Can we show that galaxies are of quantum-mechanical origin?

Jérôme Martin

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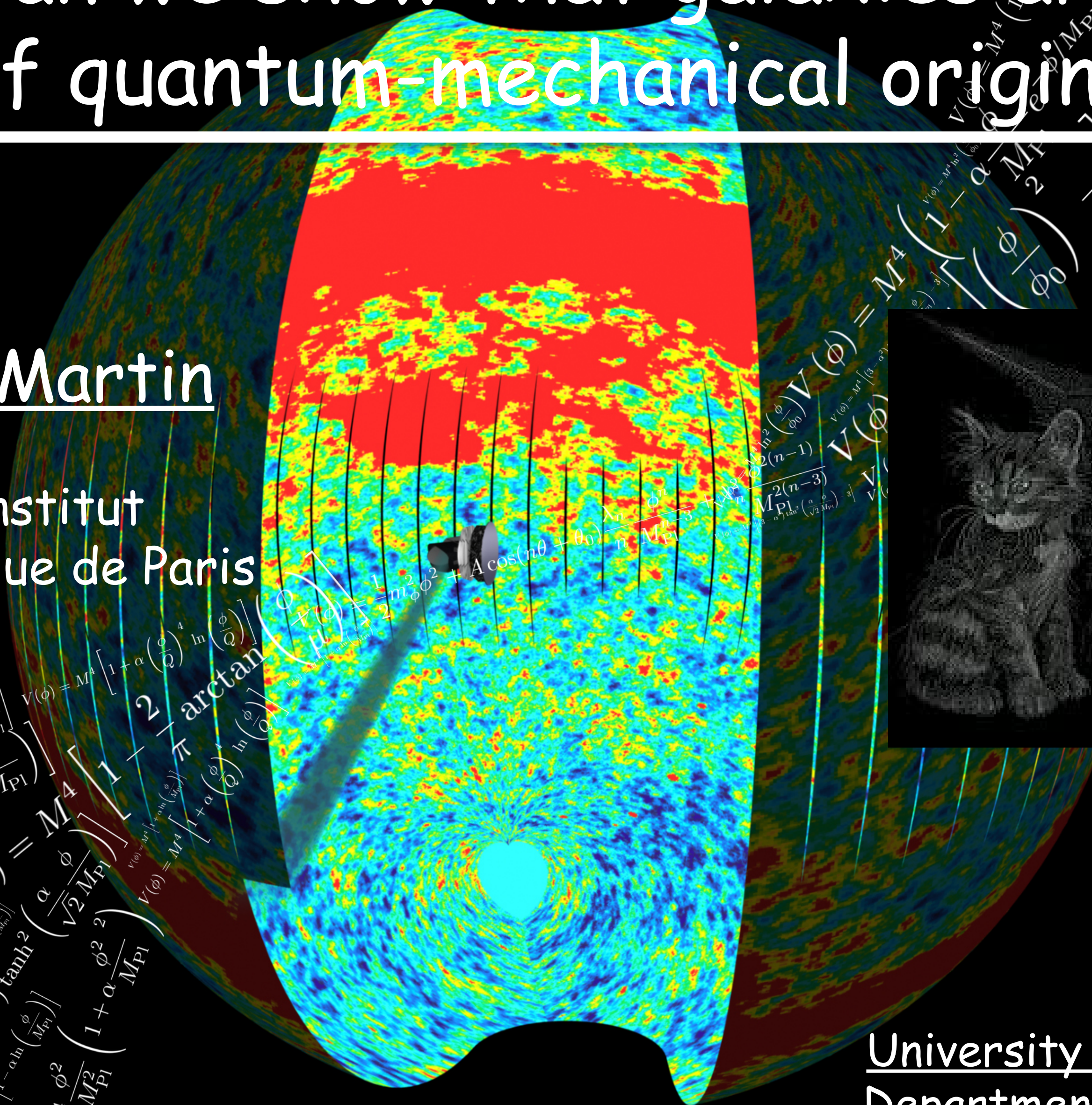

University of Helsinki, Department of Physics

March, 2024

Outline

- Introduction
- Cosmological Perturbations of Quantum-Mechanical Origin during inflation
- Measures of "quantumness"
- Measures of "classicality"
- Signature of the Quantum Origin of the Perturbations in the Sky?
- Discussion & Conclusions

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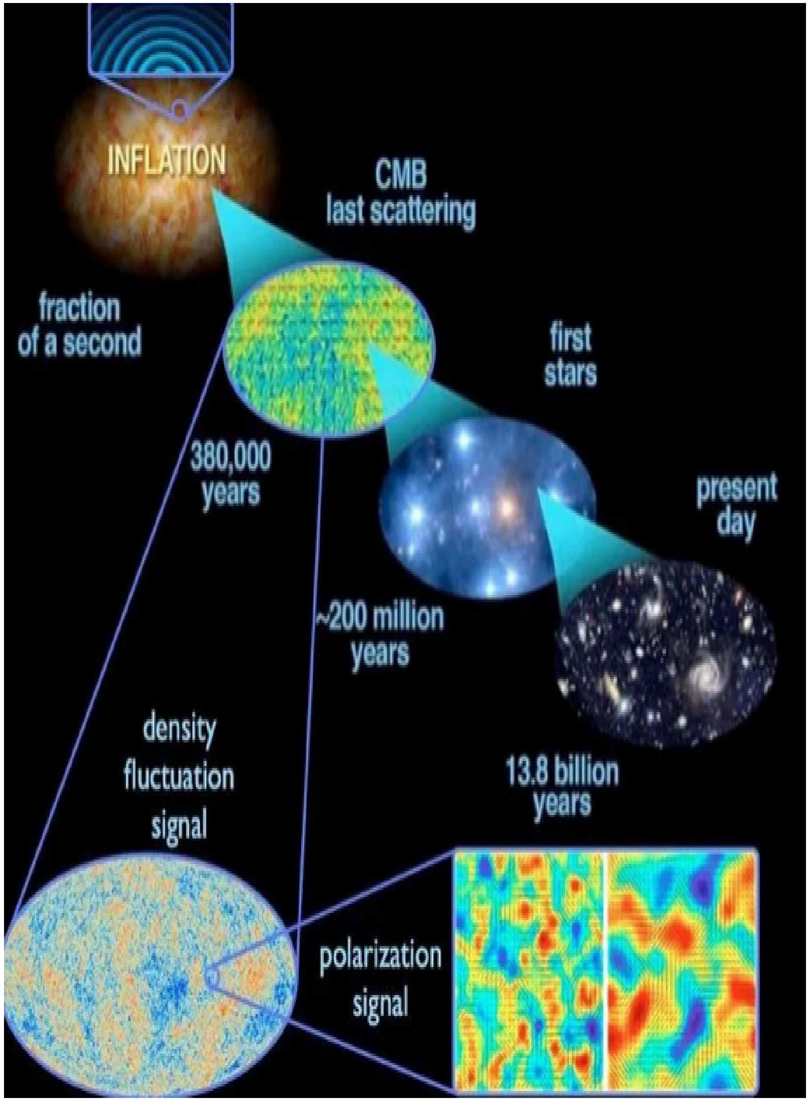
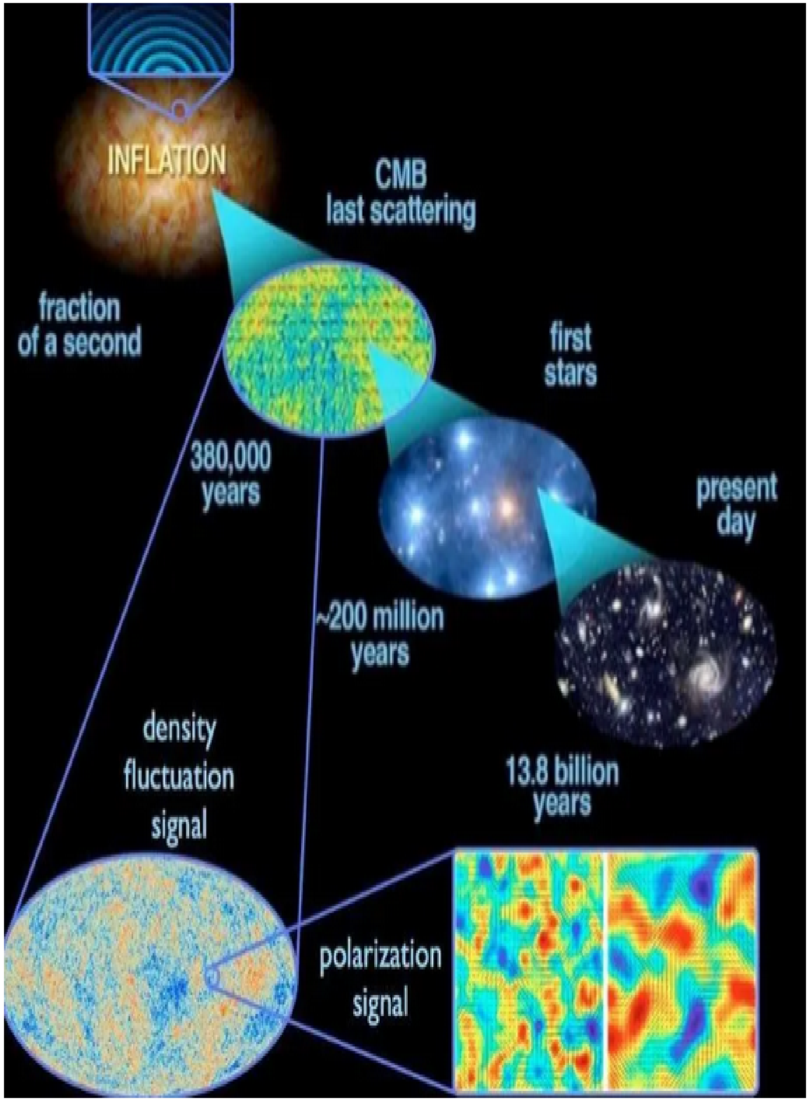
Quantum perturbations



- Inflation solves the puzzles of the standard model and also gives an explanation of the origin of the structures (galaxies, CMB anisotropies etc …) observed in the Universe

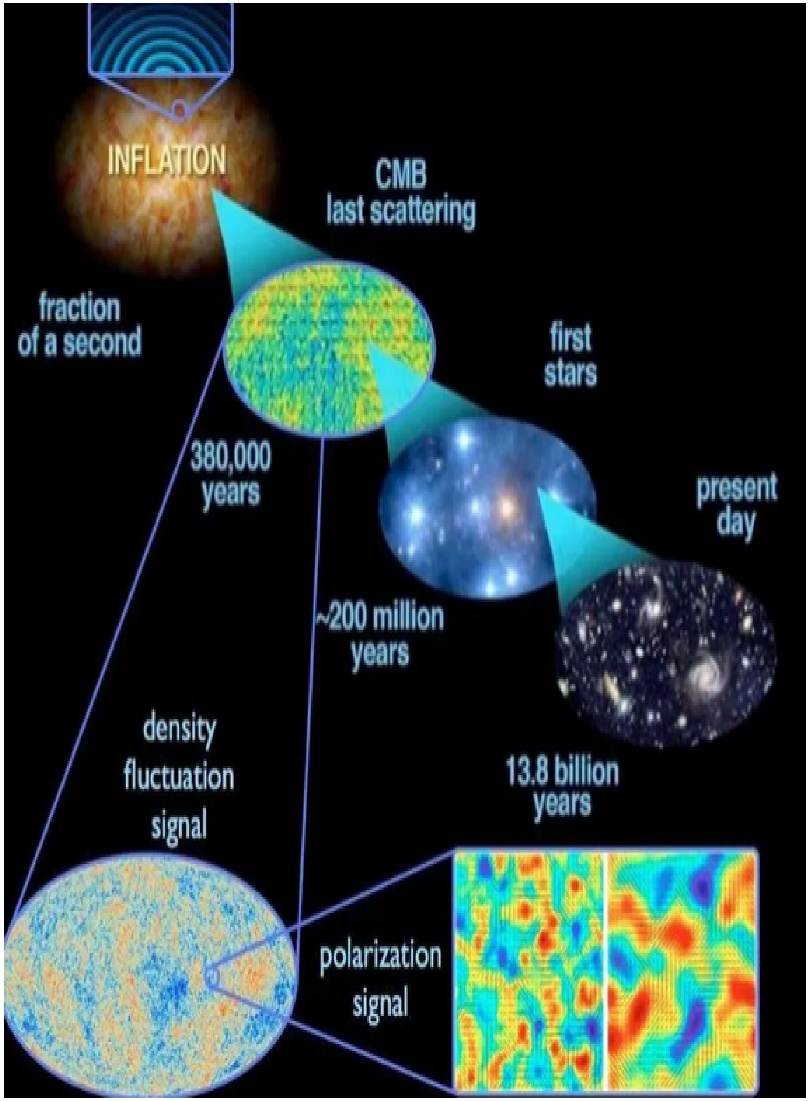


- Inflation solves the puzzles of the standard model and also gives an explanation of the origin of the structures (galaxies, CMB anisotropies etc …) observed in the Universe

- Main idea: quantum fluctuations of the gravitational and scalar (inflaton) fields during inflation amplified by gravitational instability and stretched by cosmic expansion

All structures in the Universe are of quantum-mechanical origin

Quantum perturbations



- Scalar perturbations are characterized by one quantity (a combination of metric and inflaton perturbations): <u>curvature perturbations</u>

$$g_{\mu\nu} = g_{\mu\nu}(t) + \delta g_{\mu\nu}(t, \boldsymbol{x}) + \cdots$$

$$\phi = \phi(t) + \delta\phi(t, \boldsymbol{x}) + \cdots$$

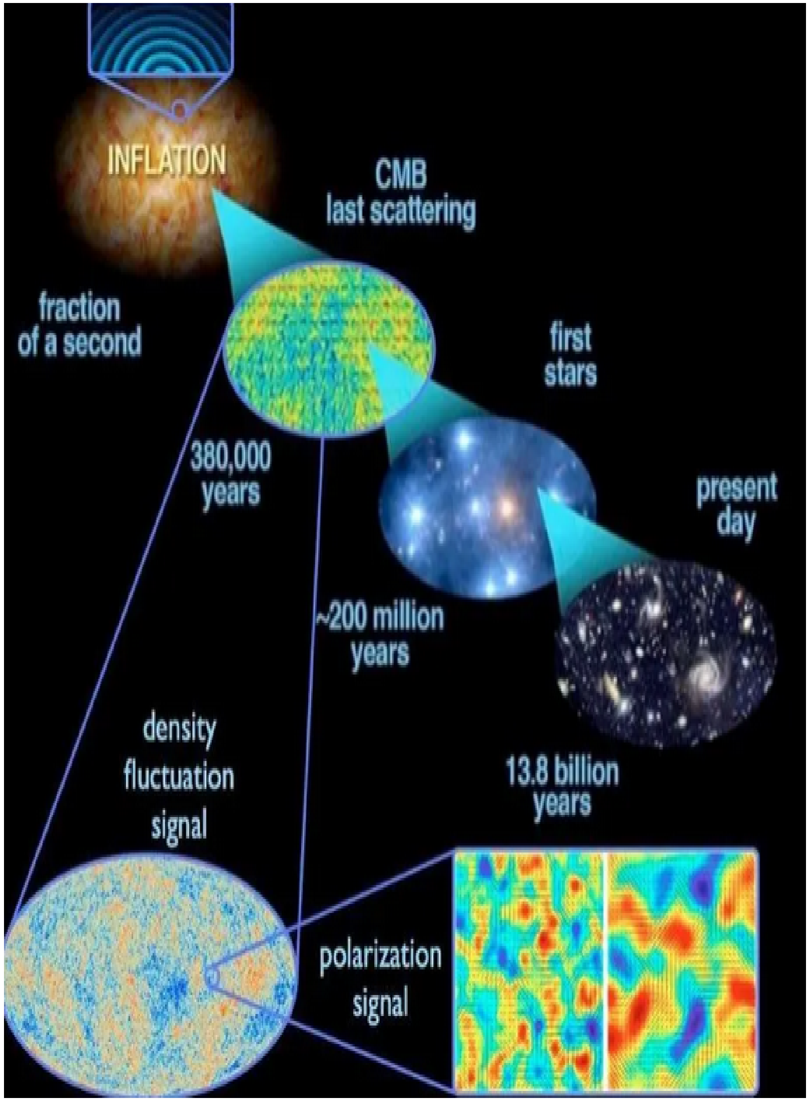
↓

$$\zeta(\eta, \boldsymbol{x}) = \frac{v(\eta, \boldsymbol{x})}{z(\eta)}$$

Mukhanov-Sasaki variable

with

$$z(\eta) = a(\eta)\sqrt{2\epsilon_1} M_{\mathrm{Pl}}\,, \quad \epsilon_1 = -\frac{\dot{H}}{H^2}$$



- Hamiltonian of the system

$$S = \frac{M_{\rm Pl}^2}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,R - \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + V(\phi)\right]$$

