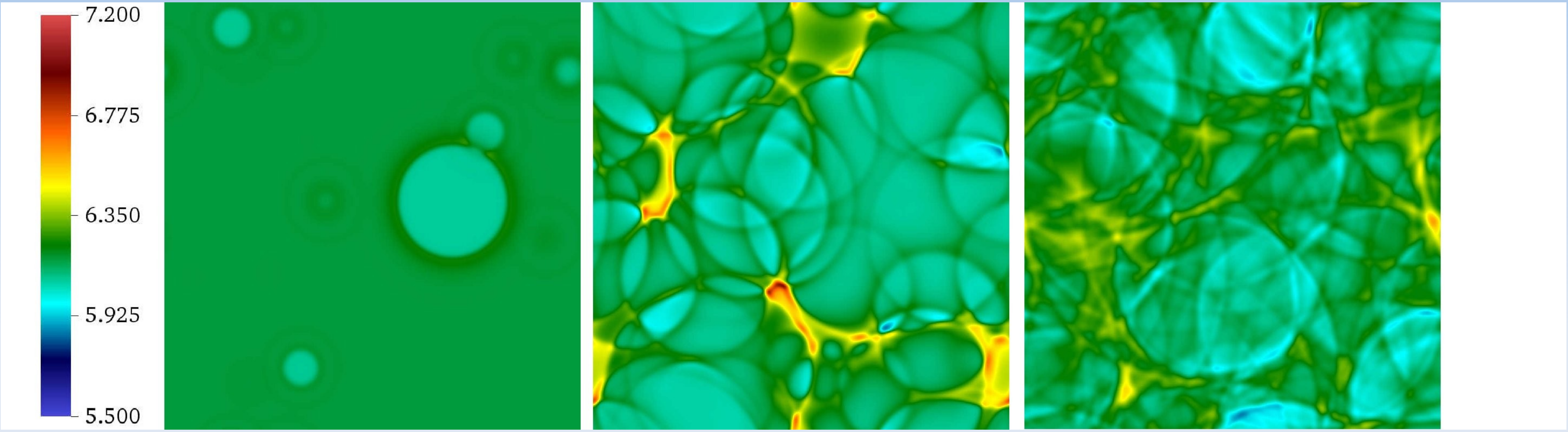
Gravitational waves from cosmological phase transitions

Thomas Konstandin

Helsinki, Feb 1, 2023

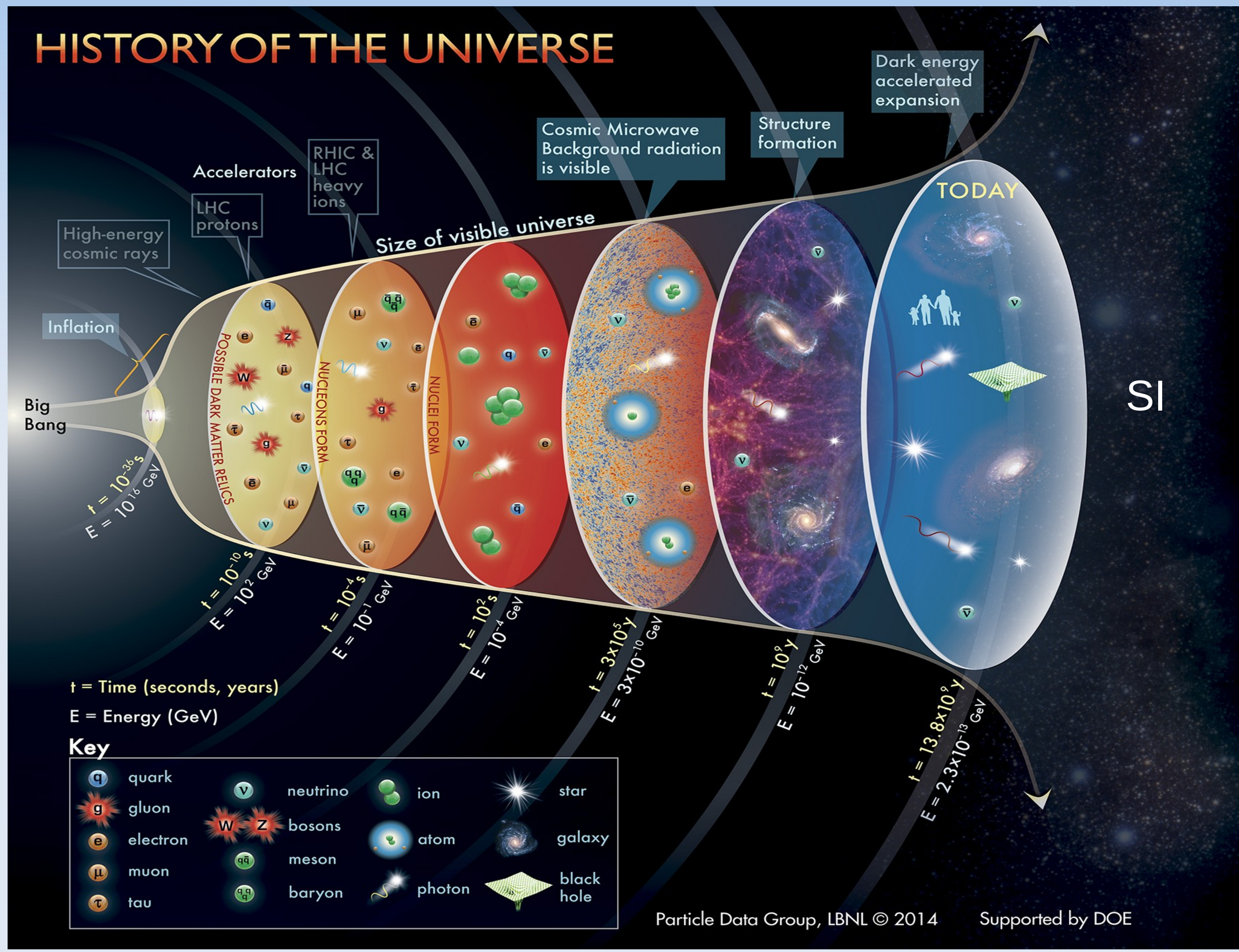
with Jinno/Rubira/Stomberg
and Giese/van de Vis

Gravitational waves from cosmological phase transitions



I. Introduction
II. Simulations
III. Extrapolations

Standard Cosmology

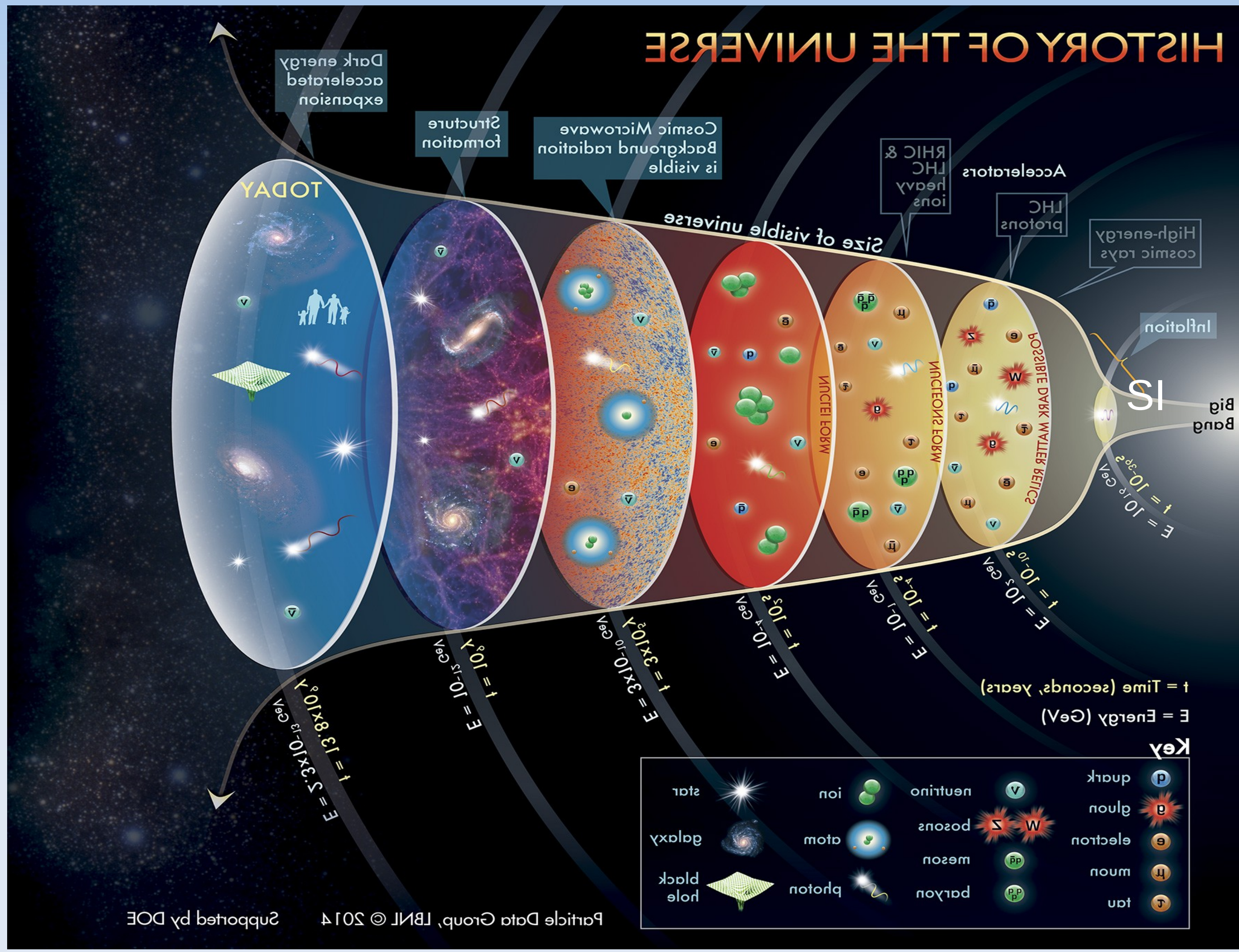


SI

time

temperature

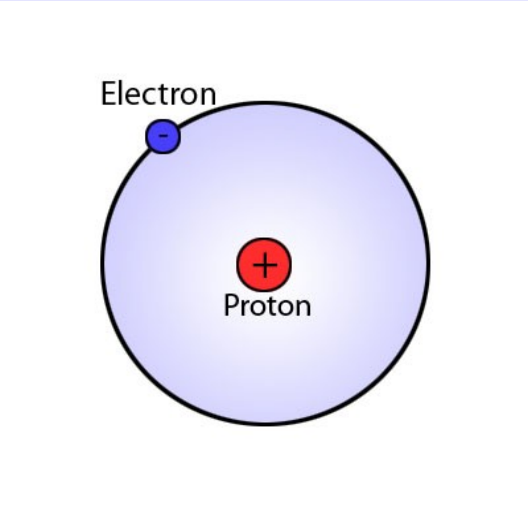
Standard Cosmology

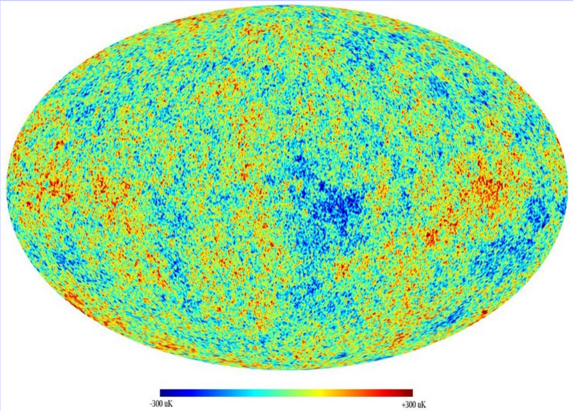


temperature →

← time

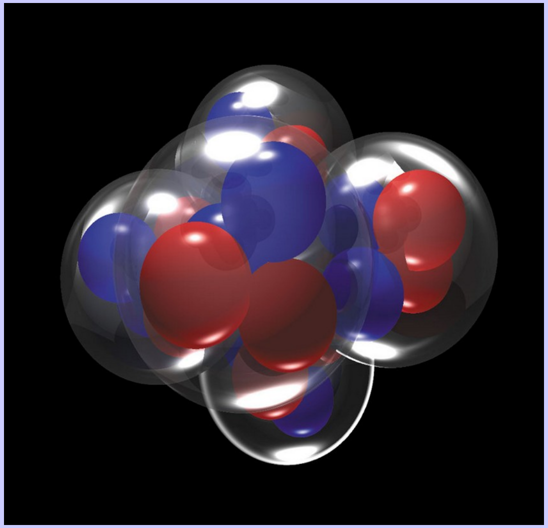
Atomic physics at T~eV

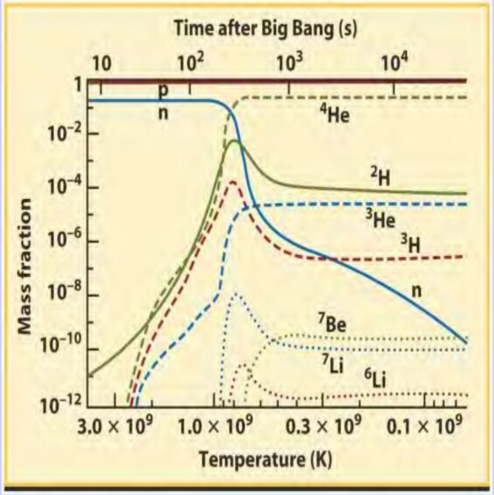




The Cosmic Microwave Background **links** atomic physics to cosmology at temperature T~eV

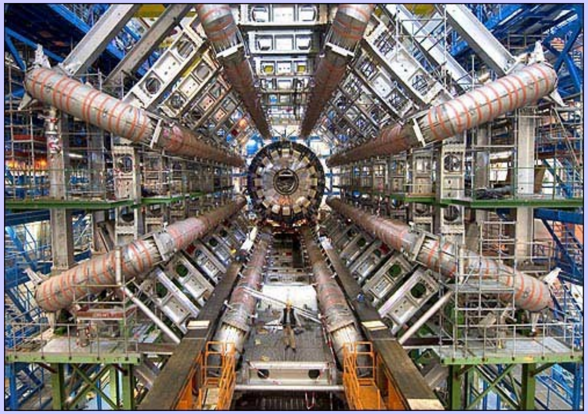
Nuclear physics at T~MeV

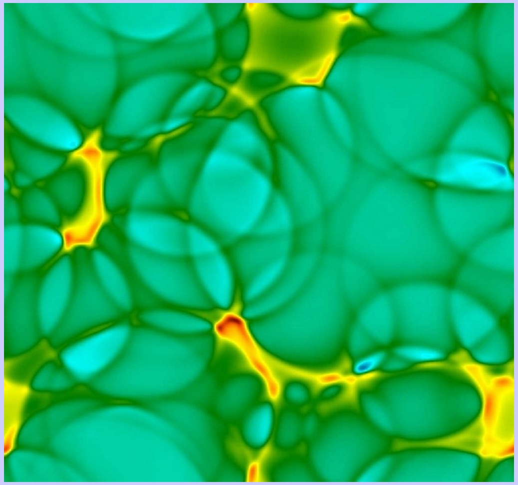




Big bang nucleosynthesis **links** nuclear physics to cosmology at temperature T~MeV

Phase transition at T~100 GeV?





