Quantum Diffusion during Cosmic Inflation

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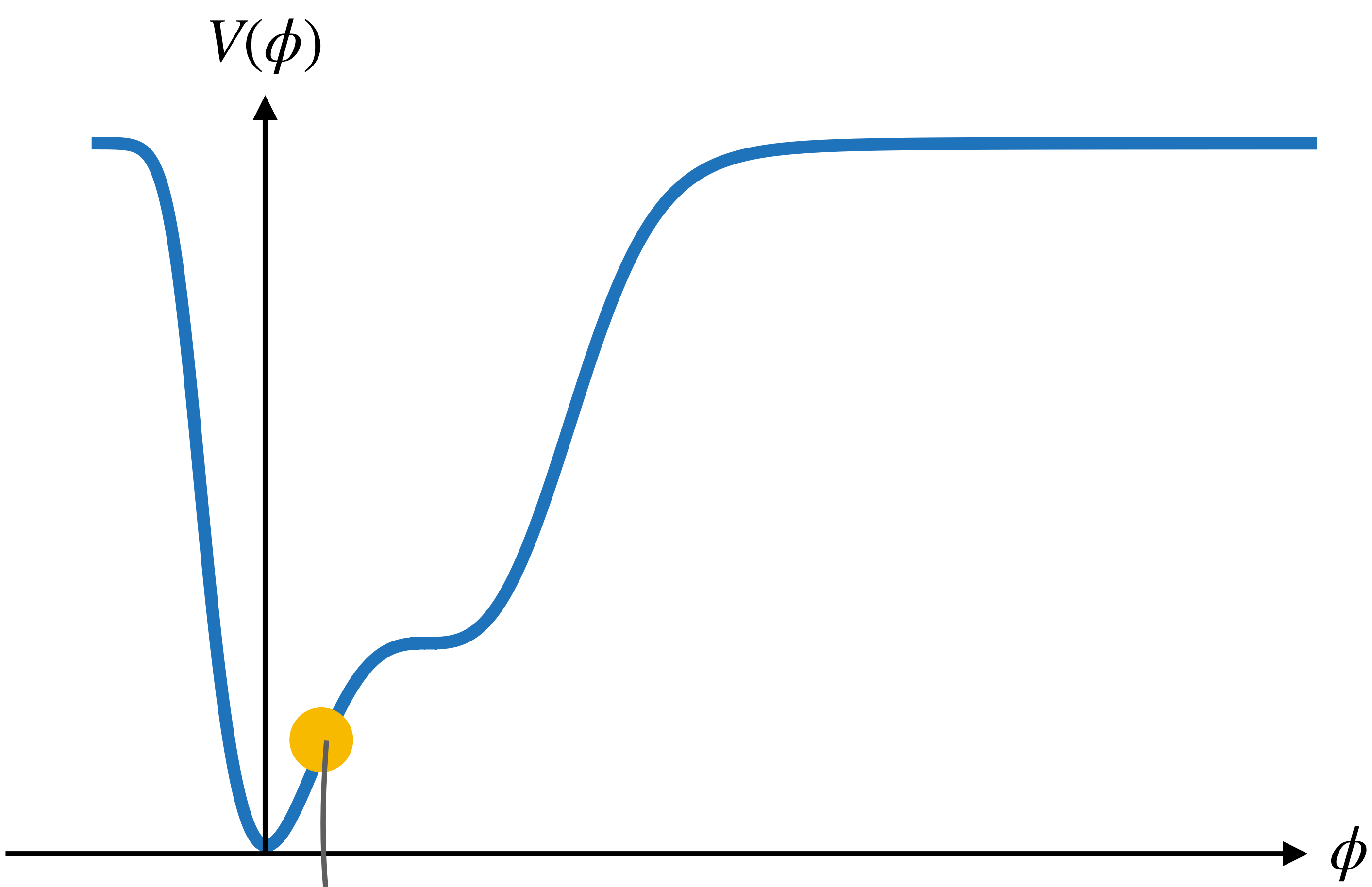
Outline

- Primordial Black Holes
- Stochastic δN formalism
- Heavy tails
- CMB probes the full potential

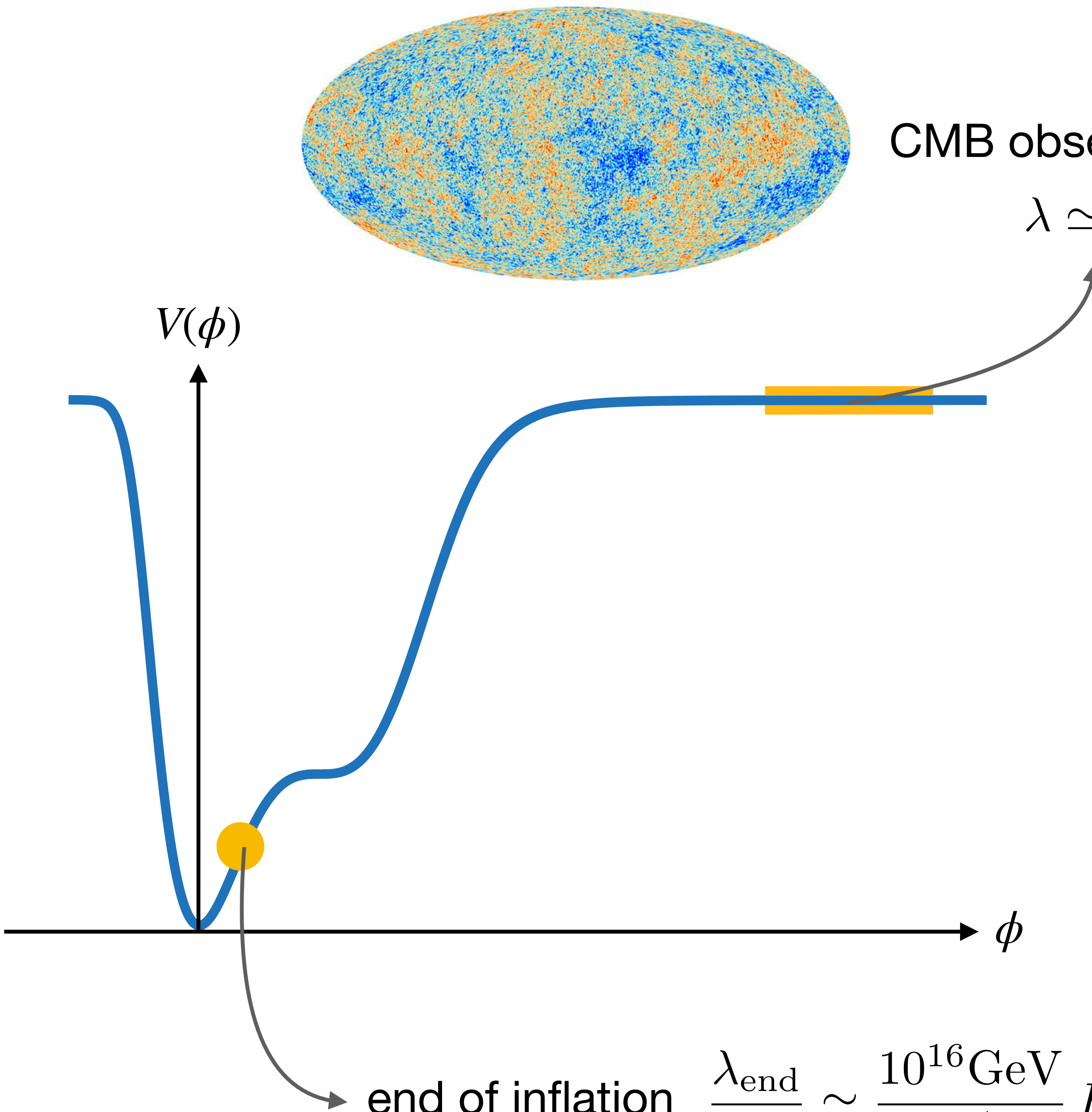
Primordial Black Holes

Probing the end of inflation



end of inflation $\frac{\lambda_{\rm end}}{\rm m} \simeq \frac{10^{16}{\rm GeV}}{\rho_{\rm end}^{1/4}} R_{\rm rad}^{-1}$

Probing the end of inflation



CMB observed scales

$\lambda \simeq \mathrm{Mpc}$

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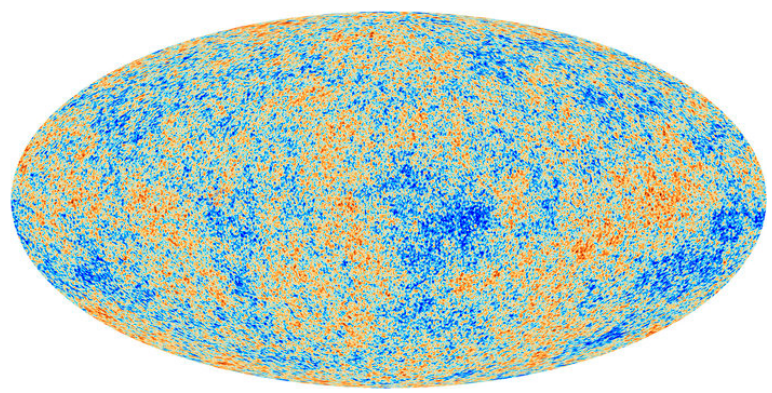

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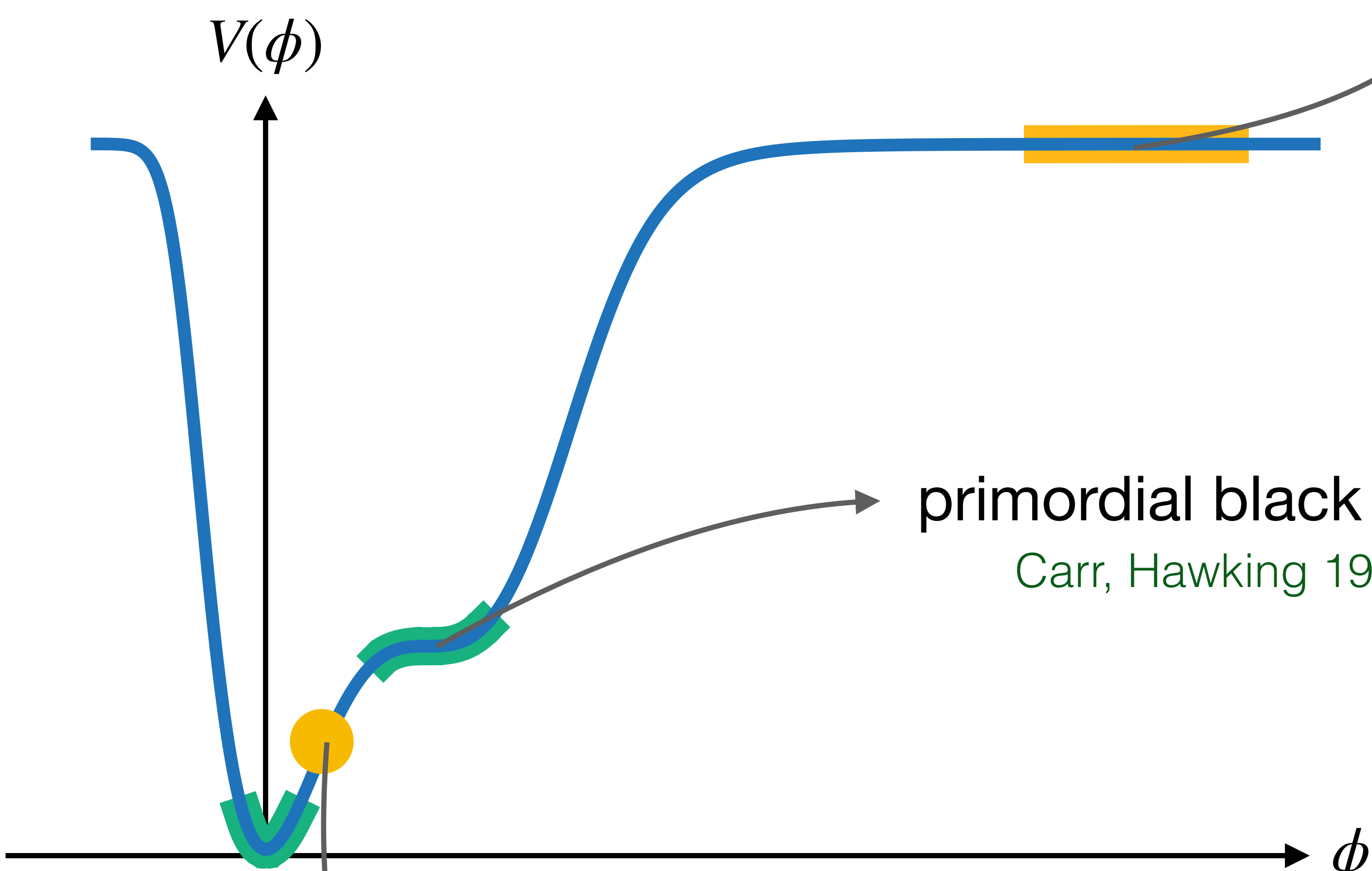
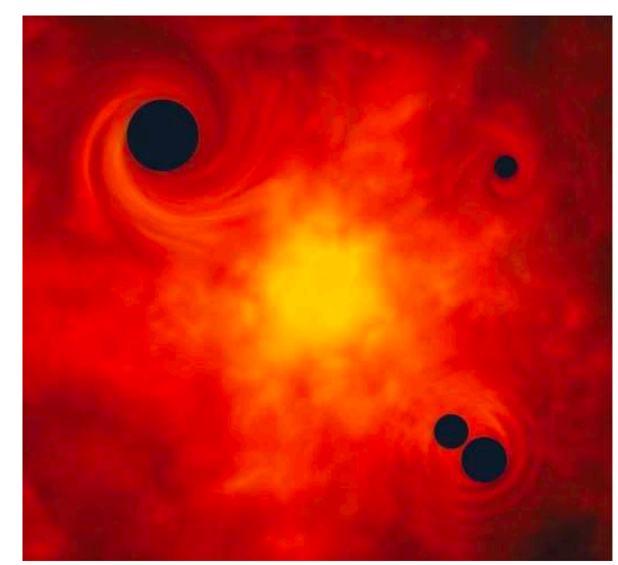


primordial black holes?

