Cosmological backreaction in simulations with numerical relativity

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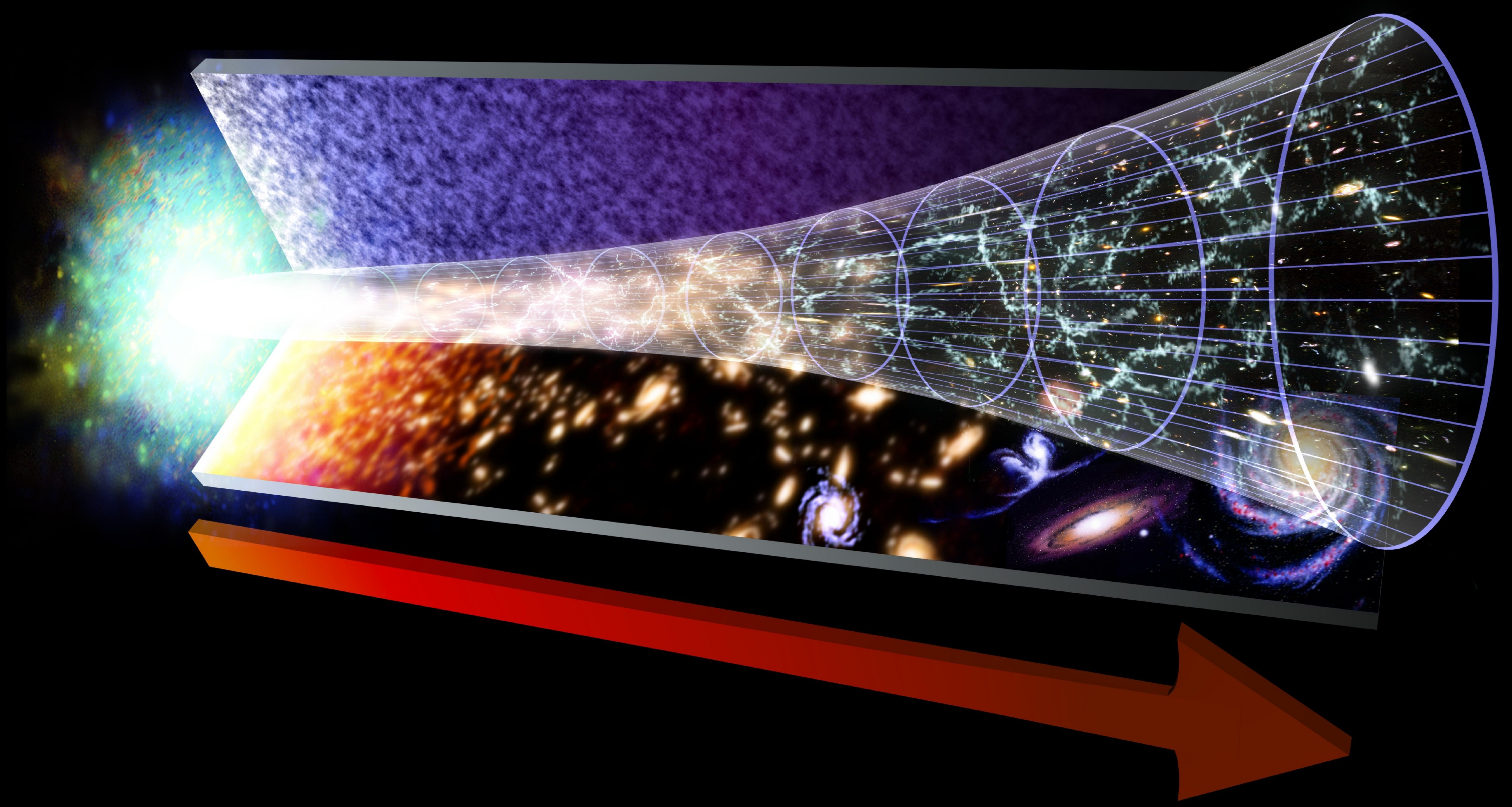
UNIVERSITY OF CAMBRIDGE

Herchel Smith Fund

The Stephen Hawking Centre for Theoretical Cosmology

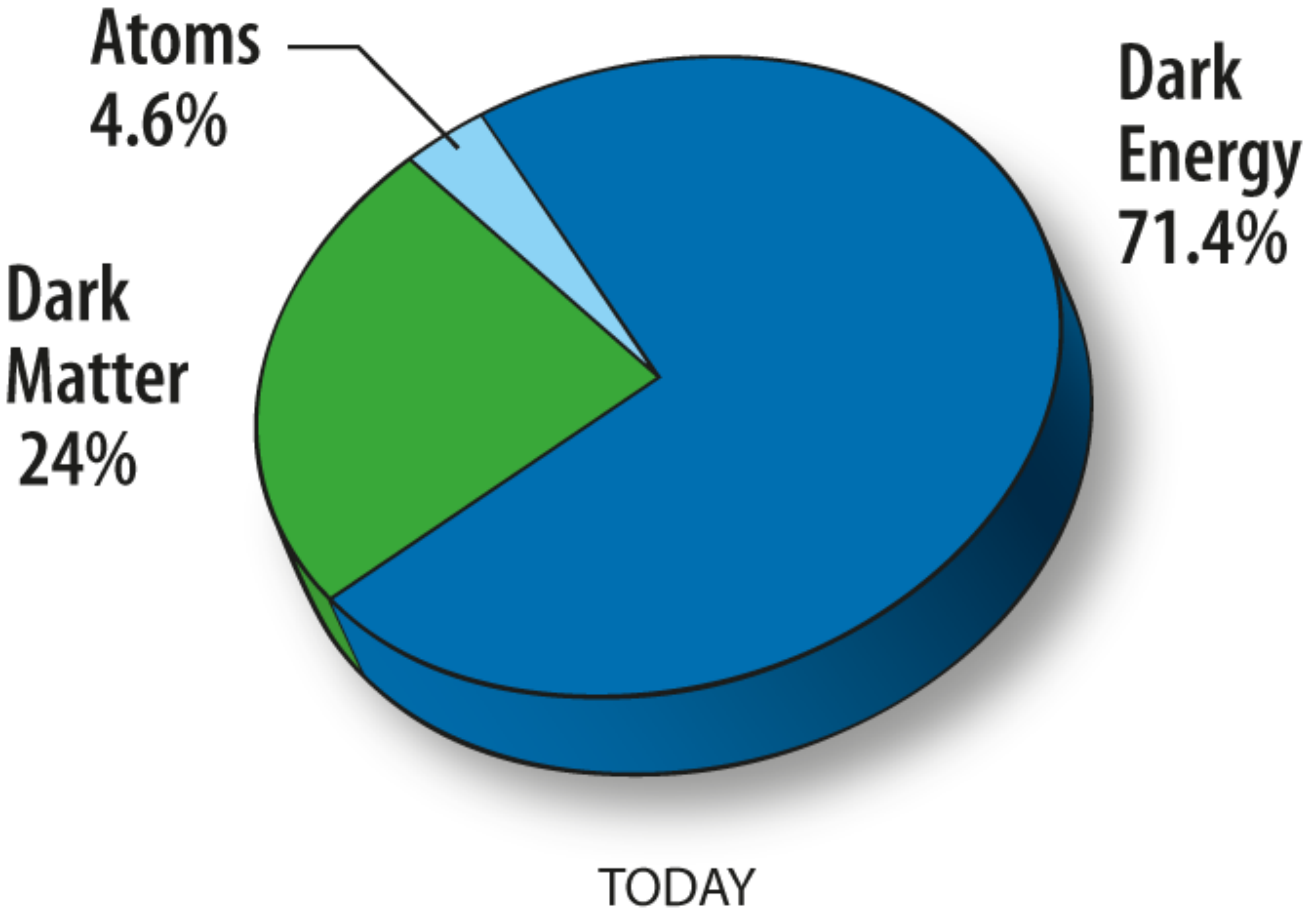
Cosmology



The standard model

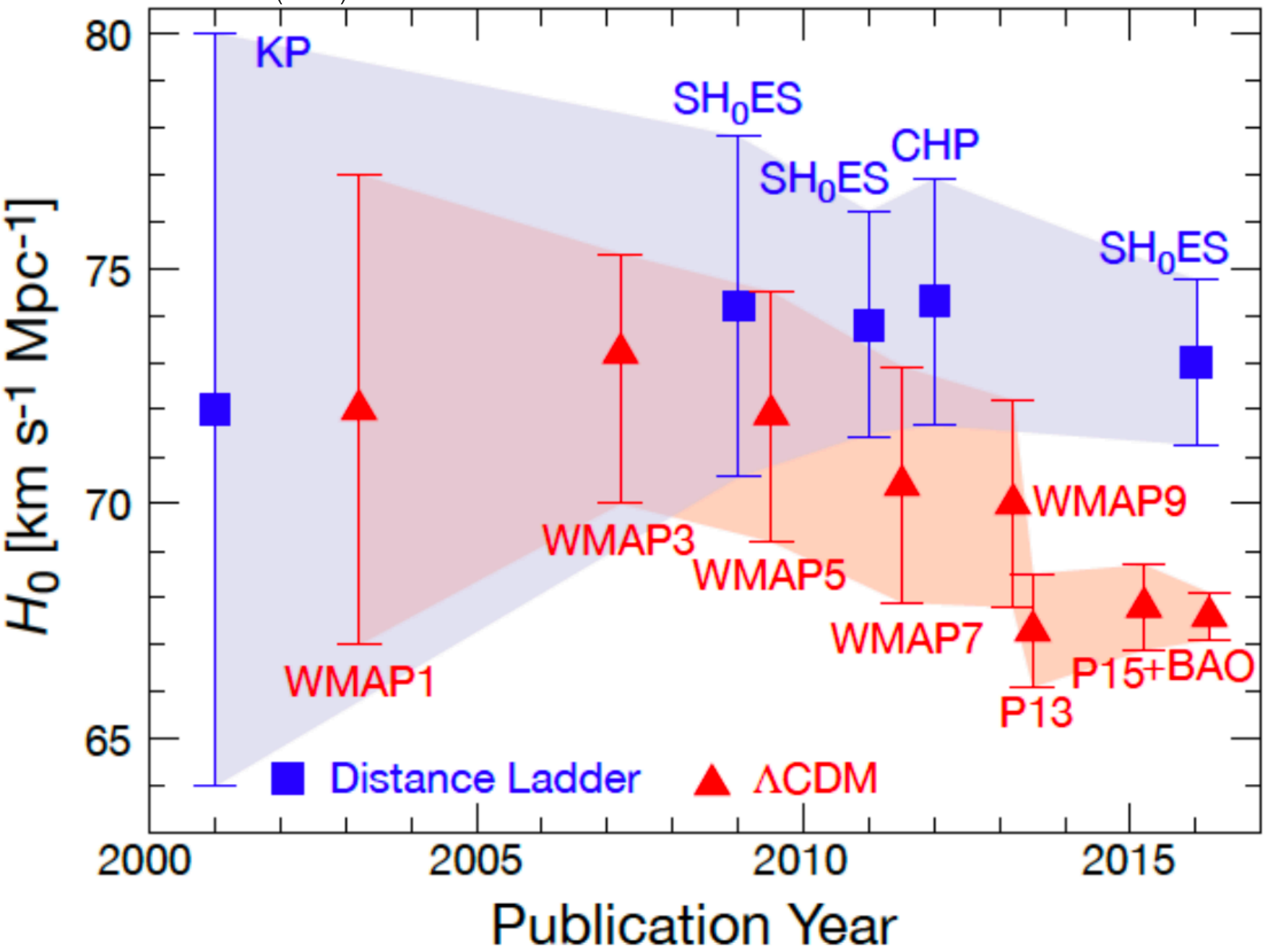
- Based on General Relativity
- Flat, Lambda Cold Dark Matter - LCDM
- Successful in explaining most cosmological observations in a surprisingly simple framework
  - But, there are some interesting tensions

e.g. the Hubble tension



Hinshaw et. al (2013)

Freedman (2017)



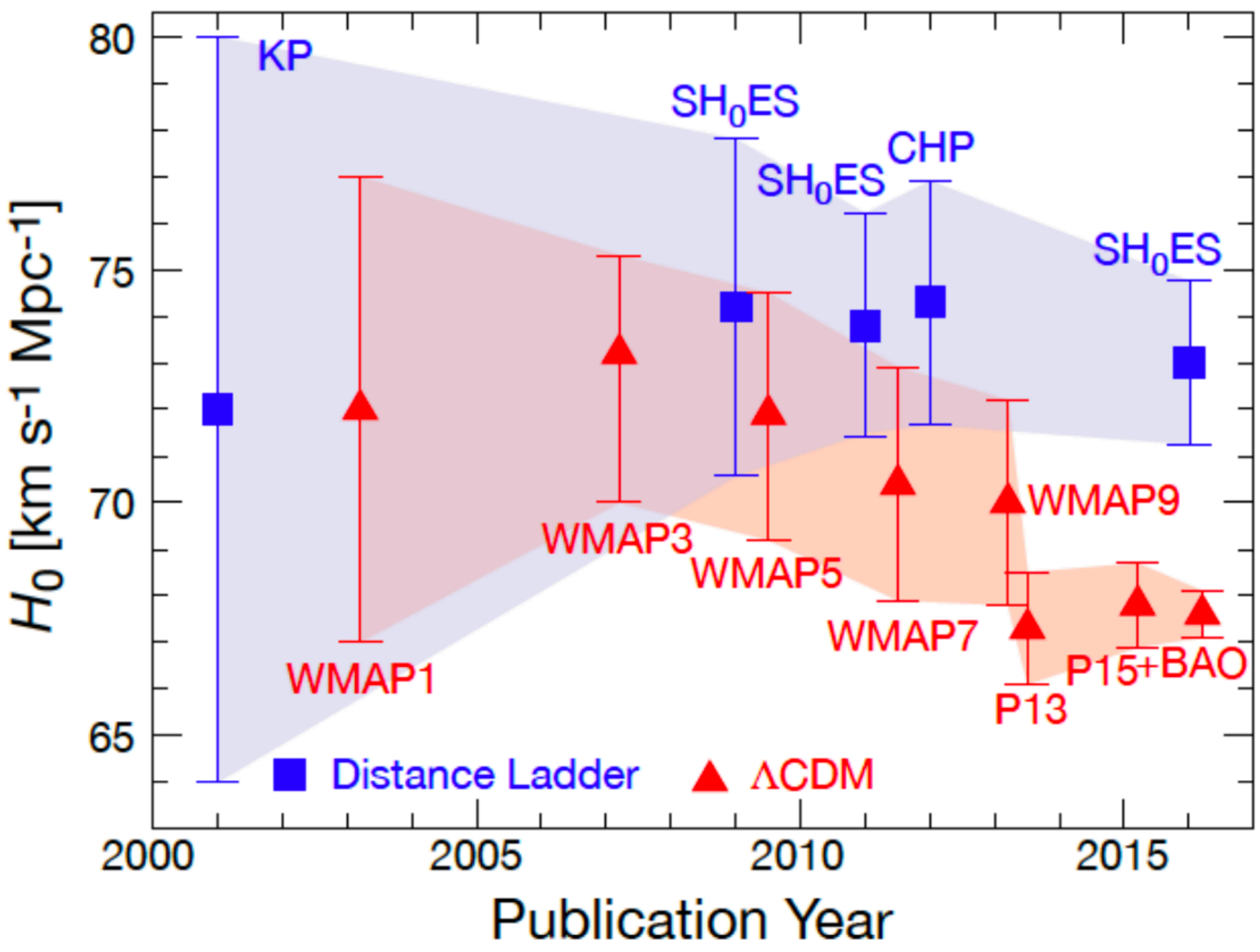
Late Universe
(supernova + cepheids)

i.e., measurements

Early Universe (CMB)

i.e., our standard model

Nobody really knows why



- Unknown systematics?
- Redshift uncertainties?
- Early Universe physics?
- Calibration of distances for SNe?
- Modified gravity?
- Early Dark Energy?
- Assumptions in LCDM?
- The list goes on…

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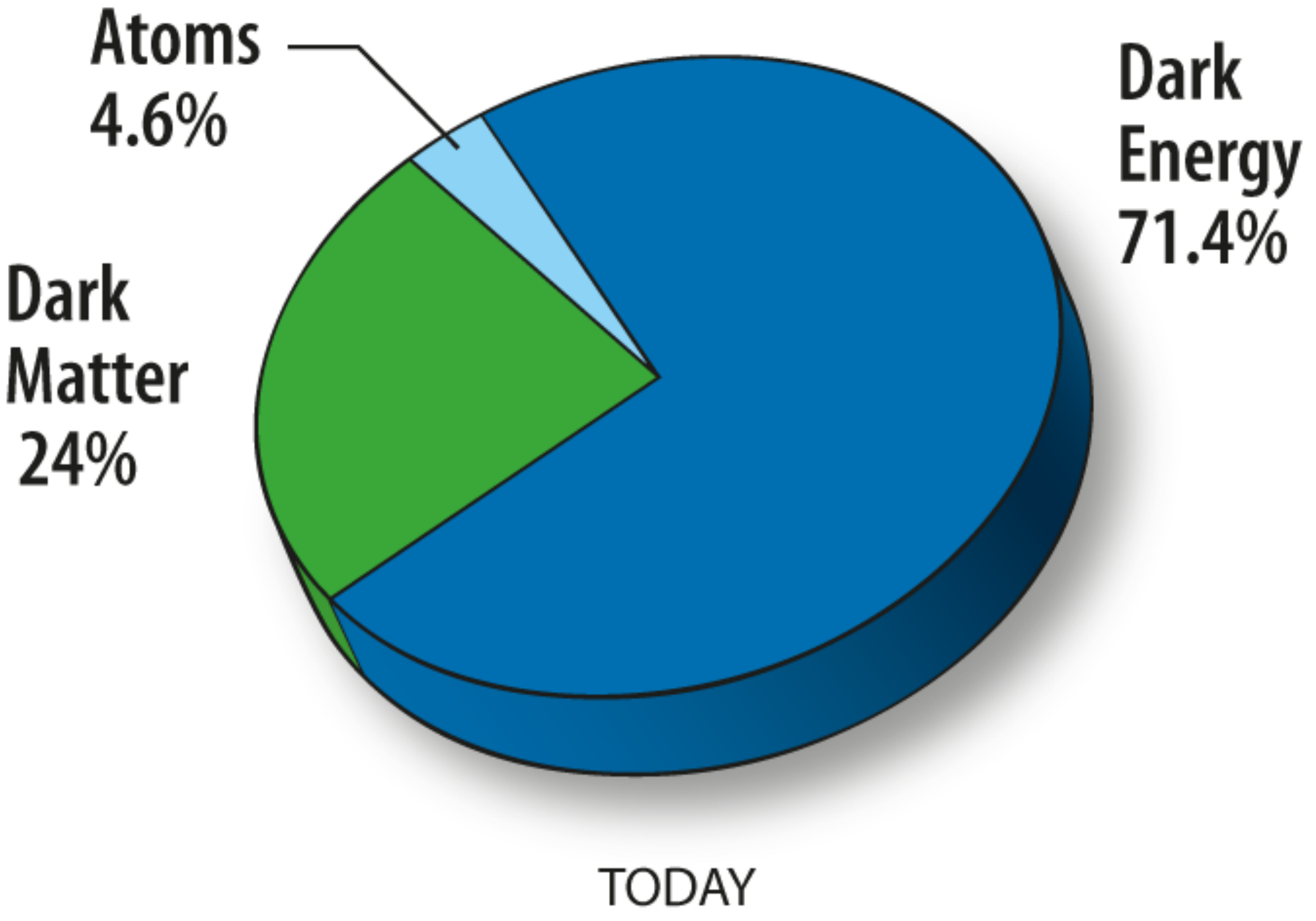
  - But, there are some interesting tensions
  - …and 95% of the Universe remains unexplained
  - ***New physics beyond LCDM?***
  - ***Modifications to GR?***



Hinshaw et. al (2013)

