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New gravitational degrees of freedom as a solution to the dark matter problem

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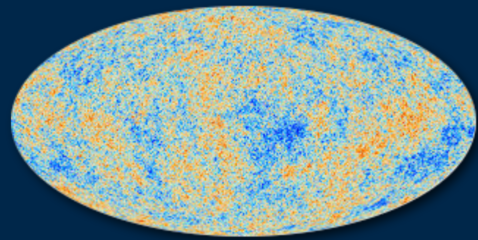
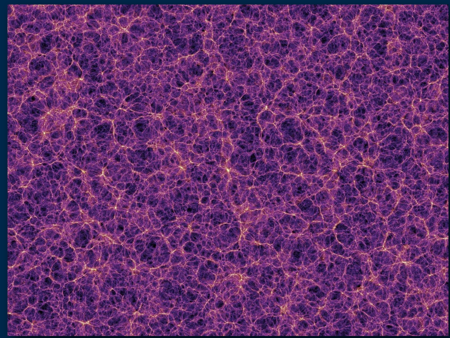
- The problem
  - Dark Matter
  - Gravity
- MOND Proposal
- New dof for MOND: scalars and time-like vectors
- Building to a new theory: requirements from cosmology
- A new relativistic MOND
    - Successful cosmology
- Healthiness and future directions
- Conclusions

Dark matter problem





Mismatch between

observed dynamics of visible matter ⟷ its gravitational influence

Solutions

GR +

Dark Matter →

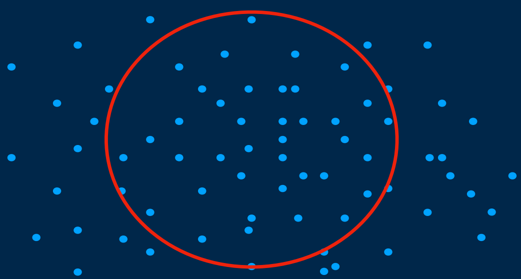
Primordial Black holes

New particle not in standard model

Wave dark matter: coherent fields — e.g. axions

Dark matter

Statistical description



Particle DM

New matter dof

Wave DM

$\langle \vec{x} | \Psi(t) \rangle$

Schrödinger (or Klein-Gordon) eq.

On galactic scales ($\sim kpc$) and above

Distribution function (d.f.) $f(t, \vec{x}, \vec{u})$

- Evolves in phase-space
- Collisionless Boltzmann (Vlasov) equation

$$\frac{\partial f}{\partial t} + \vec{u} \cdot \vec{\nabla}_x f + \vec{\nabla}_x \Phi \cdot \vec{\nabla}_u f = 0$$

Schrödinger-Vlasov correspondence

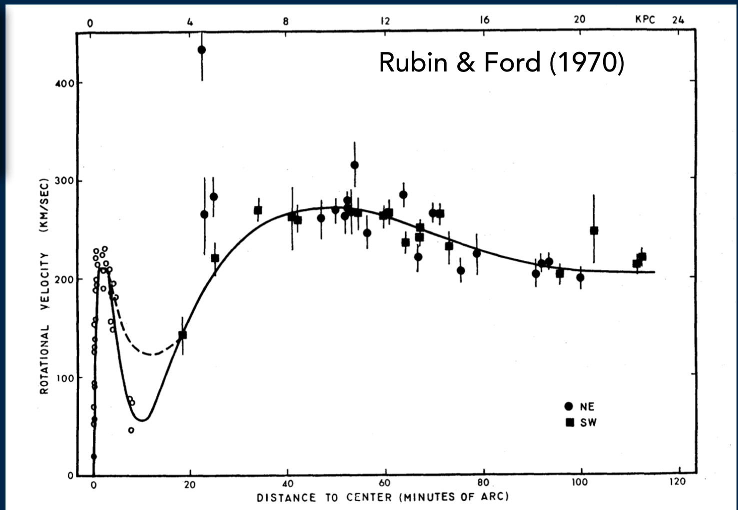
Widrow & Kaiser ApJ Lett 416, L71 (1993)
Kopp, Vattis & Skordis, PRD (2017) (and ref. therein)

Another solution?

Non-GR gravity

Modified Newtonian Dynamics

Milgrom (1983)



$v \sim$ const $\Longrightarrow a \sim \frac{v^2}{r} \sim \frac{1}{r}$

Deviation from Newton when

$a < a_0 \sim 1.2 \times 10^{-10} m/s^2$

Universal constant

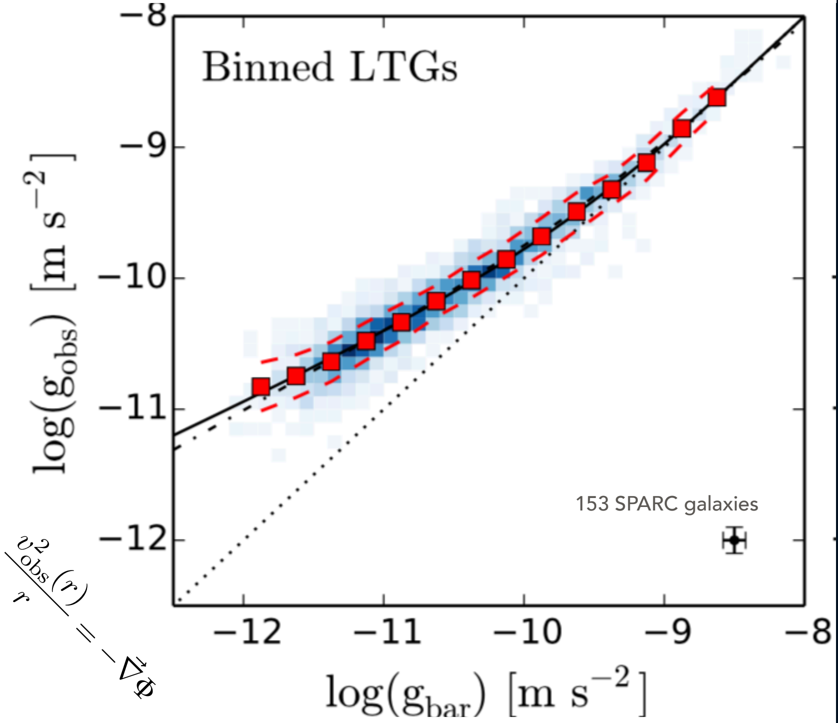
mod. Gravity $a = |\vec{\nabla}\Phi|$ $\vec{\nabla} \cdot \left(\frac{|\vec{\nabla}\Phi|}{a_0} \vec{\nabla}\Phi \right) = 4\pi G_N \rho$

mod. Inertia $\frac{a^2}{a_0} \sim |\vec{\nabla}\Phi|$ $\vec{\nabla}^2 \Phi = 4\pi G_N \rho$

MOND: regularity in galaxies

Radial Acceleration Relation

Lelli, McGaugh, Schombert & Pawlowski (2017)

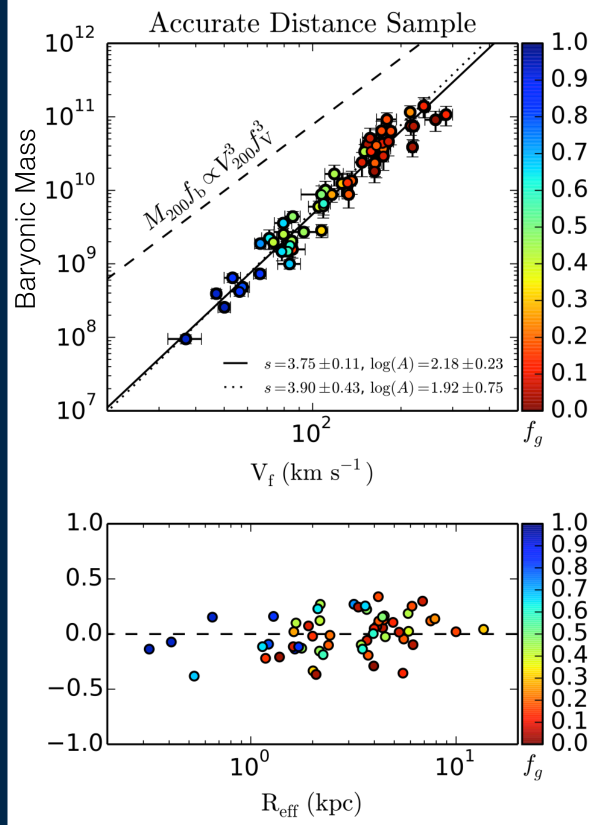


$$g_{\rm obs}(r) = \frac{v_{\rm obs}^2(r)}{r} = -\vec{\nabla}\Phi$$

$$g_{\rm bar}(r) = \frac{v_{\rm bar}^2(r)}{r} = -\vec{\nabla}\Phi_{\rm bar}$$

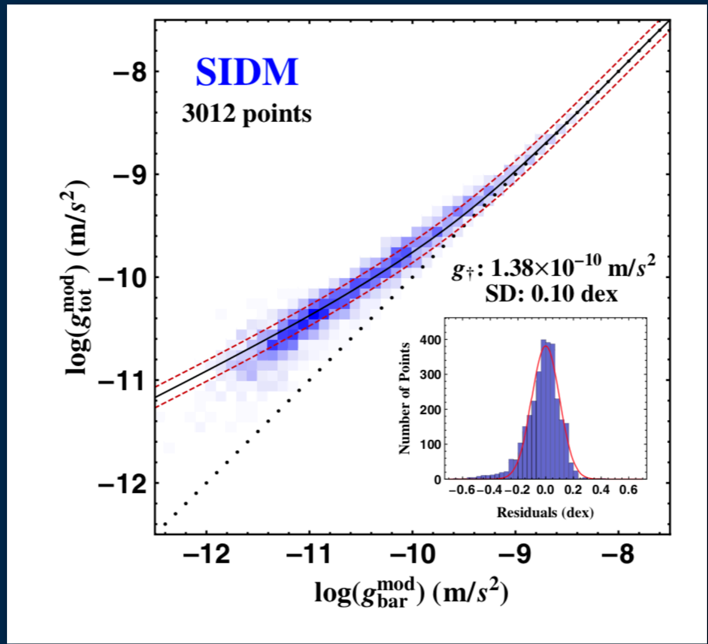
Baryonic Tully-Fisher

McGaugh et al. (2000)



DARK MATTER ++

Self-interacting Dark Matter



Spergel & Steinhardt, PRL 84, 3760 (2000)

Ren et al, PRX 9, 031020 (2019)

Superfluid dark matter (Khoury & Berezhiani, 2015)

- Axion-like particles with mass of order eV and strong self-interactions.
- Aptly described as collective excitations: phonons
- Superfluid phonons: Goldstone bosons of a spontaneously broken global U(1) symmetry.
- Lagrangian put in by hand (no fundamental theory)

Beyond GR

Lovelock's Theorem (1967) The only

- local,
- diffeomorphism invariant action,
- which leads to 2nd order field equations
- and which depends only on a metric

in 4D

is a linear combination of the Einstein-Hilbert action with a cosmological constant up to a total derivative: GR

