First-order phase transitions with large bubbles: beyond conformal fluid and flat spacetime

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JCAP 03 (2024) 059

JCAP 01 (2025) 100

ArXiv 204.08037 [gr-qc].

In collaboration with Mark Hindmarsh and Jani Dahl

Helsinki, 17.09.2025

HELSINKI INSTITUTE OF PHYSICS

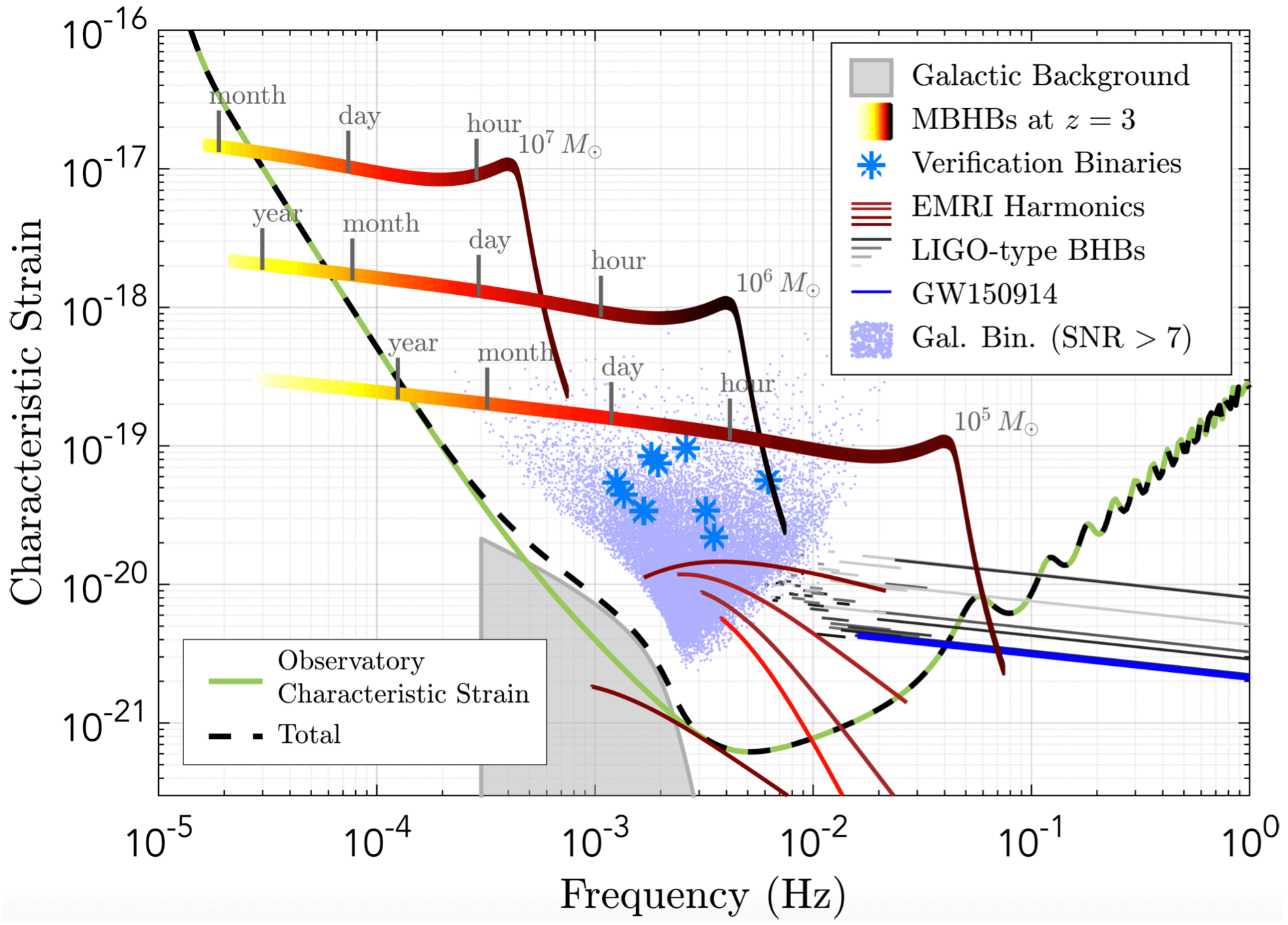
Countdown for Laser Interferometer Spasce Antenna (2035)!

Image created by Anna Kormu

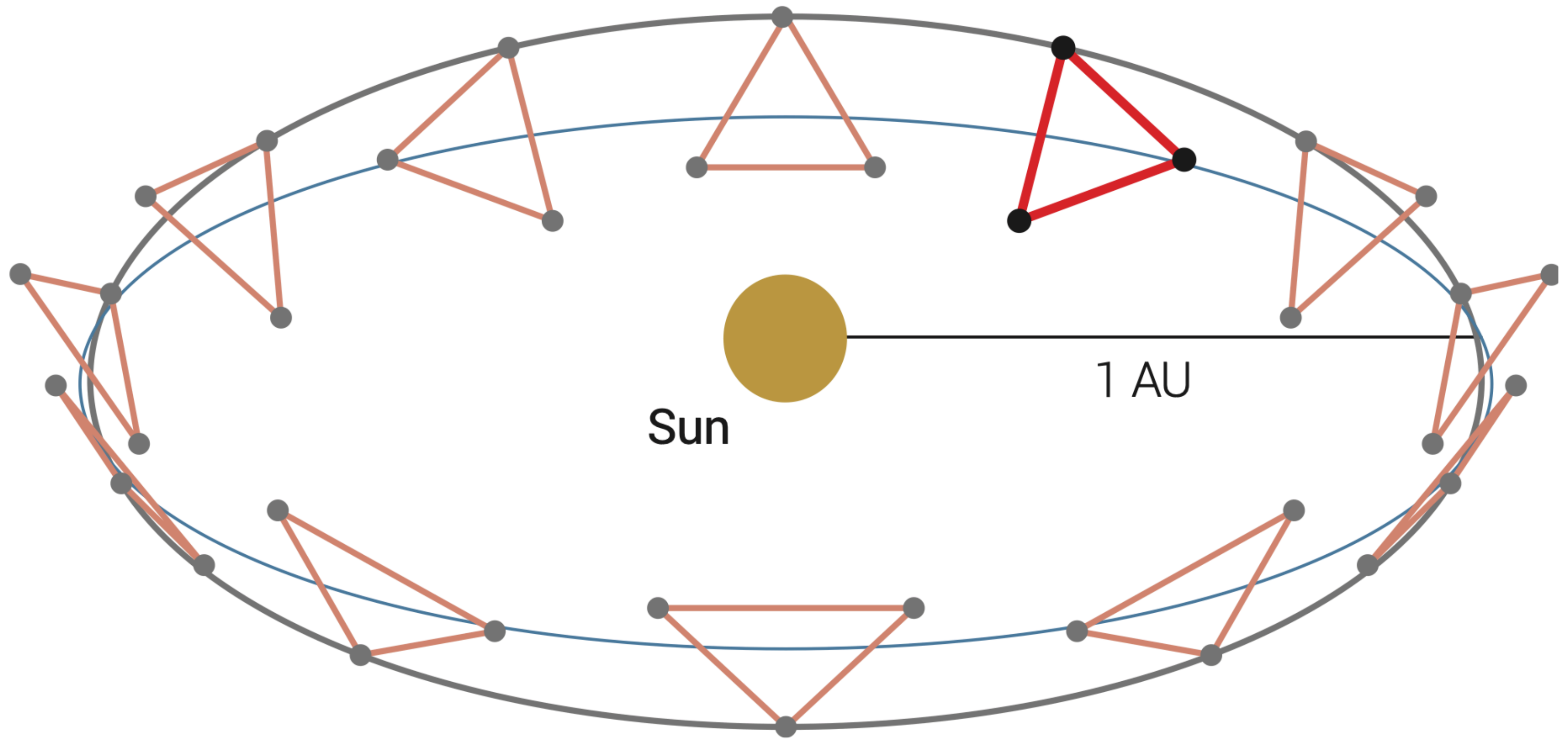
Countdown for LISA (2035)!

(Amaro-Seoane et al. 2017)

- Three free falling satellites, arm length $2.5 \times 10^9\,\mathrm{m}$
- Astrophysics sources



arXiv:1702.00786



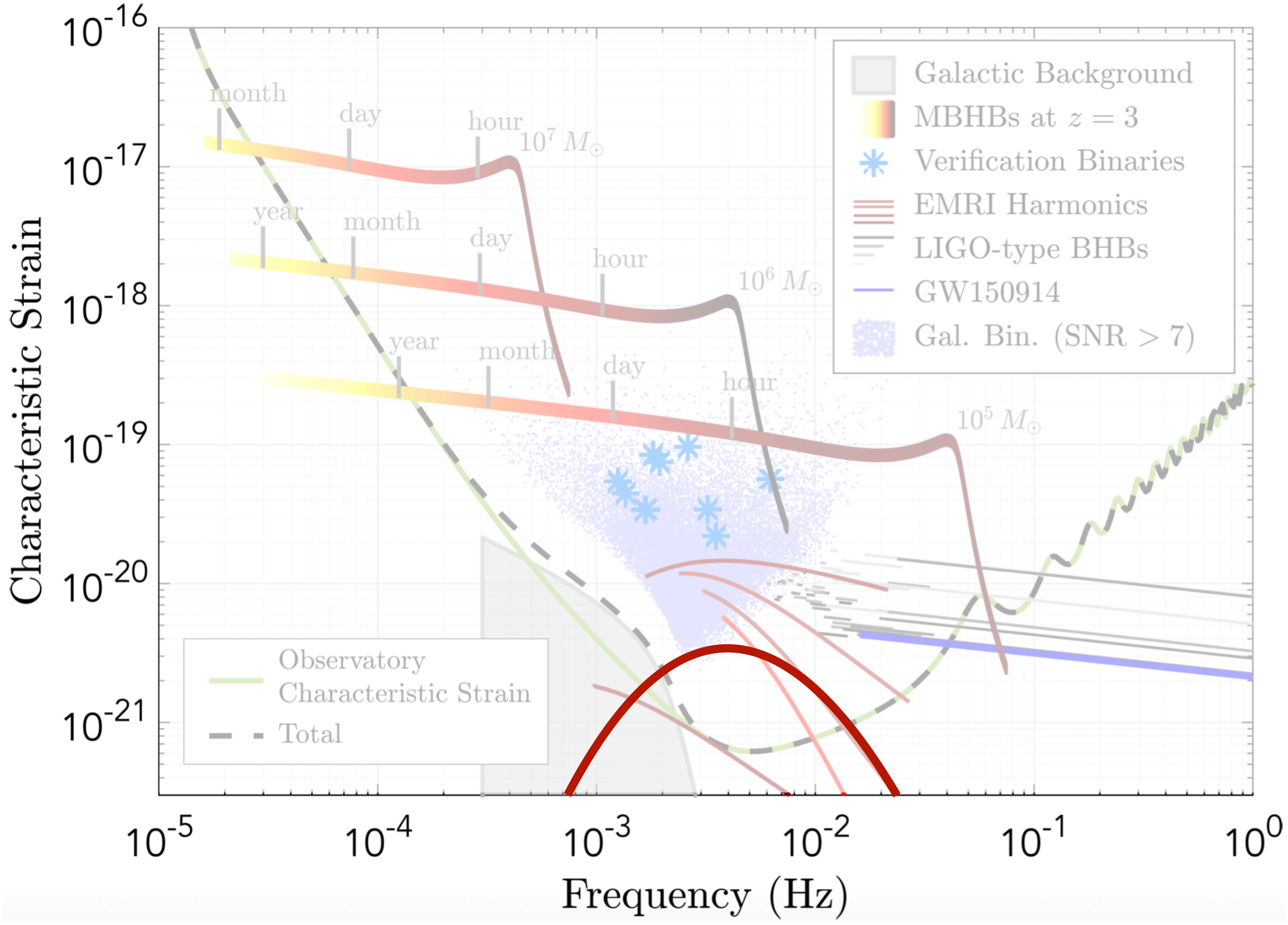
Laser Interferometer Space Antenna, arXiv:1702.00786

Countdown for LISA (2035)!

(Amaro-Seoane et al. 2017)

- Three free falling satellites, arm length $2.5 \times 10^9\,\mathrm{m}$

- Cosmology?