The standard model

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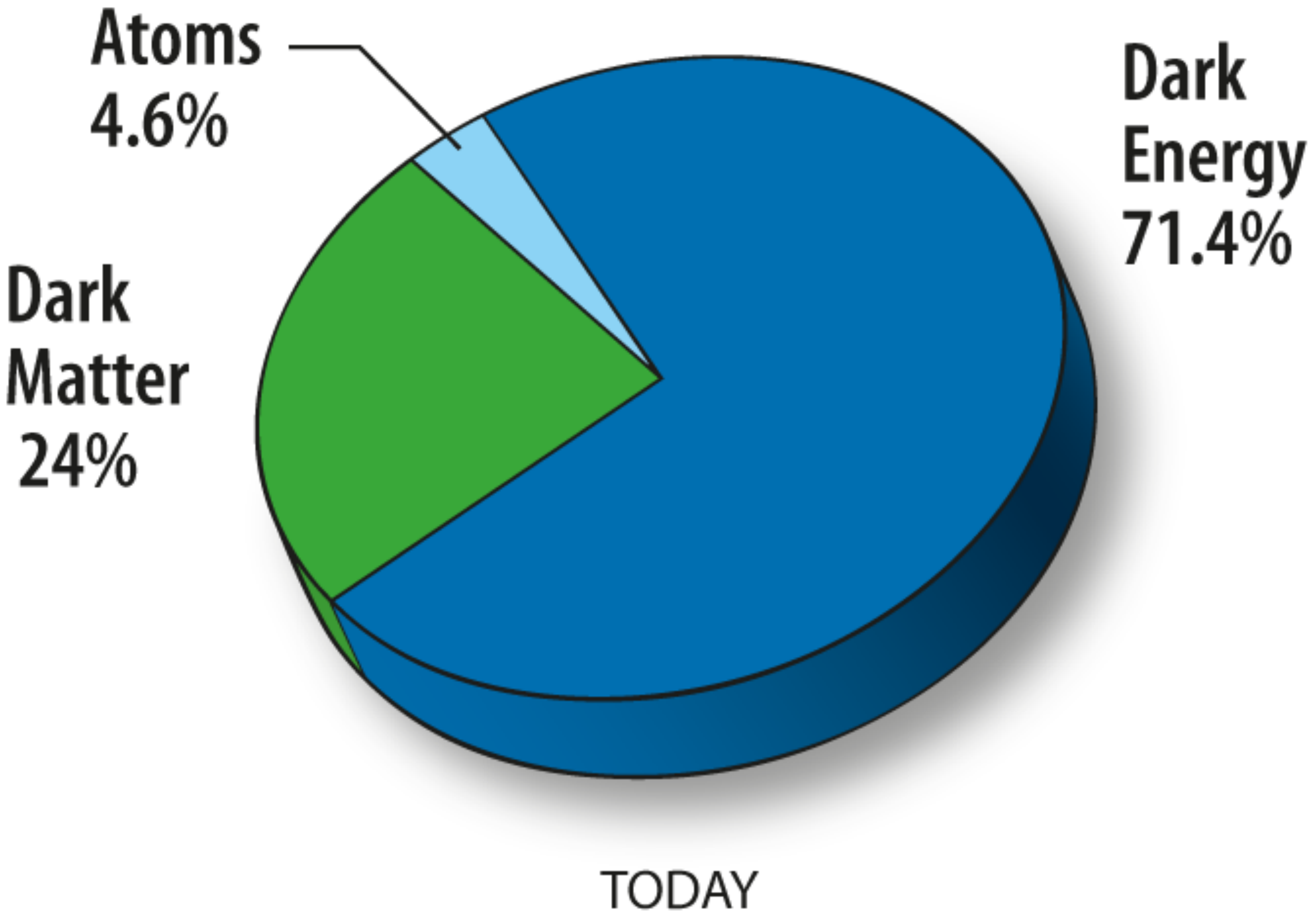
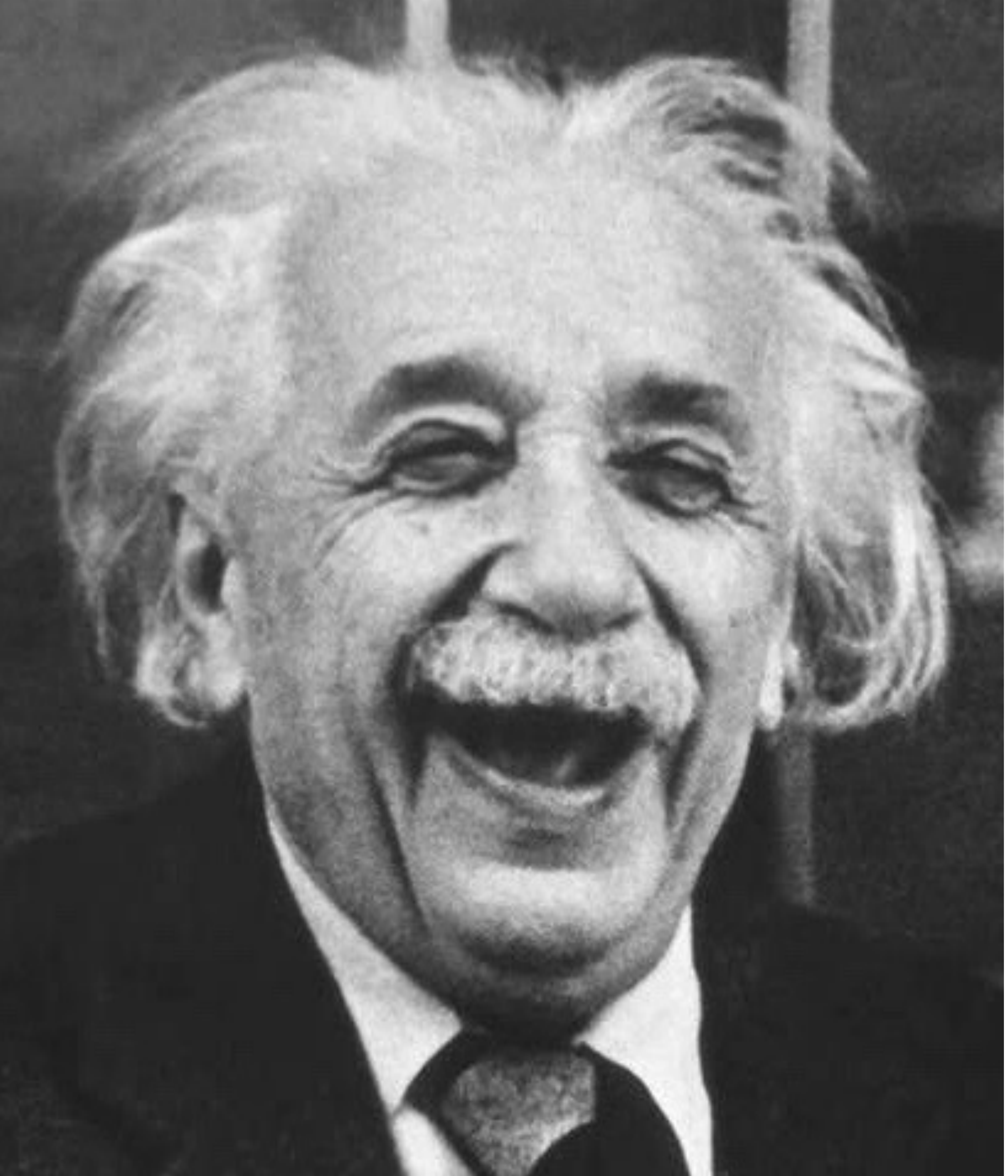
    - But, there are some interesting tensions
    - …and 95% of the Universe remains unexplained
    - ***New physics beyond LCDM?***
    - ***Modifications to GR?***
    - ***Existing physics that's neglected?***



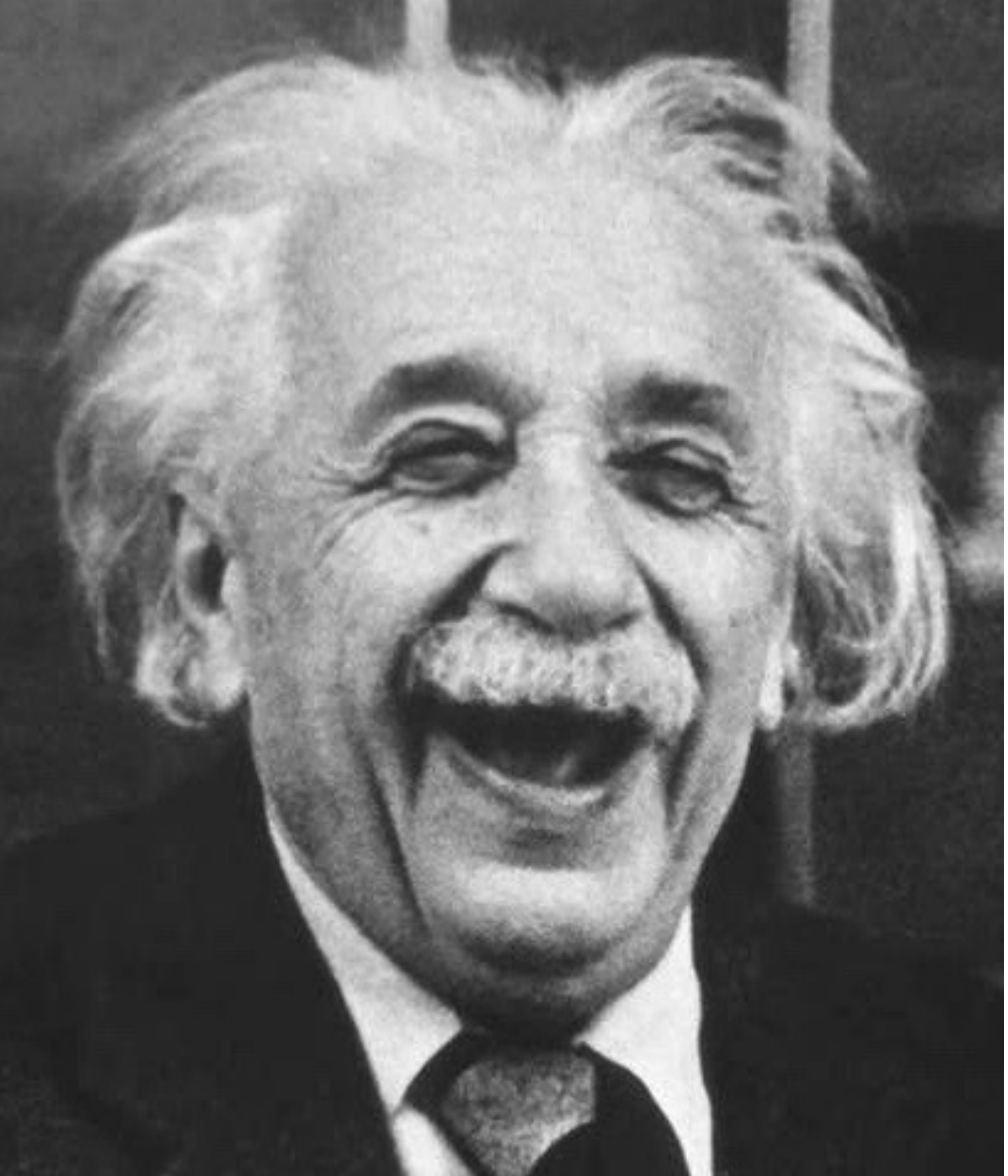
Hinshaw et. al (2013)



Modern cosmology is based on General Relativity

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Einstein tensor Stress-energy tensor



Modern cosmology is based on
General Relativity

Spacetime = Matter

(but not actually that easy...)

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

RICCI TENSOR

RICCI SCALAR

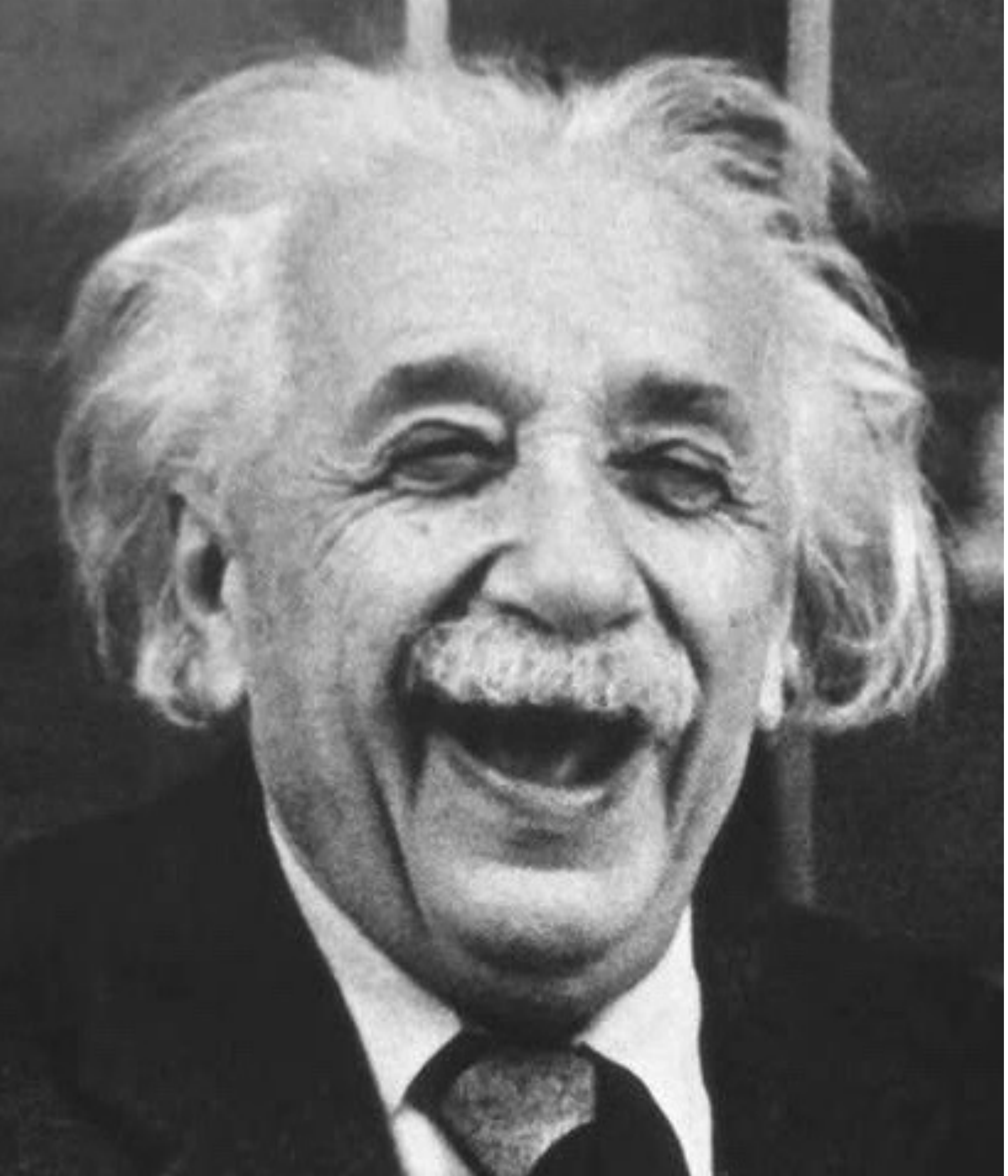
$$R_{\mu\nu} = \partial_\alpha \Gamma^\alpha_{\mu\nu} - \partial_\nu \Gamma^\alpha_{\mu\alpha} + \Gamma^\alpha_{\lambda\alpha}\Gamma^\lambda_{\mu\nu} - \Gamma^\alpha_{\lambda\nu}\Gamma^\lambda_{\mu\alpha}$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\beta}\left(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu}\right)$$

$$T_{\mu\nu} = \rho_0 h\, u_\mu u_\nu + P g_{\mu\nu}$$

(for a perfect fluid)



"Einstein's" Universe

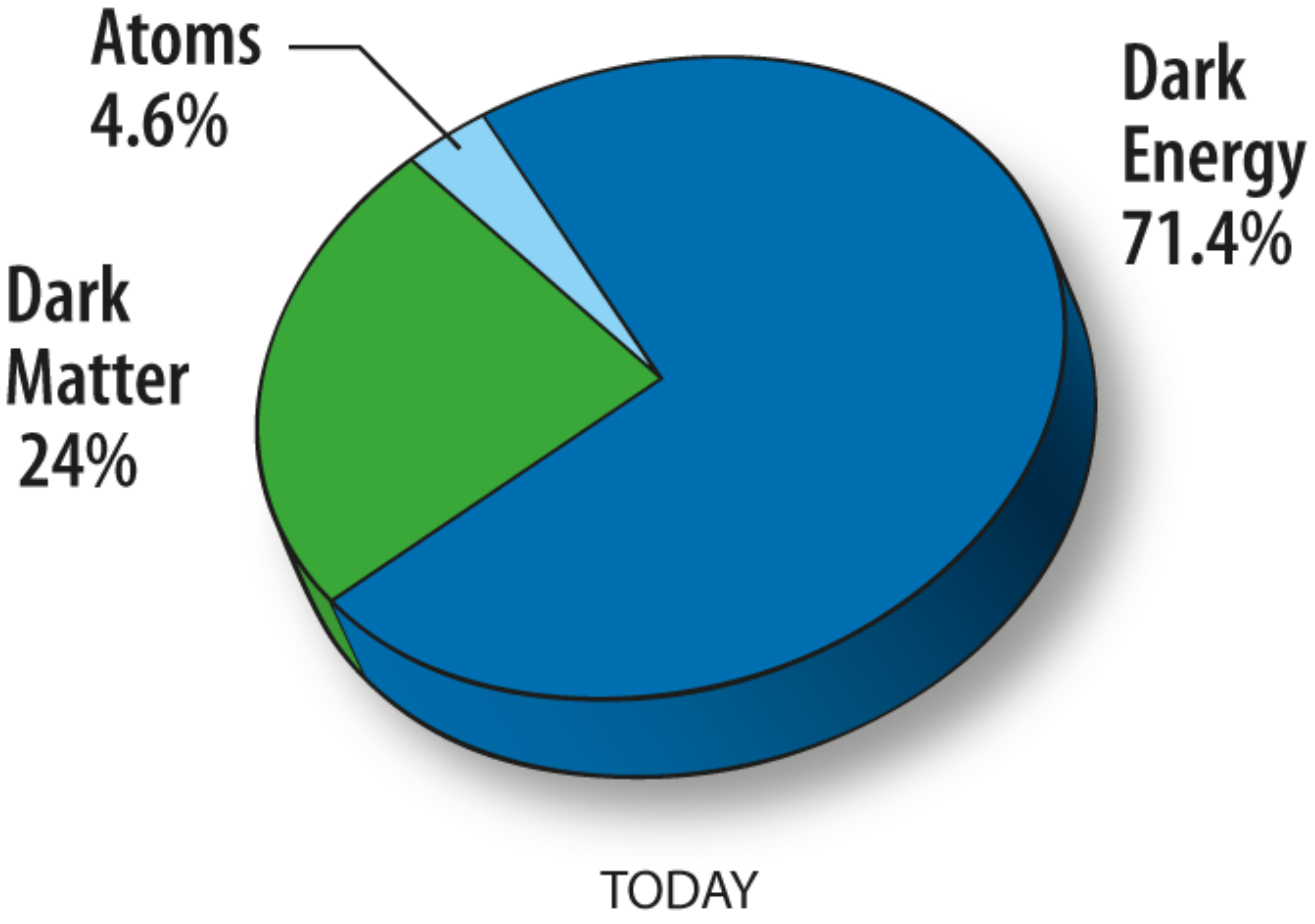
$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Simple concept; tricky equations

... so we simplify them!

The standard model

- Based on General Relativity
- Flat, Lambda Cold Dark Matter - LCDM
- Assumes the Universe is both *homogeneous* and *isotropic*



Hinshaw et. al (2013)

The standard model

Assuming **homogeneity** and **isotropy** in General Relativity gives *Friedmann-Lemaitre-Robertson-Walker (FLRW)* spacetime

$$ds^2 = a^2(\eta)\left(-d\eta^2 + \delta_{ij}dx^i dx^j\right)$$

The standard model

Solving Einstein's equations assuming **homogeneity** and **isotropy** gives the *Friedmann* equations

cosmological constant

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi G\rho a^2}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$

density

curvature (constant)

scale factor (size)

$$\frac{a''}{a} = -\frac{4\pi G\rho a^2}{3} + \frac{\Lambda c^2}{3}$$

Numerical cosmology

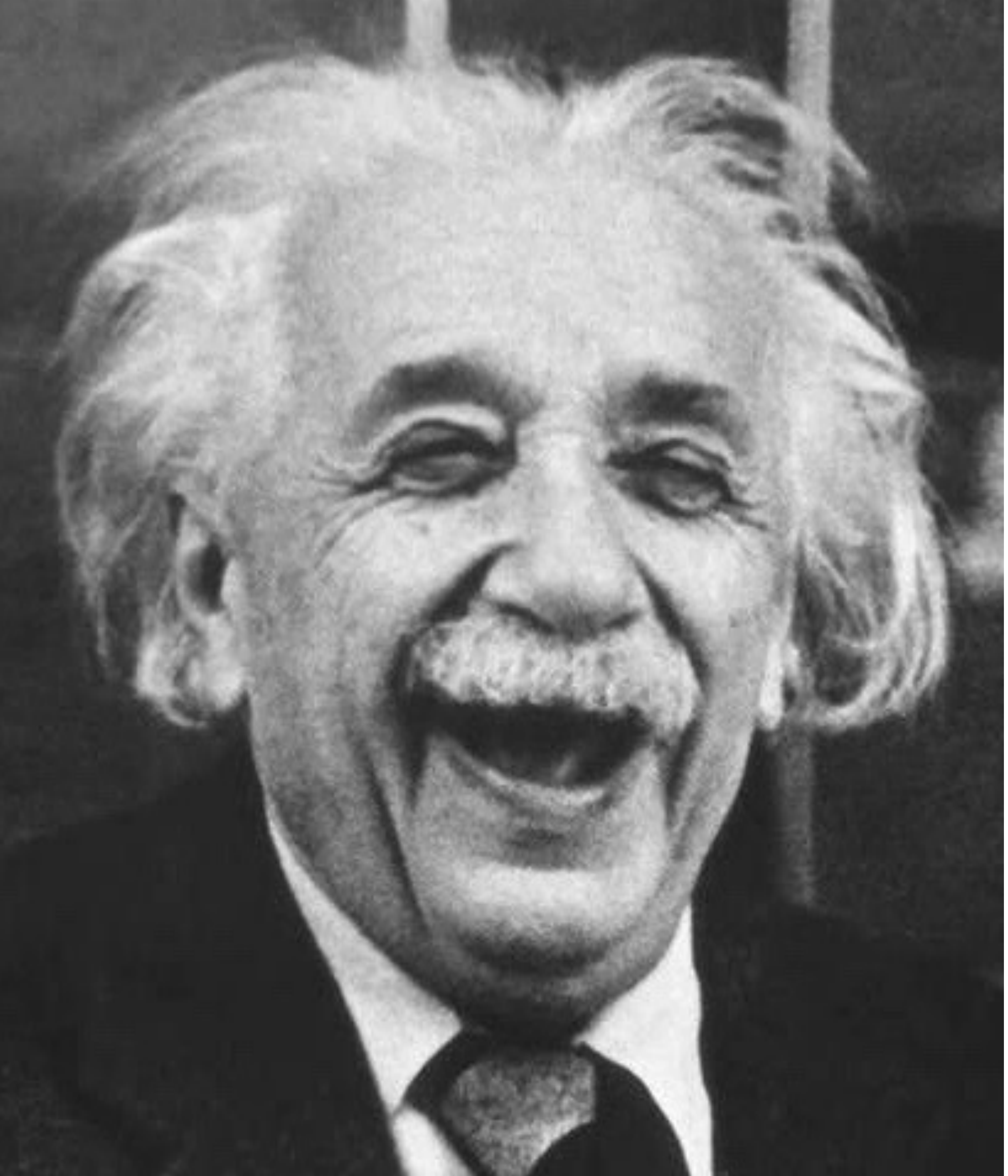
Springel et. al (2005)

- Particle methods + Newtonian gravity + FLRW expansion
- Our point of comparison for our cosmological observations
- Match many properties of our late-time Universe

Numerical cosmology

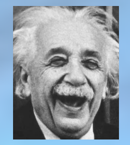
Springel et. al (2005)

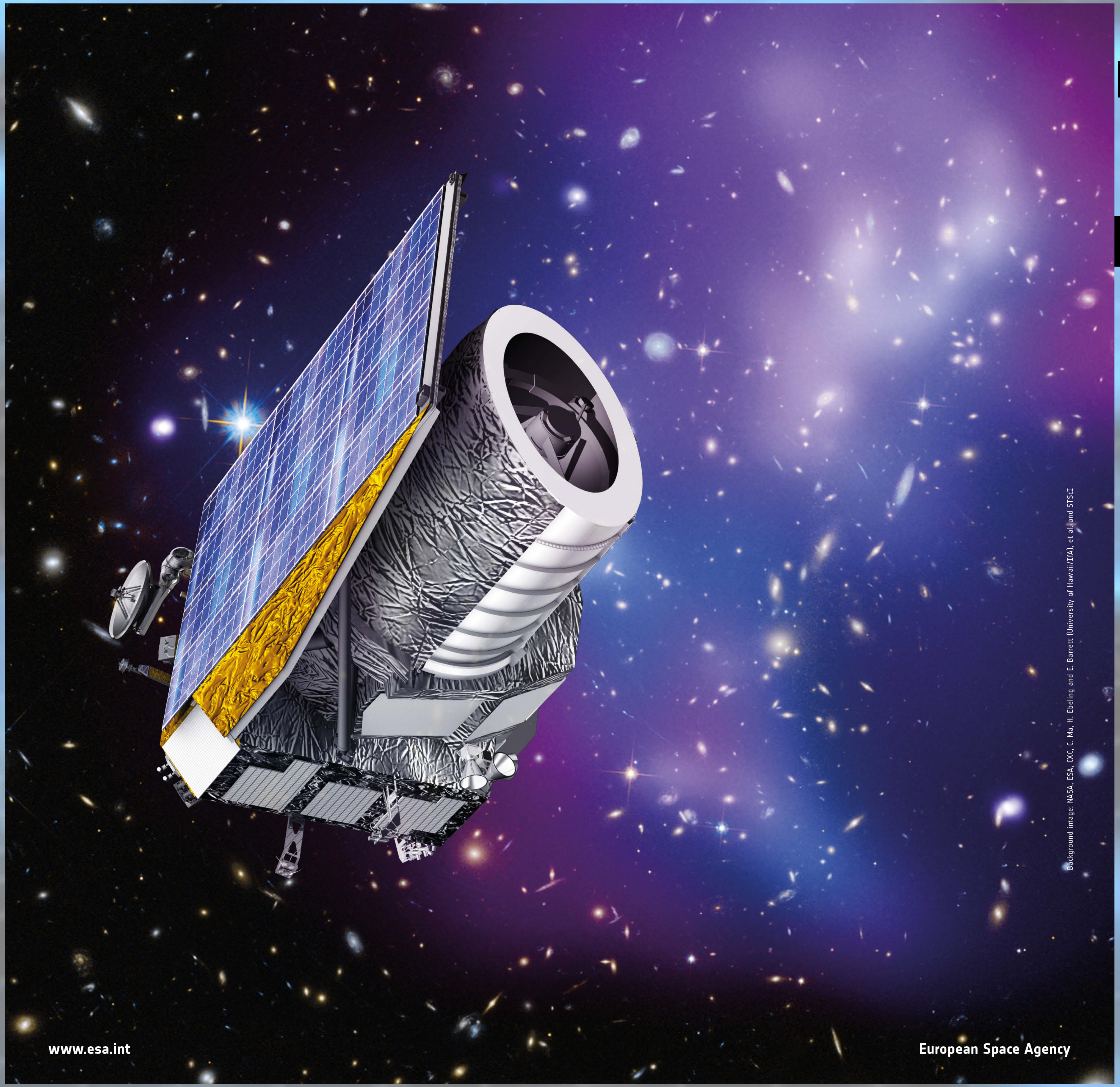
- Particle methods + Newtonian gravity + FLRW expansion
- Homogeneous background *remains* homogeneous
- **Matter cannot interact with spacetime**