New dof

$\varphi^{(I)}$ $\alpha_\mu^{(I)}$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Minkowski

$h_{\mu\nu}$

$\varphi^{(I)}$ $\alpha_\mu^{(I)}$

Matter

e.g. $\mathcal{L} \sim (\partial h)^2 + (\partial \varphi)^2$

Transformations

$h_{\mu\nu} \leftrightarrow \alpha_\mu^{(I)}$, $h_{\mu\nu} \leftrightarrow \varphi^{(I)}$, $\alpha_\mu^{(I)} \leftrightarrow \varphi^{(I)}$

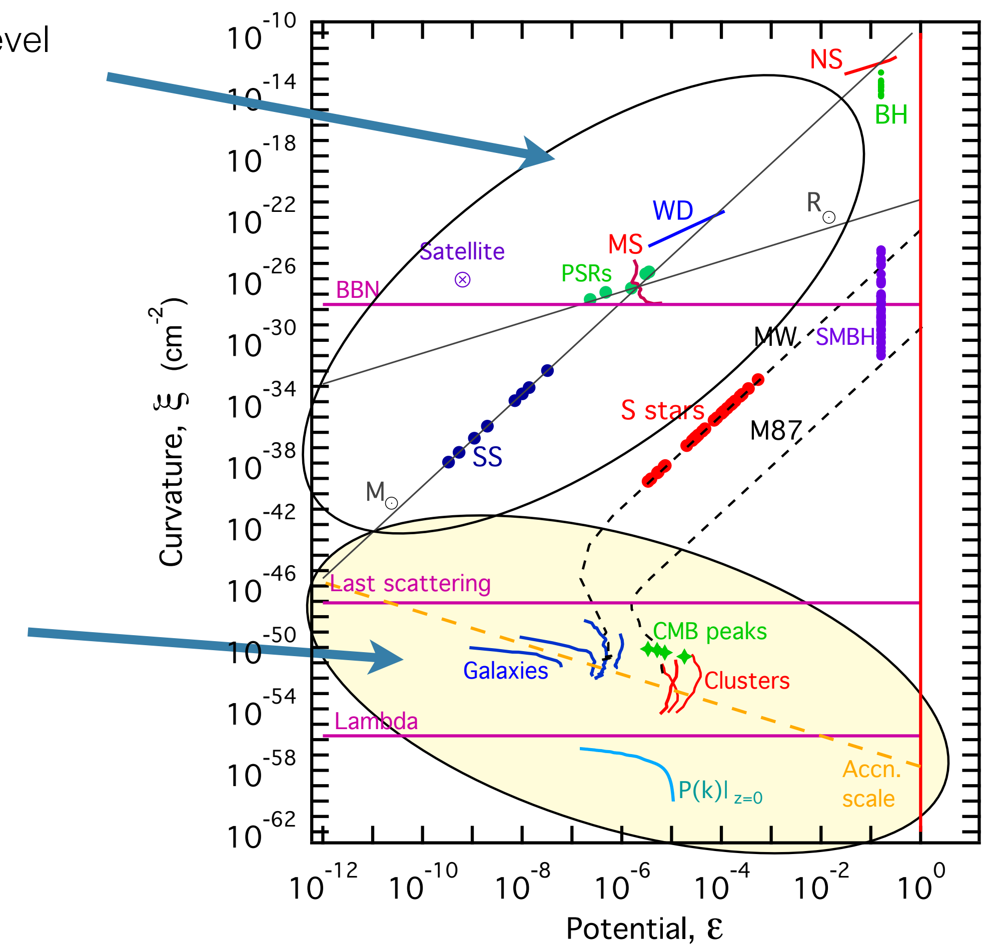
Gravity

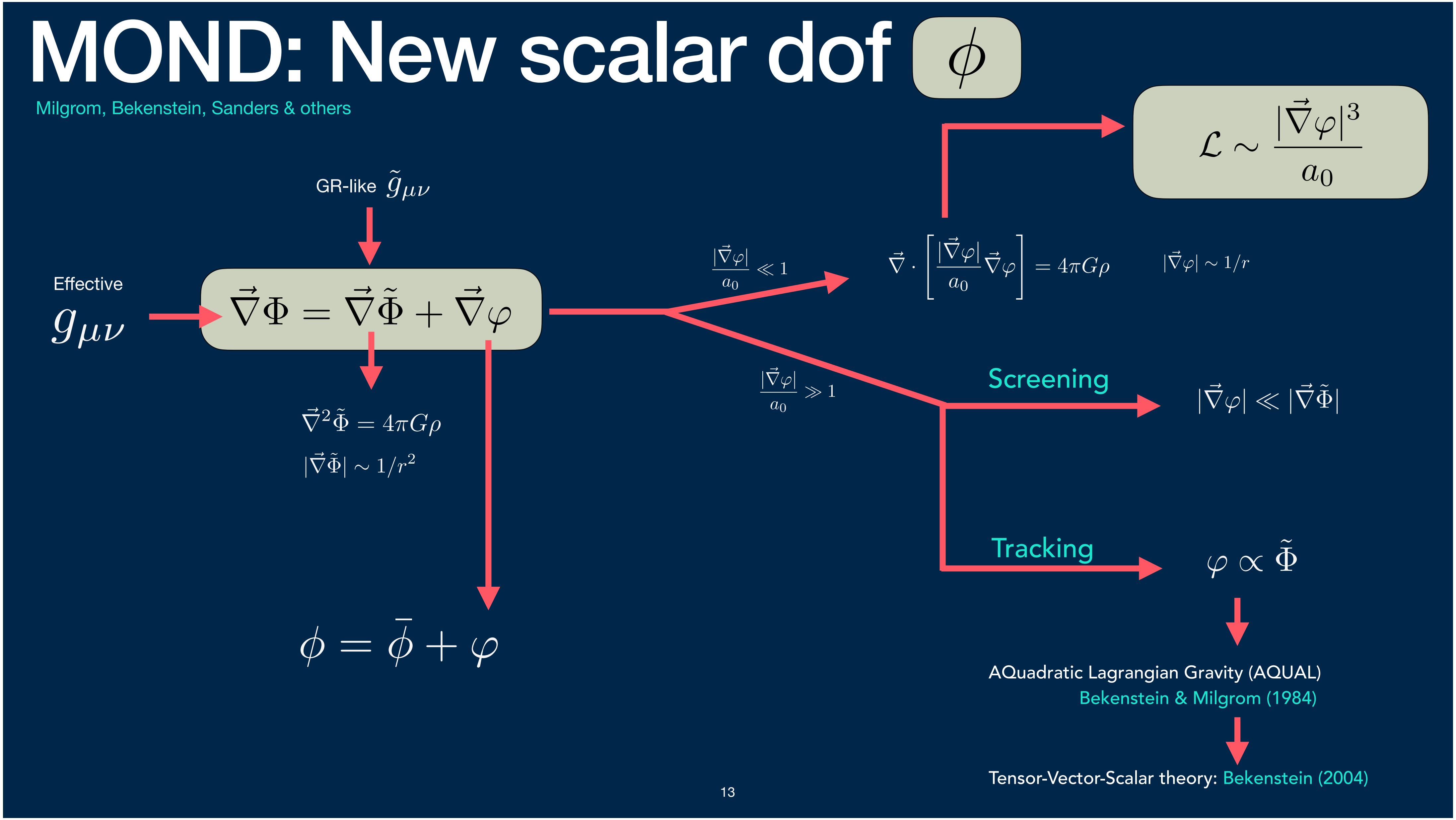
e.g. $\mathcal{L} \sim (\partial h)^2 + (\partial \varphi)^2 + (\partial h)(\partial \varphi)$

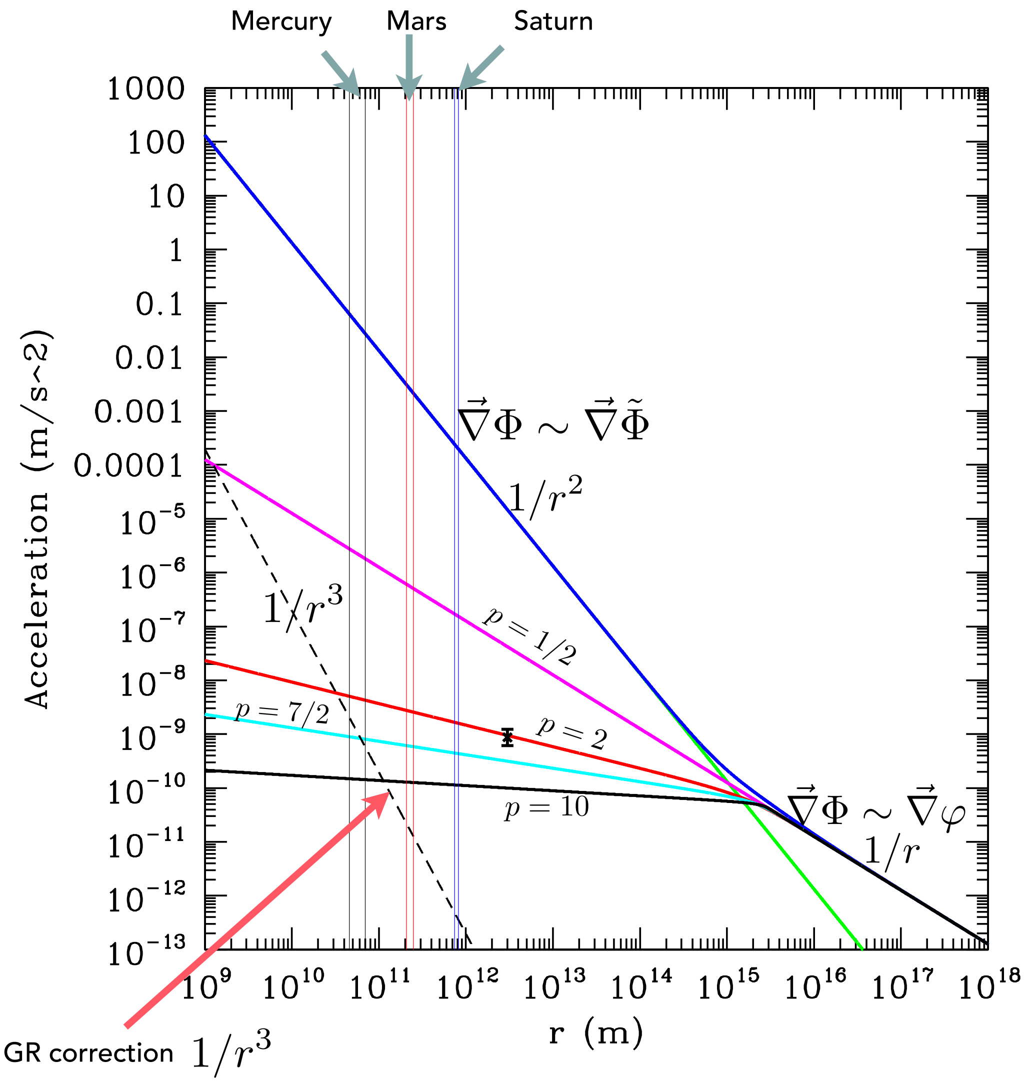
T. Baker, D. Psaltis and C.S., Astrophys. J. 802, 63 (2015)