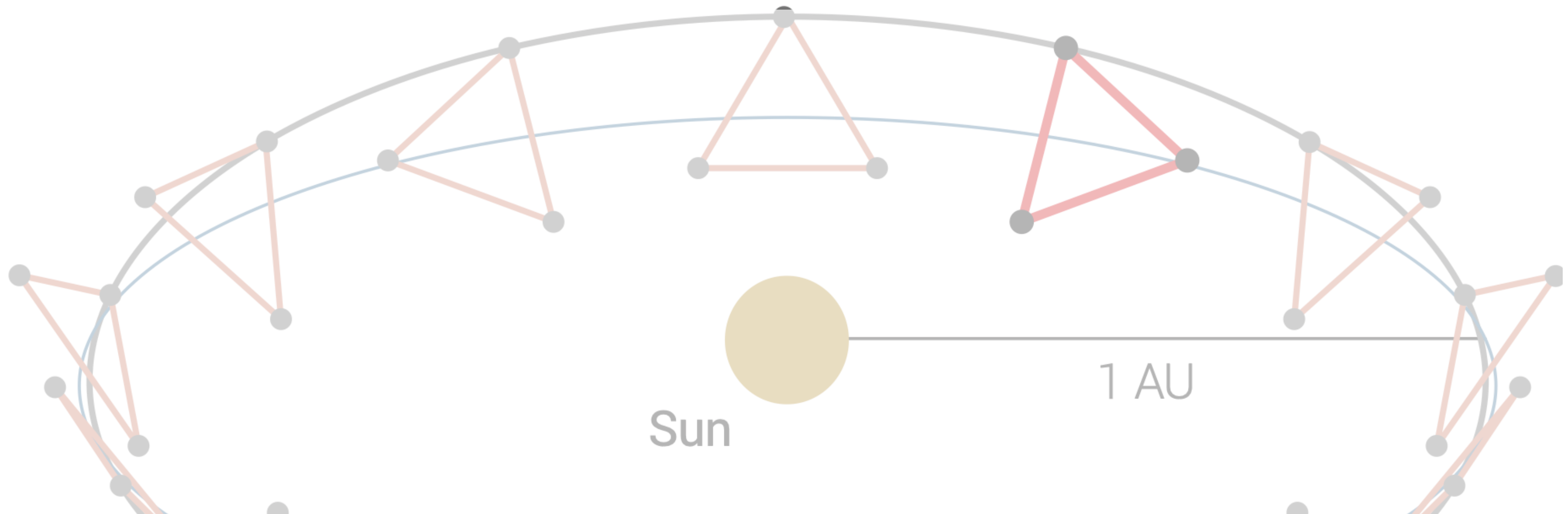
arXiv:1702.00786

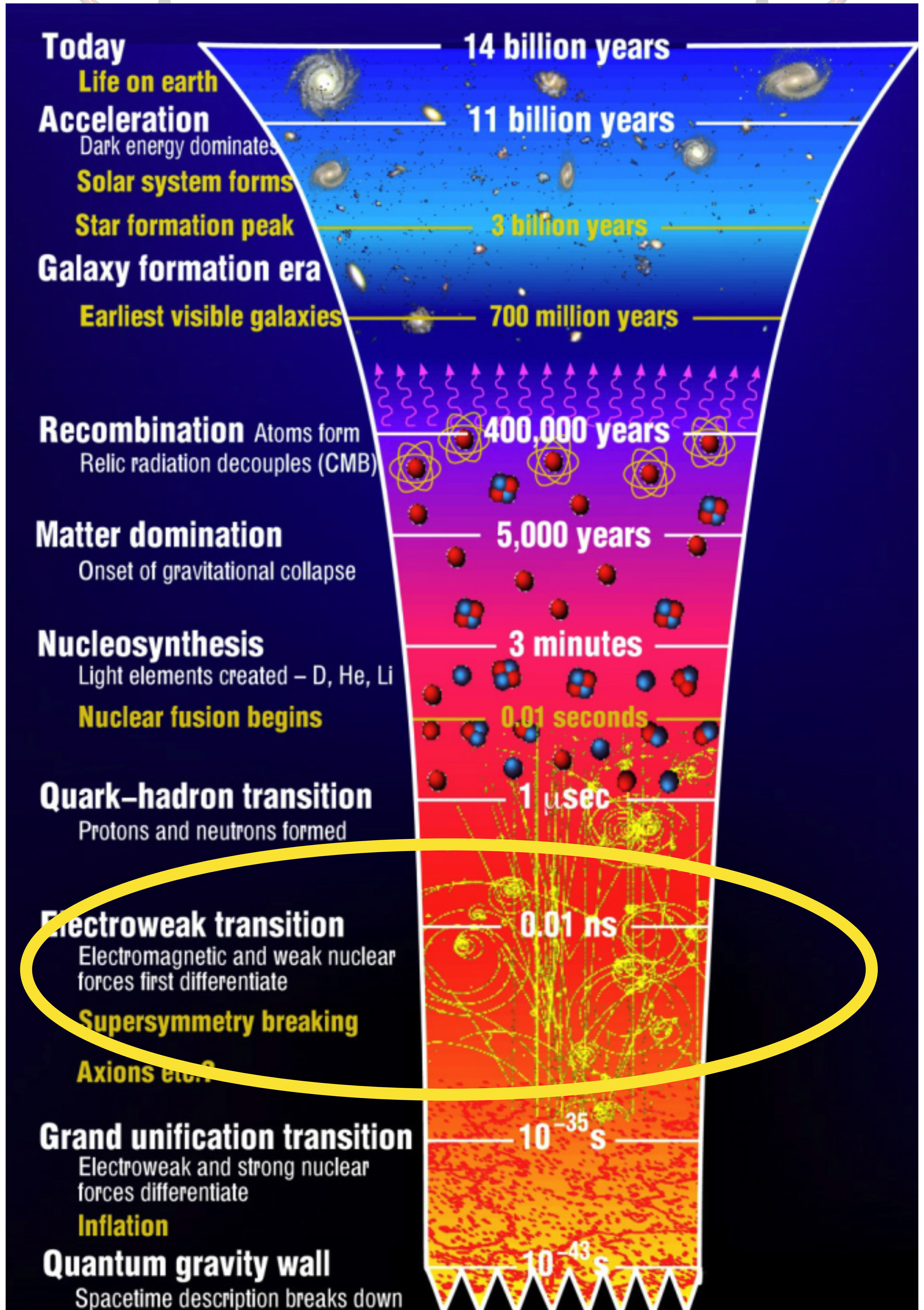
Characteristic frequency redshifted to today

$\sim 10^{-6}\,\mathrm{Hz}$

$\sim 10^{-3}\,\mathrm{Hz}$

$\gtrsim 10^{8}\,\mathrm{Hz}$





Center fot theoretical cosmology

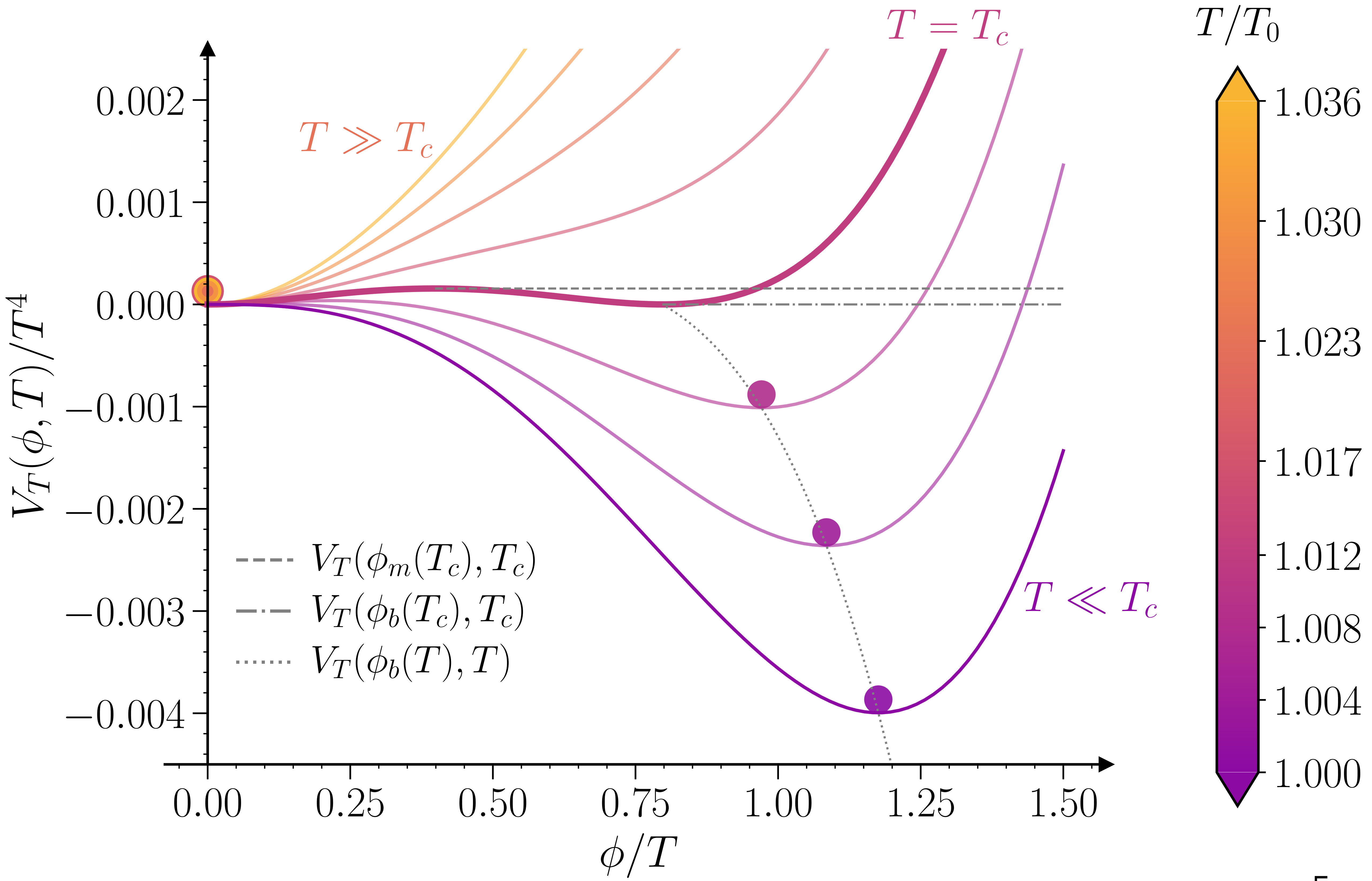
First-order phase transitions in particle theories

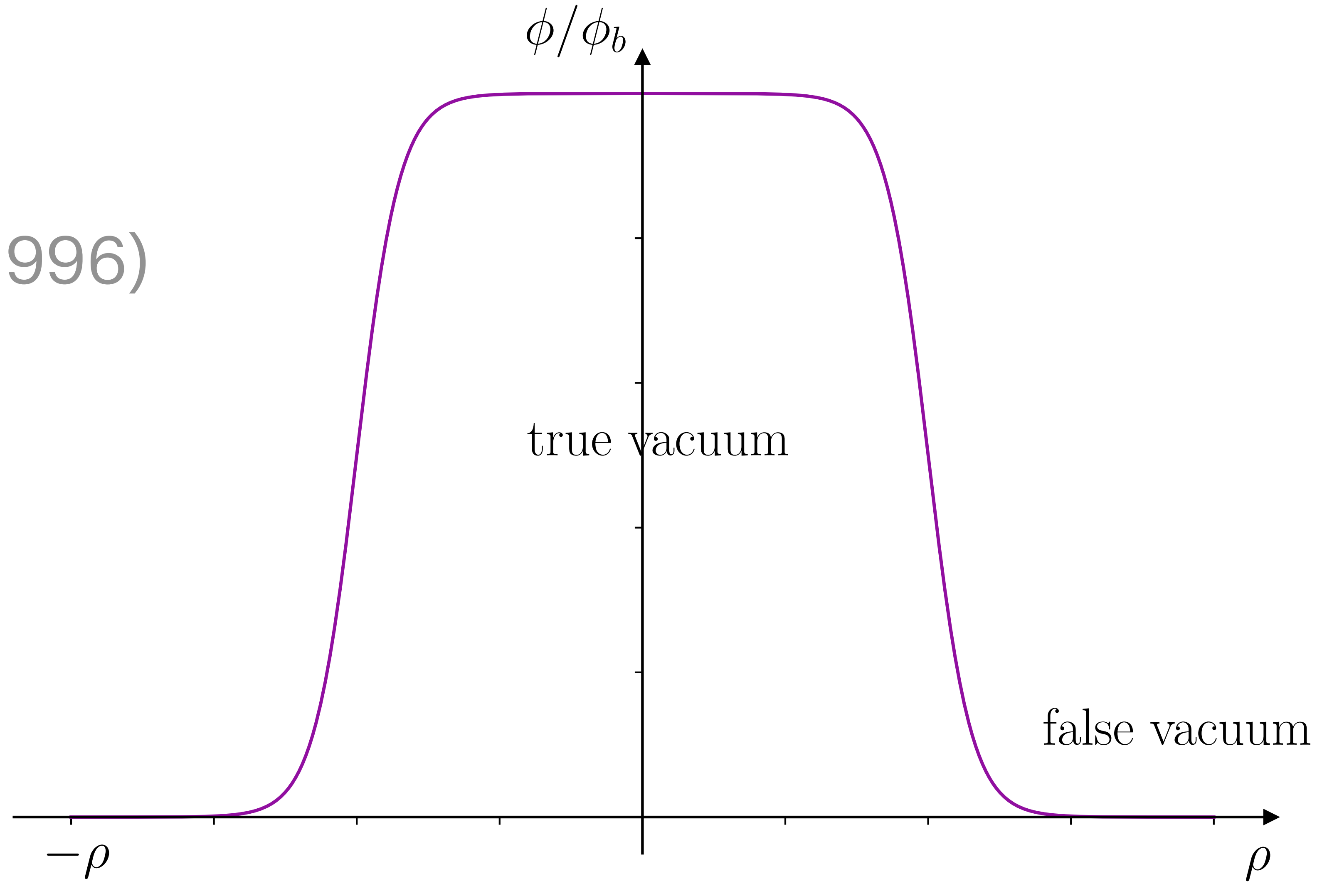
Kirzhnits 1972, Kirzhnits & Linde 1972, Dine et al. 1992, Coleman, 1977, Linde 1983