$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Matter & spacetime are intimately linked
- Our universe is ***"lumpy"***
- ***Lumpy*** matter implies ***lumpy*** spacetime

 Local matter inhomogeneities —> local curvature



ropagation, and hence our observations

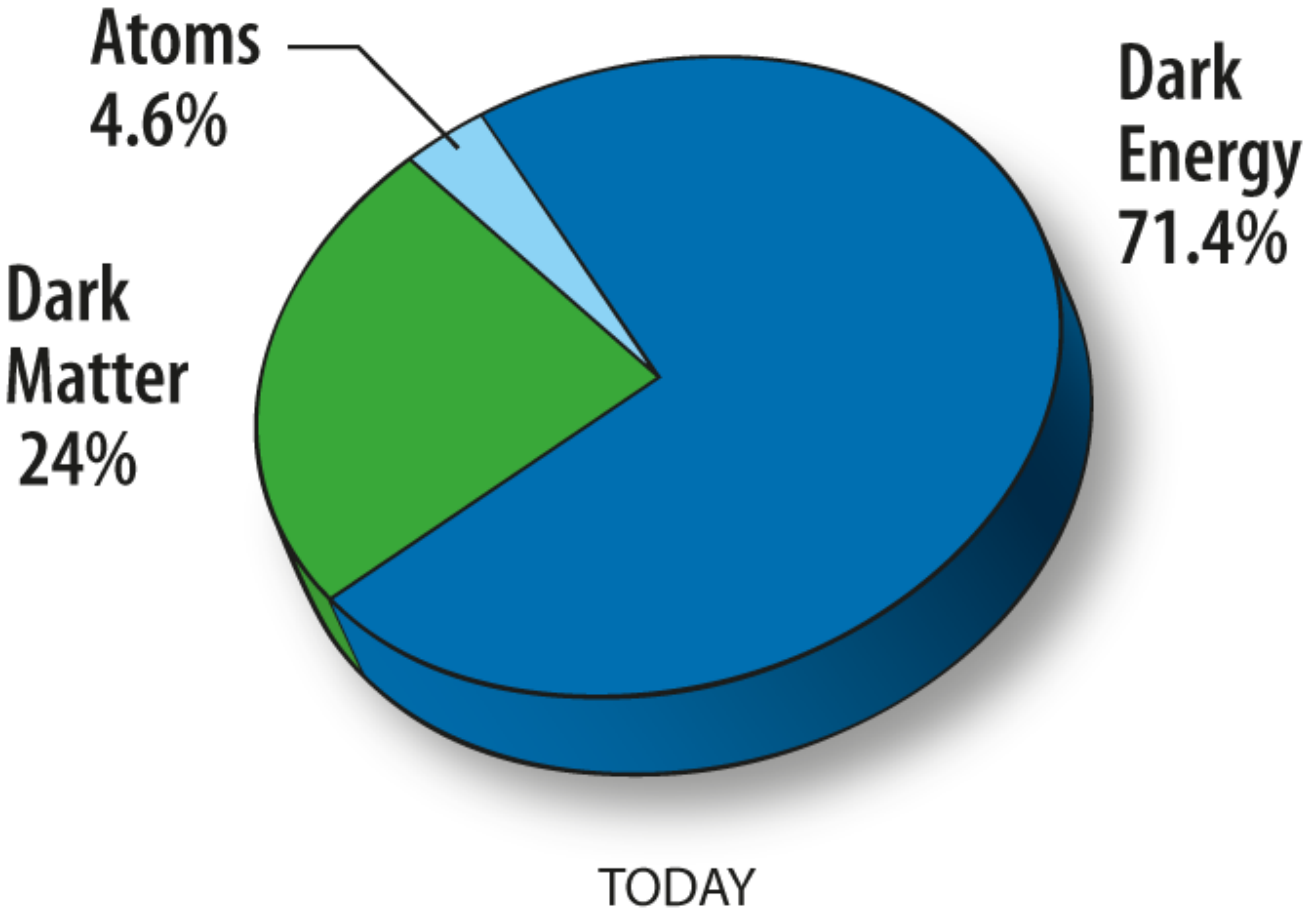
Is it significant for upcoming surveys?

Image: James Mertens

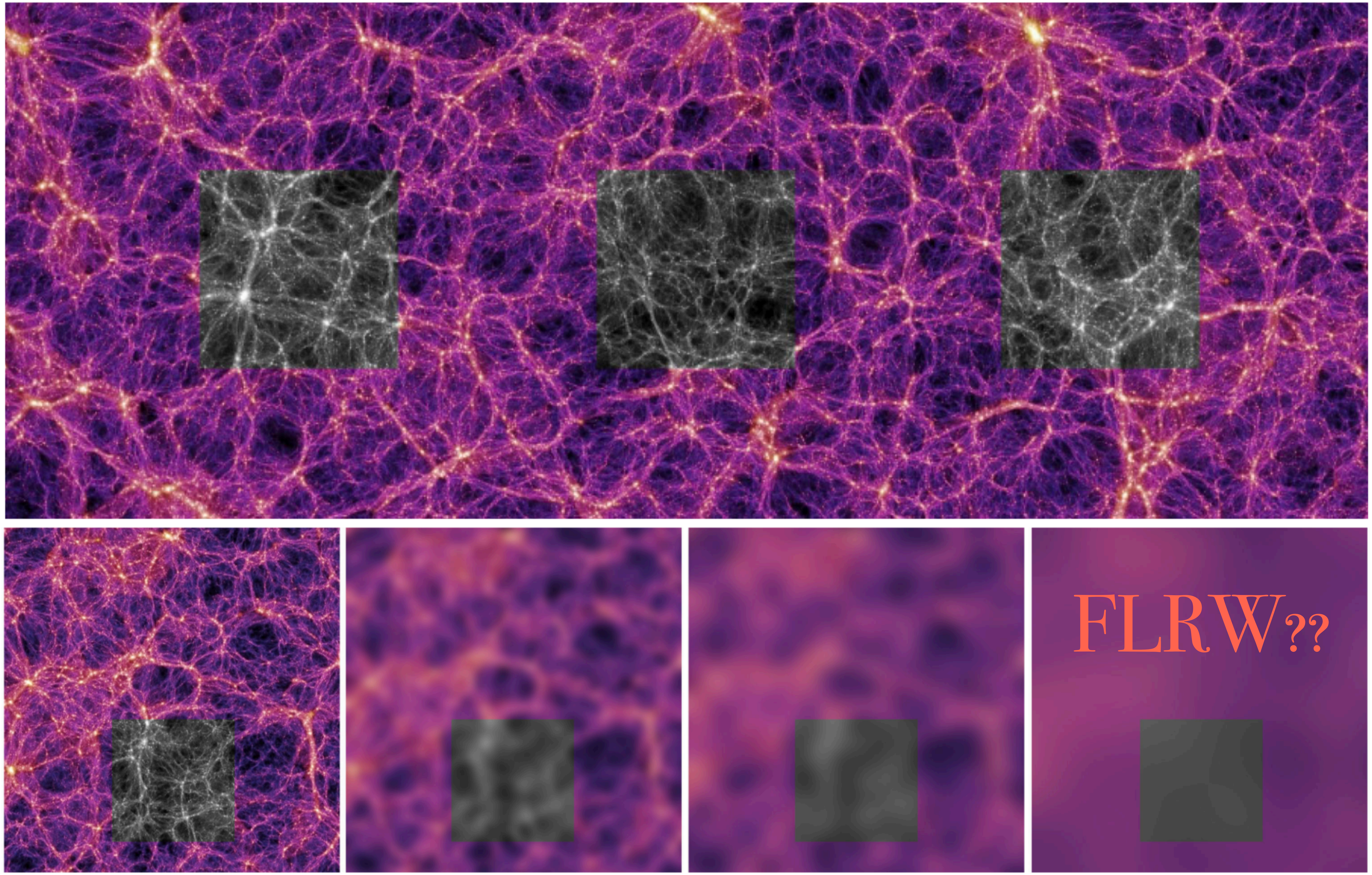
The standard model

- Based on General Relativity
- Flat, Lambda Cold Dark Matter - LCDM
- Assumes the Universe is both *homogeneous* and *isotropic*
  - ***Justification:* Universe is homogeneous on scales > 80 - 100 Mpc**

➡ **Can we really smooth over all the structure beneath this scale?**



Hinshaw et. al (2013)



(not necessarily)

FIG. 1: Structure in the Millennium simulation [40] (from [26]). Can we describe the universe as smooth on scales of order 150Mpc, shown here in the black and white boxes (top panel)? The averaging problem is shown in the bottom row: how do we go from left to right? Does this process give us corrections to the 'background', or is it the 'background' itself? How does it relate to the 'background' left at the end of inflation?

Clarkson *et al.* 2011 (arXiv:1109.2314)

FLRW

*Perfectly homogeneous &
isotropic, pressure-less spacetime*

Inhomogeneous

*Averaged evolution of fully
inhomogeneous, anisotropic spacetime
in nonlinear GR, for a family of*
comoving observers

FLRW

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G\rho}{3} + \frac{\Lambda c^2}{3}$$

Inhomogeneous

co-moving observers

$$\left(\frac{\dot{a}_D}{a_D}\right)^2 = \frac{8\pi G\langle\rho\rangle_D}{3} + \frac{\Lambda c^2}{3} - \frac{\langle\mathcal{R}\rangle_D c^2}{6} - \frac{\mathcal{Q}_D c^2}{6}$$

$$\frac{\ddot{a}_D}{a_D} = -\frac{4\pi G\langle\rho\rangle_D}{3} + \frac{\Lambda c^2}{3} + \frac{\mathcal{Q}_D c^2}{3}$$

See Buchert et. al (2000-2019)

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FLRW SCALE FACTOR REPLACED BY RATE-OF-CHANGE OF VOLUME

$$\frac{\ddot{a}_D}{a_D} = -\frac{4\pi G\langle\rho\rangle_D}{3} + \frac{\Lambda c^2}{3} + \frac{\mathcal{Q}_D c^2}{3}$$

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FLRW DENSITY REPLACED WITH AVERAGE DENSITY WITHIN DOMAIN

$$\frac{\ddot{a}_D}{a_D} = -\frac{4\pi G\langle\rho\rangle_D}{3} + \frac{\Lambda c^2}{3} + \frac{\mathcal{Q}_D c^2}{3}$$

See Buchert et. al (2000-2019)

FLRW

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G\rho}{3} + \frac{\Lambda c^2}{3}$$

Inhomogeneous

co-moving observers

$$\left(\frac{\dot{a}_D}{a_D}\right)^2 = \frac{8\pi G\langle\rho\rangle_D}{3} + \frac{\Lambda c^2}{3} - \frac{\langle\mathcal{R}\rangle_D c^2}{6} - \frac{\mathcal{Q}_D c^2}{6}$$

COSMOLOGICAL CONSTANT IS THE SAME

$$\frac{\ddot{a}_D}{a_D} = -\frac{4\pi G\langle\rho\rangle_D}{3} + \frac{\Lambda c^2}{3} + \frac{\mathcal{Q}_D c^2}{3}$$

See Buchert et. al (2000-2019)

FLRW

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Curvature is allowed to (and does) evolve

$$\frac{\ddot{a}_D}{a_D} = -\frac{4\pi G\langle\rho\rangle_D}{3} + \frac{\Lambda c^2}{3} + \frac{\mathcal{Q}_D c^2}{3}$$

See Buchert et. al (2000-2019)

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$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$

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SMALL-SCALE STRUCTURES THAT ARE SMOOTHED OVER AFFECT AVERAGED EVOLUTION

$$\frac{\ddot{a}_D}{a_D} = -\frac{4\pi G\langle\rho\rangle_D}{3} + \frac{\Lambda c^2}{3} + \frac{\mathcal{Q}_D c^2}{3}$$

See Buchert et. al (2000-2019)

FLRW

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$

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Inhomogeneous

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$$\mathcal{Q}_D \equiv \frac{2}{3}\left(\langle\Theta^2\rangle_D - \langle\Theta\rangle_D^2\right) - 2\langle\sigma^2\rangle_D$$

$$\frac{\ddot{a}_D}{a_D} = -\frac{4\pi G\langle\rho\rangle_D}{3} + \frac{\Lambda c^2}{3} + \frac{\mathcal{Q}_D c^2}{3}$$

See Buchert et. al (2000-2019)

Springel et. al (2005)

IF the average of the Universe *always* coincides with FLRW…

$$\mathcal{Q}_D = 0, \quad \langle \mathcal{R} \rangle_D \propto k$$

Springel et. al (2005)

DOES the average of the Universe *always* coincide with FLRW?

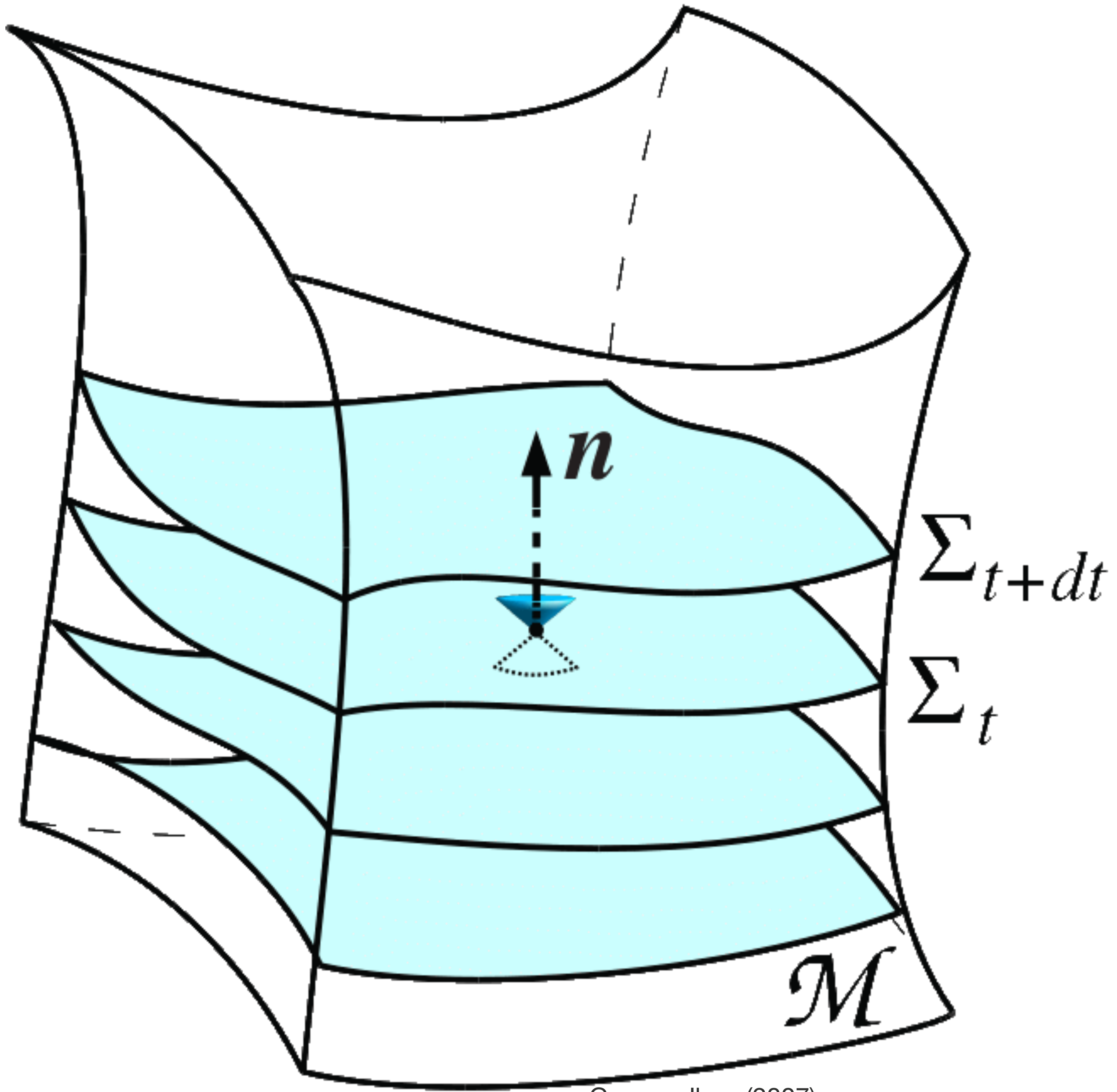
??? $\mathcal{Q}_D = 0, \quad \langle \mathcal{R} \rangle_D \propto k$???

Requires evolving and averaging an inhomogeneous universe
with no simplifying assumptions

Numerical relativity

Allows us to solve the field equations with no simplifying assumptions to gravity (i.e., not weak) or geometry (i.e., not flat locally OR globally)

We start by splitting 4D spacetime into 3D space + 1D time

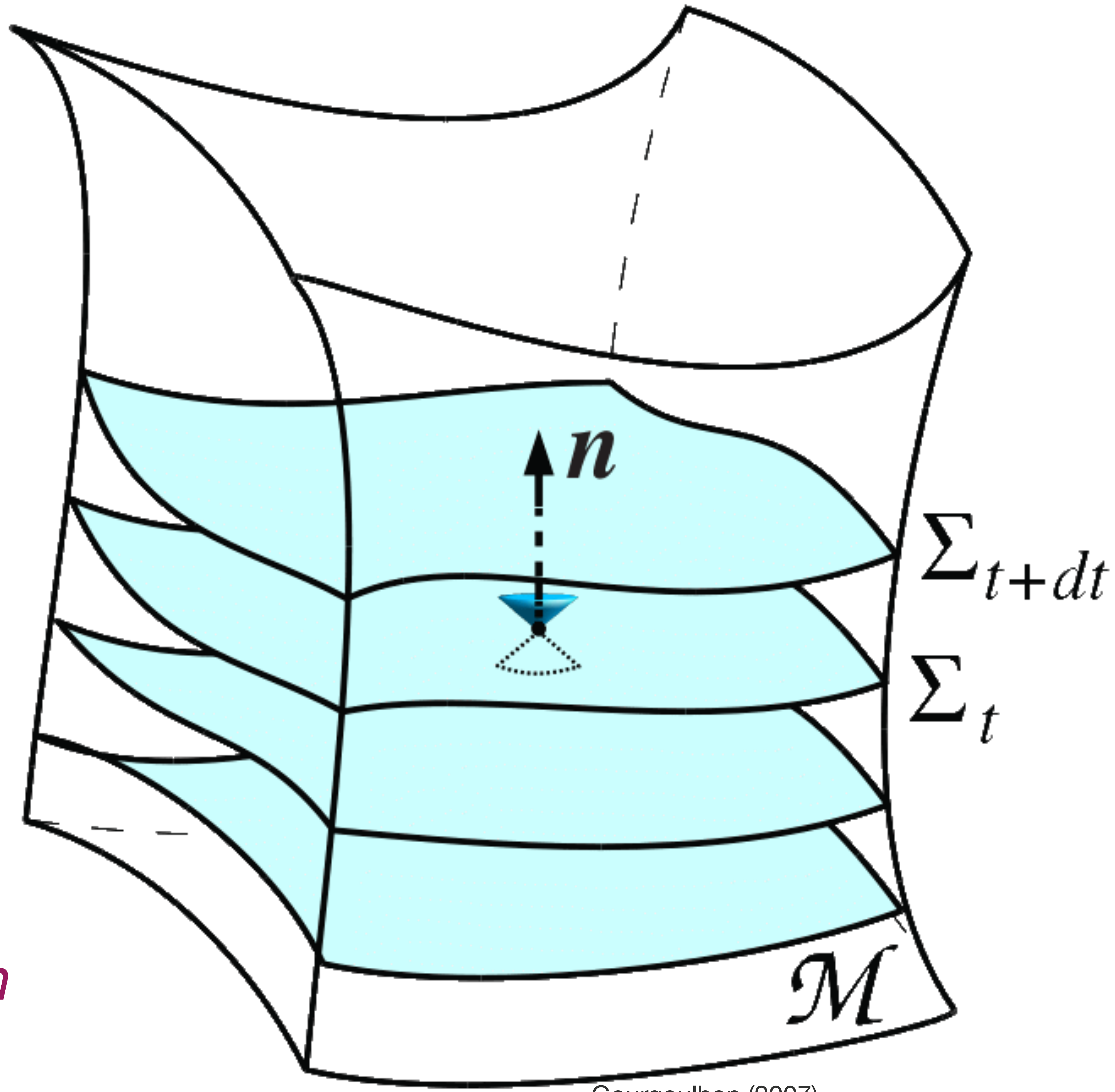


Gourgoulhon (2007)

Numerical relativity

$$\gamma_{\mu\nu} \equiv g_{\mu\nu} + n_\mu n_\nu$$

spatial metric — full 4D metric — normal vector



Gourgoulhon (2007)

projects 4D objects into the spatial surfaces

So we can write the field equations as a set of evolution eqns for purely spatial objects

Use various projections / contractions of the Riemann tensor:

$$R^{\alpha}{}_{\mu\beta\nu} \equiv \partial_\beta \, {}^{(4)}\Gamma^{\alpha}_{\mu\nu} - \partial_\nu \, {}^{(4)}\Gamma^{\alpha}_{\mu\beta} + {}^{(4)}\Gamma^{\alpha}_{\lambda\beta} \, {}^{(4)}\Gamma^{\lambda}_{\mu\nu} - {}^{(4)}\Gamma^{\alpha}_{\lambda\nu} \, {}^{(4)}\Gamma^{\lambda}_{\mu\beta}$$

$${}^{(4)}\Gamma^{\alpha}_{\mu\nu} \equiv \frac{1}{2} g^{\alpha\beta} \left(\partial_\mu g_{\beta\nu} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu} \right)$$

Use various projections / contractions of the Riemann tensor:

$$R^{\alpha}{}_{\mu\beta\nu} \equiv \partial_\beta \, {}^{(4)}\Gamma^{\alpha}_{\mu\nu} - \partial_\nu \, {}^{(4)}\Gamma^{\alpha}_{\mu\beta} + {}^{(4)}\Gamma^{\alpha}_{\lambda\beta} \, {}^{(4)}\Gamma^{\lambda}_{\mu\nu} - {}^{(4)}\Gamma^{\alpha}_{\lambda\nu} \, {}^{(4)}\Gamma^{\lambda}_{\mu\beta}$$

OUR SPATIAL/PROJECTION TENSOR

e.g….

$$\gamma^{\mu}{}_{a}\gamma^{\nu}{}_{b}\gamma^{\alpha}{}_{c}\gamma^{\beta}{}_{d}R_{\mu\nu\alpha\beta}$$

Hamiltonian constraint

$$\mathcal{R} + K^2 - K_{ij}K^{ij} - \frac{16\pi G}{c^2}\rho = 0$$

Momentum constraint

$$D_j K^j{}_i - D_i K - \frac{8\pi G}{c^3} S_i = 0$$

No time evolution here...

