Thermal effects in Ising Cosmology

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18/10/2023

Based on Eur.Phys. J. C 83 (2023) 431 and Universe 2023, 9, 434 (Nikos Irges and Antonis Kalogirou)

Norway grants

Understanding the Early Universe:
interplay of theory and collider experiments

Joint research project between the University of Warsaw & University of Bergen

The research leading to the results presented in this talk has received funding from the Norwegian Financial Mechanism for years 2014-2021, grant nr 2019/34/H/ST2/00707

WYDZIAŁ FIZYKI UNIWERSYTET WARSZAWSKI

HR EXCELLENCE IN RESEARCH

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CONTENTS

INTRO

- Inflation:

1) An elegant explanation for the homogeneity and isotropy of the Universe

2) A causal mechanism to generate the inhomogeneities

- The anisotropies leave their imprints on CMB

- Precise measurements of CMB's anisotropies and spectral index provide a test-playground for inflation

Planck Collaboration, Y. Akrami et al., Astron. Astrophys. 641 (2020)

INTRO

- Inflation works very well for a slowly rolling scalar field with

$$\dot{\phi}^2 < V(\phi)$$

$$n_{\rm S} \simeq (1 - 4\,\epsilon_{\rm H} + 2\,\delta_{\rm H})$$

- For a (not so simple) choice of potential and a (not so) small amount of fine tuning (in ϵ_H and δ_H) n_S matches its experimental value

0.9649 ± 0.0042 at 68 % *CL*

Planck Collaboration, Y. Akrami et al., Astron. Astrophys. 641 (2020)

INTRO

- What about the simplest monomial Inflaton potential?

- Our proposal

Free massive scalar field and its thermal evolution under the dS/CFT correspondence

- The FLRW metric

$$ds^2 = a^2\left(d\tau^2 - d\mathbf{x}^2\right)$$

$$a = -\frac{1}{H\tau}$$

$$H = \frac{\dot{a}}{a^2}$$

For the expanding Poincare patch of de Sitter

- QFT in dS space:

Conformally flat metric with a time-like coordinate $\tau \in (-\infty, 0]$

$$ds^2 = a^2\left(d\tau^2 - d\mathbf{x}^2\right) \quad \text{and} \quad \alpha(\tau) = -\frac{1}{H\tau}$$

$|in\rangle$ *vacuum (observer) defined at* $\tau \to -\infty$

$|out\rangle$ *vacuum (observer) defined at the horizon* $\tau = 0$

$$\langle J | \, \mathbf{\Phi}^I = \langle I | \, \mathbf{\Phi}^J$$

Bogolyubov Transformation

$$I, J = \text{in}, \text{out}$$

- The $\tau = 0$ surface is also called the Horizon of the expanding Poincare patch of dS space

THERMAL PROPAGATORS

- The action to be quantized under thermal effects

$$\mathcal{S} = \int d^4x \sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}(m^2 + \xi\mathcal{R})\phi^2\right]$$

- Consider a $d+1$ dimensional FLRW spacetime

$$ds^2 = a^2\left(d\tau^2 - d\mathbf{x}^2\right)$$

- Klein-Gordon equation for the mode $\phi_{\mathbf{k}} = \dfrac{\chi_{\mathbf{k}}}{\alpha}$

$$\boxed{\ddot{\chi}_{\mathbf{k}} + \omega^2_{|\mathbf{k}|}\chi_{\mathbf{k}} = 0}$$

$$\omega^2_{|\mathbf{k}|}(\tau) = |\mathbf{k}|^2 + m^2_{\mathrm{dS}}(\tau)$$

$$\boxed{m^2_{\mathrm{dS}}(\tau) = \frac{1}{\tau}\left(M^2 - \frac{d^2-1}{4}\right),\ M^2 = \frac{m^2}{H^2} + 12\xi}$$

THERMAL PROPAGATORS

- The solution is a combination of the Hankel functions $H^{1,2}_{\nu_{cl}}(\tau, |\mathbf{k}|)$ with weight

$$\nu_{\rm cl} = \frac{d}{2}\sqrt{1 - \frac{4M^2}{d^2}}$$

- Quantization includes time-dependent vacua and a doubled Hilbert space

- Time-dependent vacua

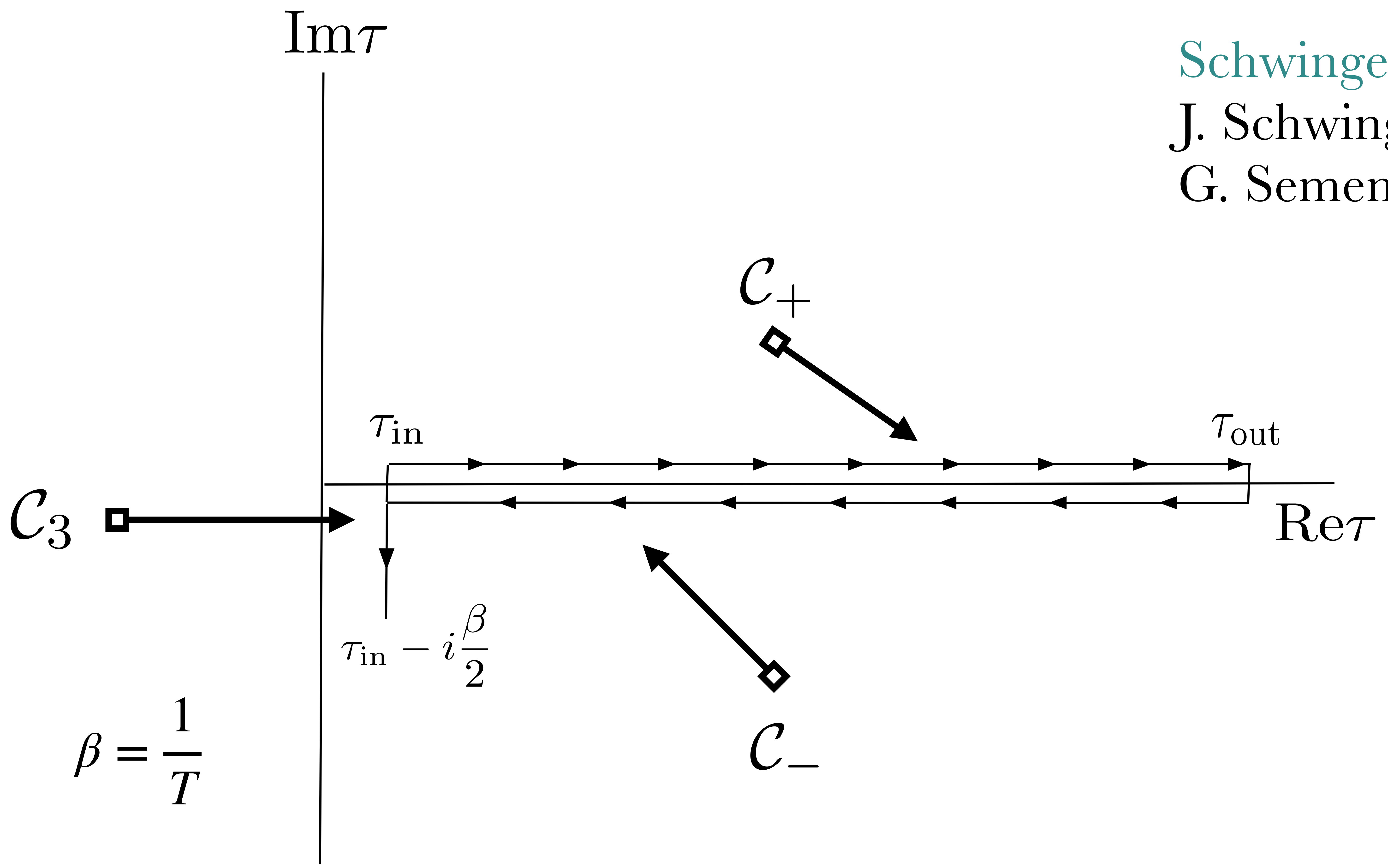
$|in\rangle$ vacuum is empty for the "in" observer
(Bunch-Davies vacuum)

