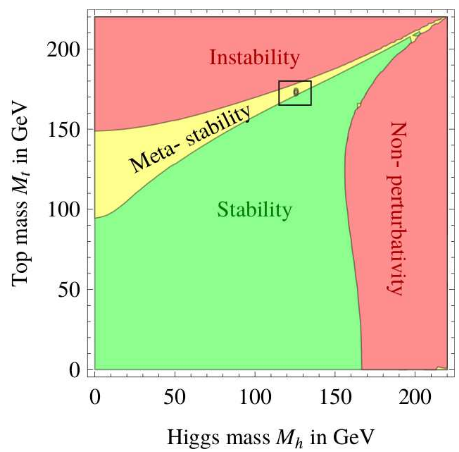
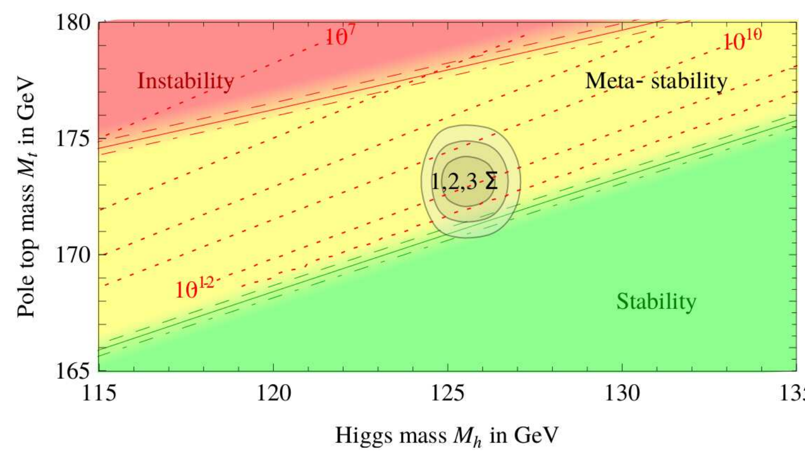
1. Motivation

Current measurements of the masses of the Higgs and the top quark, place our universe in a metastable region of the Higgs field potential

[Degrassi et al., 2012]
[Buttazzo et al., 2013]





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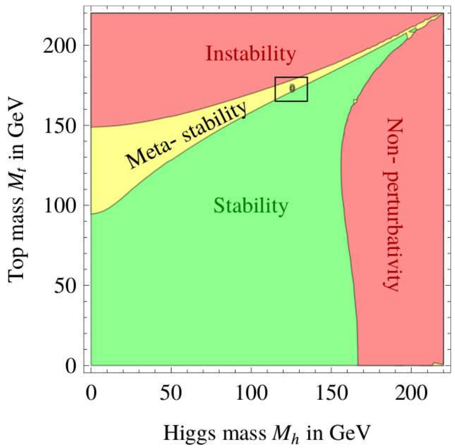

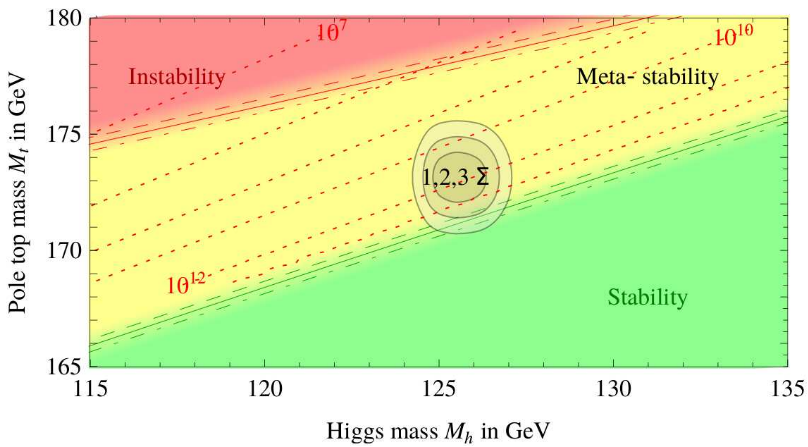

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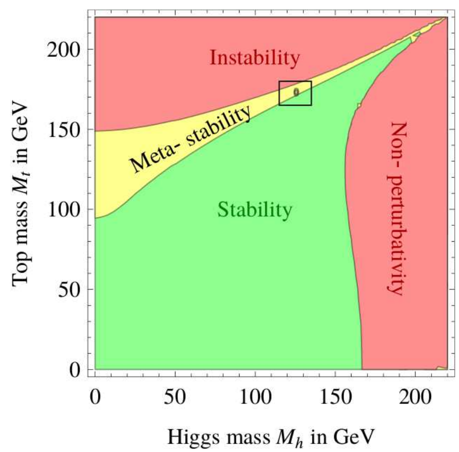

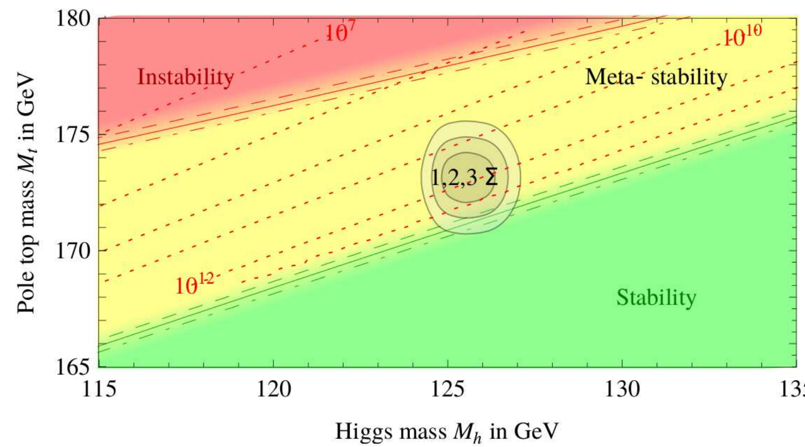

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Tachyonic regions and gradient effects are not well described by a Coleman-Weinberg potential and a new method must be sought.