Carr, Hawking 1974



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Primordial black holes

- Could constitute part or all of dark matter Chapline 1975
  $M = 10^{16} - 10^{17}\text{g},\ 10^{20} - 10^{24}\text{g},\ 10 - 10^3 M_\odot$
- Could provide progenitors for the LIGO/VIRGO events
  $M = 10 - 100\ M_\odot$
- Could provide seeds for cosmological structures
  $M > 10^3 M_\odot$
  Mészáros 1975
  Afshordi, McDonald, Spergel, 2003
- Could provide seeds for supermassive black holes in galactic nuclei
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  Carr, Rees 1984
  Bean, Magueijo 2002

Draft version September 1, 2020
Typeset using LaTeX **twocolumn** style in AASTeX63

Properties and astrophysical implications of the $150\,\mathrm{M}_\odot$ binary black hole merger GW190521

LIGO Scientific Collaboration and Virgo Collaboration

ABSTRACT

The gravitational-wave signal GW190521 is consistent with a binary black hole merger source at redshift 0.8 with unusually high component masses, $85^{+21}_{-14}\,\mathrm{M}_\odot$ and $66^{+17}_{-18}\,\mathrm{M}_\odot$, compared to previously reported events, and shows mild evidence for spin-induced orbital precession. The primary falls in the mass gap predicted by (pulsational) pair-instability supernova theory, in the approximate range $65-120\,\mathrm{M}_\odot$. The probability that at least one of the black holes in GW190521 is in that range is 99.0%. The final mass of the merger ($142^{+28}_{-16}\,\mathrm{M}_\odot$) classifies it as an intermediate-mass black hole. Under the assumption of a quasi-circular binary black hole coalescence, we detail the physical properties of GW190521's source binary and its post-merger remnant, including component masses and spin vectors. Three different waveform models, as well as direct comparison to numerical solutions of general relativity, yield consistent estimates of these properties. Tests of strong-field general relativity targeting the merger-ringdown stages of the coalescence indicate consistency of the observed signal with theoretical predictions. We estimate the merger rate of similar systems to be $0.13^{+0.30}_{-0.11}\,\mathrm{Gpc}^{-3}\,\mathrm{yr}^{-1}$. We discuss the astrophysical implications of GW190521 for stellar collapse, and for the possible formation of black holes in the pair-instability mass gap through various channels: via (multiple) stellar coalescences, or via hierarchical mergers of lower-mass black holes in star clusters or in active galactic nuclei. We find it to be unlikely that GW190521 is a strongly lensed signal of a lower-mass black hole binary merger. We also discuss more exotic possible sources for GW190521, including a highly eccentric black hole binary, or a primordial black hole binary.

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Properties and astrophysical implications of the 150 $M_\odot$ binary black hole merger GW190521

LIGO

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6.3. *Primordial BH Mergers*

Primordial BHs (PBHs; Carr & Hawking 1974; Khlopov 2010) are thought to be formed from collapse of dark matter overdensities in the very early Universe (at redshifts $z > 20$, i.e. before the formation of the first stars), and may account for a nontrivial fraction of the density of the Universe (Carr et al. 2016; Clesse & García-Bellido 2017). Since the binary components of GW190521 are unlikely both to have formed directly from stellar collapse, it is possible that they may be of PBH origin (Bird et al. 2016); however, theoretical expectations of the mass distribution and merger rate of PBH binaries have large uncertainties (e.g., Byrnes et al. 2018), so we do not attempt to quantify such scenarios. Some theories of PBH formation predict predominantly small component spins $\chi \ll 1$ (Chiba &

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For hints in favour of PBH existence, see *e.g.* García-Bellido and Clesse (1711.10458)

Primordial black holes originating from inflationary fluctuations

- Large fluctuations are required to source PBHs

- If large fluctuations are produced, they might backreact on the expansion dynamics

- During inflation, cosmological perturbations are of quantum-mechanical nature —> quantum backreaction?

- The quantum state in which cosmological perturbations are placed possesses specific features, which allow one to design an effective, stochastic theory to incorporate their backreaction

Stochastic δN formalism

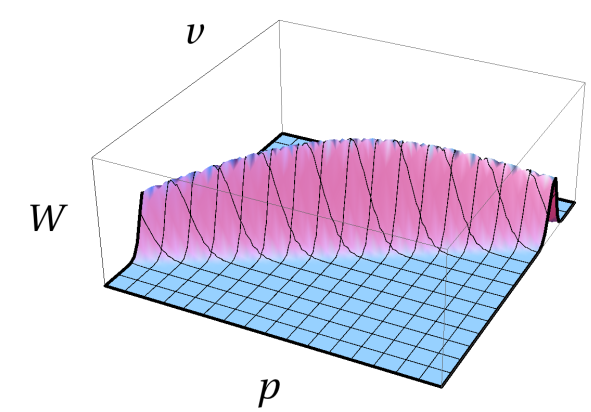
The quantum state of cosmological perturbations

- $|\Psi\rangle = \bigotimes_{\boldsymbol{k}\in\mathbb{R}^{3+}} |\Psi_{\boldsymbol{k}}\rangle$ with $|\Psi_{\boldsymbol{k}}\rangle = \frac{1}{\cosh r_k} \sum_{n=0}^{\infty} e^{2in\varphi_k} (-1)^n \tanh^n r_k |n_{\boldsymbol{k}}, n_{-\boldsymbol{k}}\rangle$

Two-mode squeezed state (Gaussian state)

- Evolution equation $\frac{\partial}{\partial t} W(v,p,t) = -\{W(v,p,t)\ ,\ H(v,p,t)\}_{\text{Poisson Bracket}}$

For quadratic Hamiltonians



- Quantum mean value and stochastic average

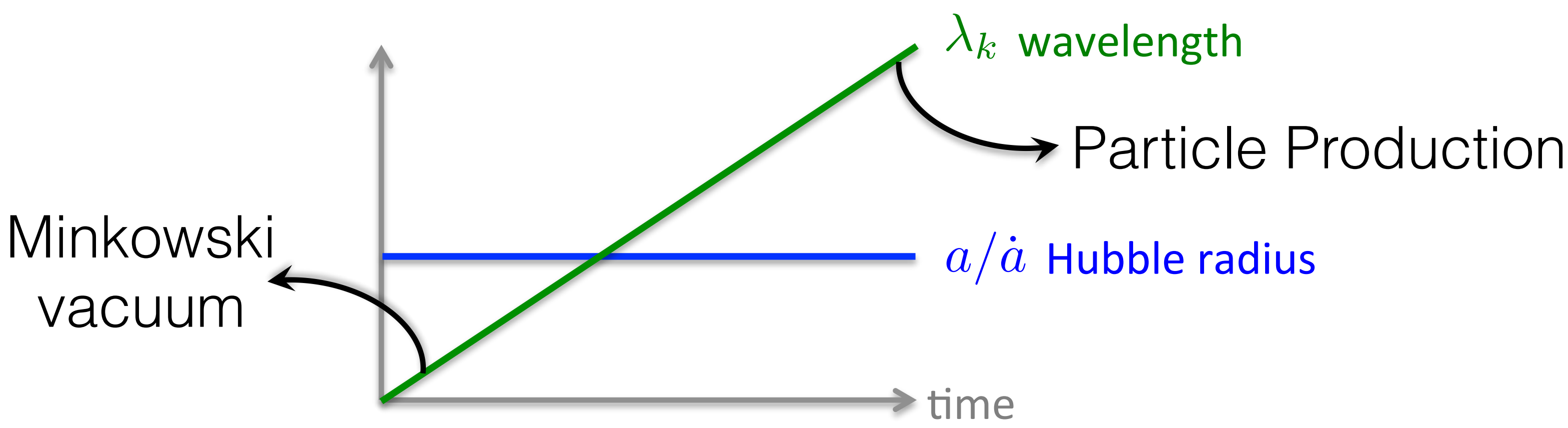
$$\left\langle \hat{\mathcal{O}}(\hat{v},\hat{p}) \right\rangle_{\text{quant}} = \int W(v,p)\, \tilde{\mathcal{O}}(v,p)\, \mathrm{d}v\, \mathrm{d}p$$

Large squeezing: $\tilde{O}(v,p) \longrightarrow O(v,p)$

Lesgourgues, Polarski, Starobinsky (1997)
Martin, VV (2016)

(at least for proper operators…) Revzen (2006); Martin, VV (2017)

Stochastic Inflation



Quantum fluctuations source the **background**

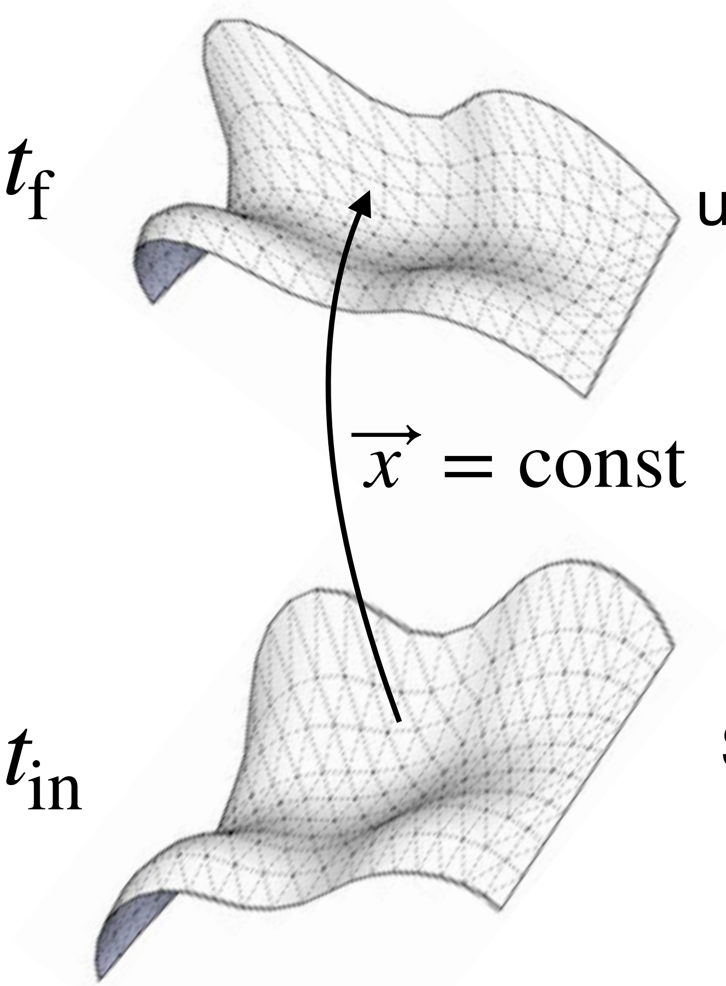
Coarse-grained field $\hat{\phi}_{\text{coarse grained}} = \int_{k<\sigma a H(N)} \mathrm{d}^3\boldsymbol{k} \left[\phi_{\boldsymbol{k}}(N)\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \hat{a}_{\boldsymbol{k}} + \phi^*_{\boldsymbol{k}}(N)\, e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \hat{a}^\dagger_{\boldsymbol{k}}\right]$

$N = \ln(a)$

At leading order in slow roll: $\frac{\mathrm{d}}{\mathrm{d}N}\phi_{\text{cg}} = -\frac{V'(\phi_{\text{cg}})}{3H^2(\phi_{\text{cg}})} + \frac{H(\phi_{\text{cg}})}{2\pi}\xi(N)$ Starobinsky, (1982) 1986

Over one e-fold: $\frac{\Delta\phi_{\text{quant}}}{\Delta\phi_{\text{class}}} \sim \zeta$

Stochastic-δN formalism



uniform density slice
$\delta\rho = 0$

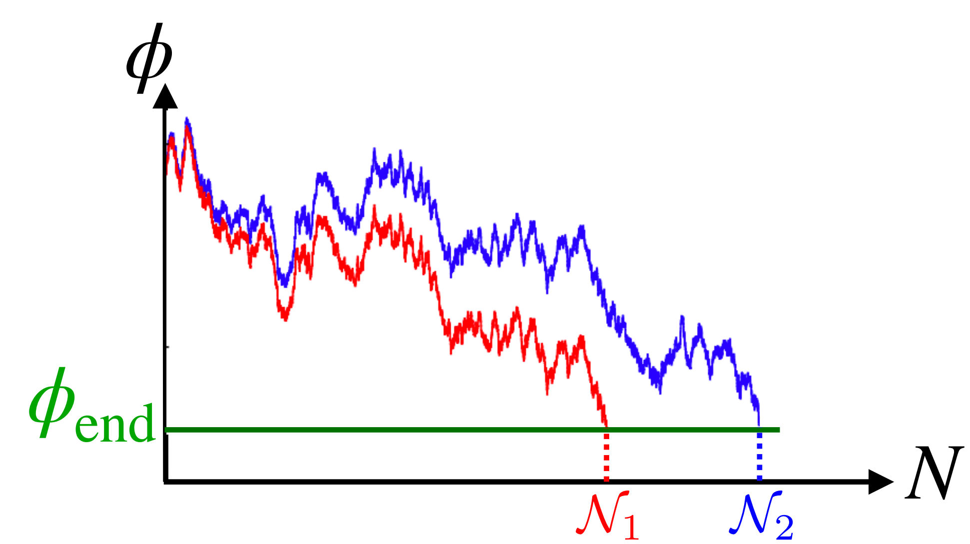
Spatially flat slice
$\psi = 0$

$$\zeta(t,x) = N(t,x) - N_0(t) \equiv \delta N$$

Lifshitz, Khalatnikov (1960)
Starobinsky (1983)
Wands, Malik, Lyth, Liddle (2000)

The realised number of e-folds
is a stochastic quantity:

$$\zeta_\text{coarse grained} = \mathcal{N} - \langle \mathcal{N} \rangle$$



Enqvist, Nurmi, Podolsky, Rigopoulos (2008) ; Fujita, Kawasaki, Tada, Takesako (2015); VV, Starobinsky (2015)