$|out\rangle$ vacuum is empty for the "out" observer

N. A. Chernikov and E. A. Tagirov, '68
B. Allen, Phys. Rev. D32 (1985) 3136

THERMAL PROPAGATORS

- Doubled Hilbert space



Schwinger-Keldysh path integral
J. Schwinger, '61, L. V. Keldysh, '65
G. Semenoff and N. Weiss, '85

THERMAL PROPAGATORS

- The $T = 0$ ($C_3 = 0$) "in-in" field propagator

$$\mathcal{D} = \begin{array}{|c|c|} \hline \langle 0|\,\mathcal{T}[\Phi^+(\tau_1)\Phi^+(\tau_2)]\,|0\rangle \;=\; \mathcal{D}_{++}(\tau_1;\tau_2) & \langle 0|\,\Phi^-(\tau_1)\Phi^+(\tau_2)\,|0\rangle \;=\; \mathcal{D}_{+-}(\tau_1;\tau_2) \\ \hline \langle 0|\,\Phi^+(\tau_2)\Phi^-(\tau_1)\,|0\rangle \;=\; \mathcal{D}_{-+}(\tau_1;\tau_2) & \langle 0|\,\mathcal{T}^*[\Phi^-(\tau_1)\Phi^-(\tau_2)]\,|0\rangle \;=\; \mathcal{D}_{--}(\tau_1;\tau_2) \\ \hline \end{array}$$

$$\mathcal{D}_{-+}(\tau_1;\tau_2) = \chi_{|\mathbf{k}|}(\tau_1)\chi^*_{|\mathbf{k}|}(\tau_2)$$

$$\mathcal{D}_{+-}(\tau_1;\tau_2) = \mathcal{D}^*_{-+}(\tau_1;\tau_2),\; \mathcal{D}_{--}(\tau_1;\tau_2) = \mathcal{D}^*_{++}(\tau_1;\tau_2)$$

$$\mathcal{D}_{++}(\tau_1;\tau_2) = \theta(\tau_1-\tau_2)\mathcal{D}_{-+}(\tau_1;\tau_2) + \theta(\tau_2-\tau_1)\mathcal{D}_{+-}(\tau_1;\tau_2)$$

- The flat space limit propagator

$$\mathcal{D}_{++} = \frac{i}{k^2-m^2+i\varepsilon}$$

$i\varepsilon$ is hidden

THERMAL PROPAGATORS

- What about the “out-out” field propagator?

- Connected with the $C_3 \neq 0$ contour

- One should make sure that the KMS condition for thermal equilibrium is satisfied