These *must* be satisfied on EVERY spatial slice!

Now project *some* indices in the direction of the normal vector…

$$\frac{d}{dt}\gamma_{ij} = -2\alpha K_{ij}$$

UNSTABLE

(for long time evolutions)

$$\frac{d}{dt}K_{ij} = \alpha\left[\mathcal{R}_{ij} - 2K_{ik}K^{k}{}_{j} + KK_{ij}\right] - D_i D_j \alpha$$
$$- \frac{8\pi G}{c^4}\alpha\left[S_{ij} - \frac{1}{2}\gamma_{ij}(S - \rho\, c^2)\right],$$

Arnowitt, Deser & Misner (ADM; 1959)

First, a conformal decomposition (for stability reasons)

$$\gamma_{ij} = e^{4\phi}\tilde{\gamma}_{ij}$$

(ADM Formalism —> BSSN Formalism)

1959 ***1999***

after some more projections and algebra…

$$\frac{d}{dt}\phi = -\frac{1}{6}\alpha K$$

conformal factor

$$\frac{d}{dt}K = \alpha\left(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2\right) - D^2\alpha + \frac{4\pi G}{c^4}\alpha\left(S + \rho\, c^2\right)$$

trace of extrinsic curvature

$$\frac{d}{dt}\tilde{\gamma}_{ij} = -2\alpha\tilde{A}_{ij}$$

trace-free extrinsic curvature

$$\frac{d}{dt}\tilde{A}_{ij} = e^{-4\phi}\left[-(D_iD_j\alpha)^{\mathrm{TF}} + \alpha\left(\mathcal{R}_{ij}^{\mathrm{TF}} - \frac{8\pi G}{c^4}S_{ij}^{\mathrm{TF}}\right)\right] + \alpha\left(K\tilde{A}_{ij} - 2\tilde{A}_{ik}\tilde{A}^{k}{}_{j}\right)$$

$$\frac{d}{dt}\tilde{\Gamma}^i = -2\tilde{A}^{ij}\partial_j\alpha + 2\alpha\left(\tilde{\gamma}^i_{jk}\tilde{A}^{kj} - \frac{2}{3}\tilde{\gamma}^{ij}\partial_j K - \frac{8\pi G}{c^3}\tilde{\gamma}^{ij}S_j + 6\tilde{A}^{ij}\partial_j\phi\right) + \frac{2}{3}\tilde{\gamma}^i\partial_j\beta^j + \frac{1}{3}\tilde{\gamma}^{li}\partial_l\partial_j\beta^j + \tilde{\gamma}^{lj}\partial_j\partial_l\beta^i$$

introduced to preserve hyperbolicity

$$\frac{d}{dt}\phi = -\frac{1}{6}\alpha K$$

conformal factor

$$\frac{d}{dt}K = \alpha\left(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2\right) - D^2\alpha + \frac{4\pi G}{c^4}\alpha\left(S + \rho\, c^2\right)$$

$$\frac{d}{dt}\tilde{\gamma}_{ij} = \ldots$$

$$\frac{d}{dt}\tilde{A}_{ij} = \ldots$$

$$+ \ldots$$

$$\frac{d}{dt}\tilde{\Gamma}^i = \ldots$$

$$+ \frac{2}{3}\tilde{\gamma}^i\partial_j\beta^j + \frac{1}{3}\tilde{\gamma}^{li}\partial_l\partial_j\beta^j + \tilde{\gamma}^{lj}\partial_j\partial_l\beta^i$$

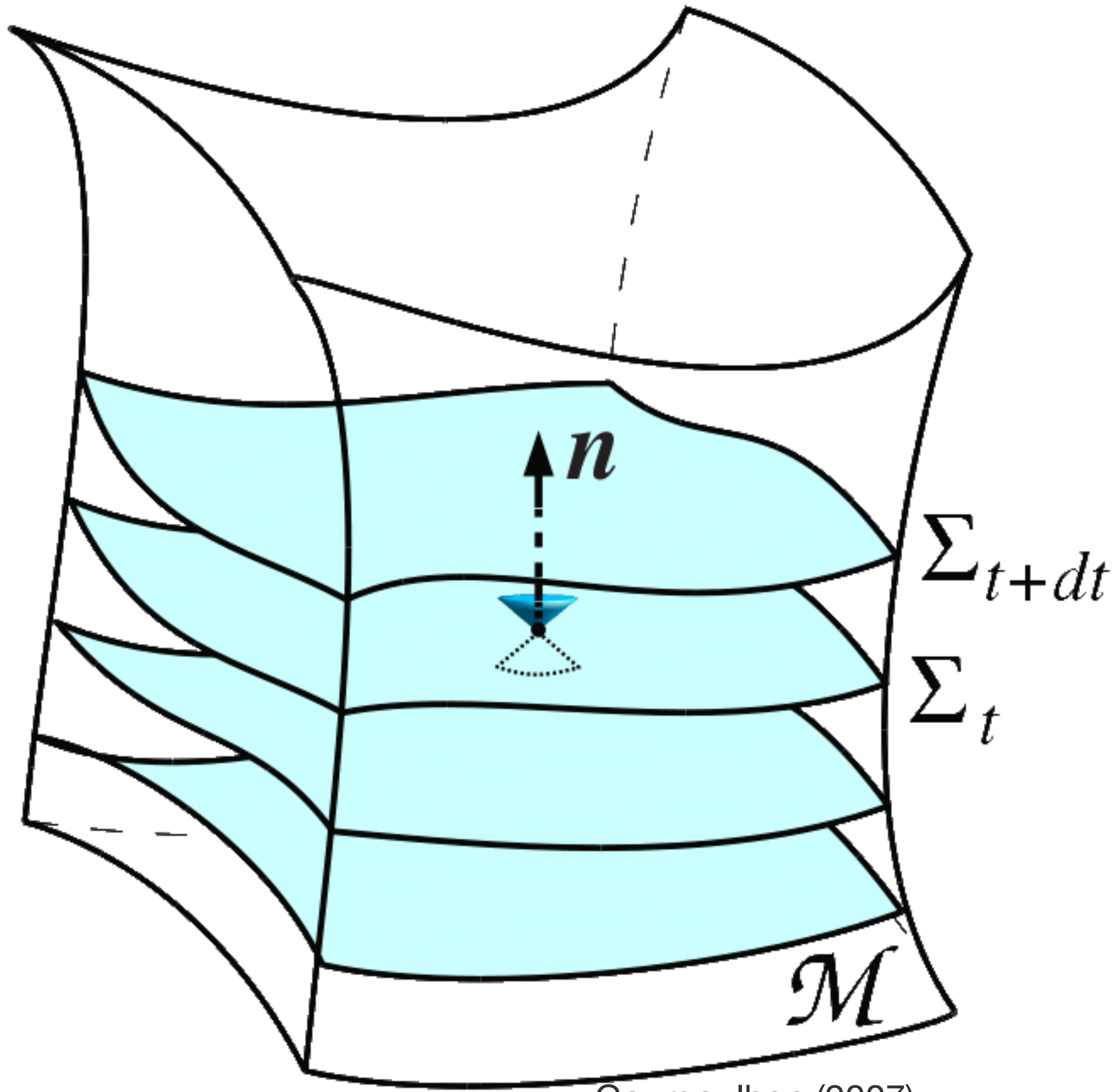
introduced to preserve hyperbolicity

Field equations in terms of purely spatial objects

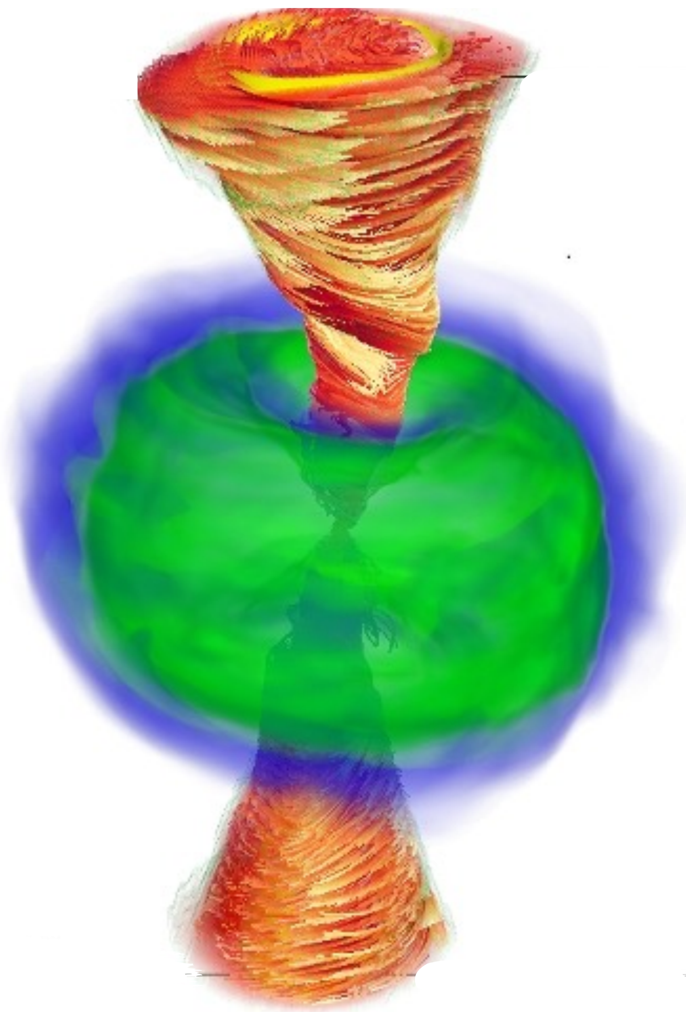
tell us how our chosen spatial slice evolves in time for a ***completely general*** *metric!*

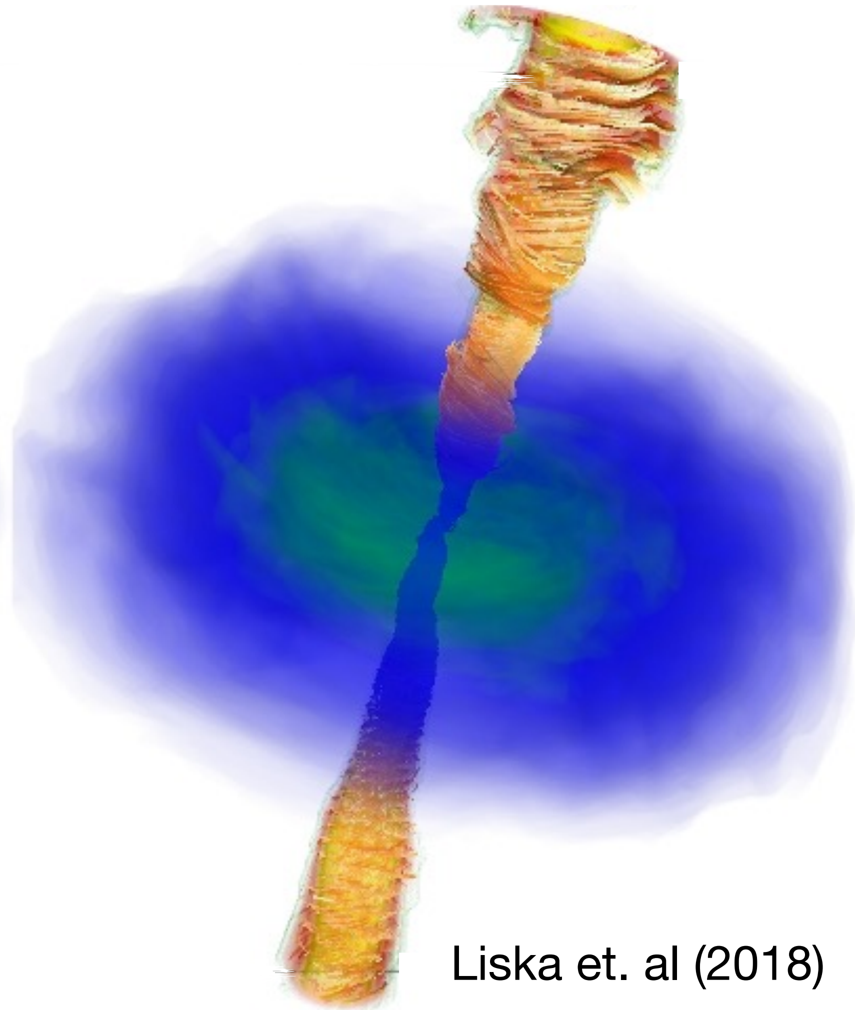
i.e.… no background or simplifications!

Numerical relativity

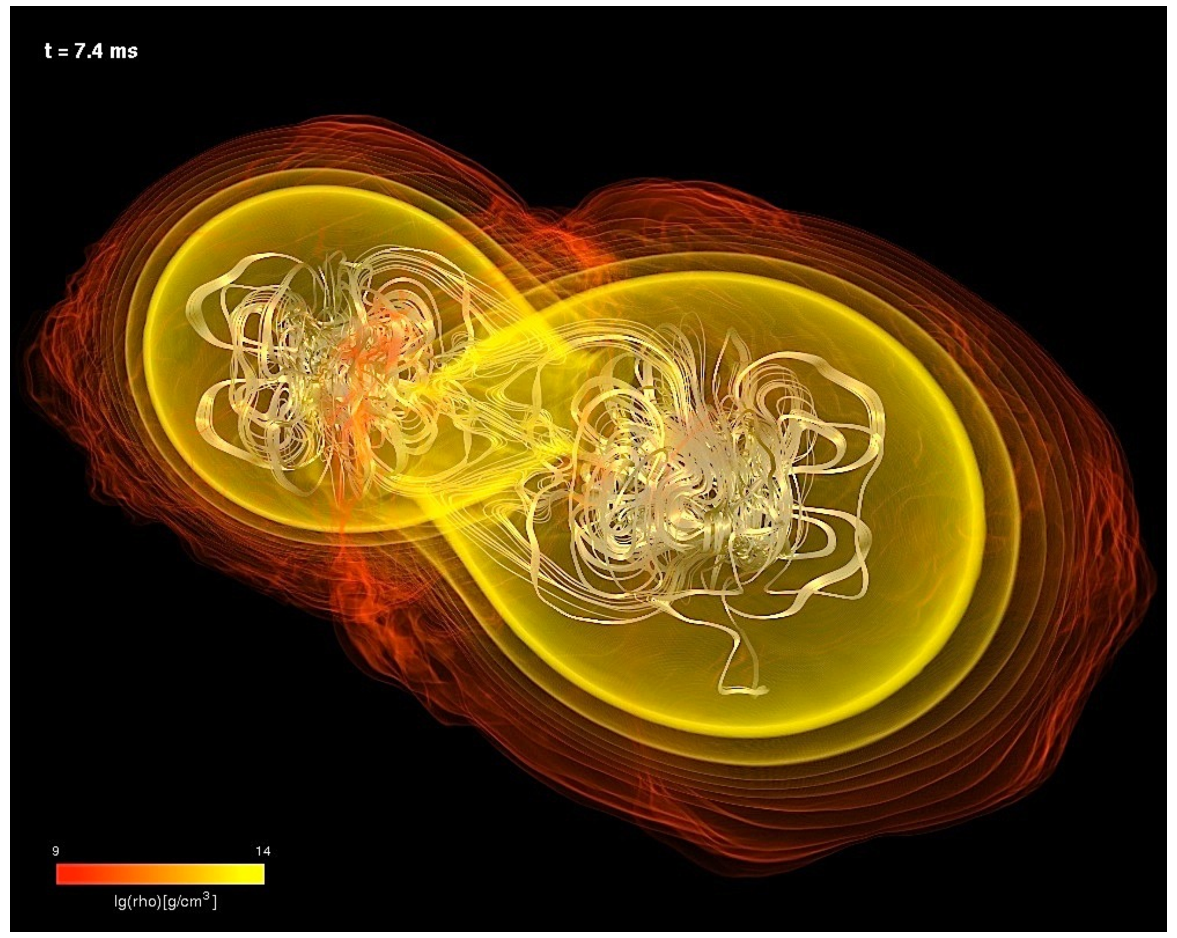


Gourgoulhon (2007)

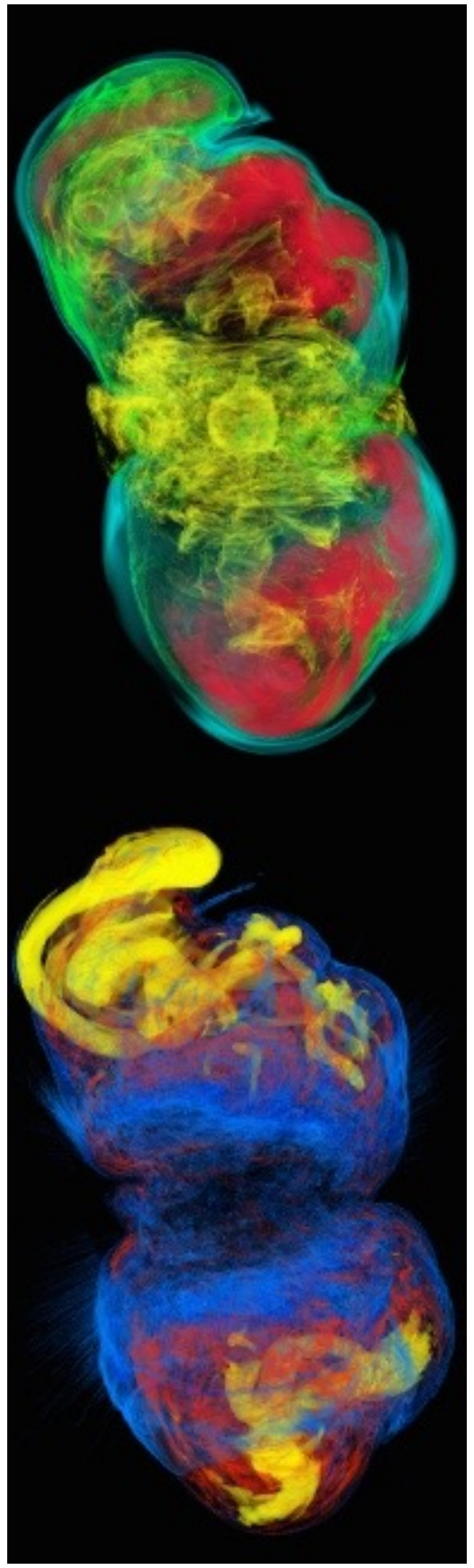


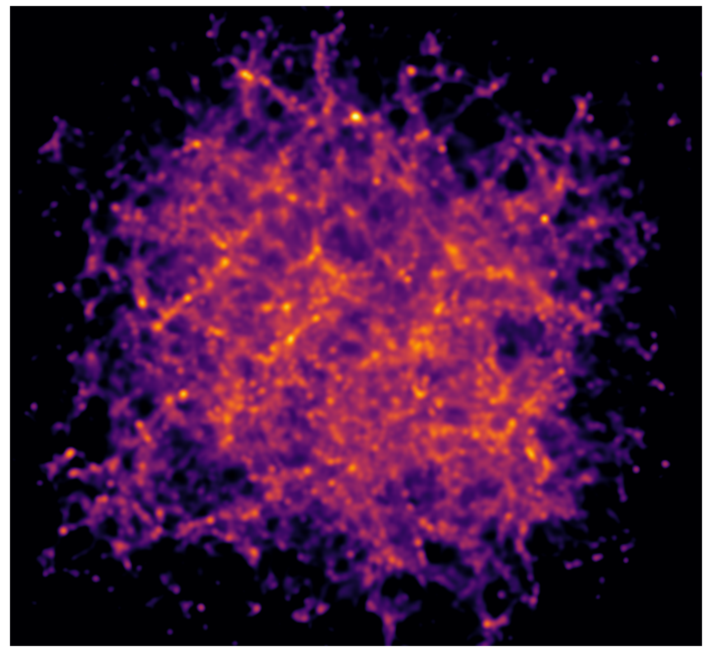


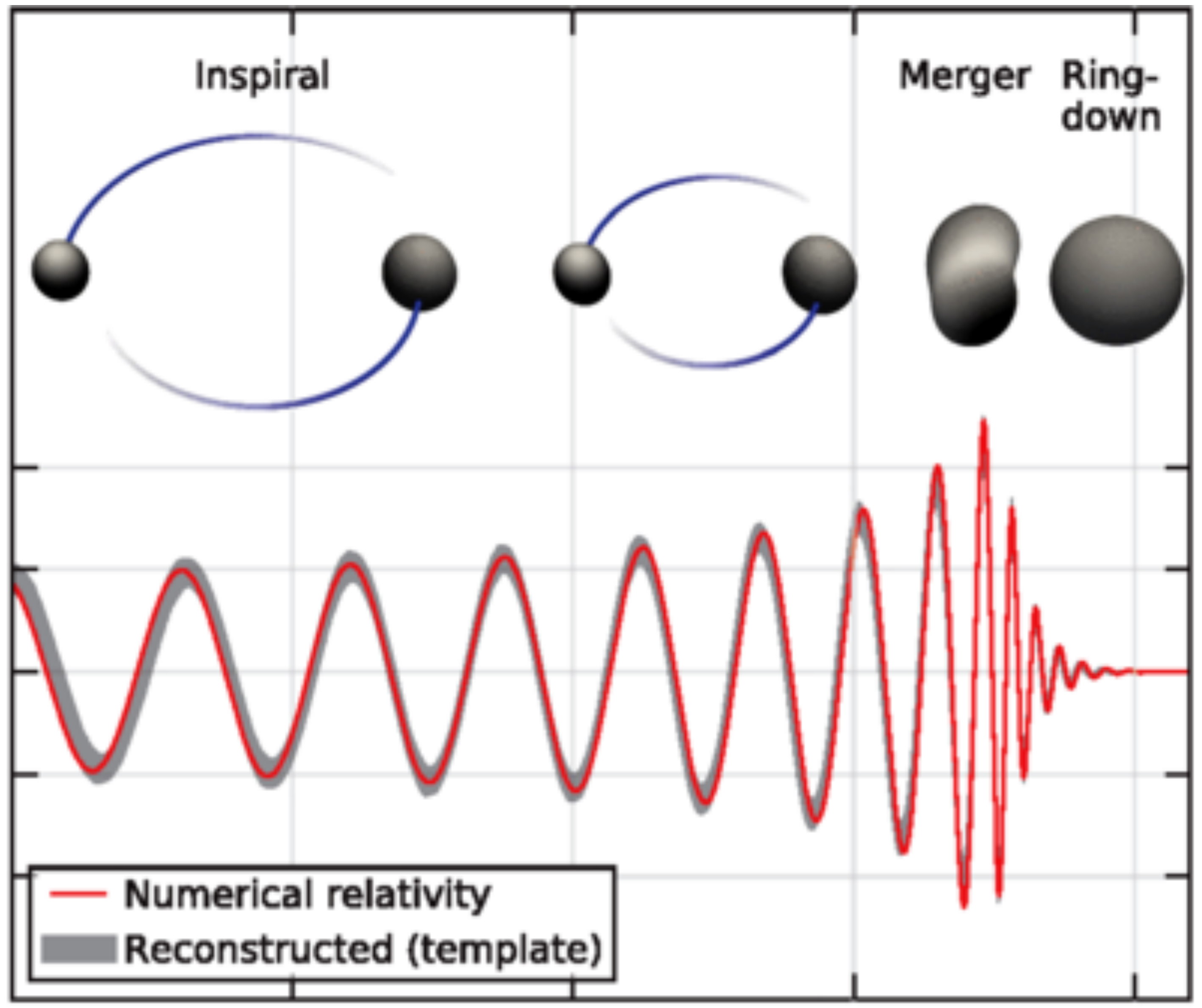
Liska et. al (2018)