Stochastic-δN formalism

$$\frac{\mathrm{d}\phi}{\mathrm{d}N} = -\frac{V'}{3H^2} + \frac{H}{2\pi}\xi(N)$$

Langevin equation

$$\frac{\mathrm{d}}{\mathrm{d}N}P(\phi, N) = \frac{\partial}{\partial\phi}\left(\frac{V'}{3H^2}P\right) + \frac{\partial^2}{\partial\phi^2}\left(\frac{H^2}{8\pi^2}P\right) = \mathscr{L}_\phi \cdot P$$

Fokker-Planck equation

Equation for the PDF of the first passage time

VV, Starobinsky (2015)
Pattison, VV, Assadullahi, Wands (2017)

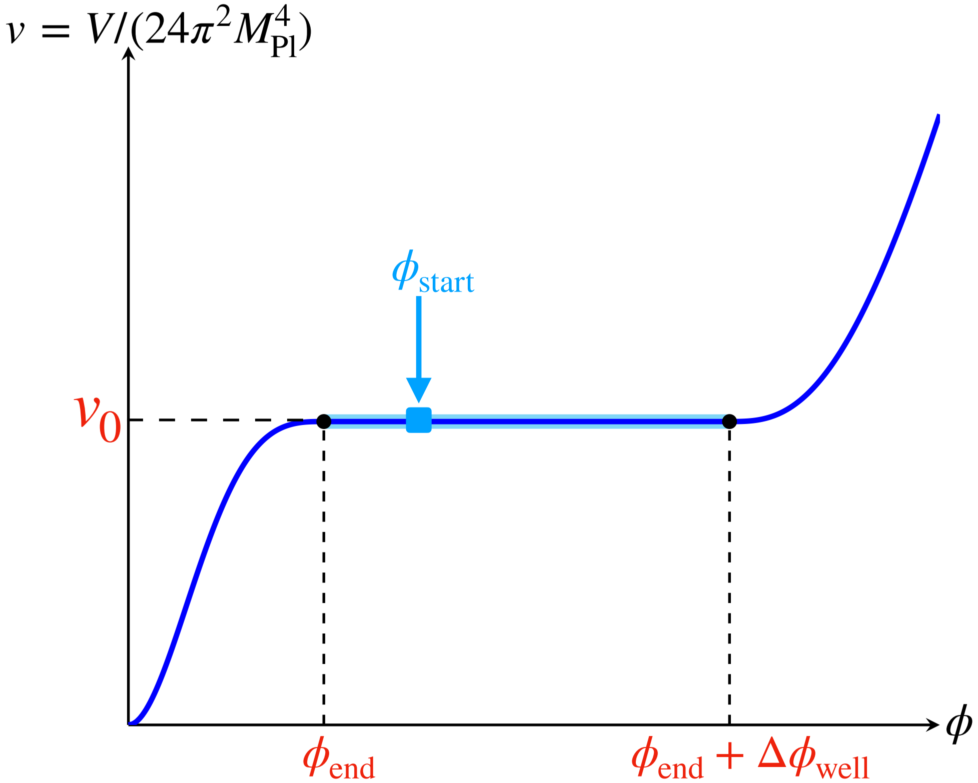
$$\frac{\mathrm{d}}{\mathrm{d}\mathcal{N}}\mathscr{P}(\mathcal{N}, \phi) = \mathscr{L}_\phi^\dagger \cdot \mathscr{P}$$

Computational program:

- Solve the first passage time problem
- Identify the PDF of the first passage time with the PDF of (coarse-grained) curvature perturbations
- Integrate that PDF above the PBH formation threshold
- Extract the PBH mass fraction

Heavy Tails

Toy model: flat potential

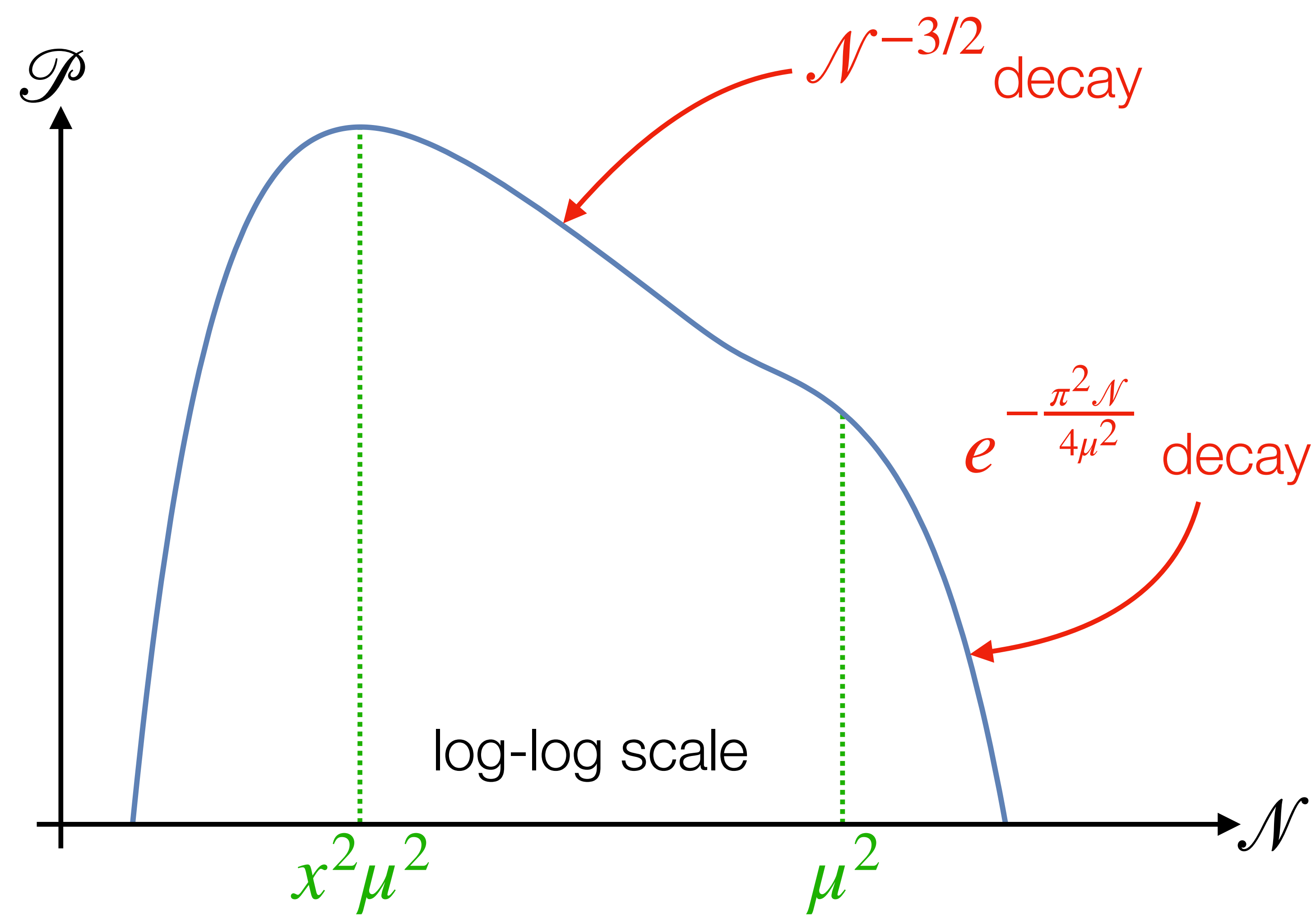


Pattison, VV, Assadullahi, Wands (2017)

$$\langle \mathcal{N} \rangle = \mu^2 x(1 - x/2)$$

$\mathcal{P}(\mathcal{N}/\mu^2; x)$ is universal

$$x = \frac{\phi - \phi_0}{\Delta\phi_{\rm well}} \qquad \mu^2 = \frac{\Delta\phi_{\rm well}^2}{v_0 M_{\rm Pl}^2}$$



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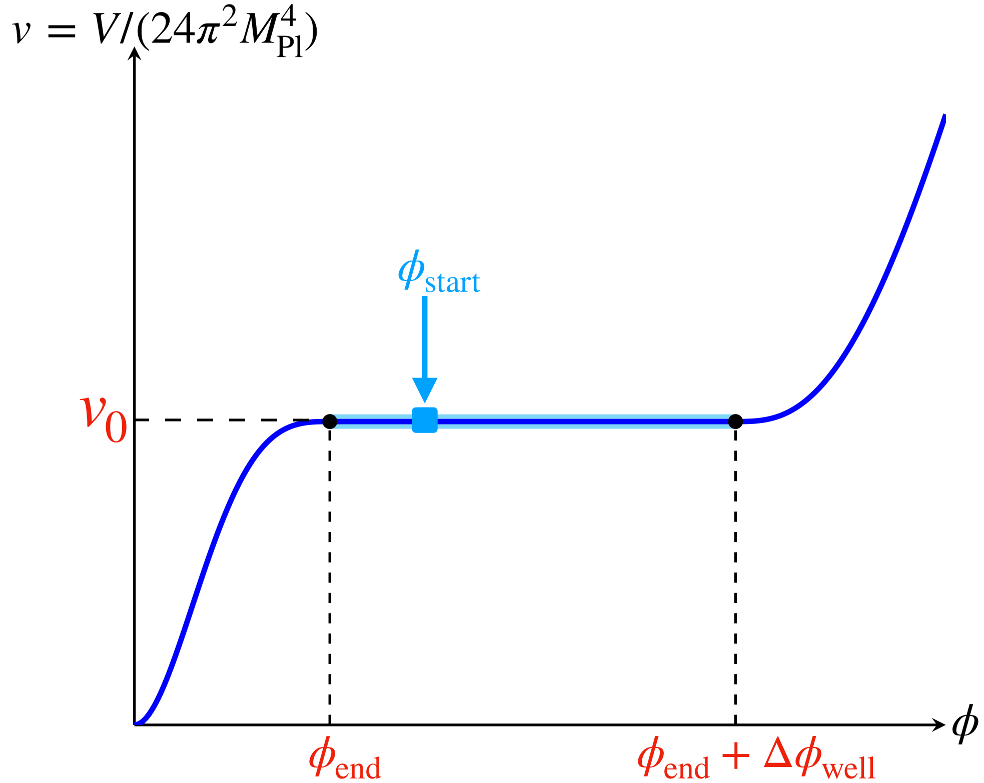

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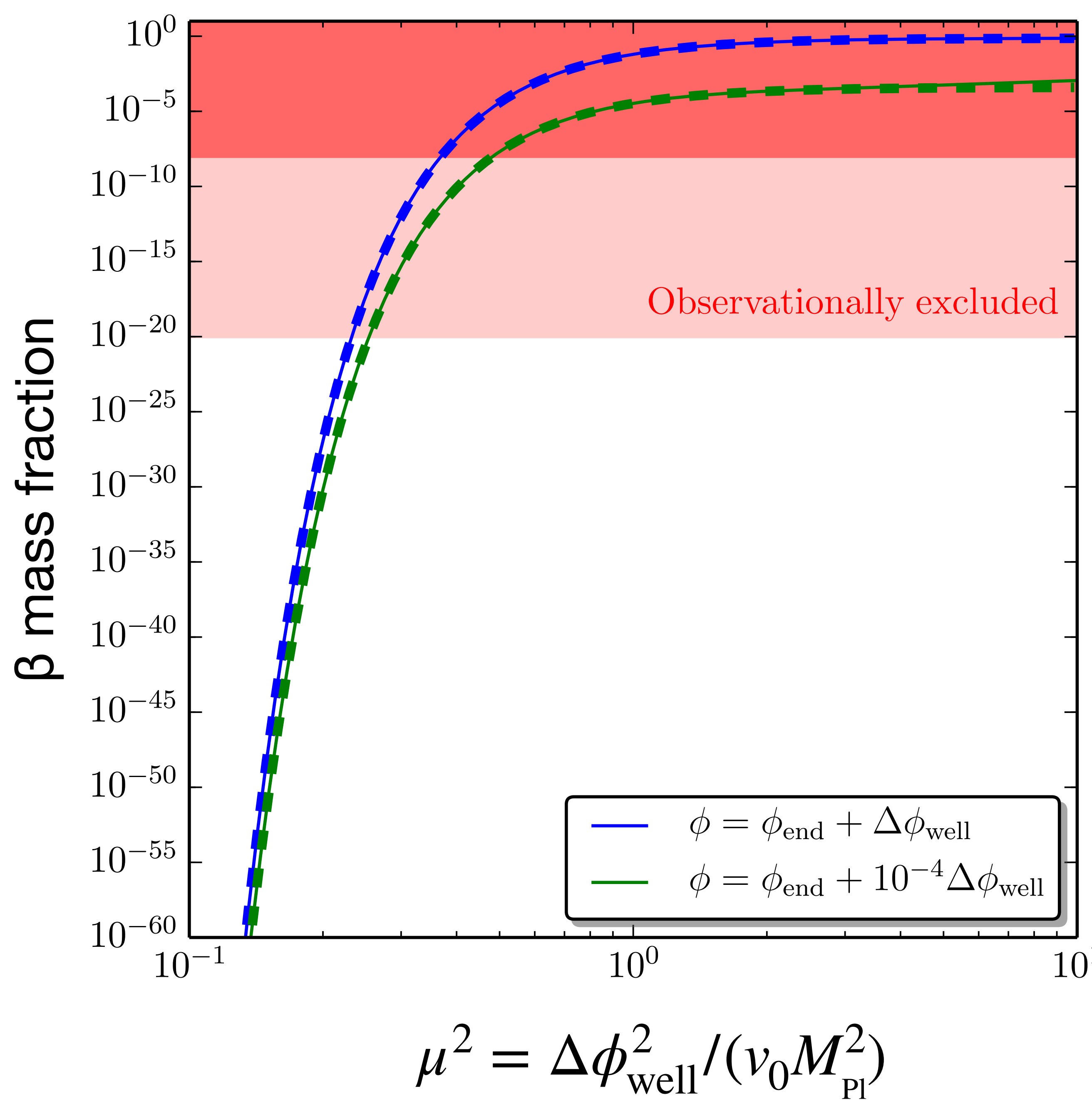