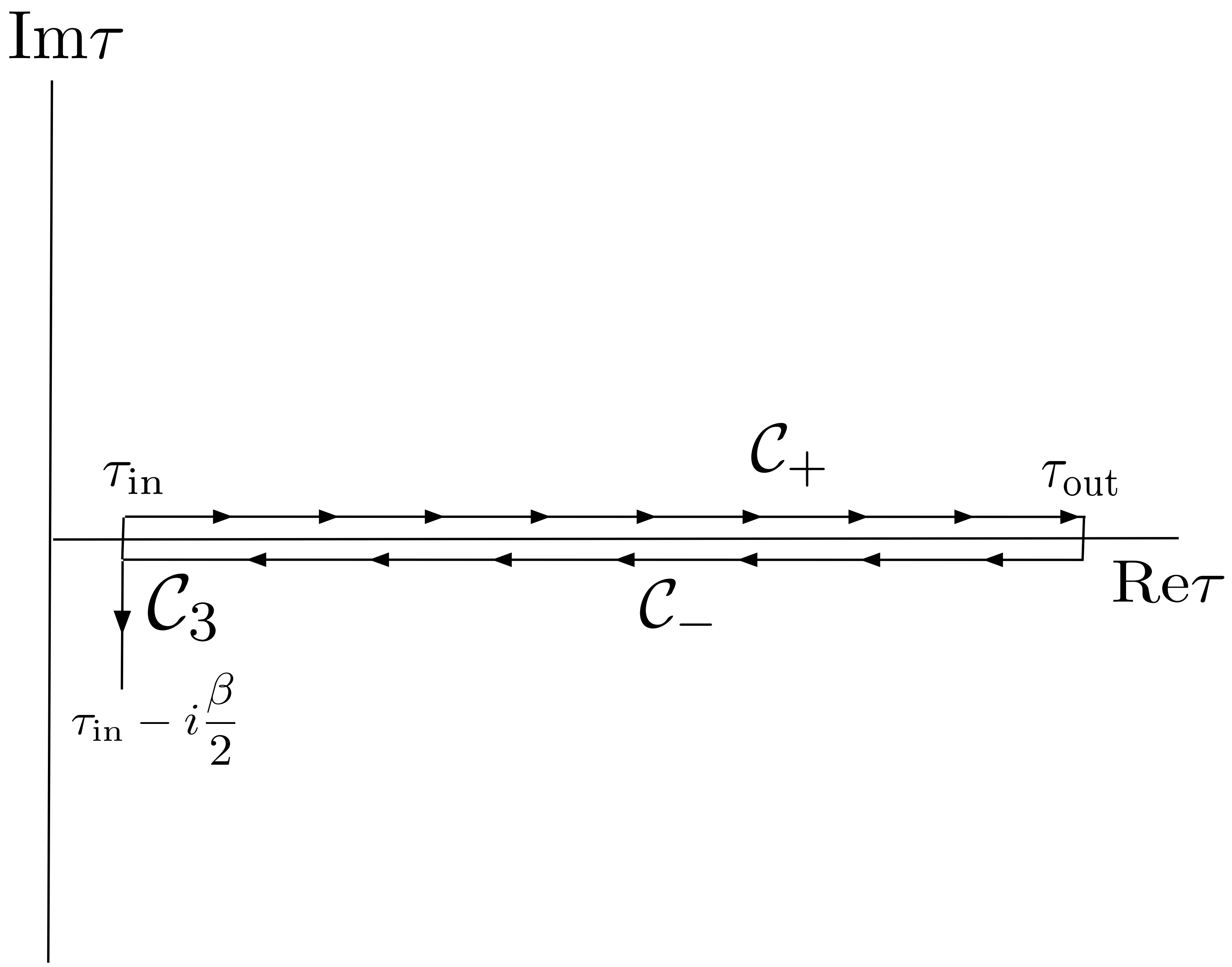
THERMAL PROPAGATORS

$\Phi^3(\tau)$ defined on C_3

$$\langle 0|\, \mathcal{T}[\Phi^3(\tau_1)\Phi^3(\tau_2)]\, |0\rangle = \mathcal{D}_{33}(\tau_1;\tau_2)$$

$$\langle 0|\, \Phi^+(\tau_1)\Phi^3(\tau_2)\, |0\rangle = \mathcal{D}_{3+}(\tau_1;\tau_2)$$

$$\langle 0|\, \Phi^-(\tau_1)\Phi^3(\tau_2)\, |0\rangle = \mathcal{D}_{3-}(\tau_1;\tau_2)$$



Satisfy the sewing conditions
for $a \in \{-,+,3\}$

$$\mathcal{D}_{a-}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{\text{in}}} = \mathcal{D}_{a3}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{\text{in}}} \qquad \frac{\partial}{\partial\tau_2}\mathcal{D}_{a-}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{\text{in}}} = \frac{\partial}{\partial\tau_2}\mathcal{D}_{a3}(\tau_1;\tau_2)\Big|_{\tau_2=\tau_{\text{in}}}$$

THERMAL PROPAGATORS

- The $T \neq 0$ ($C_3 \neq 0$) "out-out" field propagator

$$\mathcal{D}_{\beta/2} = \begin{array}{|c|c|} \hline \mathcal{D}_{++} + n_B(\beta/2)\,(\mathcal{D}_{++} + \mathcal{D}_{--}) & \mathcal{D}_{+-} + n_B(\beta/2)\,(\mathcal{D}_{++} + \mathcal{D}_{--}) \\ \hline \mathcal{D}_{-+} + n_B(\beta/2)\,(\mathcal{D}_{++} + \mathcal{D}_{--}) & \mathcal{D}_{--} + n_B(\beta/2)\,(\mathcal{D}_{++} + \mathcal{D}_{--}) \\ \hline \end{array}$$

- the Bose-Einstein distribution parameter

$$n_B(\beta) = \frac{e^{-\beta\omega_{|\mathbf{k}|}}}{1 - e^{-\beta\omega_{|\mathbf{k}|}}}$$

THERMAL PROPAGATORS

- Concrete $T \neq 0$ ($C_3 \neq 0$) "out-out" field propagator should respect the KMS condition

- Proof that Schwinger-Keldysh ≡ Thermofield dynamics for time-dependent Hamiltonian via "in-in"

$$\boxed{\mathcal{D}_\beta = U_\beta \, \mathcal{D} \, U_\beta^T}$$

$$\boxed{\beta = \frac{1}{T}}$$

$$\boxed{U_\beta = \begin{pmatrix} \cosh \theta_{|\mathbf{k}|} & \sinh \theta_{|\mathbf{k}|} \\ \sinh \theta_{|\mathbf{k}|} & \cosh \theta_{|\mathbf{k}|} \end{pmatrix}}$$

A Bogolyubov Transformation (BT) with coefficients

$$\cosh \theta_{|\mathbf{k}|} = \frac{1}{\sqrt{1 - e^{-\beta \omega_{|\mathbf{k}|}}}} \quad \textit{and} \quad \sinh \theta_{|\mathbf{k}|} = \sqrt{\cosh^2 \theta_{|\mathbf{k}|} - 1}$$

THERMAL PROPAGATORS

- All the allowed thermal transformations of $\mathcal{D}$ are correlators of the form

$$\mathcal{D}^{I}_{J,\alpha} = \langle J;\alpha|\, \mathcal{T}[\mathbf{\Phi}^{I}(\mathbf{\Phi}^{I})^{T}]\, |J;\alpha\rangle$$

- In Thermofield dynamics language $(\mathbf{\Phi}^{I})^{T} = (\Phi^{+,I}, \Phi^{-,I})$ with $I, J = \mathrm{in}, \mathrm{out}$. α is a thermal index