Possibly, the electroweak phase transition drove the Universe **out-of-equilibrium.** This would provide a link to current particle physics experiments.

Electroweak phase transition

gravitational
waves

baryogenesis

Ground based GW experiments

Observation of black hole merges put GW astrophysics and multi-messanger astronomy firmly on the physics landscape. But what can we learn in particle physics and cosmology?

Stochastic backgrounds are limited by BBN constraints (N_{eff}). Ground based experiments are barely competitive right now, but this might improve in the future.

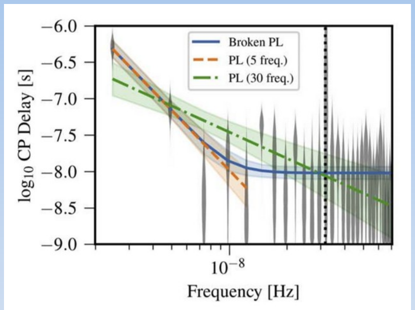
Future space telescopes

Space based experiments are sensitive to smaller frequencies where stochastic backgrounds GWs are easier to detect and can provide a link to EW physics.

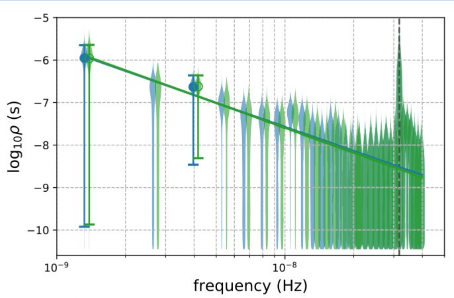
Anticipated launch in 2030s.

IPTAs: a tentative hint

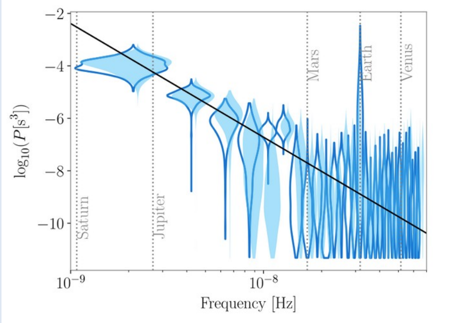
NanoGrav



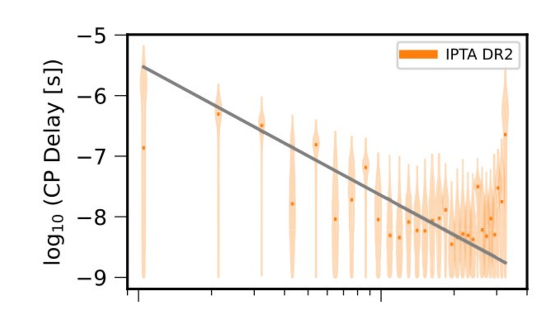
EPTA



PPTA



IPTA



There is a tentative hint of a stoachastic GW background in pulsar timing array data.

A detection would require the characteristic Hellings-Downs curve in the correlations.

An interpreation in terms of phase transitions is possible with some tension:

- shape disfavors PTs as an explanation

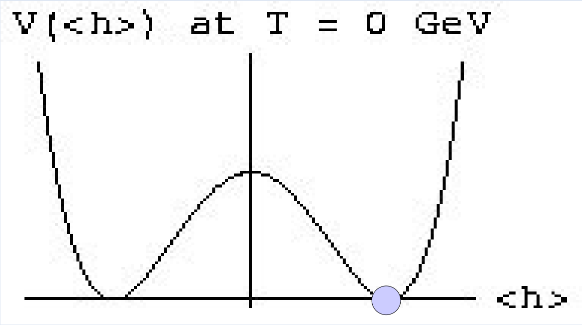
- phase transition temperature is close to BBN (~ 1 MeV)

- CMB impact through μ-distorsions

Electroweak symmetry breaking

The **Mexican hat** potential is designed to lead to a finite Higgs vacuum expectation value (VEV) and break the electroweak symmetry

$$V(h) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2$$

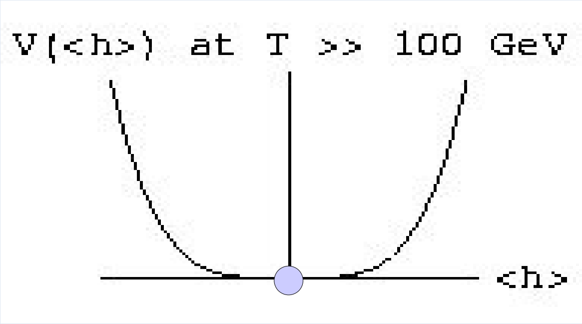


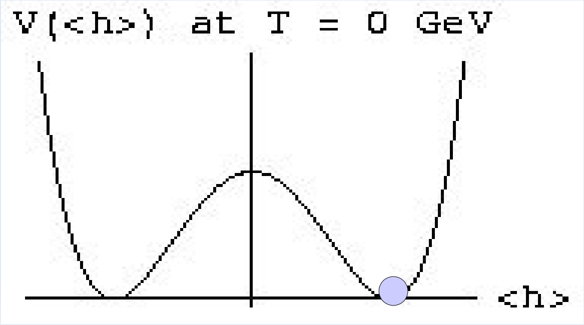
Electroweak symmetry breaking

[Weinberg '74]

At large temperatures the symmetry is restored

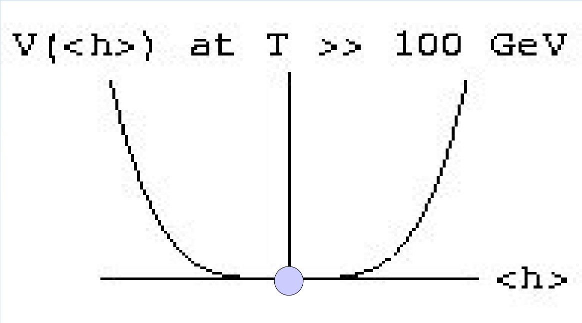
$$V(h,T) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2 + \mathrm{const}\ \times\ h^2\,T^2 + \mathrm{details}$$

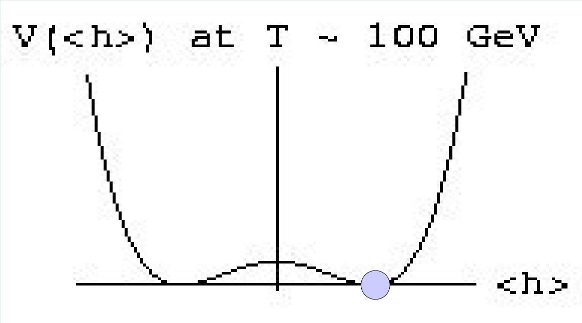




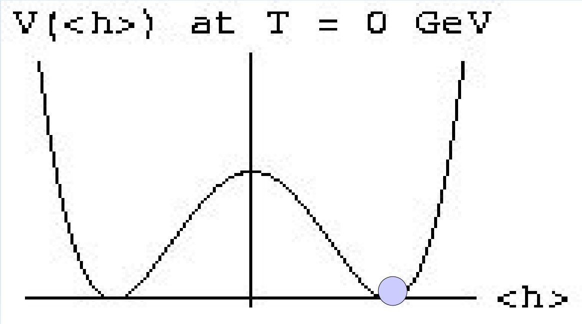
Electroweak symmetry breaking

Depending on the details, the phase transition can be very weak or even a cross over



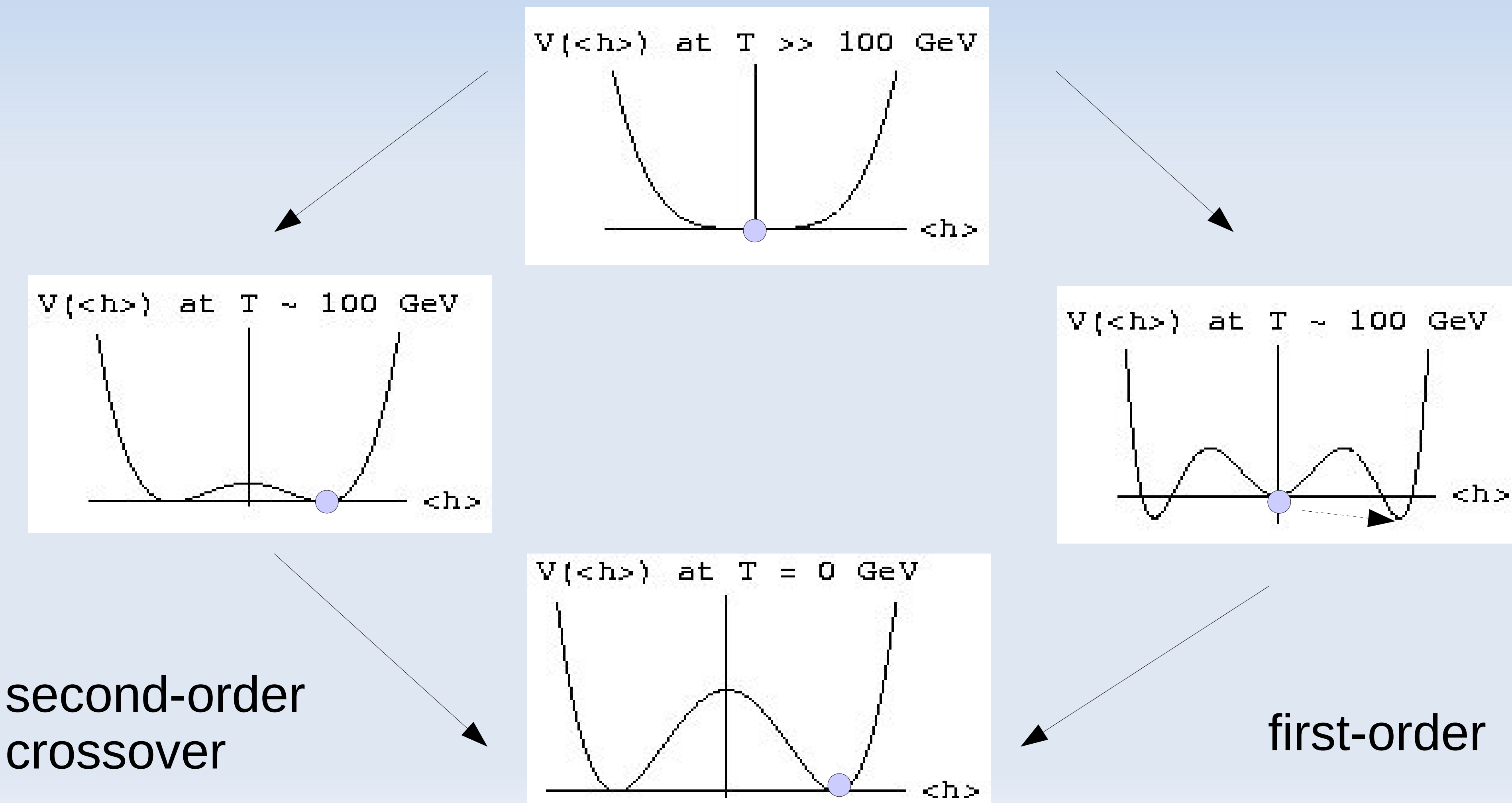


second-order
crossover



Electroweak symmetry breaking

It can also be a strong phase transition if a **potential barrier** seperates the new phase from the old phase



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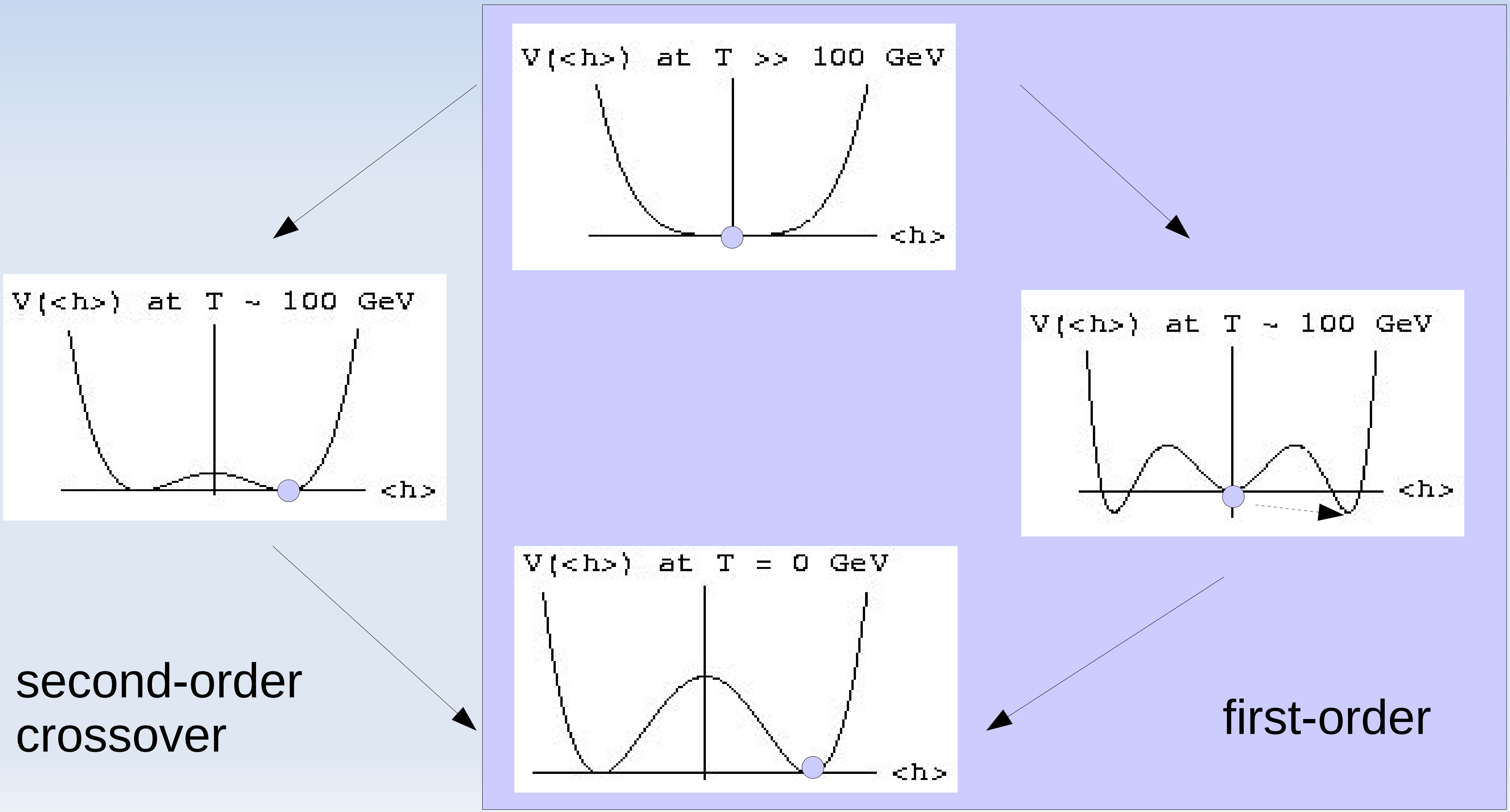
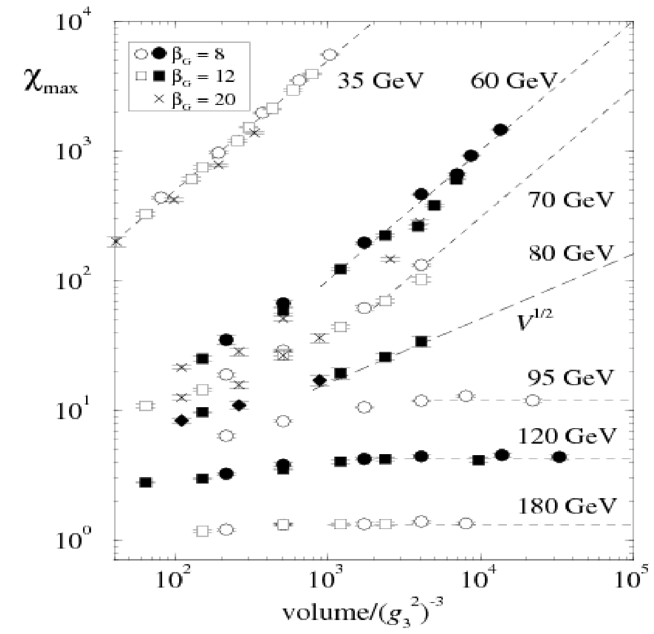


Electroweak phase transition in the SM

The effective potential is the standard tool to study phase transition at finite temperature.