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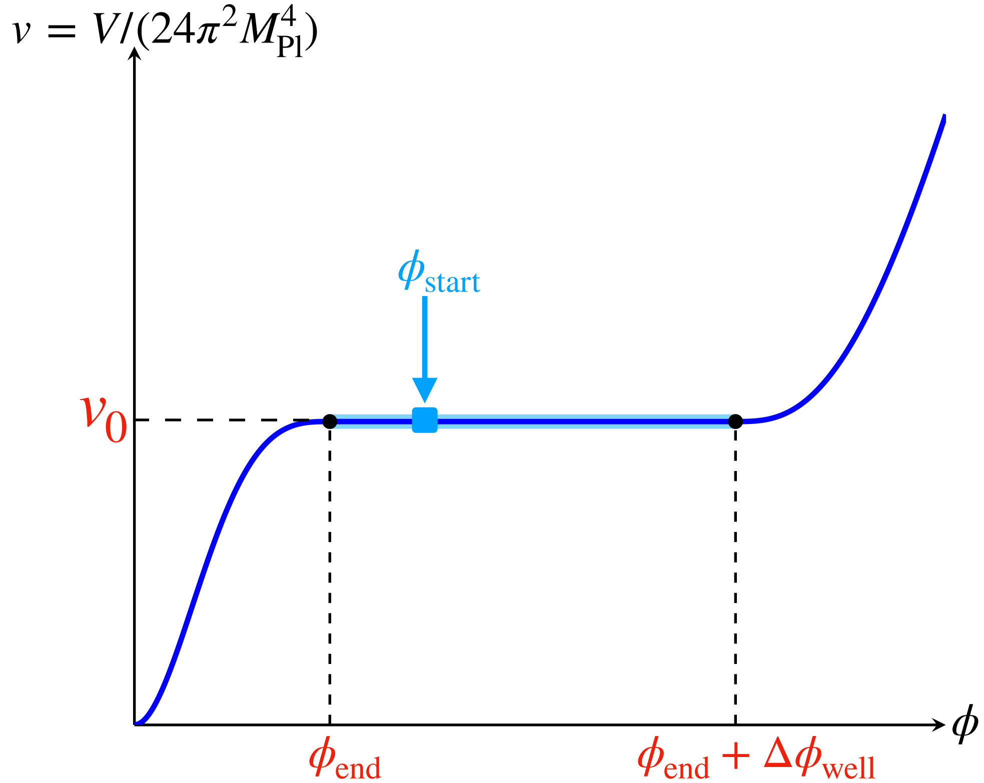

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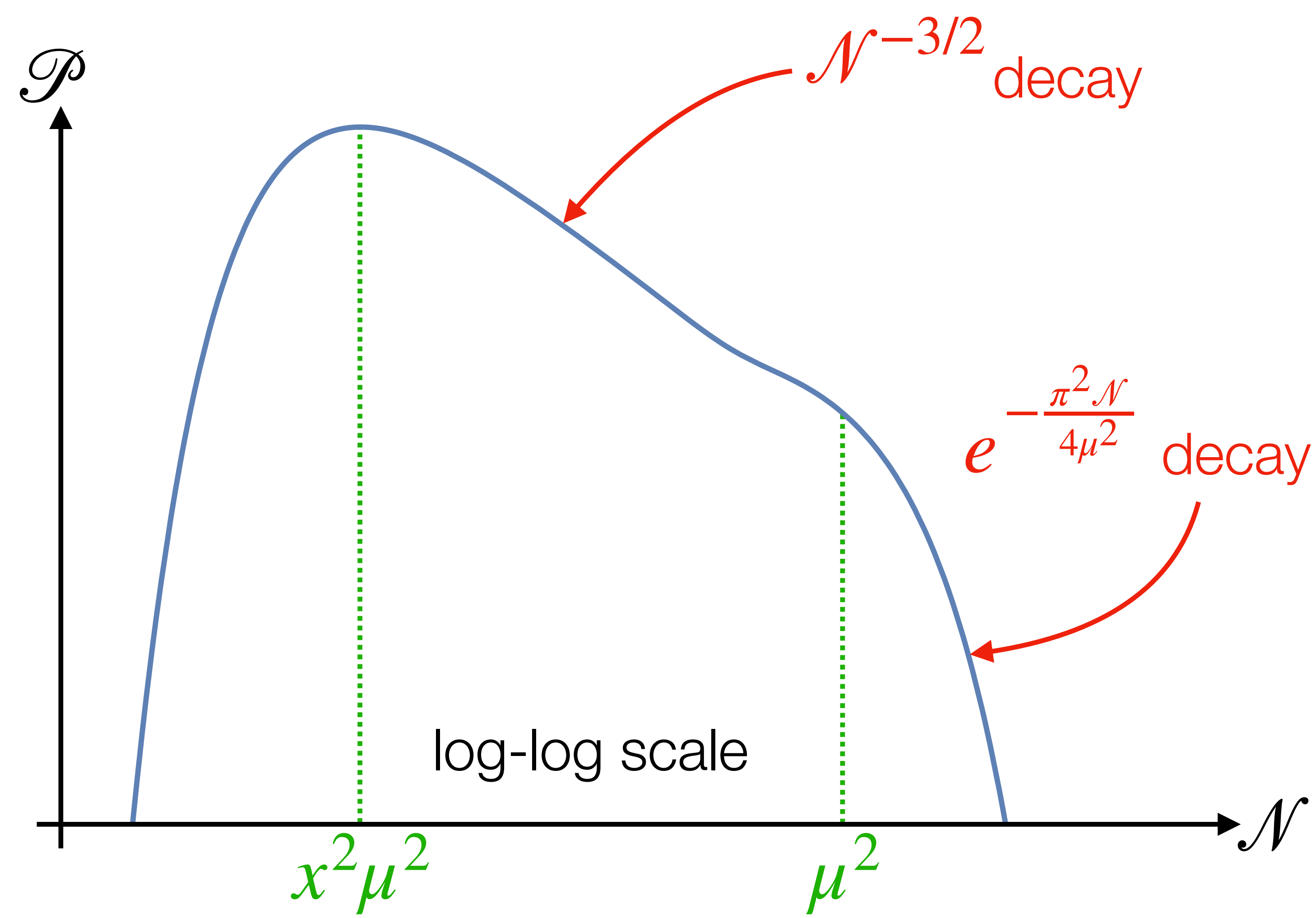

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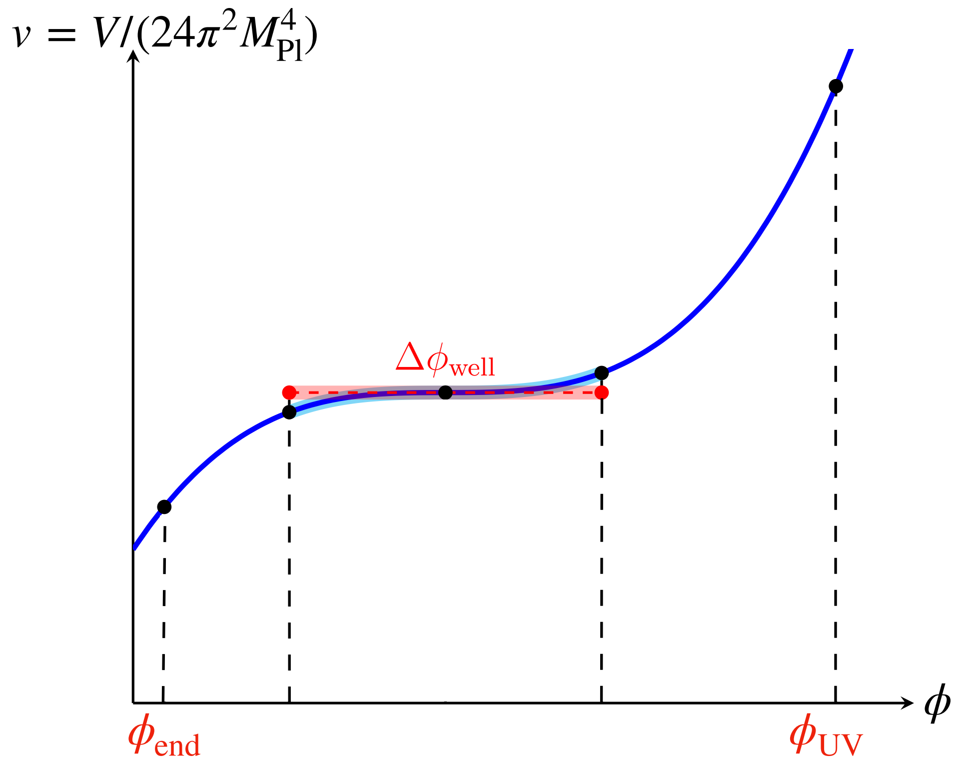

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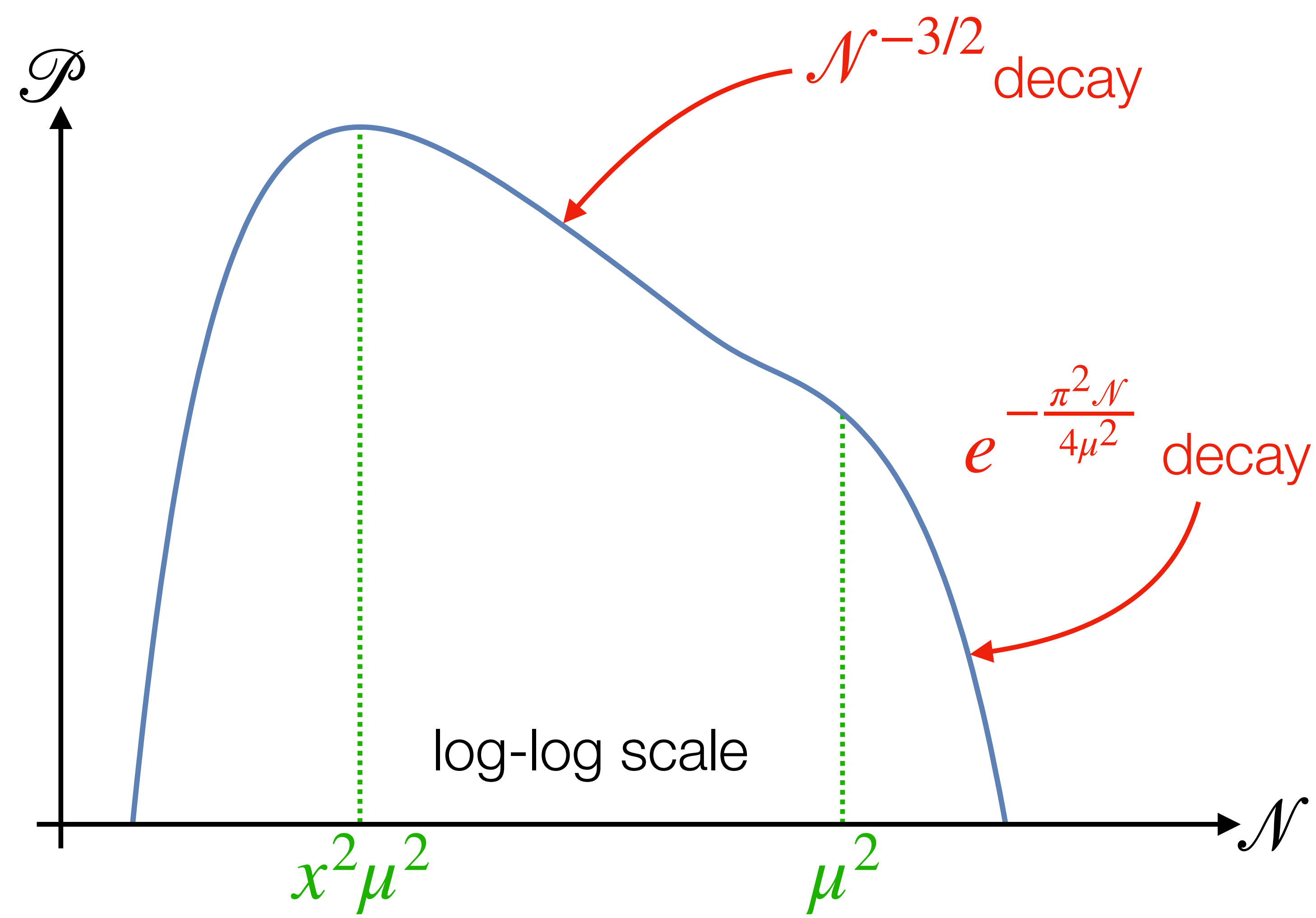

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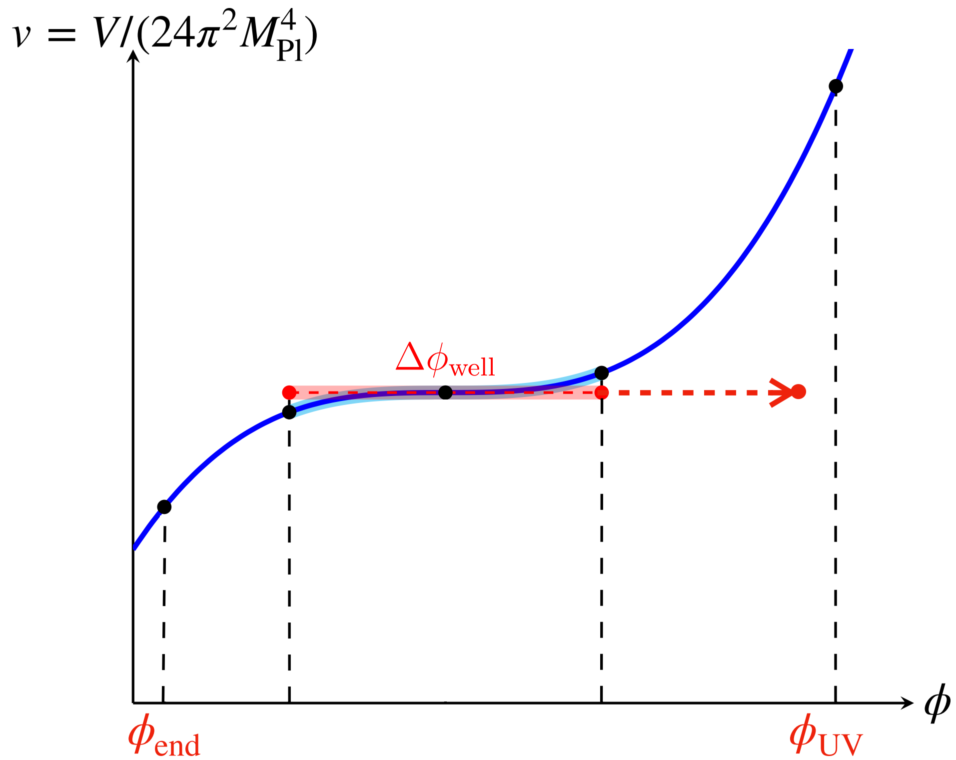

