Reconstructing the EFT of Inflation from Cosmological Data

based on arXiv:1911.05838 and arXiv:1904.00991

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in collaboration with Paul Hunt, Subodh Patil, Subir Sarkar

CEICO, Institute of Physics of the Czech Academy of Sciences

EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research,
Development and Education

MINISTRY OF EDUCATION,
YOUTH AND SPORTS

Reconstructing the EFT of Inflation from Cosmological Data

or

Finding a *precise* dictionary between the parameters of the effective theory of inflation and their primordial power spectra

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Inflation basically

- An early stage of near-exponential expansion. Many multiplications of the scale factor a, $\times \sim e^{60}$.
- Proposed to solve the horizon problem (assuming a beginning, signals only had finite distance to travel, yet observe same conditions in regions beyond this distance), the flatness problem ($\Omega_K \equiv \Omega - 1 = K/\dot{a}^2$) and also (historically) the monopole problem.
- Typically driven by a scalar field ϕ with non-zero, almost-flat potential $V(\phi)$, slowly rolling.
- Provides, in addition, *quantitative predictions* for the statistics of curvature perturbations $\mathcal{R}$, the seeds of later structure formation.
- The scalar field fluctuates quantum mechanically, and, having energy-momentum, leads to perturbations in curvature.

The primordial power spectrum

- The PPS $P(k)$ is the variance of the Fourier coefficients of curvature perturbation: $\langle \mathcal{R}_{\mathbf{k}} \mathcal{R}_{\mathbf{k}'} \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}') P(k)$.
- Dimensionless: $\mathcal{P}(k) = k^3 P(k)/2\pi^2$.
- Different inflationary scenarios produce different primordial power spectra.
- For slow-roll case $\mathcal{P}(k) = A(k/k_*)^{n_s - 1}$ where $n_s = 2\eta - 4\epsilon$, $\epsilon = -\dot{H}/H^2 = \dot{\phi}^2/(2H^2)$ and $\eta = -\ddot{\phi}/(H\dot{\phi})$.
- In the simplest case: Gaussian statistics. Suppressed non-Gaussian statistics.
- Leave imprint on the temperature fluctuations of the CMB: $\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell$ where $\Delta T(\hat{\mathbf{n}}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})$.
- Linear relation between $\mathcal{P}(k)$ and C_ℓ: $C_\ell = \int_0^\infty \mathrm{d} \log k \, \Delta_\ell^{TT}(k)^2 \mathcal{P}(k) \rightarrow \mathbf{d} = \mathbf{W}\mathbf{p}$.
- Crucially, $\mathbf{W}$ depends on the cosmological parameters.

Idea of the effective field theory of inflation

- Focus on scalar perturbations $\delta\phi(\mathbf{x}, t)$ of $\phi(x, t) = \phi_0(t) + \delta\phi(\mathbf{x}, t)$.
- Before perturbations: background.
- *Fix* the background to an FLRW background with scale factor $a(t)$.
- From it Hubble constant $H(t)$ and $\epsilon(t) = -\dot{H}(t)/H^2(t)$
- In pure de Sitter $a(t) = e^{Ht}$, or $a(\tau) = -1/(H\tau)$ where $\tau = \int \mathrm{d}t'/a(t')$.
- Background will affect perturbations through $\epsilon(t)$.
- Will consider *adiabatic* perturbations.
- These are perturbations which can be cancelled by a coordinate transformation, a space-dependent time shift, $\pi(\mathbf{x}, t)$
- $\delta\phi(t, \mathbf{x}) \to \delta\phi(t, \mathbf{x}) - \dot{\phi}_0(t)\pi(t, \mathbf{x})$
- If $\pi(\mathbf{x}, t) = \delta\phi(t, \mathbf{x})/\dot{\phi}_0(t)$ then $\phi(t, \mathbf{x}) \to \phi_0(t)$.