- GR experimentally tested
- Deviations at $10^{-3} - 10^{-20}$ level
- New dof suppressed

- Need DM
- GR not tested
- New dof active




MOND: New scalar dof ϕ
Milgrom, Bekenstein, Sanders & others
GR-like $\tilde{g}_{\mu\nu}$
Effective $g_{\mu\nu}$
$\vec{\nabla}\Phi = \vec{\nabla}\tilde{\Phi} + \vec{\nabla}\varphi$
$\vec{\nabla}^2\tilde{\Phi} = 4\pi G\rho$
$|\vec{\nabla}\tilde{\Phi}| \sim 1/r^2$
$\phi = \bar{\phi} + \varphi$
$\frac{|\vec{\nabla}\varphi|}{a_0} \ll 1$
$\vec{\nabla}\cdot\left[\frac{|\vec{\nabla}\varphi|}{a_0}\vec{\nabla}\varphi\right] = 4\pi G\rho$
$|\vec{\nabla}\varphi| \sim 1/r$
$\mathcal{L} \sim \frac{|\vec{\nabla}\varphi|^3}{a_0}$
$\frac{|\vec{\nabla}\varphi|}{a_0} \gg 1$
Screening
$|\vec{\nabla}\varphi| \ll |\vec{\nabla}\tilde{\Phi}|$
Tracking
$\varphi \propto \tilde{\Phi}$
AQuadratic Lagrangian Gravity (AQUAL)
Bekenstein & Milgrom (1984)
Tensor-Vector-Scalar theory: Bekenstein (2004)
13



Screening

$$\mathcal{L} \sim \frac{|\vec{\nabla}\phi|^3}{a_0} + \frac{\beta_p |\vec{\nabla}\phi|^{2(p+1)}}{a_0^{2p}}$$

$$p \to \infty \quad \Rightarrow \quad \vec{\nabla}\phi \to \text{const}$$

DBion — Burrage & Khoury (2014)

Quintic Galileon term works but needs a new scale:
Babichev, Deffayet & Esposito-Farese (2011)

Tracking

$$\vec{\nabla} \cdot \left[f\left(\frac{|\vec{\nabla}\varphi|}{a_0}\right) \vec{\nabla}\varphi \right] = 4\pi G\rho$$

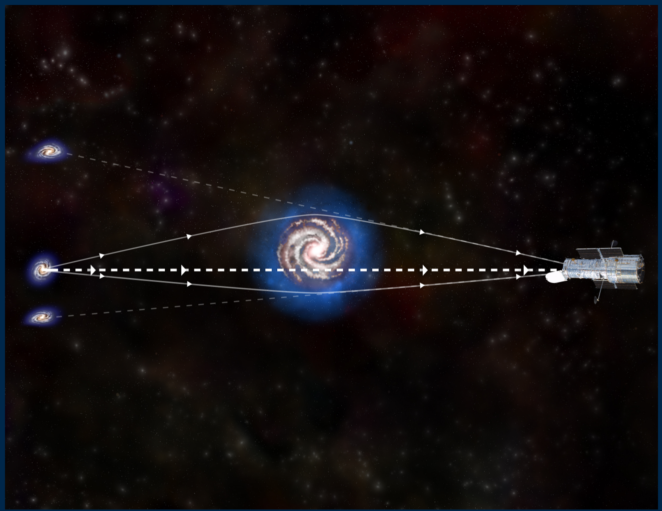
Interpolation function: $f\left(\frac{|\vec{\nabla}\varphi|}{a_0}\right)$

$\frac{|\vec{\nabla}\varphi|}{a_0} \gg 1$ → Const.

$\frac{|\vec{\nabla}\varphi|}{a_0} \ll 1$ → $\frac{|\vec{\nabla}\varphi|}{a_0}$

→ ?

New time-like vector dof A_μ



$$ds^2 = -(1+2\Phi)dt^2 + (1-2\Phi)d\vec{x}^2$$

$$\Phi = \tilde{\Phi} + \varphi$$

$\rightarrow$ $ds^2 \neq e^{\pm 2\varphi}\left[-e^{2\tilde{\Phi}}dt^2 + e^{-2\tilde{\Phi}}d\vec{x}^2\right]$

But:

$$g_{\mu\nu} = e^{-2\phi}\tilde{g}_{\mu\nu} - \left(e^{2\phi} - e^{-2\phi}\right) A_\mu A_\nu$$

$$\tilde{g}^{\mu\nu} A_\mu A_\nu = -1$$

Sanders, ApJ 480, 492 (1997)

$\downarrow$

Tensor-Vector-Scalar theory: Bekenstein, PRD 70, 083509 (2004)

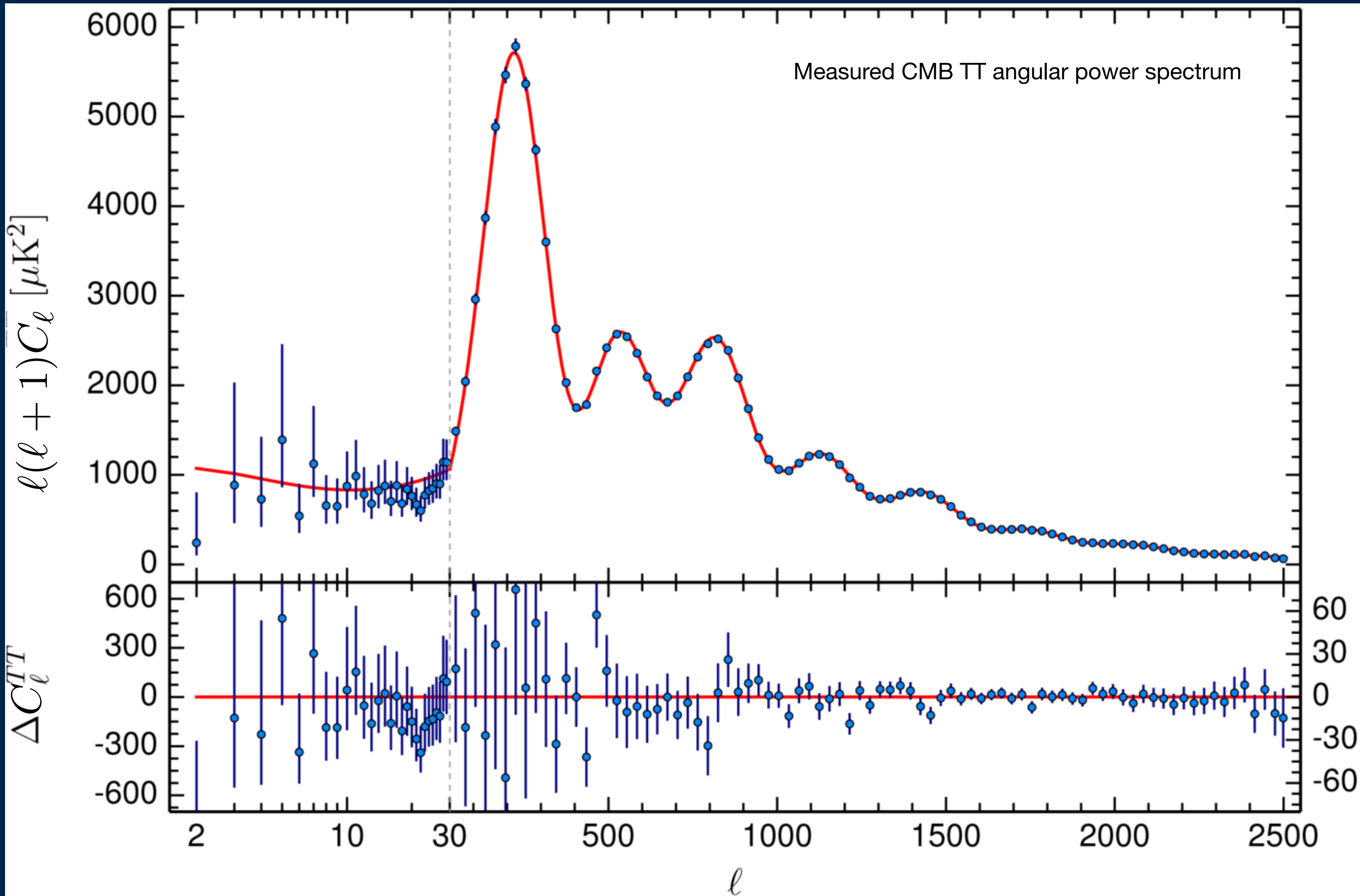
Disagreement with CMB

Agreement with matter power spectrum

Skordis, Mota, Ferreira, Boehm, PRL 96, 011301 (2006)
Dodelson & Liguori, PRL 97, 231301 (2006)

Tensor mode speed $\neq 1$

Boran et al., PRD 97, 041501 (2018)
Skordis & Zlosnik, PRD 100, 104013 (2019)



Measured CMB TT angular power spectrum
$\ell(\ell+1)C_\ell\ [\mu K^2]$
6000
5000
4000
3000
2000
1000
0
ΔC_ℓ^{TT}
600
300
0
-300
-600
60
30
0
-30
-60
2
10
30
500
1000
1500
2000
2500
ℓ