2.1. Background

Tunneling Phenomena

- Energetically forbidden regions are accessed in QM through tunneling.
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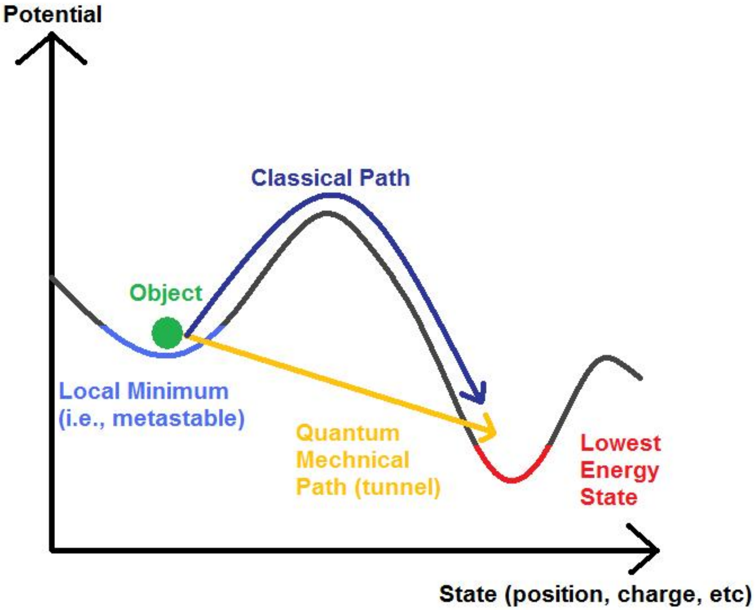
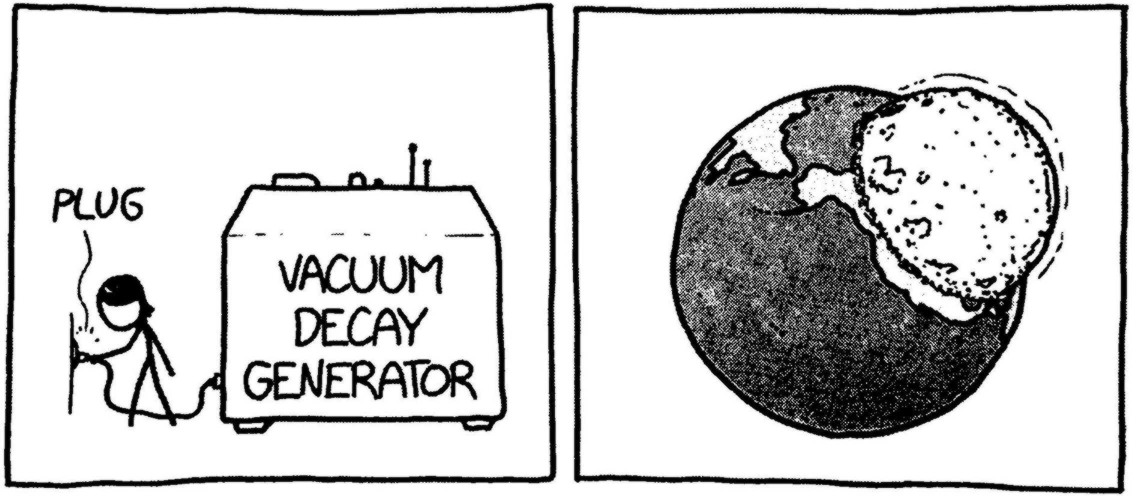


Vacuum state decay

- The expectation value of a field at tree-level corresponds to a local minimum.
- A theory with more than one local minimum presents different vacuum sectors.
- They can be connected by specific Euclidean solutions of the classical e.o.m..

2.2. Background

How to produce true vacuum?



Found in “How To: Absurd Scientific Advice for Common Real-World Problems”
by Randall Munroe author of XKCD.com Comics

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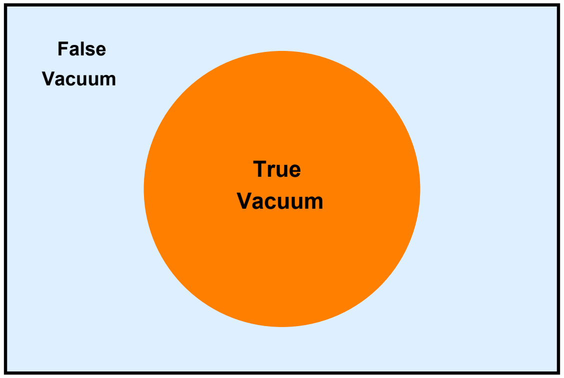
A non-homogeneous background called the bounce, denoted by $\varphi_b(r)$, is used subject to a scalar potential able to nucleate bubbles.



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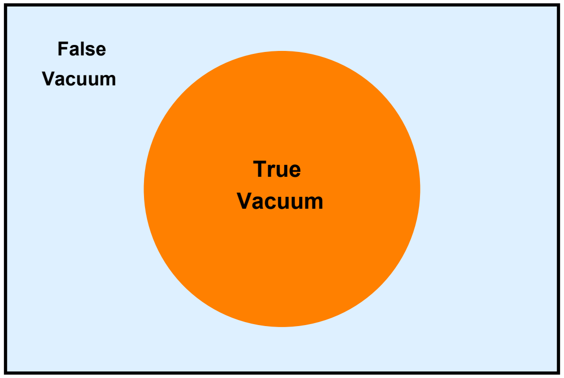

[Lee and Weinberg, 1986]
[Andreassen et al., 2017]

Gradients play a role in tunneling situations, which most treatments do not account for.

Different cases such as other geometries, EW phase transition and scale invariant potentials, etc. have been treated

[Baacke, 1990], [Baacke and Junker, 1994], [Sürig, 1998], [Garbrecht and Millington, 2015], [Garbrecht and Millington, 2017], [Ai et al., 2018].

3. Previous Studies

Semi-classical field theory

Approximate the euclidean vacuum to vacuum transition amplitude for a theory,

$$Z[0] = \langle \varphi_+ | e^{-HT/\hbar} | \varphi_+ \rangle = \mathscr{N} \int \mathscr{D}\phi \, e^{-S_E[\phi]},$$

using the extremal point of the action, which satisfies the boundary conditions at hand.

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Effective action in the standard model

Some have investigated conditions for vacuum stability.

Others assume a metastable situation and compute the decay rates to use as bounds

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There is a relation between the decay rate, $\mathit{\Gamma}$, and the effective action, $\Gamma^{(n)}$, given by

[Garbrecht and Millington, 2015]

$$\mathit{\Gamma}/V = 2\left|\mathfrak{Im}\, \mathrm{e}^{-\Gamma^{(n)}[\varphi^{(n)}]/\hbar}\right|/VT$$

In our study

$$\Gamma^{(1)}[\varphi^{(1)}] = B + \hbar B^{(1)} + \hbar^2 B^{(2)}$$

4. The Questions

Are quantities computed using effective potentials gauge independent?

