A heatwave affair: mixed Higgs–R^2 preheating on the lattice arXiv:2007.10978 [hep-ph]

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Talk outline

1. Overview of Higgs inflation
2. Healing Higgs inflation
3. New stuff

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Scalar field inflation: the big idea

- Observation: the Universe is $\sim$homogeneous and spatially flat
- Solve with inflation (accelerated expansion)
- $P = -\rho \rightarrow a(t) \propto e^{Ht};\ H^2 = \frac{1}{3M_P^3}\Lambda$
- Power spectrum has suggestive form: $\left\langle \mathcal{R}^2 \right\rangle = \int \frac{dk}{k} \mathcal{P}_{\mathcal{R}}(k)$

Scalar field inflation: the big idea

- Scalar inflaton field:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \partial_\nu \phi - V(\phi) \right]$$

- Energy-momentum tensor:

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \mathcal{L} \to \begin{cases} \rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p = \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{cases}$$

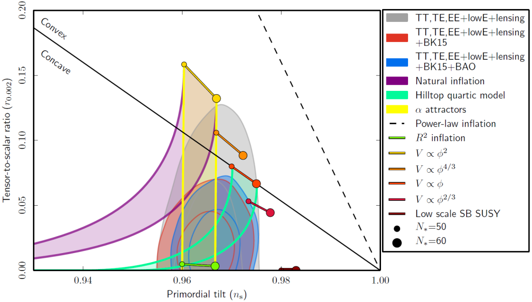
- "Slow roll" conditions:

$$\boxed{\frac{\dot{\phi}^2}{2V(\phi)} \ll 1, \quad \left| \frac{\ddot{\phi}}{3H\dot{\phi}} \right| \ll 1} \longleftrightarrow \boxed{\epsilon_v \equiv \frac{M_P^2}{2} \left(\frac{V'(\phi)}{V(\phi)} \right)^2, \; \eta_v \equiv M_P^2 \frac{V''(\phi)}{V(\phi)} \ll 1}$$

- Dynamics from *potential*, perturbations frozen by Hubble friction

What are our options?

Cosmological constraints:



Planck Collaboration, arXiv:1807.06211 (2018)

- Power laws look dubious
- Bigger problem: *what is this scalar field?*
- Only found one fundamental [a] spin-0 field. . .

[a] The jury's somewhat out on this one

What are our options?

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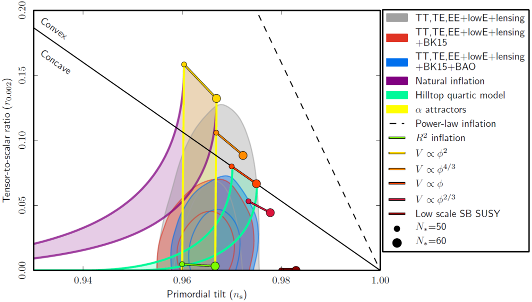

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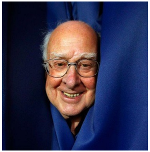

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Higgs inflation: the basic idea

- Add dimensionless Higgs-curvature coupling

$$S_J = \int d^4\tilde{x}\sqrt{-\tilde{g}}\left\{-\frac{M_P^2+\xi h^2}{2}\tilde{R}+\frac{\tilde{\partial}_\mu\tilde{\partial}^\mu h}{2}-\frac{\lambda}{4}\left(h^2-v^2\right)^2\right\}$$

- Weyl transformation into Einstein frame:

$$\boxed{g_{\mu\nu}\equiv\Omega^2\tilde{g}_{\mu\nu},\ \ \Omega^2=1+\frac{\xi h^2}{M_P^2}},\quad h\approx\frac{M_P}{\sqrt{\xi}}\exp\left(\frac{\xi}{\sqrt{6}M_P}\right)$$

- Nice inflationary action

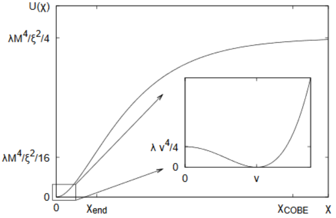
$$S_E = \int d^4x\sqrt{-g}\left\{-\frac{M_P^2}{2}R+\frac{\partial_\mu\chi\partial^\mu\chi}{2}-U(\chi)\right\},$$

$$U(\chi)=\frac{\lambda M_P^4}{4\xi^2}\left(1+\exp\left(-\frac{-2\chi}{\sqrt{6}M_P}\right)\right)^{-2}$$