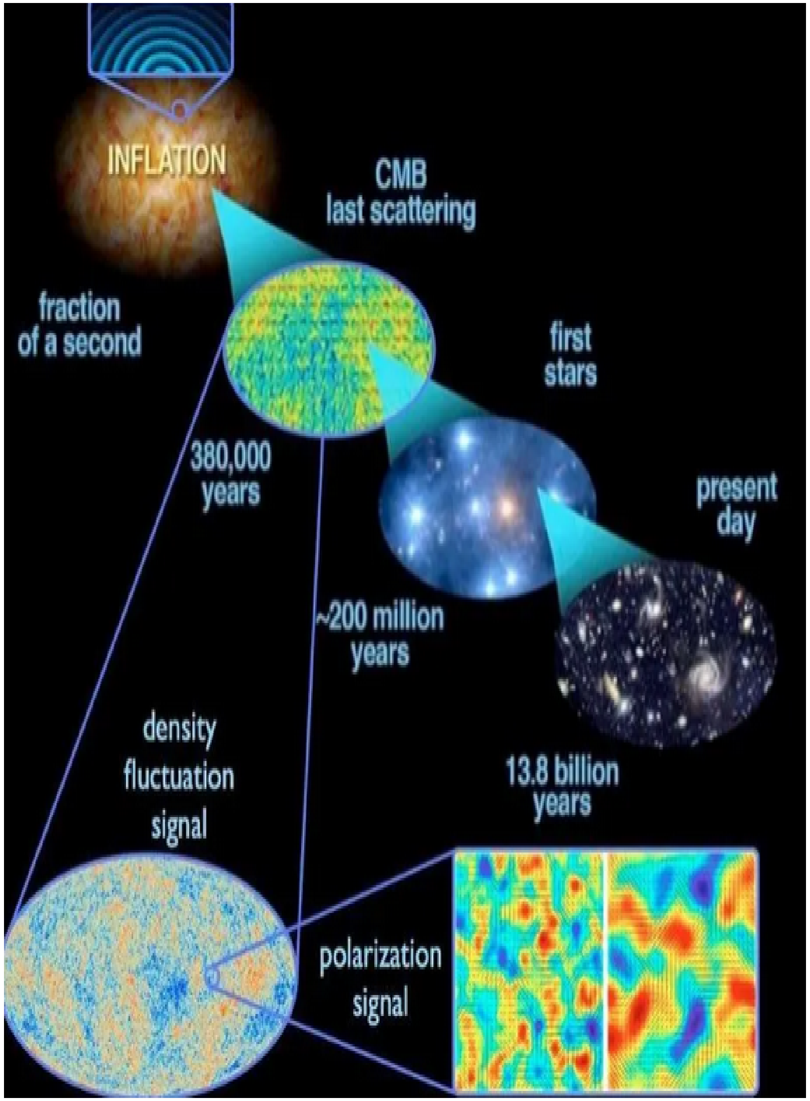
$$\downarrow$$

$$\mathcal{H} = \int_{\mathbb{R}^{3+}} \mathrm{d}^3\boldsymbol{k}\,\mathcal{H}_{\boldsymbol{k}}$$

$$\boxed{\mathcal{H}_{\boldsymbol{k}} = p_{\boldsymbol{k}}p_{\boldsymbol{k}}^\star + \left(k^2 - \frac{z''}{z}\right)v_{\boldsymbol{k}}v_{\boldsymbol{k}}^\star}$$

with

$$v(\eta,\boldsymbol{x}) = \frac{1}{(2\pi)^{3/2}}\int_{\mathbb{R}^3}\mathrm{d}^3\boldsymbol{k}\,v_{\boldsymbol{k}}(\eta)\,e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

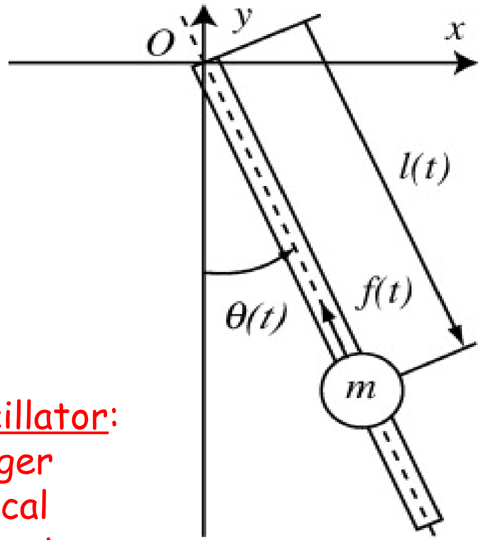


- Equation of motion:

$$v_{\boldsymbol{k}}''(\eta) + \omega^2(k,\eta)v_{\boldsymbol{k}}(\eta) = 0$$

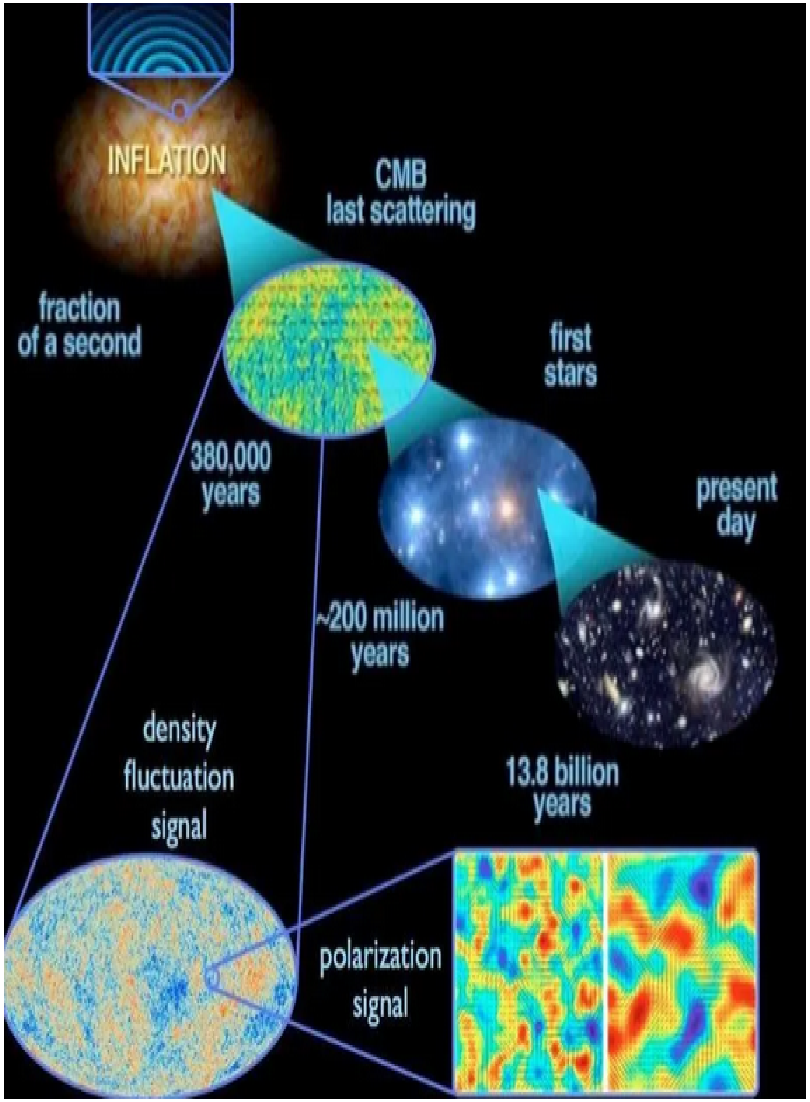
with

$$\omega^2(k,\eta) = k^2 - \frac{z''}{z}$$



Parametric oscillator: akin to Schwinger effect, dynamical Casimir effect, etc …

Quantum perturbations



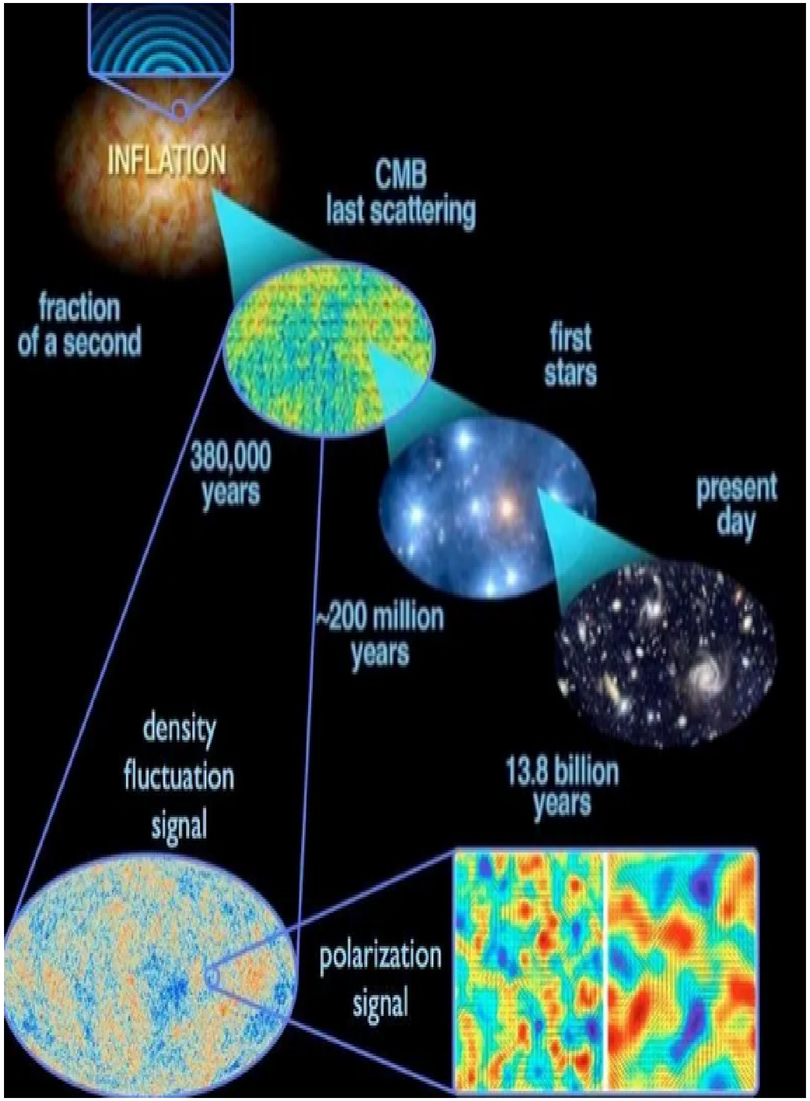
- In fact, the MS quantities mix +/- k modes and, therefore, does not exactly represents the amplitude associated to a mode k

$$v_{\boldsymbol{k}}^{\star} = v_{-\boldsymbol{k}} \longrightarrow v_{\boldsymbol{k}} = \frac{1}{\sqrt{2k}}\left(c_{\boldsymbol{k}} + c_{-\boldsymbol{k}}^{\star}\right)$$

- It is more convenient to work with real quantities for "position" and "momentum" of mode k:

$$q_{\boldsymbol{k}} = \frac{1}{\sqrt{2k}}\left(c_{\boldsymbol{k}} + c_{\boldsymbol{k}}^{\star}\right) = q_{\boldsymbol{k}}^{\star}$$

$$\pi_{\boldsymbol{k}} = -i\sqrt{\frac{k}{2}}\left(c_{\boldsymbol{k}} - c_{\boldsymbol{k}}^{\star}\right) = \pi_{\boldsymbol{k}}^{\star}$$



- Quantization of perturbations

$$\hat{q}_{\boldsymbol{k}}\Psi = q_{\boldsymbol{k}}\Psi, \qquad \hat{q}_{-\boldsymbol{k}}\Psi = q_{-\boldsymbol{k}}\Psi$$

$$\hat{\pi}_{\boldsymbol{k}}\Psi = -i\frac{\partial}{\partial q_{\boldsymbol{k}}}\Psi, \quad \hat{\pi}_{-\boldsymbol{k}}\Psi = -i\frac{\partial}{\partial q_{-\boldsymbol{k}}}\Psi$$

- Schroedinger equation

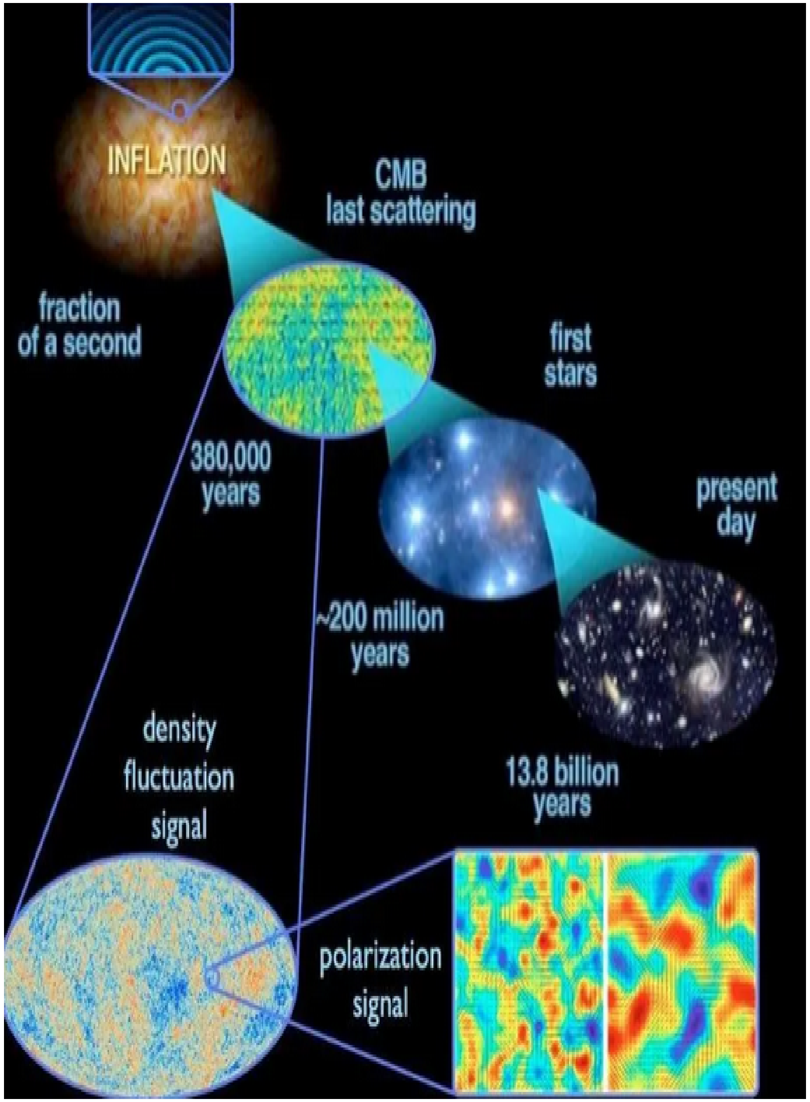
$$\Psi[v] = \prod_{\boldsymbol{k}\in \mathrm{R}^{3+}} \Psi(\eta, q_{\boldsymbol{k}}, q_{-\boldsymbol{k}})$$

$$i\frac{\partial}{\partial \eta}\Psi\left(\eta, q_{\boldsymbol{k}}, q_{-\boldsymbol{k}}\right) = \left(\hat{\mathcal{H}}_{\boldsymbol{k}} + \hat{\mathcal{H}}_{-\boldsymbol{k}} + \hat{\mathcal{H}}_{\pm\boldsymbol{k}}\right)\Psi\left(\eta, q_{\boldsymbol{k}}, q_{-\boldsymbol{k}}\right)$$

with

$$\hat{\mathcal{H}}_{\boldsymbol{k}} = \frac{1}{2}\left(1+\frac{\omega^2}{k^2}\right)\left(\frac{1}{2}\hat{\pi}_{\boldsymbol{k}}^2 + \frac{k^2}{2}\hat{q}_{\boldsymbol{k}}^2\right)$$

$$\hat{\mathcal{H}}_{\pm\boldsymbol{k}} = \left(1-\frac{\omega^2}{k^2}\right)\left(\frac{1}{2}\hat{\pi}_{\boldsymbol{k}}\hat{\pi}_{-\boldsymbol{k}} - \frac{k^2}{2}\hat{q}_{\boldsymbol{k}}\hat{q}_{-\boldsymbol{k}}\right)$$



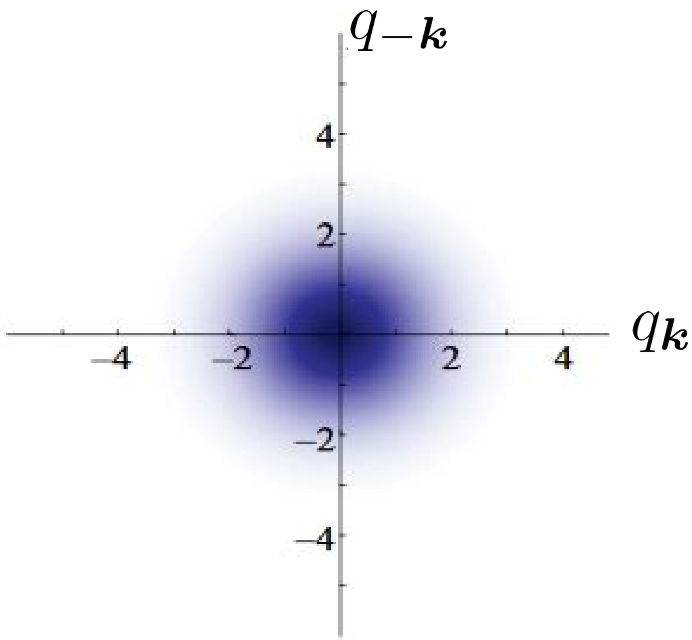
- Quadratic Hamiltonian -> Gaussian state

$$\Psi(\eta, q_{\boldsymbol{k}}, q_{-\boldsymbol{k}}) = N_k(\eta) e^{A(k,\eta)k(q_{\boldsymbol{k}}^2 + q_{-\boldsymbol{k}}^2) - B(k,\eta)k q_{\boldsymbol{k}} q_{-\boldsymbol{k}}}$$

- The state is entangled

$$\boxed{\Psi(\eta, q_{\boldsymbol{k}}, q_{-\boldsymbol{k}}) \neq \Psi(\eta, q_{\boldsymbol{k}})\Psi(\eta, q_{-\boldsymbol{k}})}$$