Idea of the effective field theory of inflation (continued)

- Now we are rid of the scalar field fluctuations.
- The same coordinate transformation changes the metric $g_{\mu\nu}$ according to

$$g_{\mu'\nu'} \to \frac{\partial x^\mu}{\partial x^{\mu'}} \frac{\partial x^\nu}{\partial x^{\nu'}} g_{\mu\nu}$$

  and so the scalar fluctuations are subsumed by the metric fluctuations.
- The general theory is built of the metric.
- More precisely, metric invariants of the remaining symmetry: spatial diffeomorphisms.

Idea of the effective field theory of inflation (continued)

- We turn to the 3+1 (ADM) formalism where metric is built out of lapse N, shift N^i and a 3-metric $h_{ij} = a^2(t)e^{2\mathcal{R}(\mathbf{x},t)}\delta_{ij}$ built as $ds^2 = -N^2dt^2 + a^2(t)e^{2\mathcal{R}}\delta_{ij}(dx^i + N^i dt)(dx^j + N^i dt)$
- The invariant terms that will be used are *powers, combinations and contractions* of $\delta g^{00} \equiv 1 + g^{00}$ and $\delta E_{ij} \equiv E_{ij} - E^0_{ij}$ where $E_{ij} = NK_{ij} = \frac{1}{2}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i)$, a subtracted, scaled extrinsic curvature.
- Write down most general action in these variables, organised by their power.
- Requiring that $\delta S/\delta g_{\mu\nu} = 0$ gives right background $H(t)$ determines the first part of the action $S_1 = M^2_{\rm Pl}\int {\rm d}^4x\sqrt{-g}\left(\frac{1}{2}R - \left(\frac{1}{N^2}\dot{H} + 3H^2 + \dot{H}\right)\right)$

Idea of the effective field theory of inflation (continued)

- The quadratic action can be written in terms of the curvature perturbation $\mathcal{R}$, the part that scales the 3-metric $h_{ij} = a^2(t)e^{2\mathcal{R}}\delta_{ij}$ after solving the constraint equations for N and N^i.
- $S_2 = M_{\rm Pl}^2 \int \mathrm{d}^4x\, a^3 \epsilon \left(\frac{\dot{\mathcal{R}}^2}{c_s^2} - \frac{(\partial\mathcal{R})^2}{a^2} + \lambda \frac{(\partial^2\mathcal{R})^2}{a^4} \right)$
- EFT built out of *powers* of *the field* $\mathcal{R}$ (coming originally from δg^{00} and δE_{ij} invariants) with undetermined *coupling constants* (Wilson coefficients).
- Three coupling constants determine the fluctuations.
- If matter theory is only constructed by powers of a scalar and its first derivatives $\mathcal{L}(\phi, \nabla\phi)$ then $\lambda = 0$

The effective field theory of inflation

- Contains only the curvature perturbation $\mathcal{R}$ field with two, in general, time-dependent coupling constants $c_s(\tau)$ and $\epsilon(\tau)$. ϵ is the expansion parameter of the EFT.
  $S_2 = M_{\mathrm{Pl}}^2 \int \mathrm{d}^3x \int \mathrm{d}\tau \, a^2(\tau)\epsilon(\tau) \left(\mathcal{R}'^2/c_s(\tau)^2 - (\partial_i \mathcal{R})^2\right)$
- A more complicated inflationary scenario is *shoehorned* into these time-dependent coupling constants.
- A time-dependence of $\epsilon(\tau)$ or $c_s(\tau)$ leads to characteristic scales, 'features', in $\langle \mathcal{R}_k \mathcal{R}_k \rangle \propto \mathcal{P}(k)$.
- Fractional changes in PPS $\Delta\mathcal{P}/\mathcal{P} \propto \Delta\epsilon/\epsilon$ or $u(\tau) = 1/c_s^2 - 1$.
- Would like to *infer* $\Delta\epsilon/\epsilon(\tau)$ or $u(\tau)$ from estimates of $\Delta\mathcal{P}/\mathcal{P}$ itself estimated from *data* C_ℓ.

How large features?