[Jackiw, 1974]
[Andreassen et al., 2014]

The effective potential is generally not gauge-independent, unless the quantities rely solely on extremal points.

(see [Nielsen, 1975, Aitchison and Fraser, 1984], [Metaxas and Weinberg, 1996, Lalak et al., 2016], [Endo et al., 2017, Plascencia and Tamarit, 2016].)

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- **How to deal with gauge fields using these methods?**
- **How big are the contributions to the rate coming from radiative corrections and gradient effects?**
- **Are higher loop corrections to decay rates over inhomogeneous backgrounds gauge dependent?**

5. Our toy model for a $U(1)$

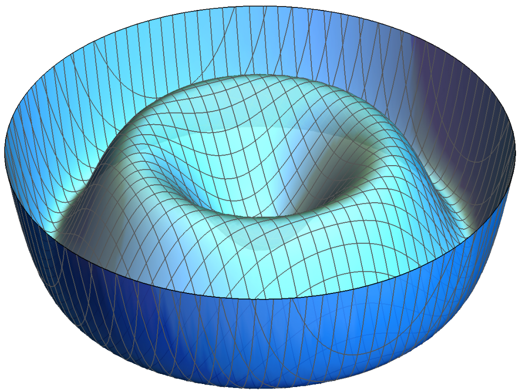
Consider the following Lagrangian in Euclidean space-time

$$\mathscr{L}_E = \frac{1}{4} F_{\mu\nu}F_{\mu\nu} + (D_\mu \phi)^*(D_\mu \phi) + U(\phi^*\phi) + \mathscr{L}_{\text{G.F.}} + \mathscr{L}_{\text{ghost}},$$

where

$$U(\phi^*\phi) = \alpha(\phi^*\phi) + \lambda(\phi^*\phi)^2 + \lambda_6(\phi^*\phi)^3,$$

$$\mathscr{L}_{\text{G.F.}} = \frac{1}{2\xi}(\partial_\mu A_\mu - \zeta\, g\, \phi\, G)^2,$$



and leads to the partition function

$$Z[J, K_\mu, \bar{\psi}, \psi] = \int \mathscr{D}[\phi, A_\mu, \eta, \bar{\eta}] \exp\left(-\frac{1}{\hbar} \int \mathrm{d}^4x \left[\mathscr{L}_E - J(x)\phi(x) - K_\mu(x) A_\mu(x) - \bar{\psi}(x)\eta(x) - \bar{\eta}(x)\psi(x) \right] \right).$$

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The field configuration $\varphi_b(r)$ is obtained from the classical e.o.m. by looking at $O(4)$ invariant solutions connecting the minima.

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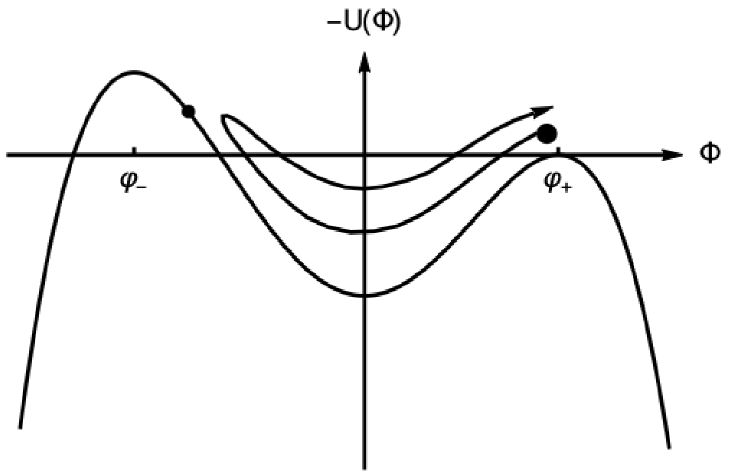

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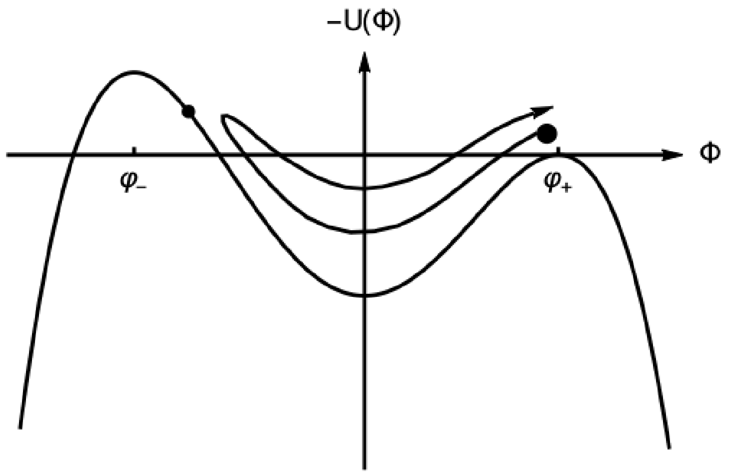

In a thin wall approximation the dissipative term is ignored, with $r^2 = \mathbf{x}^2 + x_4^2$ one has:

$$-\frac{\mathrm{d}^2\varphi}{\mathrm{d}r^2} - \frac{3}{r}\frac{\mathrm{d}\varphi}{\mathrm{d}r} + U'(\varphi) = 0,$$

alternatively in Hyper-spherical harmonics' terms we keep the lower angular momentum harmonics.