Exact dS space can only sustain the G-H temperature

$$\frac{1}{\beta} = T \to T_{\mathrm{dS}} = \frac{H}{2\pi}$$

- The thermal dS-scalar propagator admits an explicit and compact form

$$\boxed{\mathcal{D}_{\beta} = \mathcal{D} + \left(\mathcal{D}_{++} + \mathcal{D}^{*}_{++}\right)(s^2 + sc)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}}$$

$$s \equiv \sinh\theta_{|\mathbf{k}|}$$
$$c \equiv \cosh\theta_{|\mathbf{k}|}$$

THE COSMOLOGICAL OBSERVABLES

- The thermal power spectrum for $\tau_1 = \tau_2$, $H|\tau| = 1$ *and* $|\mathbf{k}\tau| \lesssim 1$

$$P_{S,\beta}\mathbf{1} = \mathcal{D}_\beta \mathbf{1}\big|_{\tau_1=\tau_2}$$

$\mathbf{1}$ *is the* 2×2 *matrix with unit elements*

$$\mathcal{D}_\beta = \mathcal{D} + \left(\mathcal{D}_{++} + \mathcal{D}^*_{++}\right)\left(s^2 + sc\right)\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$\boxed{\begin{aligned}\kappa \equiv \omega_{|\mathbf{k}|}|\tau|\Big|_{|\mathbf{k}\tau|=1} &= \sqrt{\left(|\mathbf{k}|^2 + m_{\mathrm{dS}}^2\right)|\tau|^2\Big|_{|\mathbf{k}\tau|=1}} \\ &= \sqrt{\frac{5-d^2}{4} + M^2}\end{aligned}}$$

- Defines the observables n_S, n_S', n_S'' and f_{NL}

- The picture

$$\mathcal{D} \longrightarrow \mathcal{D}_\beta \longleftrightarrow |\mathrm{in}> \longrightarrow |\mathrm{out}, \beta>$$

THE COSMOLOGICAL OBSERVABLES

- Time-independent BT

$|\,\mathrm{in}> \longrightarrow |\,\mathrm{out}, \beta_{T_{\mathrm{dS}}}> \longrightarrow$ *Exact de Sitter* $\longrightarrow d = 3, M^2 = 0 \to \kappa = i$

P. R. Anderson, C. Molina-Paris and Emil Mottola, Phys. Rev. D 72 (2005) 043515
P. R. Anderson and E. Mottola, Phys. Rev. D 89 (2014) 104038

$$P_S(\tau) = \left(\frac{H}{2\pi}\right)^2\left(1 + |\mathbf{k}\tau|\right) \Rightarrow P_S(0) = \left(\frac{H}{2\pi}\right)^2$$

scale invariant spectrum

- Time-dependent BT

$|\,\mathrm{in}> \longrightarrow |\,\mathrm{out}, \beta > \beta_{T_{\mathrm{dS}}}> \longrightarrow$ *Broken de Sitter* $\longrightarrow \Omega_{|\mathbf{k}|} = \omega_{|\mathbf{k}|}(|c|^2 + |s|^2)$

$$\kappa \to \boxed{\Lambda = \kappa\left(1 + 2\frac{e^{-2x\kappa}}{1 - e^{-2x\kappa}}\right) = \kappa\coth(x\kappa)}$$

THE COSMOLOGICAL OBSERVABLES

- $x = \dfrac{\pi H}{2\pi T}$, for $x \in [\pi, \infty)$. When $x = \pi$, admits its natural dS value where $T = T_{\rm dS}$

- The transformed state has reduced isometry than Bunch-Davies

$$\ddot{\phi} + 2\cancel{H}\dot{\phi} + \left(\mu_H^2 + \xi\frac{\mathcal{R}}{H^2}\right)a^2H^2\phi = 0\,, \qquad \dot{H} = \cancel{-\frac{1}{2a}\dot{\phi}^2} \quad H = \text{const}\,.$$

- The limiting cases

$x = \pi$	$x \gtrsim \pi$	$x \to \infty$
$T = T_{\rm dS}$, $\Lambda \to \infty$, $\kappa \to i$	$T < T_{\rm dS}$, $\Lambda \to$ finite $\kappa \to$ finite	$T \to 0$, $\Lambda \to 0$, $\kappa \to 0$
Scale invariance *(like symmetry restoration)*	*Scale invariance is slightly broken*	*Scale invariance* *(like symmetry restoration)*

THE COSMOLOGICAL OBSERVABLES

- The spectral index of scalar curvature fluctuations, n_S, is shifted due to finite temperature effects

$$P_{S,\beta} = P_S[1 + 2(s^2 + sc)]$$

- All the freedom is included in Λ which admits its natural value when $x \approx \pi$

$$n_{S,\beta} = 1 + \frac{d\ln\left(|\mathbf{k}|^3 P_{S,\beta}\right)}{d\ln|\mathbf{k}|}$$

$$\delta n_S \equiv n_{S,\beta} - 1 = \boxed{-\frac{2x}{\Lambda}\left[\frac{e^{-x\Lambda}}{1 - e^{-2x\Lambda}}\right]}$$

- The shift is extended to other observables

$$n_{S,\beta}^{(1)} = \delta n_S \left[2 - \frac{1}{\Lambda^2} - \frac{x}{\Lambda}\left(1 + \frac{2e^{-2x\Lambda}}{1 - e^{-2x\Lambda}}\right)\right]$$

$$n_{S,\beta}^{(1)} = \frac{d n_{S,\beta}}{d \ln |\mathbf{k}|}, \quad n_{S,\beta}^{(2)} = \frac{d n_{S,\beta}^{(1)}}{d \ln |\mathbf{k}|} \qquad f_{NL} = \frac{5}{6}\frac{N_{\rho\rho}}{N_\rho^2}$$

$$n_{S,\beta}^{(2)} = \frac{\left(n_{S,\beta}^{(1)}\right)^2}{\delta n_S} + \delta n_S \left[-\frac{2}{\Lambda^2} + \frac{2}{\Lambda^4} - \frac{x}{\Lambda}\left(2 - \frac{1}{\Lambda^2}\right)\left(1 + \frac{2e^{-2x\Lambda}}{1 - e^{-2x\Lambda}}\right) + \frac{4x^2}{\Lambda^2}\frac{e^{-2x\Lambda}}{(1 - e^{-2x\Lambda})^2}\right]$$

$$N_\rho = \frac{\partial N}{\partial \rho}, \; N_{\rho\rho} = \frac{\partial^2 N}{\partial \rho^2} \text{ and } \rho \equiv P_{S,\beta}$$