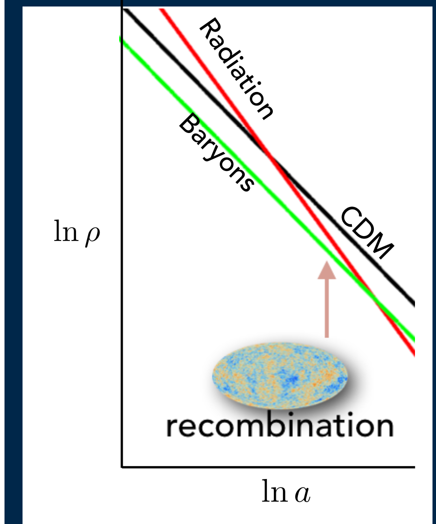
Dark matter and the CMB

$\Psi \quad \Phi$

Matter era: constant in time

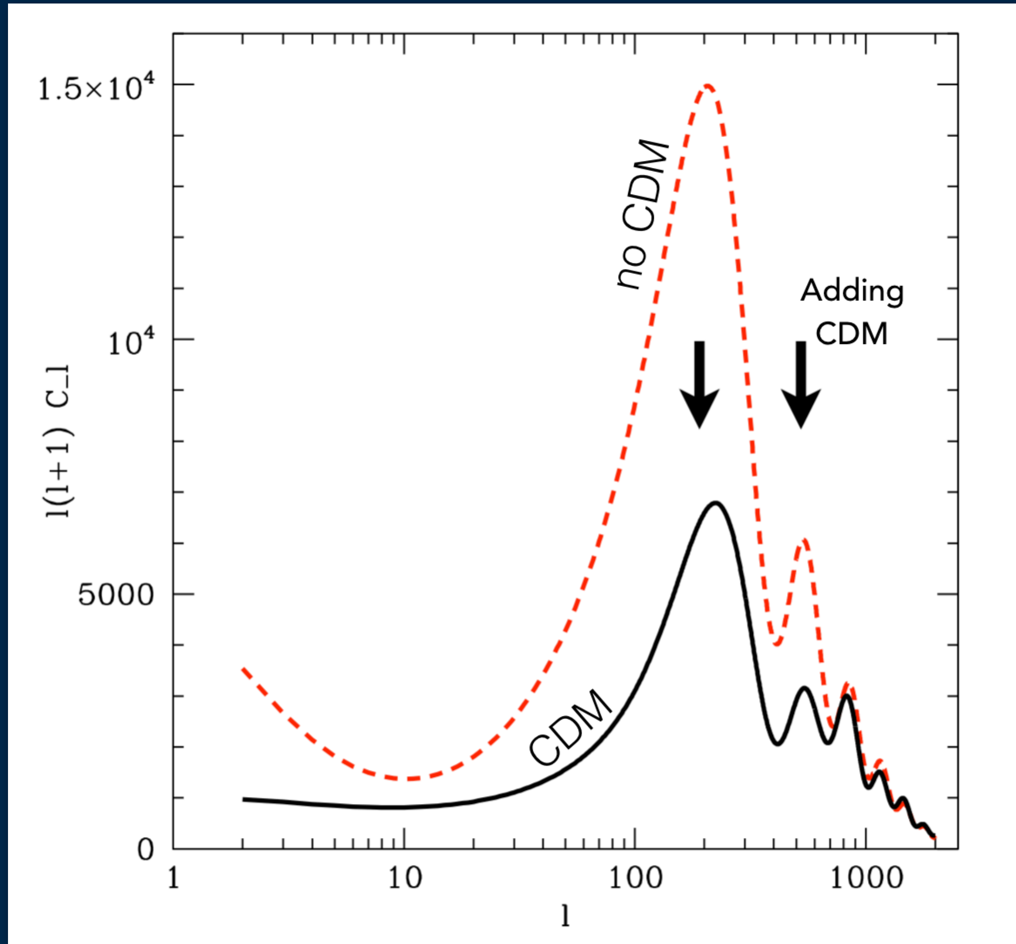
Radiation era: Oscillatory decay

"Energy transfer" to photons ⟶ Increase CMB power



More radiation during recombination

⟶ More CMB power



MOND: ϕ A_μ (timelike)

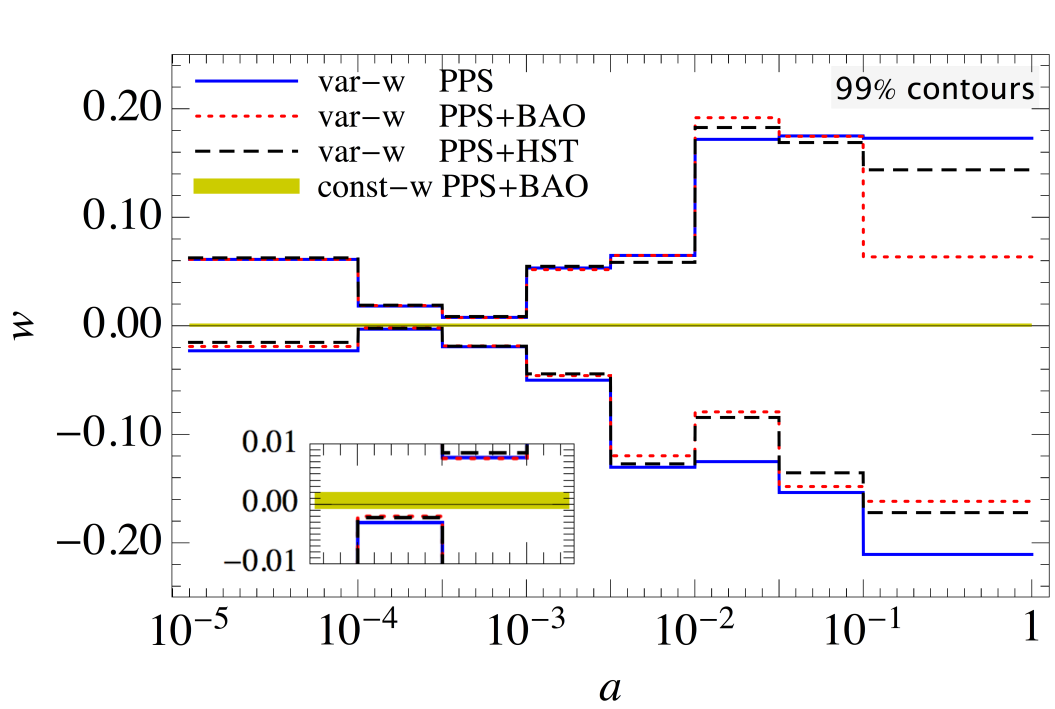
Lensing ok

Galaxies ok

$$\phi = \bar{\phi} + \varphi$$

MOND FLRW cosmology? $\bar{\phi}(t)$ → Energy density ρ_ϕ

What are the possibilities?



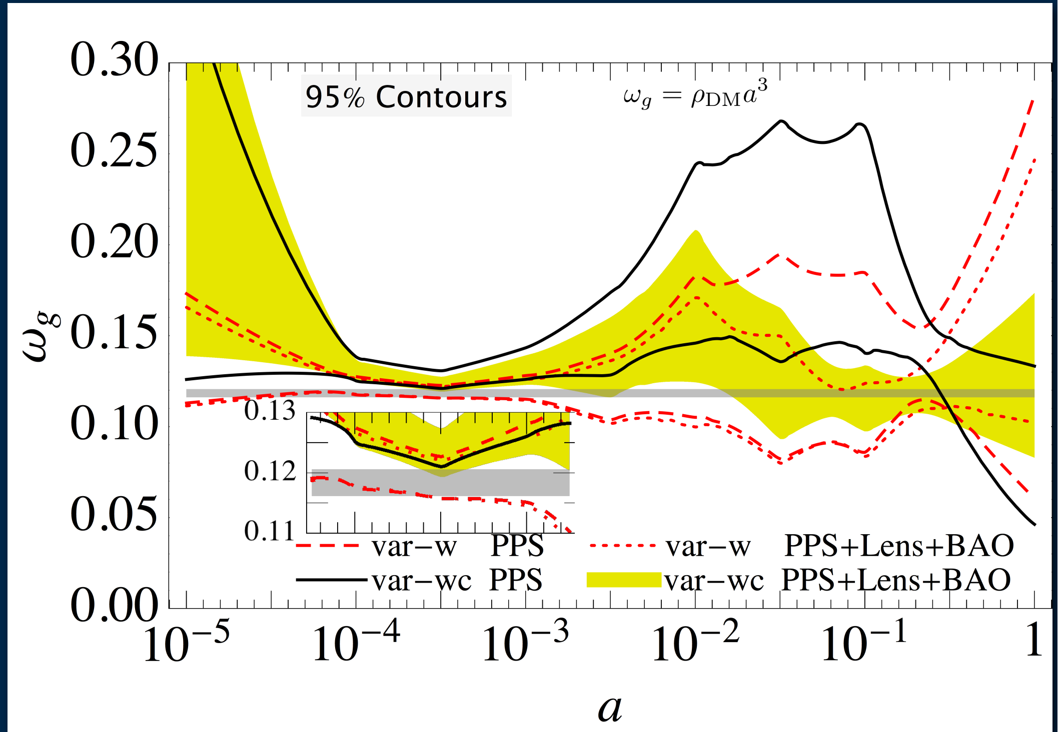
8 pixels in log-a

W. Hu, ApJ 506, 485 (1998)

Generalized Dark Matter $P = w\rho$

D. Thomas, M. Kopp, CS, S. Ilic, PRL120, 221102 (2018)

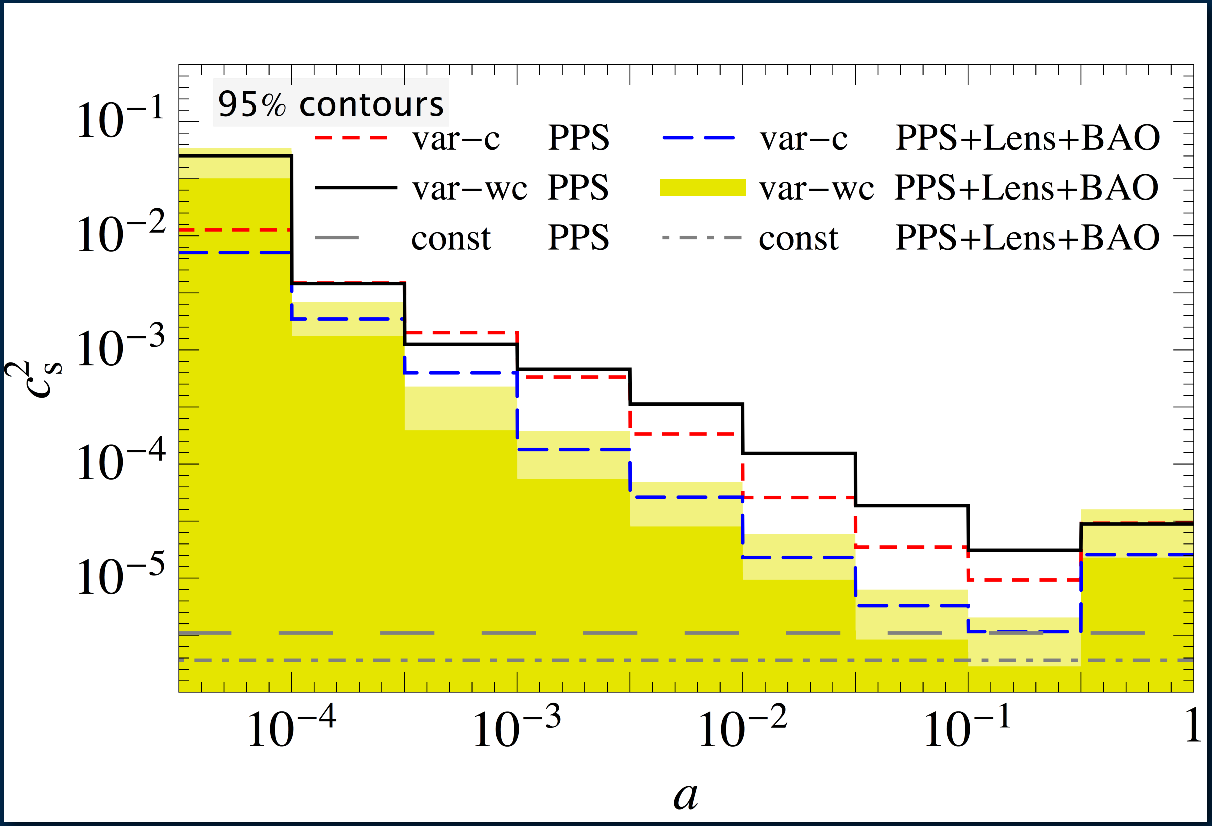
S. Ilic, M. Kopp, CS, D. Thomas, arXiv:2004.09572



From equality to recombination:

effective FLRW cosmology is GR + “dust”

9 pixels in log-a



$$\delta P \sim c_s^2 \delta \rho$$

Between 0 < z < 1000

effective linear cosmology is GR + "dust"

$$0 < k \lesssim 0.1 (hMpc)^{-1}$$

$$L \gtrsim 100 Mpc$$

S. Ilic, M. Kopp, CS, D. Thomas, arXiv:2004.09572

GRAVITY?