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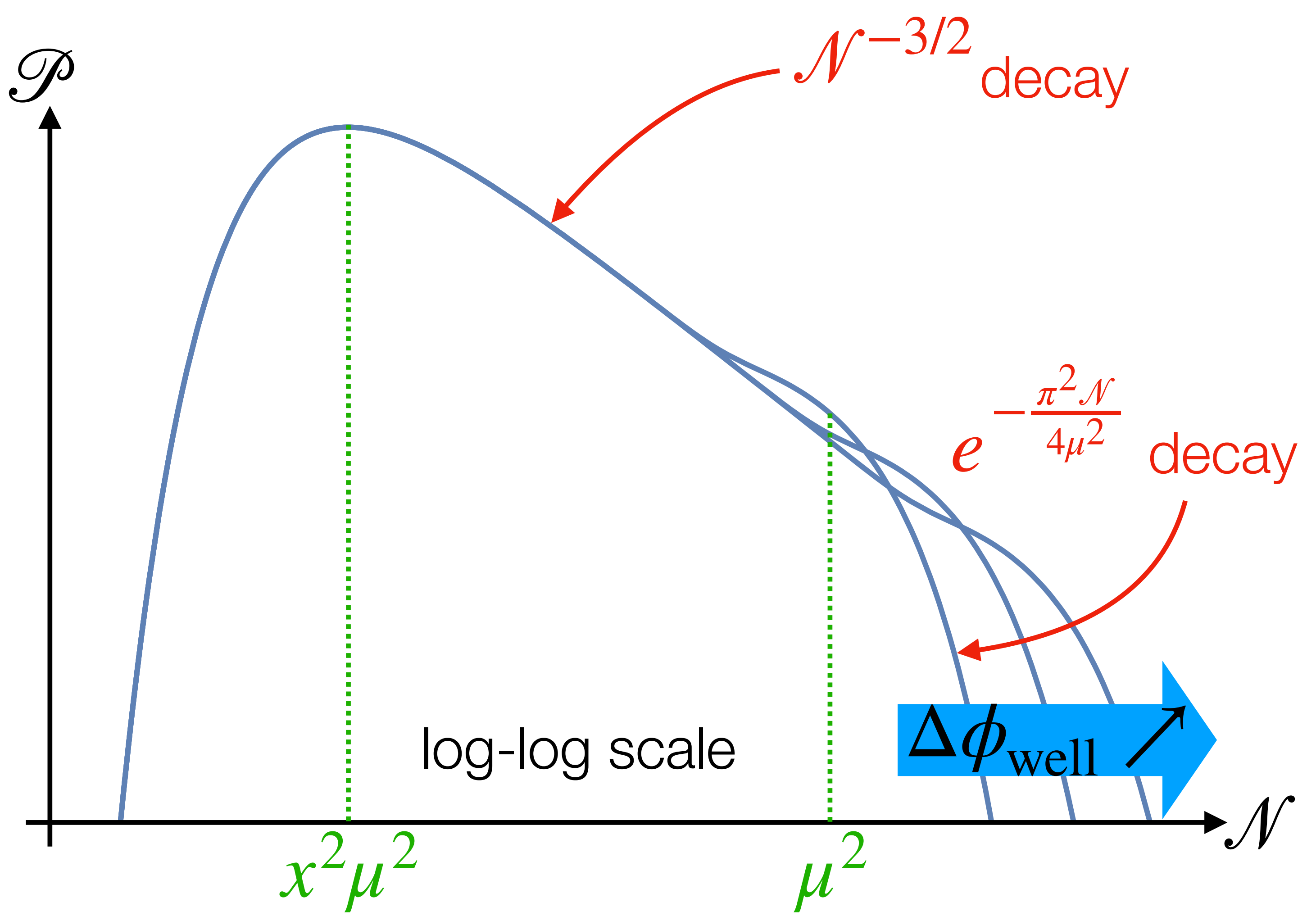
$\Delta\phi_{\rm well} \to \infty$: infinite inflation?



Exponential tails

Ezquiaga, Garcia-Bellido, VV (2020)

$$\mathscr{P}(\mathcal{N},\phi)=\sum a_n(\phi)e^{-\Lambda_n\mathcal{N}}$$

Flat well

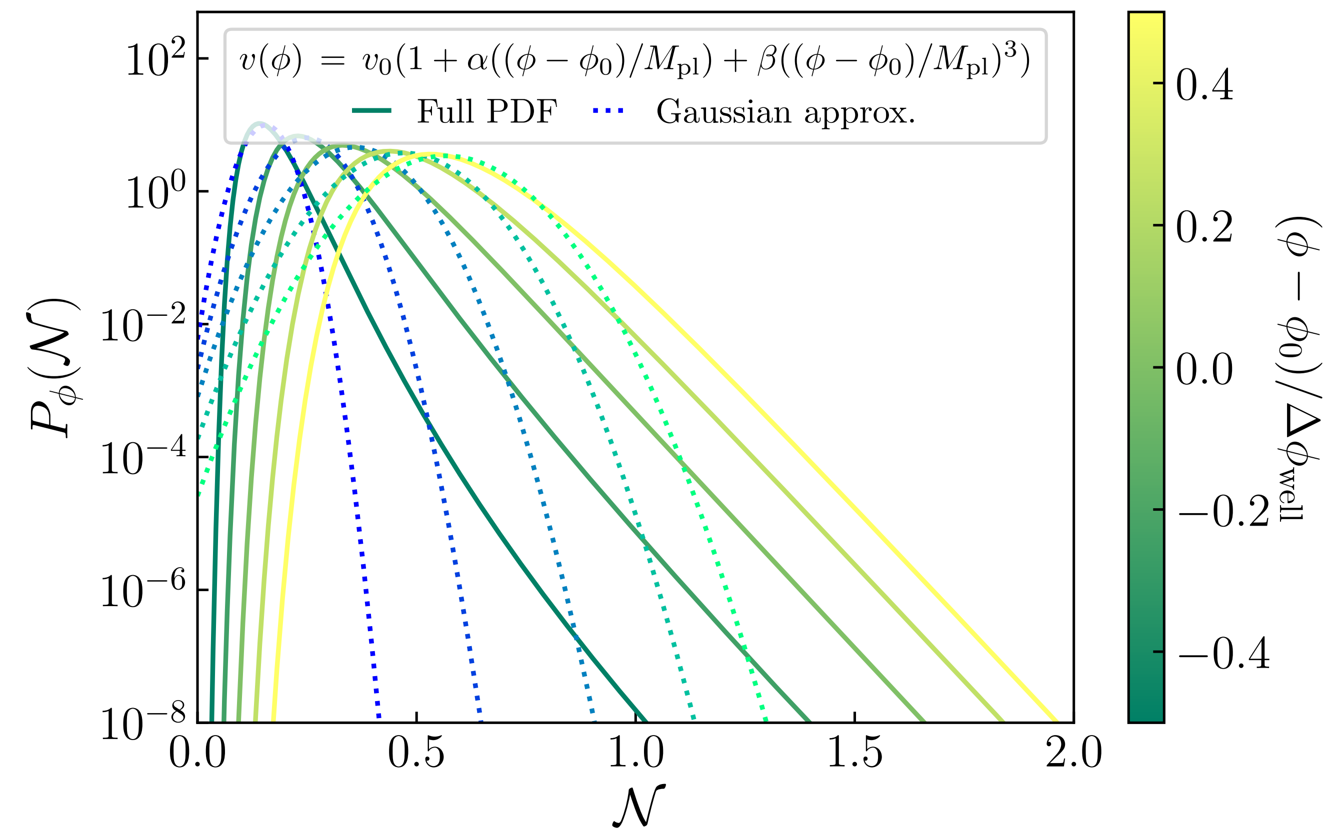
$$\Lambda_n=\frac{\pi^2}{\mu^2}\left(n+\frac{1}{2}\right)^2 \qquad \mu^2=\frac{\Delta\phi_{\rm well}^2}{v_0M_{\rm Pl}^2}$$

Constant slope well $\quad v=v_0\left(1+\alpha\frac{\phi}{M_{\rm Pl}}\right)$

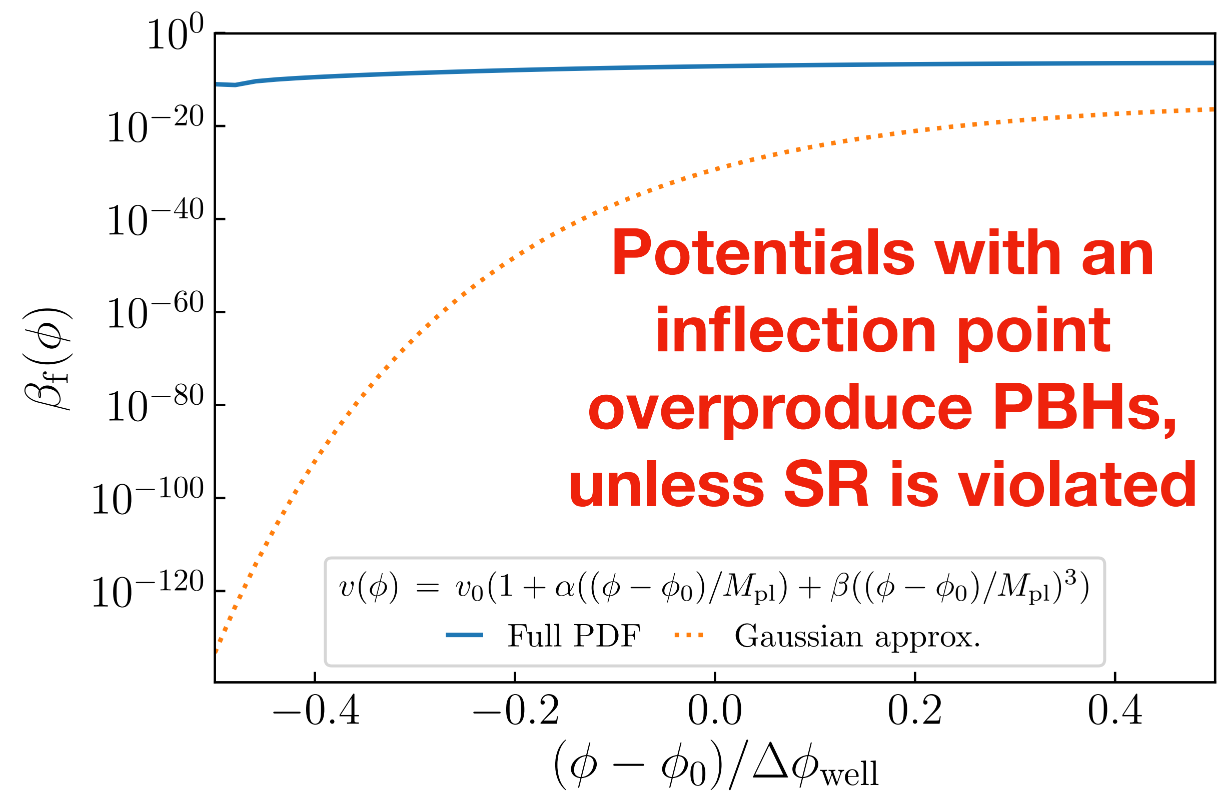
$$\Lambda_n\simeq\frac{\alpha^2}{4v_0}+\frac{\pi^2}{\mu^2}\left(n+\frac{1}{2}\right)^2$$

Cubic inflection point $\quad v=v_0\left(1+\alpha\frac{\phi^3}{M_{\rm Pl}^3}\right)$

$$\Lambda_n\simeq\left(\frac{3}{2}\right)^{2/3}\pi^2\left(v_0\alpha\right)^{1/3}\left(n+\frac{1}{2}\right)^2$$