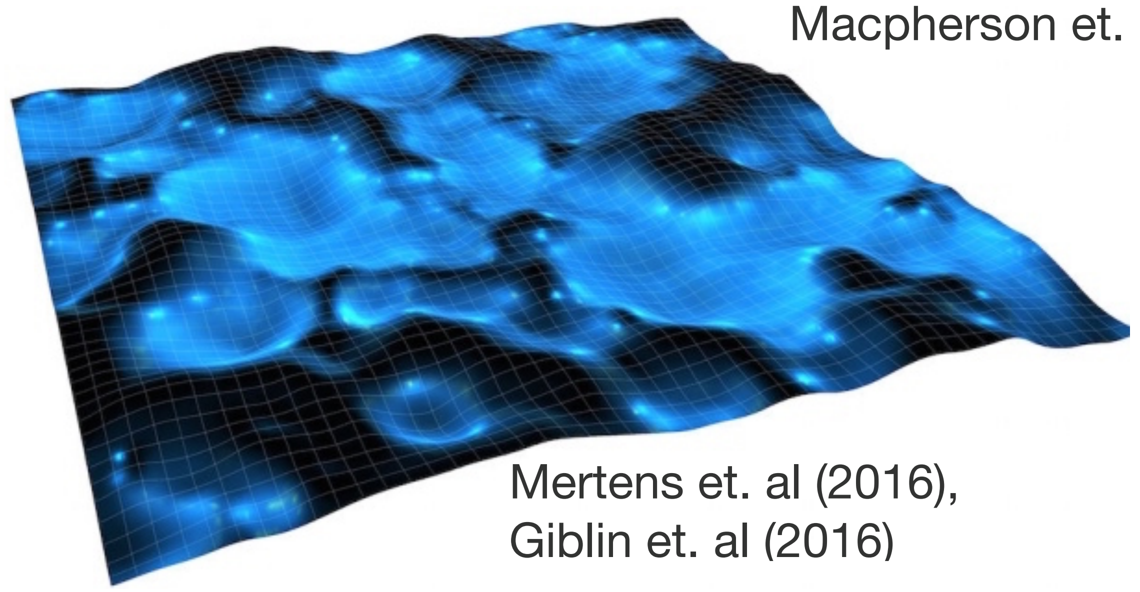
Giacomazzo et. al (2011)


Moesta et. al (2014)


Macpherson et. al (2018a)

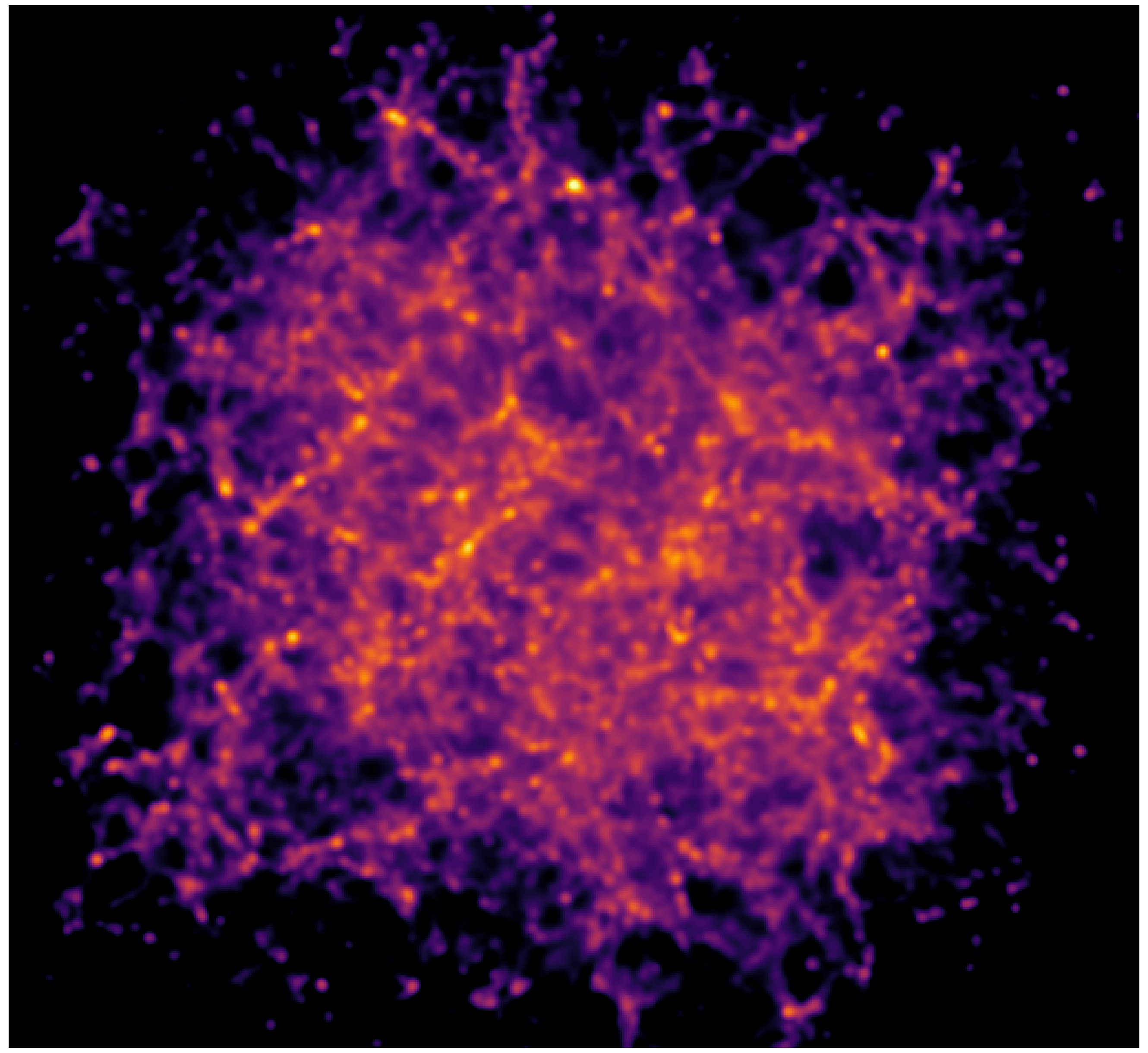


Abbott et. al (2016)


Mertens et. al (2016),
Giblin et. al (2016)

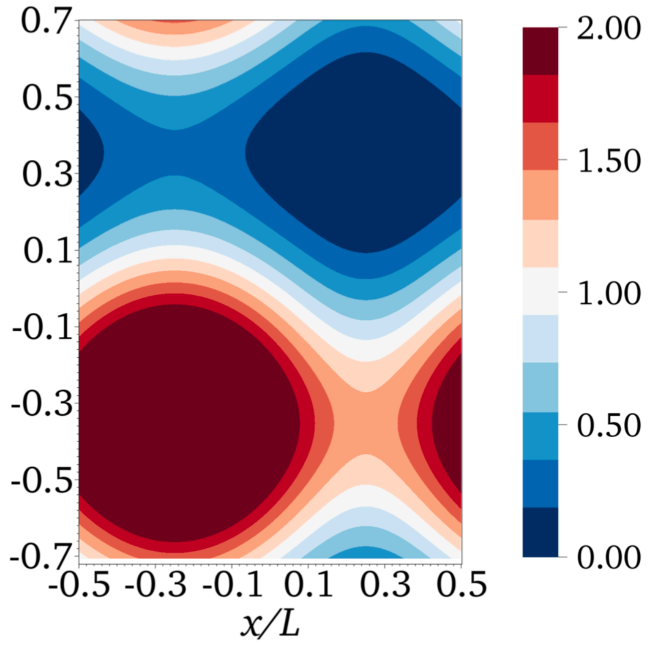
Numerical relativity

inhomogeneous cosmology edition

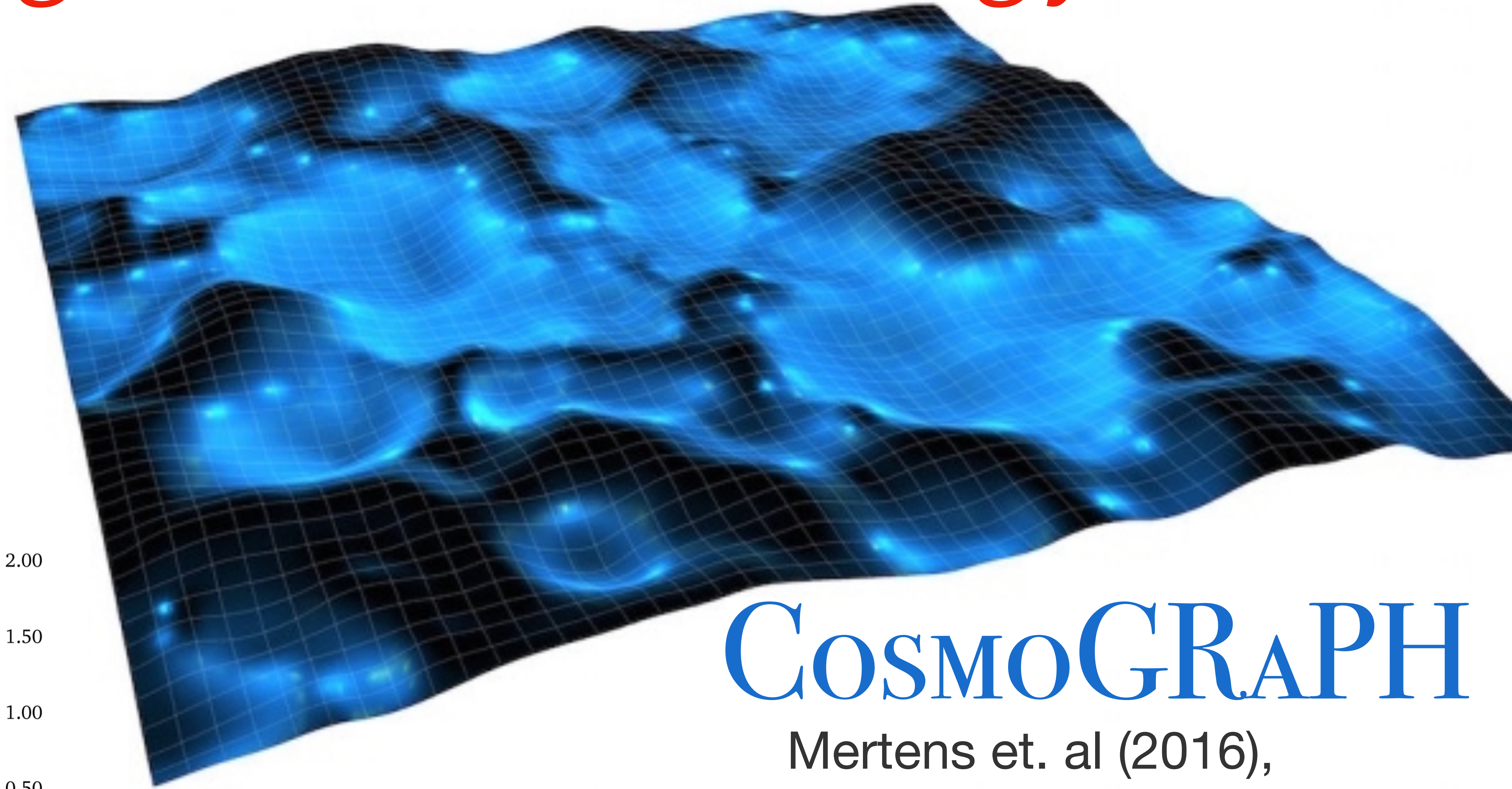


Macpherson et. al (2017,2018,2019)

einstein toolkit



Bentivegna & Bruni (2016)
Bentivegna (2016)



CosmoGRaPH

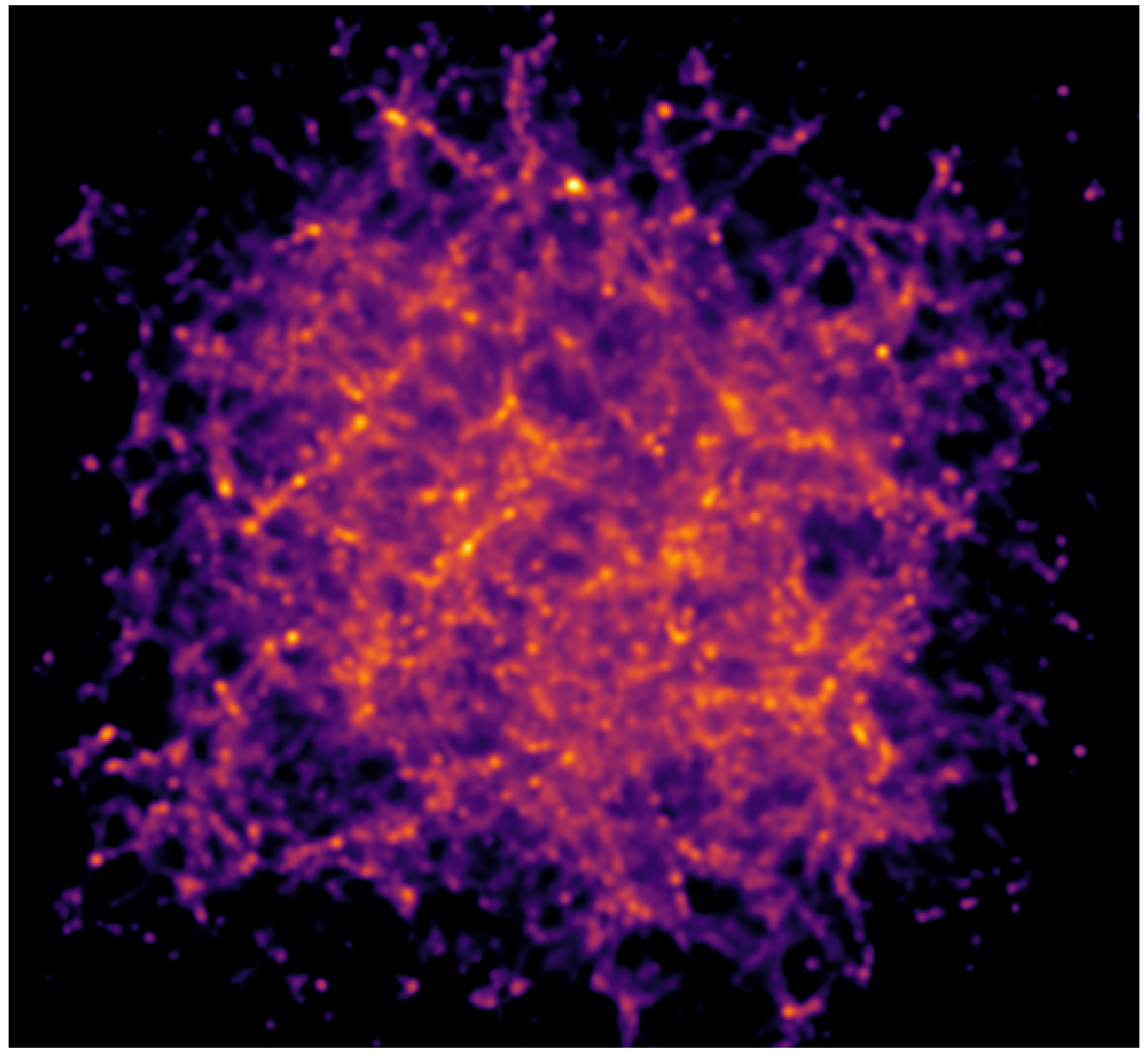
Mertens et. al (2016),
Giblin et. al (2016,2017,2018)

(some approximations for GR)

and… Daverio et. al (2017,2019), East et. al (2018), Adamek et. al (2013-2019), Barrera-Hinjosa & Li (2019)

gevolution **GRAMSES**

Numerical relativity
inhomogeneous cosmology edition



Macpherson et. al (2017,2018,2019)

einstein toolkit

Cactus / Einstein Toolkit

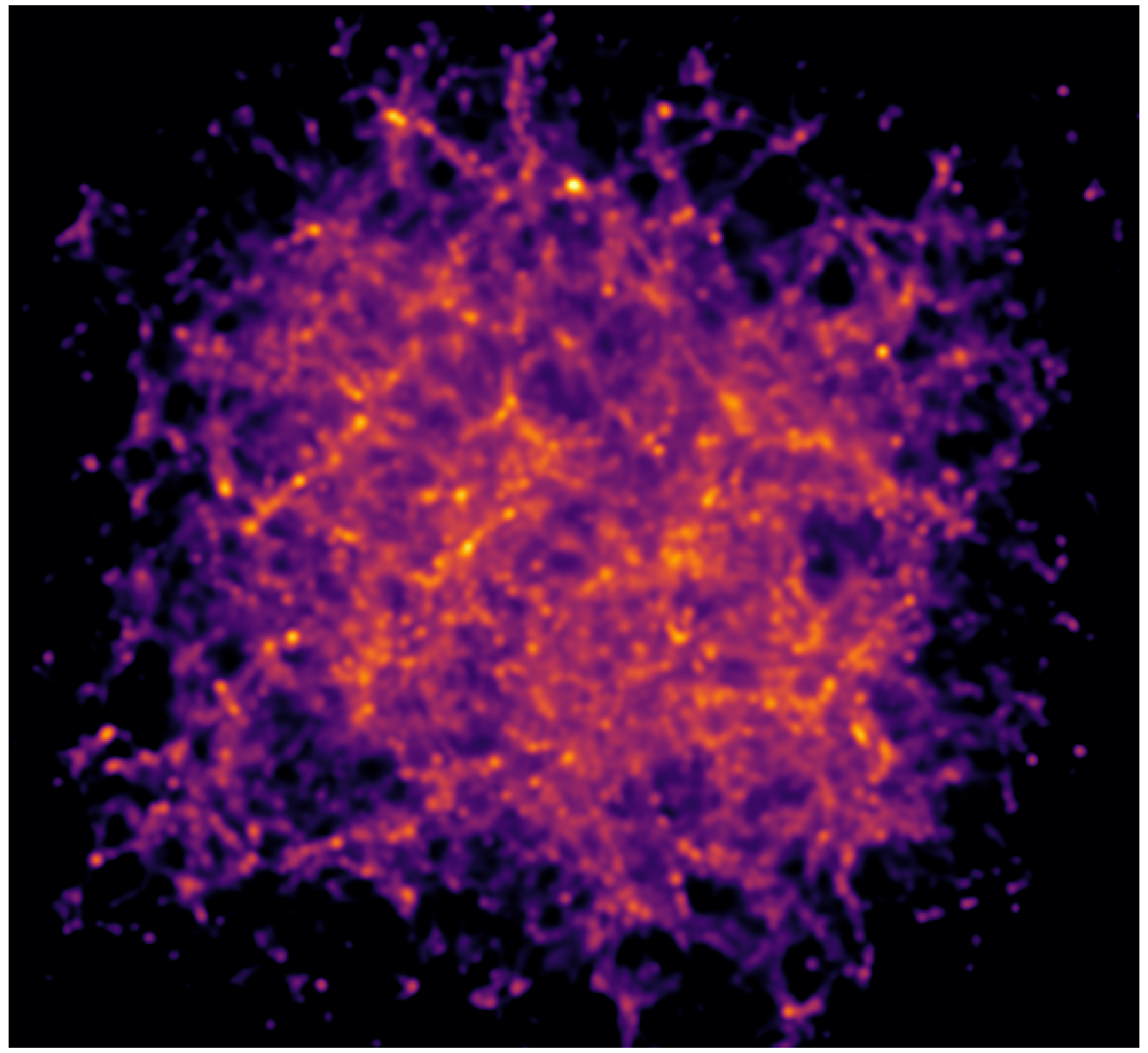
- Widely used
- Free and open-source

FLRWSolver

- A module to initialise cosmological spacetimes
- tested in arxiv:1611.05447

Numerical relativity

inhomogeneous cosmology edition



Macpherson et. al (2017,2018,2019)

Key Assumptions

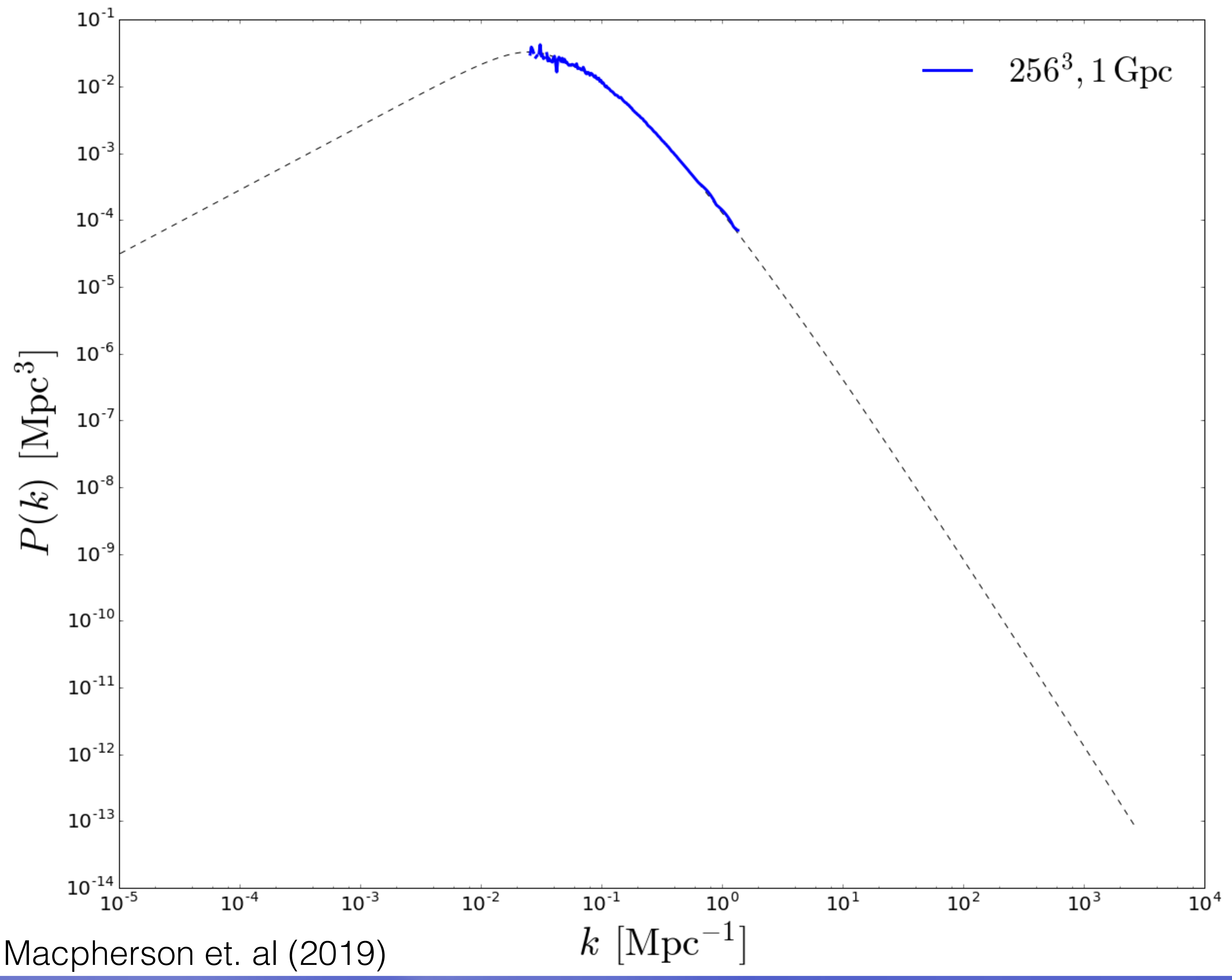
- Hydrodynamics on a grid (i.e., no particles)
- Matter dominated (no dark energy) perfect fluid
- Begin simulations with linearly perturbed FLRW (but no specified background during evolution)
- Periodic boundary conditions