P. Creminelli and M. Zaldarriaga, JCAP 10 (2004) 006

A. Kehagias and A. Riotto, Nucl. Phys. B868 (2013) 577-595

$$f_{NL} = -\frac{5\left[x(-1+\Lambda^2)^2\left(1 + x\Lambda\cot(\frac{x\Lambda}{2})\right) + 2\Lambda^3\sinh(x\Lambda)\right]}{6\Lambda^2\left[x(-1+\Lambda^2) + \Lambda\sinh(x\Lambda)\right]}$$

THE COSMOLOGICAL OBSERVABLES

- The physical case $\Lambda \to 1.5117,\ x \to \pi$

$$\Lambda = \kappa \left(1 + 2\frac{e^{-2x\kappa}}{1 - e^{-2x\kappa}}\right) = \kappa \coth(x\kappa)$$

$n_{S,\beta} \equiv n_S \approx 1 - 0.036 = 0.964$
(0.9649 ± 0.0042)

$n^{(1)}_{S,\beta} \approx 0.0186$
(0.013 ± 0.012)

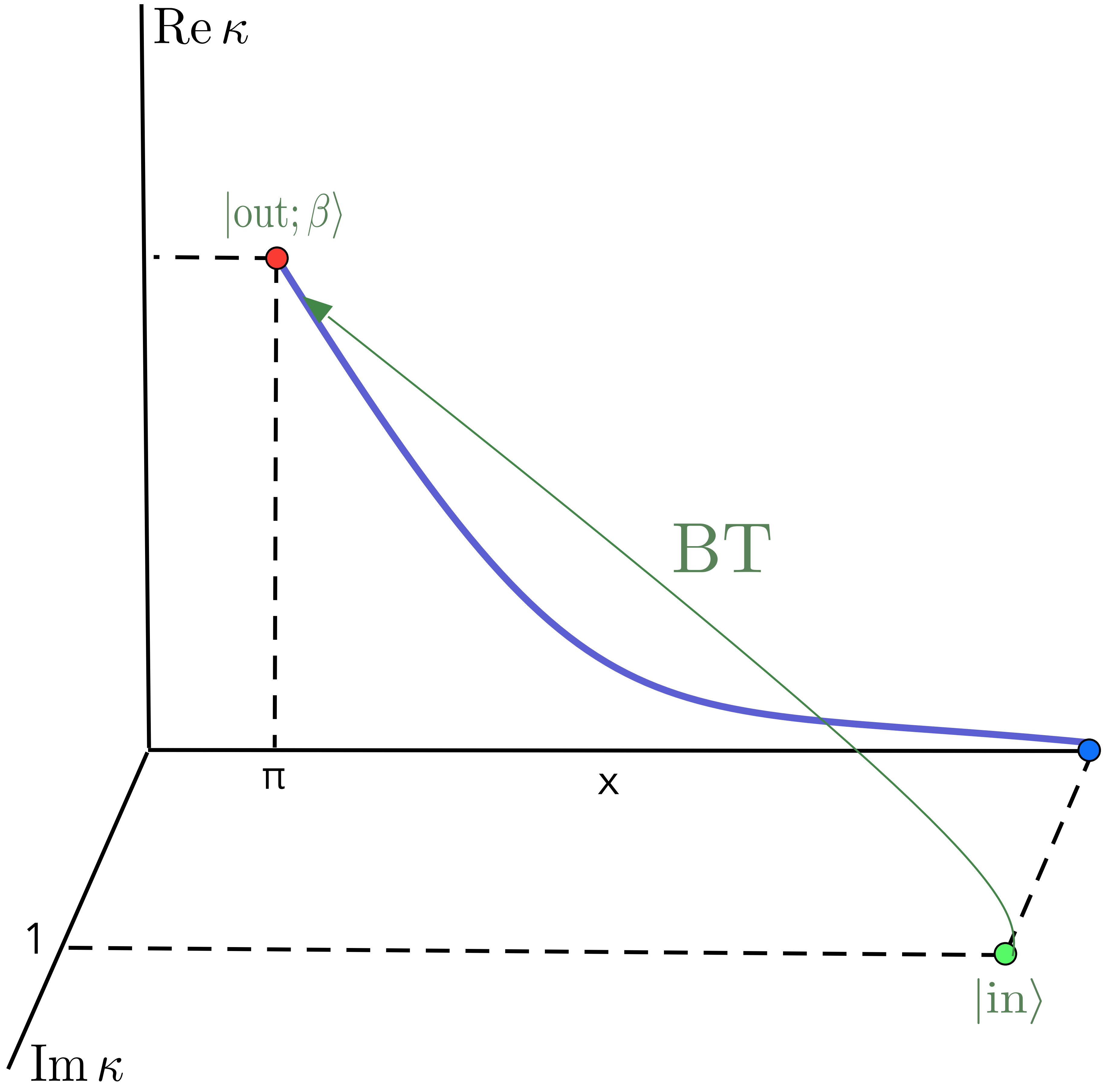
$f_{NL} \approx -1.7138$
(-0.9 ± 5.1)

?