- Compute corrections using perturbation theory.
- Consider excursions from $\epsilon = \epsilon_0$ *or* from $c_s = 1$, $\Delta\epsilon/\epsilon(\tau) \equiv (\epsilon(\tau) - \epsilon_0)/\epsilon_0$ and $u(\tau) \equiv 1/c_s^2(\tau) - 1$.
- Split action S_2 into exactly solvable part with constant ϵ (or c_s) and an interacting part $S_{\rm int}$ proportional to $\Delta\epsilon/\epsilon$ or $u(\tau)$
- $S_2 = \epsilon_0 M_{\rm Pl}^2 \int \mathrm{d}^3x \int \mathrm{d}\tau a^2((\mathcal{R}')^2 - (\partial_i \mathcal{R})^2 + u(\tau)(\mathcal{R}')^2)$
- $S_2 =$ $\epsilon_0 M_{\rm Pl}^2 \int \mathrm{d}^3x \int \mathrm{d}\tau a^2((\mathcal{R}')^2 - (\partial_i \mathcal{R})^2 + \Delta\epsilon/\epsilon((\mathcal{R}')^2) - (\partial_i \mathcal{R})^2)$
- For $\Delta\mathcal{P}/\mathcal{P} \sim 10\%$, corrections from 2nd order perturbation theory: $\sim (0.1)^2 = 1\%$.
- For features $\Delta\mathcal{P}/\mathcal{P} \sim 20\%$, error from truncation 4% so can consider 2nd order perturbation theory, in which case error will be below $(0.2)^3 \sim 0.8\%$

Intermezzo. A no-Λ agenda: Subir's gambit

- Λ is small $\sim H_0^2/(8\pi G)$. If fundamental, difficult to justify why it should know about the expansion rate *today*.
- Let us instead retain $\Lambda = 0$ and see what we can get away with.
- Effect on CMB (plotting $D_\ell \equiv \ell(\ell+1)/(2\pi)C_\ell$):

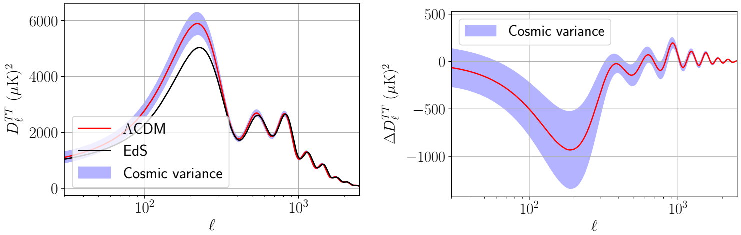


Figure: Using a power-law PPS for $\Omega_\Lambda = 0.67$ (red line) and $\Omega_\Lambda = 0$ (black line) but $H_0 = 44\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $\Omega_b = 0.09$, $\Omega_{\mathrm{CDM}} = 0.8$.

What does data suggest?

- Take CMB data, C_ℓ, from Planck and find most likely $\mathcal{P}(k)$ subject to roughness penalty assuming different cosmological parameters.
- Roughness penalty necessary (regularisation) as $\mathbf{W}^{-1}$ does not exist, so no simple relation $\mathbf{p} = \mathbf{W}^{-1}\mathbf{d}$.

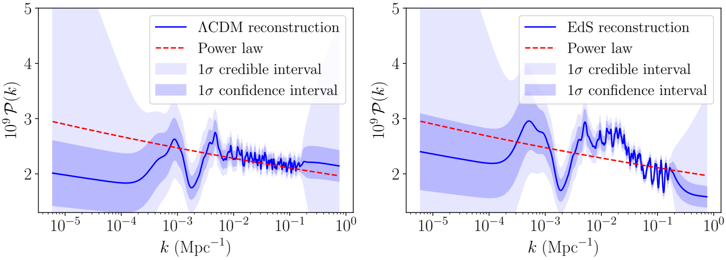


Figure: PPS reconstructing assuming different cosmological parameters.

Without Λ

- Features in the PPS: a *luxury* to ΛCDM, a *requirement* for a no-Λ (EdS) model.
- The UV physics is the most speculative: should be open to other shapes of PPS than power-law.
- If unwilling to go this far: Do the acoustic peaks have an oscillatory primordial component?
- Or can just appreciate the dictionary on its own.