Initial conditions

- Assume at early times the Universe is well-described by FLRW + small (linear) perturbations
- We can then solve Einstein's equations using *linear perturbation theory*
- Relate perturbations in density, velocity, and curvature easily
- As soon as the simulation starts, there are *no assumptions* on the size of the perturbations

einstein toolkit

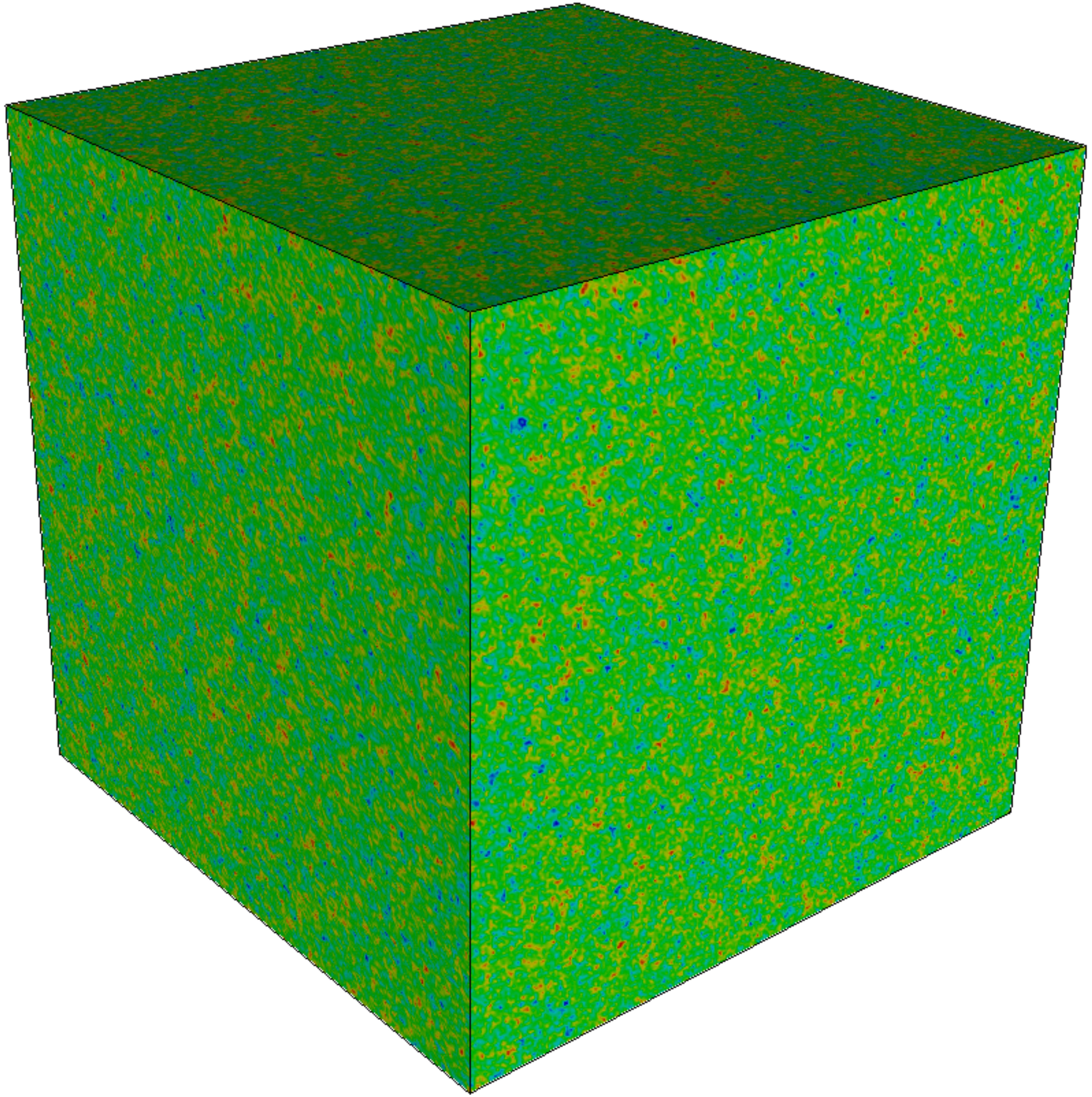
Initial conditions: a homemade CMB



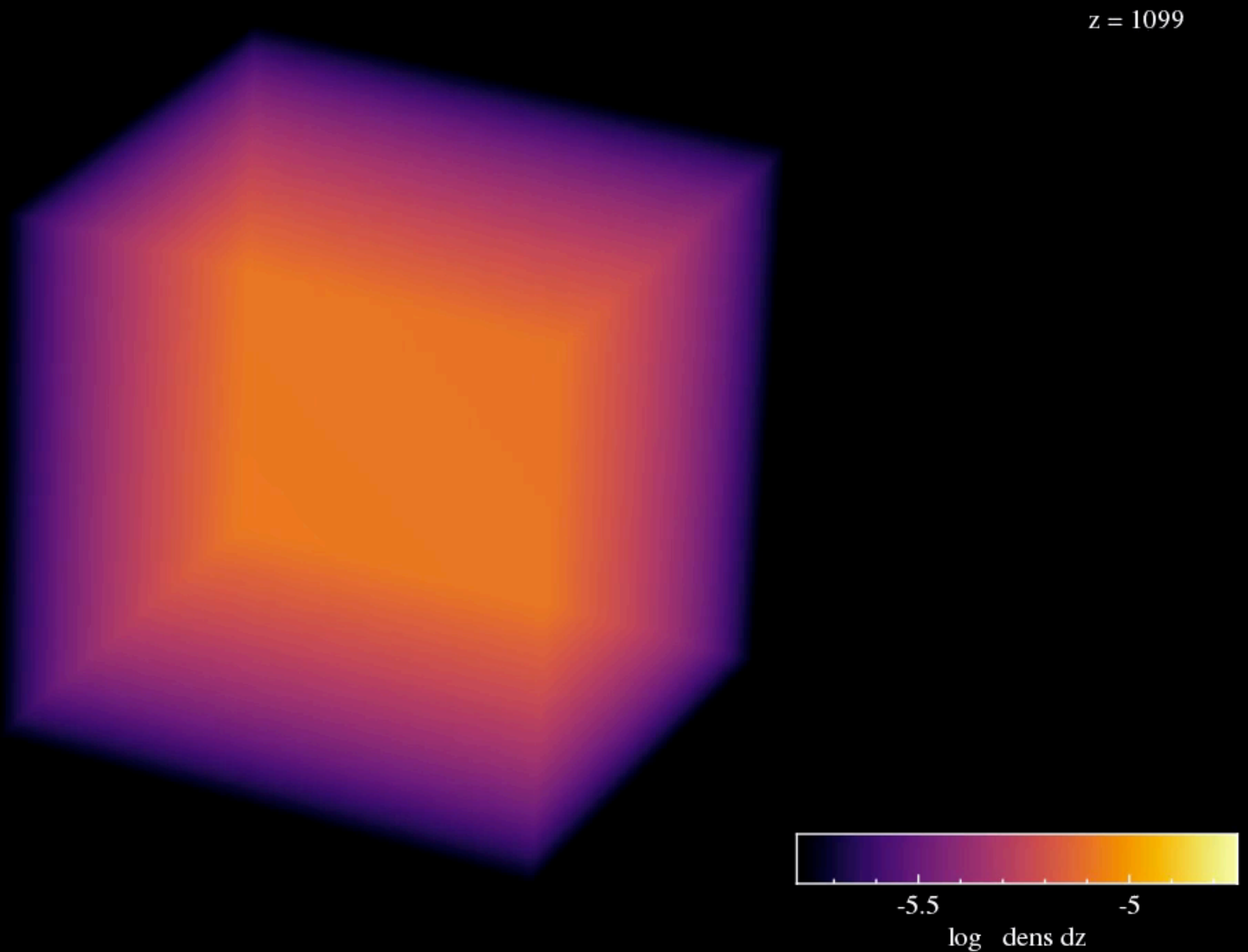
Macpherson et. al (2019)

- Create Gaussian random field using matter power spectrum at CMB (z ~ 1100)
- This is the perturbation to the density field
- Find corresponding velocity and curvature perturbations
- Assuming *linear perturbations*

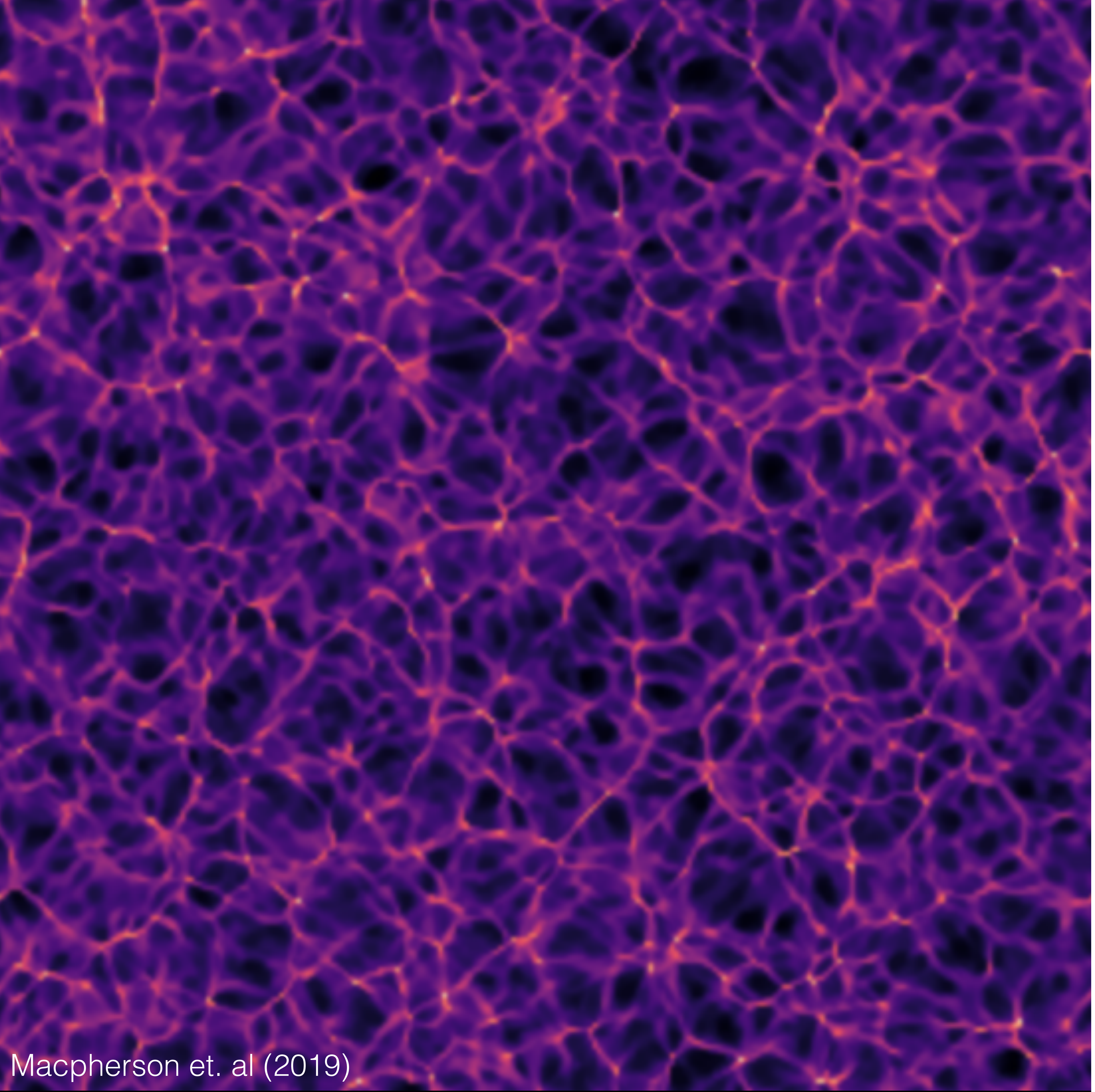
einstein toolkit



256^3 : 1 Gpc^3



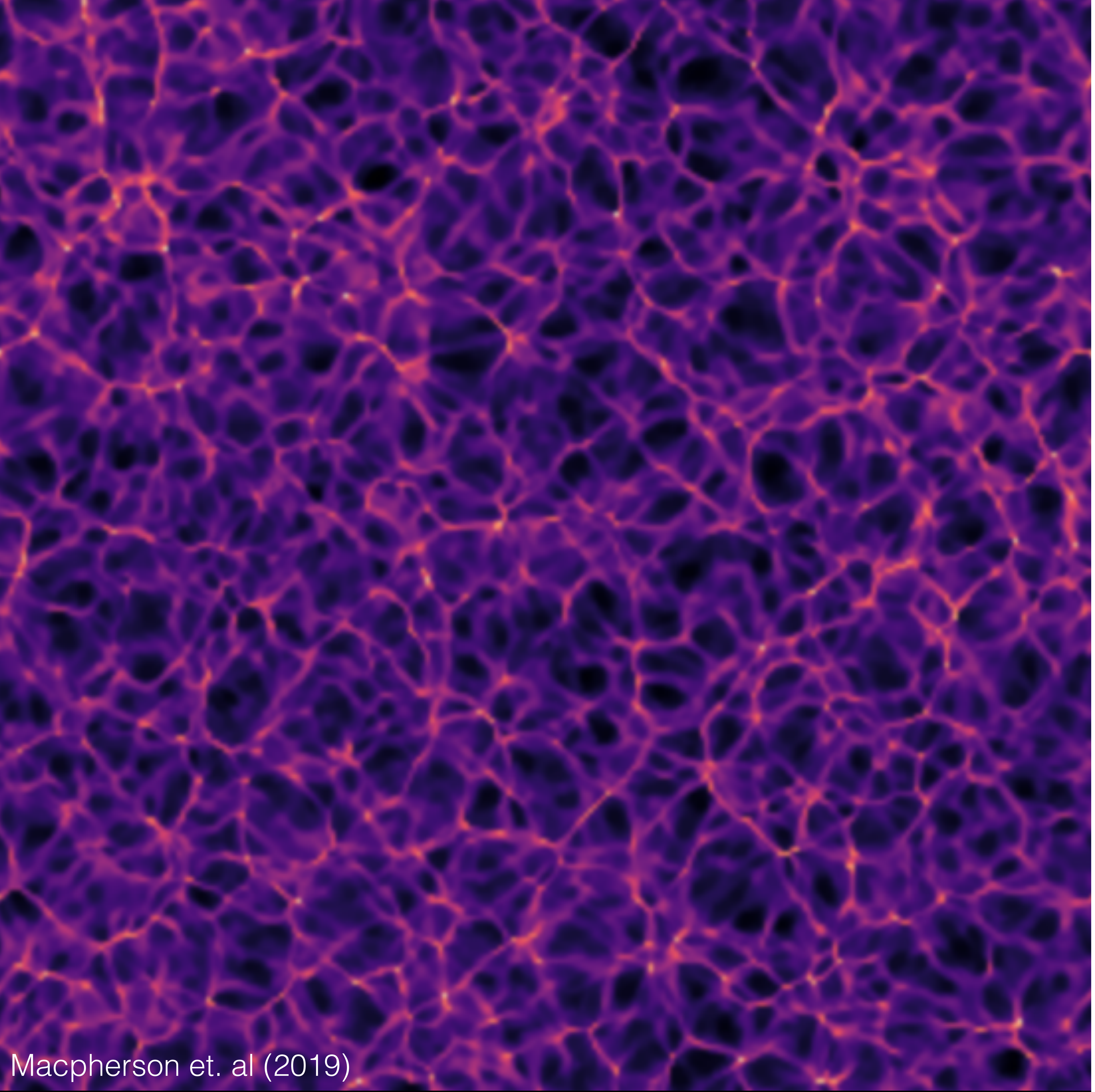
Macpherson et. al (2019)


Macpherson et. al (2019)

What we've done with this…

- Testing FLRWSolver, emergence of tensor modes and gravitational slip (in a simplified universe)
  arXiv:1611.05447
- Inhomogeneous expansion, averaged dynamics & backreaction of small-scale structures
  arXiv:1807.01711
- Local deviations in an effective Hubble parameter (in response to the H0 tension)
  arXiv:1807.01714

LOTS more to be done…



We’re interested in the averaged dynamics of this universe

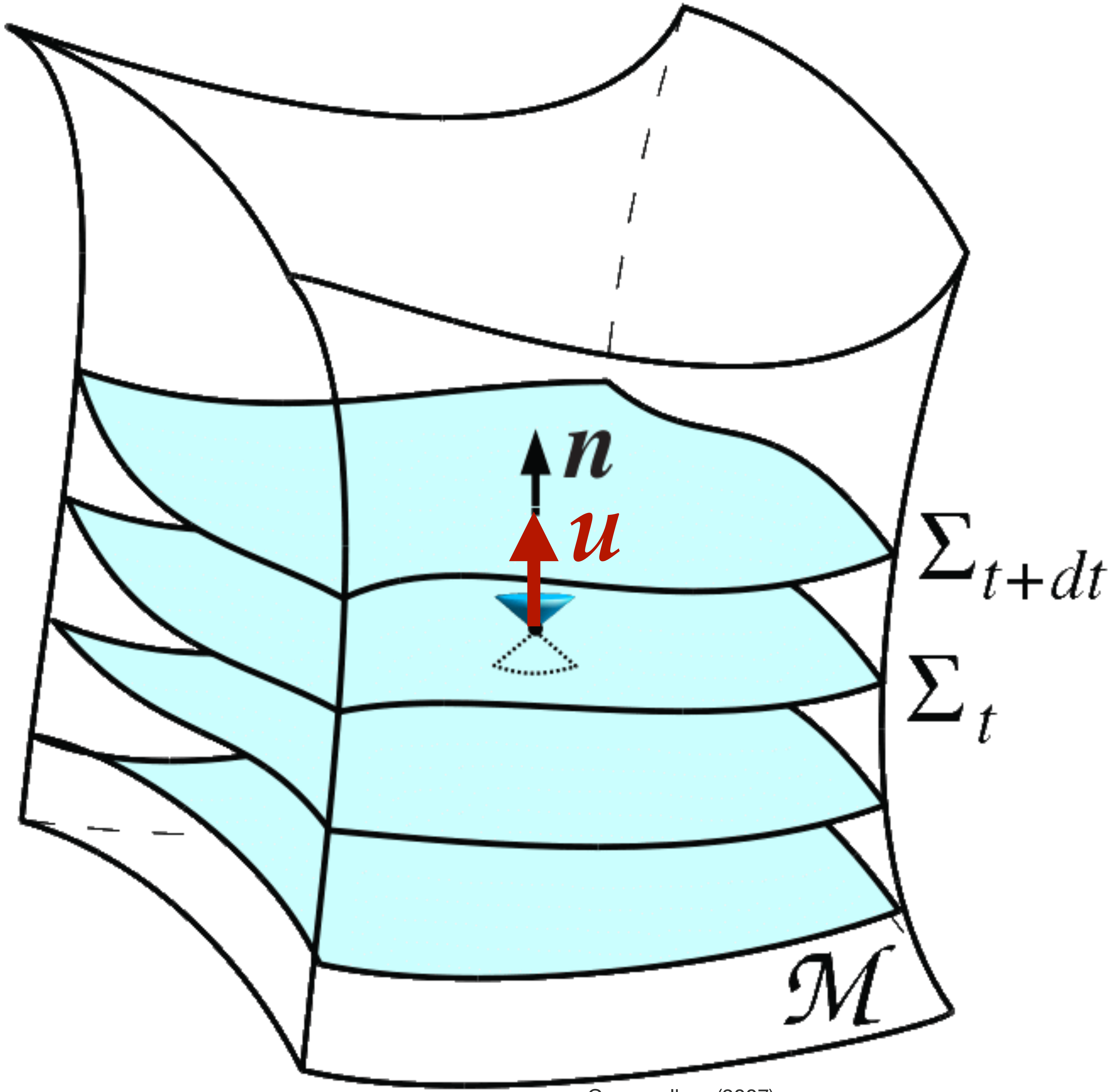
Which means we need to choose a spatial slice to average over…

The slice we used for the simulation seems like a good place to start!

comoving

$$n^\mu = u^\mu$$

tricky for nonlinear simulations!

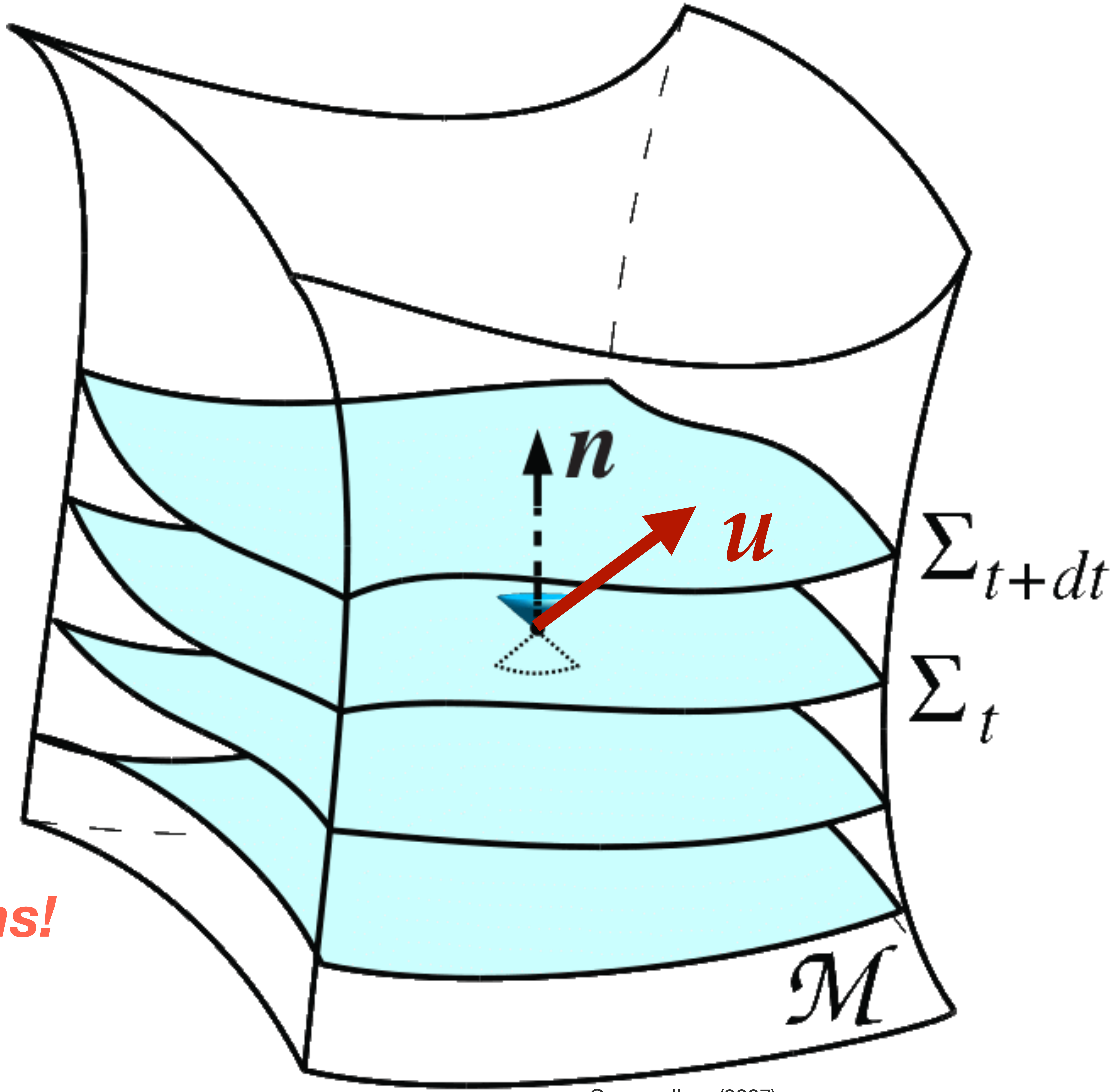


Gourgoulhon (2007)

non-comoving

$$n^\mu \neq u^\mu$$

non-tricky for nonlinear simulations!