Lattice studies show that there is a crossover in the SM.

A light Higgs would lead to a 1st-order PT.



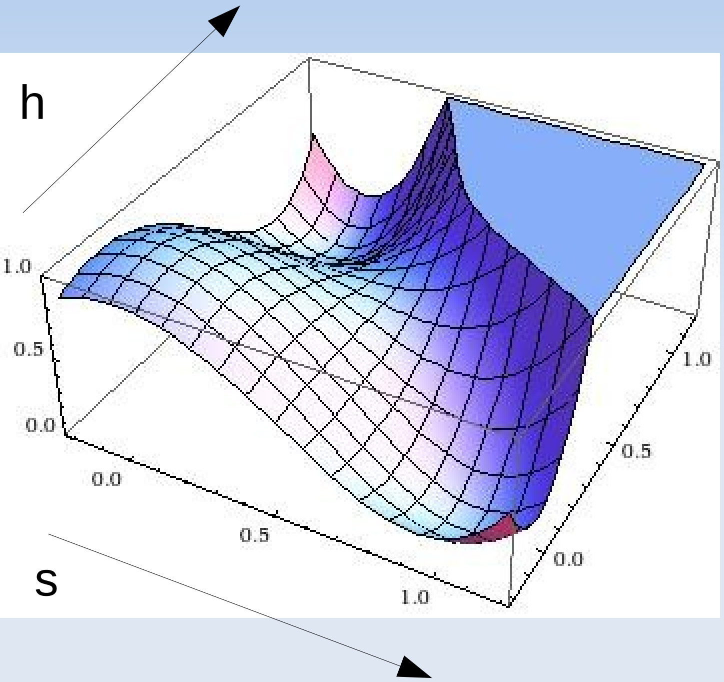
[Kajantie, Laine, Rummukainen, Shaposhnikov '96]

Singlet extension

The Standard Model only features a electroweak crossover.

A potential barrier and hence first-order phase transitions are quite common in extended scalar sectors:

$$V(h,s) = \frac{\lambda}{4}\left(h^2 - v^2\right)^2 + m_s^2 s^2 + \lambda_s s^4 + \lambda_m s^2 h^2$$



The singlet field has an additional $\mathbb{Z}_2$ symmetry and is a viable DM candidate.

The phase transition proceeds via

$$(h,s) = (0,w) \rightarrow (h,s) = (v,0)$$

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First-order phase transitions

• first-order phase transitions proceed by bubble nucleations

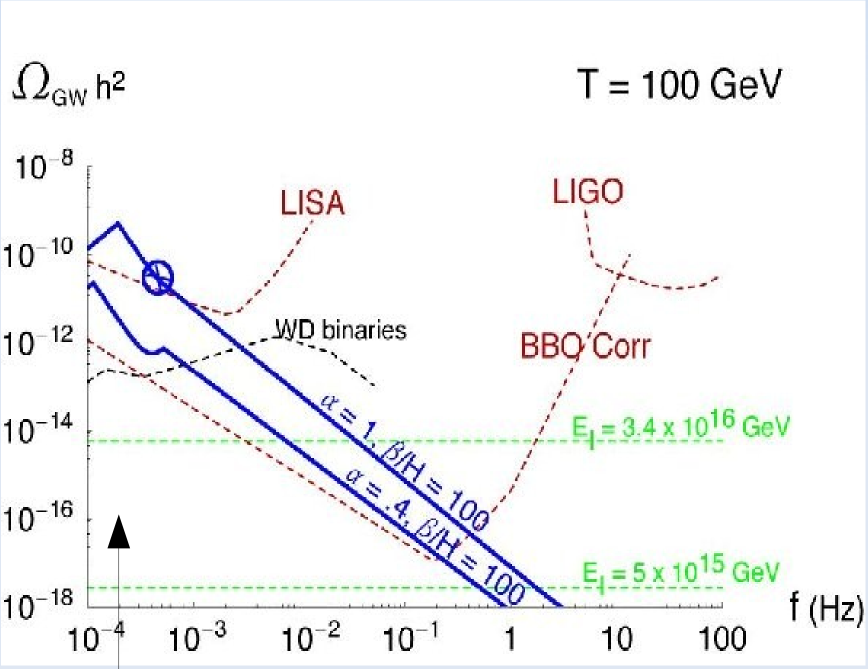
• in case of the electroweak phase transition, the ”Higgs bubble wall” separates the symmetric from the broken phase

• this is a violent process $(v_{wall} \simeq O(c))$ that drives the plasma out-of-equilibrium and sets the fluid into motion

Observation

[Grojean&Servant '06]

The produced gravitational waves can be observed with laser interferometers in space

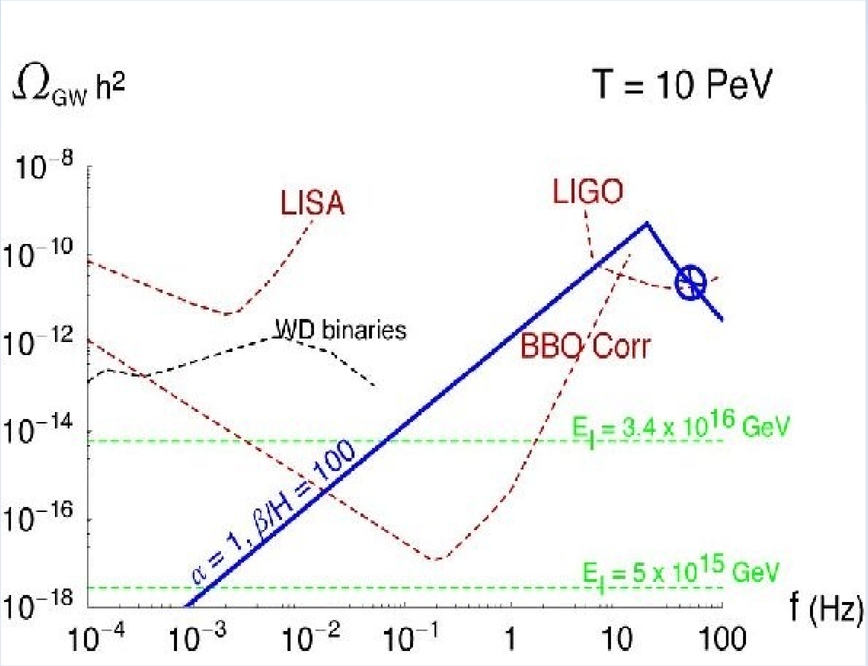


redshifted **Hubble horizon** during a phase transition at T ~ 100 GeV

Observation

[Grojean&Servant '06]

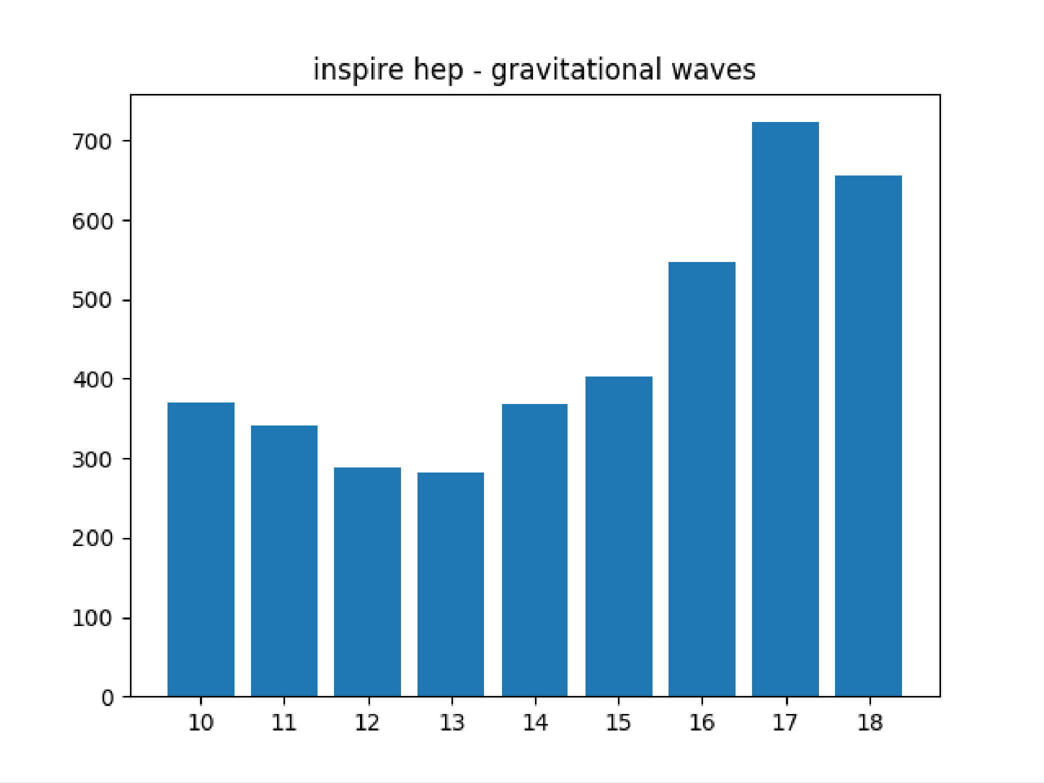
… or on the ground

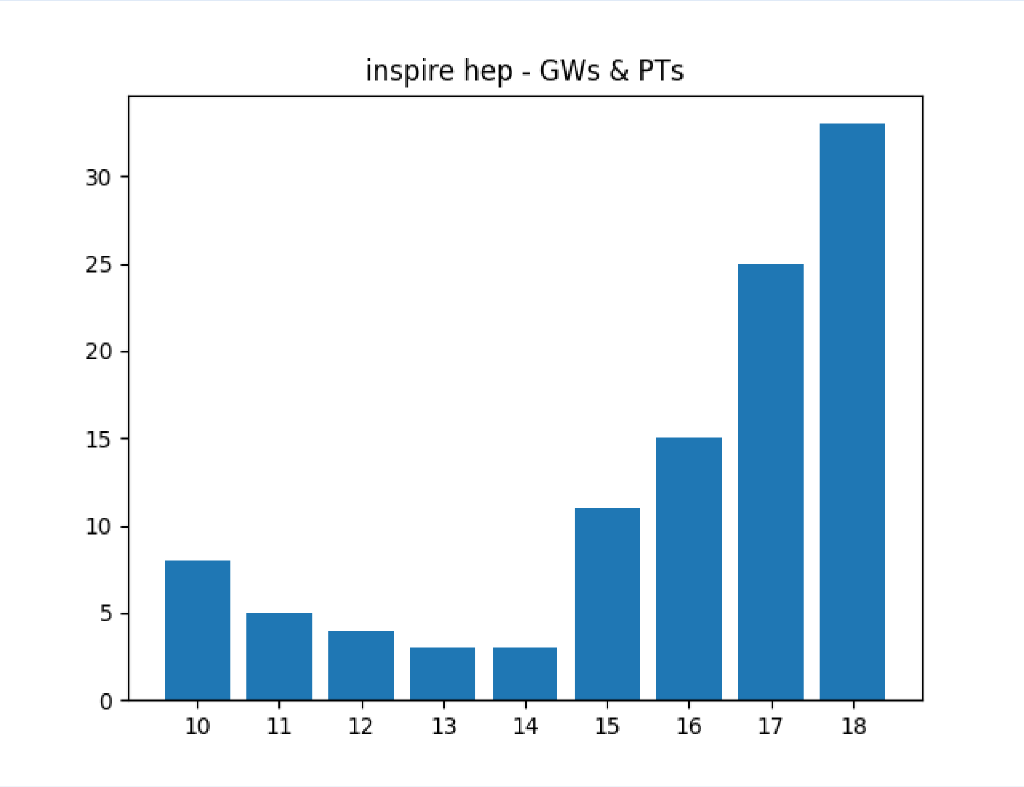


Strong phase transition at larger temperatures produce the same energy fraction of gravitational waves but at higher frequencies.