Finding relations and their inverses

- Compute change to two-point function
  $\Delta\mathcal{P} \propto \Delta\langle\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}\rangle = \Delta\langle 0|\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}|0\rangle$
- Expectation values in QFT. Use Schwinger-Keldysh formalism.
- Helped by Weinberg:

$$\langle\mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}\rangle = \sum_{n=0} i^n \int_{-\infty}^{\tau_n} \mathrm{d}\tau_{n-1} \cdots \int_{-\infty}^{0} \mathrm{d}\tau_1$$
$$\langle 0|[H_{\rm int}(\tau_1), \ldots, [H_{\rm int}(\tau_{n-1}), [H_{\rm int}(\tau_n), \mathcal{R}_{\mathbf{k}}\mathcal{R}_{\mathbf{k}'}]]]|0\rangle$$

where $\mathcal{H}_{\rm int} = -\mathcal{L}_{\rm int}$ from $S_{\rm int} = \int \mathrm{d}\tau \int \mathrm{d}^3x\, \mathcal{L}_{\rm int}$.

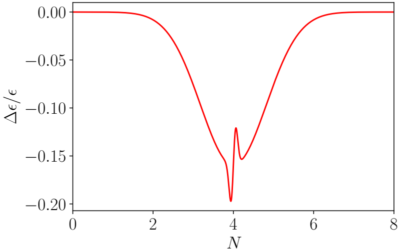
- Fourier expand $\mathcal{R}(\tau) = \int \frac{\mathrm{d}^3k}{(2\pi)^3}(\hat{a}_{\mathbf{k}}\mathcal{R}_k(\tau)e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{a}^\dagger_{\mathbf{k}}\mathcal{R}^*_k(\tau)e^{-i\mathbf{k}\cdot\mathbf{x}})$
- where $\mathcal{R}_k(\tau) = iH(1+ik\tau)e^{-ik\tau}/\sqrt{4\epsilon k^3}$
- and promote to ladder operators with
  $[a_{\mathbf{k}}, a^\dagger_{\mathbf{k}'}] = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}')$
- Then use Wick's theorem (all possible contractions).

The relations and inverse

- Schematically, $\Delta\mathcal{P} \sim \Delta\langle\mathcal{R}\mathcal{R}\rangle \sim \int \mathrm{d}\tau f(k\tau)X(\tau)$
- We find the dictionary: $\frac{\Delta\mathcal{P}}{\mathcal{P}}(k) = -k\int_{-\infty}^{0} \mathrm{d}\tau\, u(\tau) \sin(2k\tau)$ inverting to $u(\tau) = \frac{4}{\pi}\int_0^\infty \frac{\mathrm{d}k}{k}\frac{\Delta\mathcal{P}}{\mathcal{P}}(k)\sin(-2k\tau)$
- $\Delta_1\mathcal{P}/\mathcal{P}(k) =$ $\frac{1}{k}\int_{-\infty}^{0}\frac{\mathrm{d}\tau}{\tau^2}\Delta\epsilon/\epsilon(\tau)((1-2k^2\tau^2)\sin(2k\tau) - 2k\tau\cos(2k\tau))$ inverting to $\Delta\epsilon/\epsilon(\tau) = \frac{2}{\pi}\int_0^\infty \frac{\mathrm{d}k}{k}\frac{\Delta_1\mathcal{P}}{\mathcal{P}}(k)\left(\frac{2\sin^2(k\tau)}{k\tau} - \sin(2k\tau)\right)$
- Find at 2nd order that correction is the square of the 1st order correction: $\Delta\mathcal{P}_{\mathrm{rec}}/\mathcal{P}_{\mathrm{rec}}(k) = \Delta_1\mathcal{P}/\mathcal{P}(k) + (\Delta_1\mathcal{P}/\mathcal{P}(k))^2$
- Quadratic equation: $Y = X^2 + X$. So $X = \Delta_1\mathcal{P}/\mathcal{P}(k) = \frac{1}{2}\left(-1+\sqrt{1+4\frac{\Delta\mathcal{P}_{\mathrm{rec}}}{\mathcal{P}_{\mathrm{rec}}}}\right) \equiv \Delta\mathcal{P}_{\mathrm{eff}}/\mathcal{P}_{\mathrm{eff}}(k)$ and we know how $\Delta_1\mathcal{P}/\mathcal{P}(k)$ relates to c_s or ϵ (linear relation) so can isolate c_s or ϵ (by inverse transform).