- The quantum state is a two-mode squeezed state

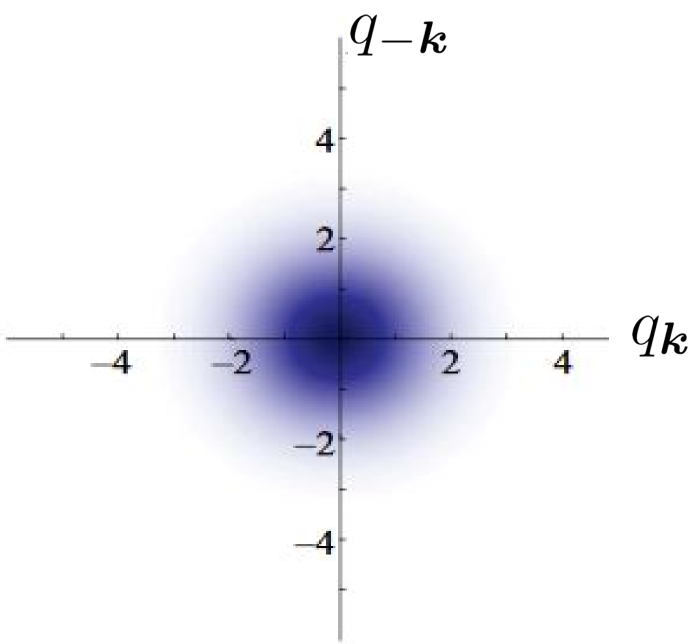


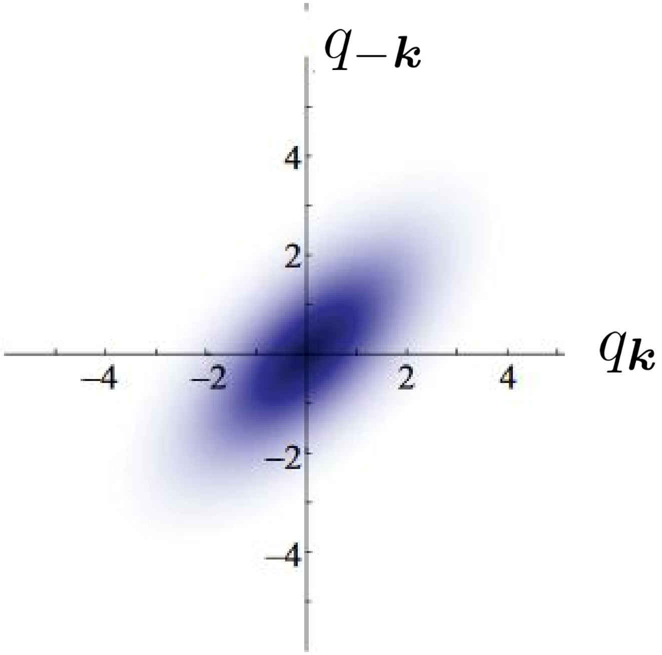
Two mode vacuum coherent state

$$\Psi_0\left(q_{\boldsymbol{k}}, q_{-\boldsymbol{k}}\right) = \frac{1}{\pi^{1/4}} e^{-q_{\boldsymbol{k}}^2/2 - q_{-\boldsymbol{k}}^2/2}$$

$$= \frac{1}{\sqrt{\pi}} e^{-(q_{\boldsymbol{k}} - q_{-\boldsymbol{k}})^2/4} e^{-(q_{\boldsymbol{k}} + q_{-\boldsymbol{k}})^2/4}$$

Quantum state of the CMB: a two-mode squeezed state





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$$\Psi_R\left(q_{\boldsymbol{k}}, q_{-\boldsymbol{k}}\right) = \frac{1}{\sqrt{\pi}} e^{-(q_{\boldsymbol{k}} - q_{-\boldsymbol{k}})^2/(4R^2)}$$

$$\times\, e^{-R^2(q_{\boldsymbol{k}} + q_{-\boldsymbol{k}})^2/4}$$

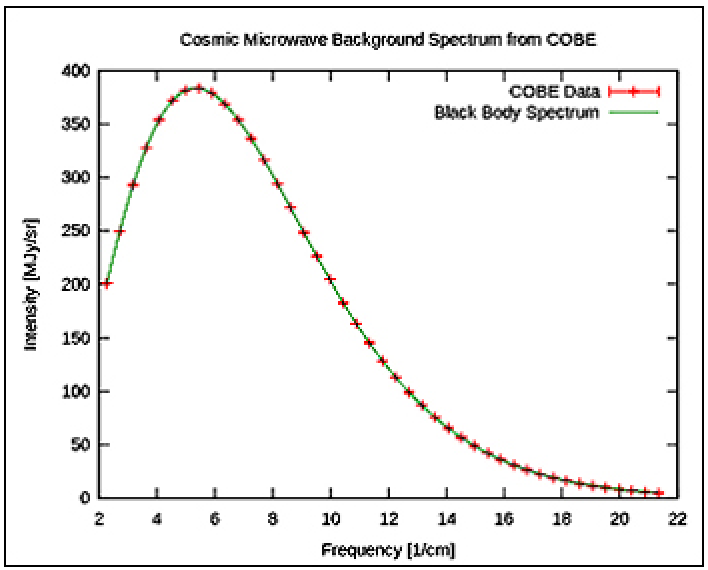
Squeezing parameter: $r = \ln R$

-> quantum correlations between modes +/- k

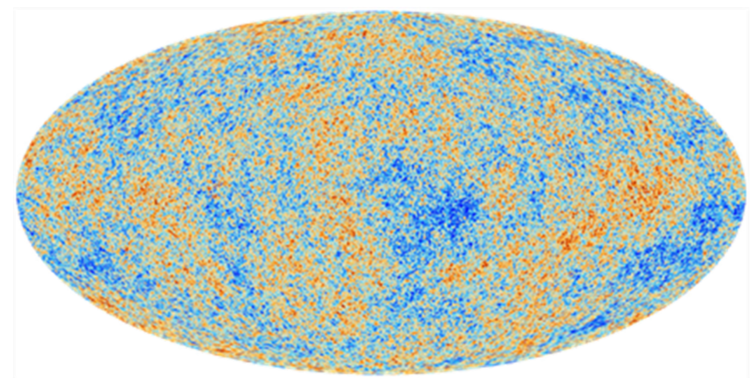
Quantum state of the CMB: a strongly two-mode squeezed state

The cosmological two-mode squeezed state is <u>(very!) strongly</u> squeezed

CMB is the most accurate black body ever produced in Nature



CMB anistropy is the strongest squeezed state ever produced in Nature



$$r_k = \mathcal{O}\left(10^2\right)$$

$$-10\log_{10}\left(e^{-2r_{\boldsymbol{k}}}\right)\ \mathrm{dB}$$

- ~ 15 dB in the lab
- > 500 dB inflation

Criterions for classicality

- Are the perturbations classical or quantum?
- What do we mean by “classical” and “quantum”?
- Usually in the literature …

Quantum

- Strong squeezing
- Non-vanishing discord
- Separability (Peres criterion)
- …

Classical

- Positive Wigner function
- WKB state
- Effect of decoherence?
- …

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- Mutual information between systems A and B

$$I(A,B) = S(A) + S(B) - S(A,B)$$

$$S(A) = -\sum_i p(a_i) \ln [p(a_i)]$$

$p(a_i)$: pdf

$$S(A,B) = -\sum_{ij} p(a_i,b_j) \ln [p(a_i,b_j)]$$

$p(a_i,b_j)$: joined pdf

$$p(a_i,b_j) = p(a_i)p(b_j) \longrightarrow I(A,B) = 0$$

- Mutual information in a different way with Bayes theorem

$p(a_i|b_j)$ = probability of a_i given b_j $\longrightarrow$ $-\sum_i p(a_i|b_j) \ln [p(a_i|b_j)]$ "entropy of A given b_j"

$$S(A|B) = \sum_j p(b_j) \left\{ -\sum_i p(a_i|b_j) \ln [p(a_i|b_j)] \right\}$$

$$J(A,B) = S(A) - S(A|B)$$

Bayes theorem: $p(a_i,b_j) = p(b_j|a_i)p(a_i) \longrightarrow S(A|B) = S(A,B) - S(B)$

$$J(A,B) \equiv S(A) - S(A|B) = S(B) + S(A) - S(A,B) = I(A,B)$$

- Mutual information I between systems A and B in Quantum Mechanics

$$S(A,B) = -\mathrm{Tr}\left(\hat{\rho}\log_2\hat{\rho}\right)$$

$$S(A) = -\mathrm{Tr}\left(\hat{\rho}_{\rm A}\log_2\hat{\rho}_{\rm A}\right) \text{ with } \hat{\rho}_{\rm A} = \mathrm{Tr}_B\hat{\rho}$$

$I(A,B)$ can easily be generalized to QM

- Mutual information J between systems A and B in Quantum Mechanics

Upon measurement of $|b_j\rangle$:

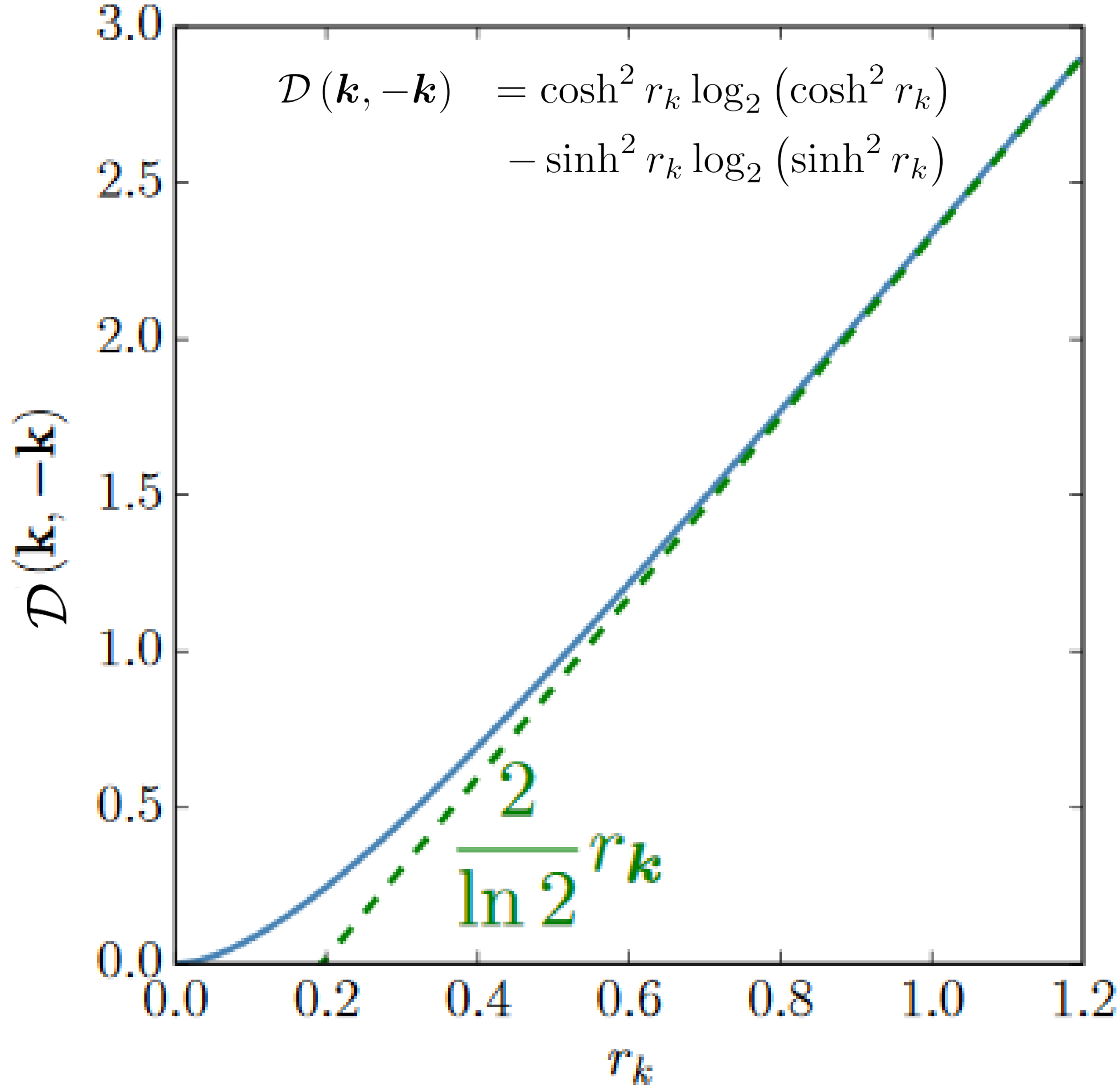
$$\hat{\rho} \to \frac{1}{\mathrm{Prob}(|b_j\rangle)}\hat{\Pi}_{|b_j\rangle}\hat{\rho}\hat{\Pi}_{|b_j\rangle}$$

$$\hat{\rho}(A|\hat{\Pi}_{|b_j\rangle}) = \mathrm{Tr}_B\left[\frac{1}{\mathrm{Prob}(|b_j\rangle)}\hat{\Pi}_{|b_j\rangle}\hat{\rho}\hat{\Pi}_{|b_j\rangle}\right]$$

$$S(A|B) = \sum_j \mathrm{Prob}(|b_j\rangle)S\left[\hat{\rho}\left(A|\hat{\Pi}_{|b_j\rangle}\right)\right]$$

This allows us to generalize J in QM but, now, crucially, I≠J

$$\mathcal{D}(A,B) \equiv I(A,B) - J(A,B)$$



J. Martin & V. Vennin, PRD 93 (2016), 1023505, arXiv:1510.04038

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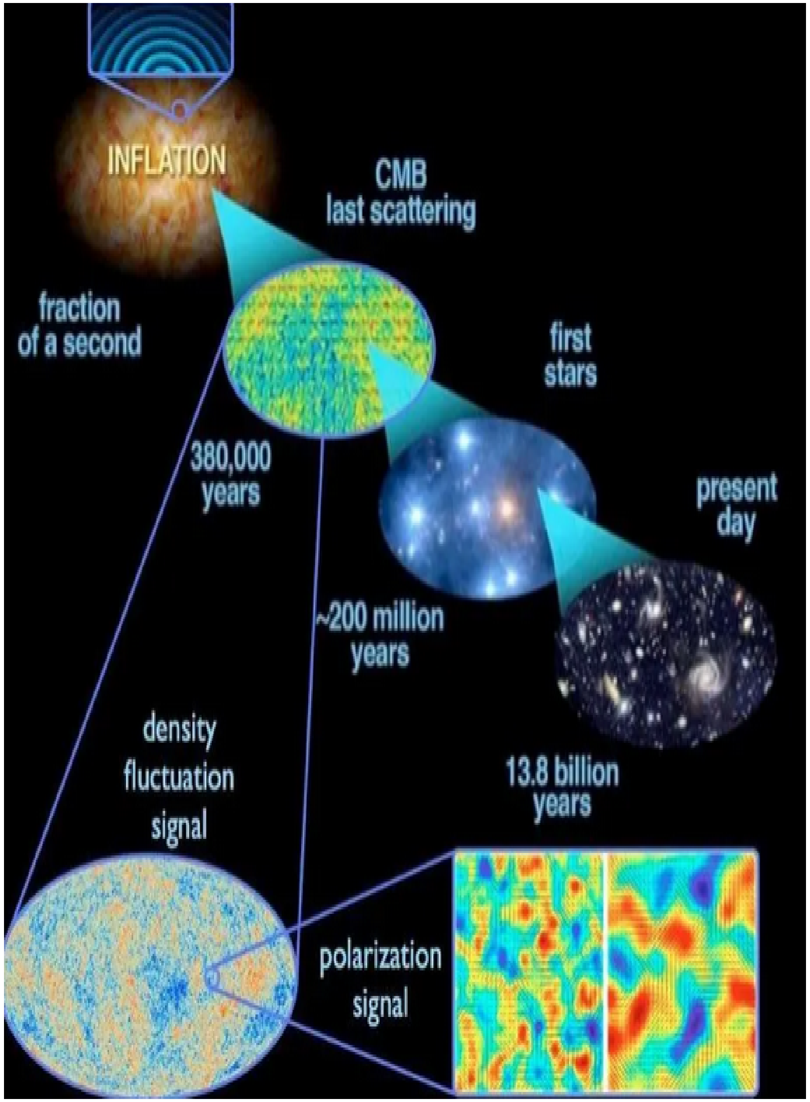
Quantum perturbations in phase space



- Weyl correspondence: operator $\longrightarrow$ phase space function

$$W(\hat{O})(p,q) = \int_{-\infty}^{+\infty} \mathrm{d}x\, e^{-ipx} \left\langle q + \frac{x}{2} \middle| O(\hat{q},\hat{p}) \middle| q - \frac{x}{2} \right\rangle$$

- Examples:

1) $W[\mathcal{F}(\hat{q})](q,p) = \mathcal{F}(q), \quad W[\mathcal{F}(\hat{p})](q,p) = \mathcal{F}(p)$