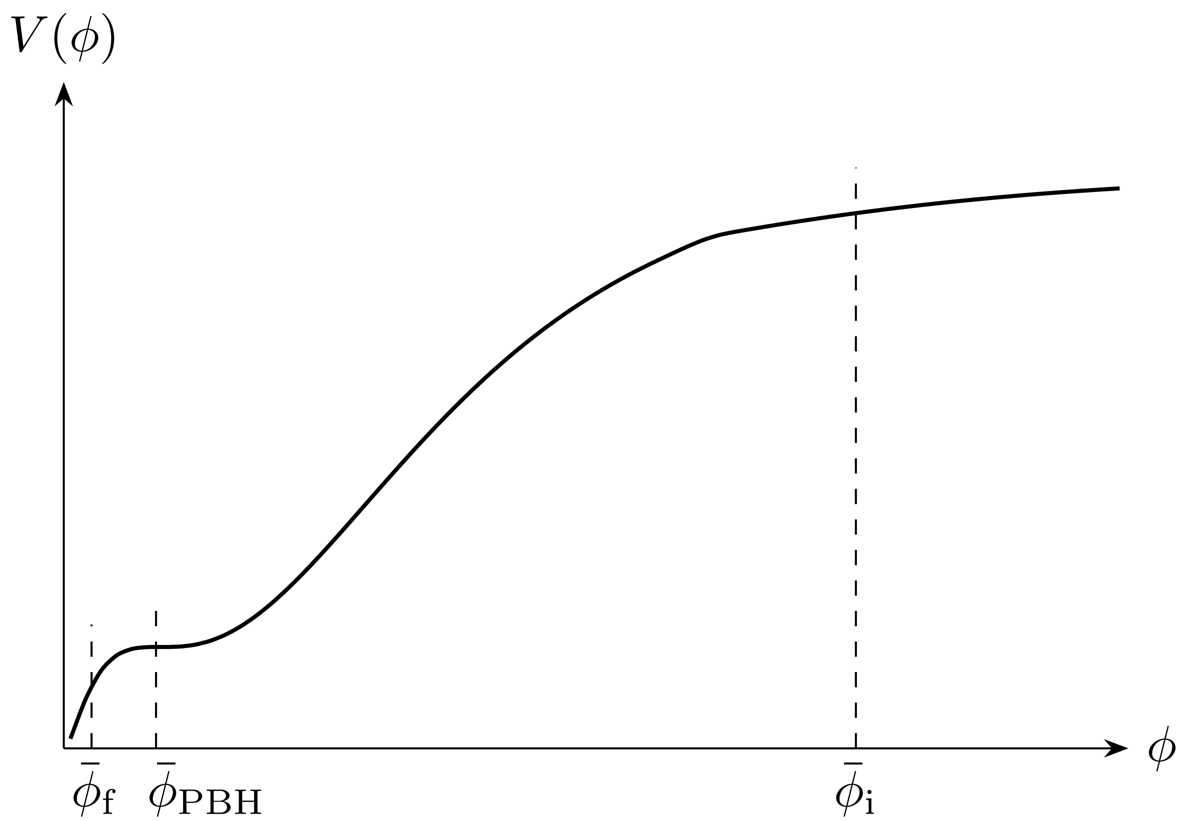


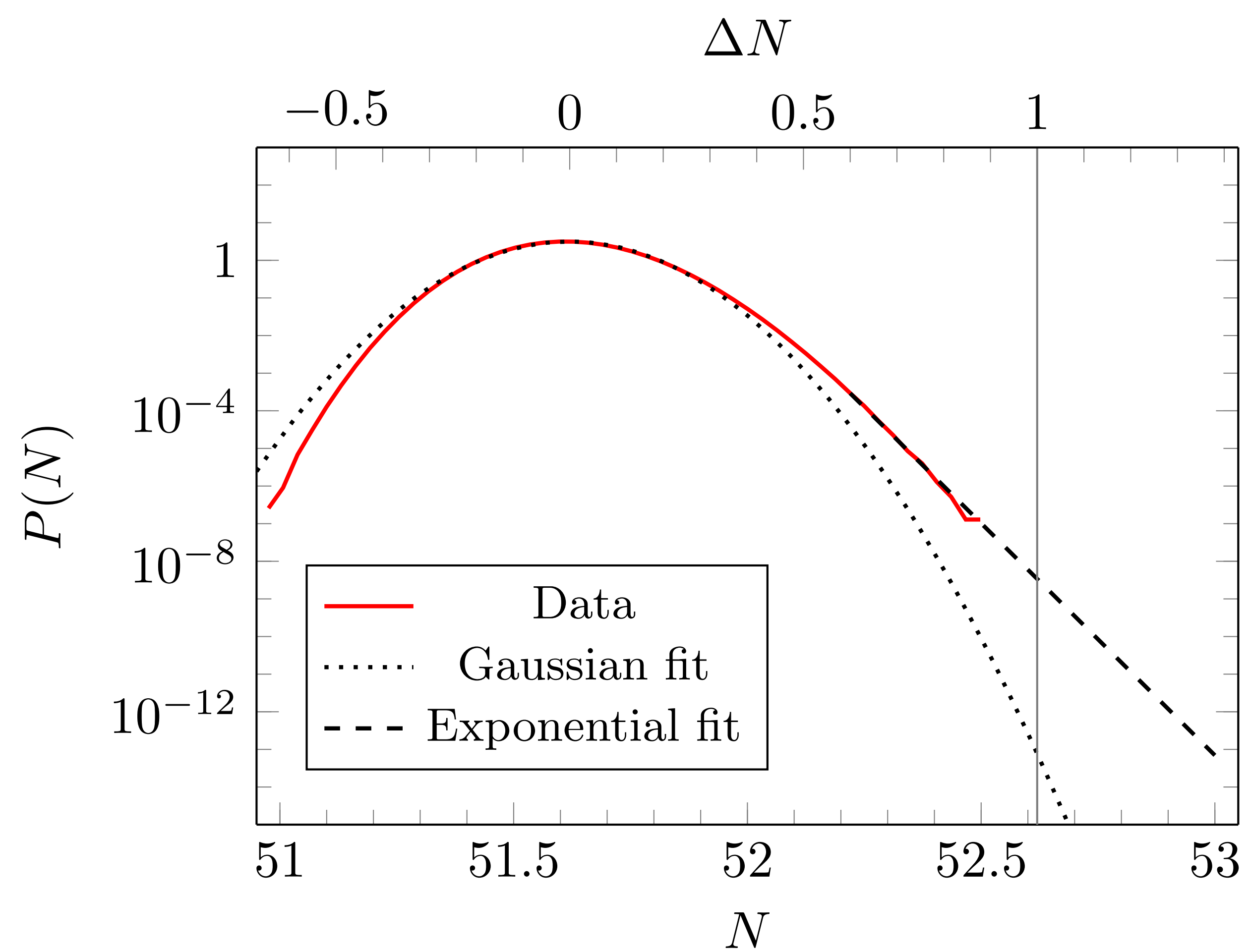
$v_0=10^{-3},\ \alpha=0.24,\ \beta=9,\ \phi_{\rm end}=9$



Exponential tails
in ultra slow roll models

D. Figueroa, S. Raatikainen, S. Räsänen, E. Tomberg (2020)





Exponential tails
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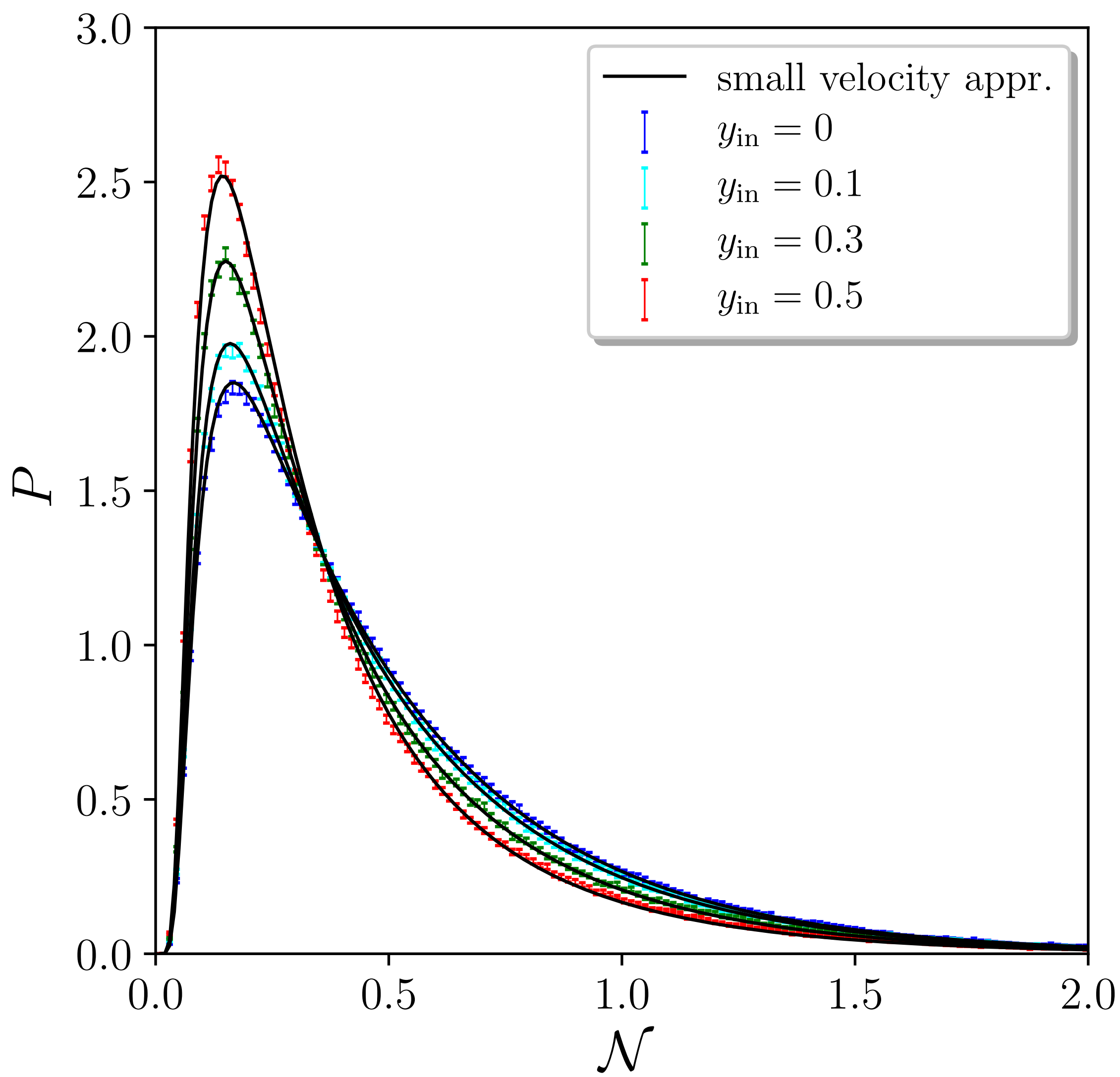
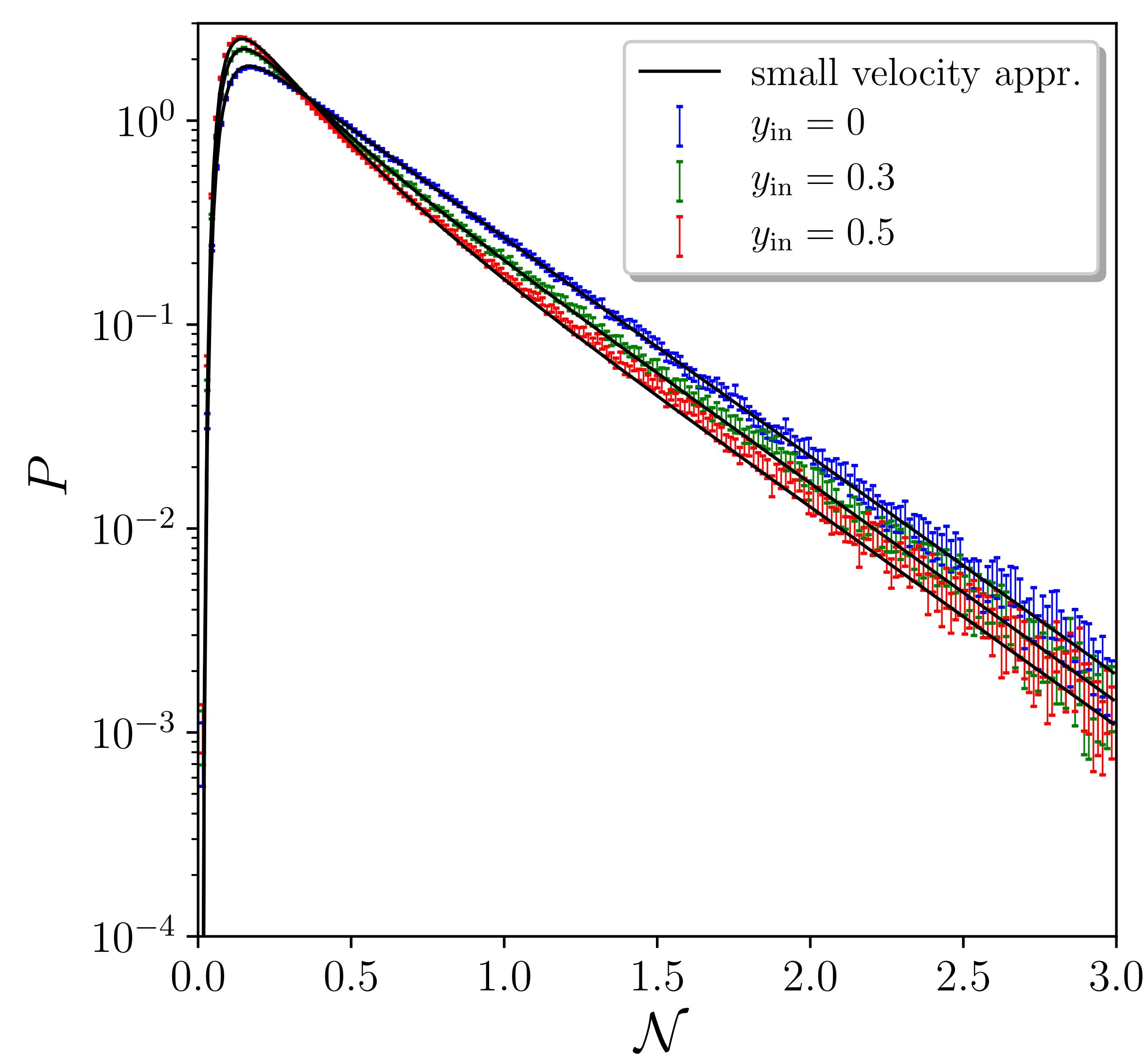
Pattison, Vennin, Wands, Assadullahi (2021)





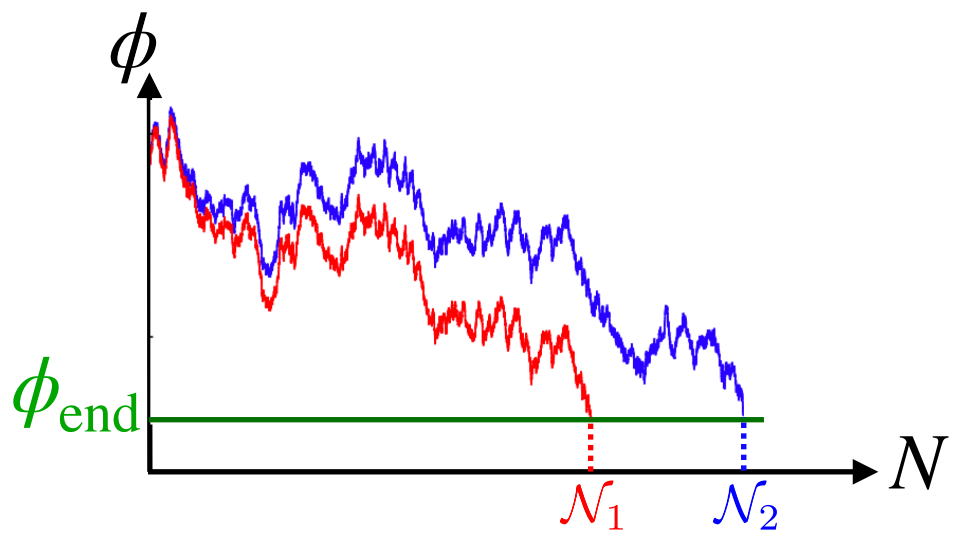
CMB probes the full potential

Kenta Ando, VV (2020)

So far we have computed the statistics of $\delta\mathcal{N}(\phi) \longrightarrow \delta\mathcal{N}(k)$?