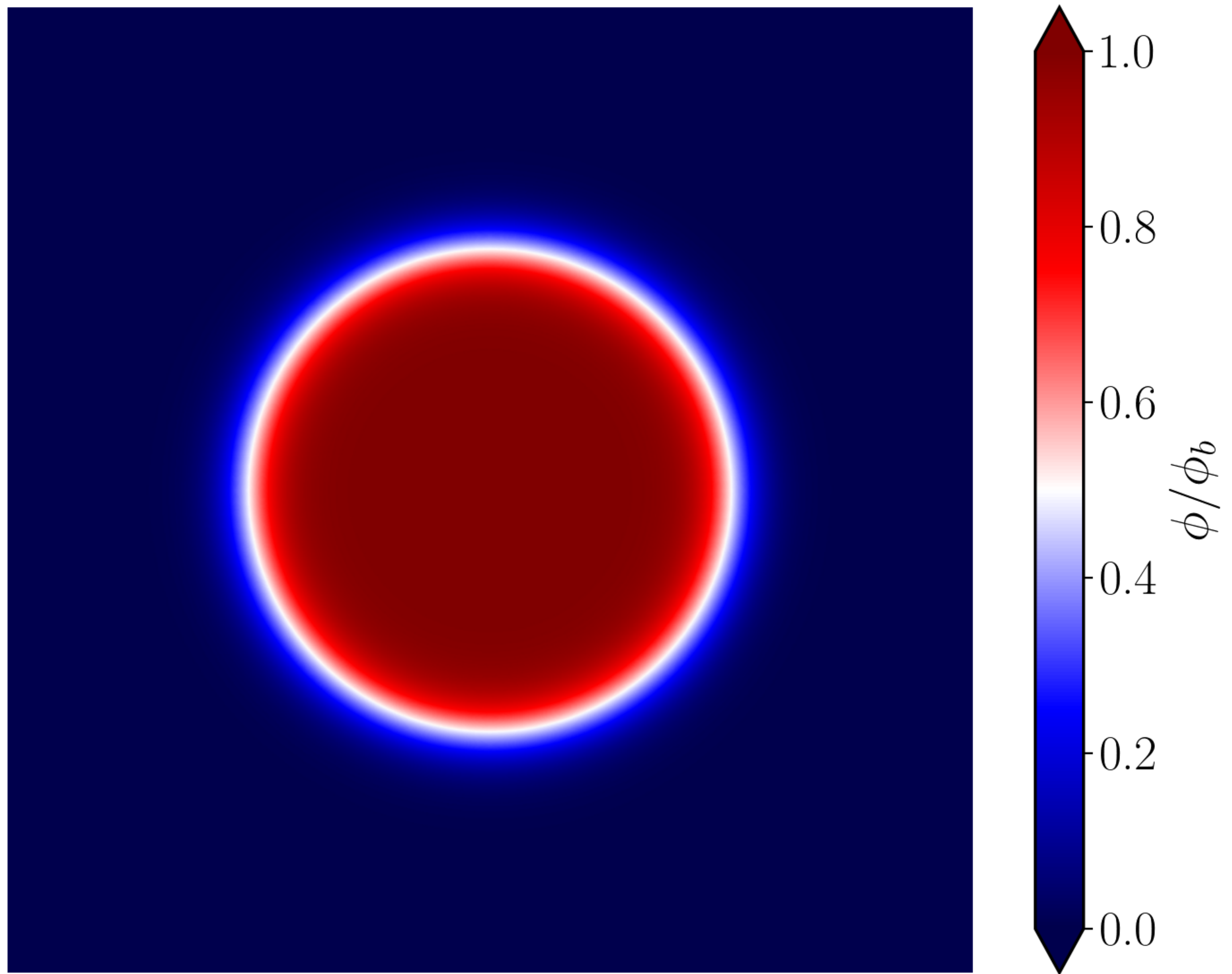
Electroweak phase transition is:

- Crossover in the Standard Model Kajantie et al., Phys.Rev.Lett. 77 (1996)
- First order in many theories beyond the Standard Model

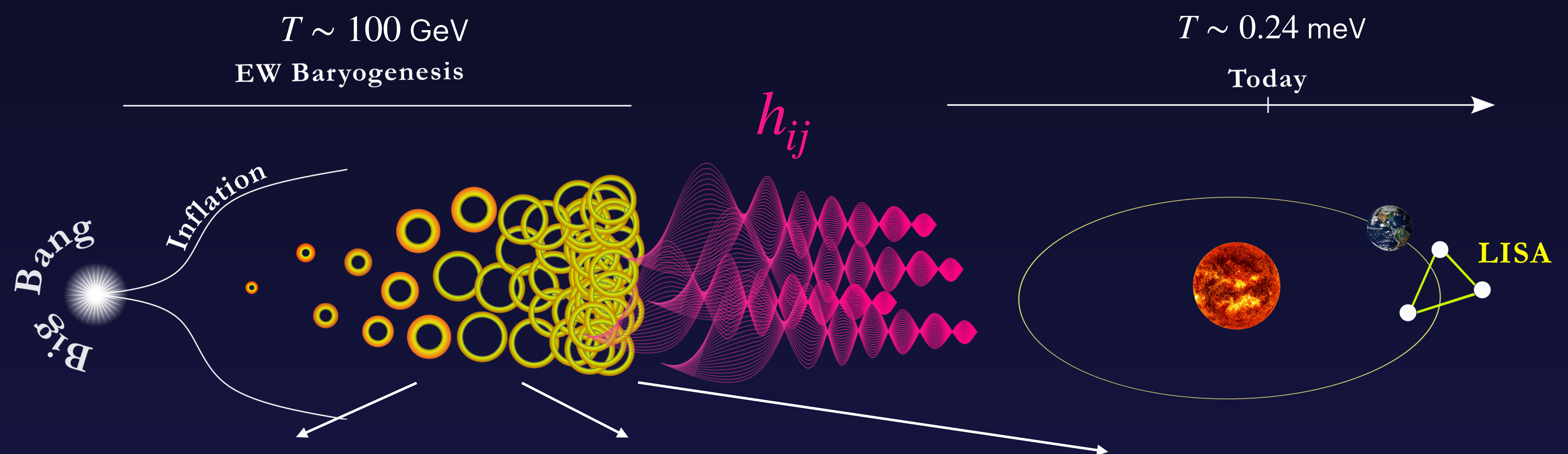
Test for physics beyond the Standard Model



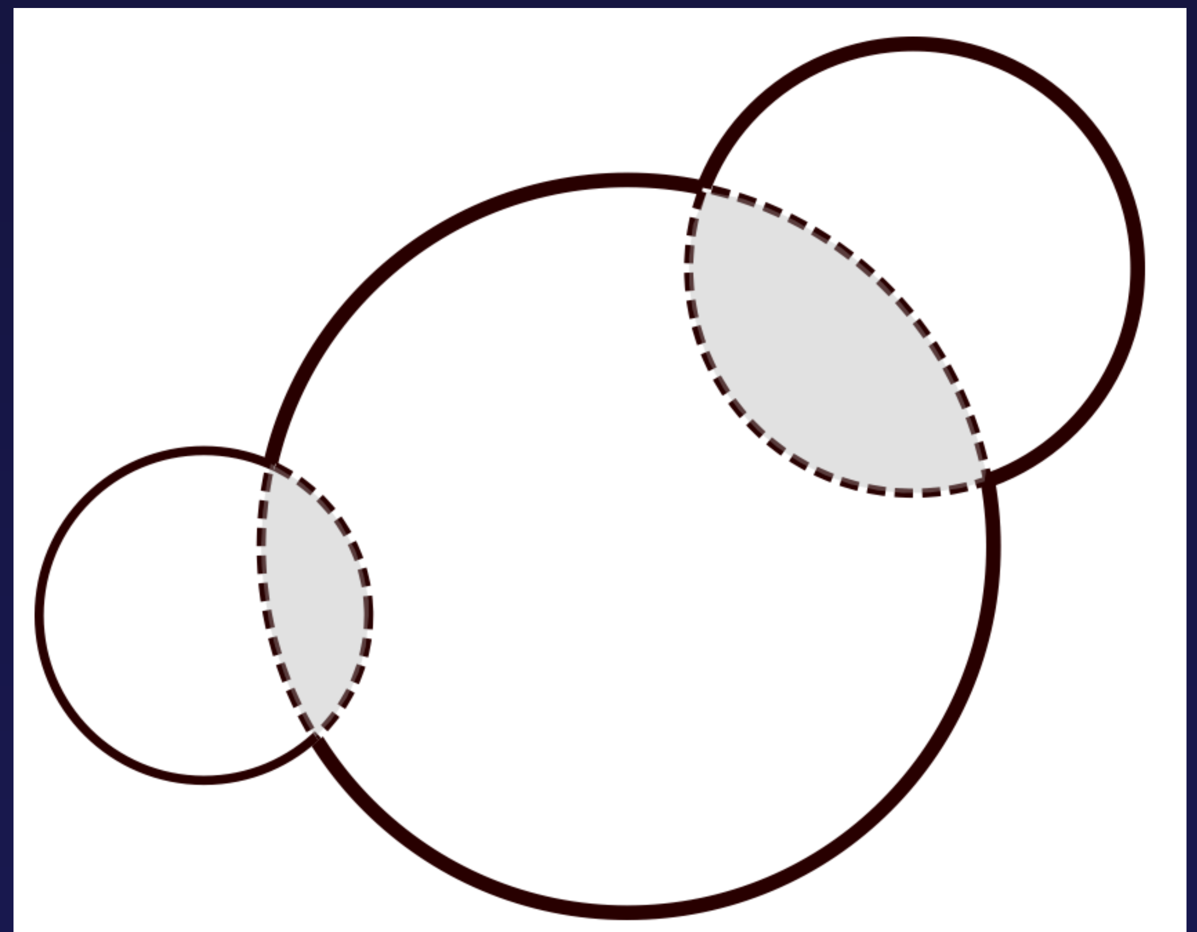




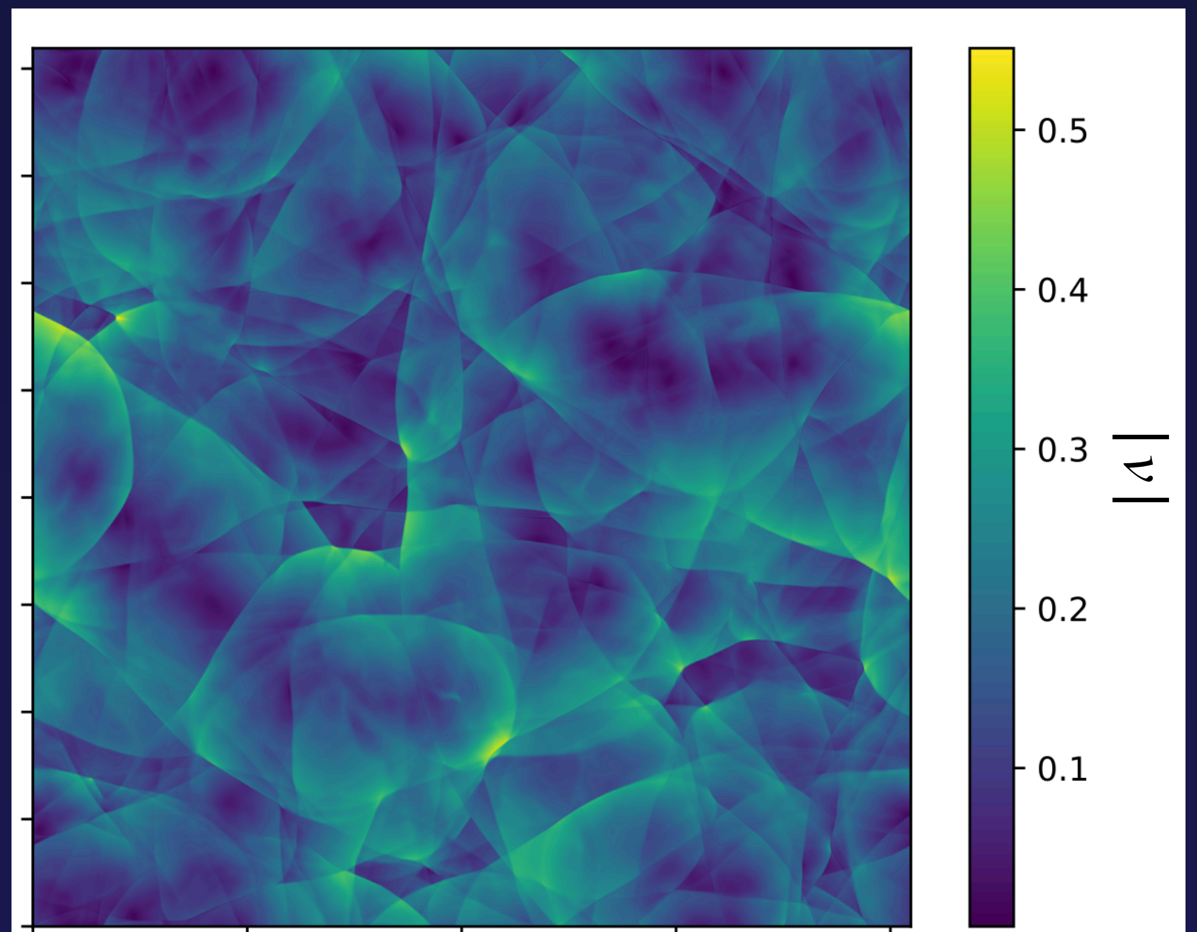
First order phase transitions in a snapshot