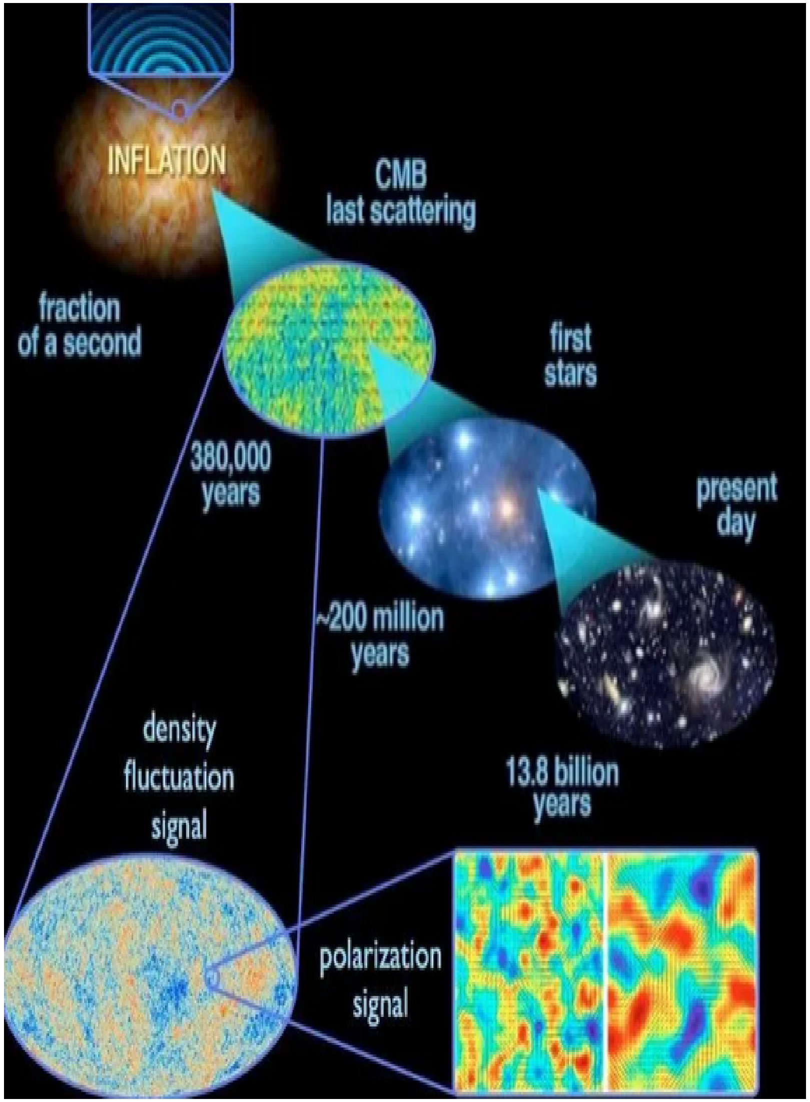
2) $W(\hat{q}\hat{p})(q,p) = qp + \frac{i}{2}, \quad W(\hat{p}\hat{q})(q,p) = pq - \frac{i}{2}$

$\hookrightarrow W(\hat{q}\hat{p} + \hat{p}\hat{q})(q,p) = 2qp$

3) $W(\hat{p}^2\hat{q}^2)(q,p) = p^2q^2 - 2ipq - \frac{1}{2}$

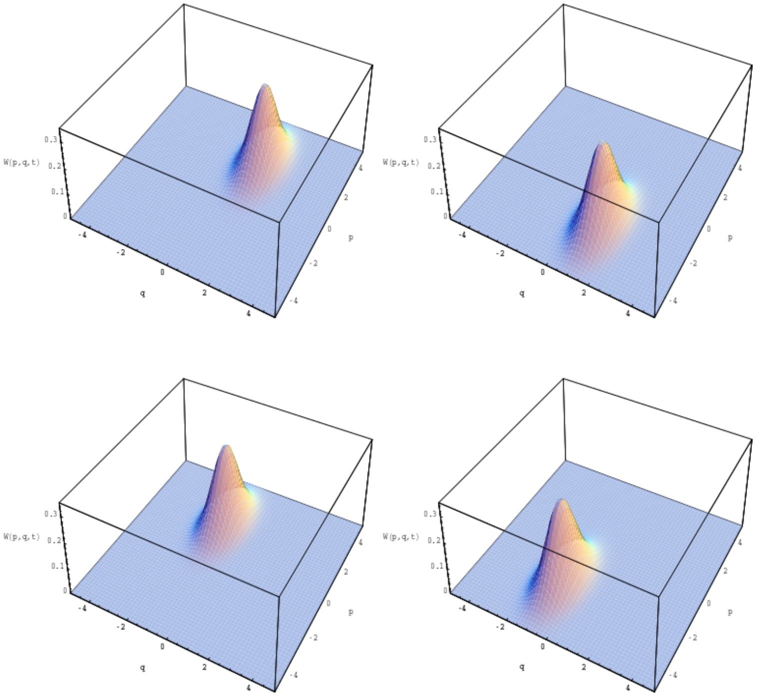
$W(\hat{q}^2\hat{p}^2)(q,p) = p^2q^2 + 2ipq - \frac{1}{2}$

$\hookrightarrow W(\hat{p}^2\hat{q}^2 + \hat{q}^2\hat{p}^2)(q,p) = 2p^2q^2 - 1$

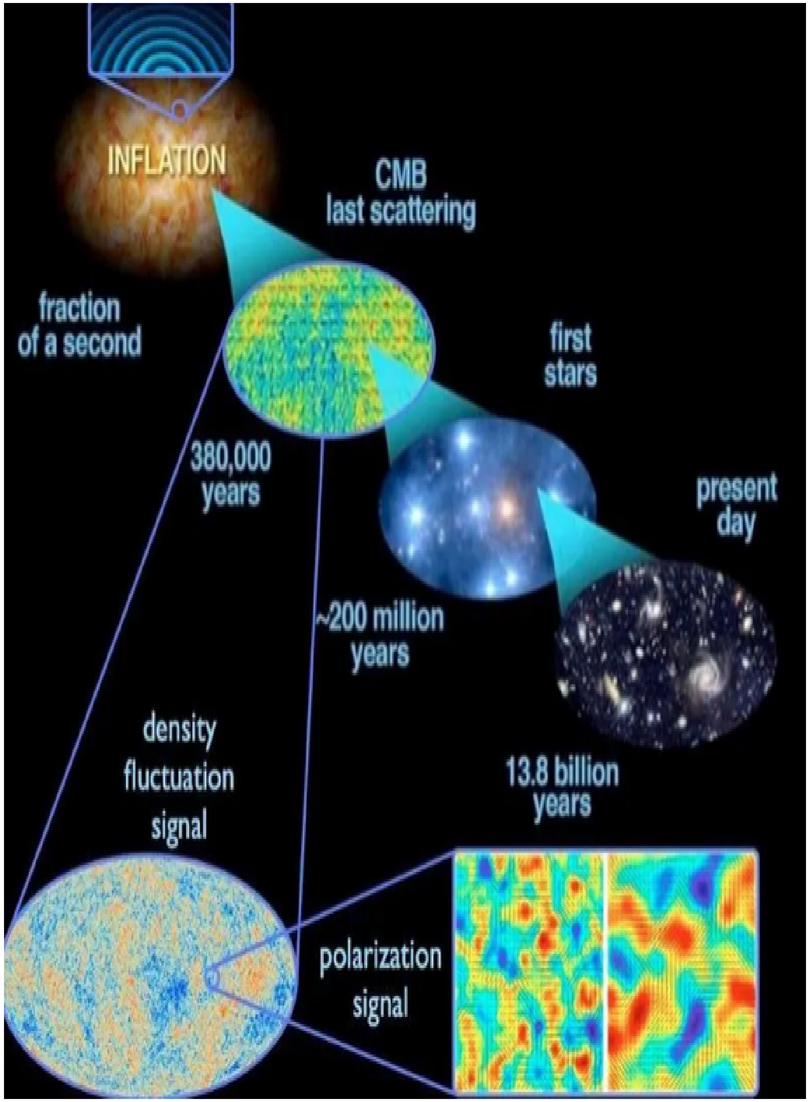


- Definition: Wigner function = Weyl transform of the density matrix

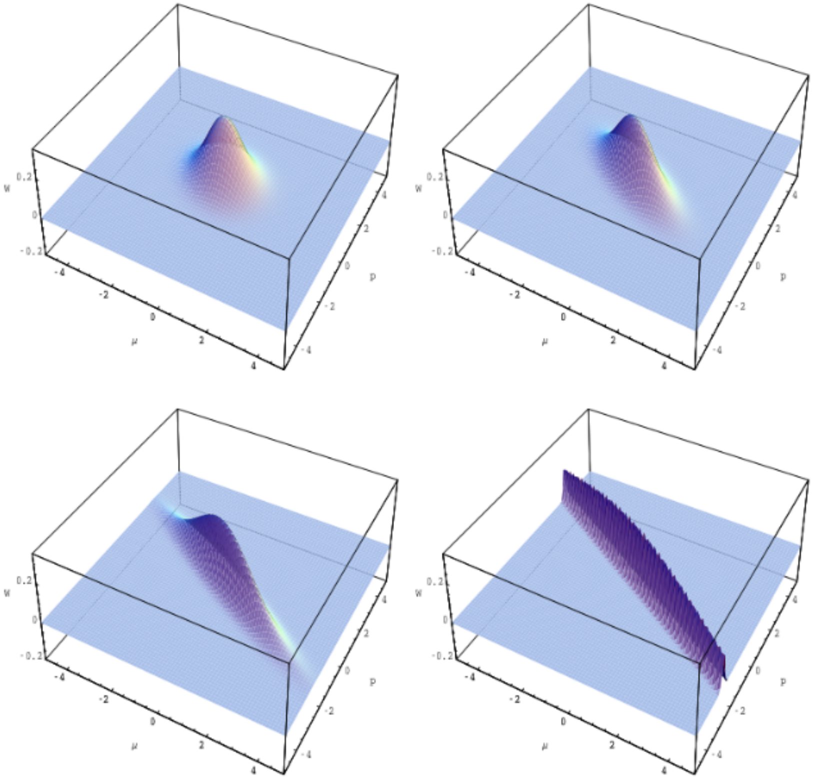
$$W(q,p) = W\left(\frac{\hat{\rho}}{2\pi}\right)(q,p)$$



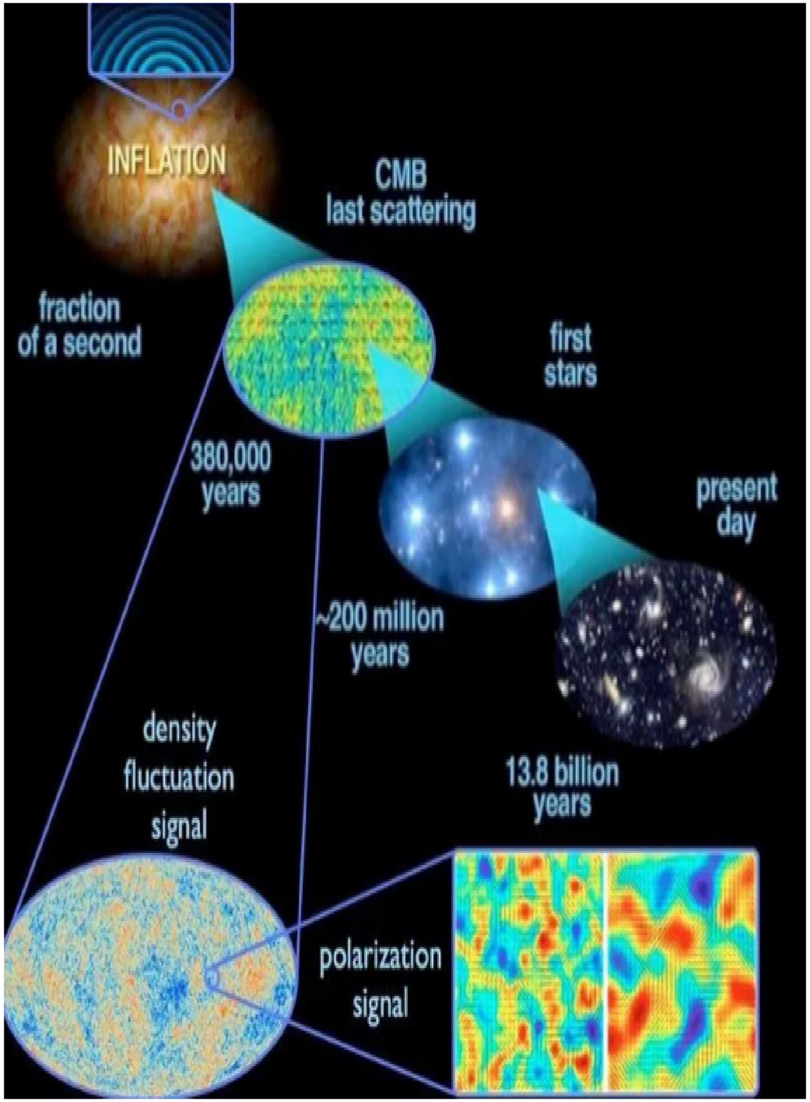
Example: coherent state



- The Wigner function of quantum perturbations is a Gaussian



Two-mode squeezed state

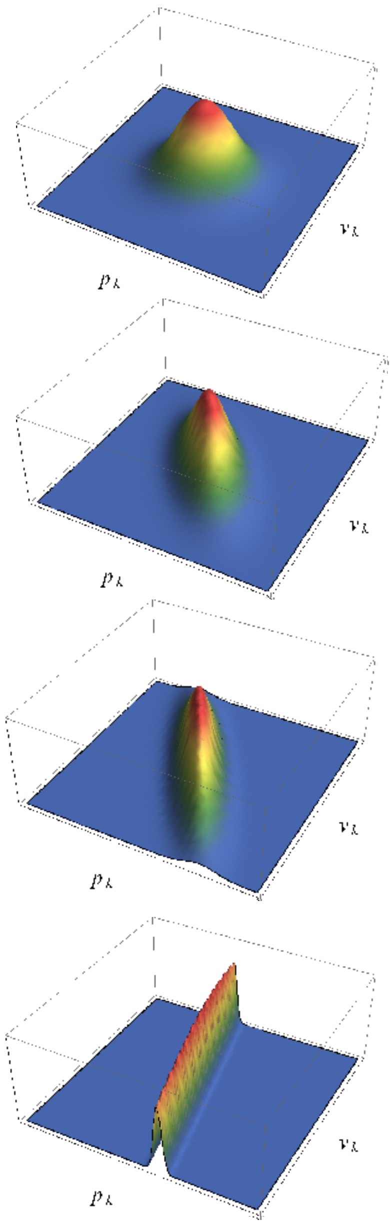


- The fundamental theorem

$$
\begin{aligned}
\left\langle \hat{A} \right\rangle &= \mathrm{Tr}\left(\hat{\rho}\hat{A}\right) \\
&= \int \mathrm{d}q\,\mathrm{d}p\, W(q,p)\, W(\hat{A})(q,p)
\end{aligned}
$$

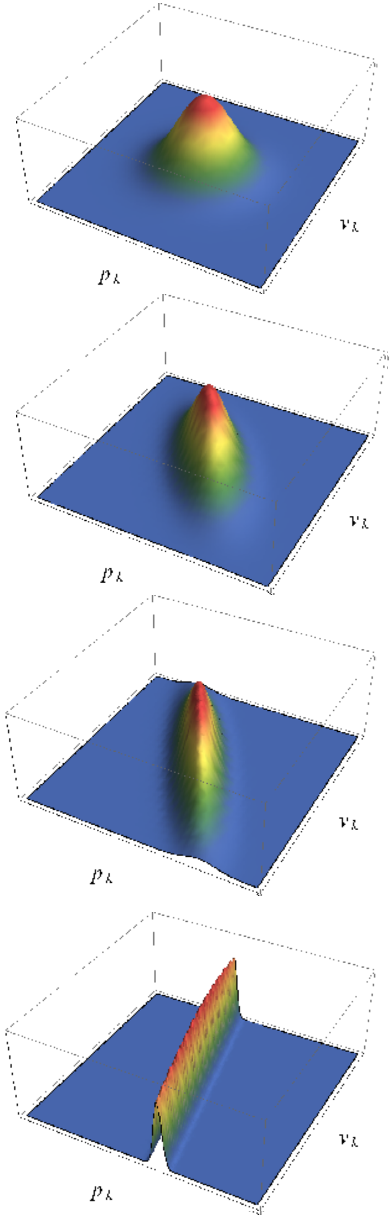
Quantum mean value of an operator = stochastic average of the Weyl transform of this operator with a pdf given by the Wigner function

The quantum to classical transition



Do we also have arguments to say that the system is classical?

$$\left\langle \hat{A} \right\rangle = \mathrm{Tr}\left(\hat{\rho}\hat{A}\right) = \int \mathrm{d}q\,\mathrm{d}p\,W(q,p)\,W(\hat{A})(q,p) \overset{?}{=} \left\langle W(\hat{A})(q,p) \right\rangle_{\mathrm{stocha}}$$



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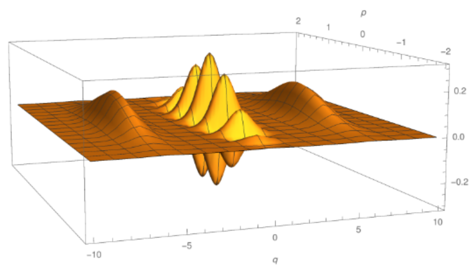
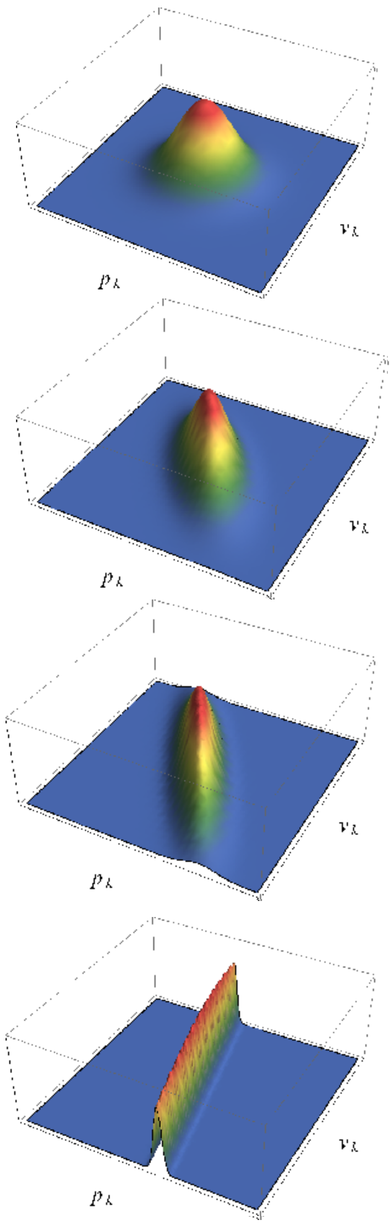
Must be positive to be a physical pdf



$$\Psi_{\mathrm{CAT}} = \frac{N_{\mathrm{CAT}}}{\sqrt{2}}\left[\Phi_+(q) + \Phi_-(q)\right]$$