$n^{(2)}_{S,\beta} \approx 0.1250$
(0.022 ± 0.012)

Λ	x
$\to 0$	$\to \infty$
10^{-6}	$3.5 \cdot 10^{7}$
0.01	1600
0.5	14.8
$\to 1.5117$	$\to \pi$

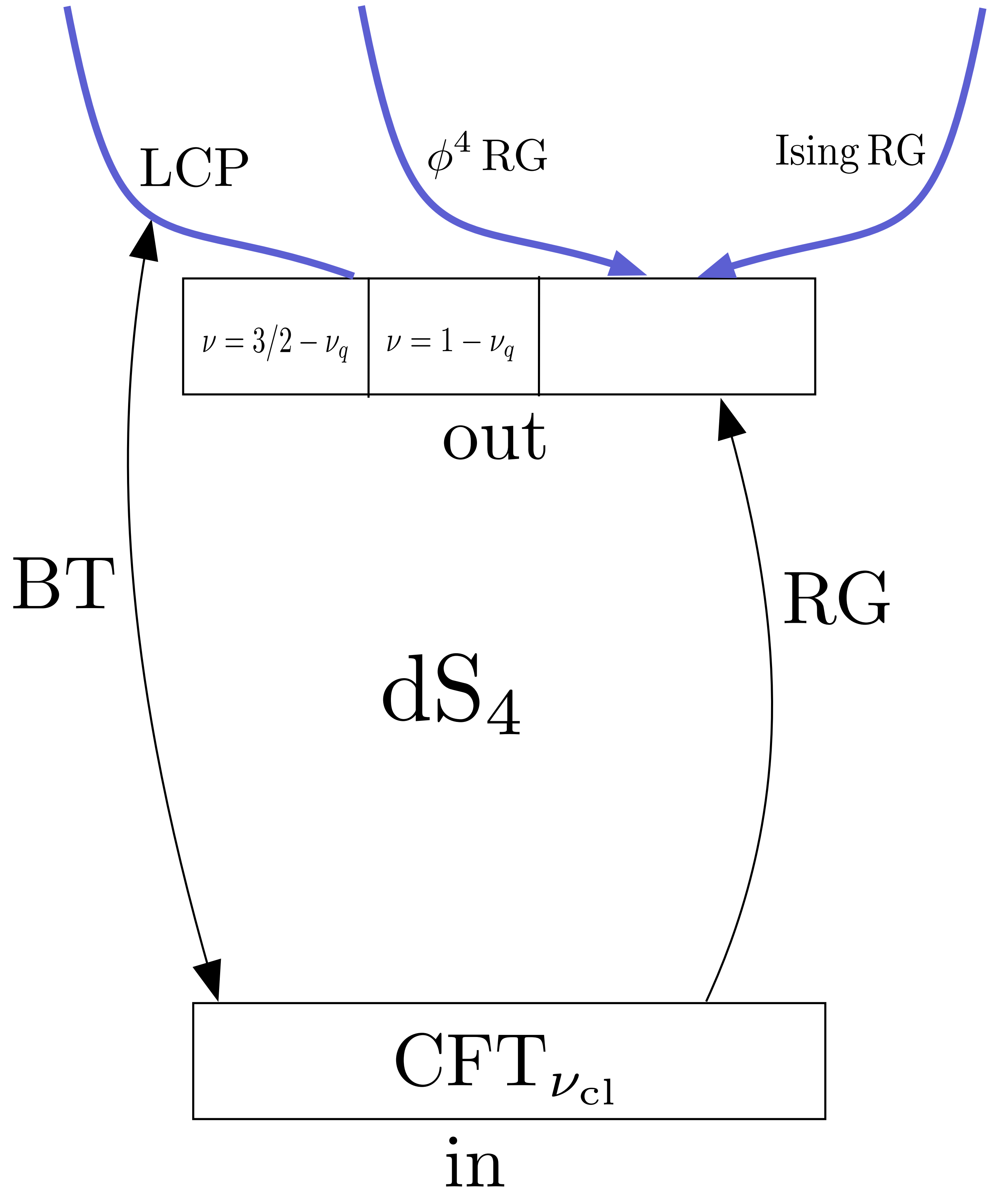
Planck Collaboration, Y. Akrami et al., Astron. Astrophys. 641 (2020)



SPECTRAL INDEX AND dS/CFT

- The dS/CFT correspondence

4d Bulk: $|\,\mathrm{in} >$ $\longrightarrow$ $|\,\mathrm{out}, \beta_{T_{\mathrm{dS}}} >$ $\longleftrightarrow$ 3d Boundary: UV to IR RG flow