GWs from PTs

ArXiv activity:





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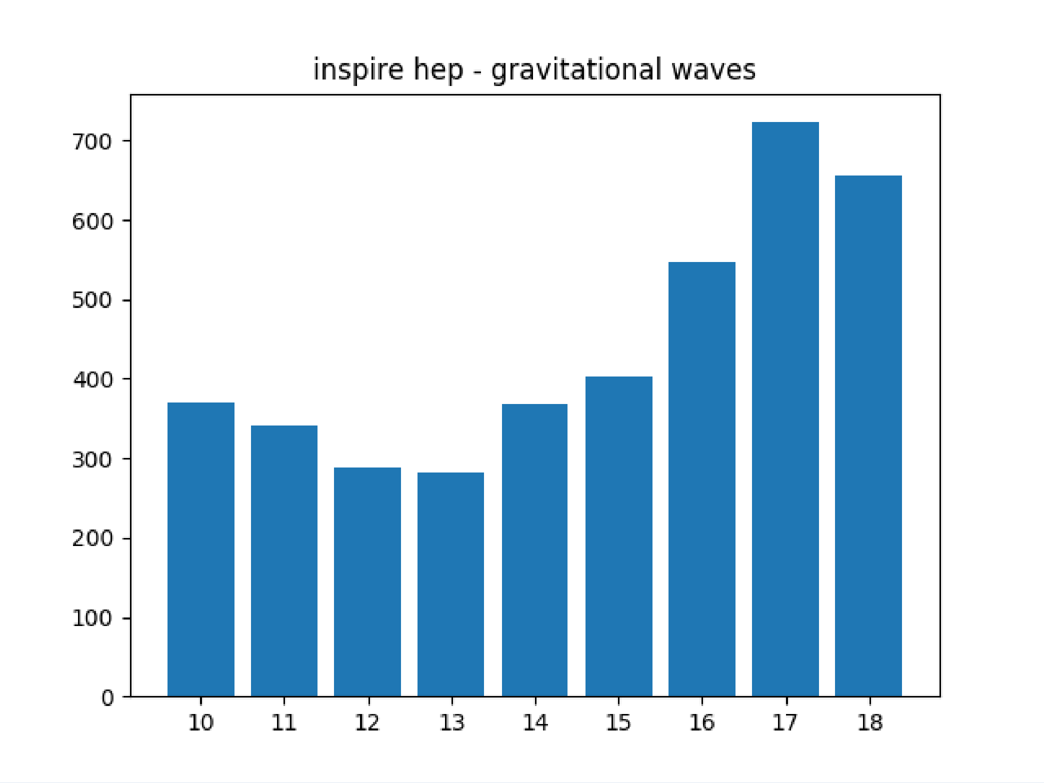
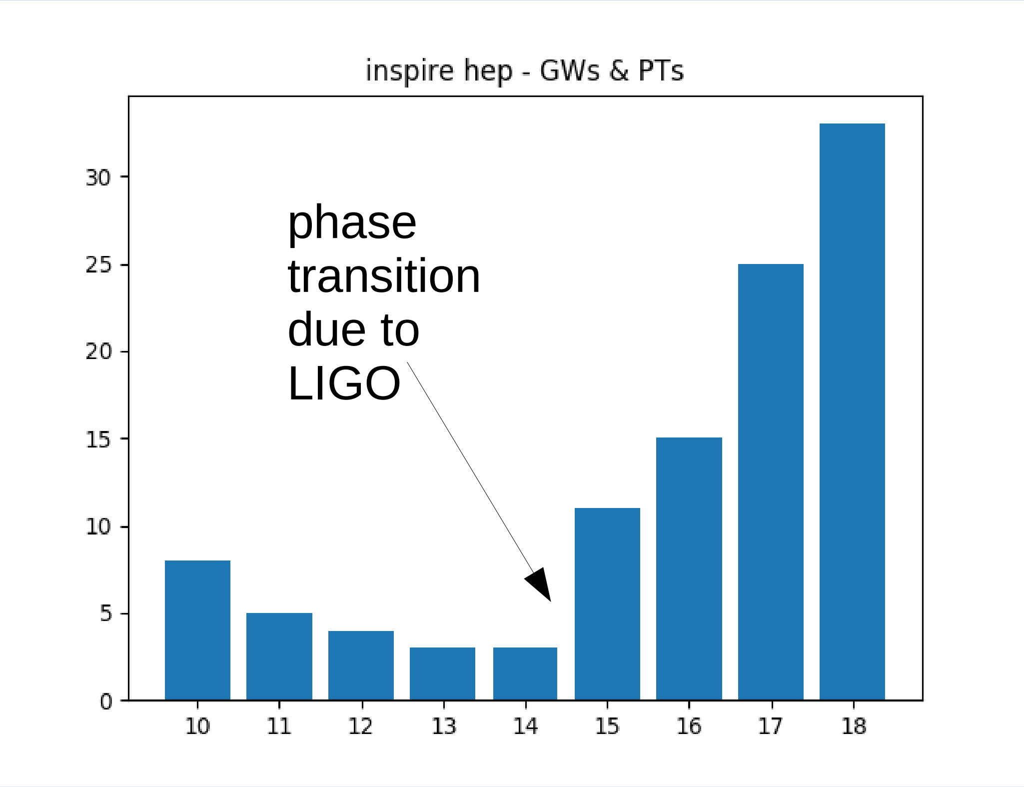




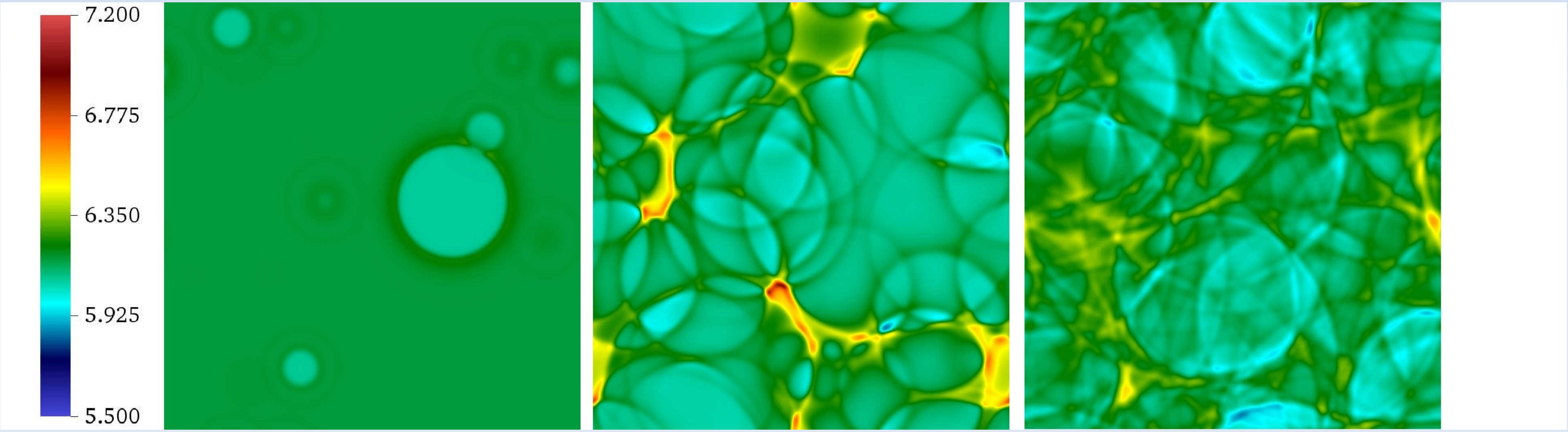
Sources of GWs from PTs

During and after the phase transition, several sources of GWs are active

- Collisions of the scalar field configurations / initial fluid shells
- Sound waves after the phase transition
  (long-lasting → dominant source)
- Turbulence
- Magnetic fields

In the last 10 years, simulations became the main tool to incorporate all these effects.

GWs from cosmological phase transitions



[Hindmarsh, Huber, Rummukainen, Weir '15]

Back of the envelope

There are several quantities that can enter in the determination of the GW spectrum:

The temperature of the phase transition T.

The (inverse) duration of the phase transition

$P \propto \exp(\beta t)$ and typically $\beta / H \sim O(100)$

The wall velocity v_w.

The amount of latent heat Λ that is transformed into kinetic energy *K* in the plasma:

$$\Lambda \to K \,, \quad \alpha = \frac{K}{\rho_{\rm tot}}$$

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Back of the envelope

The Weinberg formula determines how stochastic gravitational waves are produced

$$\frac{dE_{GW}}{d\omega d\Omega} = 2G_N\omega^2\lambda_{ij,lm}(\hat{\mathbf{k}})T^*_{ij}(\hat{\mathbf{k}},\omega)T_{lm}(\hat{\mathbf{k}},\omega),$$

And generally the energy fraction in GWs scales as

$$\Omega^*_{GW}(\omega) = \frac{dE_{GW}}{E_{tot}d\log\omega} \propto (\lambda H)^2\left(\frac{K}{\rho_{\rm tot}}\right)^2\Delta(\omega/\beta, v_w)$$

The length (time) scale λ has to be of order of the Hubble parameter H for observable GWs. This is given by the bubble size, the duration of the phase transition or the lifetime of the fluid motion.

Back of the envelope

The peak frequency at production is linked to the bubble size or the duration of the phase transition

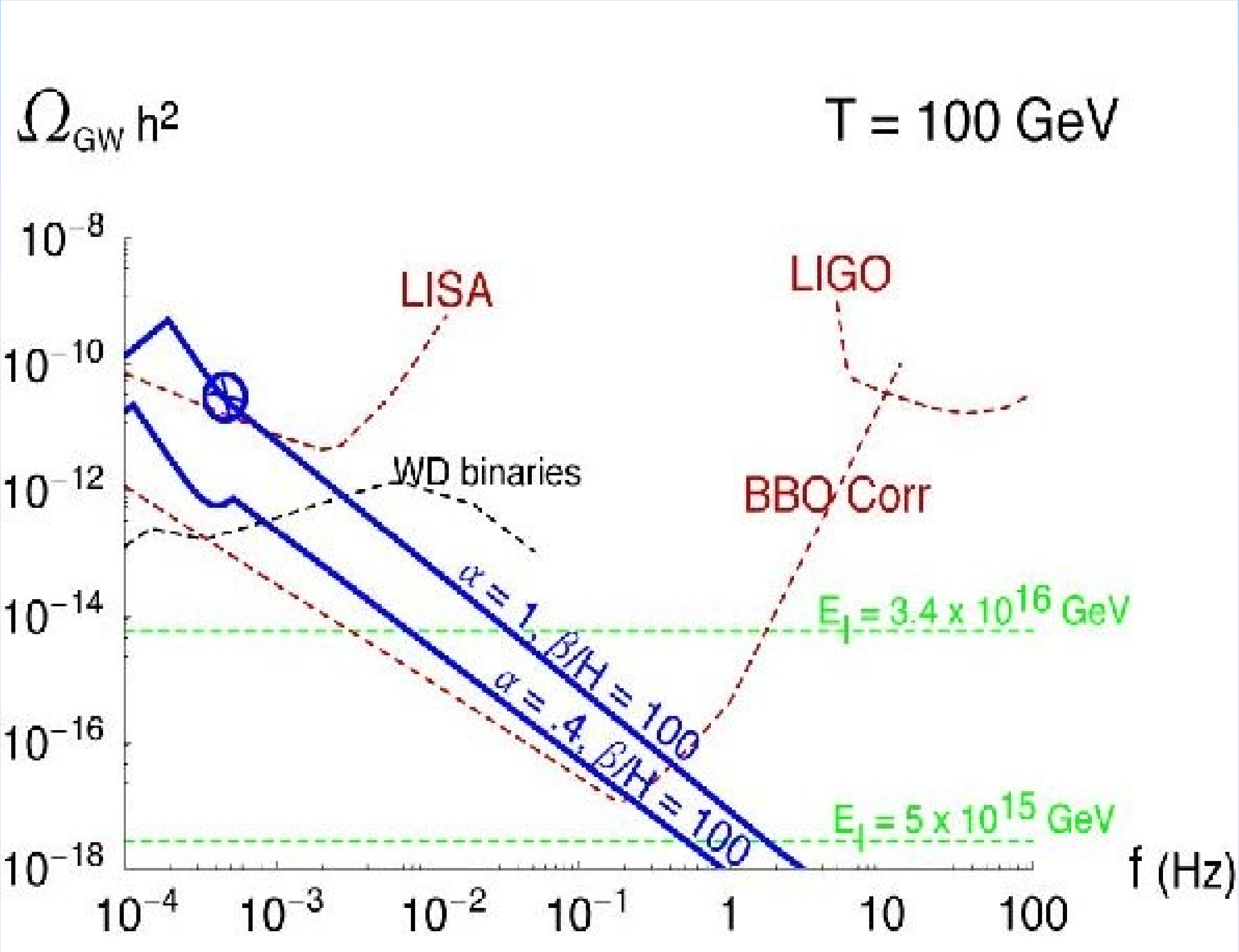
$$\omega^*_{\rm peak} \simeq \beta \simeq O(100)\, H$$

After the redshift, this amounts to

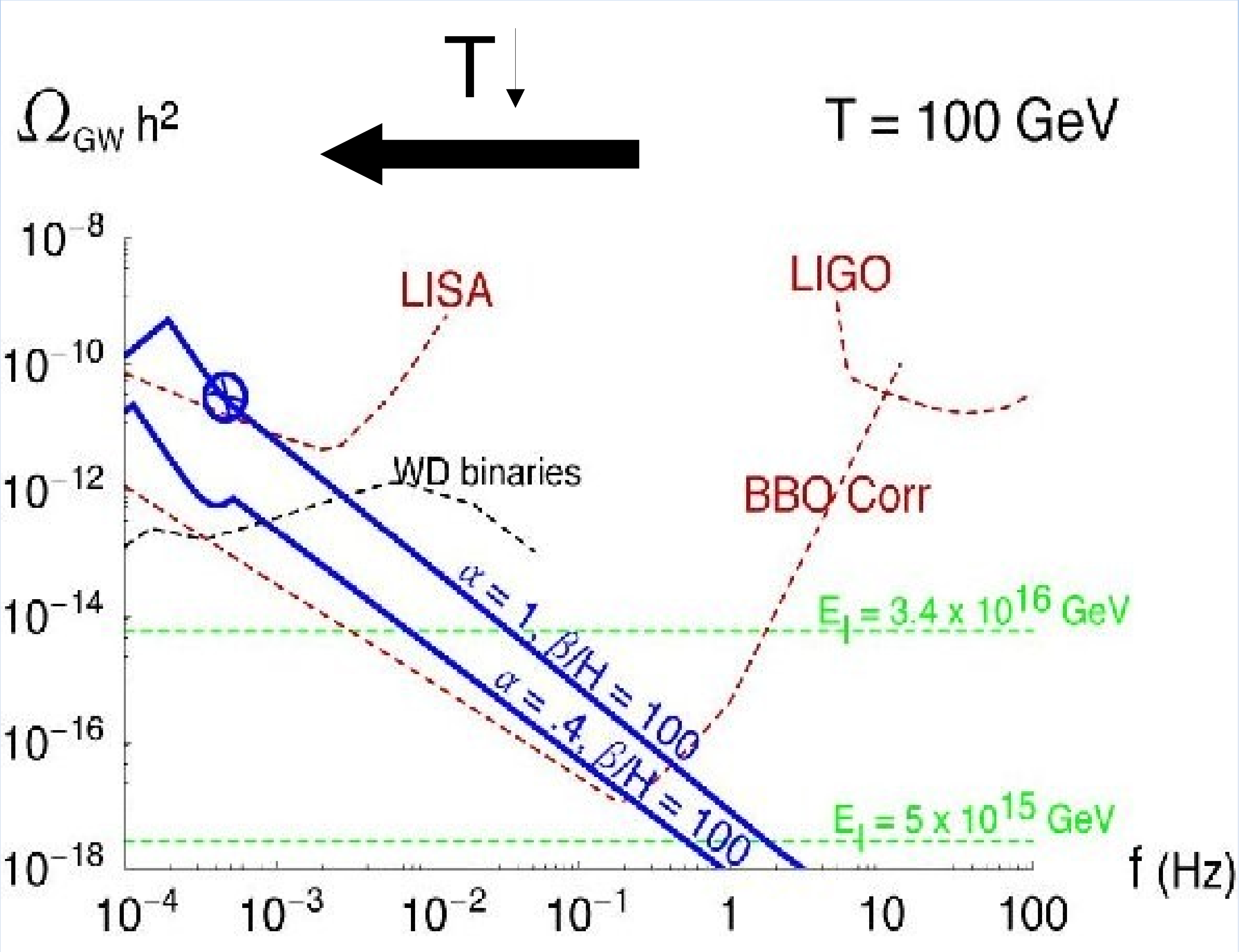
$$\omega_{\rm peak} \simeq \frac{\beta}{100\, H} \frac{T}{100 \rm GeV} \ {\rm mHz}$$

Since GWs behave as radiation, Ω_{GW} is only redshifted after the transition to matter domination.