Toy model

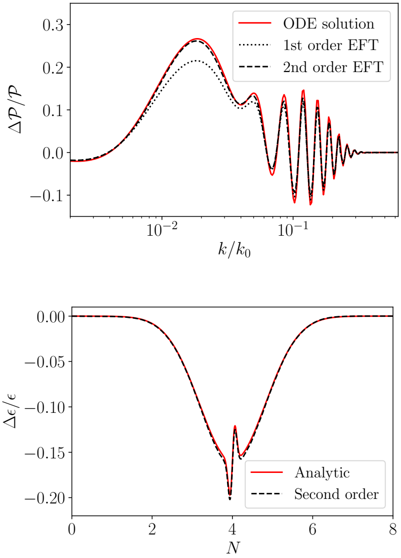
- Model with localised feature at N_0 with a fast (σ_2) and slow component (σ_1) and amplitudes c_1,c_2:
  $\Delta\epsilon/\epsilon(N) = c_1 e^{-(N-N_0)^2/\sigma_1^2} + c_2(N - N_0)e^{-(N-N_0)^2/\sigma_2^2}$.
  For $c_1 = 0.159$, $c_2 = 0.99$, $\sigma_1 = 1.16$, $\sigma_2 = 0.09$ and $N_0 = 4$:



- Can find the resulting change in the PPS using the dictionary.
- Can also solve the curvature perturbation equation numerically assuming this change.
  $\frac{\mathrm{d}^2\mathcal{R}_k}{\mathrm{d}N^2} + \left(3 - \epsilon(N) + \frac{\epsilon'(N)}{\epsilon(N)}\right)\frac{\mathrm{d}\mathcal{R}_k}{\mathrm{d}N} + \left(\frac{k}{aH}\right)^2\mathcal{R}_k = 0$

Toy model checks

- Find resulting change:



Potential reconstruction

- When writing ϵ in terms of *e*-folds: $\epsilon(N) = (\mathrm{d}\phi/\mathrm{d}N)^2/2$ from $\epsilon(t) = \dot{\phi}^2/H^2$ and $H\mathrm{d}t = \mathrm{d}N$.
- Solution: $\phi(N) = \phi_0 \pm \int_{N_0}^{N} \mathrm{d}N' \sqrt{2\epsilon(N')}$.
- Recall that $\epsilon(N) = -\mathrm{d}\log H/\mathrm{d}N$ from $\epsilon = -\dot{H}/H^2$ and $\mathrm{d}N = H\mathrm{d}t$
- Hence, $H(N) = H_0 \exp(-\int_{N_0}^{N} \mathrm{d}N' \epsilon(N'))$
- Considering the Friedmann equation $H^2 = \rho/3 \approx V/3$ we have $V(N) = H_0^2 \exp(-2\int_{N_0}^{N} \mathrm{d}N' \epsilon(N'))/3$
- Now we have $(\phi(N), V(N))$. Can also find $N = N(\phi)$ and then calculate $V(N(\phi)) = V(\phi)$

The potential

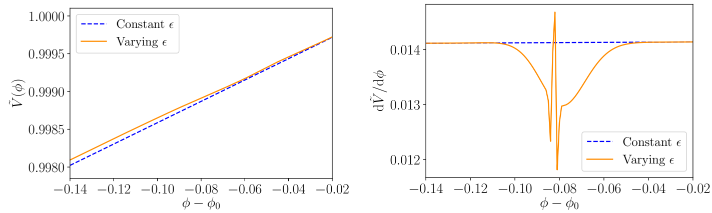
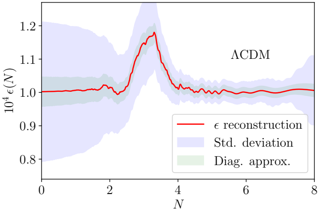
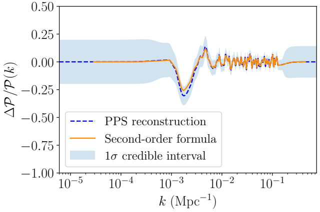


Figure: Scalar field potential (left) and derivative of potential (right) for constant $\epsilon = \epsilon_0$ and $\epsilon(\tau)$ with features.