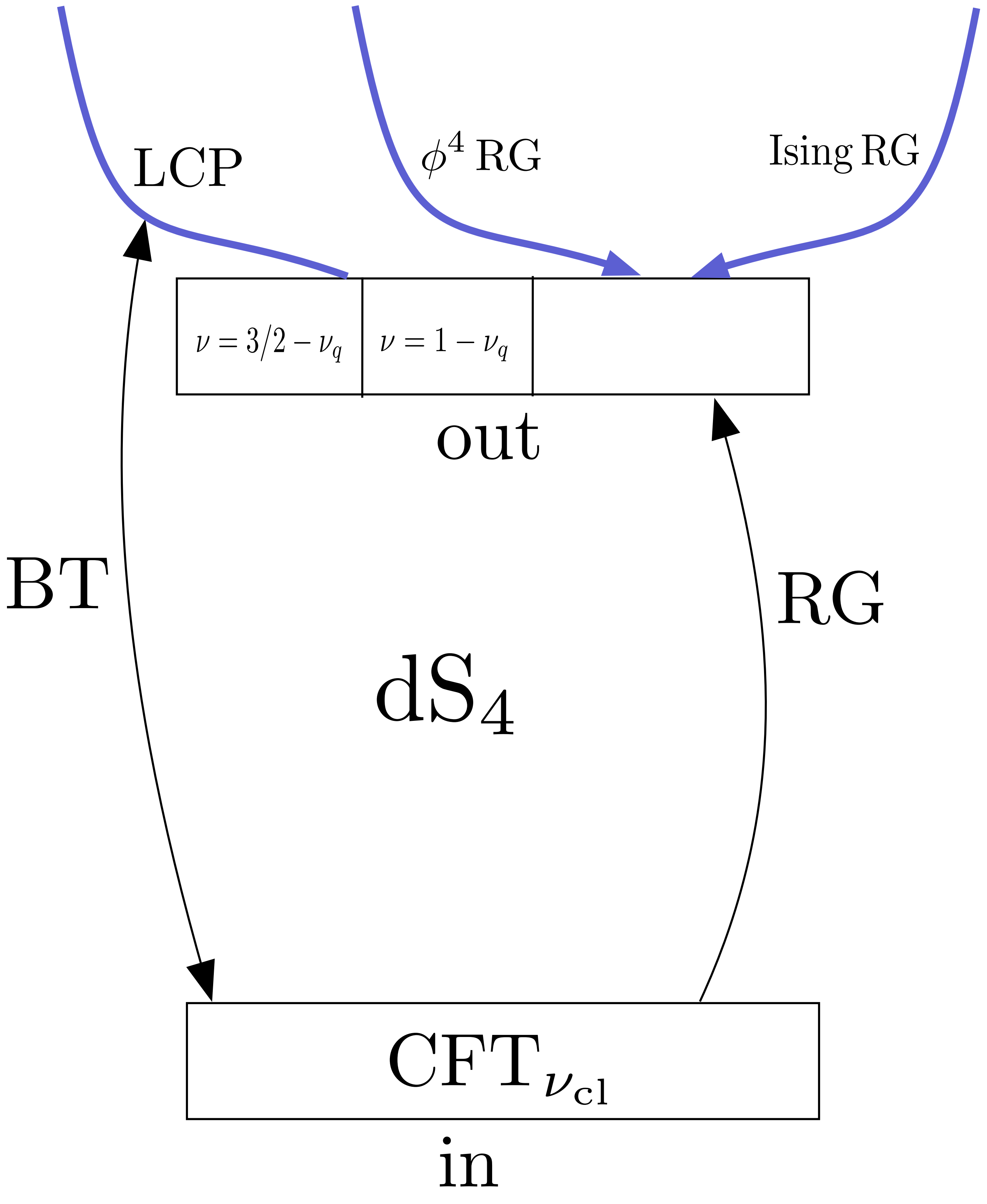


M. Bianchi, D.Z. Freedman and K. Skenderis Nucl. Phys. B 631 (2002) 159

I. Antoniadis, P. O. Mazur and E. Mottola, JCAP 09 (2012) 024

SPECTRAL INDEX AND dS/CFT

- The d = 3 scalar theory of the Ising field σ



$$\mathcal{L} = \frac{1}{2}\partial_i\sigma\partial_i\sigma - \lambda\sigma^4$$

The IR limit is a exact 3d CFT as long as the BT preserves the $SO(4)$ isometry.

Scale invariance is broken via the Coleman-Weinberg mechanism. RG brings us in the vicinity of the IR fixed point

$\updownarrow$

$|\mathrm{out};\beta\rangle$ with $\beta > \beta_{\mathrm{dS}}$

SPECTRAL INDEX AND dS/CFT

- In the dS/CFT correspondence: bulk field ϕ (Δ_-) dual to a boundary operator $\mathcal{O}$ (Δ_+)

$$\Delta_- = \frac{d}{2} - \nu \qquad\qquad \Delta_+ = \frac{d}{2} + \nu$$

$$(\Delta_-, \Delta_+)_{\mathrm{cl}} = (0, 3)$$

- Bulk and boundary propagators are related by

$$\langle \phi_{|\mathbf{k}|} \phi_{-|\mathbf{k}|} \rangle \sim \frac{1}{\langle \mathcal{O}_{|\mathbf{k}|} \mathcal{O}_{-|\mathbf{k}|} \rangle}$$

J. Maldacena, J. High Energy Phys. 05 (2003) 013
J. M. Maldacena and G. L. Pimentel, JHEP 09 (2011) 045

SPECTRAL INDEX AND dS/CFT

- There is a gauge of the metric where $\zeta_{\mathbf{k}} = z(\tau)\phi_{\mathbf{k}}$

$$< \zeta_{\mathbf{k}}\zeta_{\mathbf{k}} > \sim < \phi_{\mathbf{k}}\phi_{\mathbf{k}} >$$

J. Maldacena, J. High Energy Phys. 05 (2003) 013
J. M. Maldacena and G. L. Pimentel, JHEP 09 (2011) 045

- Then bulk and boundary propagator connection reforms to

$$\boxed{\langle \zeta_{|\mathbf{k}|}\zeta_{-|\mathbf{k}|}\rangle \sim \frac{1}{\langle \mathcal{O}_{|\mathbf{k}|}\mathcal{O}_{-|\mathbf{k}|}\rangle}}$$

SPECTRAL INDEX AND dS/CFT

- The spectral index in holography

$$
\begin{aligned}
n_S - 1 &= \frac{d}{d\ln|\mathbf{k}|}\left[\ln\left(|\mathbf{k}|^3 P_{S,\beta}\right)\right] = \frac{d}{d\ln|\mathbf{k}|}\ln\left(|\mathbf{k}|^3\langle\zeta_{|\mathbf{k}|}\zeta_{-|\mathbf{k}|}\rangle\right) \\
&= 3 - \frac{1}{\langle\mathcal{O}_{|\mathbf{k}|}\mathcal{O}_{-|\mathbf{k}|}\rangle}\left(\frac{d}{d\ln|\mathbf{k}|}\langle\mathcal{O}_{|\mathbf{k}|}\mathcal{O}_{-|\mathbf{k}|}\rangle\right)
\end{aligned}
$$

- The Callan-Symanzik

$$
\left(\frac{\partial}{\partial\ln|\mathbf{k}|} - \beta_\lambda\frac{\partial}{\partial\lambda} + (3-2\Delta_{\mathcal{O}})\right)\langle\mathcal{O}_{|\mathbf{k}|}\mathcal{O}_{-|\mathbf{k}|}\rangle = 0
$$

$$
\Delta_{\mathcal{O}} = \Delta_+ = [\Delta_{\mathcal{O}}] + \Gamma_{\mathcal{O}} \text{ and } \beta_\lambda \equiv \mu\frac{\partial\lambda}{\partial\mu}
$$

$$
\boxed{n_S = 1 - 2\Gamma_{\mathcal{O}} - \beta_\lambda\frac{\partial}{\partial\lambda}\ln\langle\mathcal{O}_{|\mathbf{k}|}\mathcal{O}_{-|\mathbf{k}|}\rangle}
$$

F. Larsen and R. McNees, JHEP 07 (2003) 051
J. P. van der Schaar, JHEP 01 (2004) 070

SPECTRAL INDEX AND dS/CFT

- The anomalous dimension

$$\boxed{\gamma_{\mathcal{O}} \equiv \mu \frac{\partial}{\partial \mu} \ln z_{\mathcal{O}}}$$

$$\boxed{\gamma_\sigma \equiv \frac{1}{2}\mu \frac{\partial}{\partial \mu} \ln Z_\sigma}$$

$$\boxed{\Gamma_{\mathcal{O}} = -\gamma_{\mathcal{O}} + 2\gamma_\sigma}$$

- For $\mathcal{O} = \sigma^4$ ($\Delta_{\sigma^4} = 3$) no shift to the spectral index

$$n_S = 1 - 2\Gamma_{\sigma^4} = 1 - 2\frac{\partial \beta_\lambda}{\partial \lambda}$$