Gourgoulhon (2007)

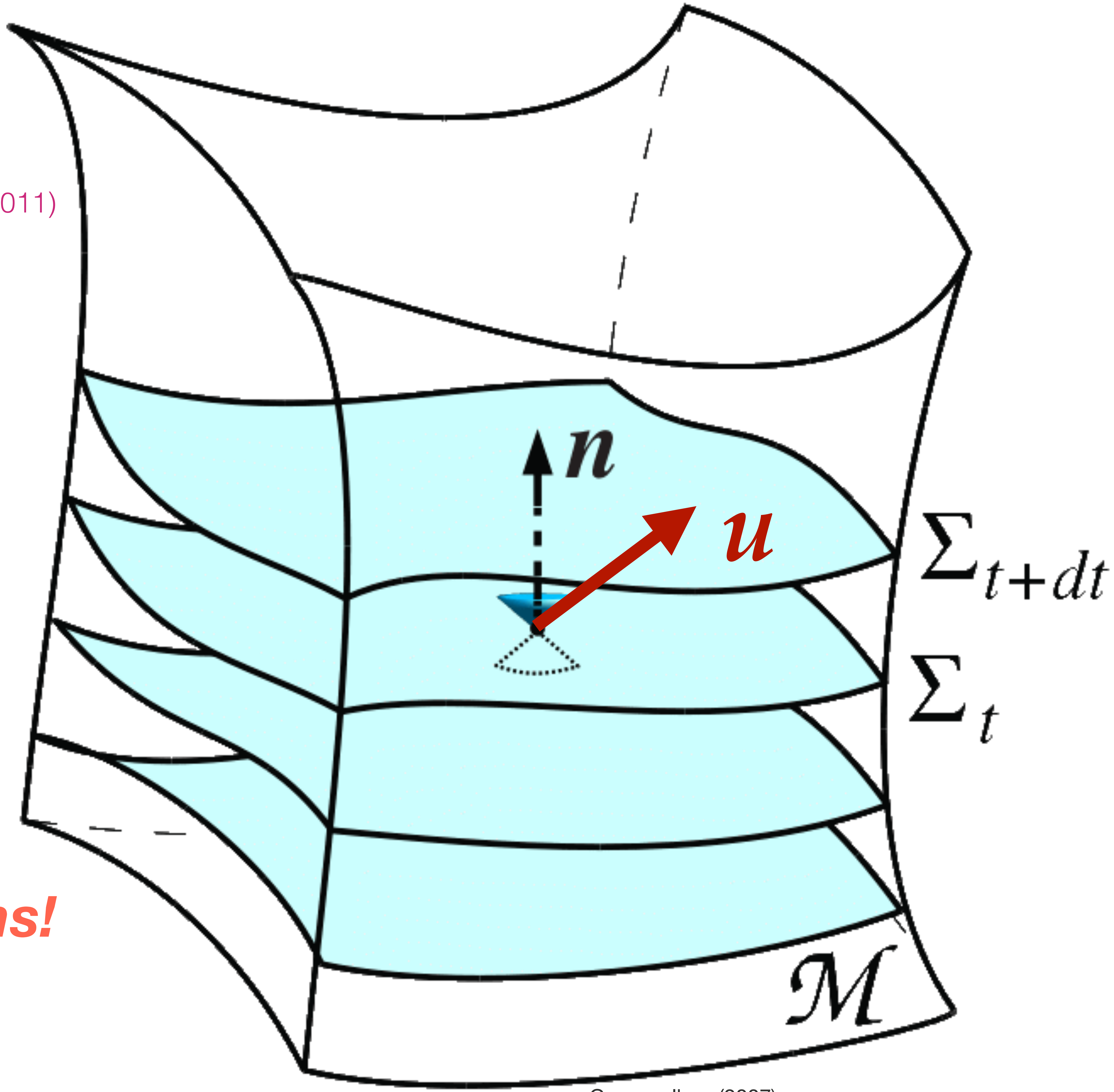
THE ORIGINAL, COMOVING FORMALISM WAS GENERALISED TO AN ARBITRARY FOLIATION

see e.g. Larena (2009), Brown+(2009), Gasperini+ (2010), Umeh+(2011)

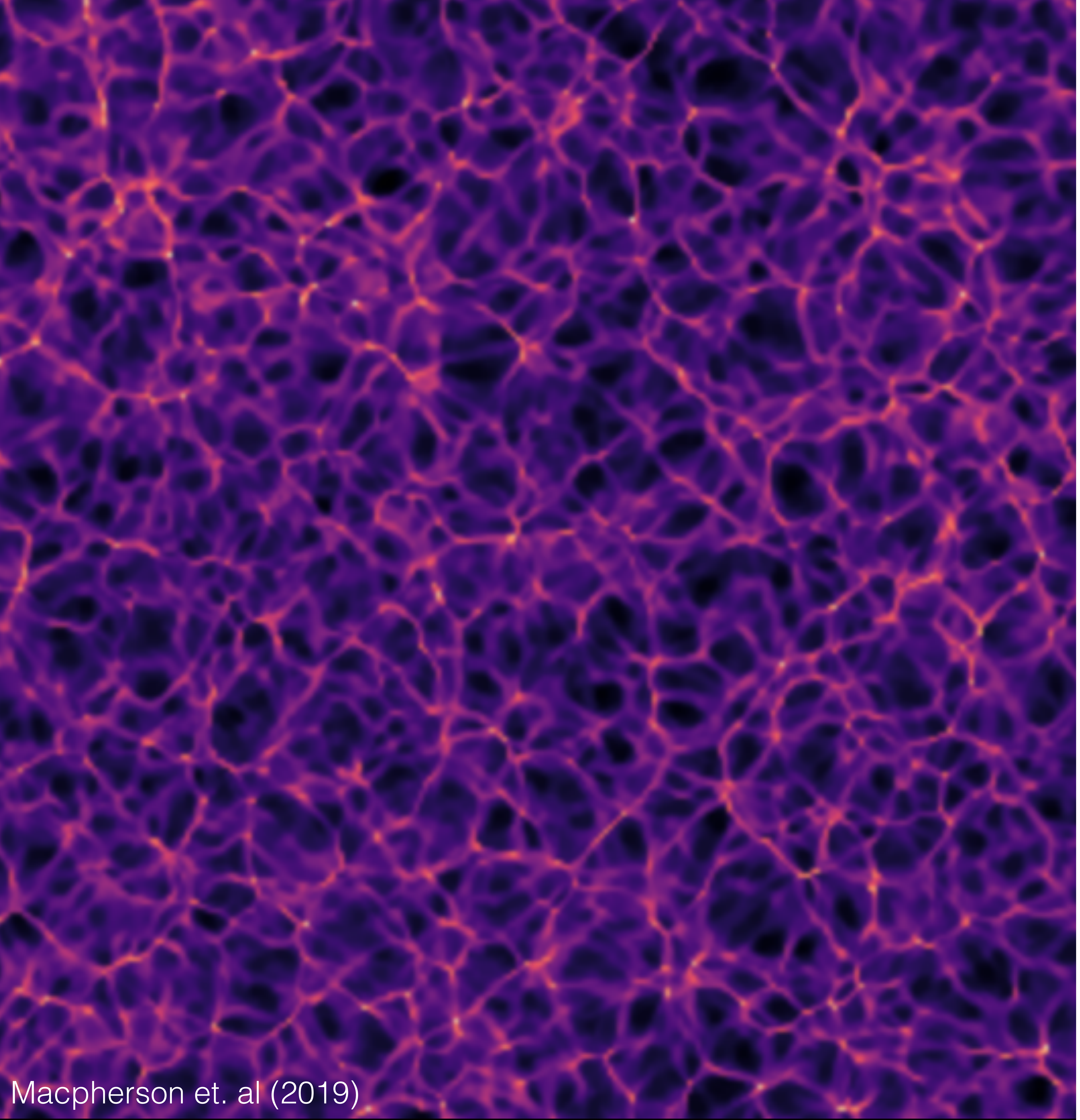
non-comoving

$$n^\mu \neq u^\mu$$

non-tricky for nonlinear simulations!



Gourgoulhon (2007)


Macpherson et. al (2019)

- We found total energy density of backreaction *and* average curvature over this whole box was ~ 1e-8
- But we found much larger effects on small scales (~ few to 10 %)

Our results: arXiv:1807.01711

- These formalisms were recently shown to be capturing properties of *the slices themselves*, rather than the fluid
- These slices are completely arbitrary
- Buchert, Mourier, & Roy (2020) released a NEW generalised averaging formalism to address this issue

arXiv:1805.10455 (*short*) and arXiv:1912.04213 (*long*)

Disclaimer:
not an expert

What's new?

- Original formalism of Buchert (2000) was based on properties *intrinsic to the fluid in comoving gauge*

- Some generalised formalisms describe average dynamics and backreaction with the *extrinsic* curvature, and therefore depend on *derivatives of the normal vector*

- *This can lead to a strong foliation dependence, which is not what we want for a cosmological model*

- In this NEW approach variables are *rescaled* to represent intrinsic properties of the fluid itself, rather than the coordinates

- This is a first step towards a fully *fluid-intrinsic* description

- By *avoiding* ***excessive*** *foliation-dependence* of the backreaction variables

See Buchert, Mourier, & Roy (2020) for more:
arXiv:1805.10455 (*short*) and arXiv:1912.04213 (*long*)

$$\tilde{\mathcal{R}} \equiv \left(\frac{\alpha}{\Gamma}\right)^2 \mathcal{R}$$

(re-scaling proper time to coordinate time)

scaled Hamiltonian constraint

$$\left(\frac{\dot{a}_D}{a_D}\right)^2 = \frac{8\pi G\langle\tilde{\rho}\rangle_D}{3} + \frac{\tilde{\Lambda}c^2}{3} - \frac{\langle\tilde{\mathcal{R}}\rangle_D c^2}{6} - \frac{\tilde{Q}_D c^2}{6}$$

Define an "effective" Hubble parameter:

$$\mathcal{H}_\mathcal{D} \equiv \frac{\dot{a}_\mathcal{D}}{a_\mathcal{D}}$$

Cosmological parameters

$$\Omega_m = \frac{8\pi G\langle\tilde{\rho}\rangle}{3\mathcal{H}_{\mathcal{D}}^2}, \quad \Omega_R = -\frac{\langle\tilde{\mathcal{R}}\rangle c^2}{6\mathcal{H}_{\mathcal{D}}^2},$$

$$\Omega_\Lambda = \frac{\tilde{\Lambda} c^2}{3\mathcal{H}_{\mathcal{D}}^2}, \quad \Omega_Q = -\frac{\tilde{\mathcal{Q}}_{\mathcal{D}} c^2}{6\mathcal{H}_{\mathcal{D}}^2}$$

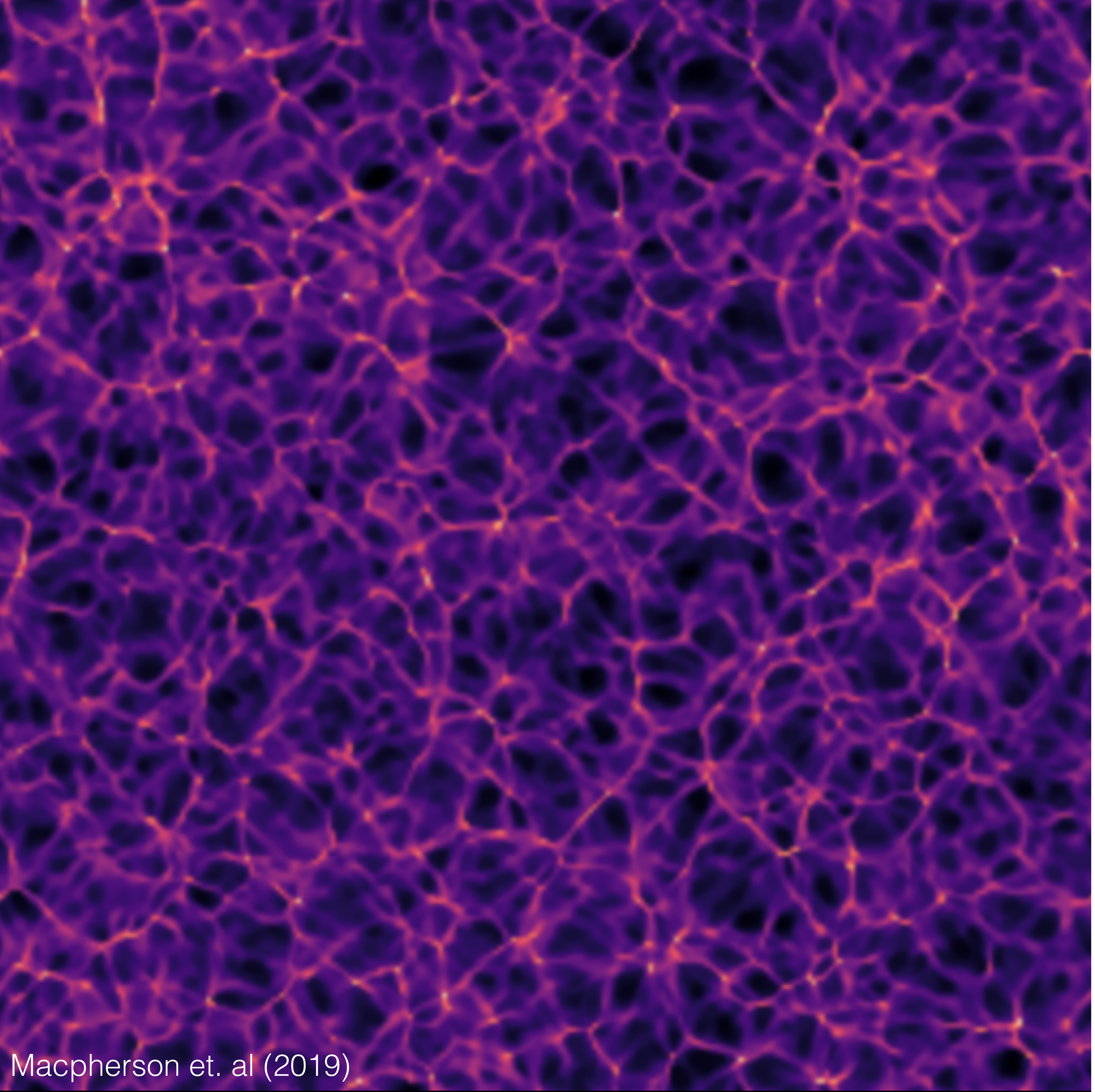
matter

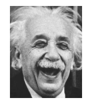
curvature

dark energy

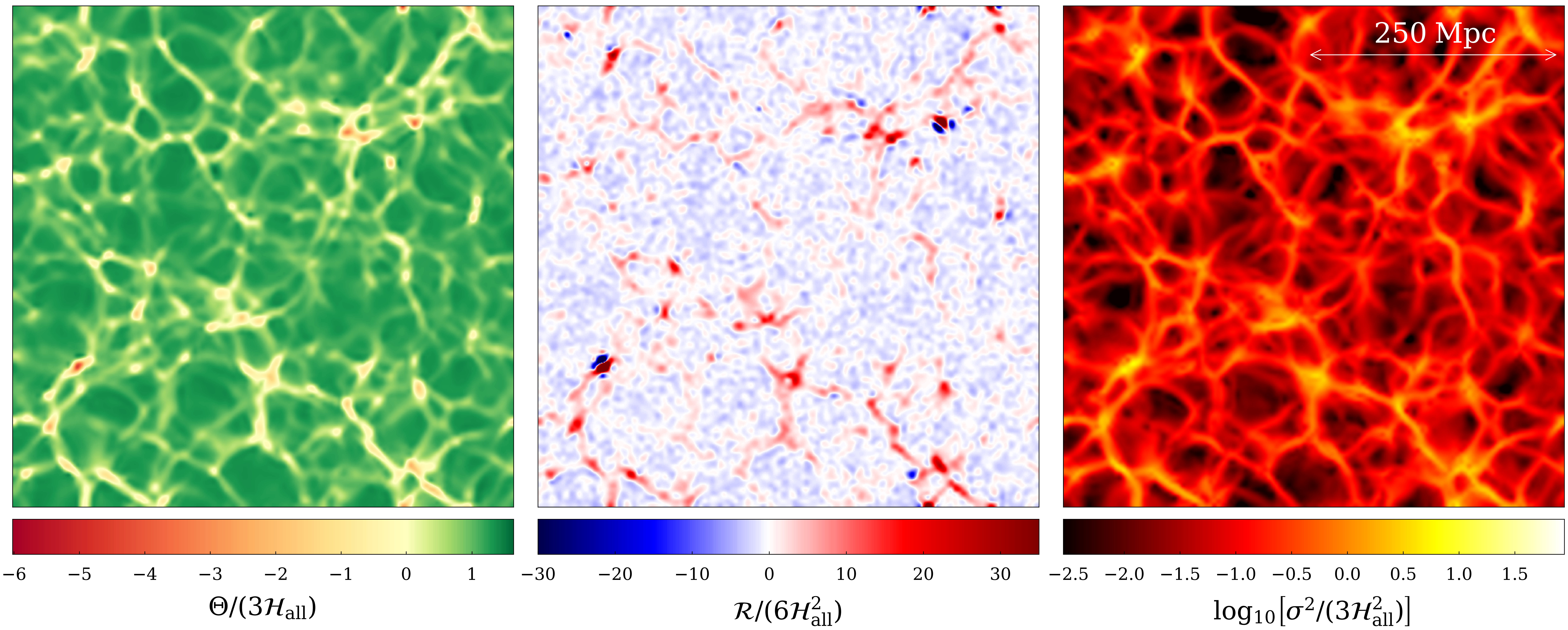
backreaction

$$\Omega_m + \Omega_R + \Omega_\Lambda + \Omega_Q = 1$$

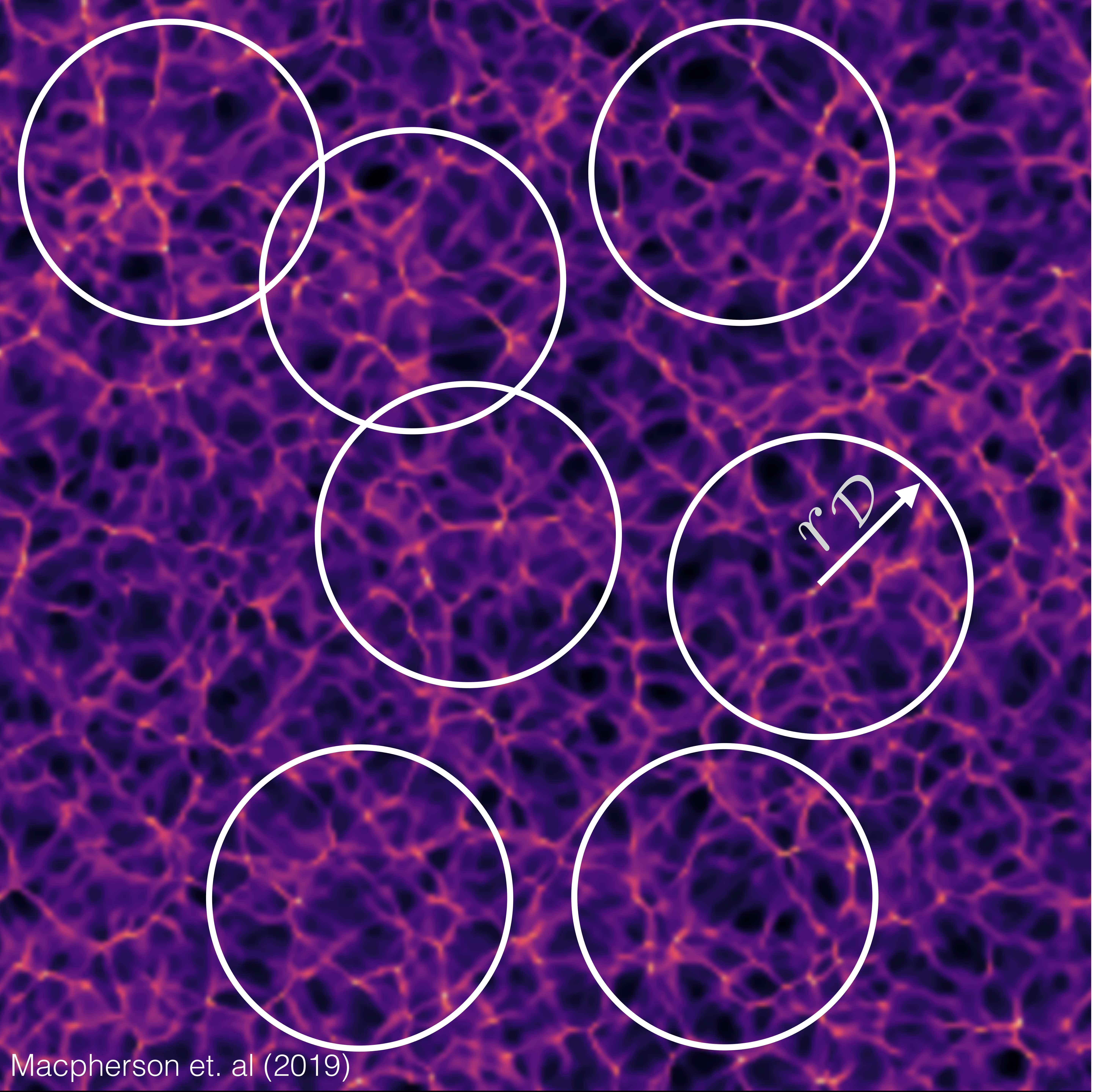




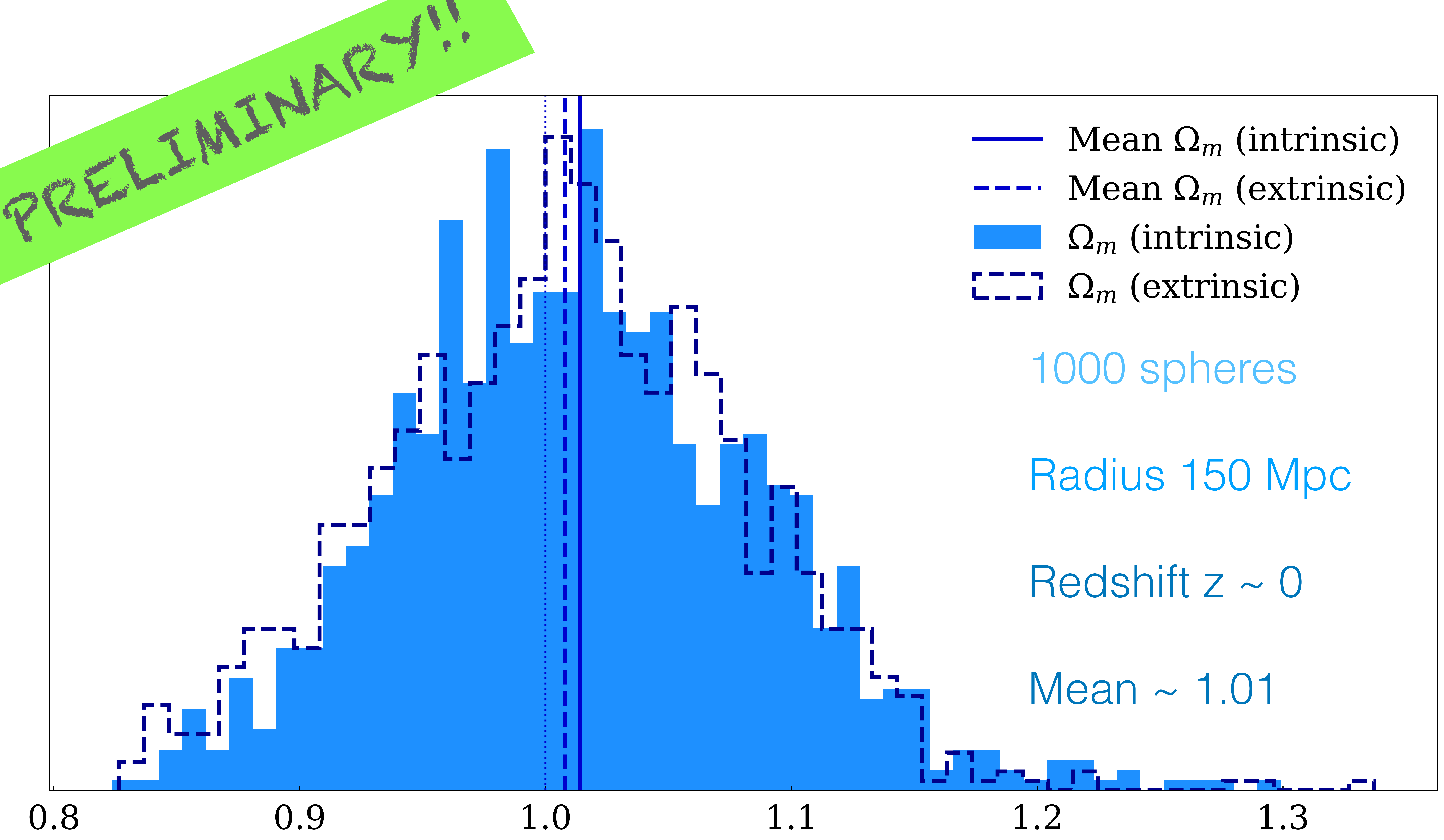
- Calculate expansion rate, shear, curvature, etc. within the domain