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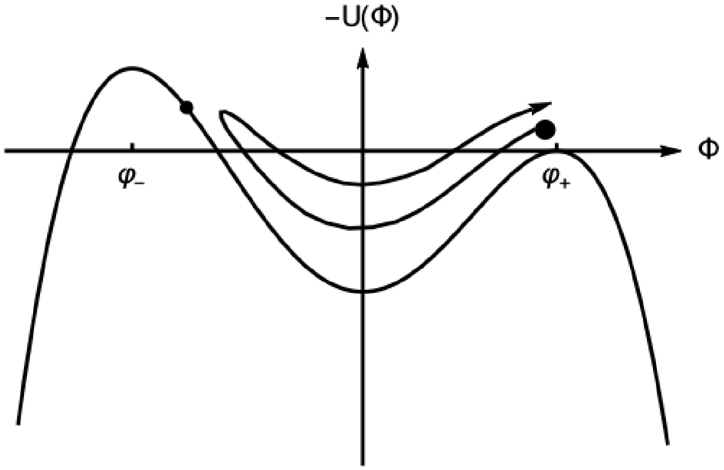

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We perform a semi-classical expansion for the theory by expanding around φ_b, e.g. making the substitution:

$$\phi = \frac{1}{\sqrt{2}}(\varphi_b + \hat{\Phi} + iG).$$

7. 1-PI Effective action

The computation of the 1-PI effective action formally gives the expression,

$$\Gamma^{(1)}[\varphi^{(1)}] = S[\varphi^{(1)}] + \frac{\hbar}{2}\log\frac{\det\mathscr{M}_{\hat{\Phi}}^{-1}(\varphi^{(1)})}{\det\mathscr{M}_{\hat{\Phi}}^{-1}(0)} + \frac{\hbar}{2}\log\frac{\det\mathscr{M}_{(A_\mu,G)}^{-1}(\varphi^{(1)})}{\det\mathscr{M}_{(A_\mu,G)}^{-1}(0)} - \hbar\log\frac{\det\mathscr{M}_{(\bar{\eta},\eta)}^{-1}(\varphi^{(1)})}{\det\mathscr{M}_{(\bar{\eta},\eta)}^{-1}(0)},$$

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where the operators are:

$$\mathscr{M}^{-1}_{\hat{\Phi}}(\varphi^{(1)}) = -\Delta + \alpha + 3\lambda\,(\varphi^{(1)})^2 + \frac{15\,\lambda_6}{4}(\varphi^{(1)})^4\,,$$

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Obs: When evaluated at constant field values, it reduces to the Coleman-Weinberg potential (CW) times a volume factor.

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aka the IDEAL Plan

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2. Solve for the tree-level Green's functions in position space

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4. The tadpole function for a field ϕ on the background φ_b,
$$\Pi(\varphi_b; x)\varphi_b(x) = \frac{\delta}{\delta\varphi(x)} \log \frac{\det M^{-1}_{\varphi_b}[\varphi(x)]}{\det M^{-1}_{\varphi_b}[0]}$$
gives the corrections to the bounce, $\delta\varphi$, according to
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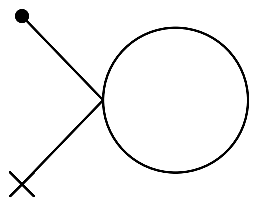
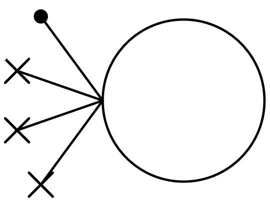
5. Substituting $\delta\varphi$ back into the action yields quadratic corrections to the decay rate.

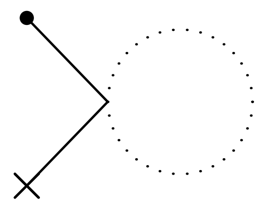
9. The slide for Feynman fans



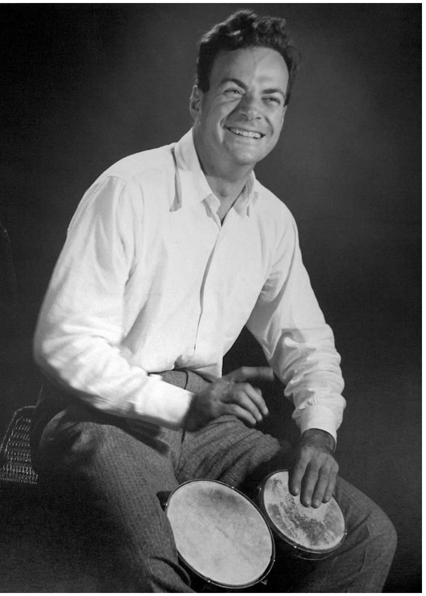
(a) First type of contributions from $\hat{\Phi}$.

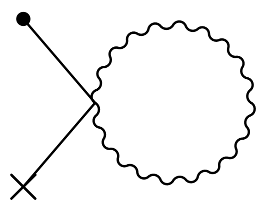


(b) Second tadpole type contribution from $\hat{\Phi}$

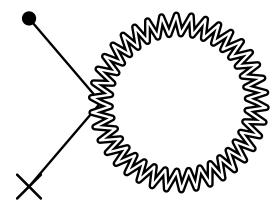


(c) Tadpole contribution coming from the ghost fields $\bar{\eta}, \eta$.

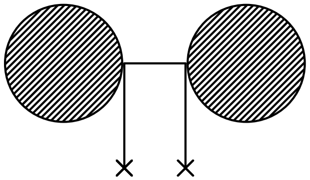




(d) Tadpole contributions from the gauge field components A_1, A_2, A_3 parallel to the bubble wall.



(e) Tadpole contribution coming from the coupled sector of A_4, G.

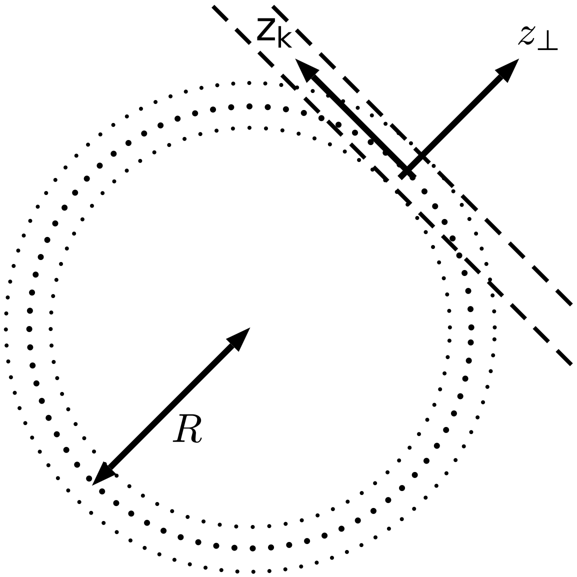


(f) Dumbbell (Bongo) diagram corresponding to the $B^{(2)}$ corrections coming from the improvements to the initial bounce.

× represents background insertions

● represents fluctuations around φ_b

10. Simplifications and Assumptions



Planar Wall (3 + 1 Decomposition)

Fourier transform the tangential directions to the bubble wall. This allows us to express Green's functions as sums over 3-momentum.