Collision

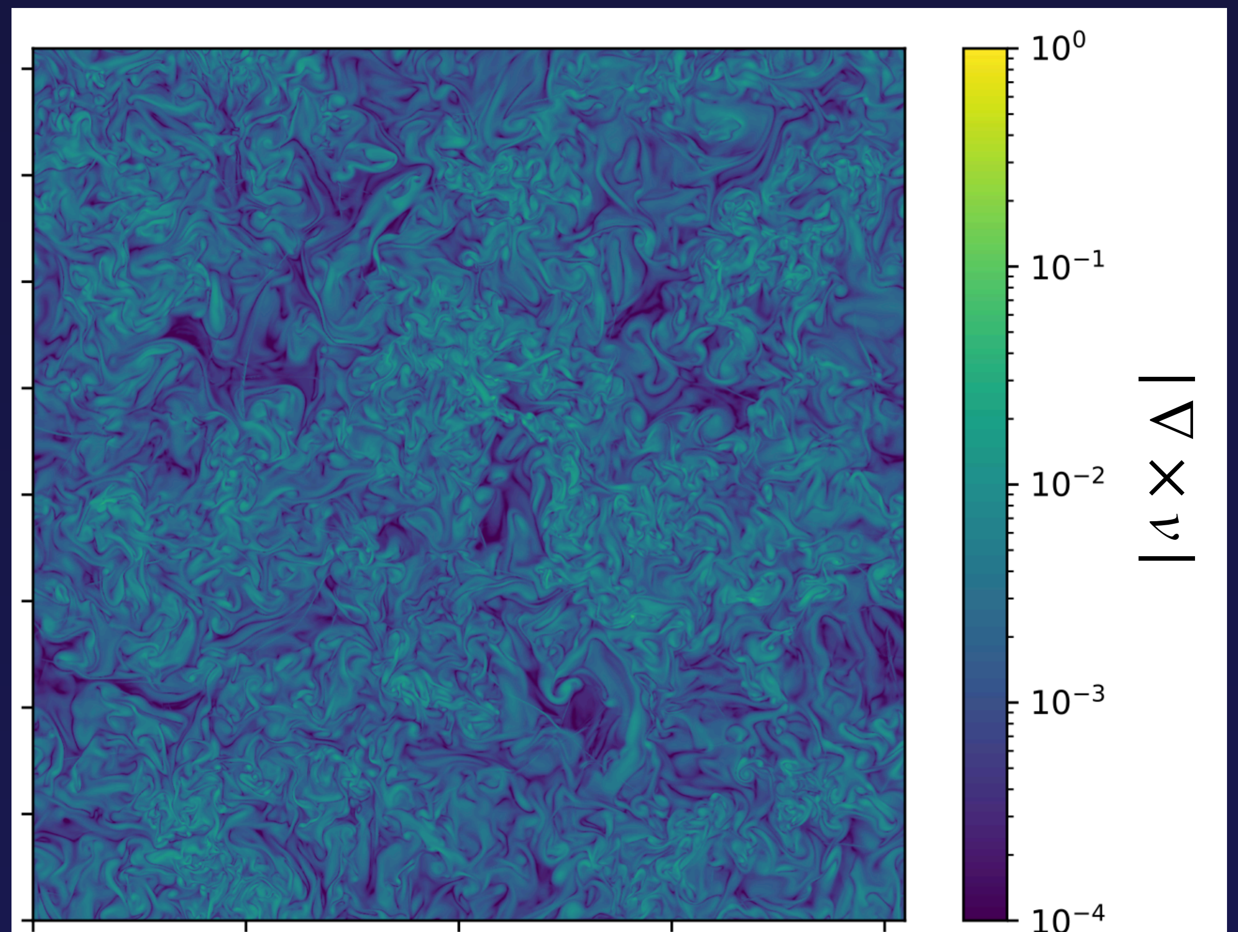


Sound waves



Correia et al 2025

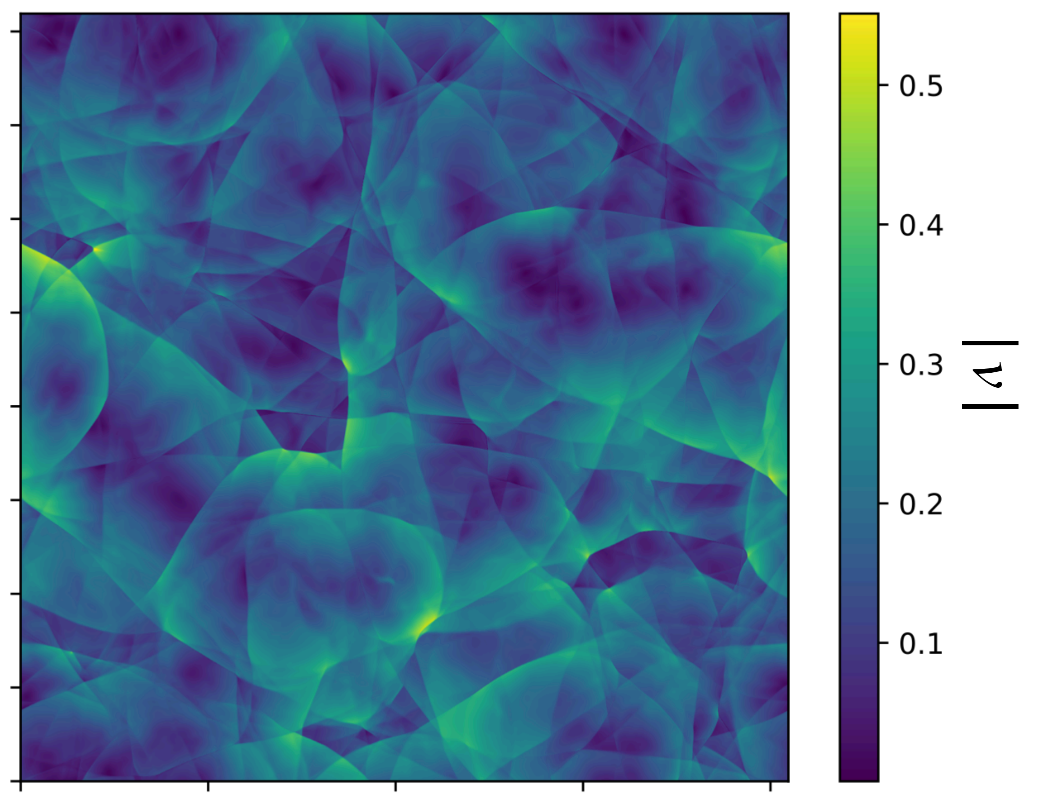
Vortical turbulence

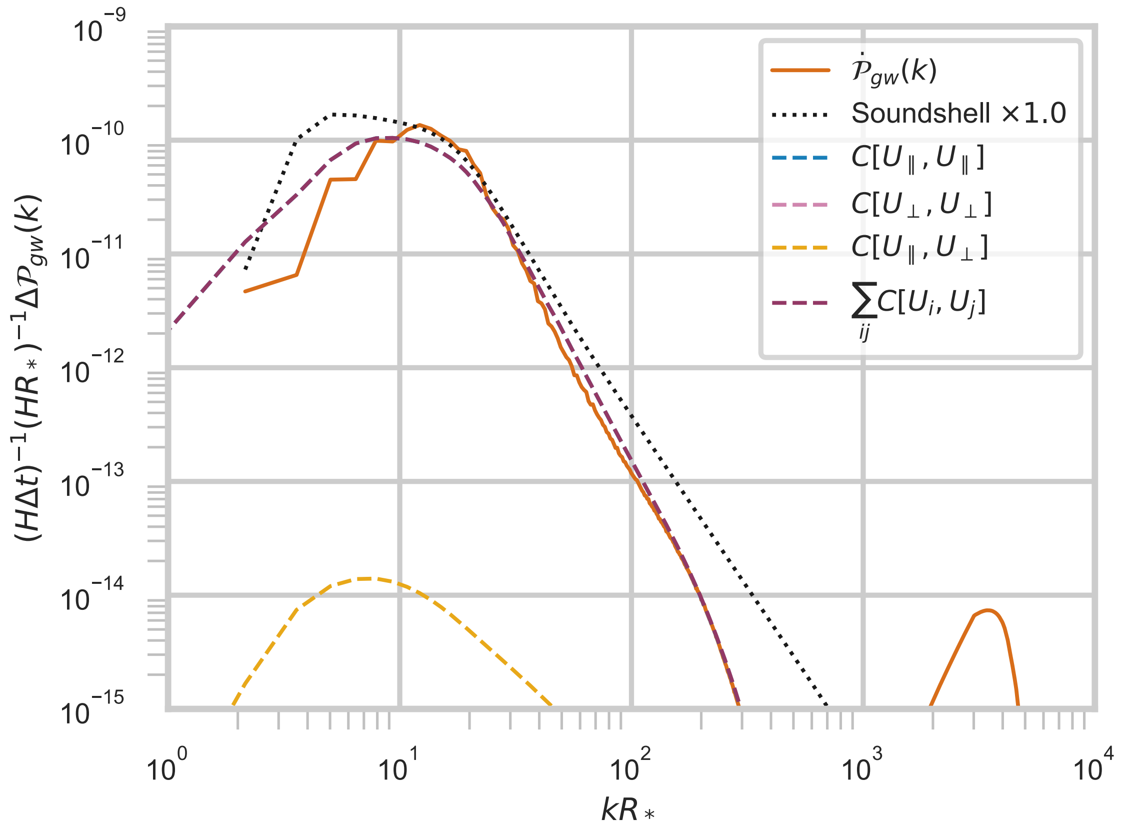


Correia et al 2025

Numerical simulations vs Analytical methods

Hindmarsh et al. 2013, 2015, 2017, Cutting et al. 2019, Caprini et al 2024, Correia et al 2025.

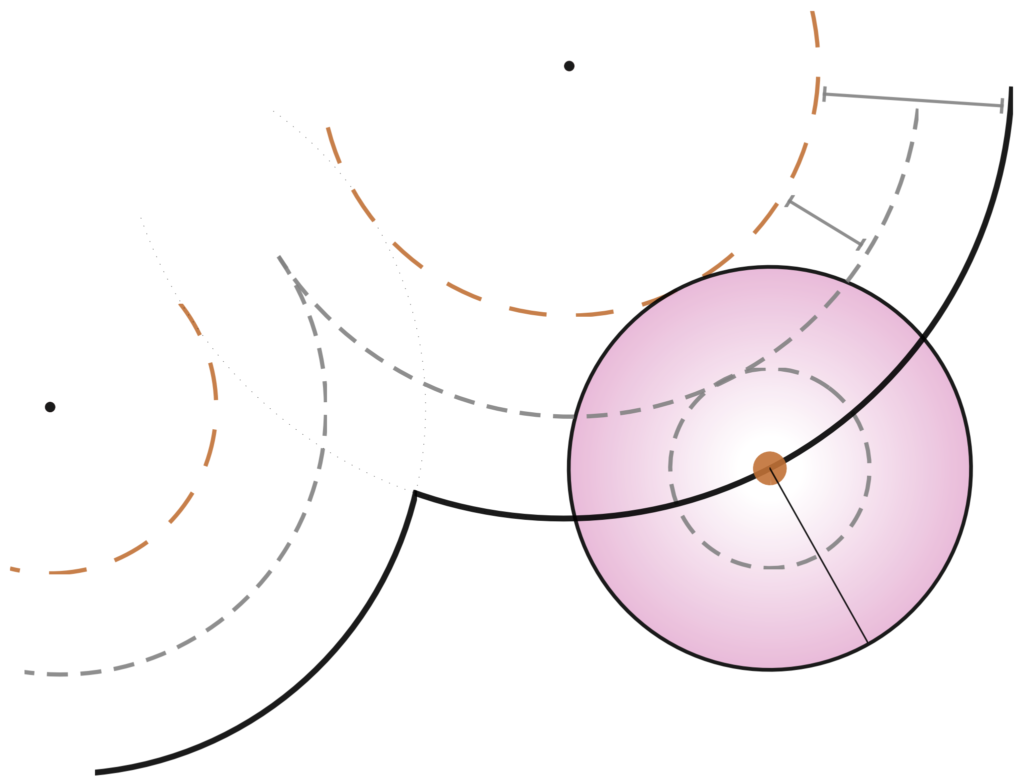




Correia et al., arXiv: 2505.17824 [astro-ph.CO]

- Most accurate
- Computationally expensive

Hindmarsh 2016, Hindmarsh & Hijazi 2019, Roper Pol et al. 2023, Guo et al. 2021.