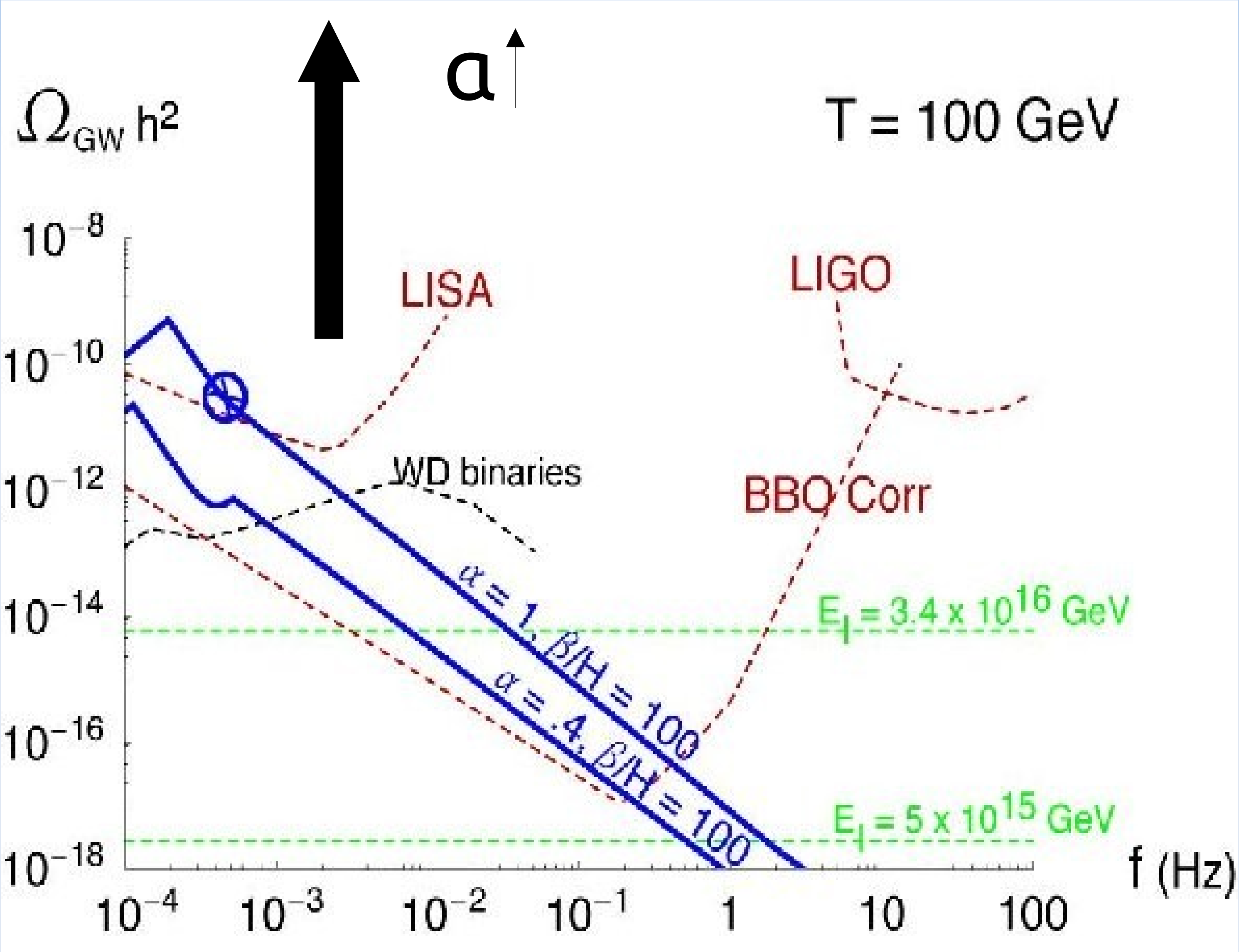
Observation



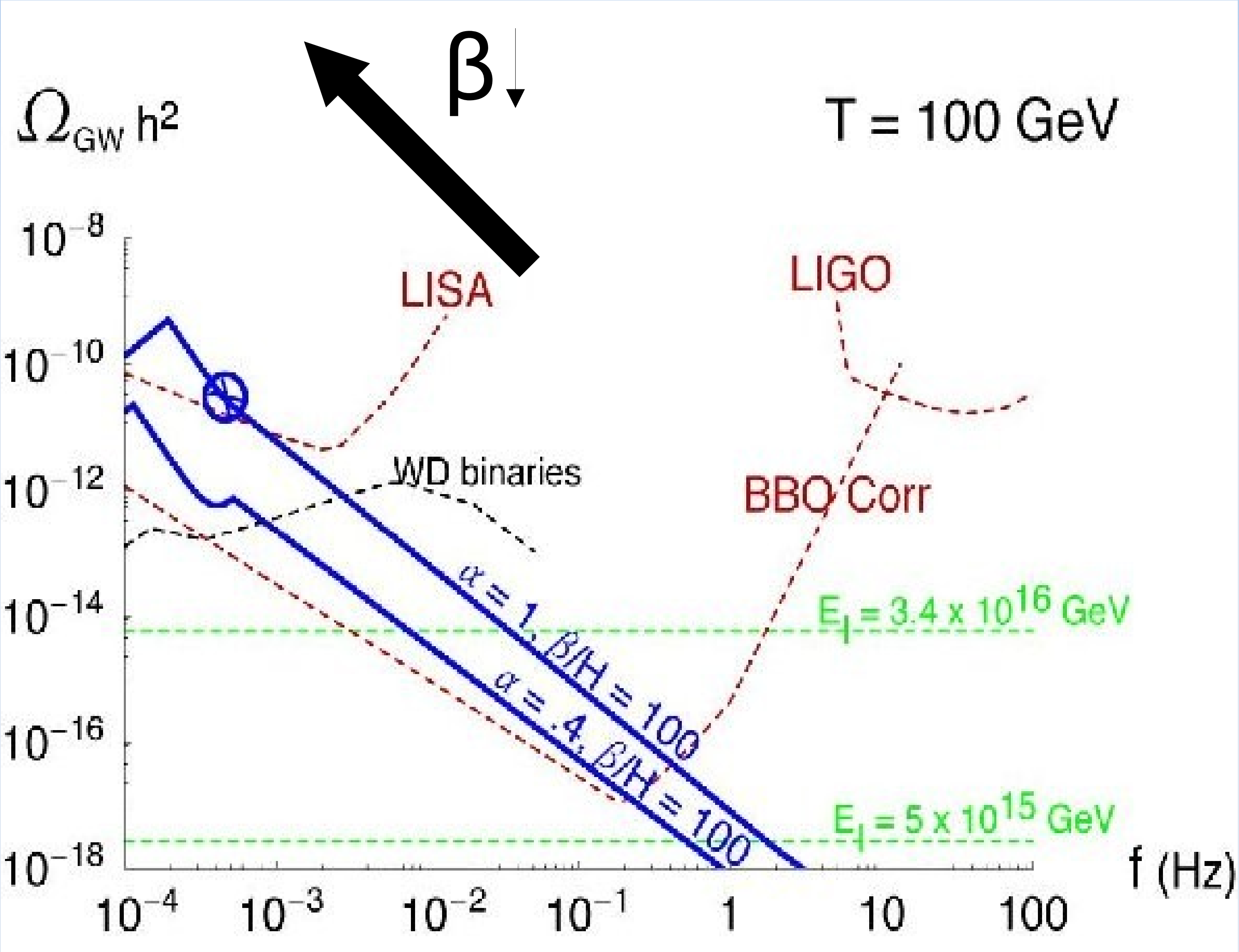
Observation



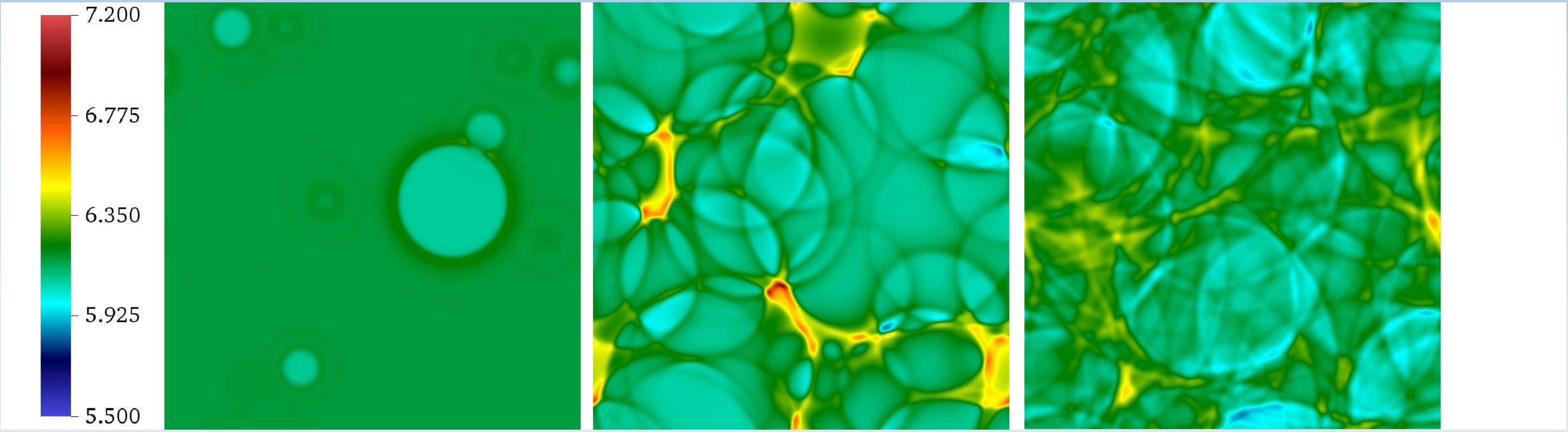
Observation



Observation



Gravitational waves from cosmological phase transitions



I. Introduction
II. Simulations
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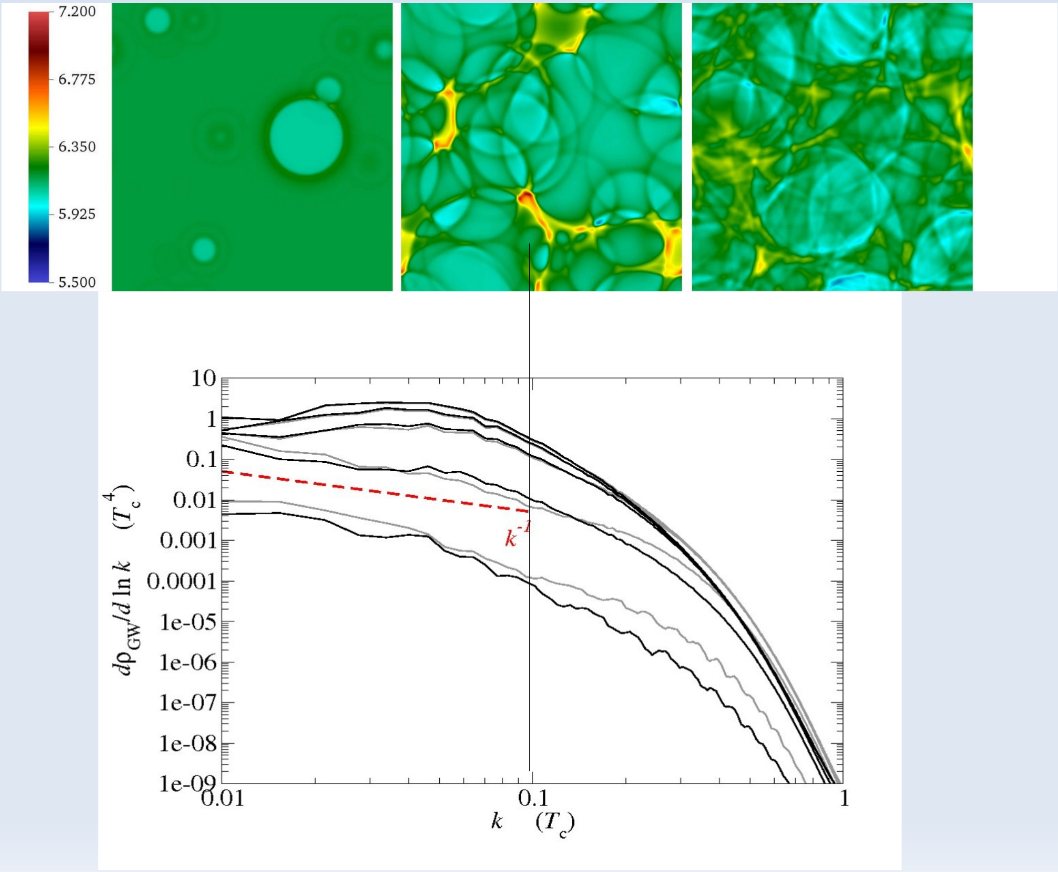
State-of-the-art: simulations

[Hindmarsh, Huber , Rummukainen, Weir '13, ’15, ’17]
[Weir ’16] [Gould, Sukuvaara,Weir ’21] [Cutting, Hindmarsh, Weir ’18&’19]
[Cutting, Escartin, Hindmarsh, Weir ’20]

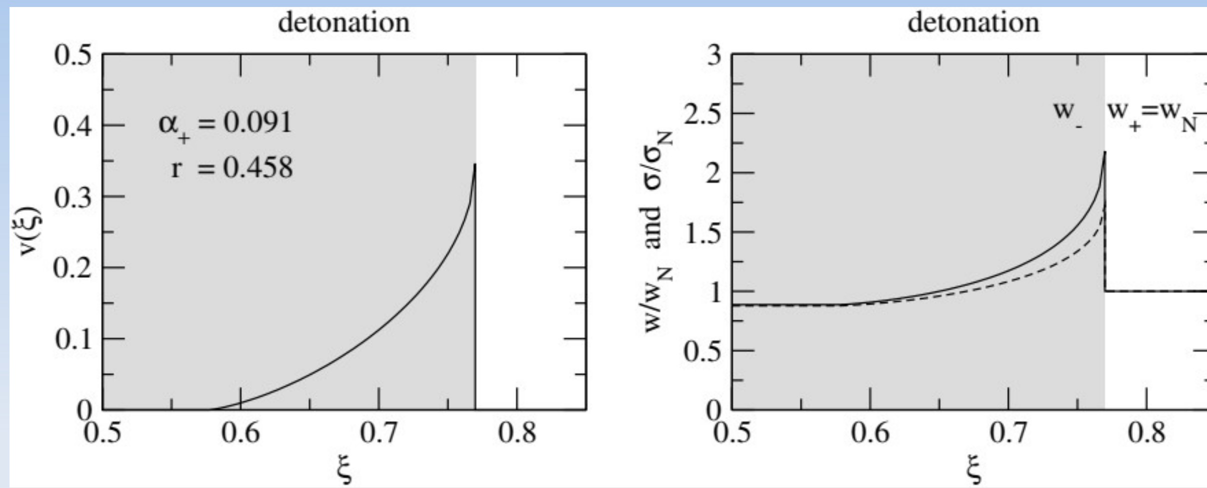
Depending on the context, the system can be descibed using hydrodynamics (fluid + Higgs) or just a scalar field



The produced GW spectrum can be read off from the simulation.