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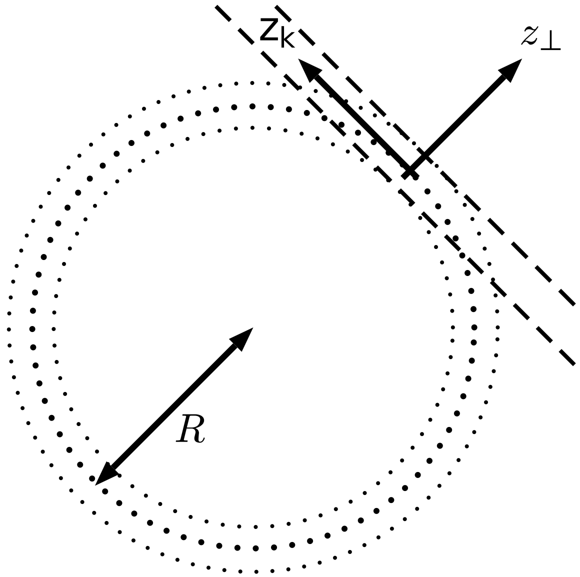

Planar Wall (3 + 1 Decomposition)

Fourier transform the tangential directions to the bubble wall. This allows us to express Green's functions as sums over 3-momentum.

Handle the easy Gauge first

$\xi = \zeta = 1$ decouples most of the components of A_μ so only A_4 and the Goldstone boson G are coupled.

11.1. Quest for a solution I: The easy components

Compactified radial coordinate $z \in (-\infty, \infty)$ to $u \in [-1, 1]$ and scan over u', to obtain the 2-point Green's function.

For every u' a splitting into an increasing and decreasing function is made:

$$G(u, u') = \Theta(u - u')G^R(u) + \Theta(u' - u)G^L(u)$$

and properly accounting for the gluing conditions, the system is solved numerically.

For the coupled block (A_4, G)

We have an extra parameter, **k**, for $|\mathbf{k}| \lesssim 2g\partial\varphi$ the gradients dominate the fluctuation operator and the system must be solved numerically using a matrix version of the splitting above. Now 8 functions to be found.

11.2. Quest for a solution II: larger $|\mathbf{k}|$

Iterative treatment to obtain the Green's functions

Split the operator into a diagonal and off-diagonal part,

$$\mathscr{M}^{-1}_{(A_4,G);\mathbf{k}}(\varphi_b;z) = \mathscr{M}_0^{-1}(z) + \delta\mathscr{M}^{-1}(z) = \begin{pmatrix} M_{\mathbf{k}}^{-1}(\varphi_b(z)) & 0 \\ 0 & N_{\mathbf{k}}^{-1}(\varphi_b(z)) \end{pmatrix} + \begin{pmatrix} 0 & 2g(\partial_z\varphi_b) \\ 2g(\partial_z\varphi_b) & 0 \end{pmatrix},$$

iterate over the solution to the diagonal part to include gradient corrections to all orders.

$$\mathscr{M}^{(n+1)}(z,z') = -\int \mathrm{d}z \mathscr{M}_0(z)\delta\mathscr{M}^{-1}(z)\mathscr{M}^{(n)}(z,z')$$

This simultaneously expands on the coupling g and the gradients of the background .

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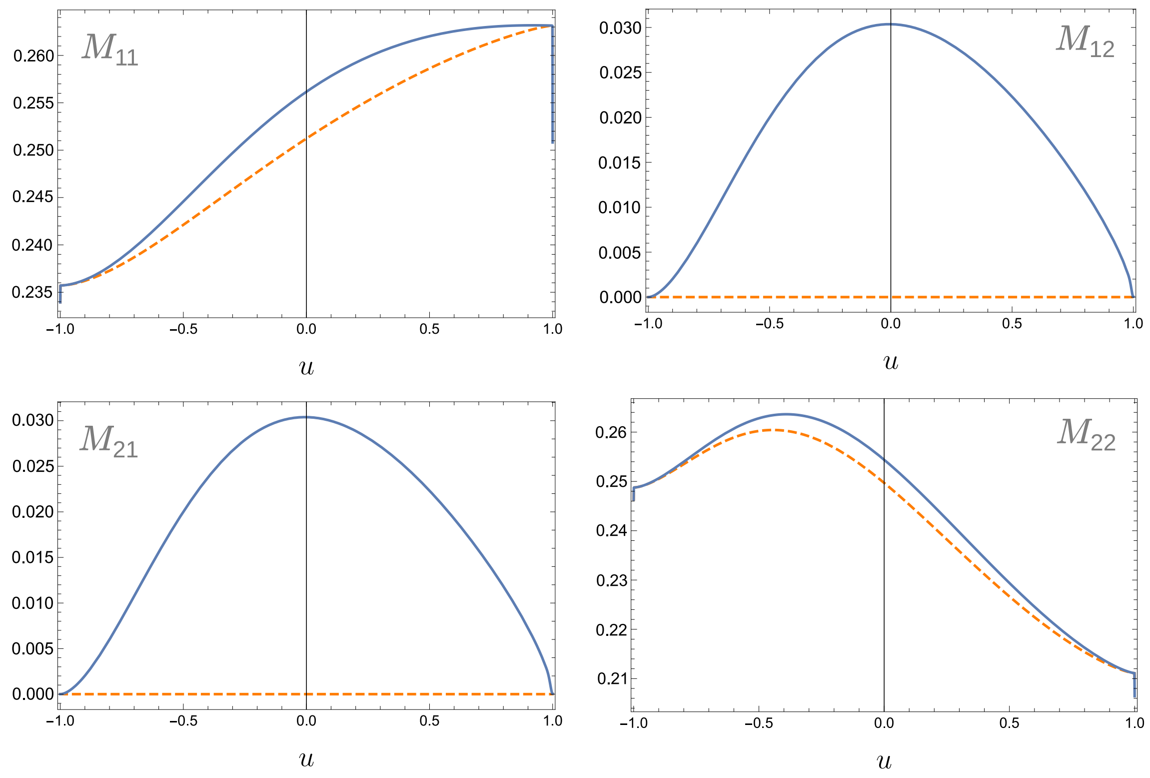
This simultaneously expands on the coupling g and the gradients of the background .

11.3. Quest for a solution III: larger **k** values

Numerical perturbative solution

For a value of $|\mathbf{k}|$, solve diagonals numerically for $G^{(0)}(u,u')$ (orange dashed) and iterate for corrections (solid blue).

Compute quantities of interest such as determinants and tadpole contributions by reconstructing functions for a range of $|\mathbf{k}|$.