Classically, there is a one-to-one correspondence

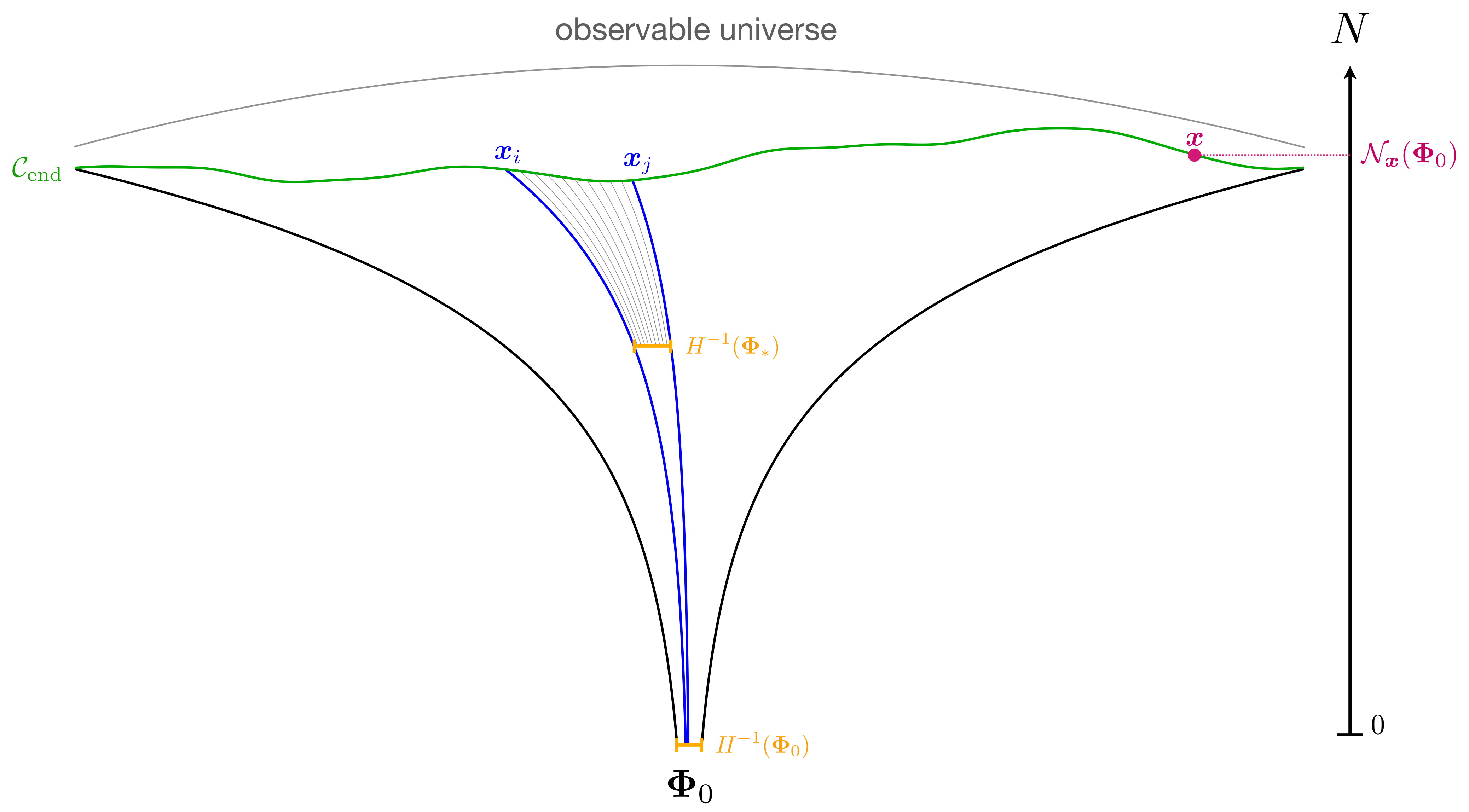
$\phi(k)$ is the value of ϕ when $k = aH$ during inflation



In the stochastic world,
this one-to-one correspondence is lost!

CMB probes the full potential

Kenta Ando, VV (2020)



The distance between two patches is encoded in the time at which they become statistically independent

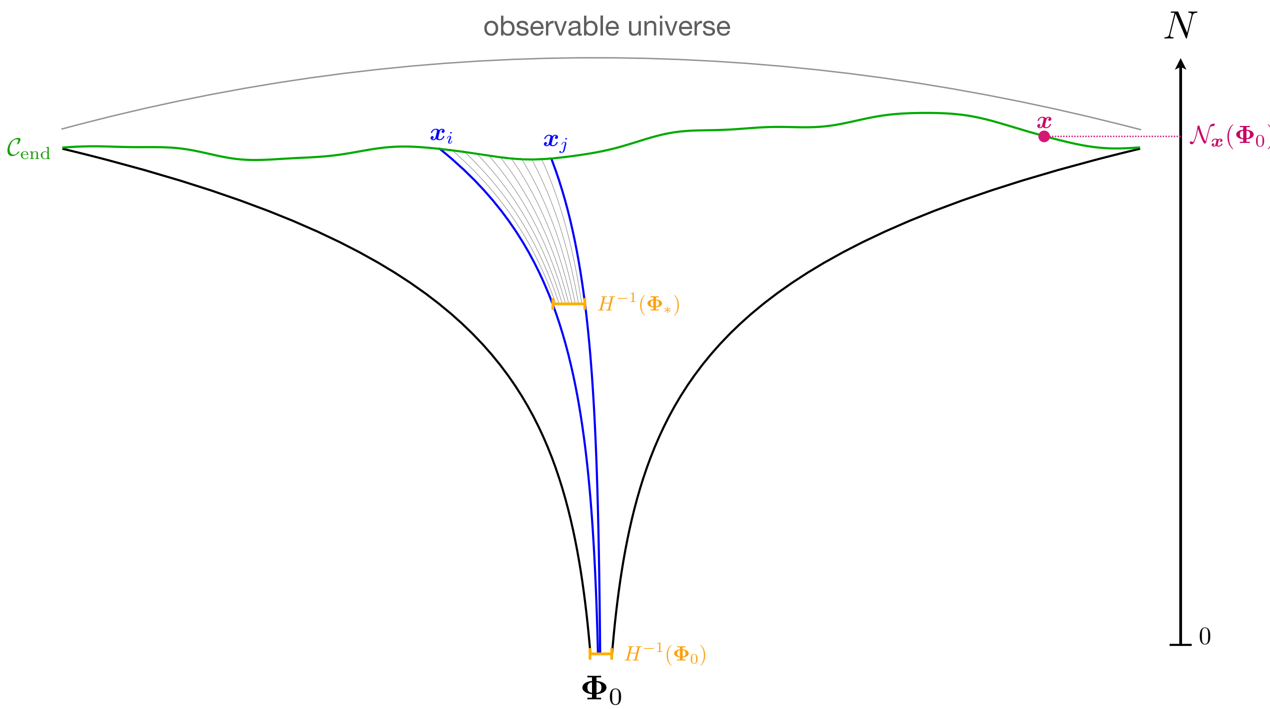
—> the two-point function is contained in the one-point dynamics of the Langevin equation

CMB probes the full potential

Kenta Ando, VV (2020)

$$\mathcal{P}_\zeta(k) = \frac{1}{k}\int \mathrm{d}\boldsymbol{\Phi}_* \left.\frac{\partial P_r\left(\boldsymbol{\Phi}_*\right)}{\partial r}\right|_{r=1/k}\left\langle\delta\mathcal{N}^2\right\rangle\left(\boldsymbol{\Phi}_*\right)$$

Integration over the full inflating domain



$$P_r\left(\boldsymbol{\Phi}_*\right) \simeq P_{\rm bw}\left[\boldsymbol{\Phi}_*, N_{\rm bw}(r)\right]$$

$$P_{\rm bw}\left(\boldsymbol{\Phi}_*, N_{\rm bw}\right) = P_{\rm FPT}\left(N_{\rm bw}, \boldsymbol{\Phi}_*\right)\frac{\displaystyle\int_0^\infty \mathrm{d}N P(\boldsymbol{\Phi}_*, N|\boldsymbol{\Phi}_0, 0)}{\displaystyle\int_{N_{\rm bw}}^\infty \mathrm{d}N_{\rm tot} P_{\rm FPT}\left(N_{\rm tot}, \boldsymbol{\Phi}_0\right)}$$

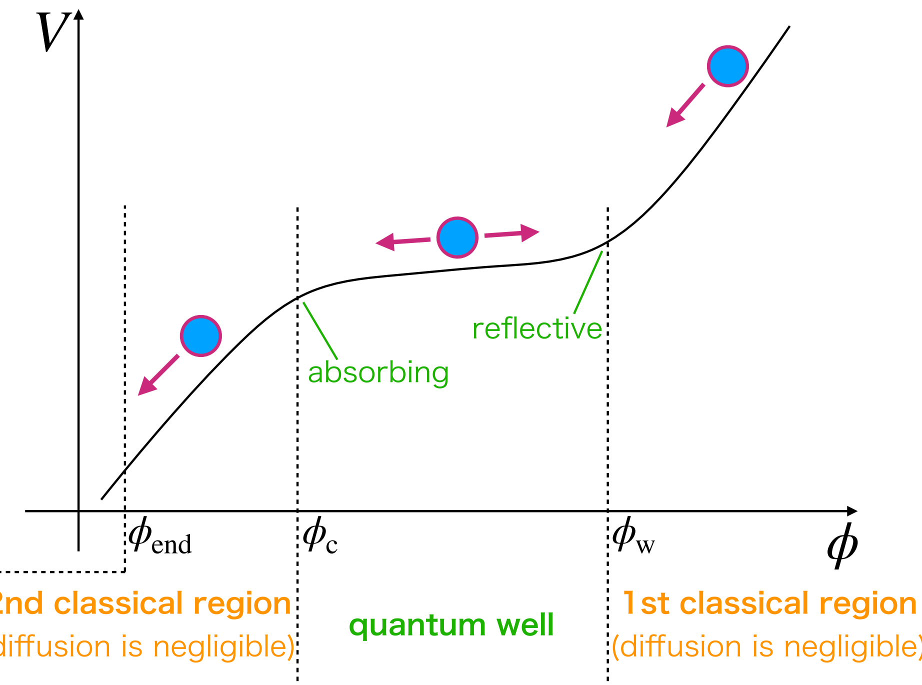
In the limit of low quantum diffusion

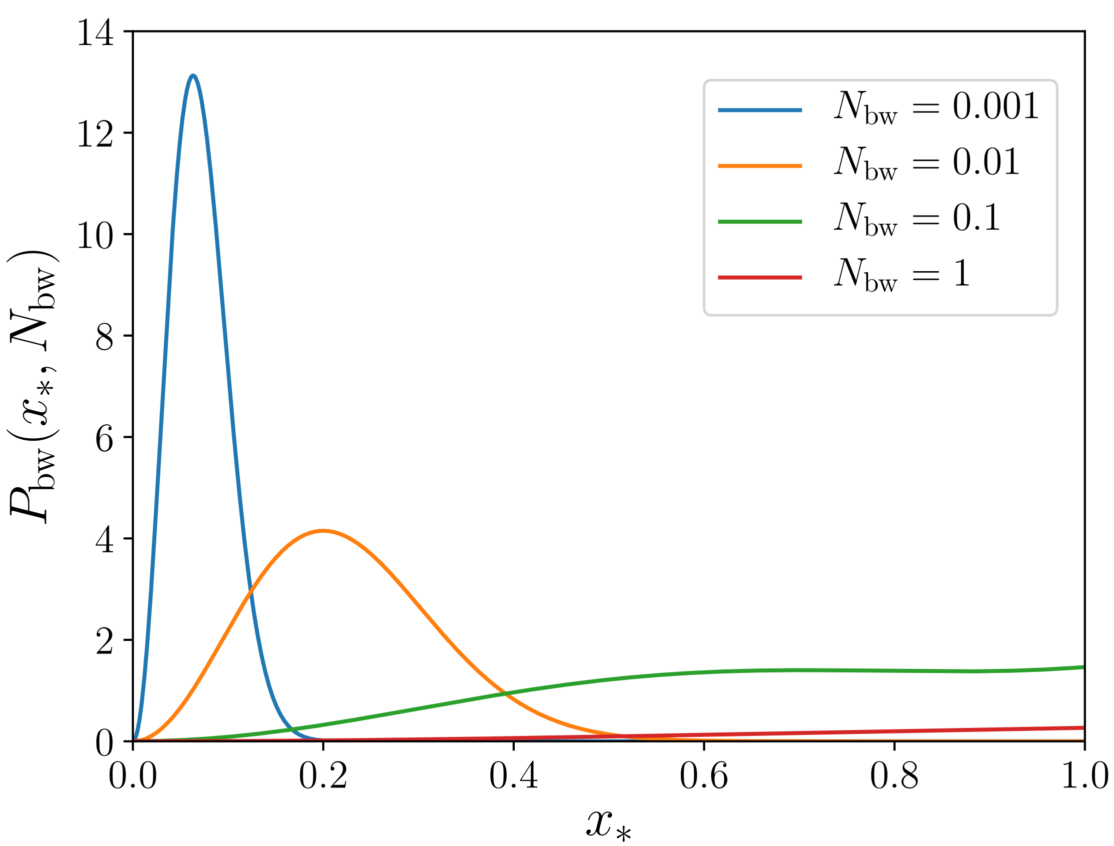
$$\mathcal{P}_\zeta(k) \simeq \int \mathrm{d}\boldsymbol{\Phi}_* P_{\rm bw}\left[\boldsymbol{\Phi}_*, N_{\rm bw}(k)\right]\mathcal{P}_{\zeta,\rm cl}(\boldsymbol{\Phi}_*)$$