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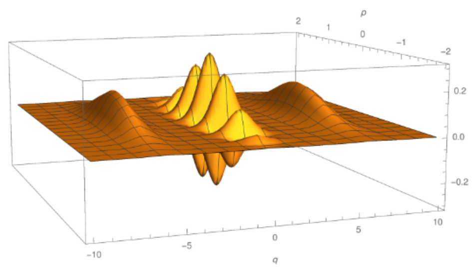
Must be positive to be a physical pdf

$$W(\hat{A})(q,p) = A(q,p)$$



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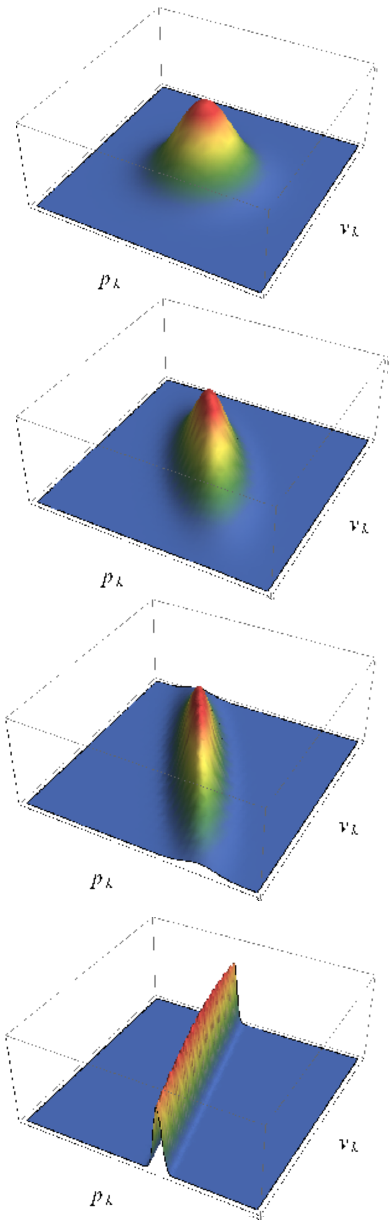
The quantum to classical transition



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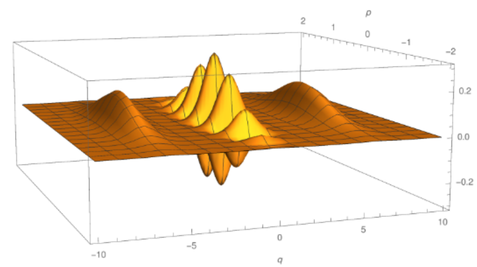

$$\Psi_{\mathrm{CAT}} = \frac{N_{\mathrm{CAT}}}{\sqrt{2}}\left[\Phi_+(q) + \Phi_-(q)\right]$$

- Quantum and classical averages for two-point function are always equal regardless of squeezing

$$\left\langle \hat{q}_{\boldsymbol{k}}^2 \right\rangle = \left\langle q_{\boldsymbol{k}}^2 \right\rangle_{\mathrm{stocha}}$$

$$\left\langle \hat{p}_{\boldsymbol{k}}^2 \right\rangle = \left\langle p_{\boldsymbol{k}}^2 \right\rangle_{\mathrm{stocha}}$$

$$\left\langle \hat{q}_{\boldsymbol{k}}\hat{p}_{\boldsymbol{k}} + \hat{p}_{\boldsymbol{k}}\hat{q}_{\boldsymbol{k}} \right\rangle = 2\left\langle q_{\boldsymbol{k}} p_{\boldsymbol{k}} \right\rangle_{\mathrm{stocha}}$$

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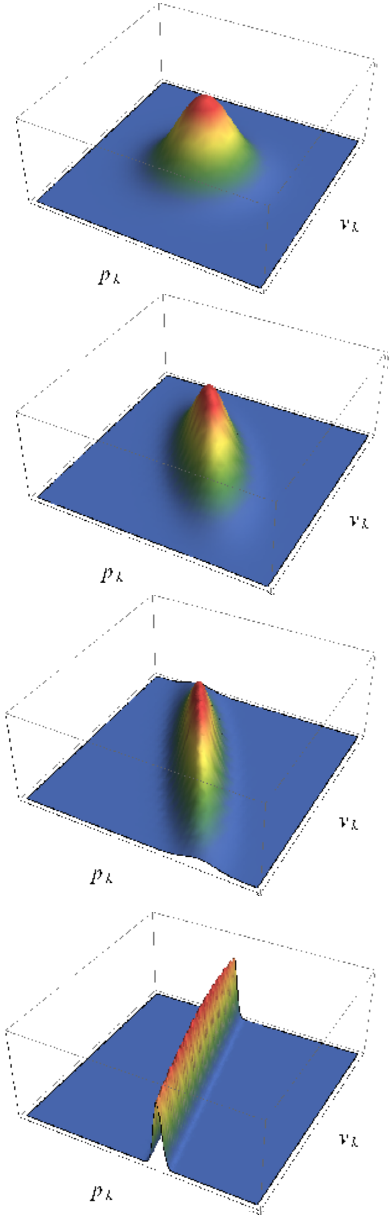

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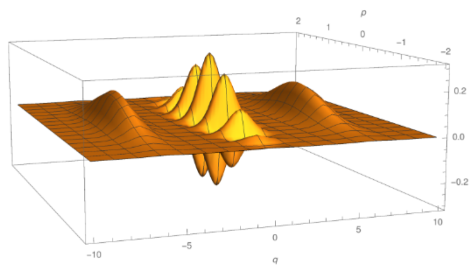
Must be positive to be a physical pdf

$$W(\hat{A})(q,p) = A(q,p)$$



$$\Psi_{\mathrm{CAT}} = \frac{N_{\mathrm{CAT}}}{\sqrt{2}}\left[\Phi_+(q) + \Phi_-(q)\right]$$

- Quantum and classical averages for two-point function are always equal regardless of squeezing

$$\left\langle \hat{q}_{\boldsymbol{k}}^2 \right\rangle = \left\langle q_{\boldsymbol{k}}^2 \right\rangle_{\mathrm{stocha}}$$

$$\left\langle \hat{p}_{\boldsymbol{k}}^2 \right\rangle = \left\langle p_{\boldsymbol{k}}^2 \right\rangle_{\mathrm{stocha}}$$

$$\left\langle \hat{q}_{\boldsymbol{k}}\hat{p}_{\boldsymbol{k}} + \hat{p}_{\boldsymbol{k}}\hat{q}_{\boldsymbol{k}} \right\rangle = 2\left\langle q_{\boldsymbol{k}} p_{\boldsymbol{k}} \right\rangle_{\mathrm{stocha}}$$

Cosmological perturbations are thus "classical" …

"decoherence without decoherence"

Are the perturbations quantum or classical?

So are the perturbations “classical” or “quantum” … ?

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- The quantum properties of the system must simply be stored in operators that are not the "simplest ones" usually considered (such as the square of the MS operator). In fact, these non-trivial operators must have non-analytical Weyl transform (Revzen 2006).

So are the perturbations "classical" or "quantum" … ?

- Not a "good" question … in fact there is no contradiction in having a discordant state with a positive Wigner function …
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- The quantum properties of the system must simply be stored in operators that are not the "simplest ones" usually considered (such as the square of the MS operator). In fact, these non-trivial operators must have non-analytical Weyl transform (Revzen 2006).
- But, how to observationnally reveal those quantum aspects?

Outline

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Detecting the quantum origin of the perturbations?

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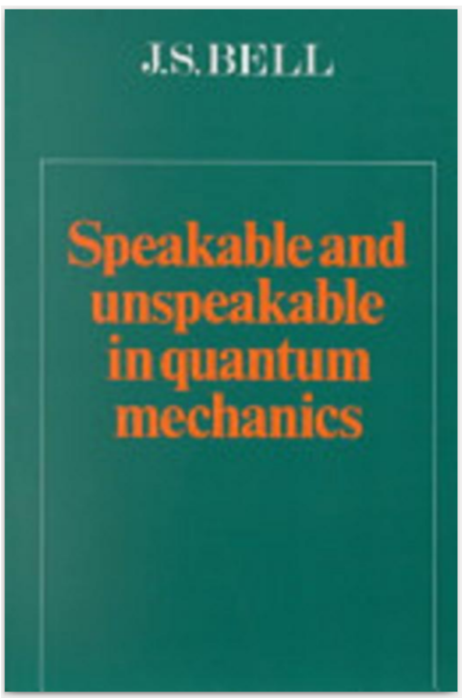
- One standard way to experimentally prove that a system is quantum is to reveal correlations that cannot be reproduced by classical probability theory
- The most well-known example of such a strategy is <u>Bell inequality violation</u>
- <u>Historical aside:</u> Whether we can have Bell inequality violation for a system with positive Wigner function has been the subject of a long-standing debate …



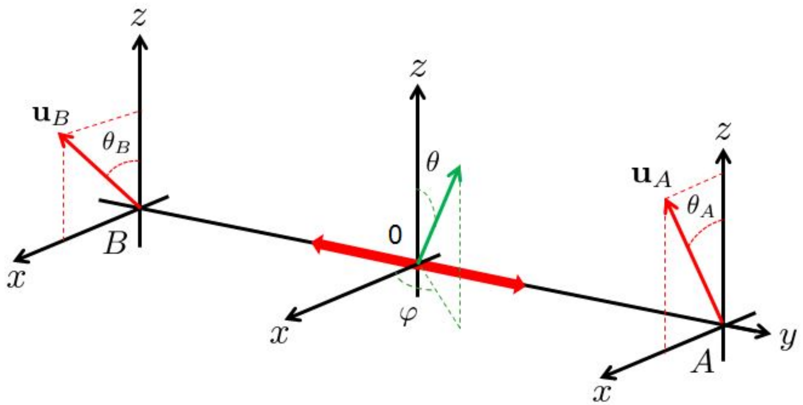
21

EPR correlations and EPW distributions

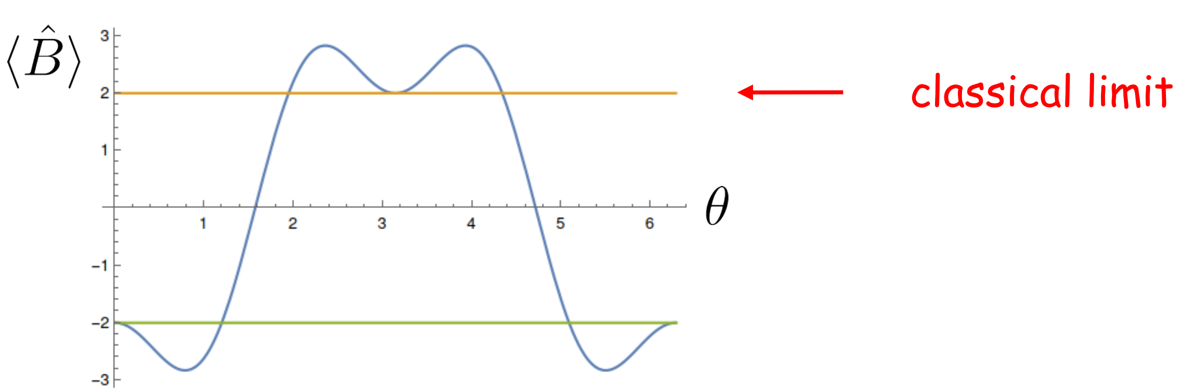
Dedicated to Professor E. P. Wigner

It is known that with Bohm's example of EPR correlations, involving particles with spin, there is an irreducible non-locality. The non-locality cannot be removed by the introduction of hypothetical variables unknown to ordinary quantum mechanics. How is it with the original EPR example involving two particles of zero spin? Here we will see that the Wigner phase space distribution[1] illuminates the problem.

"Cosmic inflation, quantum information and the pioneering role of John Bell in Cosmology", Universe 5 (2019), arXiv:1904.00083



- Bipartite system: two spins ½ travelling in opposite directions
- Quantum state: Bell State $|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|+-\rangle - |-+\rangle\right)$
- Bell operator: $\hat{B} = \boldsymbol{u}_{\rm A}\cdot\hat{S}_{\rm A}\otimes\boldsymbol{u}_{\rm B}\cdot\hat{S}_{\rm B} + \boldsymbol{u}_{\rm A}\cdot\hat{S}_{\rm A}\otimes\boldsymbol{u}'_{\rm B}\cdot\hat{S}_{\rm B} + \boldsymbol{u}'_{\rm A}\cdot\hat{S}_{\rm A}\otimes\boldsymbol{u}_{\rm B}\cdot\hat{S}_{\rm B} + \boldsymbol{u}'_{\rm A}\cdot\hat{S}_{\rm A}\otimes\boldsymbol{u}'_{\rm B}\cdot\hat{S}_{\rm B}$
- Polarizers orientation: $\theta_{\rm A}-\theta_{\rm B}=\theta,\ \theta'_{\rm A}-\theta_{\rm B}=-\theta,\ \theta_{\rm A}-\theta'_{\rm B}=-\theta,\ \theta'_{\rm A}-\theta'_{\rm B}=-3\theta$



Bell inequality in Cosmology?

- Bipartite system: two pseudo spins ½ corresponding to modes +/- k

Definitions:

$$\hat{S}_x^{\boldsymbol{k}} = \int_{-\infty}^{\infty} \mathrm{d}q_{\boldsymbol{k}}\, \mathrm{sgn}(q_{\boldsymbol{k}})\, |q_{\boldsymbol{k}}\rangle\langle q_{\boldsymbol{k}}|,$$

$$\hat{S}_y^{\boldsymbol{k}} = -i \int_{-\infty}^{\infty} \mathrm{d}q_{\boldsymbol{k}}\, \mathrm{sgn}(q_{\boldsymbol{k}})\, |q_{\boldsymbol{k}}\rangle\langle -q_{\boldsymbol{k}}|,$$

$$\hat{S}_z^{\boldsymbol{k}} = \int_{-\infty}^{\infty} \mathrm{d}q_{\boldsymbol{k}}\, |q_{\boldsymbol{k}}\rangle\langle -q_{\boldsymbol{k}}|$$

Properties:

$$(\hat{S}_x^{\boldsymbol{k}})^2 = (\hat{S}_y^{\boldsymbol{k}})^2 = (\hat{S}_z^{\boldsymbol{k}})^2 = \mathbf{1}$$

$$[\hat{S}_i^{\boldsymbol{k}}, \hat{S}_j^{\boldsymbol{k}}] = i\varepsilon_{ijk}\hat{S}_k^{\boldsymbol{k}}$$

$$W(\hat{S}_x^{\boldsymbol{k}})(q,p) = \mathrm{sign}(q)$$

$$W(\hat{S}_z^{\boldsymbol{k}})(q,p) = -\pi\delta(q)\delta(p)$$

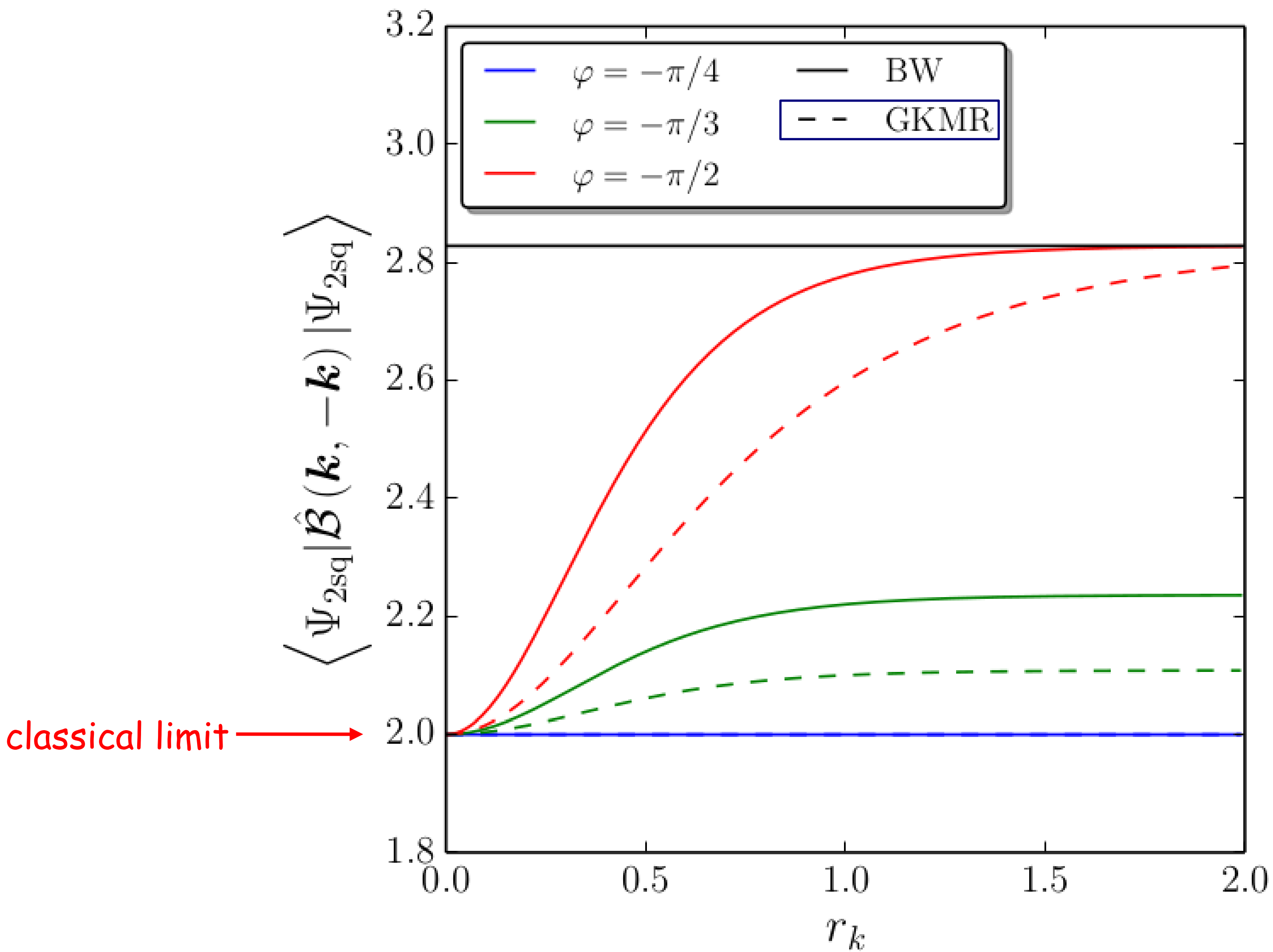
- Quantum state: two-mode squeezed state