F. Larsen and R. McNees, JHEP 07 (2003) 051
J. P. van der Schaar, JHEP 01 (2004) 070

- For us $\mathcal{O} = \Theta \equiv \delta^{ij} T_{ij}$ ($\Delta_\Theta = 3$) and the spectral index

a conserved current

$$\boxed{\Gamma_\Theta = 0} \longrightarrow \quad n_S = 1 + \frac{\partial}{\partial \ln \mu} \ln \langle \Theta(x_1)\Theta(x_2)\rangle = 1 - \beta_\lambda \frac{\partial}{\partial \lambda} \ln \langle \Theta(x_1)\Theta(x_2)\rangle$$

SPECTRAL INDEX AND dS/CFT

- Vanishing of the anomalous dimension of Θ

$$\langle \Theta\Theta \rangle = c_\Theta / |x|^{2d}$$

K. G. Wilson, Phys. Rev. 179, 1499
S. E. Derkachov, J.A. Gracey, A.N. Manashov, Eur. Phys. J. C2 569-579
C. Coriano, L. Delle Rose and K. Skenderis, Eur. Phys. J. C 81 2, 174
J. Henriksson, ArXiv: 2201.09520

- Non-trivial vanishing for $\Gamma_\Theta = 0 \rightarrow \gamma_\Theta = 2\gamma_\sigma$

A sunset-like diagram with $\sigma \Box \sigma$ insertion cancels the usual sunset

- Near the IR Wilson-Fisher fixed point $2\gamma_\sigma = \eta$ the Ising field critical exponent

SPECTRAL INDEX AND dS/CFT

- Rewrite the Callan- Symanzik equation for Θ and $\Gamma_\Theta = \eta - \eta$

$$\boxed{\left[\left(\frac{\partial}{\partial \ln \mu} + \eta\right) + \left(\beta_\lambda \frac{\partial}{\partial \lambda} - \eta\right)\right] \langle \Theta(x_1) \Theta(x_2) \rangle \simeq 0}$$

- Very close to the IR Wilson-Fisher fixed point $\beta_\lambda^2 << \dfrac{\partial \beta_\lambda}{\partial \lambda}$

$$\left(\beta_\lambda \frac{\partial}{\partial \lambda} - \eta\right) \langle \Theta(x_1) \Theta(x_2) \rangle \simeq 0$$

- The c_Θ-coupling satisfies the scaling equation

$$\beta_\lambda \partial_\lambda c_\Theta = \eta c_\Theta$$

Meaningful only outside the IR fixed point

SPECTRAL INDEX AND dS/CFT

- The approximate conformal 2-point function

$$\langle \Theta\Theta \rangle = c_\Theta / |x|^{2d} \qquad\qquad c_\Theta \sim \left(\frac{16\pi^2 - 3\lambda}{\lambda} \right)^\eta$$

- The critical exponent η non-perturbatively admits the numerical value $\eta \approx 0.036$ (MC simulation)

$$n_S = 1 - \beta_\lambda \frac{\partial}{\partial \lambda} \ln \langle \Theta(x_1)\Theta(x_2) \rangle$$

$$\boxed{n_S \simeq 1 - \eta} \qquad \boxed{n_S \simeq 1 - 0.036 = 0.964}$$

- So $\Lambda \approx 1.5117$ is indeed fixed independently (without connection to the inflationary characteristics)

SPECTRAL INDEX AND dS/CFT

- Z_σ cannot be decoupled justifying the existence of the eigenvalue equation

- The justification from boundary arguments

Renormalized Θ *comes from*

$\Theta \equiv -\beta_\lambda \mu^\varepsilon \sigma^4$

$$Z_\Theta = 1$$

$$\Theta = \Theta_0 z_\Theta^{-1/2} \longrightarrow (\mu\partial/\partial\mu + \gamma_\Theta)\langle\Theta\Theta\rangle = 0$$

- From bulk arguments

Connect bulk and boundary w/o forgetting Z_σ $\longrightarrow$ $\mu = aH$ and $\lambda = \phi$ $\longrightarrow$ $\lambda = \phi - \frac{2\gamma_\sigma}{\beta_\lambda} Ht \simeq \phi + \ln(H|\tau|)^{\frac{2\gamma_\sigma}{\beta_\lambda}}$

$$\downarrow$$

$$\left(\beta_\lambda \partial/\partial\lambda - 2\gamma_\sigma + O(\beta_\lambda^2)\right)\langle\Theta\Theta\rangle = 0$$

CONCLUSIONS

- We considered a thermal scalar in de Sitter background. Starting from the Bunch-Davies $|\mathrm{in}>$ vacuum, a Bogolyubov Transformation placed us somewhere in the interior of the finite temperature phase diagram.

- The BT rotation is considered in such a way that instead of returning to the BD vacuum we landed on the $|\mathrm{out}>$ vacuum, which is connected to an interacting IR CFT, in the universality class of the 3d Ising model.

- This interacting CFT is rather special, in the sense that the boundary operator that couples to the scalar curvature perturbations in the bulk has a classical scaling dimension. The critical exponent η is the order parameter of the breaking of the scale invariant spectrum of curvature fluctuations

- η fixes the parametric freedom in the dS scalar theory, yielding the prediction $n_{S,\beta} \approx 0.964$, up to errors associated with its lattice Monte Carlo measurements.

- Heating up the system $T \lesssim T_{\mathrm{dS}}$ numerically in a controlled way we evaluated additional cosmological observables $n^{(1)}_{S,\beta}$, f_{NL} and $n^{(2)}_{S,\beta}$. We finally note that our predicted values of $n_{S,\beta}$, $n^{(1)}_{S,\beta}$ and f_{NL} are well within current experimental bounds while $n^{(2)}_{S,\beta}$ exceeds them.

THANK YOU