Really robust results, not many a priori assumptions.
But very costly.
How to extrapolate to other models and parameters?

State-of-the-art: semi-analytic methods



Semi-analytical approaches:

Try to understand the dynamics of the scalar field / fluid.

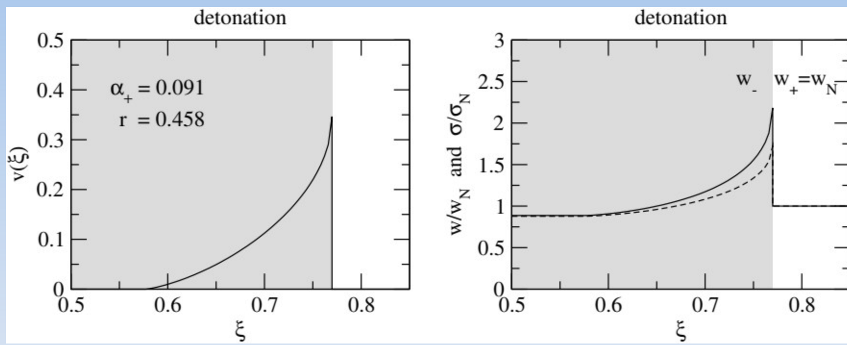
model the system in different regimes:

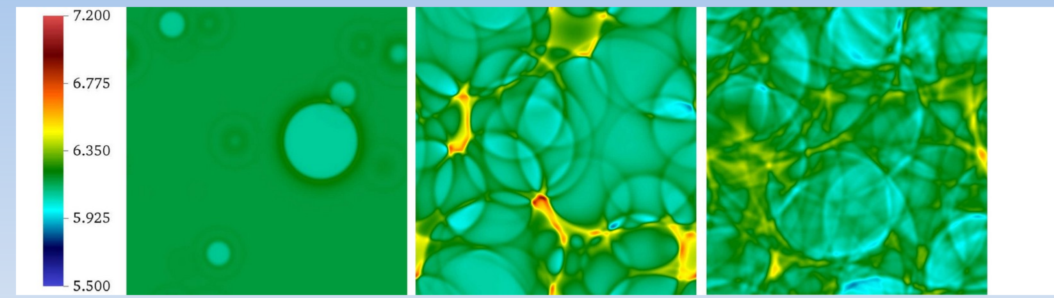
- envelope approximation
- bulk flow model
- sound shell model
- ….

For example:

[Kosowsky, Turner and Watkins ’92]
[Kosowsky and Turner ’93]
[Huber and TK ’08]
[Hindmarsh ’16]
[TK ’17]
[Jinno and Takimoto ’17, ’19]
[Hindmarsh and Hijazi ’19]
[Lewicki, Pujolas and Vaskonen ’21]
[Megevand and Membiela ’21]
…..

State-of-the-art





Semi-analytical approaches:

Pros:

- fast, many models & parameters can be studied
- better analytical understanding of the resulting spectrum

Cons:

- relies on assumptions (e.g. importance of sound waves underestimated for a long time)

Hydrodynamic simulations:

Pros:

- less *a priori* assumptions
- robust numerical results

Cons:

- costly, only few selected simulations
- model dependence (Higgs potential)
- extrapolation of the wall thickness

Bubble wall thickness

The main challenge in the hydrodynamic simulation is to cover very different length scales.

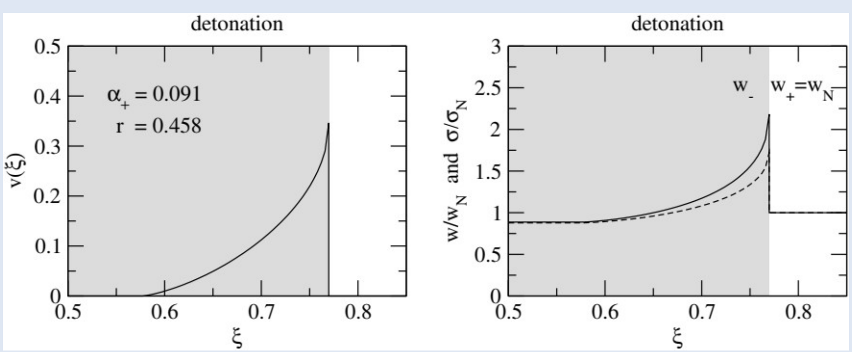
In the physical phase transition

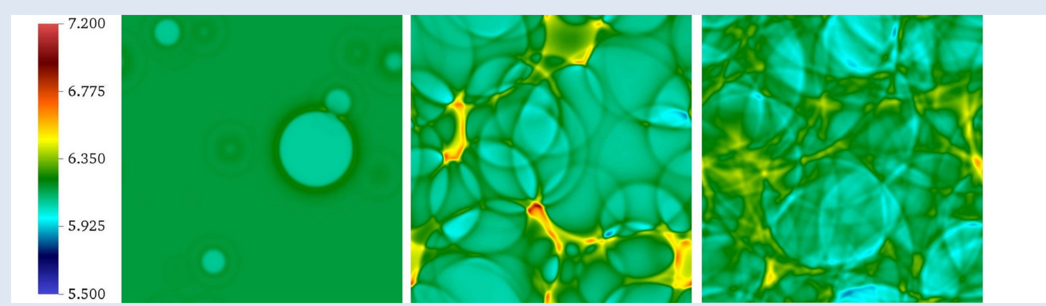
wall thickness <<<<<<< fluid shell thickness < bubble size

1/100GeV % of Hubble radius

In simulations:

grid spacing < (wall thickness < fluid shell thickness < bubble size) < box size





Higgsless simulations

In order to avoid this issue, we want to perform simulations that are agnostic about the wall thickness. This would resemble an *EFT* where the Higgs field was integrated out.

So the Higgs field acts as a background. Bubble nucleation times and locations are predetermined and the change in equation of state sets the fluid in motion.

$$p_\pm = \frac{1}{3} a_\pm T^4 - \epsilon_\pm(t, \vec{x})$$

However, this requires a hydrodynamic numerical framework that can deal with *shocks* and other discontinuities:

New High-Resolution Central Schemes for Nonlinear Conservation Laws and Convection–Diffusion Equations

Alexander Kurganov* and Eitan Tadmor†

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Central schemes may serve as universal finite-difference methods for solving nonlinear convection–diffusion equations in the sense that they are not tied to the specific eigenstructure of the problem, and hence can be implemented in a straightforward manner as black-box solvers for general conservation laws and related equations governing the spontaneous evolution of large gradient phenomena. The first-order Lax–Friedrichs scheme (P. D. Lax, 1954) is the forerunner for such central schemes. The central Nessyahu–Tadmor (NT) scheme (H. Nessyahu and E. Tadmor, 1990) offers higher resolution while retaining the simplicity of the Riemann-solver-free approach. The numerical viscosity present in these central schemes is of order $\mathcal{O}((\Delta x)^{2r}/\Delta t)$. In the convective regime where $\Delta t \sim \Delta x$, the improved resolution of the NT scheme and its generalizations is achieved by lowering the amount of numerical viscosity with increasing r. At the same time, this family of central schemes suffers from excessive numerical viscosity when a sufficiently small time step is enforced, e.g., due to the presence of degenerate diffusion terms.

In this paper we introduce a new family of central schemes which retain the sim-

Numerical issues

Hydrodynamics is an advective system
(fluids move over large distances)

This leads to issues in numerical solutions
Godunov's theorem -- for linear diff. equations there is a tradeoff between:

Excessive numerical viscosity (shocks are smeared out)

OR

Gibbs phenomenon (unphysical oscillations, $\rho < 0$)

Semi-discrete scheme
→Runge-Kutta integration

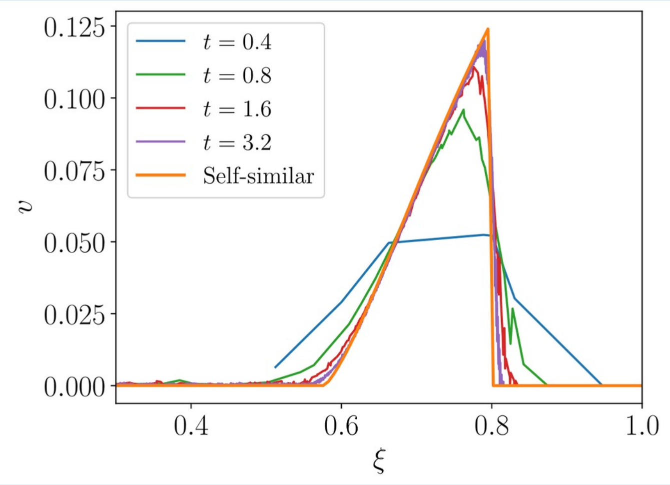
total variation diminishing (TVD) limiters on fluxes → hybridization (=non-linearities)

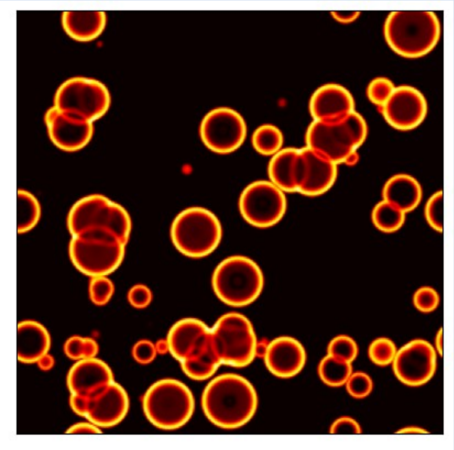
So Kurganov-Tadmore is a semi-discrete high-resolution central schemes for nonlinear conservation laws and convection–diffusion equations based on hybridization with flux-limiters

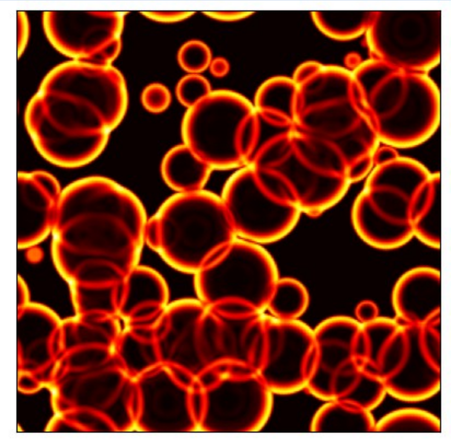
Simulation of cosmological phase transitions

We used the KT scheme to simulate relativistic hydrodynamics during cosmological first-order phase transitions.

One spherical bubble:



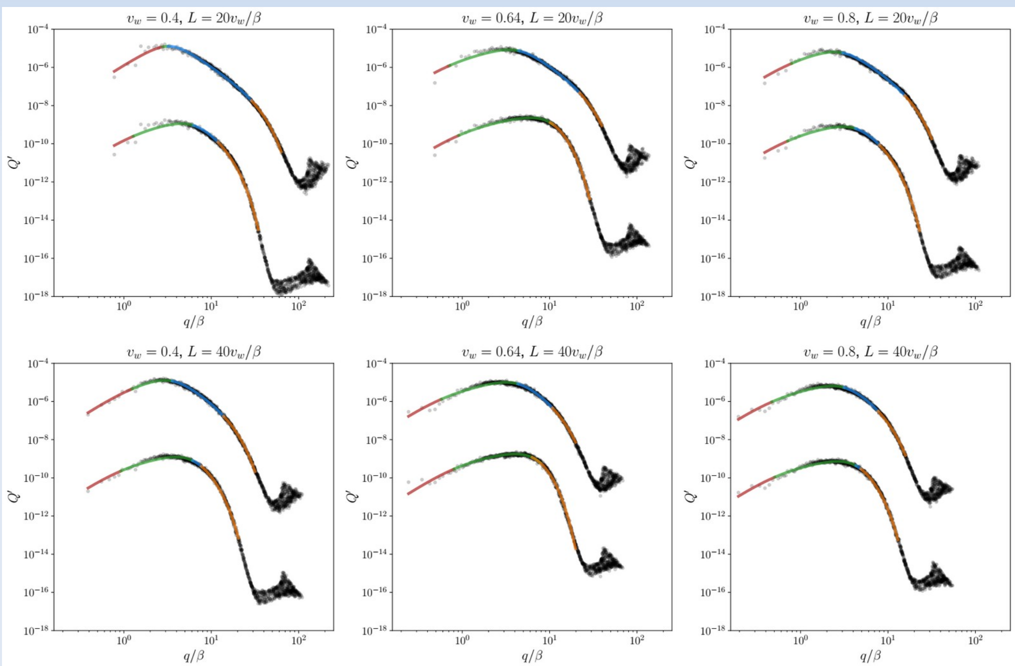


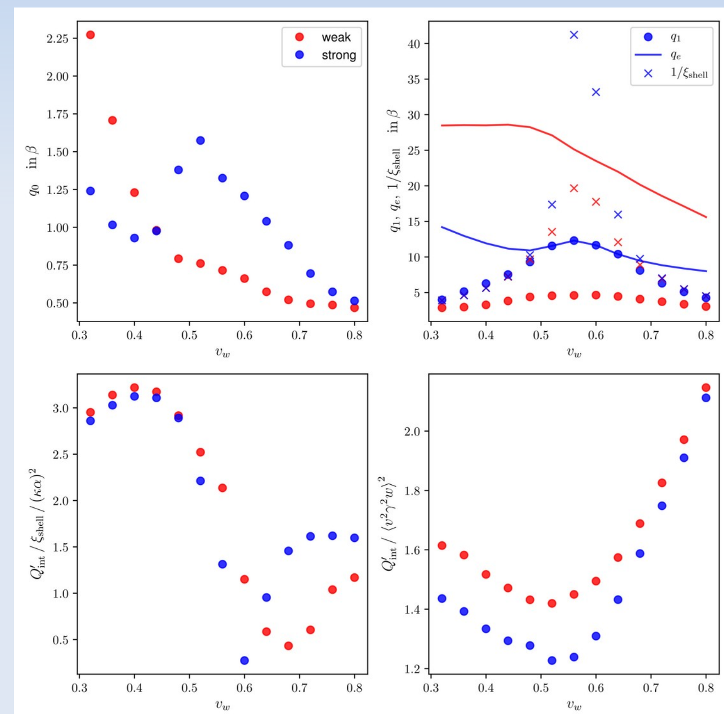


These simulations allow to extract GW spectra from the phase transition in a few hours instead of weeks (factor 2000 speed improvement compared to former approaches)

Simulation of cosmological phase transitions

The setup allows to run many simulations a day and to extract the GW spectra as functions of the PT properties: wall velocity v_w, PT strength α





The spectra have two features due to the bubble size and the shell thickness.