- Bell operators:

$$\hat{\mathcal{B}}(\boldsymbol{k},-\boldsymbol{k}) = \boldsymbol{u}^{\boldsymbol{k}} \cdot \hat{S}^{\boldsymbol{k}} \otimes \boldsymbol{u}^{-\boldsymbol{k}} \cdot \hat{S}^{-\boldsymbol{k}} + \boldsymbol{u}^{\boldsymbol{k}} \cdot \hat{S}^{\boldsymbol{k}} \otimes \boldsymbol{u}^{-\boldsymbol{k}\prime} \cdot \hat{S}^{-\boldsymbol{k}} + \boldsymbol{u}^{\boldsymbol{k}\prime} \cdot \hat{S}^{\boldsymbol{k}} \otimes \boldsymbol{u}^{-\boldsymbol{k}} \cdot \hat{S}^{-\boldsymbol{k}} + \boldsymbol{u}^{\boldsymbol{k}\prime} \cdot \hat{S}^{\boldsymbol{k}} \otimes \boldsymbol{u}^{-\boldsymbol{k}\prime} \cdot \hat{S}^{-\boldsymbol{k}}$$

- Polarisers orientation: not clear how to “interpret” but can always be chosen to optimize Bell operator average

Bell inequality in Cosmology?



J. Martin & V. Vennin, PRA93, 062117 (2016); PRD 96, 063501 (2017)

Obstructions …

- Obstruction 1:
  - Bell inequality is violated because, classically, $|B|<2$
  - This is the case because, due to the locality assumption, P(Alice, Bob) =P(Alice) P(Bob). But why should it be true in Fourier space, i.e. should $P(S_k,S_{-k})=P(S_k)\,P(S_{-k})$?
  - This should be formulated in real space.
  - In real space, the discord is strongly damped and the Bell inequality violation disappears [J. Martin & V. Vennin, PRD94, 085012 (2021); JCAP 10, 036 (2021)]

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So Bell equality violation does not work … other ways? Still an open question at this stage …

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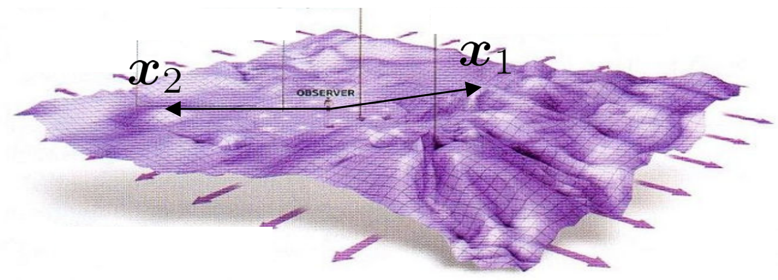
Recap

- According to cosmic inflation, the structures in our Universe originate from quantum fluctuations in the early Universe, amplified by gravitational instability and stretched by cosmic expansion
- These primordial perturbations were placed in a strongly two- mode squeezed state which is a discordant and entangled state, suggesting that quantum signatures of the perturbations should be present in the sky
- However, these signatures seems to be hidden and it remains a challenge to design a test which could reveal the origin of the primordial fluctuations
- And, in addition, one should take into account decoherence!!!
- Inflation is the only situation in Physics where GR and QM are needed to understand the theory and derive predictions and where, at the same time, we have high accuracy data
- Take away message: inflation is not only a successful scenario of the early Universe, it is also a very interesting playground for foundational issues of quantum mechanics

Real scalar field



$$\hat{z}(t,\boldsymbol{x}) = \left(\hat{\phi}(t,\boldsymbol{x}), \hat{\pi}(t,\boldsymbol{x})\right)^{\mathrm{T}}$$

$$\left[\hat{\phi}(t,\boldsymbol{x}_1), \hat{\pi}(t,\boldsymbol{x}_2)\right] = i\delta\left(\boldsymbol{x}_1 - \boldsymbol{x}_2\right)$$

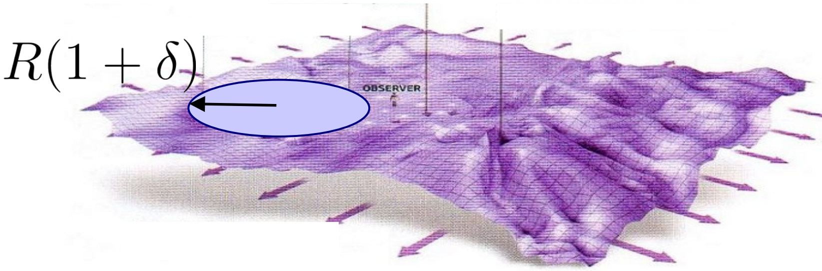
$$\langle\{\hat{z}_i(\boldsymbol{x}_1), \hat{z}_j(\boldsymbol{x}_2)\}\rangle = \int_0^{+\infty} \frac{\mathrm{d}k}{k} \mathcal{P}_{ij}(k)\,\mathrm{sinc}\left(k\left|\boldsymbol{x}_1 - \boldsymbol{x}_2\right|\right)$$

with

$$\hat{z}_i(t,\boldsymbol{x}) = \frac{1}{(2\pi)^{3/2}} \int \mathrm{d}^3\boldsymbol{k}\, \hat{z}_i(t,\boldsymbol{k})\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\left\langle\left\{\hat{z}_i^\dagger(\boldsymbol{k}_1), \hat{z}_j(\boldsymbol{k}_2)\right\}\right\rangle = \frac{2\pi^2}{k_1^3}\mathcal{P}_{ij}(\boldsymbol{k}_1)\delta\left(\boldsymbol{k}_1 - \boldsymbol{k}_2\right)$$

Coarse-grained real scalar field



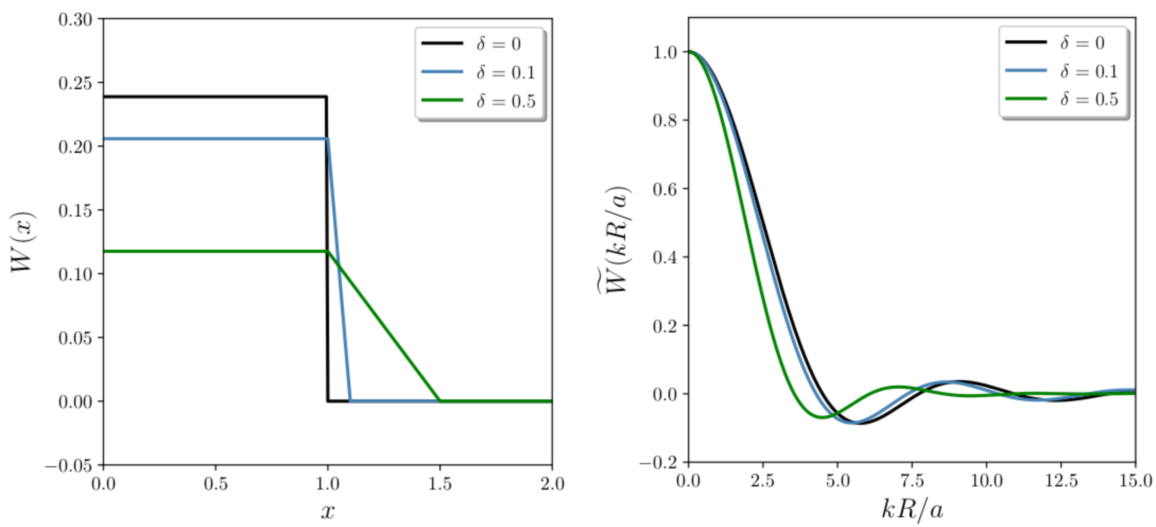
$$\hat{z}_{R,i}(\boldsymbol{x}) = \left(\frac{a}{R}\right)^3 \int \mathrm{d}^3\boldsymbol{y}\, \hat{z}_i(\boldsymbol{y}) W\left(\frac{a|\boldsymbol{y}-\boldsymbol{x}|}{R}\right)$$

Window function

$$W(x) = \frac{3}{4\pi \mathcal{F}(\delta)} \begin{cases} 1, & x \leq 1 \\ -\frac{1}{\delta}(x-1)+1, & 1 < x \leq 1+\delta \\ 0, & x > 1+\delta \end{cases}$$

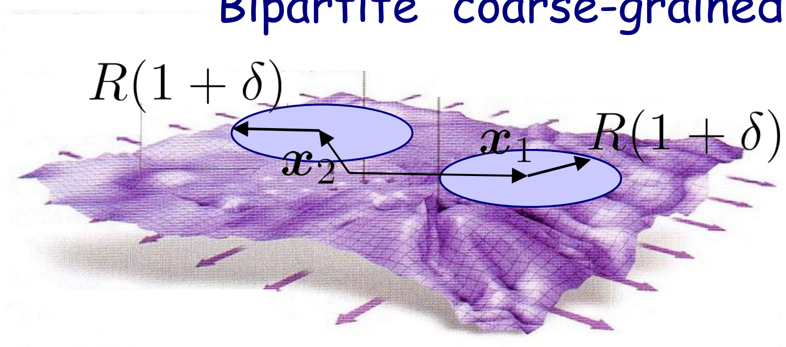
with

$$\mathcal{F}(\delta) = \frac{1}{4}(\delta+2)(\delta^2+2\delta+2)$$



Discord in real space

"Bipartite" coarse-grained real scalar field



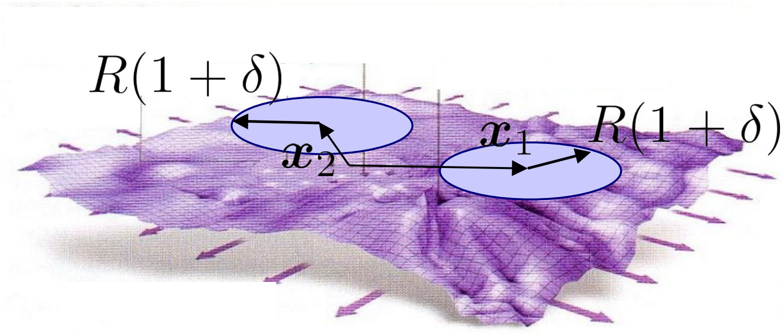
$$\hat{Z}_R(\boldsymbol{x}_1, \boldsymbol{x}_2) = \begin{pmatrix} \hat{\phi}_R(\boldsymbol{x}_1) \\ \hat{\pi}_R(\boldsymbol{x}_1) \\ \hat{\phi}_R(\boldsymbol{x}_2) \\ \hat{\pi}_R(\boldsymbol{x}_2) \end{pmatrix}$$

$$\boxed{\gamma_{ab} = \left\langle \left\{ \hat{\hat{Z}}_{R,a}(\boldsymbol{x}_1, \boldsymbol{x}_2), \hat{\hat{Z}}_{R,b}(\boldsymbol{x}_1, \boldsymbol{x}_2) \right\} \right\rangle}$$

$$\gamma = \frac{8\pi}{3G(\delta)} \left(\frac{R}{a}\right)^3 \int \frac{\mathrm{d}k}{k} \tilde{W}^2 \left(\frac{R}{a}k\right) \begin{pmatrix} \mathcal{P}_{\phi\phi}(k) & \mathcal{P}_{\phi\pi}(k) & \mathcal{P}_{\phi\phi}\,\mathrm{sinc}\left(\frac{kd}{a}\right) & \mathcal{P}_{\phi\pi}\,\mathrm{sinc}\left(\frac{kd}{a}\right) \\ - & \mathcal{P}_{\pi\pi}(k) & \mathcal{P}_{\phi\phi}(k)\,\mathrm{sinc}\left(\frac{kd}{a}\right) & \mathcal{P}_{\pi\pi}(k)\,\mathrm{sinc}\left(\frac{kd}{a}\right) \\ - & - & \mathcal{P}_{\phi\phi}(k) & \mathcal{P}_{\phi\pi}(k) \\ - & - & - & \mathcal{P}_{\pi\pi}(k) \end{pmatrix}$$

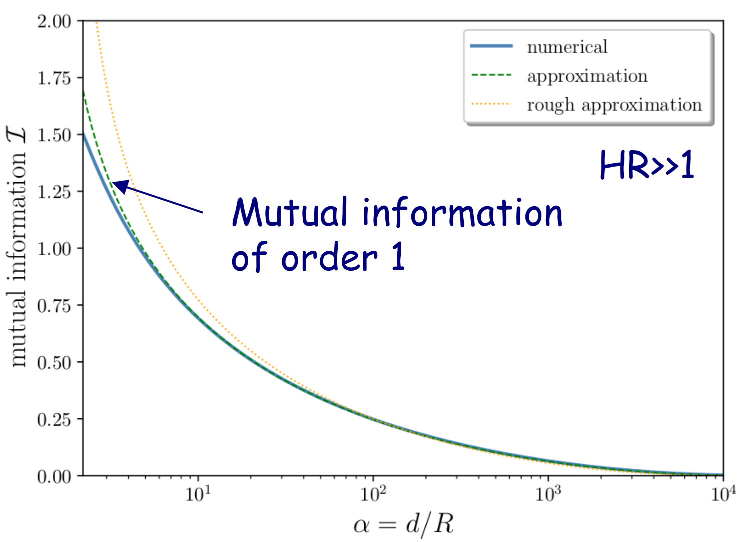
- Covariance matrix of a Gaussian system -> completely characterizes the system
- Can be computed from the mode functions of the field

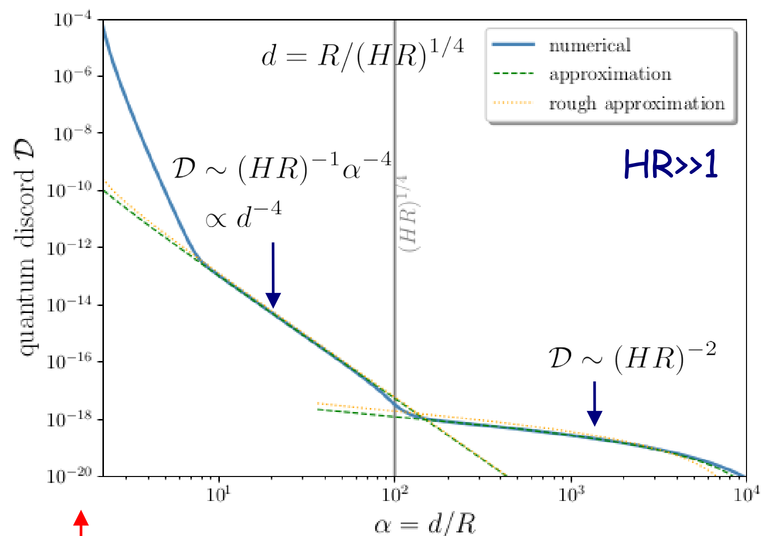
Discord in real space



- Since the system is Gaussian and its covariance matrix known, we can use the standard techniques to calculate discord