Higgs inflation: the basic idea

$$U(\chi) = \frac{\lambda M_P^4}{4\xi^2}\left(1+\exp\left(-\frac{-2\chi}{\sqrt{6}M_P}\right)\right)^{-2}$$

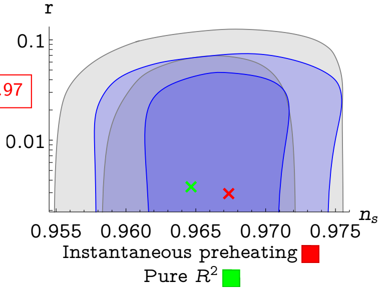
- Flat potential for super-Planckian fields



F. Bezrukov and M. Shaposhnikov, Phys. Lett. B 659 (2008) 703–706

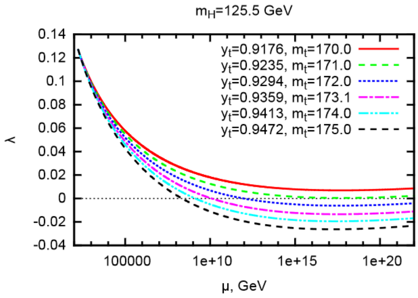
$n_s = 1 - 6\epsilon + 2\eta \approx 0.97$

$r = 16\epsilon \approx 0.0033$



What's the catch?

- Central LHCb measurements of m_H, m_T, g_{EW}, $y_T \rightarrow \lambda_H$ is negative at inflationary scales!



Bezrukov et al, Phys. Rev.D92, 083512 (2015)

- Large dimensionless coupling $\xi \sim 10^4$
- Quantize fluctuations on top of homogeneous background $\rightarrow$ cutoff $\sim \frac{M_P}{\sqrt{\xi}}$ during inflation, $\sim \frac{M_P}{\xi}$ during reheating
- Reheating in strongly coupled, decouples UV/IR

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Healing with R^2

- Add R^2 term required for renormalizability

$$S_J = \int d^4x\sqrt{-\tilde{g}}\left(-\frac{M_P^2+\xi h^2}{2}\tilde{R} + \frac{\beta}{4}\tilde{R}^2 + \frac{(\partial_\mu h)^2}{2} - \frac{\lambda}{4}\left(h^2 - v^2\right)^2\right)$$

- Lagrange multiplier and auxiliary scalar

$$S_J = \int d^4x\sqrt{-\tilde{g}}\left(\frac{\left(\tilde{\partial}_\mu h\right)^2}{2} - \frac{\lambda}{4}\left(h^2 - v^2\right)^2 - \frac{M_P^2+\xi h^2}{2}\mathcal{R} + \frac{\beta}{4}\mathcal{R}^2 - L\mathcal{R} + L\tilde{R}\right)$$

- Weyl transformation $\Omega^2 = \frac{2L}{M_P^2}$, $\phi \equiv M_P\sqrt{\frac{2}{3}}\log\Omega^2$: "scalaron"

$$S_E = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \frac{(\partial_\mu\phi)^2}{2} + e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}\frac{(\partial_\mu h)^2}{2} - V(\phi, h)\right),$$

$$V(\phi, h) = -\frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}\left(\lambda h^4 + \frac{M_P^4}{\beta}\left(e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} - 1 - \xi\frac{h^2}{M_P^2}\right)^2\right)$$

Healing Higgs inflation

- Important: Higgs is $SU(2)$ doublet
- Interested in scalar/Goldstone sectors, broken/unbroken symmetry
- Parametrise using four real fields: $h = (h_1, h_2, h_3, h_4)^T$
- $|h| \equiv \sqrt{\frac{H^\dagger H}{2}} \equiv \sqrt{h_1^2 + h_2^2 + h_3^2 + h_4^2}$