[Jinno, TK, Rubira, Stomberg 2022]

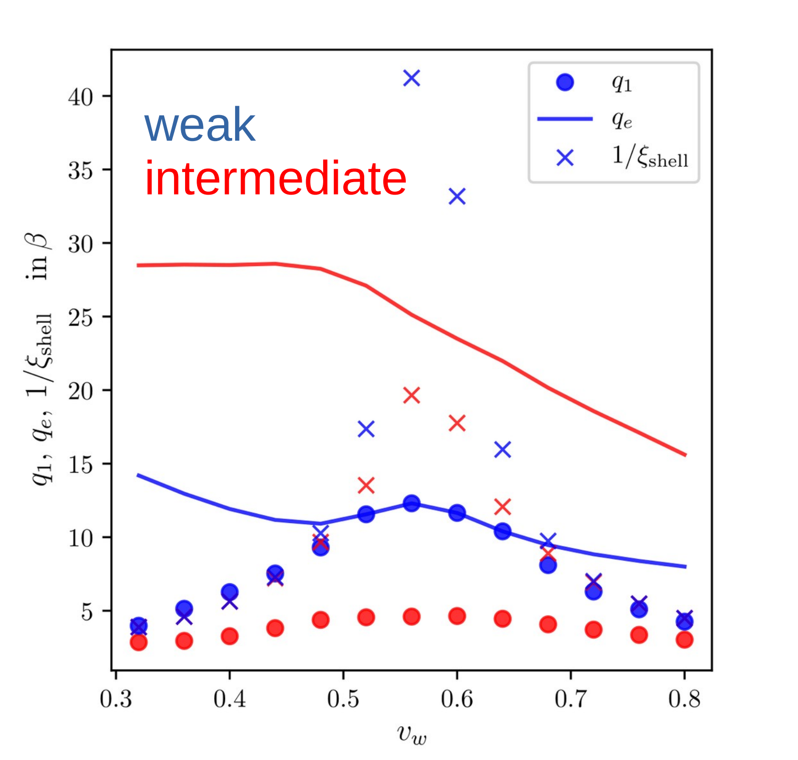
Some findings:

- Overall good agreement with all findings by the Helsinki-Sussex simluations

- some effects are not included by construction (slow down of the wall in some regions of parameter space)

- for weak phase transitions, the peak comes from the shell thickness in the spherical simulations

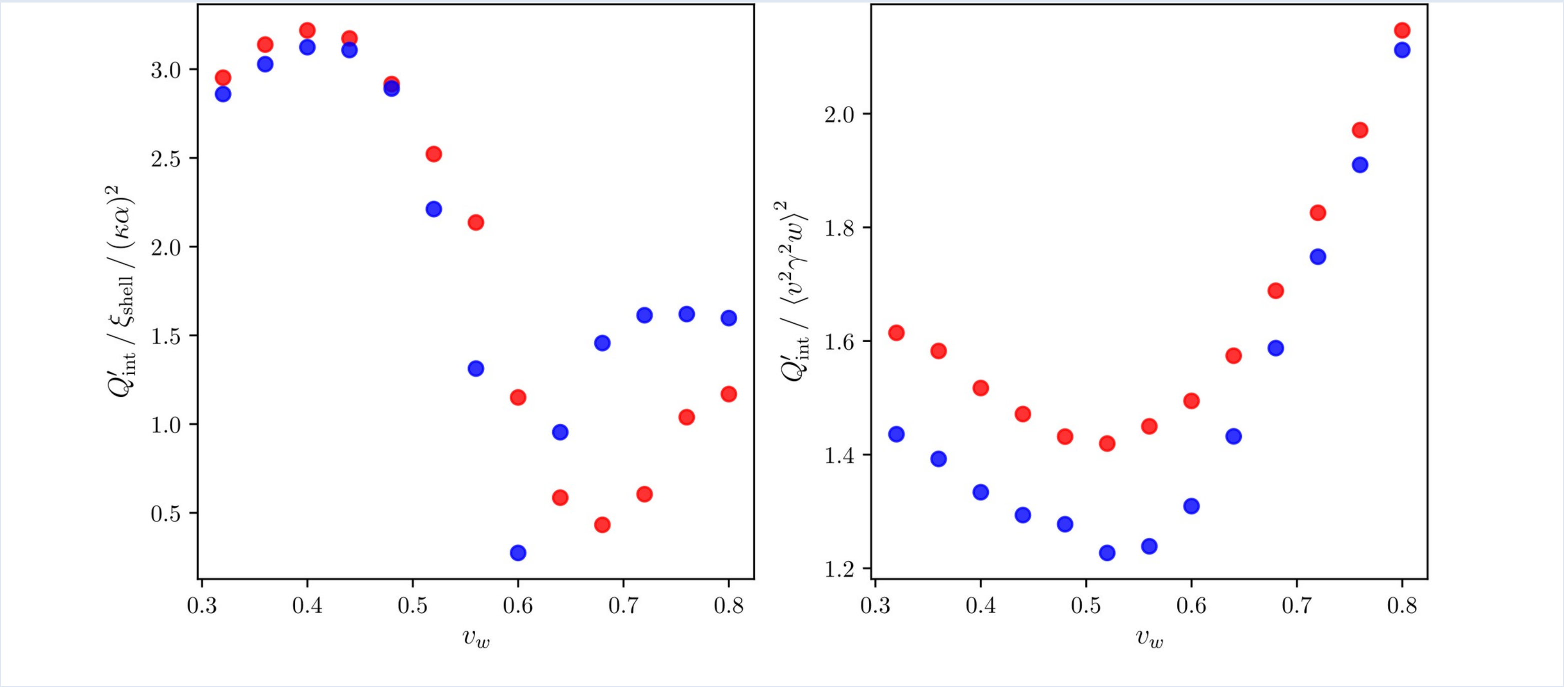
- for stronger phase transitions, it tends to be at lower freqencies (probably non-linearities in the first collision)



Some findings

- the integrated GW power spectrum relates well to the kinetic energy in the fluid (which is hard to estimate)

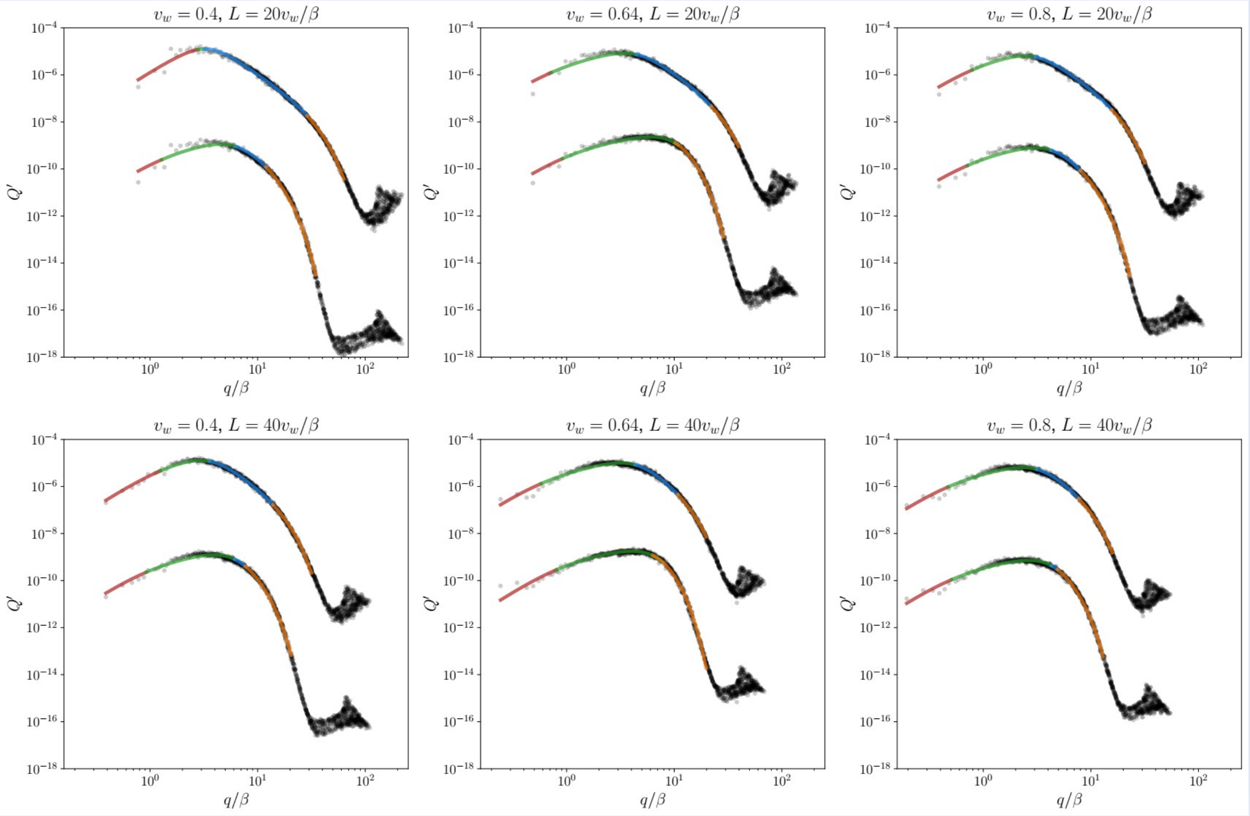
- less so for the kinetic energy in the spherical solutions (even including a fudge factor from the thickness)



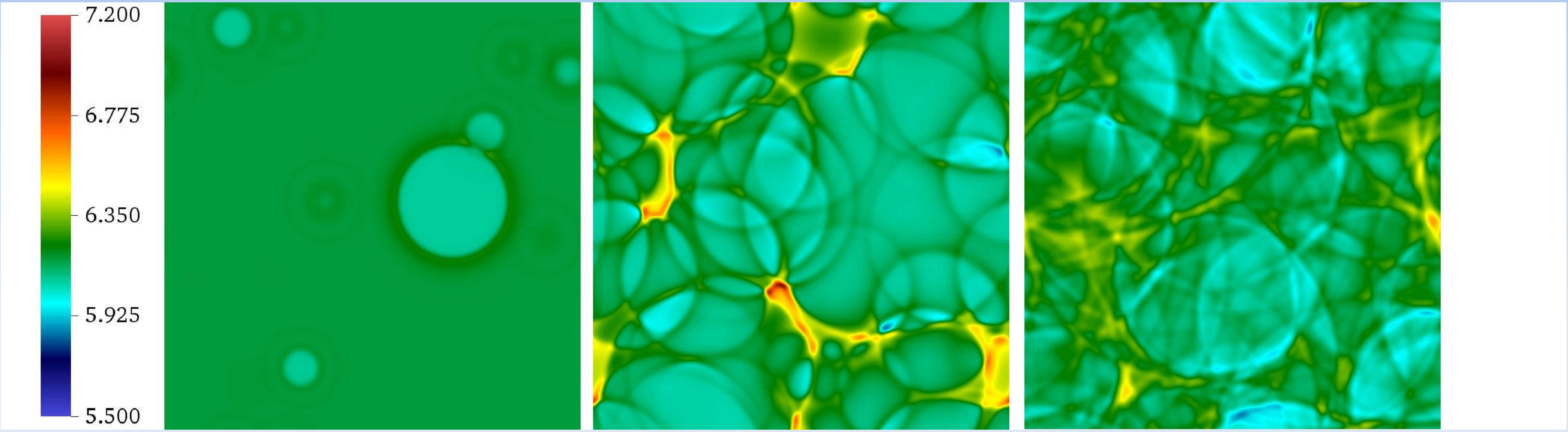
Some findings

- the intermediate and UV slode are definitely 1 and -3 in the simulations.

- the IR slope is harder to extract but there is no indication that it is much steeper than 3 (c.f. sound shell model)



Gravitational waves from cosmological phase transitions



I. Introduction
II. Simulations
III. Extrapolations

Model-dependence

The Weinberg master formula determines how stochastic gravitational waves are produced

$$\frac{dE_{GW}}{d\omega d\Omega} = 2G\omega^2 \Lambda_{ij,lm}(\hat{\mathbf{k}}) T^*_{ij}(\hat{\mathbf{k}}, \omega) T_{lm}(\hat{\mathbf{k}}, \omega),$$

And generally the energy fraction in GWs scales as

$$\Omega_{GW*}(f) \propto K^2$$

where **K** denotes the kinetic energy fraction in the fluid after the phase transition that is where the model-dependence will enter for most parts.

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rest of the talk

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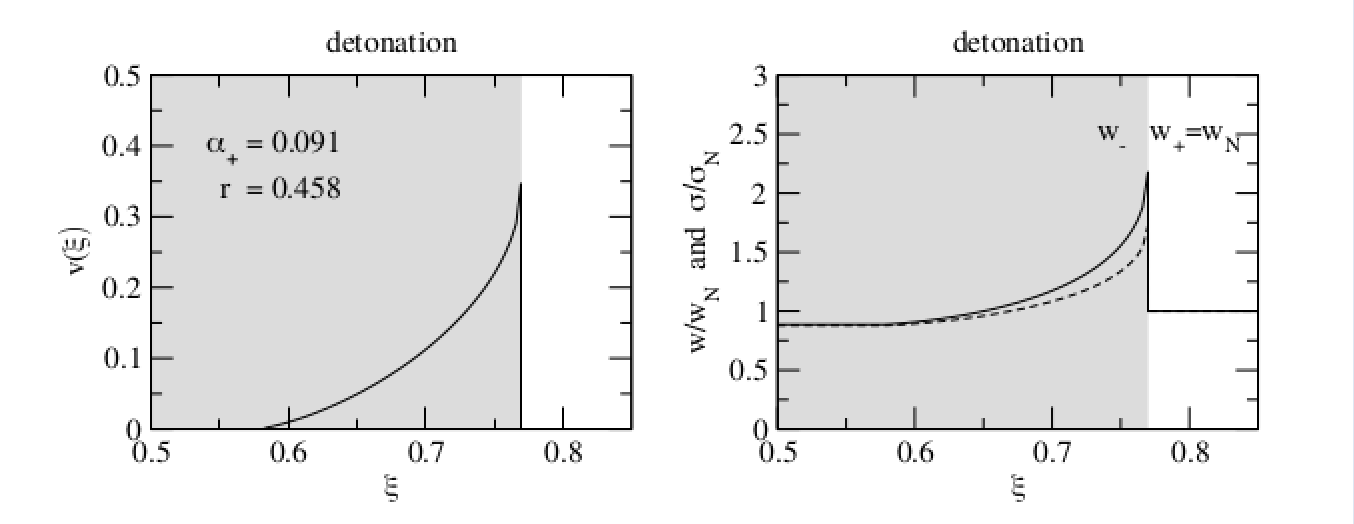
Kinetic energy

The bulk kinetic energy depends on the enthalpy *w* and the fluid velocity *v* and can be determined from an isolated spherical bubble before collision

$$K \equiv \frac{\rho_{kin}}{e}\,, \qquad \rho_{kin} = \frac{1}{V}\int dV\, v^2\gamma^2\, w\,.$$



$$\xi = r/t$$

Bag model

[Kosowsky, Turner , Watkins, ’92]
[Espinosa, TK, No, Servant ’20]