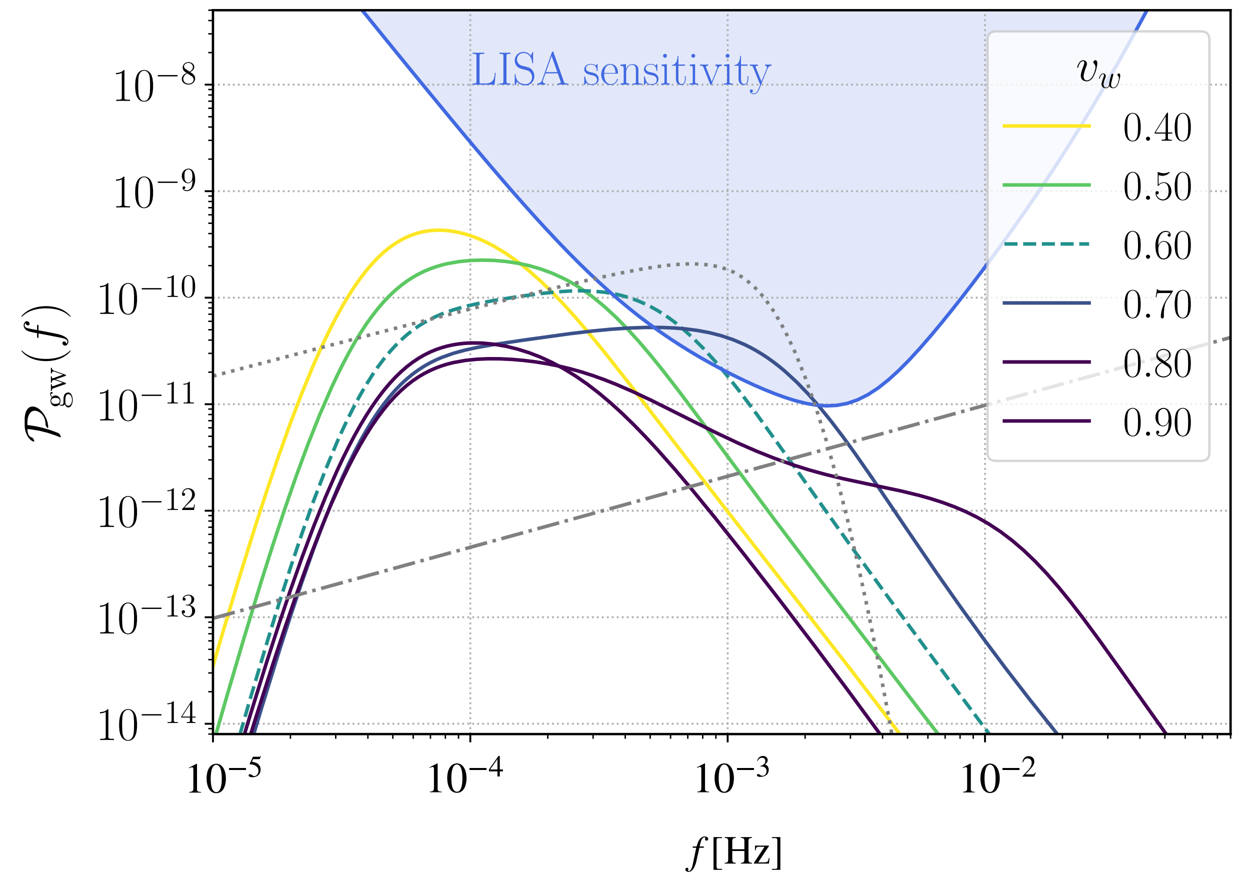


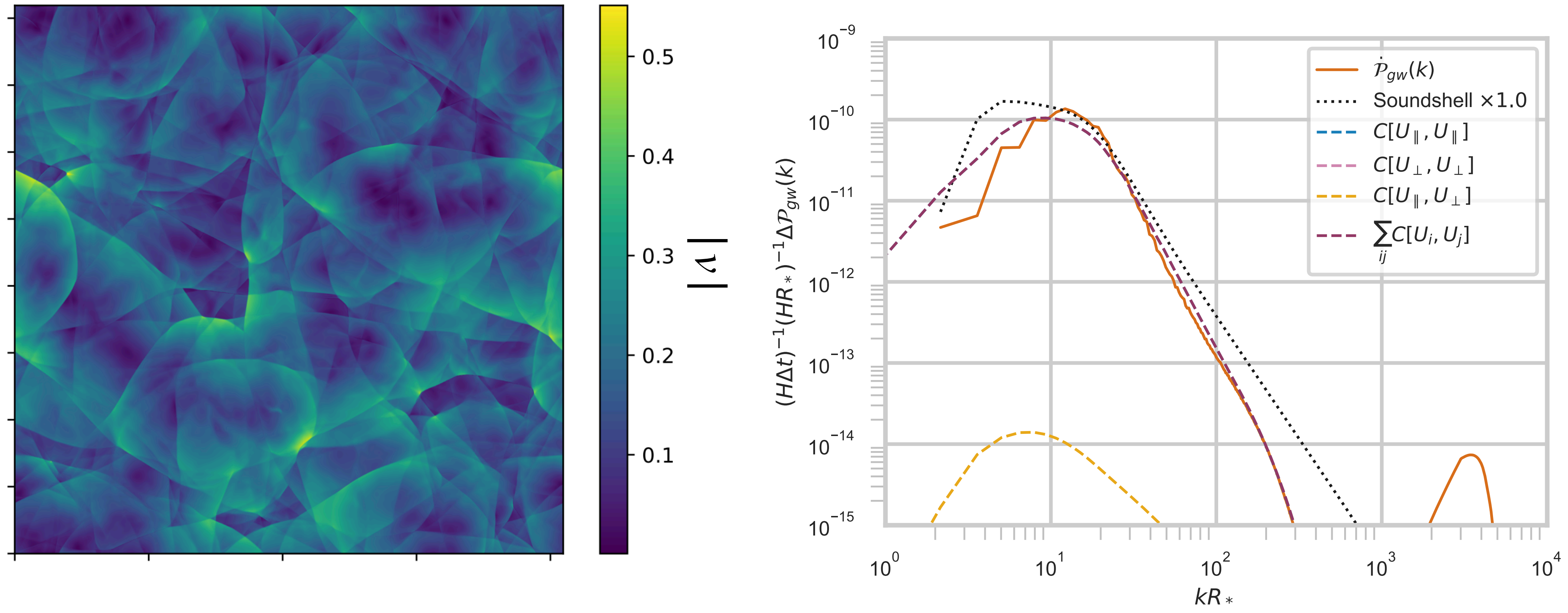
Image adapted to Gowling & Hindmarsh *JCAP* 10 (2021) 039

- Faster and cheaper
- Study the transition in limits not achievable by simulations:
  - Large scales (wavenumber)
  - Universe expansion
  - Other effects

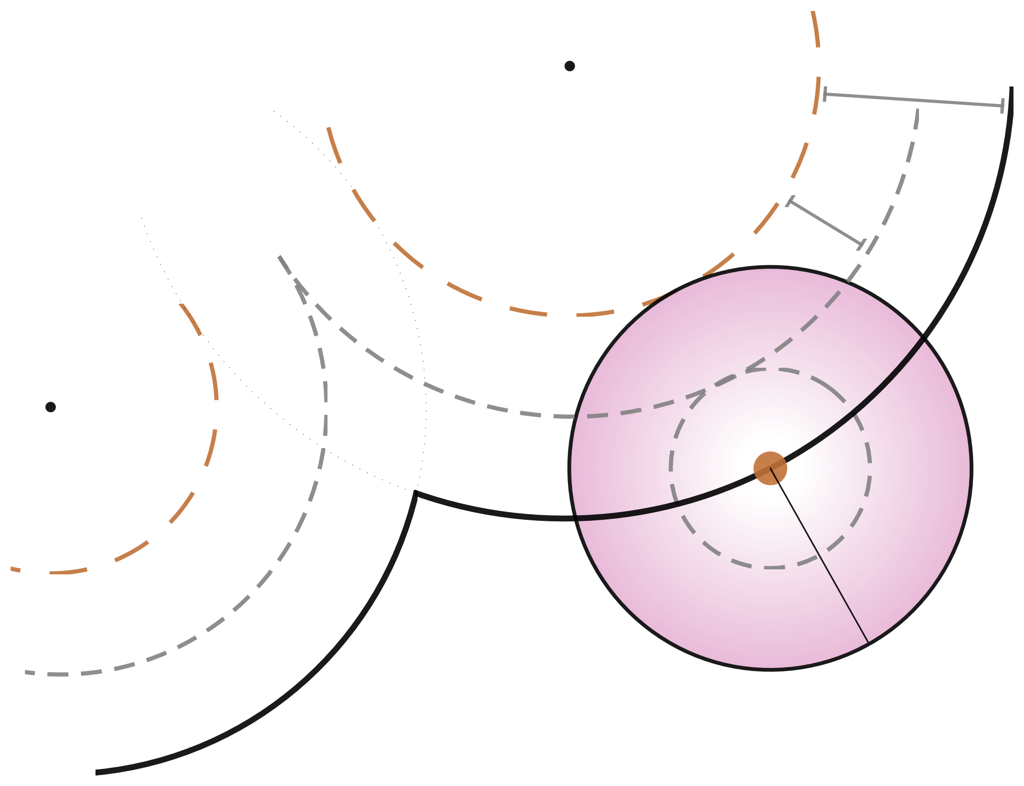
Numerical simulations vs Analytical methods

Hindmarsh et al. 2013, 2015, 2017, Cutting et al. 2019, Caprini et al 2024, Correia et al 2025.

Hindmarsh 2016, Hindmarsh & Hijazi 2019, Roper Pol et al. 2023, Guo et al. 2021.



Correia et al., arXiv: 2505.17824 [astro-ph.CO]



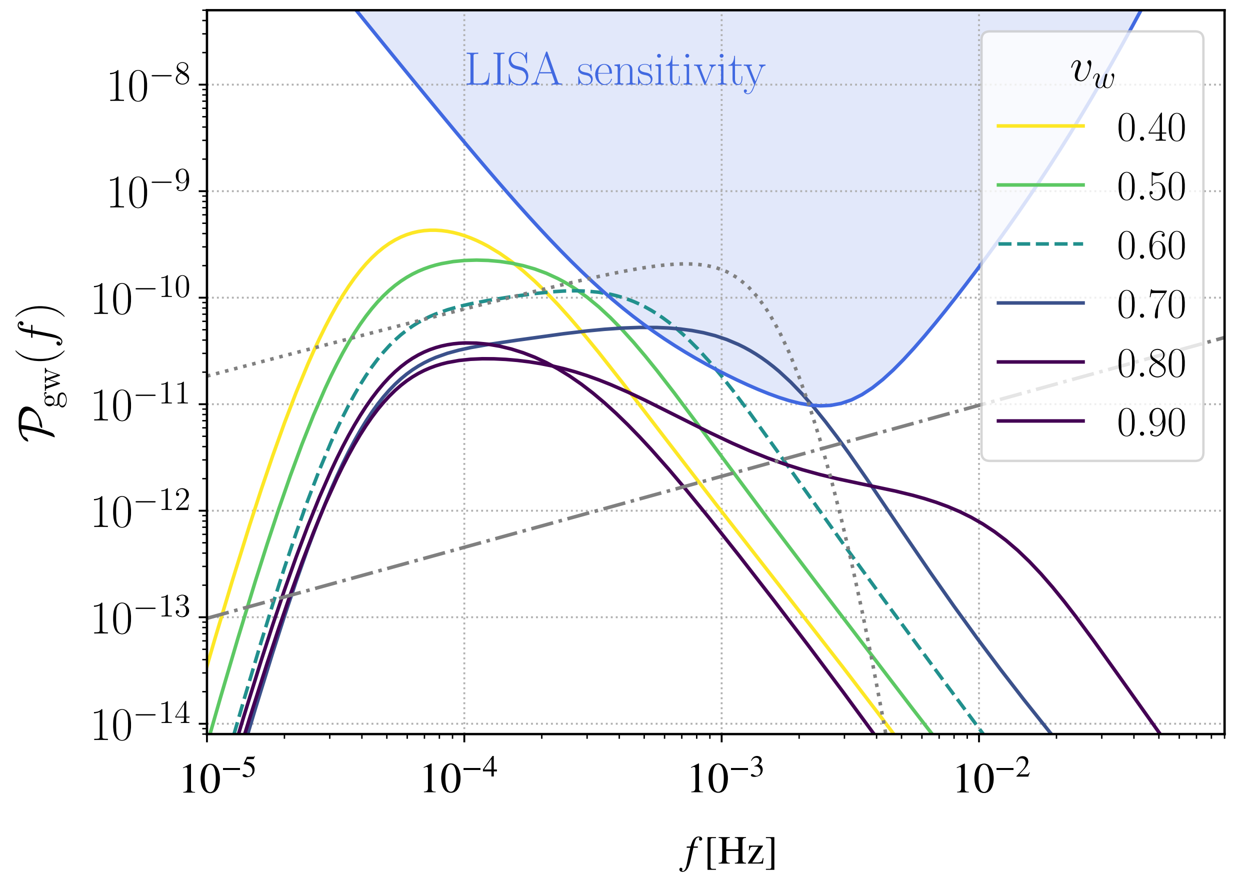


Image adapted to Gowling & Hindmarsh *JCAP* 10 (2021) 039

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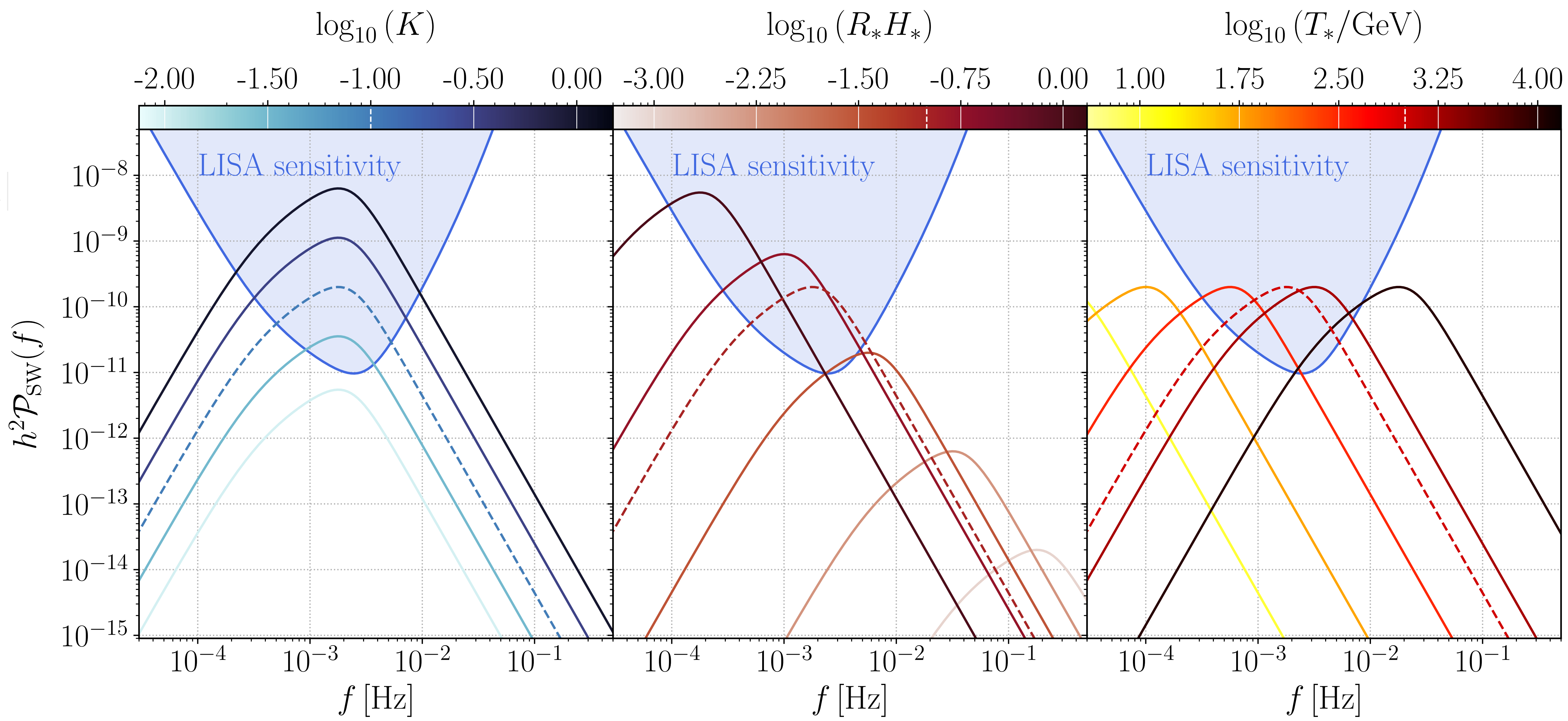