Non-relativistic regime

AQUAL: Bekenstein & Milgrom 1984

Correct lensing: Sanders vector field (1997)
TeVeS: Bekenstein (2004)

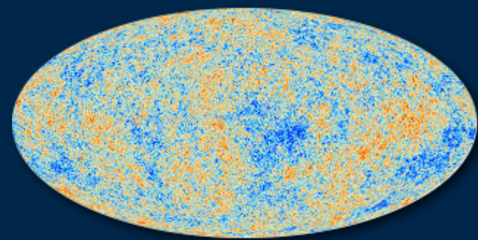
?

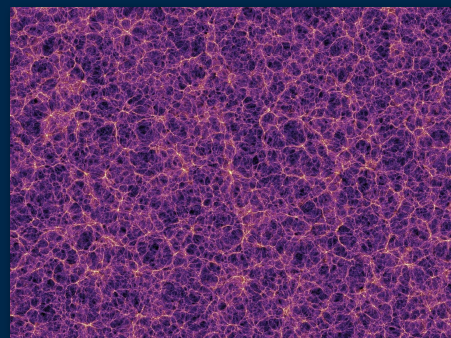
aLIGO/Virgo + EM (2017)

Tensor speed = 1

Relativistic (FRW + linear) regime

Effective description: dust (e.g. CDM)





Cosmological dust density from a scalar?

Shift-symmetric k-essence: $\mathcal{L} \sim K_2 \left(X - X_0\right)^2$ With $X = \dot{\phi}^2$

Scherrer, Phys.Rev.Lett. 93, 011301 (2004)

FLRW Limit of Ghost condensate

Arkani-Hamed et al., JHEP 05, 074 (2004)

New scalar dof mixing with metric:

$$\phi = \mathcal{Q}_0 t + \varphi$$

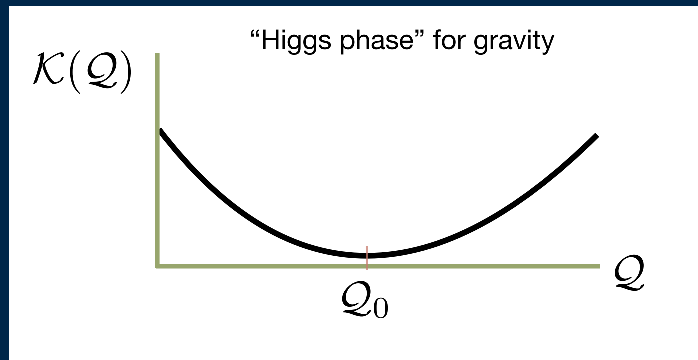
$$h_{00} \to h_{00} - 2\dot{\xi}$$
$$\varphi \to \varphi + \mathcal{Q}_0 \xi$$

FLRW: scalar acts as effective "dust"

$$\mathcal{Q} = \hat{A}^\mu \nabla_\mu \phi \to \dot{\phi}$$

PROPOSAL

Cosmological MOND analogue (Lagrangian): $\mathcal{K}(\mathcal{Q}) = \mathcal{K}_2 \left(\mathcal{Q} - \mathcal{Q}_0\right)^2 + \ldots$

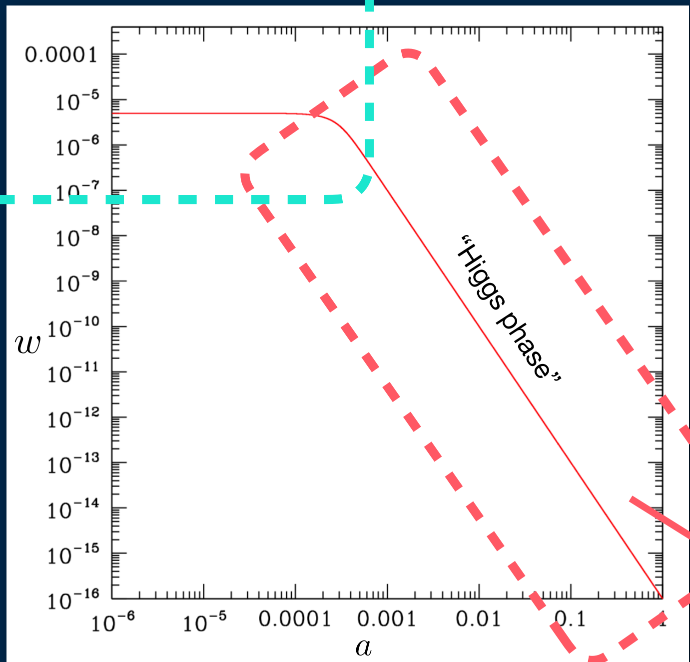


FLRW EOM: $\frac{d}{dt}\left(\frac{d\mathcal{K}}{d\mathcal{Q}}\right) = 0 \qquad \Rightarrow \dot{\bar{\phi}} = \mathcal{Q}_0 + \frac{I_0}{a^3}$

Initial condition

Early region: depends on form of $\mathcal{K}(\bar{\mathcal{Q}})$

Equation of state w(t)



Higgs phase: "effective dust"

$\rho = \frac{\mathcal{Q}_0 I_0}{a^3} + \ldots$ Density

$w = \frac{w_0}{a^3} + \ldots$ Equation of state

$c^2_{\rm ad} = \frac{2w_0}{a^3} + \ldots$ Adiabatic sound speed

Late region: Universal

$$\mathcal{K} = -2\Lambda + \mathcal{K}_2(\bar{\mathcal{Q}} - \mathcal{Q}_0)^2 + \ldots$$

Simple relativistic MOND

Skordis & Zlosnik, astro-ph/2007.00082v2

Ingredients: $g_{\mu\nu}$ $\hat{A}_\mu$ ϕ

Tensor speed = 1

Skordis & Zlosnik (2019)

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi\tilde{G}}\left[R - \frac{K_B}{2}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu} + 2(2-K_B)\hat{J}^\mu\nabla_\mu\phi - (2-K_B)\mathcal{Y} - \mathcal{F}(\mathcal{Y},\mathcal{Q}) - \lambda(\hat{A}^\mu\hat{A}_\mu + 1)\right] + S_m[g]$$

$$\hat{F}_{\mu\nu} = \nabla_\mu\hat{A}_\nu - \nabla_\nu\hat{A}_\mu$$

$$\hat{J}_\mu = \hat{A}^\nu\nabla_\nu\hat{A}_\mu$$

$$\mathcal{Y} = \left(g^{\mu\nu} + \hat{A}^\mu\hat{A}^\nu\right)\nabla_\mu\phi\nabla_\nu\phi \to |\vec{\nabla}\varphi|^2$$ Spatial dependence

$$\mathcal{Q} = \hat{A}^\mu\nabla_\mu\phi \to \dot{\phi}$$ Time dependence

FLRW cosmology $\phi = \bar{\phi}(t)$

Skordis & Zlosnik, astro-ph/2007.00082v2

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi\tilde{G}}\left[R - \frac{K_B}{2}\hat{F}^{\mu\nu}F_{\mu\nu} + 2(2-K_B)\hat{J}^{\mu}\nabla_{\mu}\phi - (2-K_B)\mathcal{Y} - \mathcal{F}(\mathcal{Y},\mathcal{Q}) - \lambda(\hat{A}^{\mu}\hat{A}_{\mu}+1)\right] + S_m[g]$$

$$\mathcal{K}(\bar{\mathcal{Q}}) = -\frac{1}{2}\mathcal{F}(0,\bar{\mathcal{Q}})$$

$$S = \frac{1}{8\pi\tilde{G}}\int d^4x a^3\left[-3H^2 + \mathcal{K}(\bar{\mathcal{Q}})\right] + S_m[g]$$

$$\mathcal{K} = -2\Lambda + \mathcal{K}_2(\bar{\mathcal{Q}} - \mathcal{Q}_0)^2 + \ldots$$

Dust solutions

Quasistatic weak-field limit $\phi = \mathcal{Q}_0 t + \varphi(\vec{x})$

Skordis & Zlosnik, astro-ph/2007.00082v2

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi\tilde{G}}\left[R - \frac{K_B}{2}\hat{F}^{\mu\nu}\hat{F}_{\mu\nu} + 2(2-K_B)\hat{J}^\mu\nabla_\mu\phi - (2-K_B)\mathcal{Y} - \mathcal{F}(\mathcal{Y},\mathcal{Q}) - \lambda(\hat{A}^\mu\hat{A}_\mu + 1)\right] + S_m[g]$$

$A_0 = -1 - \Psi$

$$\mathcal{J}(\mathcal{Y}) = \frac{1}{2}\mathcal{F}(\mathcal{Y},\mathcal{Q}_0)$$

Constraint: $\Psi = \Phi$

Lensing ok

$$\rightarrow \frac{\mathcal{Y}^{3/2}}{a_0} = \frac{|\vec{\nabla}\varphi|^3}{a_0}$$

MOND → Galaxies ok

$$S = \int d^4x \left\{ (2-K_B)\left[|\vec{\nabla}\Phi|^2 - 2\vec{\nabla}\Phi\vec{\nabla}\varphi + |\vec{\nabla}\varphi|^2 - \mu^2\Phi^2\right] + 2\mathcal{J}(\mathcal{Y}) + 16\pi\tilde{G}\Phi\rho \right\}$$

$$\mu^2 = \frac{2\mathcal{K}_2}{2-K_B}\mathcal{Q}_0^2$$

$$\mu^{-1} \gtrsim Mpc \quad (\mu \lesssim 6\times 10^{-30} eV)$$

MOND compatibility

$$\mu^2 = \frac{2\mathcal{K}_2}{2-K_B}\mathcal{Q}_0^2$$

$$\mu^{-1} \gtrsim Mpc$$

Higgs phase:

$$w \approx \frac{w_0}{a^3} + \dots$$

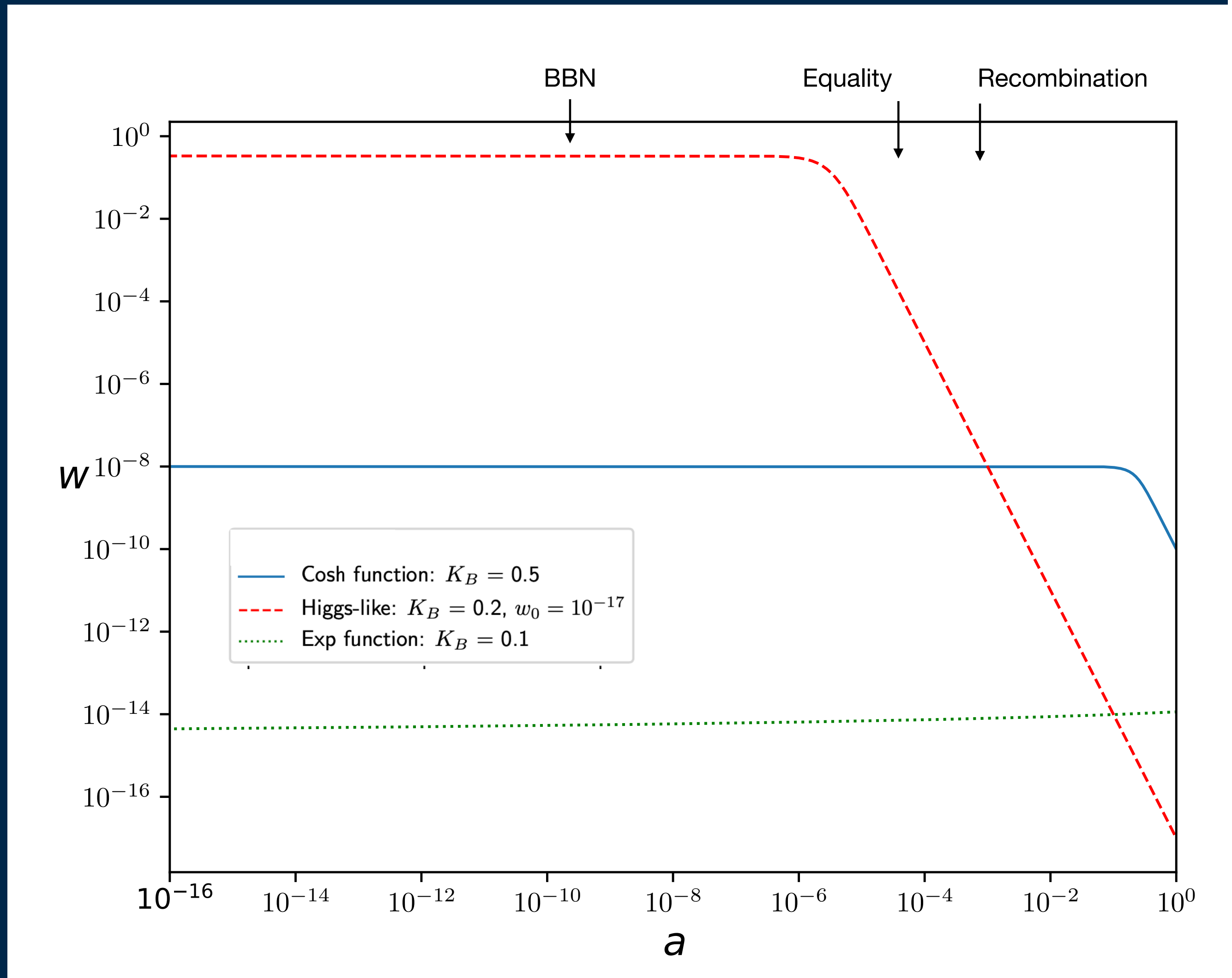
$$w_0 = \frac{3H_0^2\Omega_{\mathcal{Q}}}{4\mathcal{Q}_0^2\mathcal{K}_2} = \frac{3H_0^2\Omega_{\mathcal{Q}}}{2(2-K_B)\mu^2}$$

$$w_0 \gtrsim 10^{-8}$$

$$w_{rec} \sim O(1)$$

Data: $w_{rec} \lesssim 10^{-4}$

✗	Higgs-like	$\mathcal{K} \sim (\mathcal{Q}^2 - \mathcal{Q}_0^2)^2$
✓	Cosh	$\mathcal{K} \sim \cosh\left(\frac{\mathcal{Q}-\mathcal{Q}_0}{\mathcal{Z}_0}\right)$
✓	Exp	$\mathcal{K} \sim e^{\left(\frac{\mathcal{Q}-\mathcal{Q}_0}{\mathcal{Z}_0}\right)^2}$



Perturbations

$$\phi = \bar{\phi} + \varphi$$

$$A_i = \vec{\nabla}_i \alpha$$

→

$$E = \dot{\alpha} + \Psi$$

$$\chi = \varphi + \dot{\bar{\phi}}\alpha$$

$$\gamma = \dot{\varphi} - \dot{\bar{\phi}}\Psi$$

→

Density contrast $\delta = \frac{1+w}{\dot{\bar{\phi}} c_{\rm ad}^2}\gamma + \frac{1}{8\pi G a^2 \bar{\rho}}\vec{\nabla}^2 \left[K_B E + (2-K_B)\,\chi\right]$

Velocity divergence $\theta = \frac{\varphi}{\dot{\bar{\phi}}}$

Pressure contrast $\Pi = c_{\rm ad}^2 \delta - \frac{c_{\rm ad}^2}{8\pi G a^2 \bar{\rho}}\vec{\nabla}^2 \left[K_B E + (2-K_B)\,\chi\right]$

Fluid-like

$$\dot{\delta} = 3H\,(w\delta - \Pi) + (1+w)\left(3\dot{\Phi} - \frac{k^2}{a^2}\theta\right)$$

$$\dot{\theta} = 3c_{\rm ad}^2 H\theta + \frac{\Pi}{1+w} + \Psi$$

Field

$$K_B\left(\dot{E} + HE\right) = \frac{d\mathcal{K}}{d\mathcal{Q}}\chi - (2-K_B)\left[\frac{\dot{\bar{\phi}}}{1+w}\Pi + \left(H + \dot{\bar{\phi}}\right)\chi - 3c_{\rm ad}^2 H\dot{\bar{\phi}}\alpha\right]$$