12. Renormalization

Counter terms for the theory are found through homogeneous terms coming from CW analogue computations

$$\mathscr{L}_{\mathrm{ct}}[\varphi] = \frac{1}{2}\delta Z(\partial\varphi)^2 + \frac{\delta\alpha}{2}\varphi^2 + \frac{\delta\lambda}{4}\varphi^4 + \frac{\delta\lambda_6}{8}\varphi^6 + \frac{\delta\lambda_8}{16}\varphi^8.$$

the homogeneous term for the scalar field for example, looks like:

$$I_1 \equiv \frac{\hbar}{2}\int_{B_\Lambda}\frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}k_4}{2\pi}\log\frac{k_4^2+\mathbf{k}^2+U''(\phi)}{k_4^2+\mathbf{k}^2+U''(\phi_{fv})},$$

which together with similar terms for the other fields are enough to determine the coupling counter-terms.

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which together with similar terms for the other fields are enough to determine the coupling counter-terms.

To remove the remaining divergences one needs another method to determine the wave-function renormalization δZ. (You are encouraged to ask about the details)

13. Problems

- **Tuning the potential** One must ensure the validity of the thin wall approximation by picking potentials that present degenerate vacua.
- **Tuning the potential** To obtain finite functional determinants one requires degeneracy at the CW-level.

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- **Renormalizing** The CW-potential allows to extract coupling counterterms, the wavefunction one is obtained through a mixed representation gradient expansion technique (See [Chan, 1985, Cheyette, 1985, Gaillard, 1986] and [Henning et al., 2016]).

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- **Numerical obstacles:** Corrections need sampling in two dimensions for numerical integrals with interpolation steps in between (one iteration needs around 15k sec with 6 cores., it takes long)
- **Numerical obstacles:** A large region in $|\mathbf{k}|$ must be scanned.

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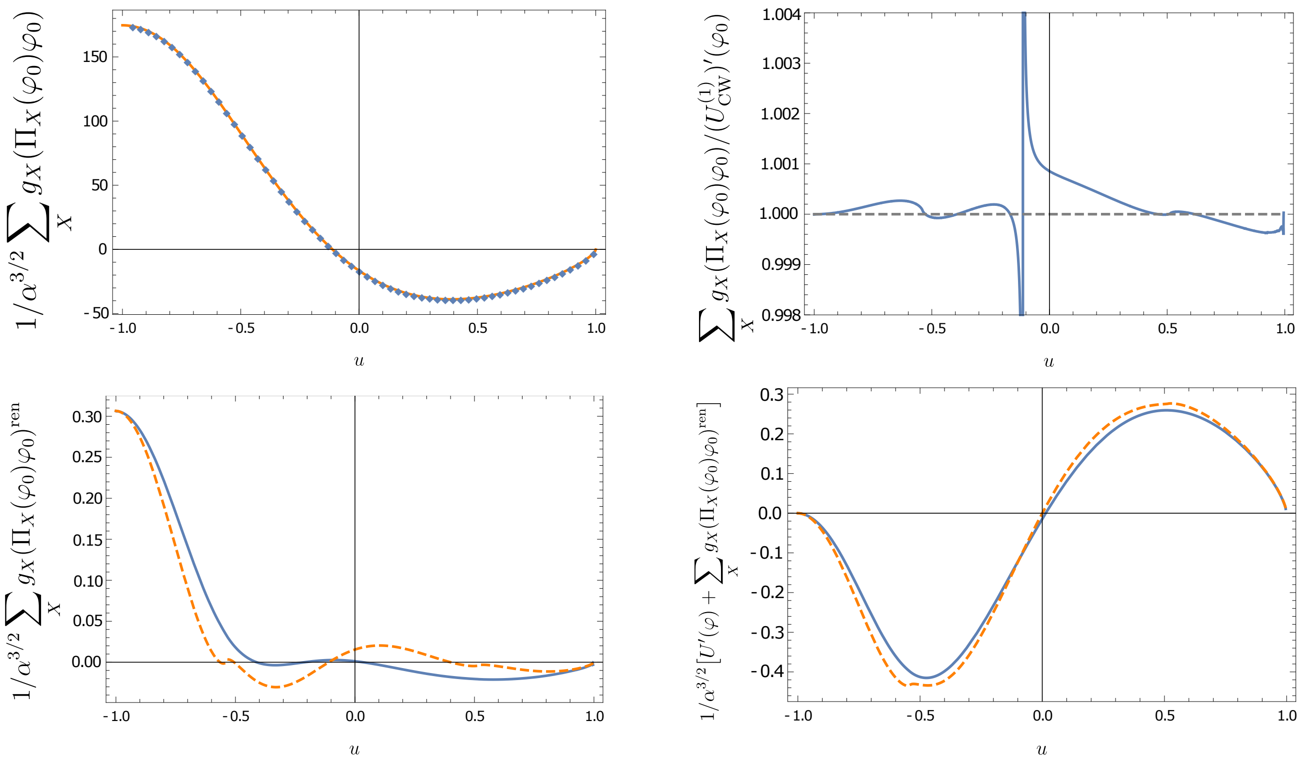
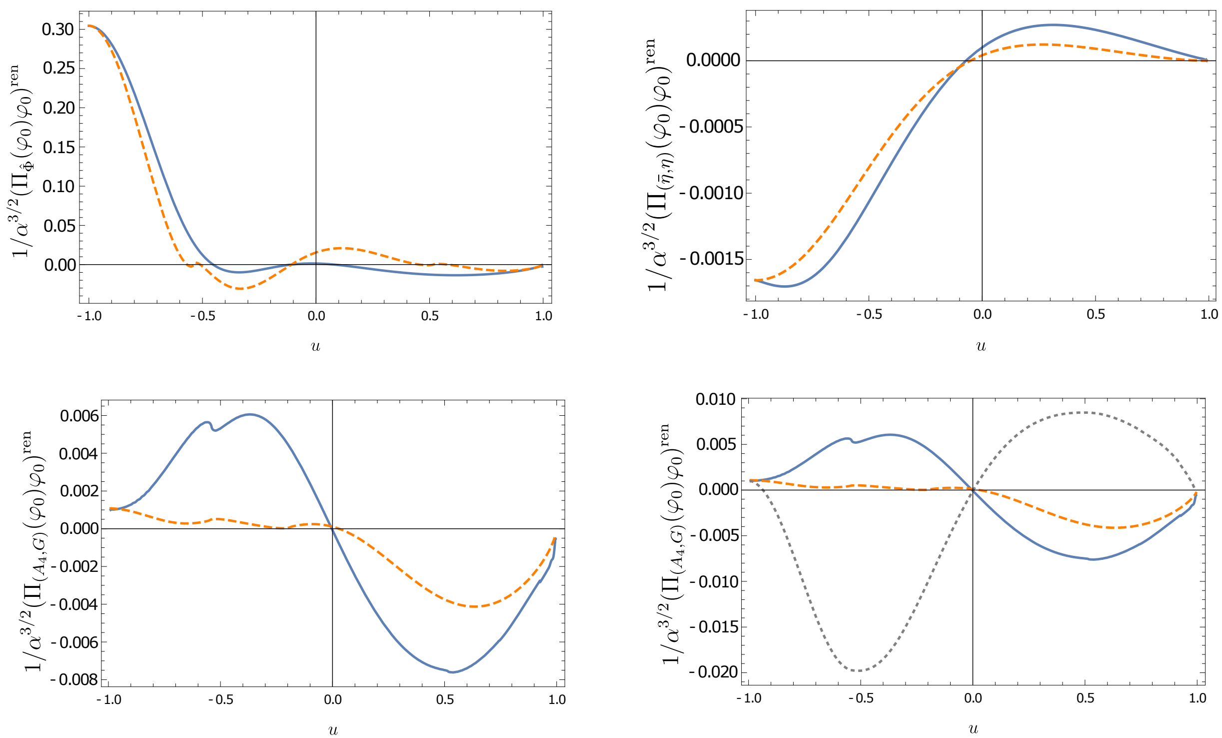
14.1. Results