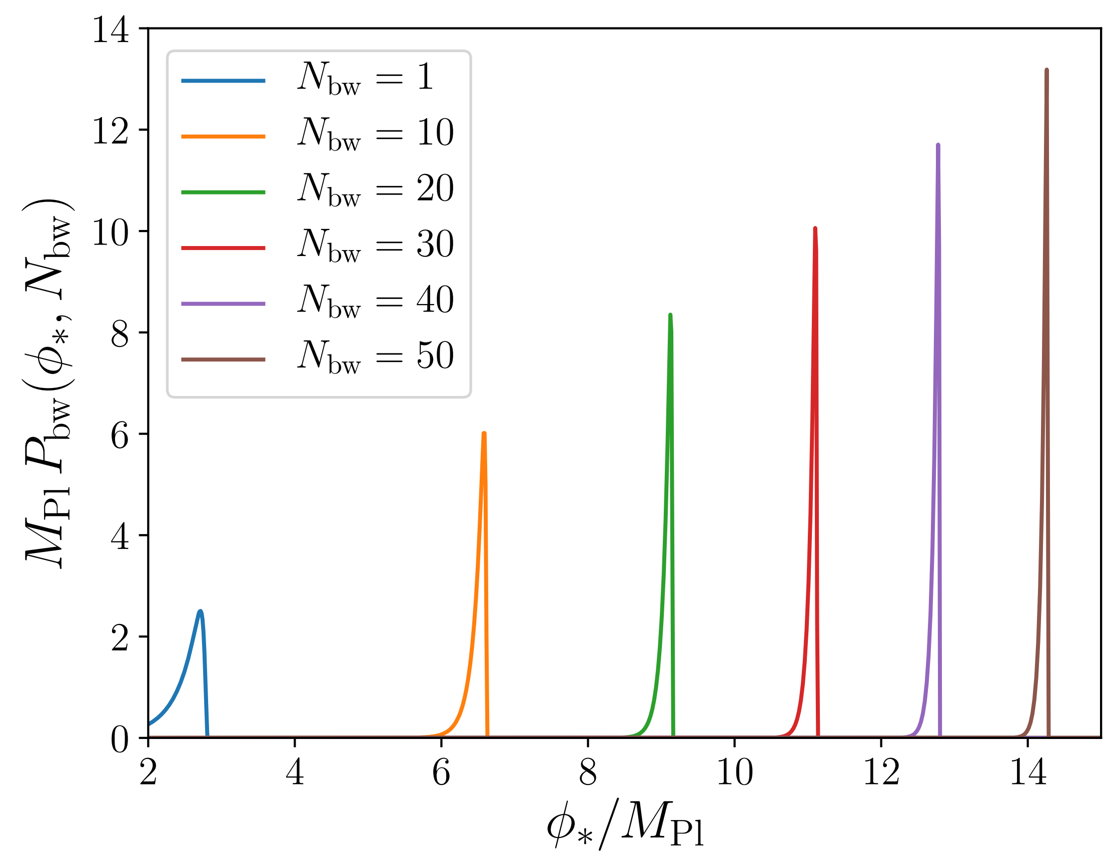
Integral of the standard result against the backward probability

CMB probes the full potential

Kenta Ando, VV (2020)







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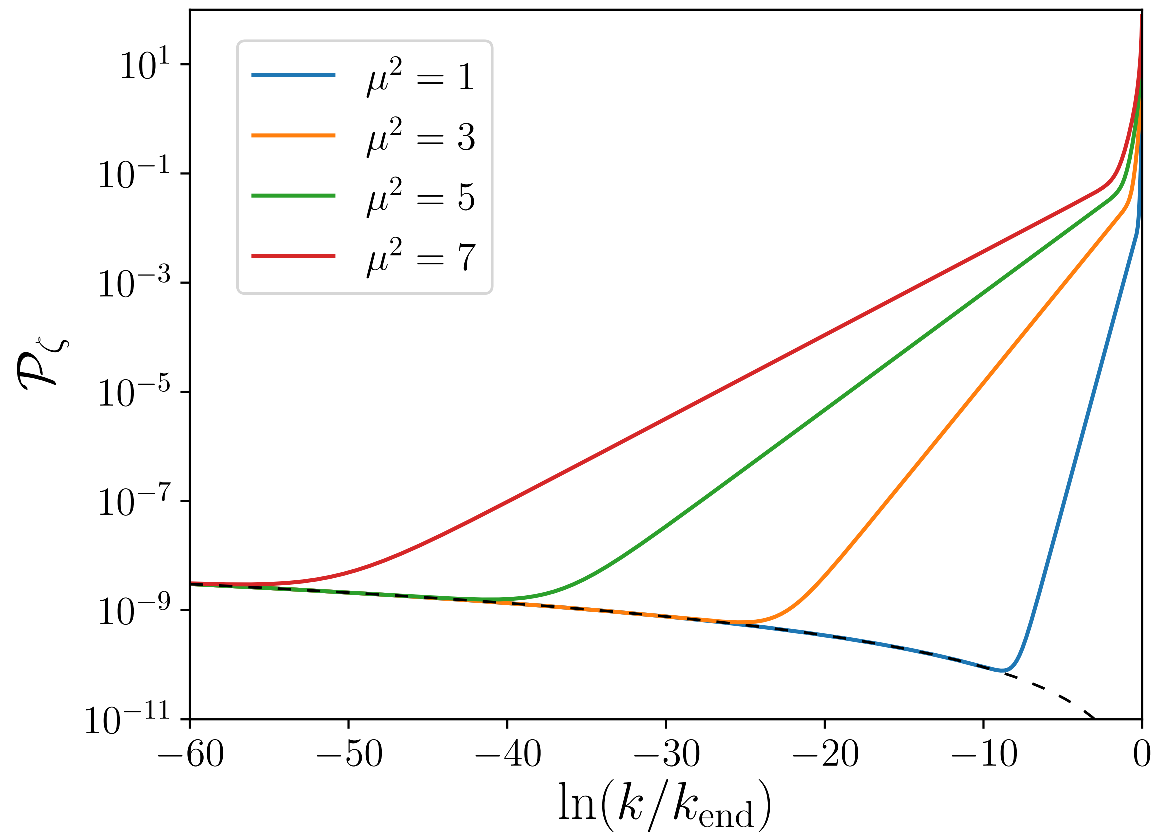

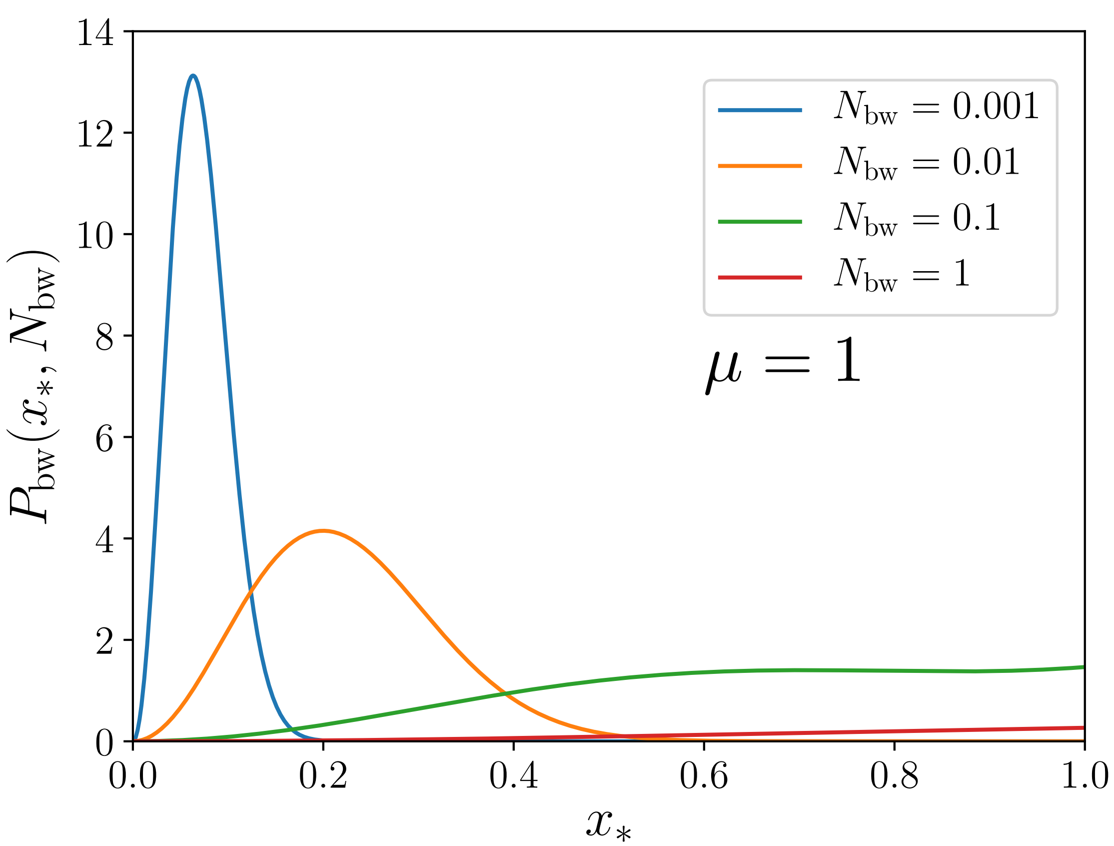

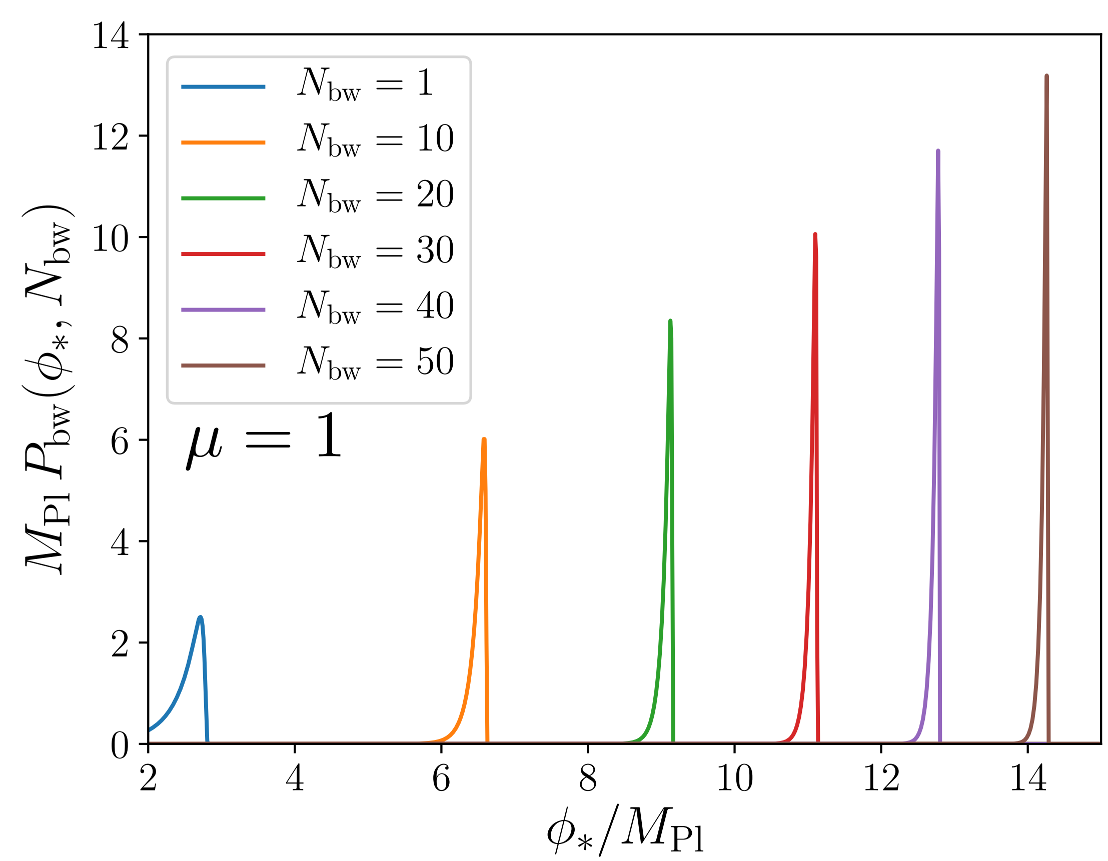

Conclusions

- Primordial black holes can be seeded by large density perturbations that form during inflation
- When this happens, quantum diffusion, that is, the back-reaction of vacuum quantum fluctuations on the background dynamics as they get amplified and stretched to large distances, may play an important role
- Quantum diffusion leads to exponential tails
- Regions of the potential dominated by stochastic diffusion in slow roll can be identified with a simple criterion: $v'^2 < v^2 v''$
- In stochastically-dominated regions, the system must spend less than one e-fold for PBHs not to be too abundantly produced

Conclusions

- Inflection-point potentials overproduce primordial black holes … unless they violate slow roll

- Quantum diffusion makes the CMB probe the whole potential: models leading to PBHs are constrained by the CMB, even if those two sets of scales are well separated

- The systematic presence of non-Gaussian tails may have implications for other rare astrophysical objects, such as ultra compact mini halos, and for alternative strategies for looking for nG signals

Thank you for your attention