The kinetic energy fraction has been calculated in the bag model

$$p_s = \frac{1}{3} a_+ T^4 - \epsilon, \quad p_b = \frac{1}{3} a_- T^4 \, .$$

$$e_s = a_+ T^4 + \epsilon, \quad e_b = a_- T^4 \, ,$$

bag
constant

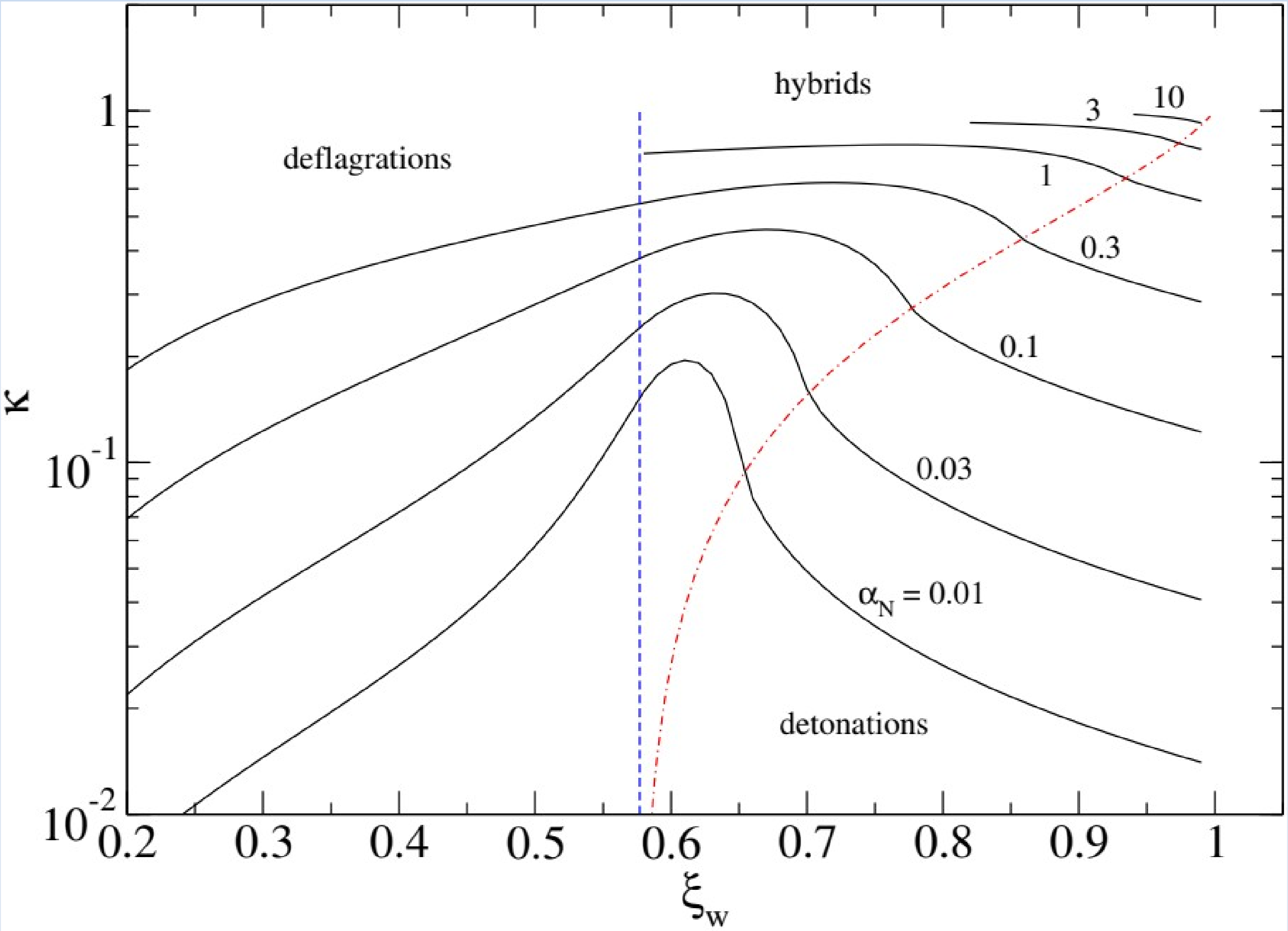
The strength of the phase transition is characterized by

$$\alpha = \frac{\epsilon}{a_+ T^4}$$

Kinetic energy fraction and efficiency coefficient

[Espinosa, TK, No, Servant '10]

$$K = \frac{\alpha}{\alpha+1}\kappa$$



wall velocity

How to match to other models?

Fitting functions of these results are used in phenomenological analysis but what is the strength parameter in a general models? In particular if only quantities at nucleation temperature are used?

$$DX = (X_s(T_n) - X_b(T_n))$$

$\alpha \propto Dp$ If the pressure difference vanishes, the bubble becomes static

$\alpha \propto De$ The energy difference fuels the kinetic motion of the bulk fluid

$\alpha \propto D\theta \propto (De - 3Dp)$ The trace difference is the bag constant in the bag model

A model comparison

[Giese, TK, van de Vis '20]

model/method	M1	M2	M3	M4	M5	M6
SM_1	0.00143		4.99 %	3.55 %	-88.45 %	713.34 %
SM_2	0.00401		1.70 %	-0.72 %	-66.69 %	351.90 %
SM_3	0.00014		1.37 %	0.94 %	-89.16 %	779.35 %
SM_4	0.00039		0.42 %	-0.32 %	-67.85 %	405.11 %
$2step_1$	0.00036		13.61 %	17.39 %	-89.52 %	945.17 %
$2step_2$	0.00563		15.68 %	21.90 %	-50.01 %	366.20 %
$2step_3$	0.00070		35.97 %	47.28 %	-89.85 %	1235.34 %
$2step_4$	0.01576		40.05 %	58.29 %	-41.80 %	485.16 %

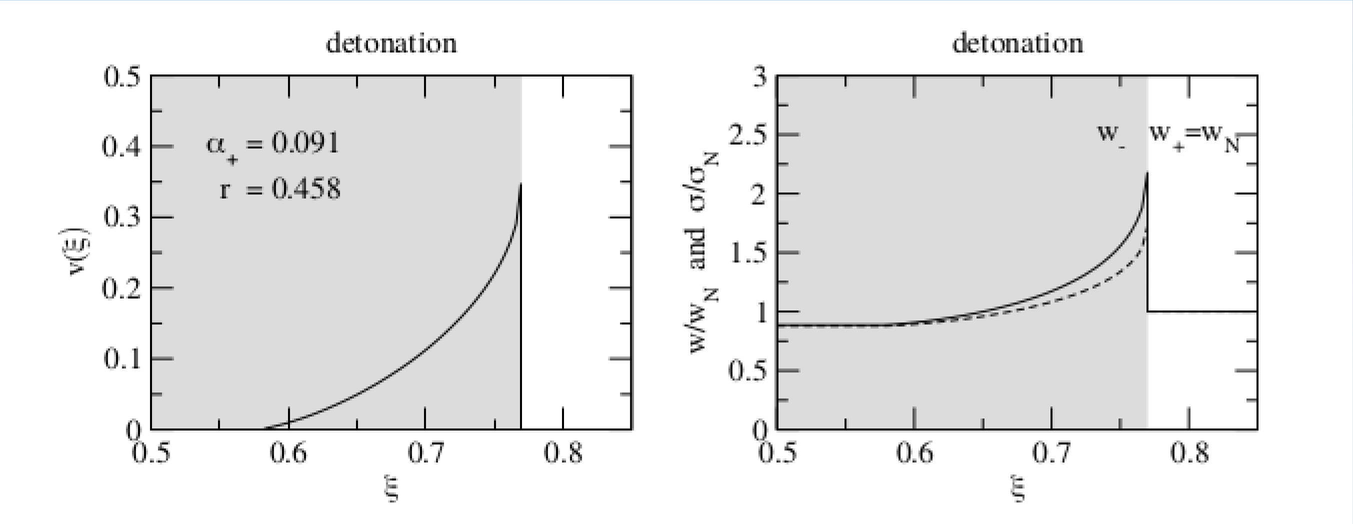
Table 4: Relative errors of the methods M2-M6 compared to the fully numerical result M1. The model parameters are given in Table 1 and 2 and a wall velocity of $\xi_w = 0.9$ was used.

$\Delta\theta$ $\Delta\theta$ Δp Δe

new approach

methods used in the literature

The matching equation



The matching equations result from the fact that the energy-momentum tensor is conserved across the bubble wall:

$$\frac{v_+}{v_-} = \frac{e_b(T_-) + p_s(T_+)}{e_s(T_+) + p_b(T_-)}, \; v_+ v_- = \frac{p_s(T_+) - p_b(T_-)}{e_s(T_+) - e_b(T_-)},$$

These equations determine T_- and v_- as functions of $v_+ = v_w$ and $T_+ = T_{nucleation}$

The matching equation

[Giese, TK, van de Vis '20]

The temperature T- can be eliminated using

$$\frac{p_b(T_+) - p_b(T_-)}{e_b(T_+) - e_b(T_-)} \simeq \left.\frac{dp_b/dT}{de_b/dT}\right|_{T_n} \equiv c_s^2 \,.$$

This then leads to

$$\frac{v_+}{v_-} \simeq \frac{(v_+ v_-/c_s^2 - 1) + (De - Dp/c_s^2)/w_+}{(v_+ v_-/c_s^2 - 1) + v_+ v_-(De - Dp/c_s^2)/w_+} \,.$$

$$DX = (X_s(T_n) - X_b(T_n))$$

This motivates the following definition of the strength parameter in terms of the *pseudotrace*

$$\bar{\theta} \equiv e - p/c_s^2 \,, \quad \alpha_{\bar{\theta}} \equiv \frac{D\bar{\theta}}{3w_+} \,,$$

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$$\bar\theta \equiv e - p/c_s^2 \, , \quad \alpha_{\bar\theta} \equiv \frac{D\bar\theta}{3 w_+} \, ,$$

K should only depend on these two quantities!

A sound argument to go beyond the bag model

[Leitao and Megevand '14] ν-model

$$p_s = \frac{1}{3} a_+ T^4 - \epsilon \,, \qquad e_s = a_+ T^4 + \epsilon \,, \qquad c_s^2 = \frac{1}{\nu - 1}$$

$$p_b = \frac{1}{3} a_- T^\nu \,, \qquad e_b = \frac{1}{3} a_- (\nu - 1) T^\nu \,,$$

model/method	M1	M2
SM_1	0.00143	0.45 %
SM_2	0.00401	0.43 %
SM_3	0.00014	0.04 %
SM_4	0.00039	0.04 %
$2step_1$	0.00036	-0.21 %
$2step_2$	0.00563	-0.80 %
$2step_3$	0.00070	-0.77 %
$2step_4$	0.01576	-3.52 %

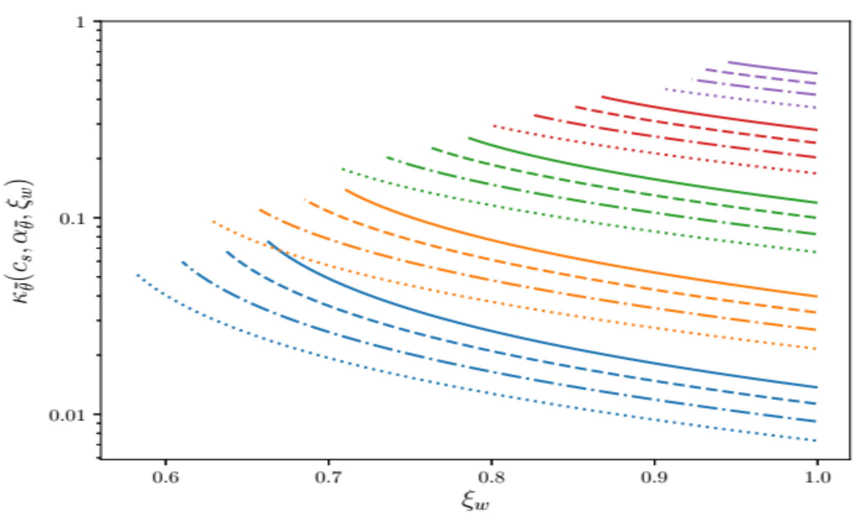

Table 4: Relative errors of the methods M2-M6 compared to the fully numerical result M1. The model parameters are given in Table 1 and 2 and a wall velocity of $\xi_w = 0.9$ was used.

Coding the kinetic energy fraction

```
import numpy as np
from scipy.integrate import odeint
from scipy.integrate import simps

def kappaNuModel(cs2,al,vp):
  nu = 1./cs2+1.
  tmp = 1.-3.*al+vp**2*(1./cs2+3.*al)
  disc = 4*vp**2*(1.-nu)+tmp**2
  if disc<0:
    print("vp too small for detonation")
    return 0
  vm = (tmp+np.sqrt(disc))/2/(nu-1.)/vp
  wm = (-1.+3.*al+(vp/vm)*(-1.+nu+3.*al))
  wm /= (-1.+nu-vp/vm)

  def dfdv(xiw, v, nu):
    xi, w = xiw
    dxidv = (((xi-v)/(1.-xi*v))**2*(nu-1.)-1.)
    dxidv *= (1.-v*xi)*xi/2./v/(1.-v**2)
    dwdv = nu*(xi-v)/(1.-xi*v)*w/(1.-v**2)
    return [dxidv,dwdv]

  n = 501 # change accuracy here
  vs = np.linspace((vp-vm)/(1.-vp*vm), 0, n)
  sol = odeint(dfdv, [vp,1.], vs, args=(nu,))
  xis, ws = (sol[:,0],-sol[:,1]*wm/al*4./vp**3)

  return simps(ws*(xis*vs)**2/(1.-vs**2), xis)
```

Table 5: Python code to calculate $\kappa_{\bar\theta}$ in the ν-model as a function of the speed of sound squared c_s^2, the strength of the phase transition $\alpha_{\bar\theta}$ and the wall velocity ξ_w.

Summary I

The observation of Gravitational Waves started a new era in astro physics.

The main appeal of these observations in cosmology is that one can probe the era before electromagnetic decoupling.

In principle, laser interferometers as LISA/LIGO/DECIGO allow to test phase transitions (and hence particle physics) from EW scales up to very high scales ~ 10^6 GeV.

LISA will fly in the 2030s and cover a large range of cosmological phase transitions in terms of strength and temperatures close to electroweak scales.

Summary II

Most robust predictions for GWs from PTs come from simulations and Higgsless simulations are very cost efficient.

To extrapolate the results from hydrodynamic simulations to other models one needs the energy fraction of a single expanding bubble.

In the literature this is typically done by matching the bag model where the energy fraction is known (as a fit). This leads to errors of order O(1) or O(10).

A model-independent approach suggests to use the *speed-of-sound* in the broken phase and the *pseudo-trace* as the strength parameter of the matching. This reduces the error to O(few %).