$w \to 0$

$c_{\rm ad} \to 0$

→

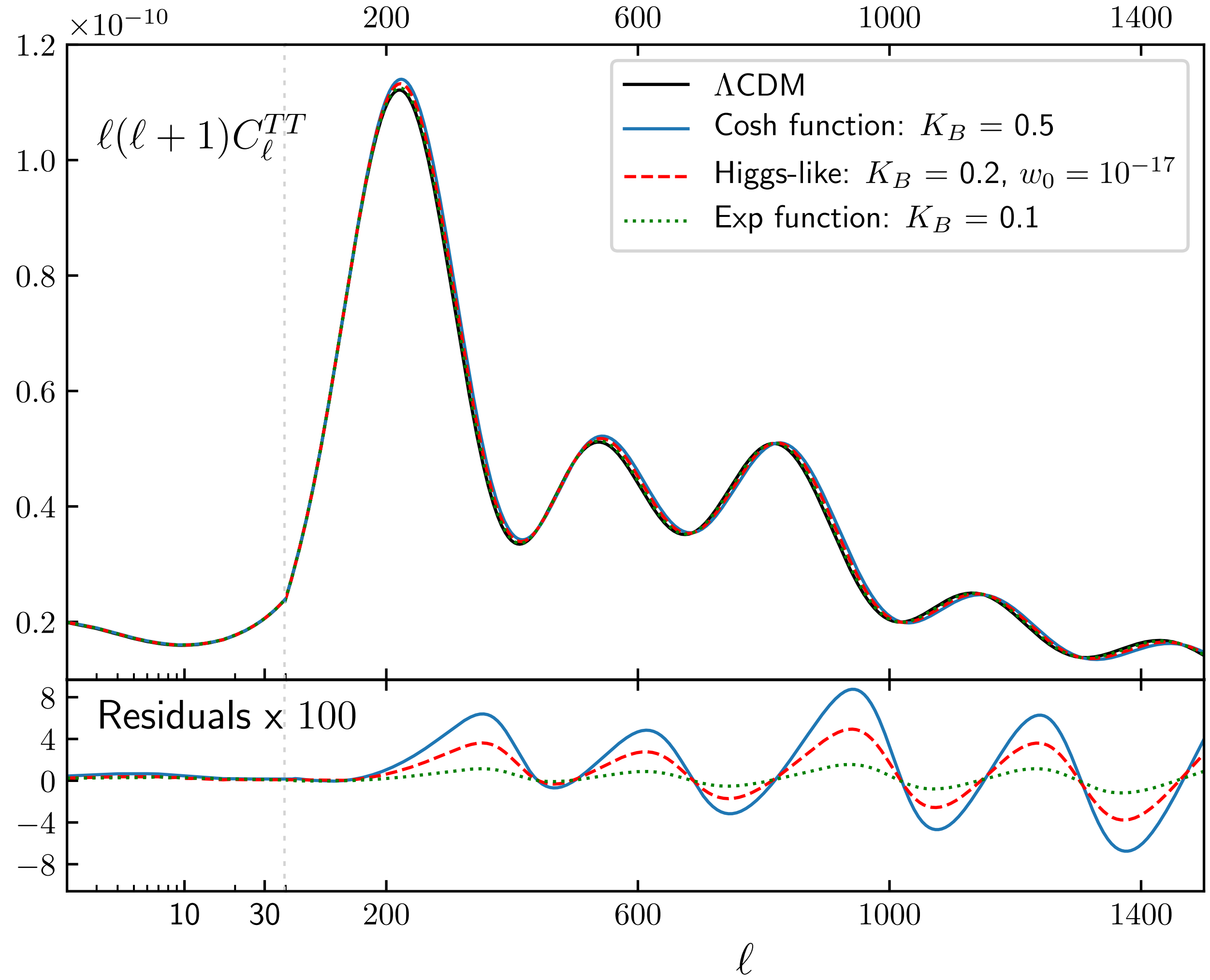
CDM-like

$$\dot{\delta} \approx 3\dot{\Phi} - \frac{k^2}{a^2}\theta$$

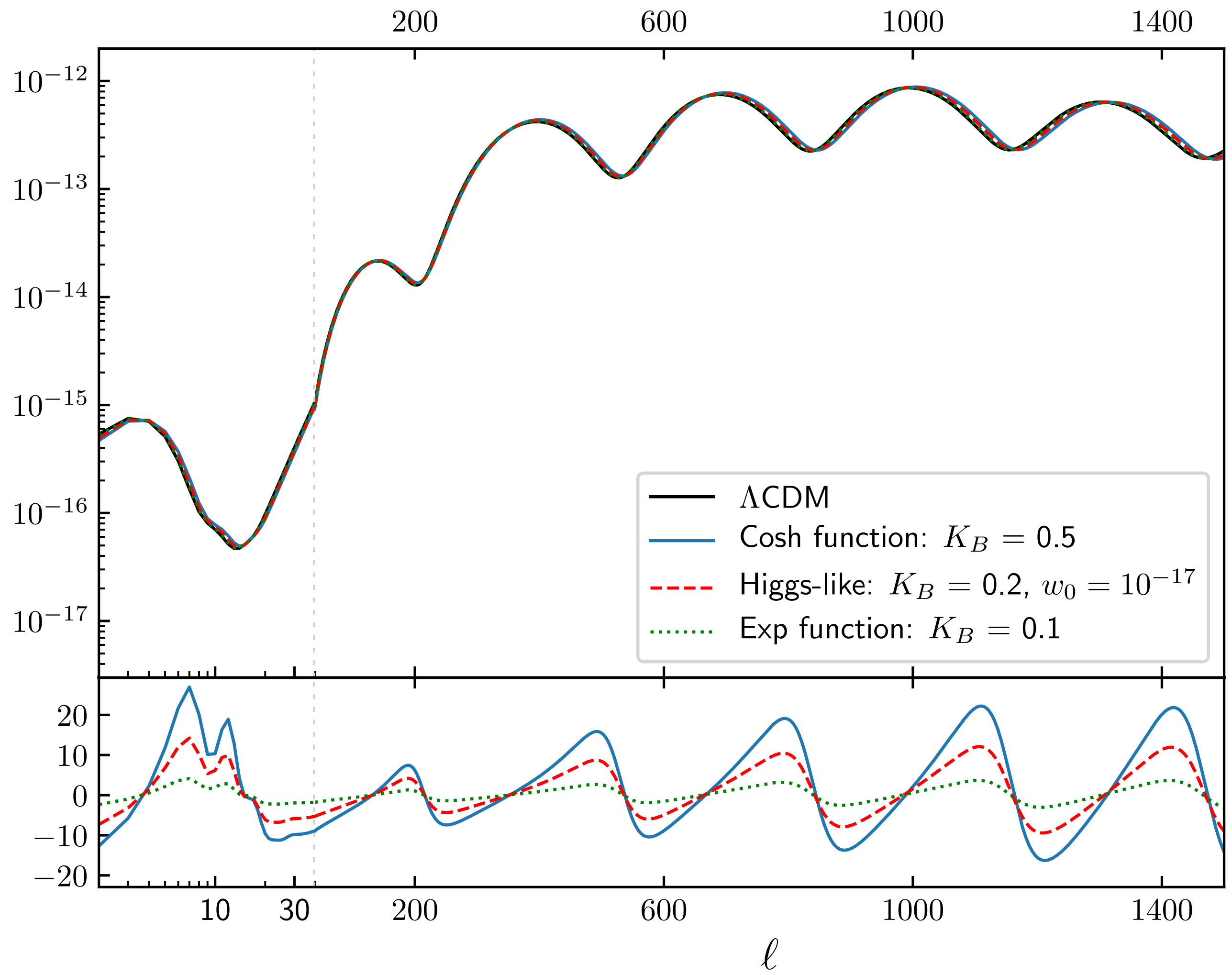
$$\dot{\theta} \approx \Psi$$

Field (decoupled)

$$K_B\left(\dot{E} + HE\right) \approx \left[\frac{3H_0^2\Omega_{0\mathcal{Q}}}{a^3} - (2-K_B)H\mathcal{Q}_0\right](\theta + \alpha)$$


$\times 10^{-10}$
200
600
1000
1400
1.2
1.0
0.8
0.6
0.4
0.2
$\ell(\ell+1)C_\ell^{TT}$
ΛCDM
Cosh function: $K_B = 0.5$
Higgs-like: $K_B = 0.2$, $w_0 = 10^{-17}$
Exp function: $K_B = 0.1$
Residuals x 100
8
4
0
−4
−8
10
30
200
600
1000
1400
ℓ

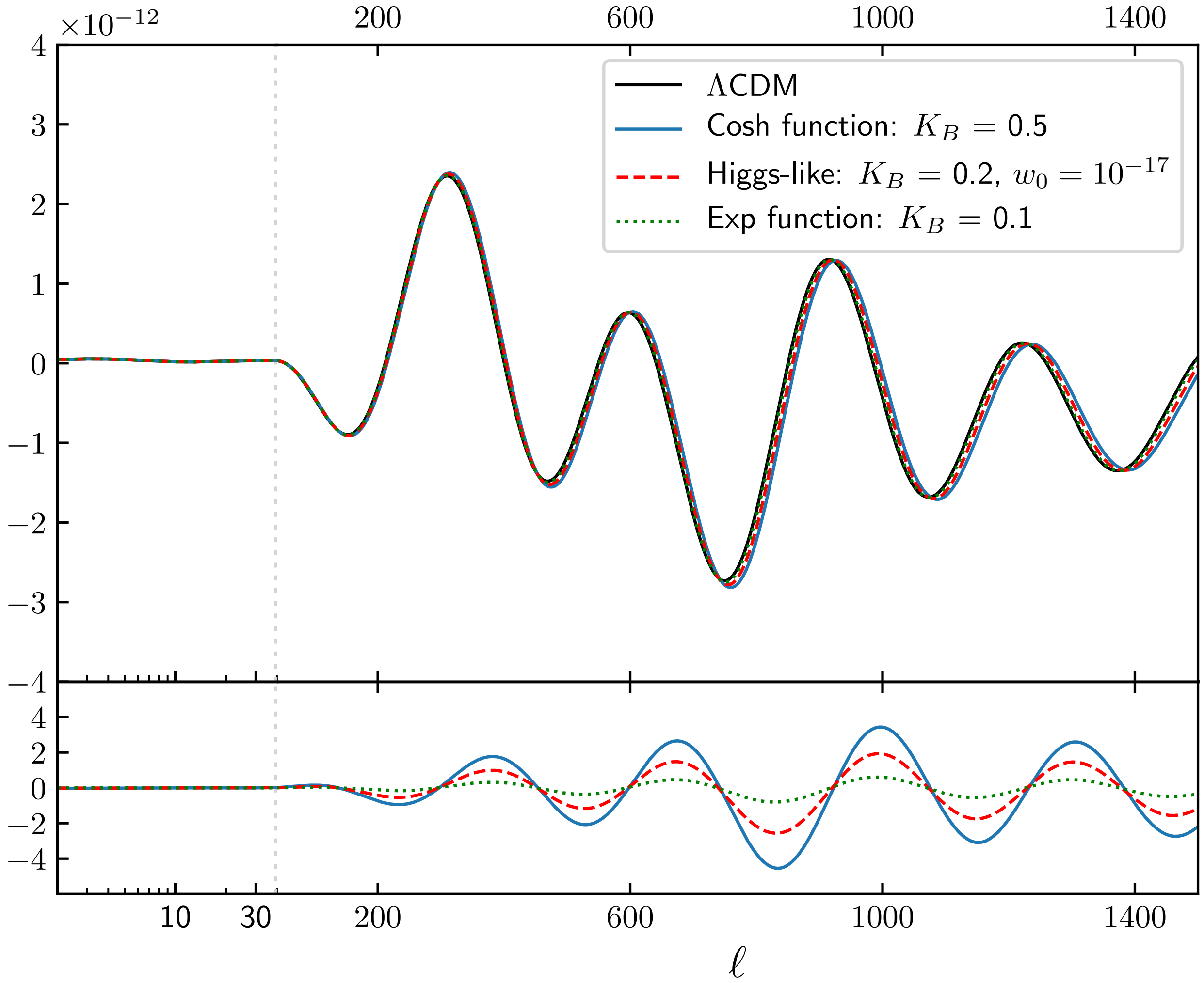
% - Residuals



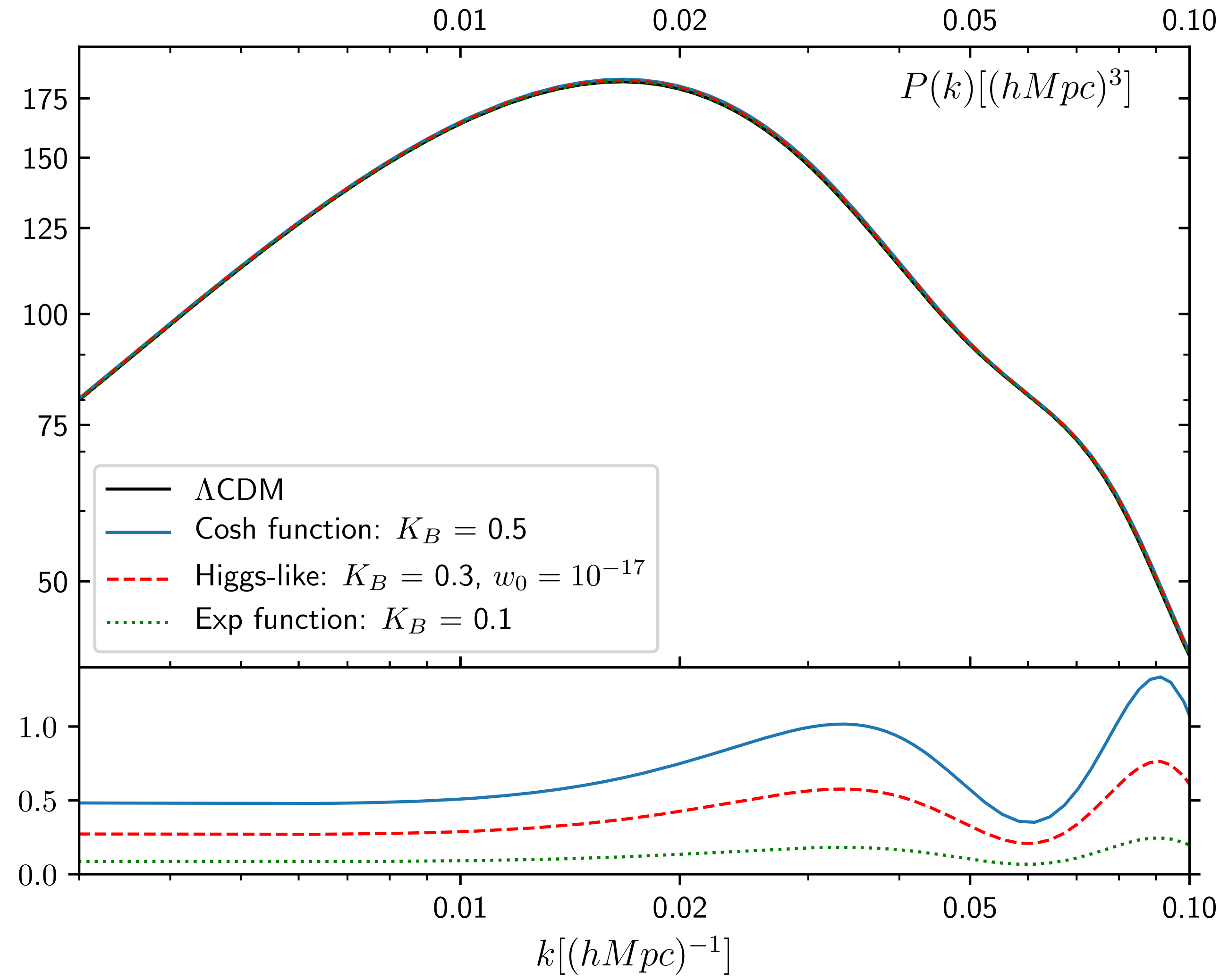
200
600
1000
1400
10^{-12}
10^{-13}
10^{-14}
10^{-15}
10^{-16}
10^{-17}
ΛCDM
Cosh function: $K_B = 0.5$
Higgs-like: $K_B = 0.2$, $w_0 = 10^{-17}$
Exp function: $K_B = 0.1$
20
10
0
−10
−20
10
30
ℓ

% - Residuals

Residuals



×10^−12
200
600
1000
1400
4
3
2
1
0
−1
−2
−3
−4
ΛCDM
Cosh function: K_B = 0.5
Higgs-like: K_B = 0.2, w_0 = 10^−17
Exp function: K_B = 0.1
4
2
0
−2
−4
10
30
200
600
1000
1400
ℓ


$P(k)[(hMpc)^3]$
175
150
125
100
75
50
ΛCDM
Cosh function: $K_B = 0.5$
Higgs-like: $K_B = 0.3$, $w_0 = 10^{-17}$
Exp function: $K_B = 0.1$
1.0
0.5
0.0
0.01
0.02
0.05
0.10
$k[(hMpc)^{-1}]$

%-Residuals

Fluctuations about Minkowski

Background

Perturbations

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$$

$$A_0 = -1 + \frac{1}{2}h^{00}$$

$$A_i = \hat{A}_i$$

$$\dot{\phi} = \mathcal{Q}_0 + \dot{\varphi}$$

$$\vec{\nabla}_i \phi = \vec{\nabla}_i \varphi$$

Gauge transformations

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$$

Mix

$$\varphi \to \varphi - \mathcal{Q}_0 \xi_0$$ (e.g. ghost condensate)

$$\hat{A}_i \to \hat{A}_i + \vec{\nabla}_i \xi_0$$ (e.g. gauged ghost condensate)

N.B.