LISA FIT

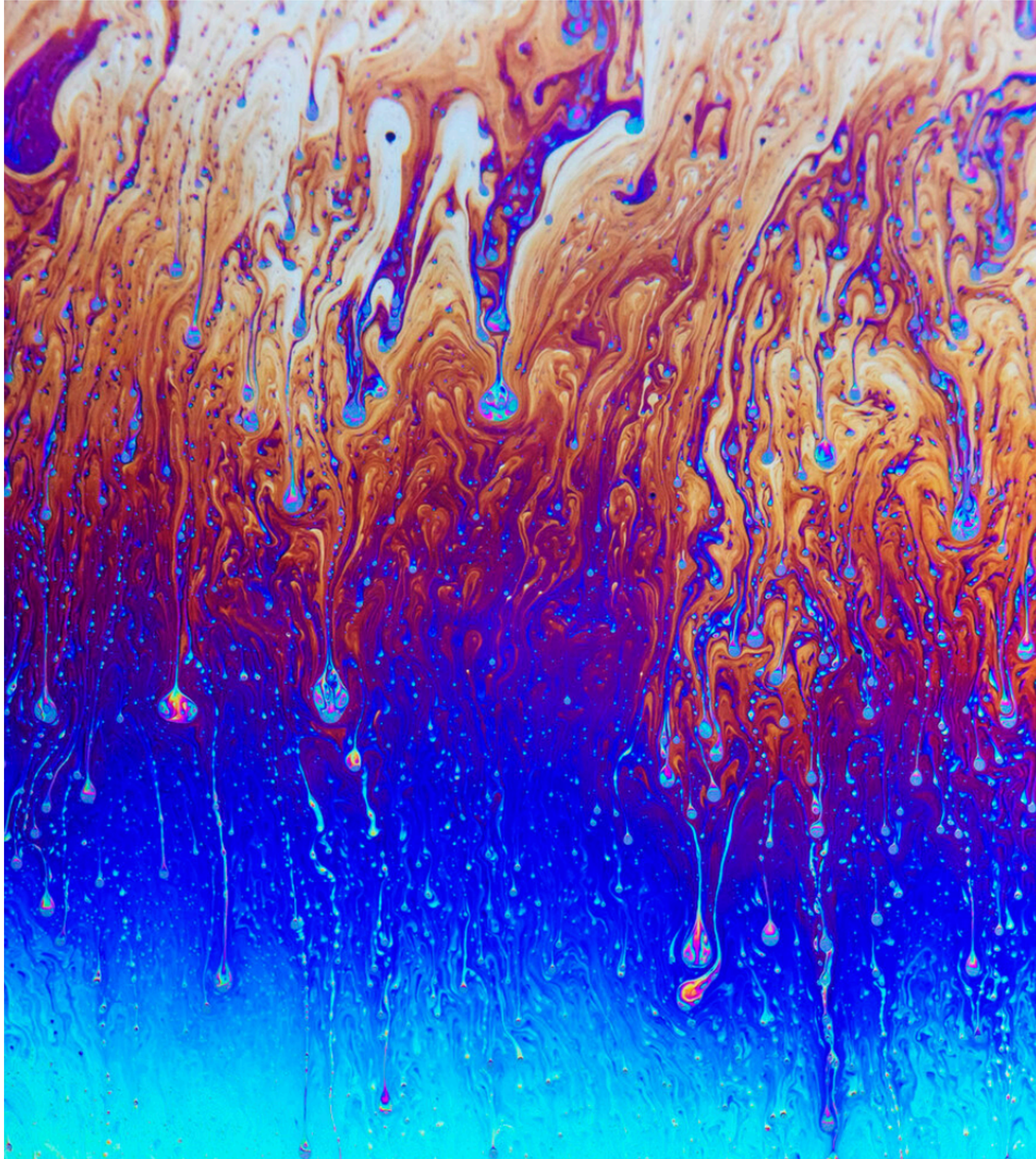
Image adapted to LISA CosmoWG, *JCAP* 10 (2024)

LISA FIT

Image adapted to LISA CosmoWG, *JCAP* 10 (2024)

I. Sound waves and the sound shell model for the acoustic source

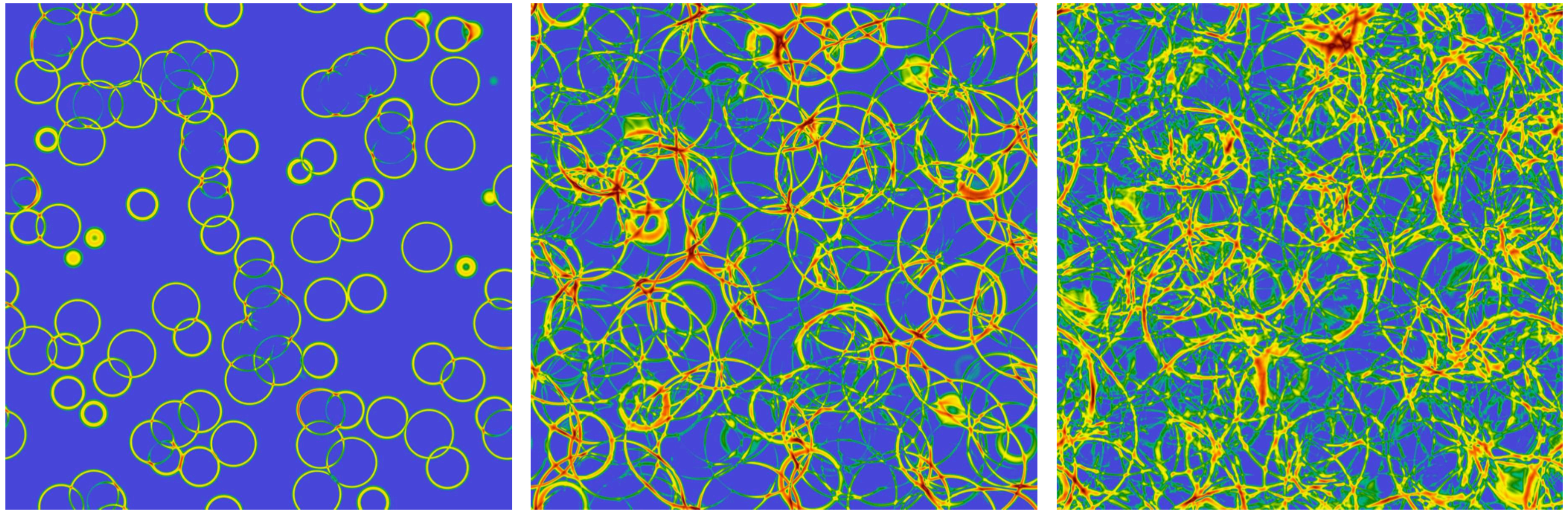
Macro photography of a soap bubble
Ingredients: 2 parts water, 1 part soap, sugar to taste.
Picture from PetaPixel



Sound waves in the plasma

Hindmarsh et al Phys.Rev.Lett. 112 (2014), Phys.Rev.D 92 (2015), Phys.Rev.D 96 (2017)

- Non-linear dynamics ⟶ Numerical simulations



Hindmarsh et al., Phys.Rev.D 92 (2015)

- Non-relativistic flow

$$|\vec{v}| \ll 1$$

- Weak phase transition

$$\alpha \ll 1$$

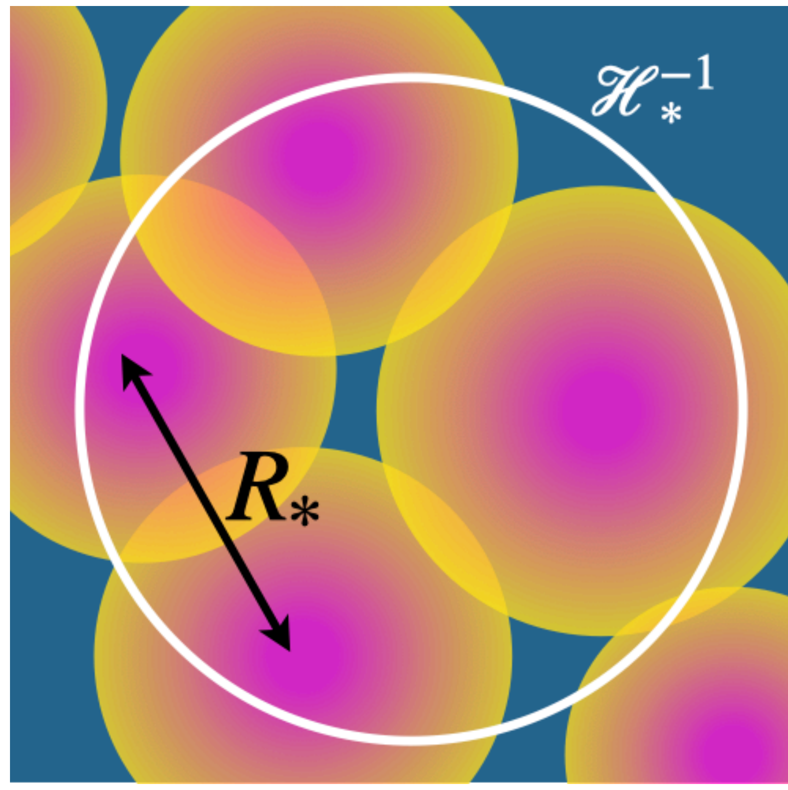
⟶ Fluid flow dominated by linear compressional motion