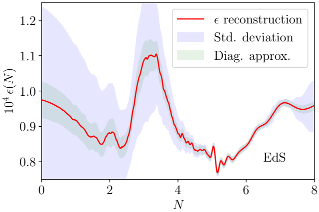
From Planck data

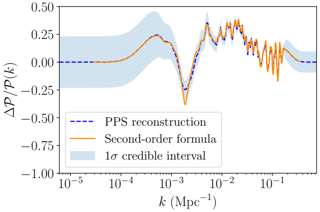
- For ΛCDM:





- For EdS:





Note on degeneracy

- Features can come from a combined change in c_s and ϵ.
- Not possible to invert unless feature is exclusively from one of the EFT parameters.
- However, theoretically c_s should not exceed 1, hence $u(\tau) \equiv 1/c_s^2 - 1 > 0$, so it is not possible to generate *any* given PPS subject to this constraint.
- Contributions to n-point functions (prescribed by EFT) from changes in ϵ and c_s do differ, so one way to disentangle.

Note on degeneracy

- The third-order action S_3 of the curvature perturbation $\mathcal{R}$ depends on $c_s(\tau)$ and $\epsilon(\tau)$.

$$S_3 = M_{\rm Pl}^2 \int \mathrm{d}^4x \Bigg(-\epsilon a \mathcal{R}(\partial\mathcal{R})^2 + \frac{3\epsilon}{{c_s}^2} a^3 \mathcal{R}\dot{\mathcal{R}}^2 - \epsilon a^3 \frac{\dot{\mathcal{R}}^3}{H}\left(2{c_s}^{-2} - 1 + \frac{4}{3}\frac{{M_3}^4}{M_{\rm Pl}^2 \dot{H}}\right) - 2a^3 \partial_i\theta\partial^2\theta\partial_i\mathcal{R} + \frac{a^3}{2}\left(3\mathcal{R} - \frac{\dot{\mathcal{R}}}{H}\right)\left(\partial_i\partial_j\theta\partial_i\partial_j\theta - \partial^2\theta\partial^2\theta\right)\Bigg) \tag{1}$$

where $N_T^i = \partial_i\theta$ and

$$\partial^2\theta = -\frac{\partial^2\mathcal{R}}{a^2 H} + \frac{\epsilon}{{c_s}^2}\dot{\mathcal{R}} \tag{2}$$

- Complicated by the fact that new (Wilson) function appear at higher order that may reintroduce degeneracy.

Summary

- Features: a luxury for ΛCDM. Necessary for other cosmological models.
- Computed corrections to PPS due to changes in ϵ and c_s, parameters in the EFTI.
- Inverted those relations to get the EFT parameters as transforms of the desired change in the PPS.
- Performed the inversion to second order in EFT parameters.
- Inversion precise to $\sim 1\%$ even when the features are $\sim 20\%$.
- Found simple realisations by reconstructing potential from the EFT parameter ϵ.
- Can reconstruct these parameters from cosmological data sets.

Outlook: future directions

- Combine constraints from CMB and LSS.
- Embrace the fine-tuning.
- In very non-standard cosmological models these EFT parameters are highly constrained.
- Hence the non-Gaussianity should be very specific: easy to look for. From preliminary investigations, non-Gaussianity is still too weak for Planck.
- Strictly, it is not necessary to use estimates of the PPS to get EFT parameters.
- Can go from CMB data, C_ℓ, directly to the EFT parameters $c_s(\tau)$ and $\epsilon(\tau)$. There is a linear relation ($\mathbf{W}_{\ell k}$) between $\mathcal{P}(k)$ and C_ℓ, and a linear relation between EFT parameters and $\mathcal{P}(k)$ ($\mathbf{W}_{k\tau}$). So can multiply matrices.
- Formalism should allow for studies of adiabatic fluctuations for contracting solutions. Duality exists for power spectrum between inflating and contracting solutions. Simply exchange $a(\tau)$.

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