J. Martin & V. Vennin, PRD94, 085012 (2021); JCAP 10, 036 (2021)



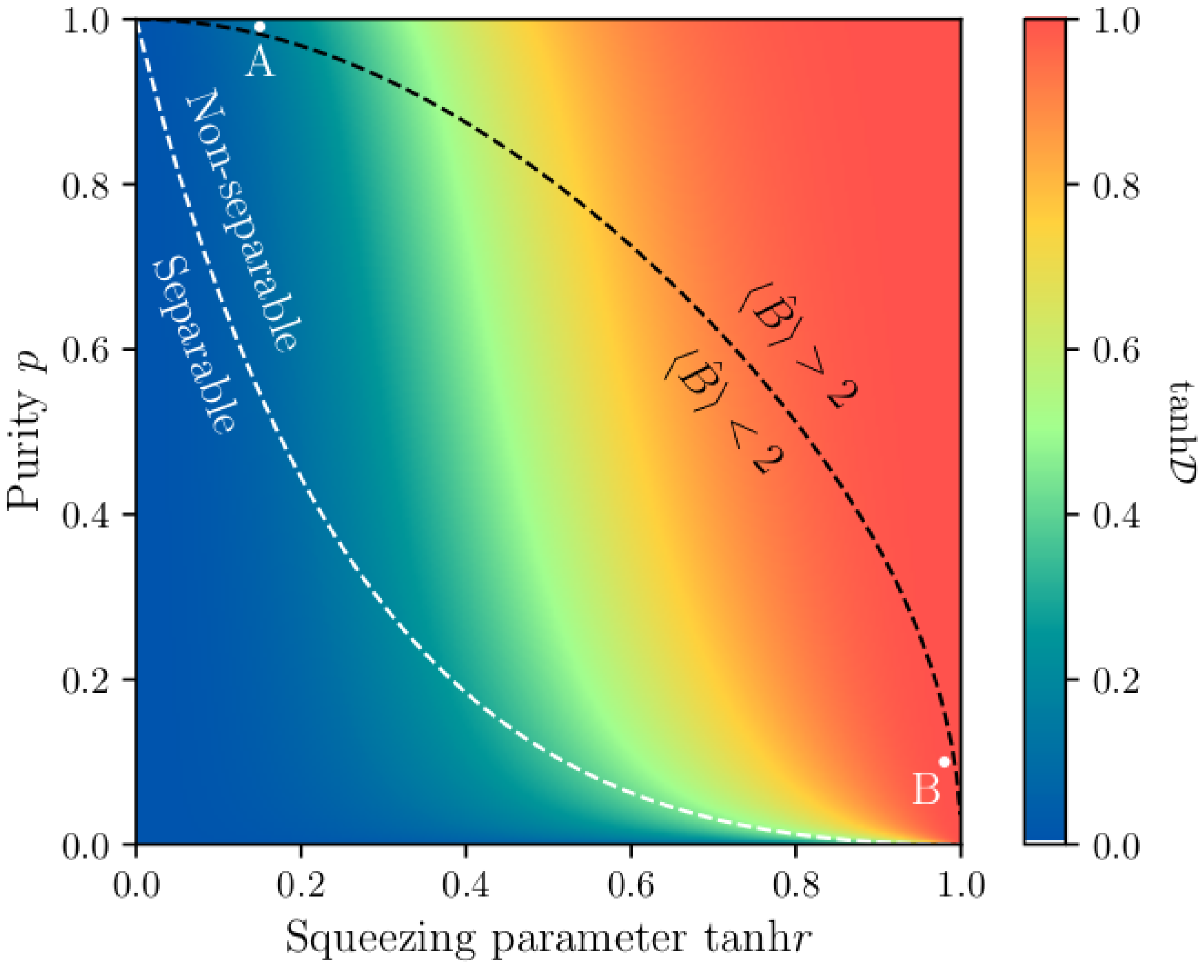


very mild dependence in d while, in flat spacetime, $\mathcal{I} \propto d^{-4}$

Discord is strongly suppressed in real space

Criterions for quantumness are not all equivalent

$$\gamma = \frac{\gamma^{\mathrm{TMSV}}}{\sqrt{p}} \longrightarrow \textbf{purity: } p = \mathrm{Tr}\left(\rho^2\right) = \frac{1}{\sqrt{\det \gamma}}$$



J. Martin, A. Micheli & V. Vennin, EPL 142, 18001 (2023)

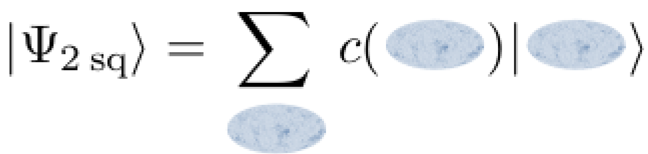
Open issues

Other important questions

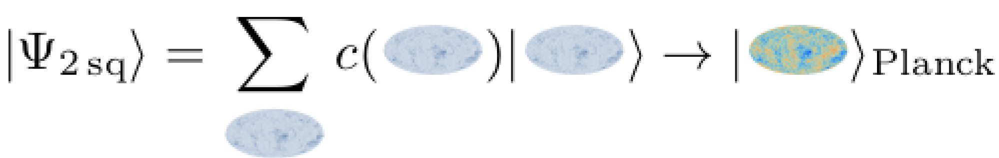
- The role of decoherence. Attempts to write a master equation for cosmological perturbations, impact for the quantum to classical transition …

  C. Burgess, R. Holman, G. Kaplanek, J. Martin & V. Vennin, arXiv:2211.11046

- The quantum measurement problem in Cosmology. The quantum state of perturbations is homogeneous and isotropic (e.g. it is invariant under the translation operator)



So the process

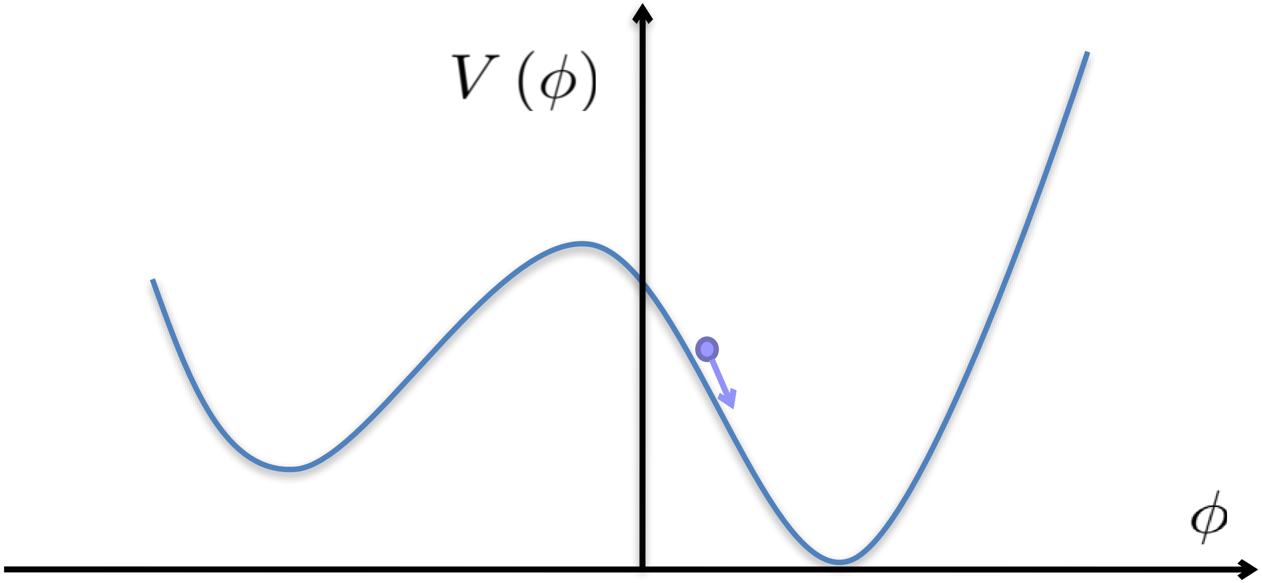


took place in the early Universe (and in absence of any observers). How?

In the early Universe matter is described by QFT. Simplest QFT model compatible with FLRW symetries = scalar field (inflaton field)

$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi)$$

$$p = \frac{\dot{\phi}^2}{2} - V(\phi)$$



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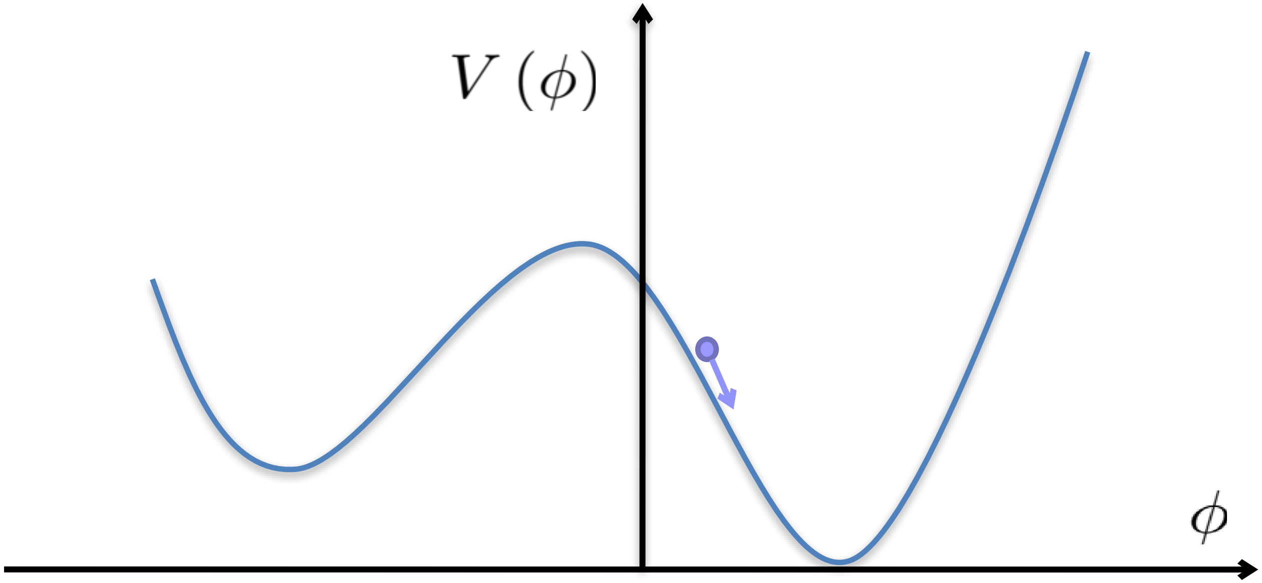
$$\left.\begin{aligned} \rho &= \frac{\dot{\phi}^2}{2} + V(\phi) \\ p &= \frac{\dot{\phi}^2}{2} - V(\phi) \end{aligned}\right\} \quad V(\phi) \gg \frac{\dot{\phi}^2}{2} \Rightarrow p < 0$$



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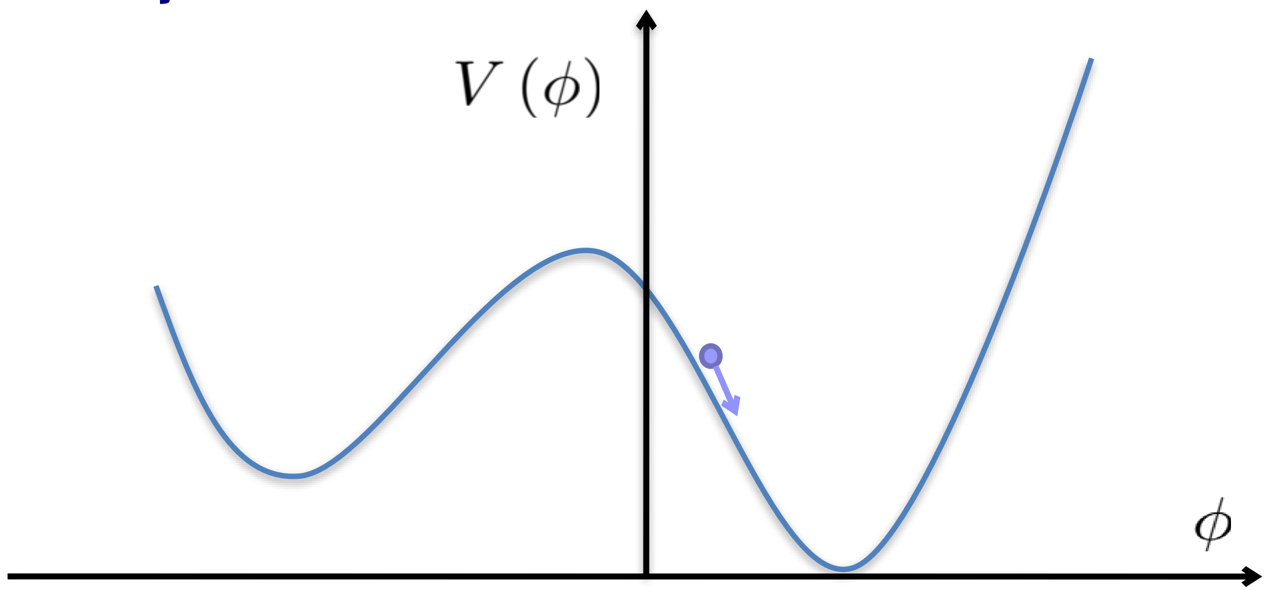
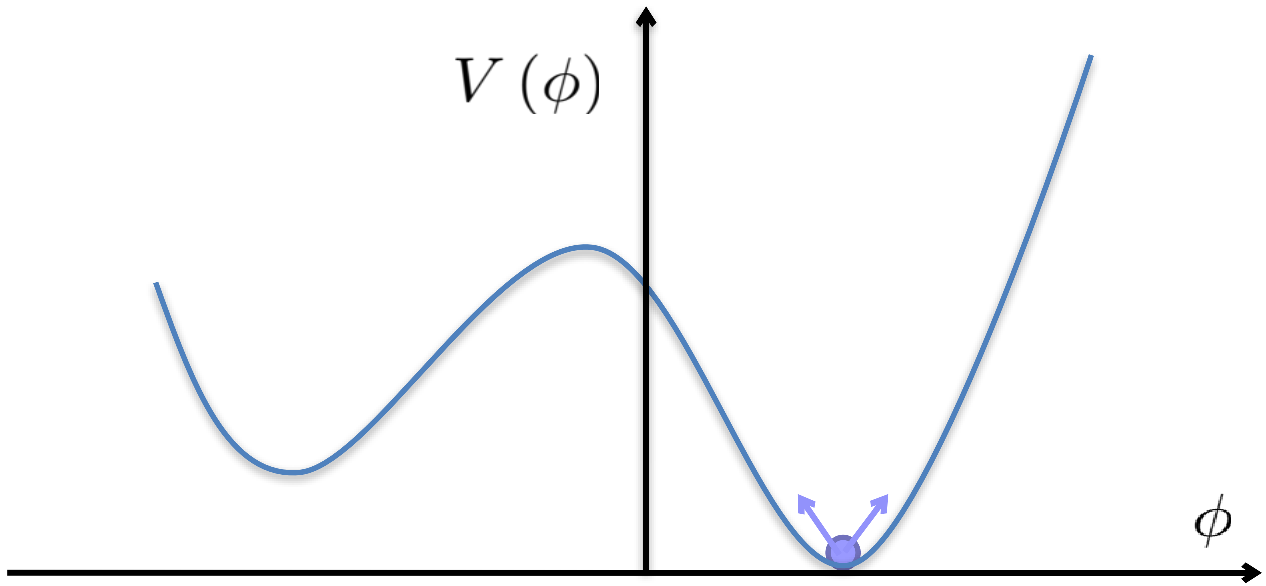
$$\left.\begin{aligned} \rho &= \frac{\dot{\phi}^2}{2} + V(\phi) \\ p &= \frac{\dot{\phi}^2}{2} - V(\phi) \end{aligned}\right] \quad V(\phi) \gg \frac{\dot{\phi}^2}{2} \Rightarrow p < 0 \quad \longleftrightarrow \quad \frac{\ddot{a}}{a} = -\frac{1}{6M_{\mathrm{Pl}}^2}(\rho + \underbrace{3p}_{\text{Relativistic term}}) > 0$$



- If the scalar field moves slowly (the potential is flat), then pressure is negative which, in the context of GR, means accelerated expansion and, hence, inflation takes place.
- During inflation, the Universe expands exponentially: $a(t) \propto \exp(Ht), \quad H = \dot{a}/a$

Inflation (usually) stops when the field reaches a part of the potential which is no longer flat enough to support inflation; this happens in the vicinity of the minimum of the potential



The field oscillates, decays and the decay products thermalize …then the radiation dominated era starts …

- Inflationary power spectrum

$$\langle\Psi|\hat{v}\left(\eta,\boldsymbol{x}\right)\hat{v}\left(\eta,\boldsymbol{x}+\boldsymbol{r}\right)|\Psi\rangle = \int\prod_{\boldsymbol{k}\in\mathbb{R}^{3+}}\mathrm{d}v_{\boldsymbol{k}}^{\mathrm{R}}\mathrm{d}v_{\boldsymbol{k}}^{\mathrm{I}}\Psi^{\star}(\eta,v_{\boldsymbol{k}}^{\mathrm{R}},v_{\boldsymbol{k}}^{\mathrm{I}})v(\eta,\boldsymbol{x})v(\eta,\boldsymbol{x}+\boldsymbol{r})\Psi(\eta,v_{\boldsymbol{k}}^{\mathrm{R}},v_{\boldsymbol{k}}^{\mathrm{I}})$$

$$= \frac{1}{2\pi}\int_0^{+\infty}\frac{\mathrm{d}p}{p}\frac{\sin(pr)}{pr}\left|f_p(\eta)\right|^2$$