Dark fields (e.g. DM)

$$\chi \to \chi$$

No mixing with $h_{\mu\nu}$

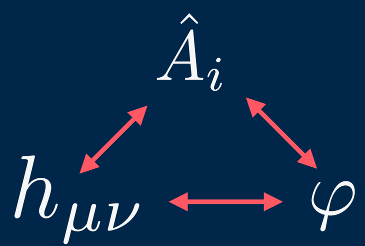
Quadratic action on Minkowski

Tensor mode graviton $c_T = 1$

$$S = \int d^4x \Big\{ -\frac{1}{2}\partial_\mu h \partial_\nu h^{\mu\nu} + \frac{1}{4}\partial_\rho h \partial^\rho h + \frac{1}{2}\partial_\mu h^{\mu\rho}\partial_\nu h^{\nu}{}_{\rho} - \frac{1}{4}\partial^\rho h^{\mu\nu}\partial_\rho h_{\mu\nu} + K_B|\dot{\vec{\hat{A}}} - \frac{1}{2}\vec{\nabla}h^{00}|^2 \; -2K_B\vec{\nabla}_{[i}\hat{A}_{j]}\vec{\nabla}^{[i}\hat{A}^{j]}$$

$$+ (2-K_B)\left[2(\dot{\vec{\hat{A}}} - \frac{1}{2}\vec{\nabla}h^{00})\cdot(\vec{\nabla}\varphi + \mathcal{Q}_0\vec{\hat{A}}) - (1+\lambda_s)|\vec{\nabla}\varphi + \mathcal{Q}_0\vec{\hat{A}}|^2\right] + 2\mathcal{K}_2\left|\dot{\varphi} + \frac{1}{2}\mathcal{Q}_0 h^{00}\right|^2 + \frac{1}{\tilde{M}_p^2}T_{\mu\nu}h^{\mu\nu}\Big\}$$

Gauge Invariant terms



Mixing: genuine modification of gravity

N.B.

Dark fields (e.g. DM)

$\sim \chi^2 \qquad \sim (\partial\chi)^2$

No mixing with $h_{\mu\nu}$

Vector modes

Choose: $h^{00} = h^{ij} = \varphi = 0$

Transverse: $\vec{\nabla}_i \hat{A}^i = \vec{\nabla}_i h^{0i} = 0$

Decouple: not produced by sources

$$S = \int d^4x \left\{ \frac{1}{2} \vec{\nabla}_i h_{0j} \vec{\nabla}^i h^{0j} + K_B \left(|\dot{\vec{\hat{A}}}|^2 - \vec{\nabla}_i \hat{A}_j \vec{\nabla}^i \hat{A}^j - \mathcal{M}^2 |\vec{\hat{A}}|^2 \right) + \frac{1}{\tilde{M}_p^2} T_{0i} h^{0i} \right\}$$

Travel at speed of light

Massive: $\mathcal{M}^2 = \dfrac{(2 - K_B)(1 + \lambda_s)\mathcal{Q}_0^2}{K_B}$

Healthy:

$0 < K_B < 2$

$\lambda_s > -1$

Scalar modes

$h^{00} = -2\Psi$ $\quad \hat{A}_i = \vec{\nabla}_i \alpha$

$h^{0i} = 0$

$h^{ij} = -2\Phi\gamma^{ij}$ $\quad \chi = \varphi + \mathcal{Q}_0 \alpha$

$$S = \int d^4x \Big\{ -6\dot{\Phi}^2 + 2|\vec{\nabla}\Phi|^2 - 4\vec{\nabla}\Phi \cdot \vec{\nabla}\Psi + K_B|\vec{\nabla}(\dot{\alpha} + \Psi + \frac{2-K_B}{K_B}\chi)|^2$$
$$+ 2\mathcal{K}_2\,|\dot{\varphi} - \mathcal{Q}_0\Psi|^2 - \frac{(2-K_B)(2+\lambda_s K_B)}{K_B}|\vec{\nabla}\chi|^2 - \frac{2}{\tilde{M}_p^2}\,(\rho\Psi + 3P\Phi)\Big\}$$

$$Z = \{\Psi, \Phi, \alpha, \varphi\} \quad \Longrightarrow \quad S = \int dt \int \frac{d^3k}{(2\pi)^3}\,[Z^\dagger \cdot \mathbf{N} \cdot Z + h.c.]$$

Normal modes:

$$\det \mathbf{N} = 0$$

$$\omega^2 = \frac{(2-K_B)}{\mathcal{K}_2 K_B}\left(1 + \frac{1}{2}K_B\lambda_s\right)k^2 + \mathcal{M}^2$$

$$\omega^2 = 0$$

Healthy:

$0 < K_B < 2$

$\lambda_s > -2/K_B$

$\mathcal{K}_2 > 0$

Scalar modes: Hamiltonian

(With T. Zlosnik, in preparation)

(Covariant Hamiltonian: with M. Bataki & T. Zlosnik, in preparation)

$$S=\int d^4x\Big\{-6\dot\Phi^2+2|\vec\nabla\Phi|^2-4\vec\nabla\Phi\cdot\vec\nabla\Psi+K_B|\vec\nabla(\dot\alpha+\Psi+\frac{2-K_B}{K_B}\chi)|^2$$
$$+2\mathcal{K}_2\,|\dot\chi-\mathcal{Q}_0\,(\dot\alpha+\Psi)|^2-\frac{(2-K_B)(2+\lambda_sK_B)}{K_B}|\vec\nabla\chi|^2-\frac{2}{\tilde M_p^2}\,(\rho\Psi+3P\Phi)\Big\}$$

$\Longrightarrow$

Canonical momenta

$$P_\Psi=0$$
$$P_\Phi=-12\dot\Phi$$
$$P_\chi=4\mathcal{K}_2\,[\dot\chi-\mathcal{Q}_0(\dot\alpha+\Psi)]$$
$$P_\alpha=-4\mathcal{Q}_0\mathcal{K}_2[\dot\chi-\mathcal{Q}_0(\dot\alpha+\Psi)]-2K_B\vec\nabla^2(\dot\alpha+\Psi)-2(2-K_B)\vec\nabla^2\chi$$

$$\dot P_\alpha=0$$

Constraints (in vacuum):

$$P_\Phi\approx 0$$
$$\vec\nabla^2\Phi\approx-\frac{1}{4}P_\alpha$$
$$\Psi\approx\Phi$$

$\Longrightarrow$

$$H=\int\frac{d^3k}{(2\pi)^3}\Big\{\frac{1}{8\mathcal{K}_2}|P_\chi|^2+\frac{1}{4K_Bk^2}\,|P_\alpha+\mathcal{Q}_0P_\chi|^2-\frac{2-K_B}{K_B}\chi(P_\alpha+\mathcal{Q}_0P_\chi)-\frac{1}{8k^2}|P_\alpha|^2$$
$$+\frac{(2-K_B)(2+\lambda_SK_B)}{K_B}k^2|\chi|^2\Big\}$$

$$\omega^2=\frac{(2-K_B)}{\mathcal{K}_2K_B}\left(1+\frac{1}{2}K_B\lambda_s\right)k^2+\mathcal{M}^2$$

$$H>0$$

$$\omega^2=0$$

$\alpha\sim\alpha_0$ $\quad H>0$

$\alpha\sim\alpha_1 t$:

- $k>\sim\mu\lesssim Mpc^{-1}$ $\quad H>0$
- $k<\sim\mu\lesssim Mpc^{-1}$ $\quad H<0$

Thoughts for further refinements

Gauge ghost condensate

Cheng et al., JHEP 05, 076 (2006)

Health gauge theory of Lorentz violation

$$S = S_{EH} + \int d^4x\sqrt{-g}\left\{-\frac{1}{4g_{\rm ggc}^2}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}(\mathcal{D}^\mu\sigma\mathcal{D}_\mu\sigma + M_{\rm ggc}^2)^2\right\} + \dots$$

Decoupling limit

$M_{\rm ggc} \to \infty$

$\hat{A}_\mu = A_\mu / M_{\rm ggc}$

$$S = S_{EH} + \int d^4x\sqrt{-g}\left\{-\frac{K_B}{2}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} - \lambda(\hat{A}^2+1)\right\} + \dots$$

Covariant derivative

$$\mathcal{D}_\mu\sigma = \nabla_\mu\sigma / M_{\rm ggc} + A_\mu$$

$$J^\mu\nabla_\mu\phi \to F^{\mu\nu}\mathcal{D}_\mu\sigma\mathcal{D}_\nu\phi$$

$$\mathcal{Y} \to \left(g^{\mu\nu} + \mathcal{D}^\mu\sigma\mathcal{D}^\nu\sigma / M_{\rm ggc}^4\right)\mathcal{D}_\mu\phi\mathcal{D}_\nu\phi$$

$$\mathcal{Q} - \mathcal{Q}_0 = \hat{A}^\mu\left(\nabla_\mu\phi + \mathcal{Q}_0\hat{A}_\mu\right) \to \mathcal{D}^\mu\sigma\mathcal{D}_\mu\phi$$

...

We have discussed

- Solutions Dark matter or Gravity?
- MOND proposal implies new gravitational fields
- Difference between new (dark) matter dof and gravitational dof
- Current proposals for relativistic MOND: 1 scalar + 1 time-like vector

We have shown

- Only healthy relativistic MOND theory with
  - Tensor-speed = 1
  - Correct lensing on galactic scales
  - Correct CMB
  - Correct matter power spectrum
  - Same scalar/vector fields leading to MOND also give good cosmology
  - Potential for refinements

Future & upcoming work: Hamiltonian, PPN, non-linear cosmology

Current collaborators: T. Zlosnik (CEICO), S. Ilic (former CEICO, current APC), M. Bataki (PhD student, U. Cyprus & CEICO)