Macpherson & Mourier (in prep)

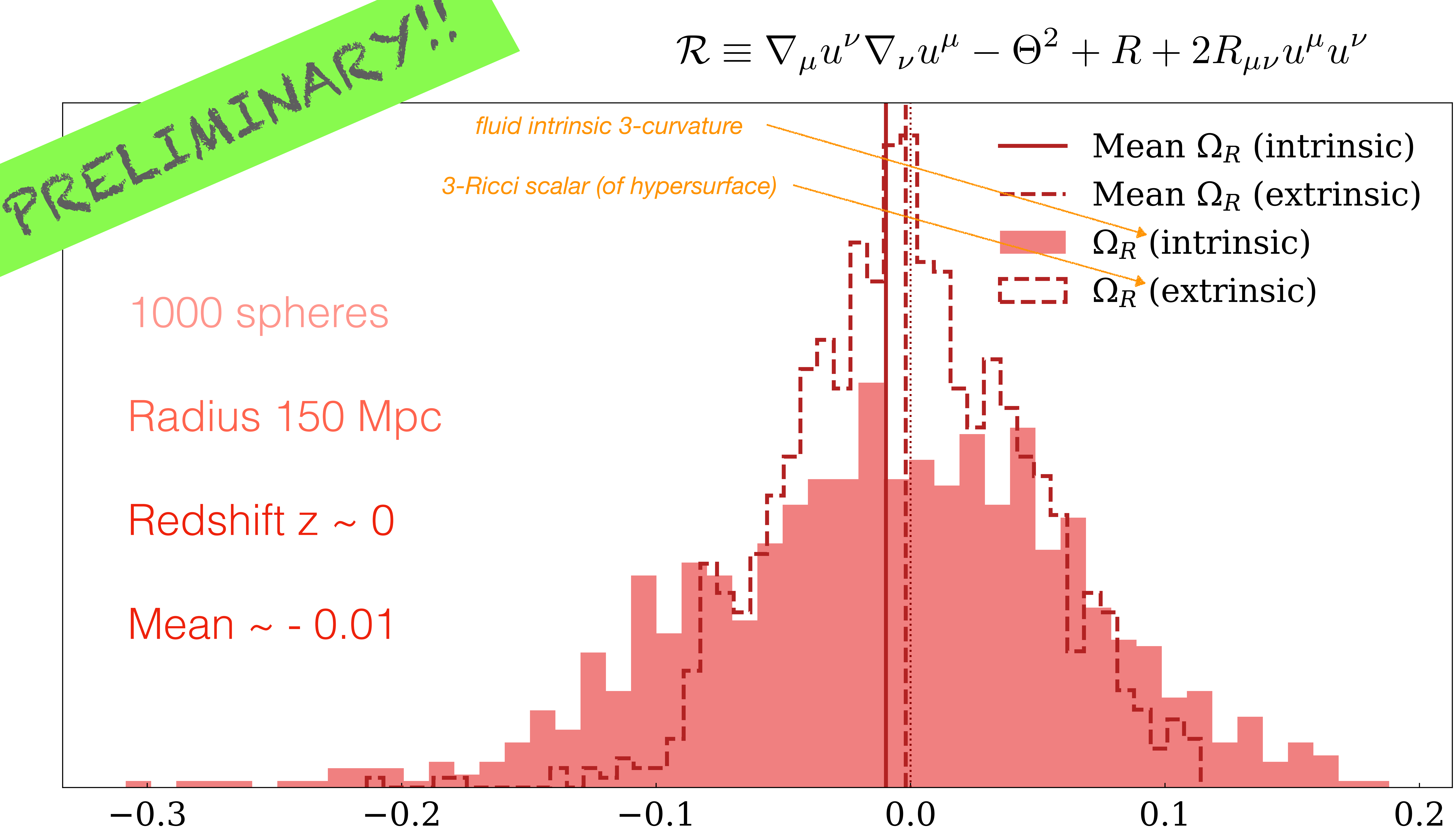


- Calculate expansion rate, shear, curvature, etc. within the domain
- Randomly place N spheres of a given radius
- Take averages over these domains
- Measure cosmological parameters
- Look at *extrinsic* vs *intrinsic* formalisms (in a non-comoving foliation)

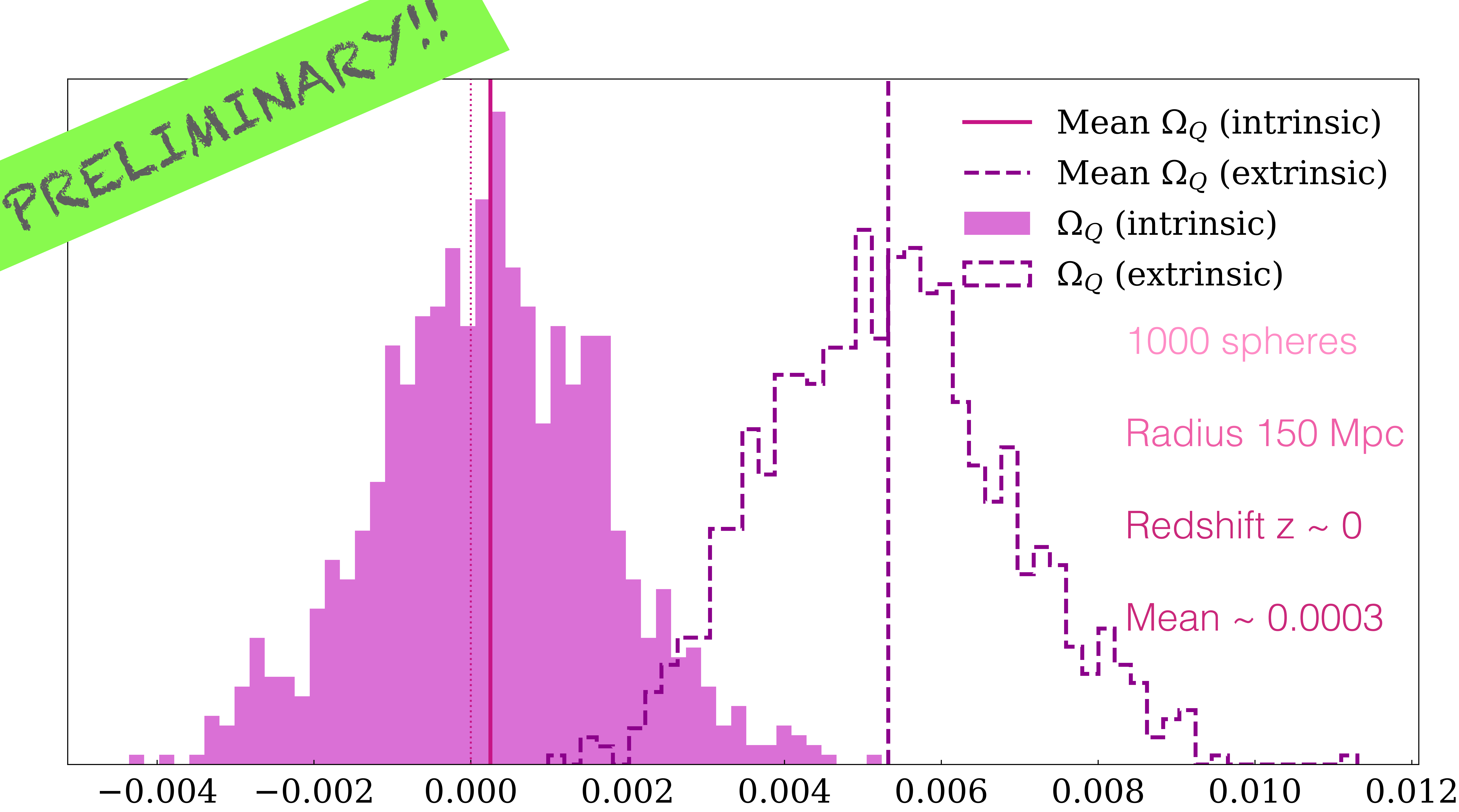


Macpherson & Mourier (in prep)

$$\mathcal{R} \equiv \nabla_\mu u^\nu \nabla_\nu u^\mu - \Theta^2 + R + 2R_{\mu\nu}u^\mu u^\nu$$

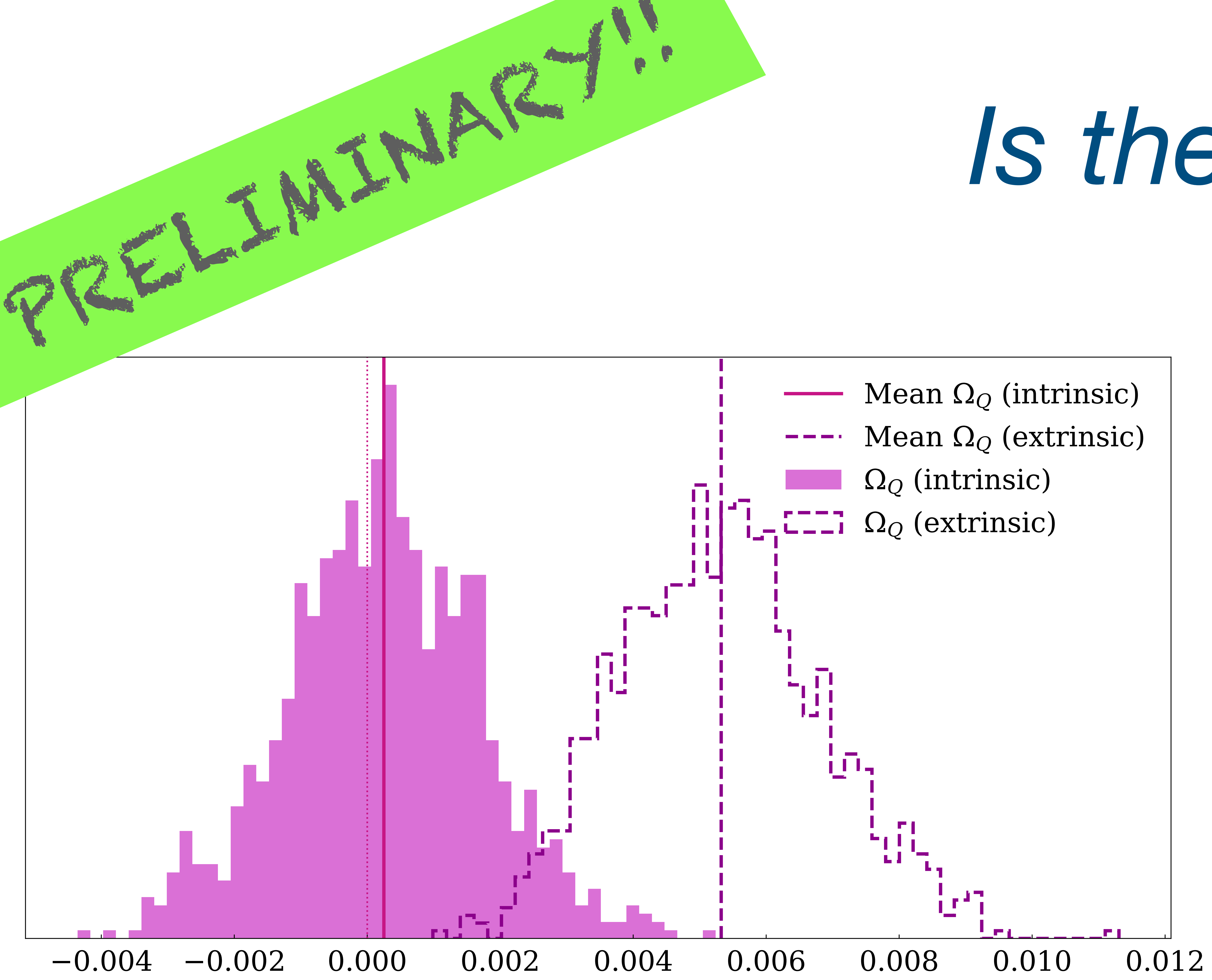


Macpherson & Mourier (in prep)



Macpherson & Mourier (in prep)

Is the difference vorticity?



Extrinsic formalism

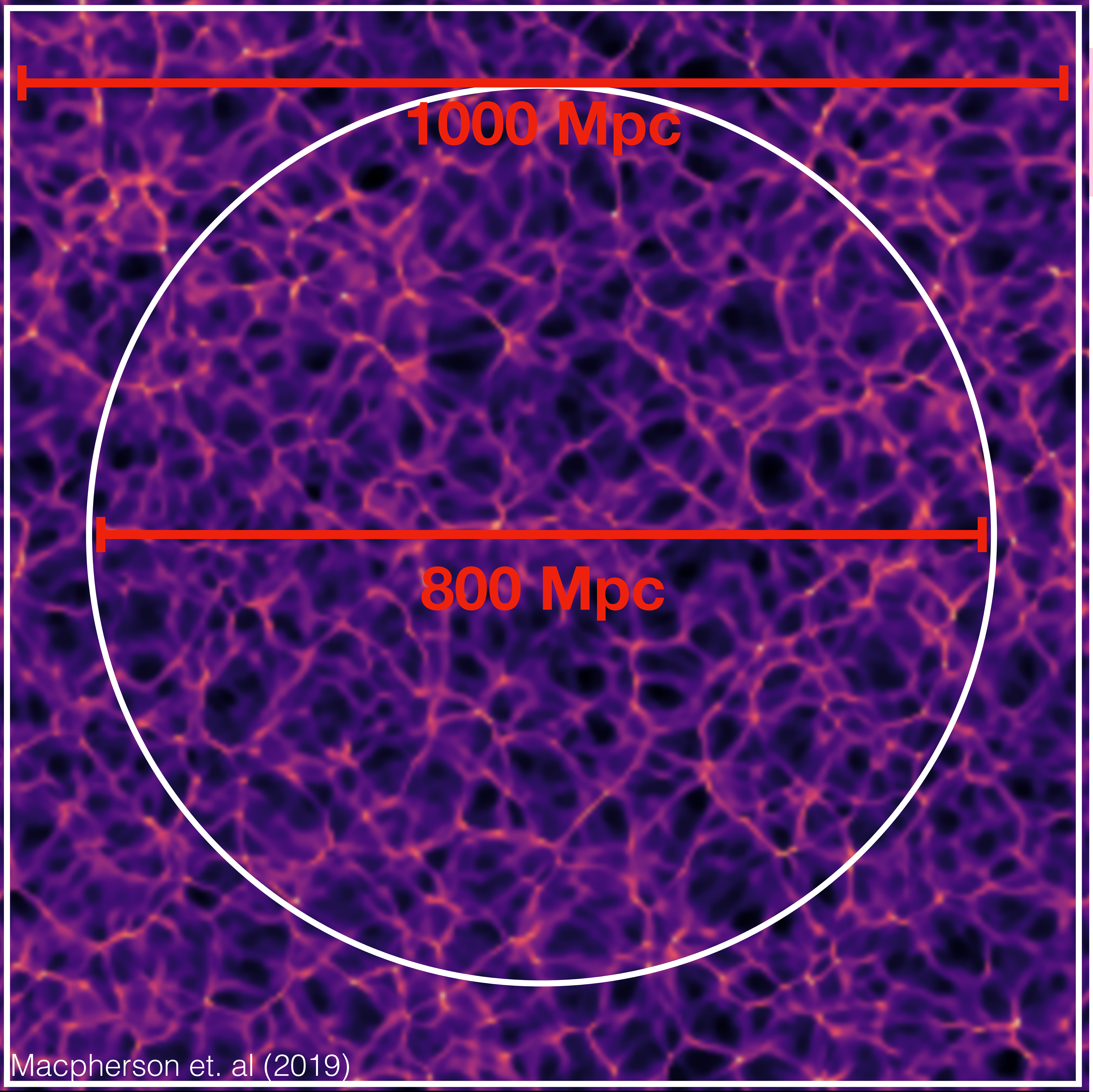
- *Backreaction variables measure properties of the* ***slices***
- *Vector field describing these slices (normal) is irrotational*
  —> Vorticity does not explicitly enter the backreaction equations
  —> BUT, vorticity still implicitly affects the fluid (if present)

Intrinsic formalism

- *Backreaction variables measure properties of the* ***fluid***
- *Vector field describing this (4-velocity) is not necessarily irrotational*
  —> Vorticity does enter the general backreaction equations
  —> It is therefore accounted for in the equations, and we get a more accurate measure of backreaction

(we have a small amount of vorticity in ICs due to linear assumption)

Macpherson & Mourier (in prep)

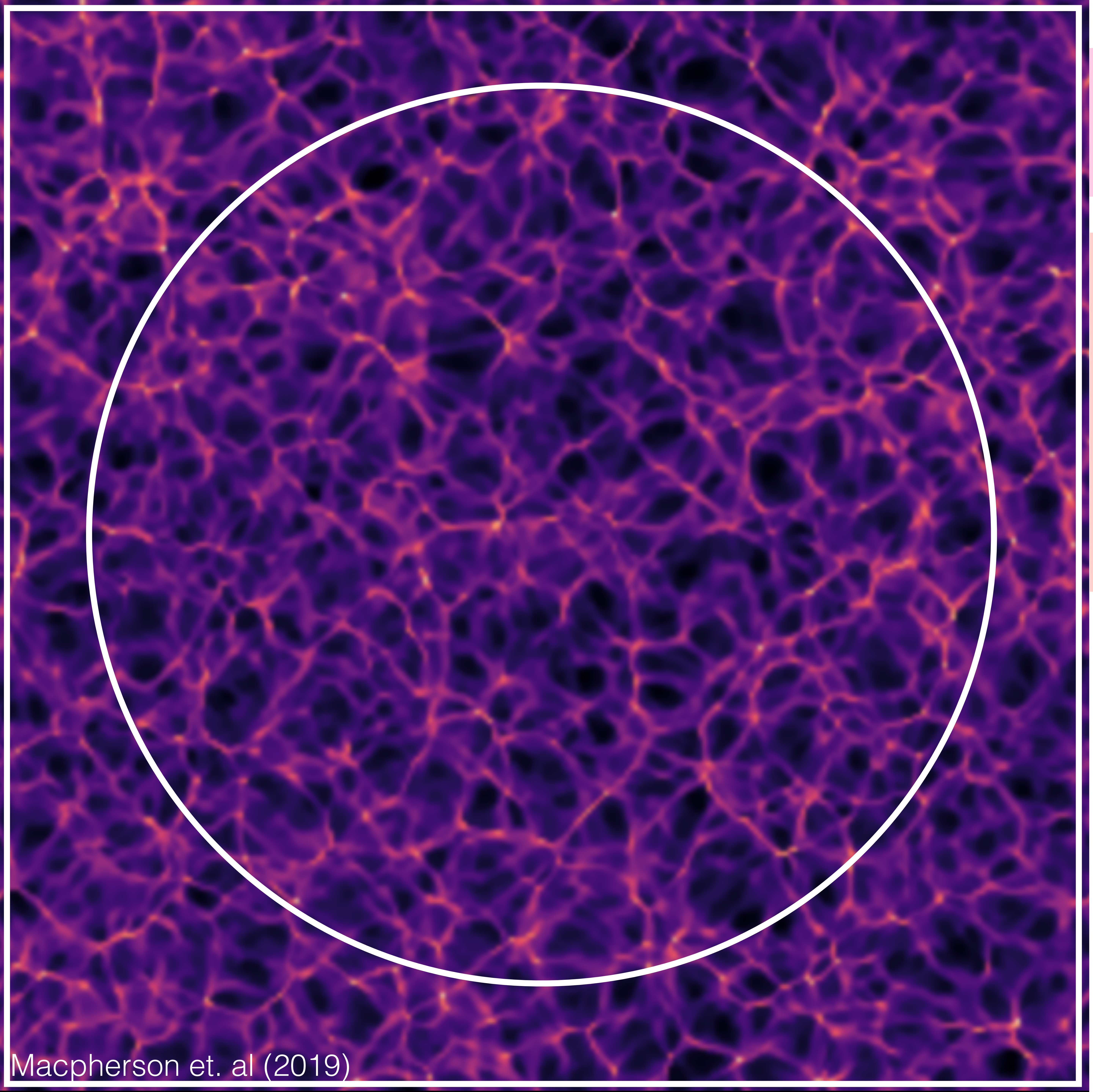


Whole box average

	Whole (cubic) domain	Sub (spherical) domain
Matter	100.5%	99.7%
Curvature	1e-6%	0.8%
Backreaction	6e-7%	0.04%

Huge jump in curvature and backreaction when moving from whole box average to (a tiny bit) smaller sub-domain

PRELIMINARY!!



Whole box average

It seems something is "forcing" our simulation to be FLRW on the whole - but not on sub-domains

periodic boundaries?

fluid approx?

the fact we started near FLRW?

Conclusions

- We do cosmological simulations considering **full GR in numerical relativity**
- The results we find with the new *intrinsic* averaging formalism are largely similar to those using the *extrinsic* averaging formalism

  —> *This is due to non-relativistic velocities and very close to homogeneous lapse*

- Even on the homogeneity scale (150 Mpc), can still get significant effects — worth looking into in more detail!
- We find potential evidence that periodic boundaries may be limiting our measurement of backreaction & curvature over the whole domain
- Still some foliation dependence in averaging, so…
  - **Very near future**: ray tracing and observables!

(ALL *PRELIMINARY* & subject to caveats)

Caveats…

- We treat Dark Matter as a fluid
  - This means we can't form virialised structures, this could have a big impact on the size of the backreaction effect!
  - Collisionless particles are better
- Periodic boundary conditions — effect on global curvature is unclear
- We assume averages over a purely spatial volume (no light cones)
  - i.e. averages all at z=0, an observer would actually look *back in time*
- Averaging is still foliation dependent (although minimised here) — ray tracing is *in progress!*