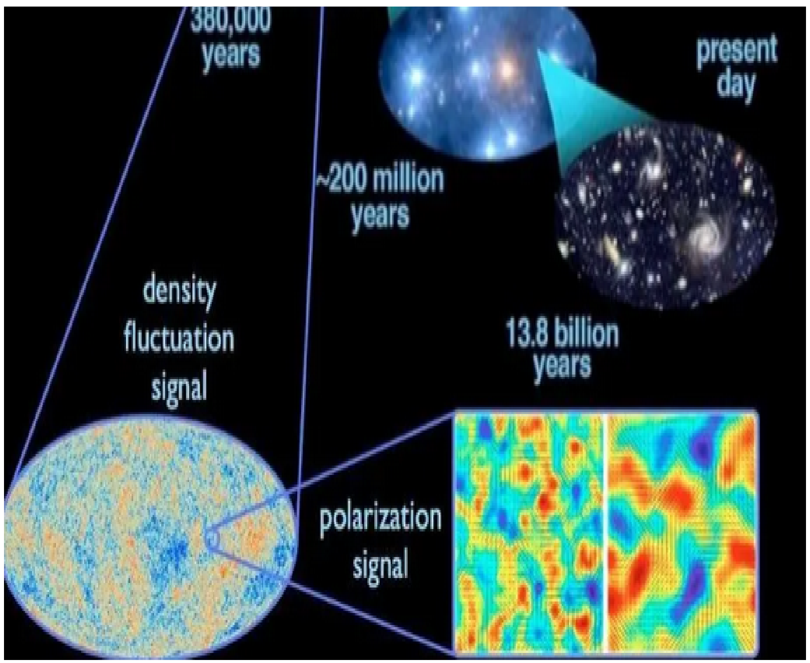
$$P_\zeta(k) = \frac{p^3}{2\pi^2}|f_{\boldsymbol{k}}|^2 = \frac{H^2}{8\pi^2\epsilon_1 M_{\mathrm{Pl}}^2}$$

$$\times\left[1-2(C+1)\epsilon_1 - C\epsilon_2 - \underbrace{(2\epsilon_1+\epsilon_2)}\ln\frac{k}{k_*}\right]$$

$$= n_{\mathrm{S}} - 1 \ll 1$$

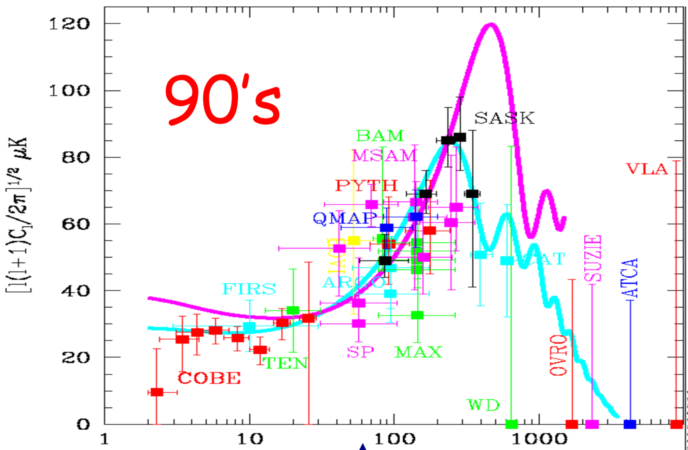
$$\epsilon_1 = -\frac{\dot{H}}{H^2} = \frac{M_{\mathrm{Pl}}^2}{2}\left(\frac{V_\phi}{V}\right)^2$$

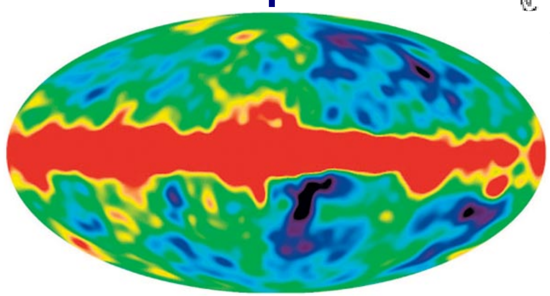
$$\epsilon_2 = 2M_{\mathrm{Pl}}^2\left[\left(\frac{V_\phi}{V}\right)^2 - \frac{V_{\phi\phi}}{V}\right]$$



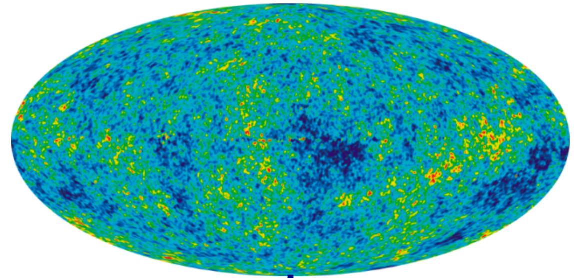
Prediction: the spectral index should be close to one but different from one

CMB temperature anisotropies: from COBE to Planck

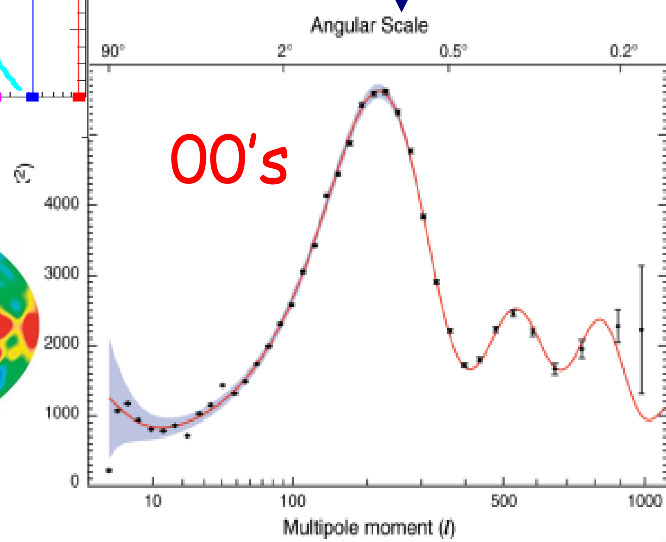




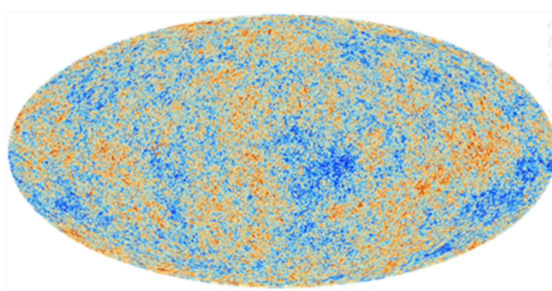
COBE (1992)

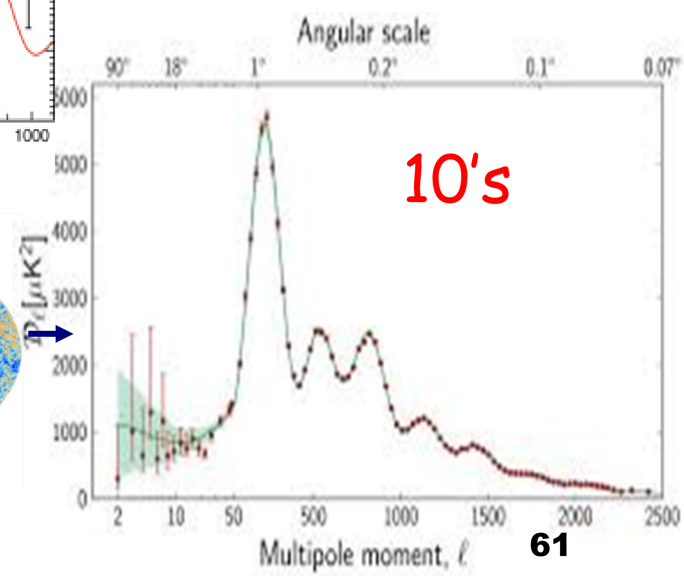


WMAP (2003)



Planck (2013 & 2015 & 2018)





Planck Measurements

- Universe spatially flat $\Omega_{\mathcal{K}} = -0.040^{+0.038}_{-0.041}$

- Adiabatic perturbations $\alpha^{(2,2500)}_{\mathcal{R}\mathcal{R}} \in [0.985, 0.999]$

- Gaussian perturbations $f_{\rm NL}^{\rm loc} = 0.8 \pm 5$, $f_{\rm NL}^{\rm eq} = -4 \pm 43$

- Almost scale invariant power spectrum $n_{\rm S} = 0.9645 \pm 0.0049$

- Background of quantum gravitational waves $r < 0.07$

Planck Measurements

- Universe spatially flat $\Omega_{\mathcal{K}} = -0.040^{+0.038}_{-0.041}$

- Adiabatic perturbations $\alpha^{(2,2500)}_{\mathcal{R}\mathcal{R}} \in [0.985, 0.999]$

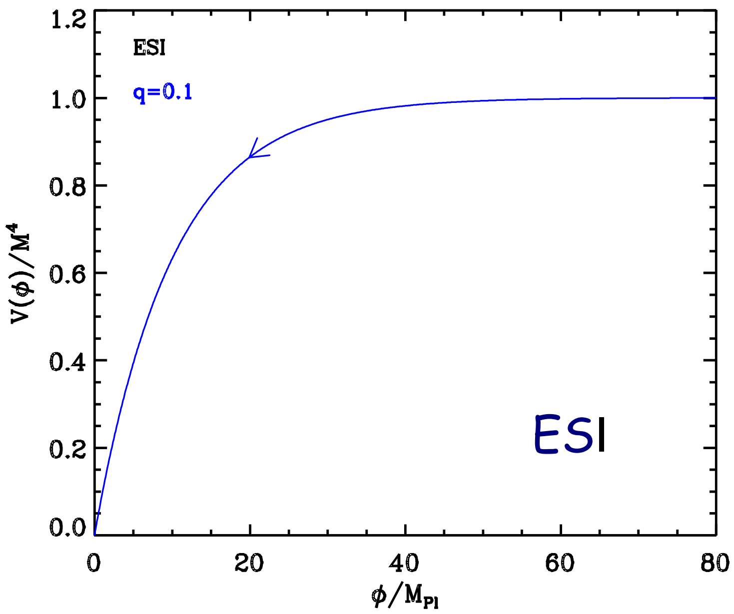
- Gaussian perturbations $f^{\rm loc}_{\rm NL} = 0.8 \pm 5$ $f^{\rm eq}_{\rm NL} = -4 \pm 43$

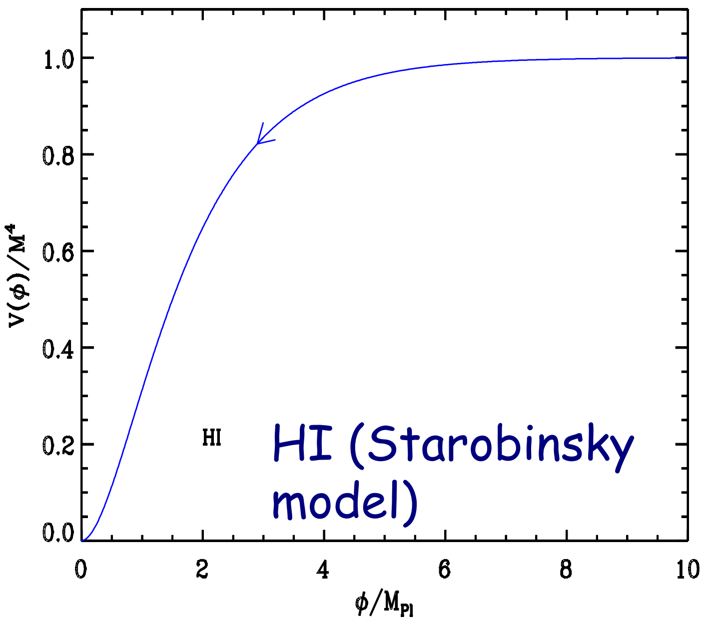
- Almost scale invariant power spectrum $n_{\rm S} = 0.9645 \pm 0.0049$

- Background of quantum gravitational waves $r < 0.07$

Indirect proof that perturbations are quantum!

Plateau inflationary models are the winners!





$$V(\phi) = M^4 \left(1 - \mathrm{e}^{-q\phi/M_{\mathrm{Pl}}}\right)$$

$$V(\phi) = M^4 \left(1 - \mathrm{e}^{-\sqrt{2/3}\phi/M_{\mathrm{Pl}}}\right)^2$$

J. Martin, C. Ringeval and V. Vennin, Phys. Dark Univ. 5-6 (2014) 75, arXiv:1303.3787

J. Martin, C. Ringeval, R. Trotta and V. Vennin, JCAP 1403 (2014) 039, arXiv1312.3529

There are two ways of treating the interactions

$$S[\phi,\chi] = -\int \mathrm{d}^4x \left(\underbrace{\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \frac{1}{2}\partial^\mu\chi\partial_\mu\chi + \frac{1}{2}m^2\phi^2}_{\text{Free fields}}\right)$$

Interactions in QFT are usually described by non-linear terms

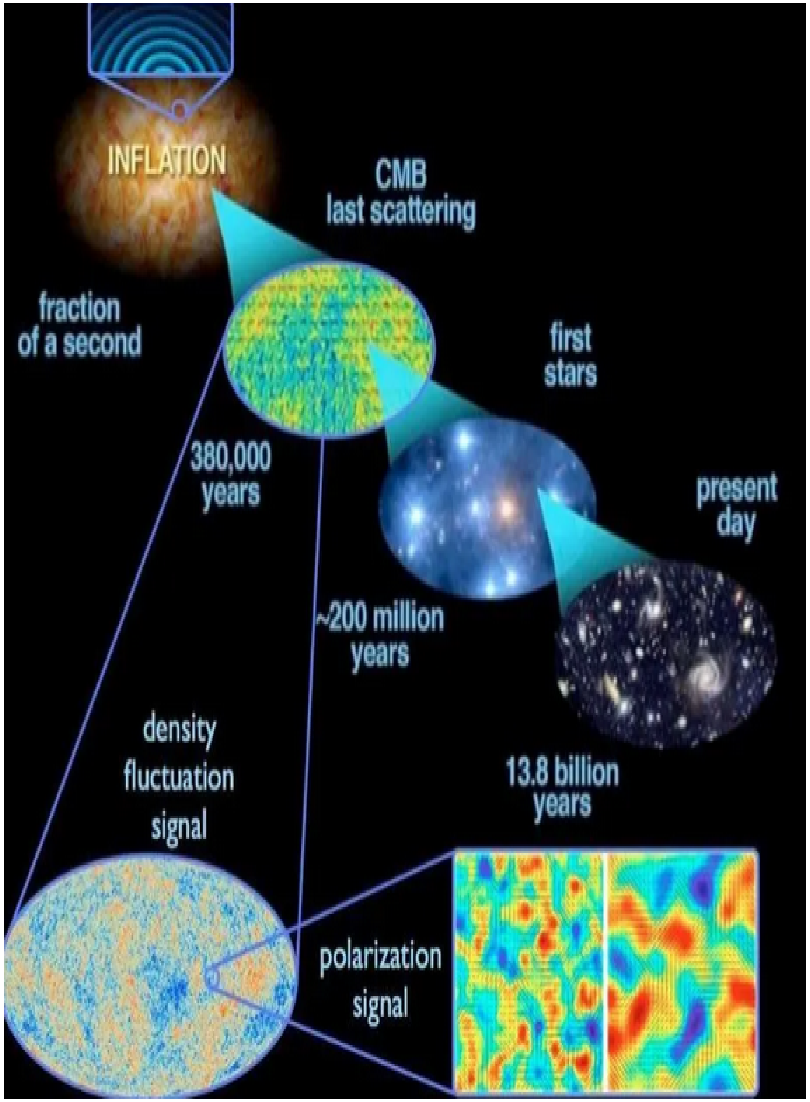
$$S[\phi,\chi] = -\int \mathrm{d}^4x \left(\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \frac{1}{2}\partial^\mu\chi\partial_\mu\chi + \frac{1}{2}m^2\phi^2 + \underbrace{\frac{\lambda}{4}\phi^4 + g\phi^2\chi^2 + \cdots}_{\text{interactions}}\right)$$

Interactions with a classical source

$$S[\phi,\chi] = -\int \mathrm{d}^4x \left[\frac{1}{2}\partial^\mu\phi\partial_\mu\phi + \frac{1}{2}\partial^\mu\chi\partial_\mu\chi + \frac{1}{2}\boxed{m^2(t)}\phi^2\right]$$

The system remains Gaussian:

- Inflationary structure formation
- Schwinger effect
- Dynamical Casimir effect …



- Interpretation of squeezing parameters

$$k^{1/2}\hat{v}_{\boldsymbol{k}}^{-\varphi_{\boldsymbol{k}}} = \cos\varphi_{\boldsymbol{k}}\left(k^{1/2}\hat{v}_{\boldsymbol{k}}^{\mathrm{R}}\right) + \sin\varphi_{\boldsymbol{k}}\left(k^{-1/2}\hat{p}_{\boldsymbol{k}}^{\mathrm{R}}\right)$$

$$k^{-1/2}\hat{p}_{\boldsymbol{k}}^{-\varphi_{\boldsymbol{k}}} = -\sin\varphi_{\boldsymbol{k}}\left(k^{1/2}\hat{v}_{\boldsymbol{k}}^{\mathrm{R}}\right) + \cos\varphi_{\boldsymbol{k}}\left(k^{-1/2}\hat{p}_{\boldsymbol{k}}^{\mathrm{R}}\right)$$

$$\downarrow$$

$$\left\langle k^{-1}\left(v_{\boldsymbol{k}}^{-\varphi_{\boldsymbol{k}}}\right)^2\right\rangle = \frac{1}{2}e^{-2r_{\boldsymbol{k}}}$$

$$\left\langle k^{-1}\left(p_{\boldsymbol{k}}^{-\varphi_{\boldsymbol{k}}}\right)^2\right\rangle = \frac{1}{2}e^{2r_{\boldsymbol{k}}}$$

Quantum state of the CMB: a strongly two-mode squeezed state

- In an <u>inflationary background</u>, using the equation of motion of the Mukhanov-Sasaki variable, one can infer the time <u>behavior of the squeezing parameters</u>

- On large scales:

$$r_{\boldsymbol{k}} \sim -2\ln(-k\eta) = 2\ln\left(\frac{a}{a_*}\right)$$

$$\varphi_{\boldsymbol{k}} \sim k\eta = -\frac{a_*}{a}$$