$$\vec{\nabla} \times \vec{v} = 0$$

↓

Shocks in the plasma form after

$$\tau_{sh} \sim \frac{R_*}{\langle \vec{v} \rangle}$$

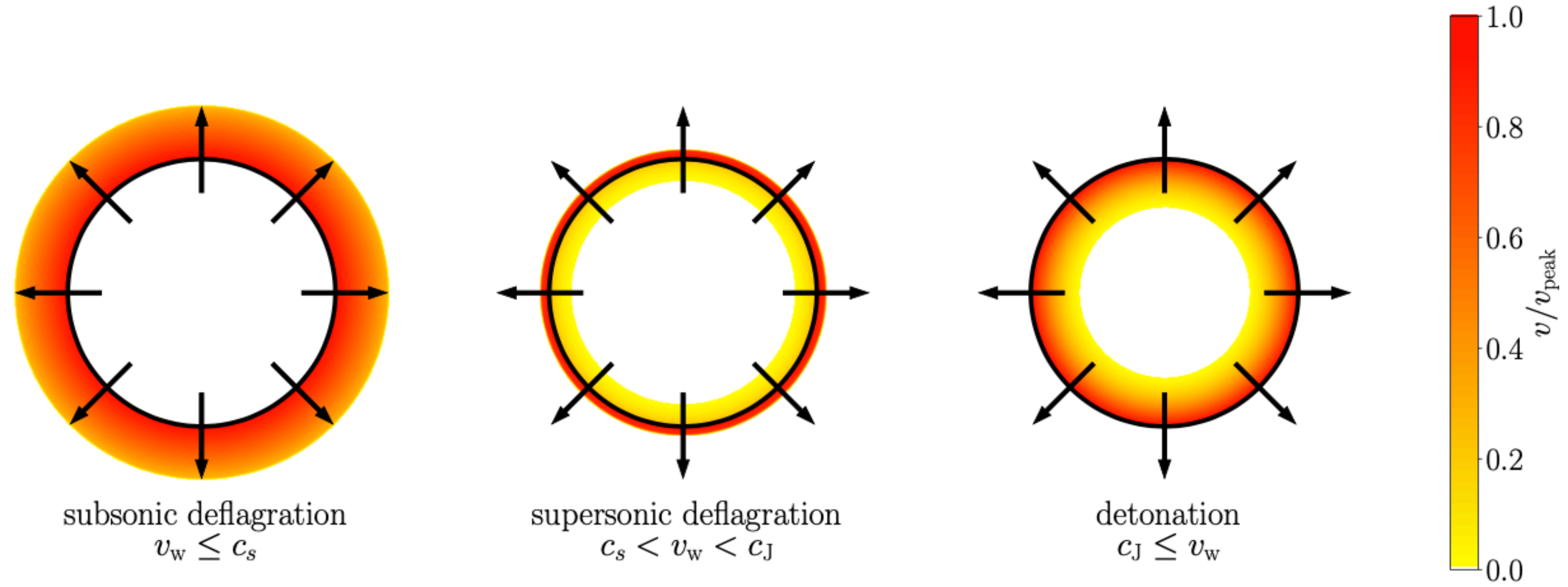


Energy of sound waves dissipates into turbulence

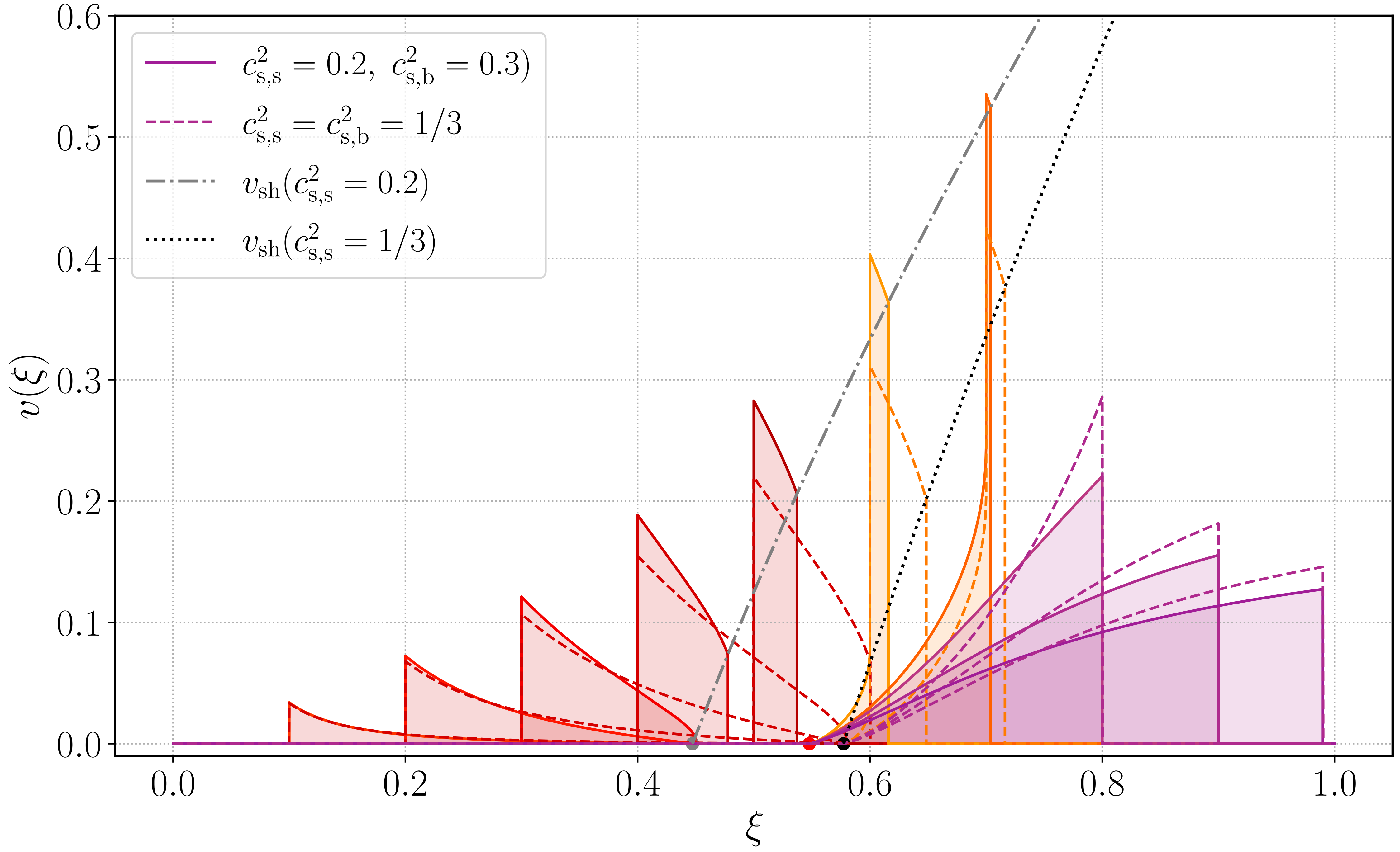
The Sound Shell Model

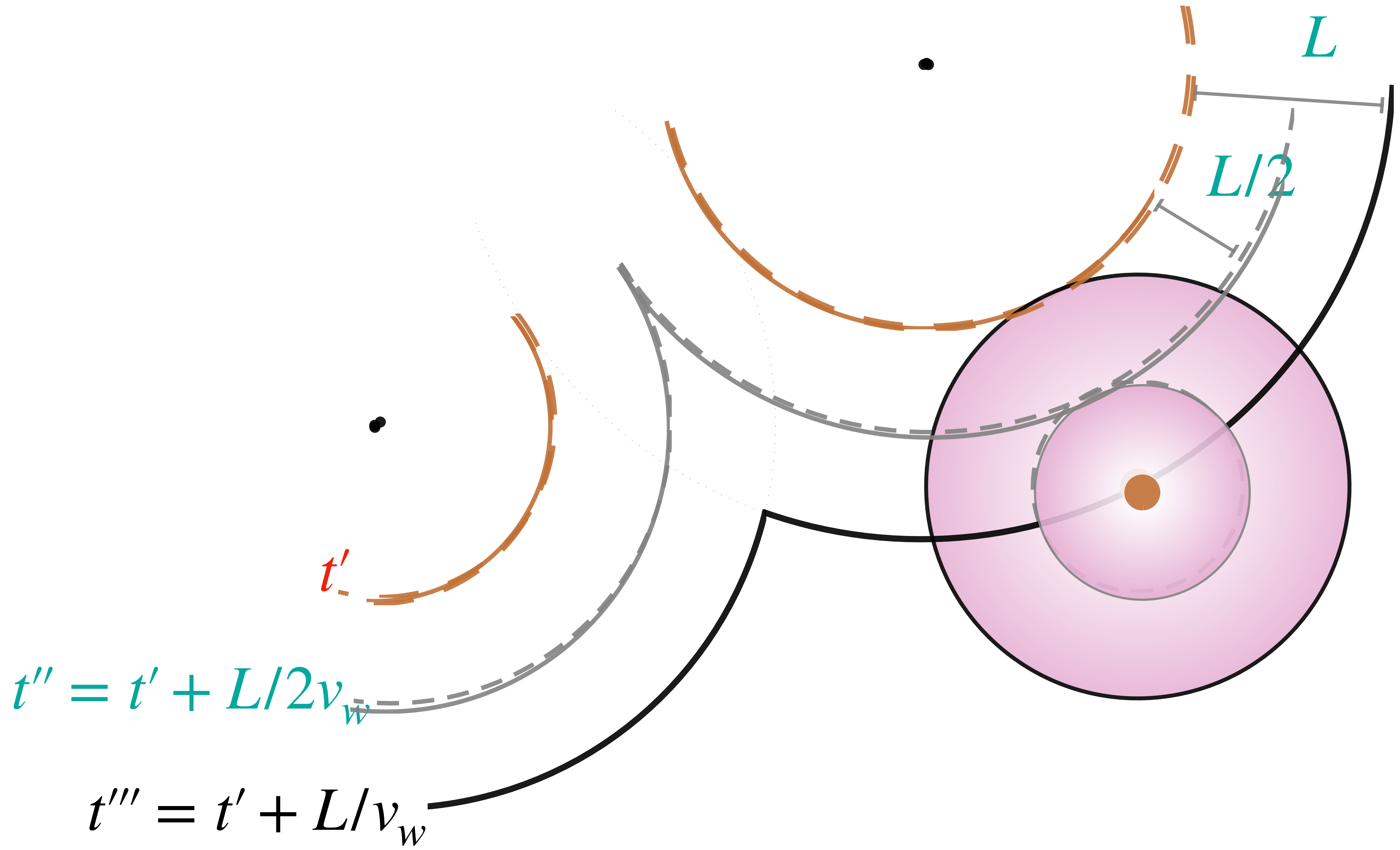
Hindmarsh Phys.Rev.Lett. 120 (2018), Hindmarsh & Hijazi *JCAP* 12 (2019)

Universal self-similar profile, $\xi = |\vec{x}|/t$



Hindmarsh et al., SciPost Phys.Lect.Notes 24 (2021)





Linear superposition of contributions from individual bubbles

$$v_i(\vec{x}, t) = \sum_{n=1}^{N_b} \frac{x_i - x_i^{(n)}}{|x_i - x_i^{(n)}|} v_{\mathrm{ip}}(\xi)$$

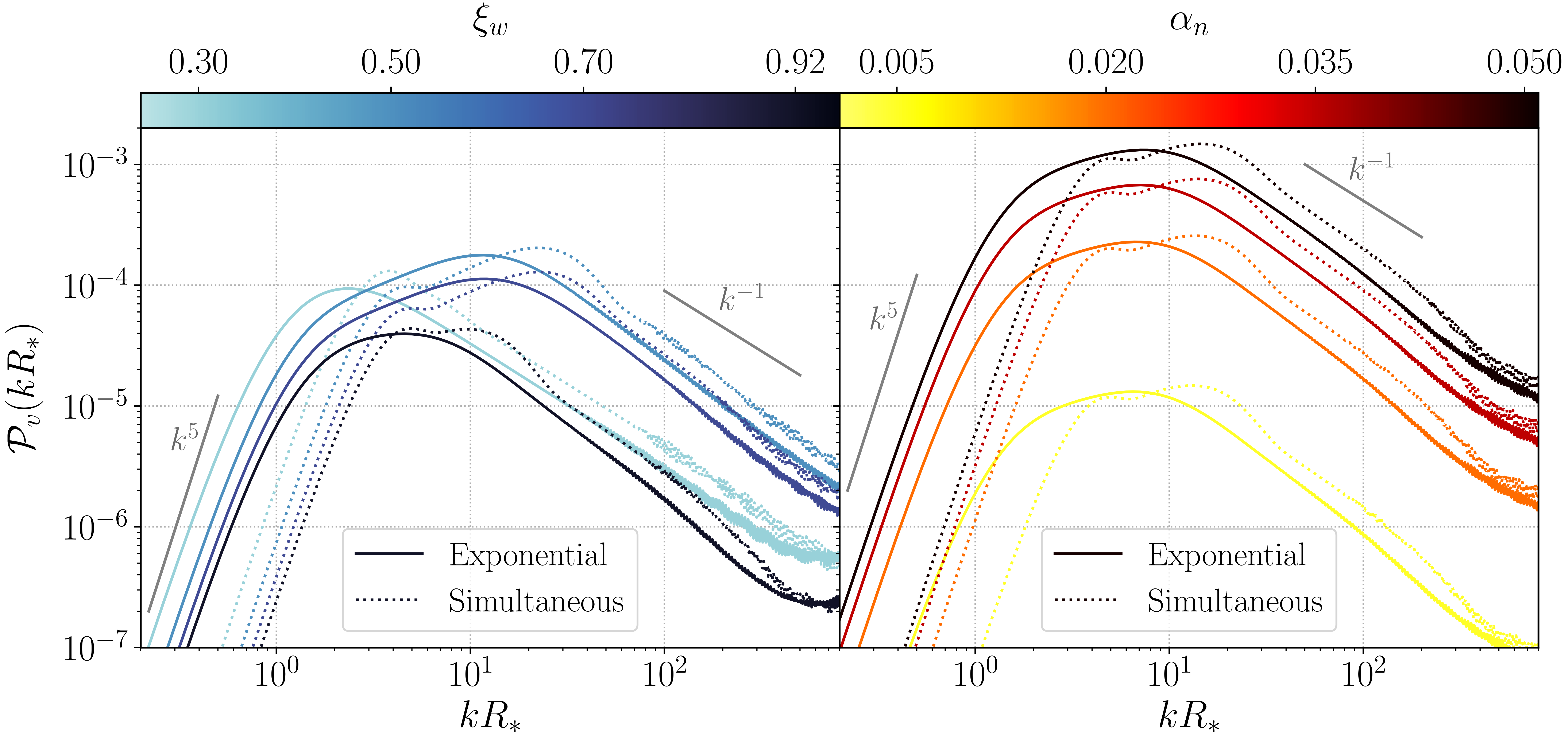
The Sound Shell Model: spectral density of plane wave amplitudes

Hindmarsh Phys.Rev.Lett. 120 (2018), Hindmarsh & Hijazi *JCAP* 12 (2019)

- Gaussian $\langle v_q^i \rangle = 0,$
  $\langle v_{q_1}^i v_{q_2}^{*j} \rangle \sim P_v(q_1)\, \delta(\vec{q}_1 - \vec{q}_2)$

Average over:

- nucleation sites $x^{(n)}$
- bubbles lifetime T_i



Plot obtained with PTtools https://doi.org/10.5281/zenodo.15268219

The Sound Shell Model: spectral density of gravitational waves

Hindmarsh Phys.Rev.Lett. 120 (2018), Hindmarsh & Hijazi *JCAP* 12 (2019)

- Assume the transition is much faster than Hubble rate $H_* R_* \ll 1$
- Linearised General Relativity

$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G \mathcal{S}_{ij} \qquad\qquad \mathcal{S}_{ij} = (e+p) v_i v_j$$

- Energy density of gravitational waves & power spectrum

$$e_{\rm gw} = T^{00}_{\rm gw} = \frac{\langle \nabla^0 h_{ij} \nabla^0 h^{ij} \rangle}{32\pi G}$$

Un-Equal Tim
of shear stre

4-dimensional integration

$$\mathscr{P}_{\rm gw}(k,t) \equiv \frac{1}{\bar{e}} \frac{{\rm d}e_{\rm gw}}{{\rm d}\ln k} \sim (16\pi G)^2 \frac{k^3}{2\pi^2} \iint_0^t {\rm dt_1 dt_2}\, \dot{\rm G}_{\rm k}({\rm t,t_1}) \dot{\rm G}_{\rm k}({\rm t,t_2})\, \Lambda_{\rm ij,kl}(\vec{\rm k}) \langle \mathcal{S}_{\rm ij}({\rm k,t_1}) \mathcal{S}_{\rm kl}({\rm k,t_2}) \rangle$$

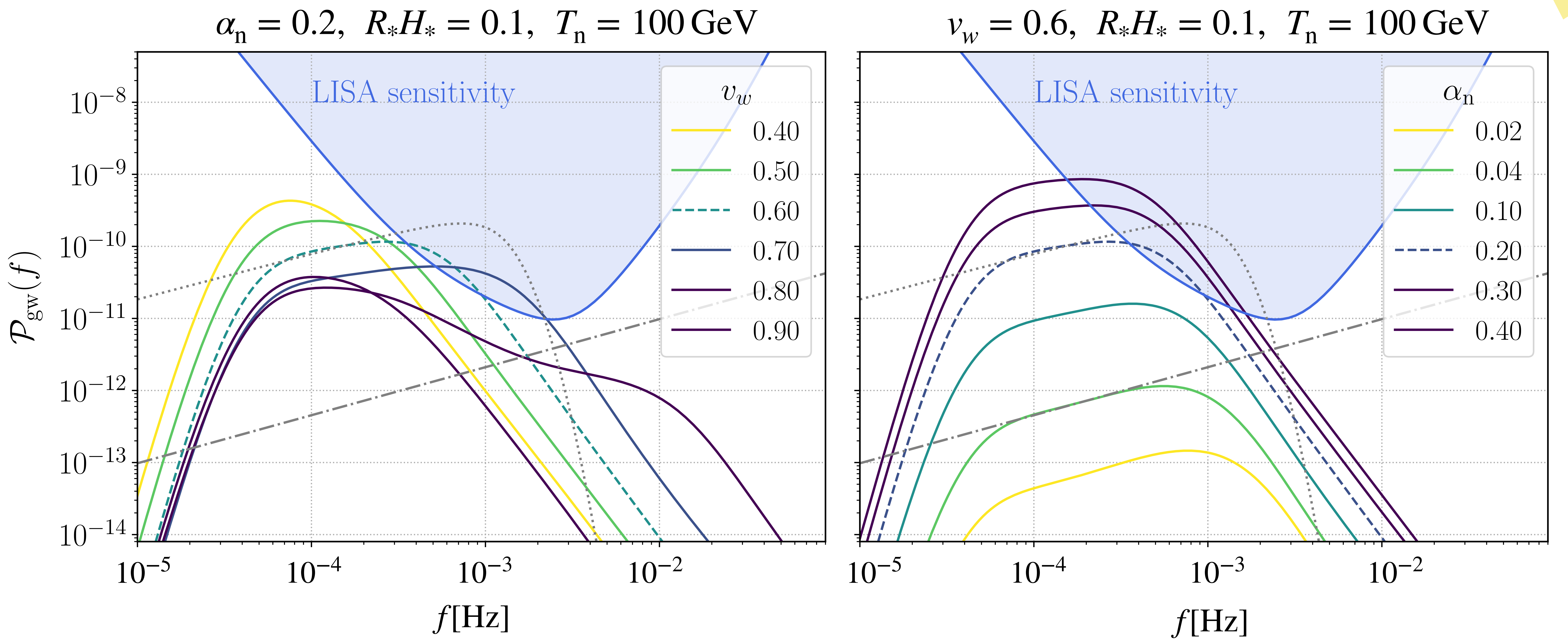
Green's function of h_{ij}

The Sound Shell Model: spectral density of gravitational waves

Hindmarsh Phys.Rev.Lett. 120 (2018), Hindmarsh & Hijazi *JCAP* 12 (2019)