Gradient effects on the total tadpole functions are shown, intermittent curves represent the old result ignoring gradients, while the solid line includes the corrections. The bottom row displays the results after renormalizing and including the tree-level contributions respectively.



14.2. Results

Gradient effects on the tadpole functions are shown per field, the dashed line is the CW-result while the solid line includes gradient corrections:



The scalar field suffers the largest corrections due to the background, followed by the $A_4 - G$ sector.

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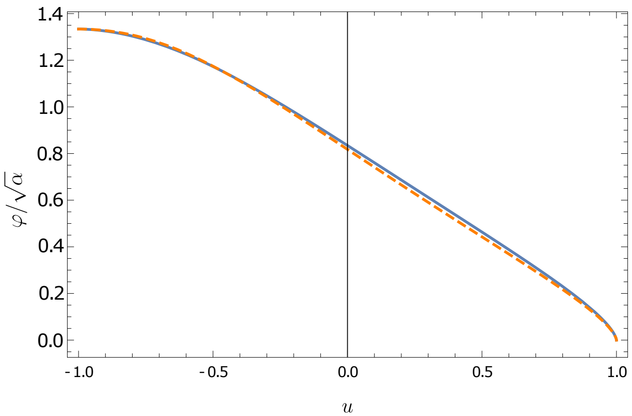
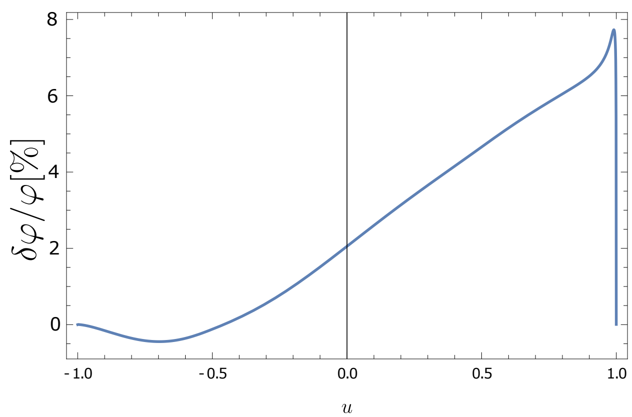
14.3. Results





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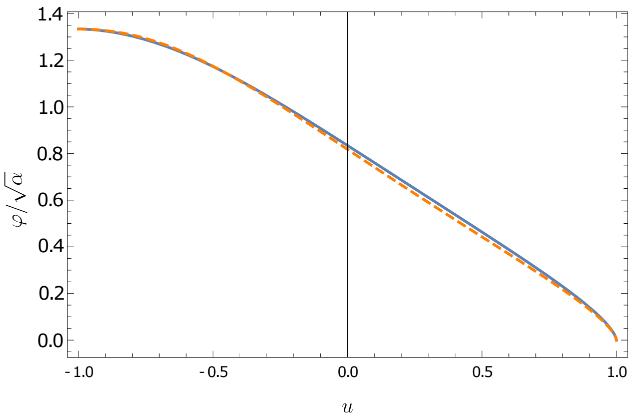

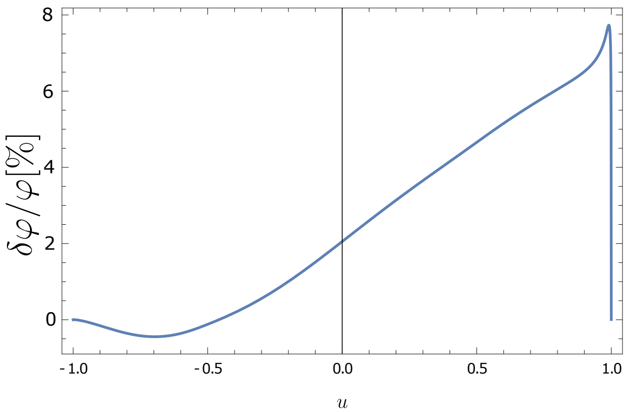

Gradient effects of the different fields to the effective action:

	Value $[\alpha^{-3/2}]$
$(\mathscr{B}^{(0)} + \mathscr{B}^{(1)\mathrm{ren}})/V$	0.473
$\mathscr{B}^{(2)\mathrm{ren}}/V$	-0.000345
$(\mathscr{B}^{(0)} + \mathscr{B}^{(1)\mathrm{ren}} + \mathscr{B}^{(2)\mathrm{ren}})/V$	0.474

The plots above show the bounce (dashed) and its corrections (solid) as well as the relative variation in the bottom plot.

15. Conclusions and Outlook

- The decay rate for this model is computed together with corrections coming from gradients of the background using a self-consistent prescription, indicating contributions from gradients comparable to 1-loop.
- A numerical treatment has been developed to compute the quantities involved which can be applied to other cases.
- We renormalize the theory to 1-loop through the use of the CW potential and a gradient expansion.