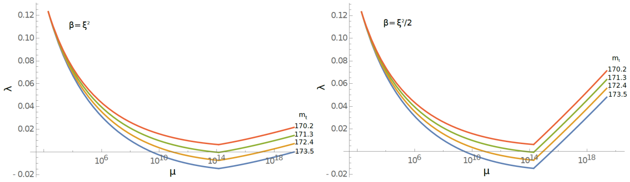
$$S_E = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \frac{(\partial_\mu\phi)^2}{2} + e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}\frac{(\partial_\mu h_i)^2}{2} - V(\phi, h)\right),$$

$$V(\phi, |h|) = -\frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}\left(\lambda|h|^4 + \frac{M_P^4}{\beta}\left(e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} - 1 - \xi\frac{|h|^2}{M_P^2}\right)^2\right)$$

Healing with R^2

$$V(\phi, |h|) = -\frac{1}{4} e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} \left(\lambda |h|^4 + \frac{M_P^4}{\beta} \left(e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} - 1 - \xi \frac{|h|^2}{M_P^2} \right)^2 \right)$$

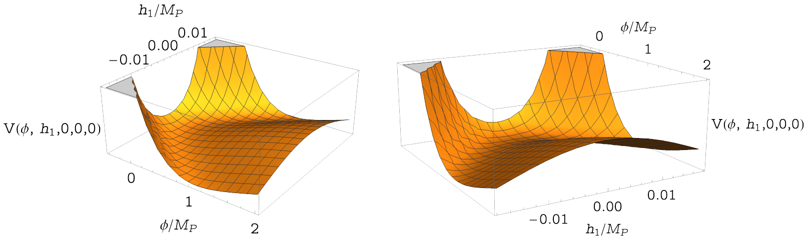
- Quartic couplings: λ, $\frac{\xi^2}{\beta}$
- Perturbative couplings only if $\beta \geq \frac{\xi^2}{4\pi}$
- Bonus: RG stabilisation of λ



D. Gorbunov and A. Tokareva, Phys. Lett. B 788 (Jan, 2019) 3741

Scalar potential

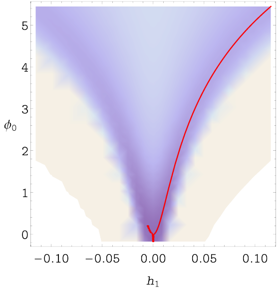
$$V(\phi, h) = -\frac{1}{4} e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} \left(\lambda |h|^4 + \frac{M_P^4}{\beta} \left(e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} - 1 - \xi \frac{|h|^2}{M_P^2} \right)^2 \right)$$



- Family of degenerate minima $|h|^2 = \frac{\xi M_P^2}{\xi^2 + \lambda\beta} \left(e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} - 1 \right)$

Inflationary dynamics

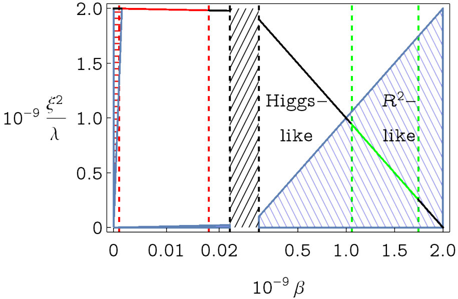
- Attractor solution $\rightarrow V_{\rm inf}(\phi) = \frac{\lambda M_P^4}{4(\xi^2+\lambda\beta)}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}\right)^2$
- Inflation: for $\xi > \mathcal{O}(1)$, arbitrary trajectories arrive at attractor $\mathcal{O}(10)$ e-foldings before pivot scale exits horizon



- Normalization of CMB power spectrum: $\boxed{\beta + \frac{\xi^2}{\lambda} \approx 2 \times 10^9}$

$$\beta + \frac{\xi^2}{\lambda} \approx 2 \times 10^9 \text{ (CMB)}, \qquad \beta \geq \frac{\xi^2}{4\pi} \text{ (perturbativity)}$$

- Still some freedom in parameters:



- $\xi^2 > \lambda\beta$: "Higgs–like"
- $\xi^2 < \lambda\beta$: "R^2–like"

Post–inflationary homogeneous dynamics

- Simplify potential:

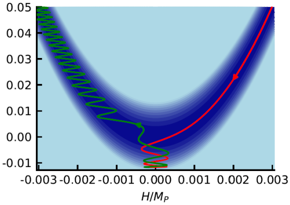
$$V(\phi,|h|) = -\frac{1}{4}e^{-2\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}\left(\lambda|h|^4 + \frac{M_P^4}{\beta}\left(e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} - 1 - \xi\frac{|h|^2}{M_P^2}\right)^2\right)$$

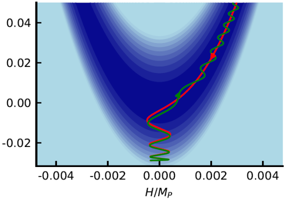
$$\rightarrow V(\phi,|h|) = \frac{1}{4}\left(\lambda + \frac{\xi^2}{\beta}\right)|h|^4 + \frac{M_P^2}{6\beta}\phi^2 - \frac{\xi M_P}{\sqrt{6}\beta}\phi|h|^2 + \frac{7}{108\beta}\phi^4 - \frac{M_P}{3\sqrt{6}\beta}\phi^3 + \frac{\xi}{6\beta}\phi^2|h|^2 + \dots$$

- Scalaron arrives at zero from attractor: one Higgs DoF only (e.g. h_1)

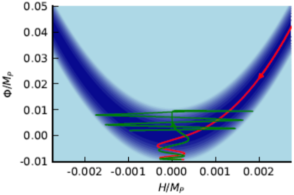
Post–inflationary homogeneous dynamics

- Scalaron crosses to negative → turn in field space





- Bifurcation between the two solutions: Higgs spends prolonged period at local maximum



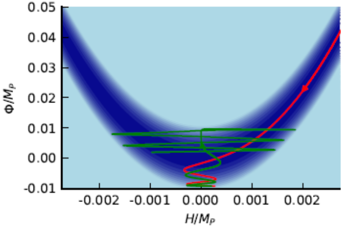
F. Bezrukov, D. Gorbunov, C. Shepherd, and A. Tokareva, Phys. Lett. B 795 (2019) 657–665

Particle production

- Slow scalaron oscillation, fast homogeneous Higgs oscillations
- Higgs and Goldstone sectors
- Parametrisation $h = (h_1, h_2, h_3, h_4)^T$

Inhomogeneous Higgs/Goldstone mass

$$m_{h,\text{eff}}^2 = -\sqrt{\frac{2}{3}}\frac{\xi}{\beta}M_P\,\langle\phi\rangle + \left(\lambda + \frac{\xi^2}{\beta}\right)\left(3\,\langle|h|\rangle^2 + \left\langle(h_i - \langle h_i\rangle)^2\right\rangle\right) + \dots$$



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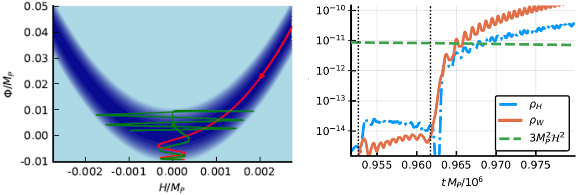
$$m_{h,\text{eff}}^2 = -\sqrt{\frac{2}{3}}\frac{\xi}{\beta} M_P \langle\phi\rangle + \left(\lambda + \frac{\xi^2}{\beta}\right)\left(3\langle|h|\rangle^2 + \left\langle\left(h_i - \langle h_i\rangle\right)^2\right\rangle\right) + \dots$$

- Previously studied using linearised approach (start $n_k = 1/2$, calculate Bogoliubov coefficients)



F. Bezrukov, D. Gorbunov, C. Shepherd, and A. Tokareva, Phys. Lett. B 795 (2019) 657–665

- Effect of backreaction
- Expansion: do initially noncritical dynamics become tachyonic?

Talk outline

Our simulation

If preheating dynamics take place with $n_k \gg 1$, one can analyse the particle production using semiclassical methods

[1]

- Solve equations

$$\ddot{\phi} - \frac{\nabla^2}{a^2}\phi + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{\partial V}{\partial \phi} + \frac{1}{\sqrt{6}M_P} e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}} \sum_i \left(\dot{h}_i^2 - \left(a^{-1}\nabla h_i\right)^2 \right) = 0,$$

$$\ddot{h}_i - \frac{\nabla^2}{a^2}h_i + 3\frac{\dot{a}}{a}\dot{h}_i - \sqrt{\frac{2}{3}}\frac{1}{M_P}\left(\dot{\phi}\dot{h}_i - a^{-2}\nabla\phi \cdot \nabla h_i\right) + e^{\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}\frac{\partial V}{\partial h_i} = 0,$$

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv \mathcal{H}^2 = \frac{1}{3M_P^2}\left\langle \rho \right\rangle,$$

$$\rho = \frac{1}{2}\left[\dot{\phi}^2 + a^{-2}\left(\nabla\phi\right)^2 + e^{-\sqrt{\frac{2}{3}}\frac{\phi}{M_P}}\left(\dot{h}_i^2 + a^{-2}\left(\nabla h_i\right)^2\right)\right] + V$$

[1] S. Y. Khlebnikov and I. I. Tkachev, Phys. Rev. Lett. 77 no. 2 (Jul, 1996) 219222

Our simulation

- Initial conditions for Fourier mode f_k from Gaussian distribution:

$$\boxed{\left\langle |f_{\mathrm{k}}|^2 \right\rangle = \frac{1}{2a^3\omega_k}}$$

where

$$\omega_k^2 = \frac{k^2}{a^2} + m^2$$

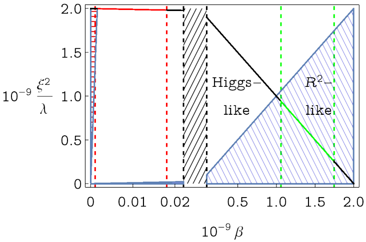
- Sufficiently large $\int d^3k\,(\ldots)$ to capture all tachyonically–growing Higgs modes

$$m_{h,\mathrm{eff}}^2 = -\sqrt{\frac{2}{3}}\frac{\xi}{\beta} M_P \,\langle\phi\rangle + \left(\lambda + \frac{\xi^2}{\beta}\right)\left(3\,\langle|h|\rangle^2 + \left\langle\left(h_i - \langle h_i\rangle\right)^2\right\rangle\right)$$

Results

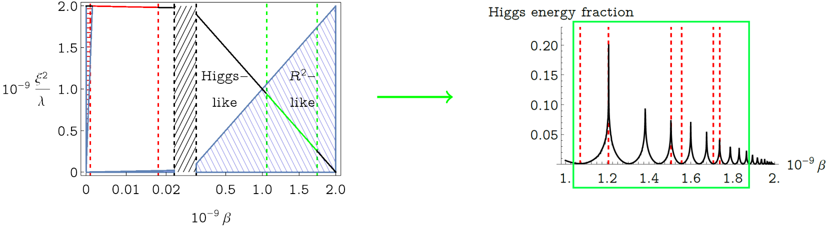
$$m_{h,\text{eff}}^2 = -\sqrt{\frac{2}{3}}\frac{\xi}{\beta}M_P\,\langle\phi\rangle + \left(\lambda + \frac{\xi^2}{\beta}\right)\left(3\,\langle|h|\rangle^2 + \left\langle(h_i - \langle h_i\rangle)^2\right\rangle\right)$$

- Higgs-like parameters $\xi^2 > \lambda\beta$: more energy in initialised Higgs modes
- Zero-point energy competes with homongeneous energy
- Reheating will be complete without $n_k \gg 1$, requires *quantum* calculation using 2PI formalism (good luck!)

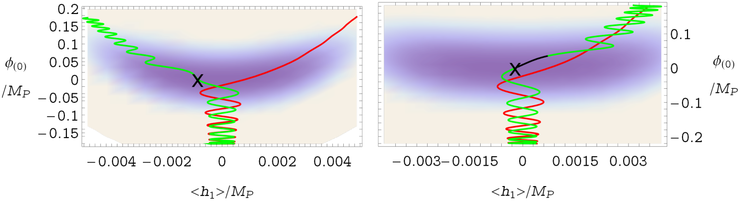


Results

- Six values of (R^2-like) β: three pairs of critical/noncritical

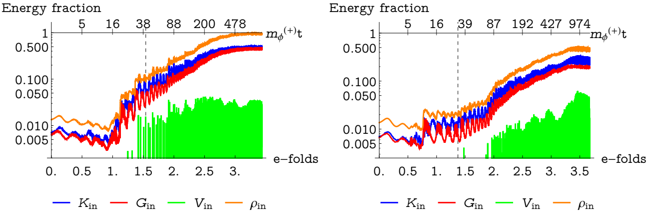


- Compare most Higgs–like noncritical to most R^2–like critical
- Trajectories:

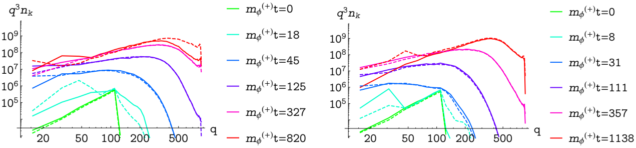


Results

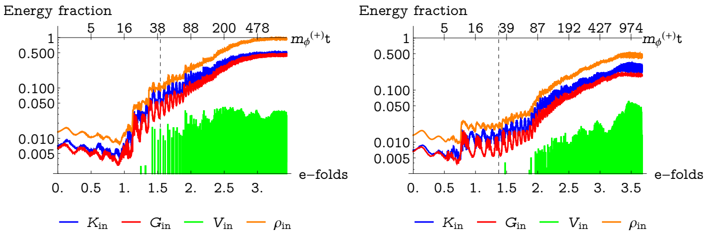
- Energy growth:



- Tachyonic growth, followed by rescattering
- Spectra:



- Efficient movement towards UV, particle production via rescattering



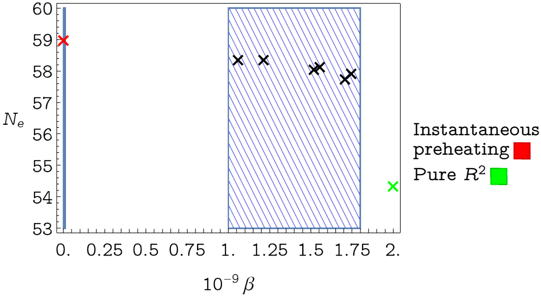
- R^2–like parameters: rescattering becomes less efficient with expansion, incomplete preheating?
- Once scalaron decay kinematically allowed and decay width Γ_ϕ exceeds Hubble, efficient perturbative decay

$$V(\phi, |h|) = \frac{1}{4}\left(\lambda + \frac{\xi^2}{\beta}\right)|h|^4 + \frac{M_P^2}{6\beta}\phi^2 - \frac{\xi M_P}{\sqrt{6}\beta}\phi|h|^2 + \dots$$

$$\to \Gamma_\phi \sim \frac{1}{8\pi m_\phi^{(-)}}\left(M_P \frac{\xi}{\beta}\right)^2$$

The ACTUAL results

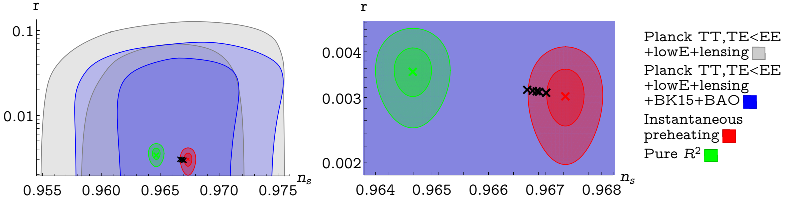
- Pivot-scale exit: $N_e(k)$ sensitive to duration of reheating



- Higgs-like "end" nearly saturates instantaneous preheating: generalise to Higgs-like parameters!

The ACTUAL ACTUAL results

- n_s, r against Planck results



- Pure R^2, instantaneous preheating indistinguishable at the moment
- Δn_s from FFTT, Δr from EPIC-2m: may be distinguished in the future

Conclusions/ future prospects

Conclusions

- The unitarity problems of Higgs inflation can be healed by adding an R^2 term to the action
- Preheating is fast and multifield in nature, instantaneous for Higgs–like parameters
- Need more accurate measurements of n_s and r to constrain the parameter space

Future prospects

- Production of primordial black holes: sufficient abundance for majority of Dark Matter?
- Gravitational waves?
- Palatini formalism

QUESTIONS