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- A numerical treatment has been developed to compute the quantities involved which can be applied to other cases.
- We renormalize the theory to 1-loop through the use of the CW potential and a gradient expansion.

What's next...

- Consider different gauge-parameters and a bigger parameter space.
- Study cases abandoning the planar and/or thin wall approximations.
- Consider more realistic models, where metastable vacua appear through radiative corrections.


Thank you for your attention!

A.1. Details for the computation of the bounce

Boundary conditions for the bounce

$$t = \pm\infty, \phi(t, \mathbf{x}) = \phi_{fv} \qquad \text{and} \qquad |\mathbf{x}| \to \infty, \phi \to \phi_{fv}$$

Find a bounce solution for tuned CW-potential, center the wall and extend values to infinity.
Conclusions from the previous studies:

- Scalar loops increase B and cause faster decay.
- Fermion loops decrease B and prolong the decay.

Parameters used for the current study:

$$\alpha = 2 \quad \lambda = -2.02546 \quad \beta = \frac{1}{2} \quad g = \frac{1}{2}$$

A.2. Extracting the zero modes

The fluctuation operator usually contains 0 modes corresponding to certain symmetries of the bounce solution as for example the location of the wall and translations.

$$\hbar B^{(1)}_{\hat{\Phi};\mathrm{dis}} = \frac{\mathrm{i}\pi\hbar}{2} - \frac{\hbar}{2}\log\left(\frac{(V\mathscr{T})^2\alpha^5}{4|\lambda_0|}\left(\frac{B}{2\pi\hbar}\right)^4\right)$$

In order to compute the functional integrals, one extracts such 0 modes, meaning that the determinants that appear in the presentation do not actually contain zero-modes but the instead are included as pre-factors coming out of the integration in the associated collective coordinate.

A.3. Implementation II: Functional determinants

The s-parameter

We deform the operator through an auxiliary parameter s, as done by [Baacke and Junker, 1994].

Given a differential operator $\mathscr{M}^{-1}$ then its deformation is $\mathscr{M}_s^{-1} = \mathscr{M}^{-1} + s$.

If G is a Green's function for $\mathscr{M}^{-1}$ with spectral decomposition $\sum_n \frac{f_n(x)f_n^*(y)}{\lambda_n}$ then the spectral decomposition for the deformed operator is $\sum_n \frac{f_n^*(y)f_n(x)}{\lambda_n+s}$ and one can write

$$\log \frac{\det M^{-1}(\varphi)}{\det M^{-1}(\chi)} = -\int_0^\infty \mathrm{d}s \int \mathrm{d}^4x\, G_s(\varphi; x, x) - G_s(\chi; x, x)$$

We take the 3+1 decomposition and numerically integrate up to some cutoff in tangential momentum and the s-parameter in our case:

$$\log \frac{\det M^{-1}(\varphi_b)}{\det M^{-1}(0)} = -\int_0^\infty \mathrm{d}s \int_{-\infty}^\infty \mathrm{d}z \int \mathrm{d}^3\mathbf{x} \int_0^{\Lambda_k} \mathrm{d}k\, \frac{k^2}{2\pi^2} \mathrm{tr}\left(G_{s,\mathbf{k}}(\varphi_b; z, z) - G_{s;\mathbf{k}}(0; z, z)\right)$$

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A.4. Wavefunction renormalization

One adds and subtracts a kernel obtained from a covariant gradient expansion to the effective action, as well as a counter term for just the divergences:

$$\Gamma|_{\mathrm{ren}} \supset \frac{1}{2}\mathrm{tr}\int_0^{\Lambda_s^2} ds \int d^4x \, (\mathscr{M}_{\varphi_b,s}(x,x) - \mathscr{M}^{\mathrm{hom}}_{\varphi_b,s}(x,x) + K_s(x)(\partial\varphi_b)^2)$$
$$- \int d^4x \, (V^{\mathrm{ren}}_{\mathrm{CW}}(\varphi_b) - V^{\mathrm{ren}}_{\mathrm{CW}}(v)) - \frac{1}{2}\int d^4x (\partial\varphi_b)^2$$
$$- \frac{1}{2}\int d^4x \, (K(x) + \delta Z)(\partial\varphi_b)^2.$$

This technique ensures that the integrand in the first line only has terms that are finite when to cutoff is taken to infinity. K is then obtained from an expansion of the form:

$$\Gamma \supset -\frac{1}{4}\int \mathrm{d}^4x \frac{\mathrm{d}^4p}{(2\pi)^4}\left(\frac{\partial^2\Delta_{A,\mu\mu}}{\partial p_\rho \partial p_\sigma}\partial_\rho\partial_\sigma m_A^2 + \frac{\partial^2\Delta_G}{\partial p_\rho \partial p_\sigma}\partial_\rho\partial_\sigma m_G^2\right)$$

where an expansion like the one below was used

$$\mathrm{tr}\log \tilde{\mathscr{M}}^{-1}_{(A_\mu,G,p)}(x) = \mathrm{tr}\log \tilde{\mathscr{M}}^{-1}_{0(A_\mu,G,p)} - \mathrm{tr}\sum_{m=1}^{\infty}\frac{(-1)^m}{m}\left(\tilde{\mathscr{M}}_{0(A_\mu,G,p)}(\tilde{\mathscr{M}}^{-1}_{1(A_\mu,G,p)} + \tilde{\mathscr{M}}^{-1}_{2(A_\mu,G,p)})\right)^m$$

A.5. Contact with Phenomenology

Our theory although picks specific higher dimensional operators, it is consistent and can be thought of as an effective field theory coming from a UV-theory having heavy Dirac fermions Ψ, χ, in which Ψ is a gauge singlet and χ has charge -1,:

$$\mathscr{L}_{\text{heavy}} = -y\bar{\psi}\Phi\chi + h.c.$$

One-loop diagrams have interactions $\lambda_{2m}|\Phi|^{2m}$ with

$$\lambda_{2m} \sim \frac{y^{2m}}{16\pi^2 M^{2(m-2)}}$$

At the benchmark point, this means each higher order interaction after 6 will be suppressed by a factor of $1/10$ for a coupling y within the perturbative regime.

$$\left.\frac{\lambda_n|\Phi|^{2m}}{\lambda_6|\Phi|^6}\right|_{|\Phi|^2=\alpha=2} = \left(\frac{4\pi}{y^2}\right)^{2(m-3)}.$$

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