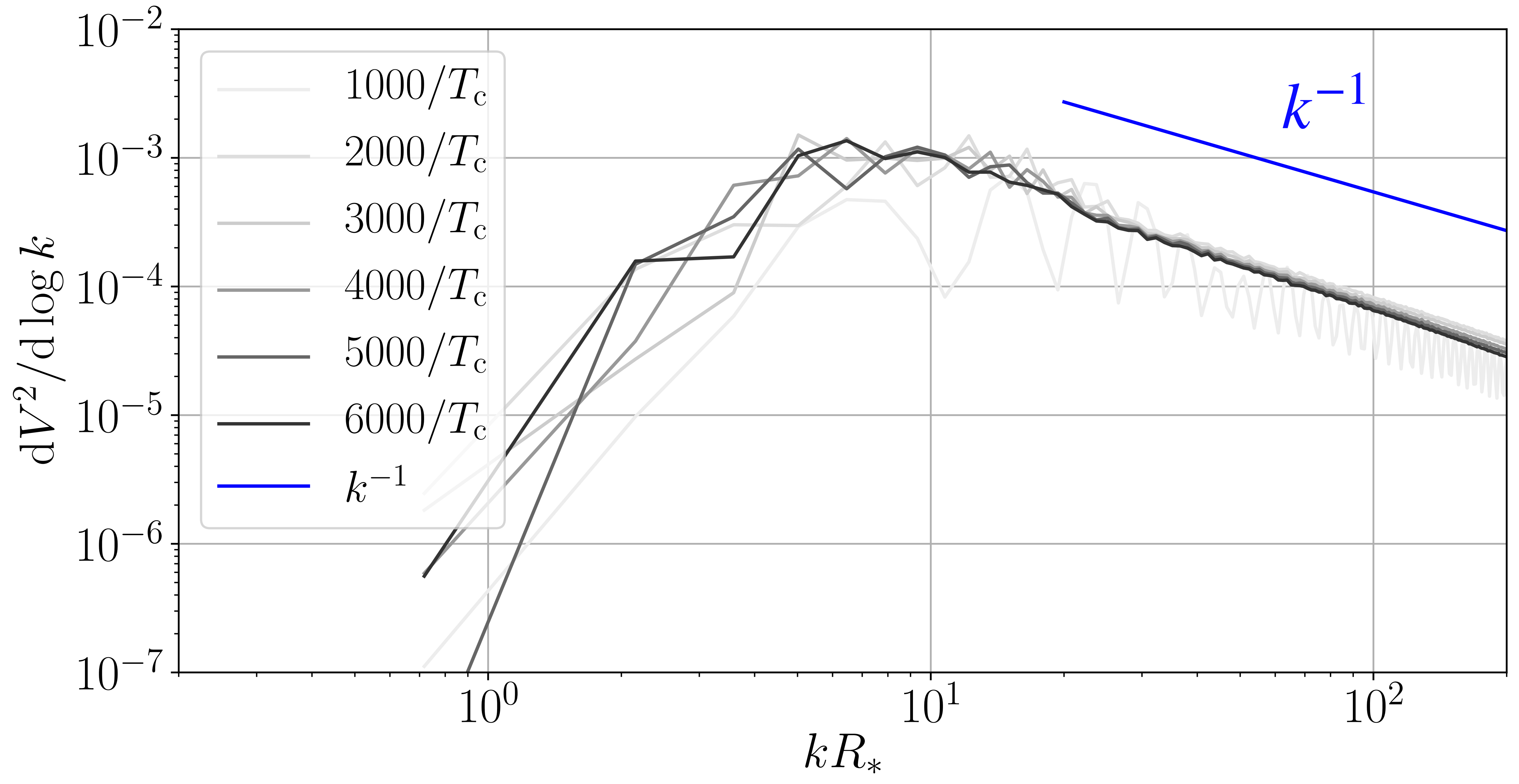
- Stationary sound waves $\tau_v \gg \tau_{ac} \sim k^{-1}$ $\longrightarrow$ $\iint_0^t dt_1 dt_2 \ldots \to \int_0^t dt_- \ldots \quad t_- = t_1 - t_2$
- High frequency approximation $kR_* \gtrsim 1$ $\longrightarrow$ $\int_0^t dt_- \ldots \to \delta(k - c_s p - c_s q)$

1-dimensional integration

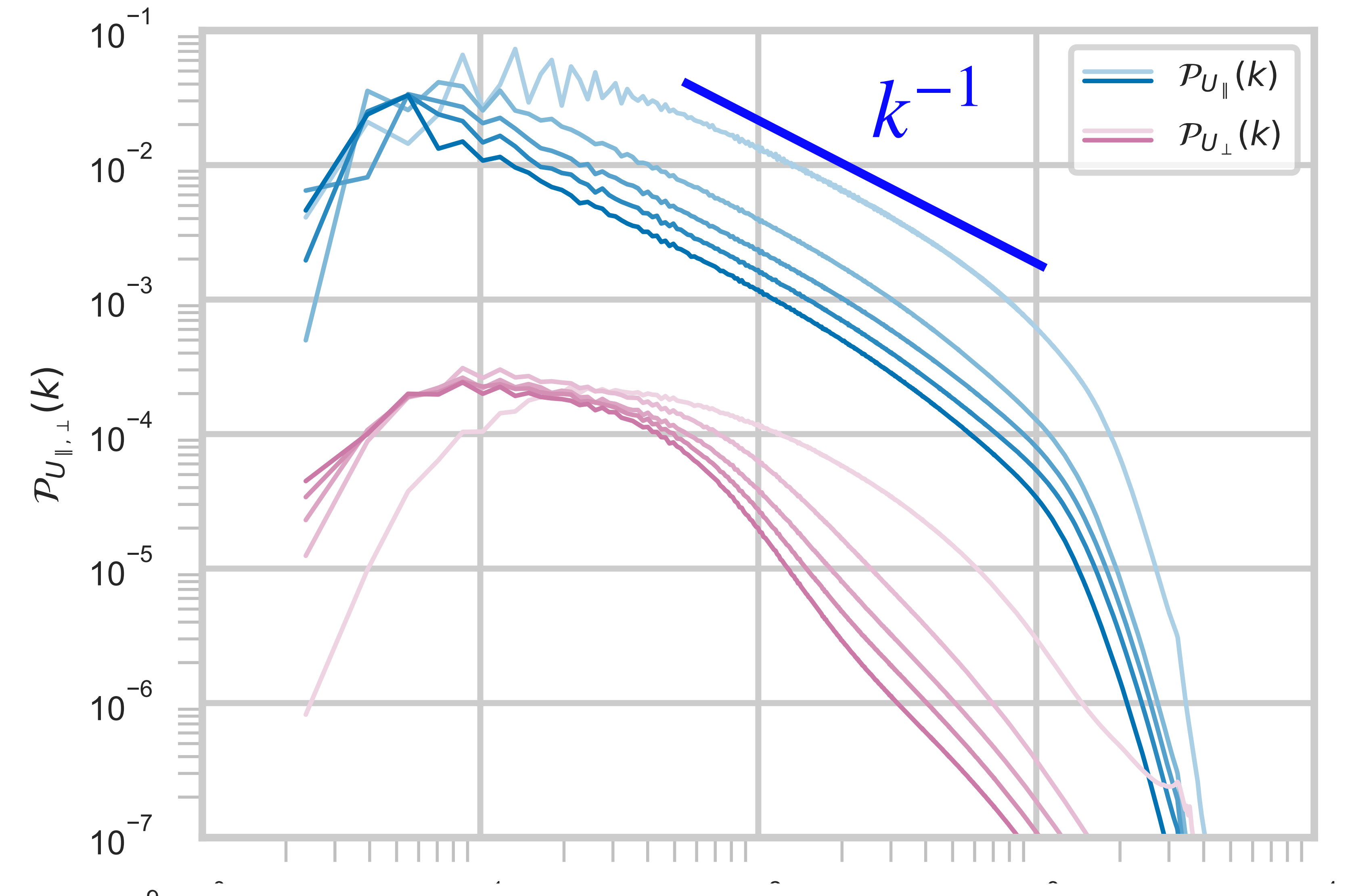


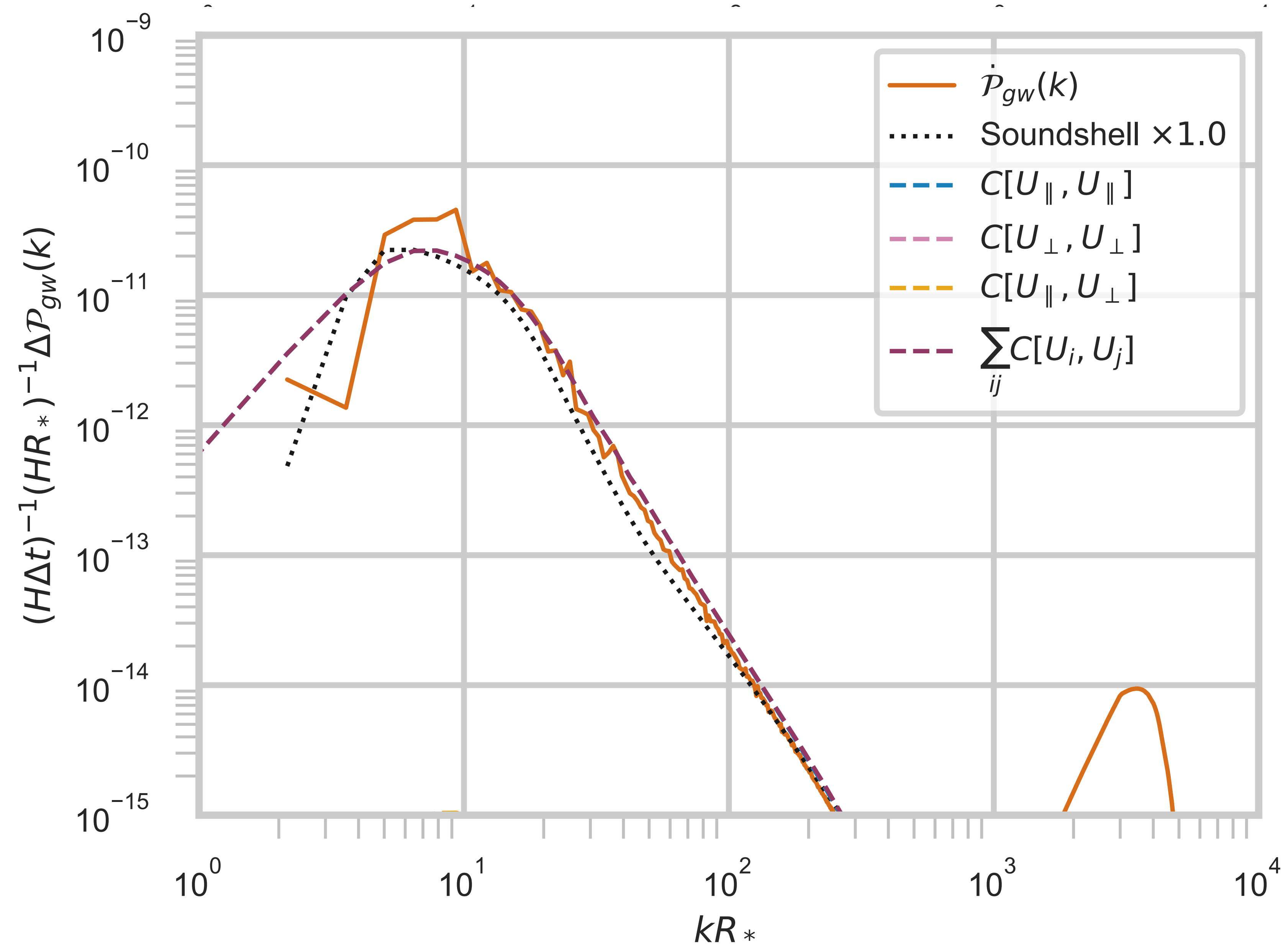
Plot obtained with PTtools https://doi.org/10.5281/zenodo.15268219

The Sound Shell Model: comparison to numerical simulations



Hindmarsh et al., Phys.Rev.D 96 (2017)





Correia et al., arXiv: 2505.17824 [astro-ph.CO]

- Ansatz (fit to simulations):

$$P_v \sim KR_*^2 \frac{(kR_*)^2}{1+(kR_*)^6} \begin{cases} \propto k^2, & k \ll R_* \quad \text{causality} \\ \propto k^{-4}, & k \gg R_* \quad \text{shocks} \end{cases}$$

Shortcomings of the Sound Shell Model

Long wavelengths $kR_* \lesssim 1$

High frequency approximation
only for $kR_* \gtrsim 1$

Roper Pol et al, *Phys.Rev.D* 109 (2024)

Sharma et al *JCAP* 12 (2023)

Universe expansion

- Conformal fluid $c_s = 1/\sqrt{3}$

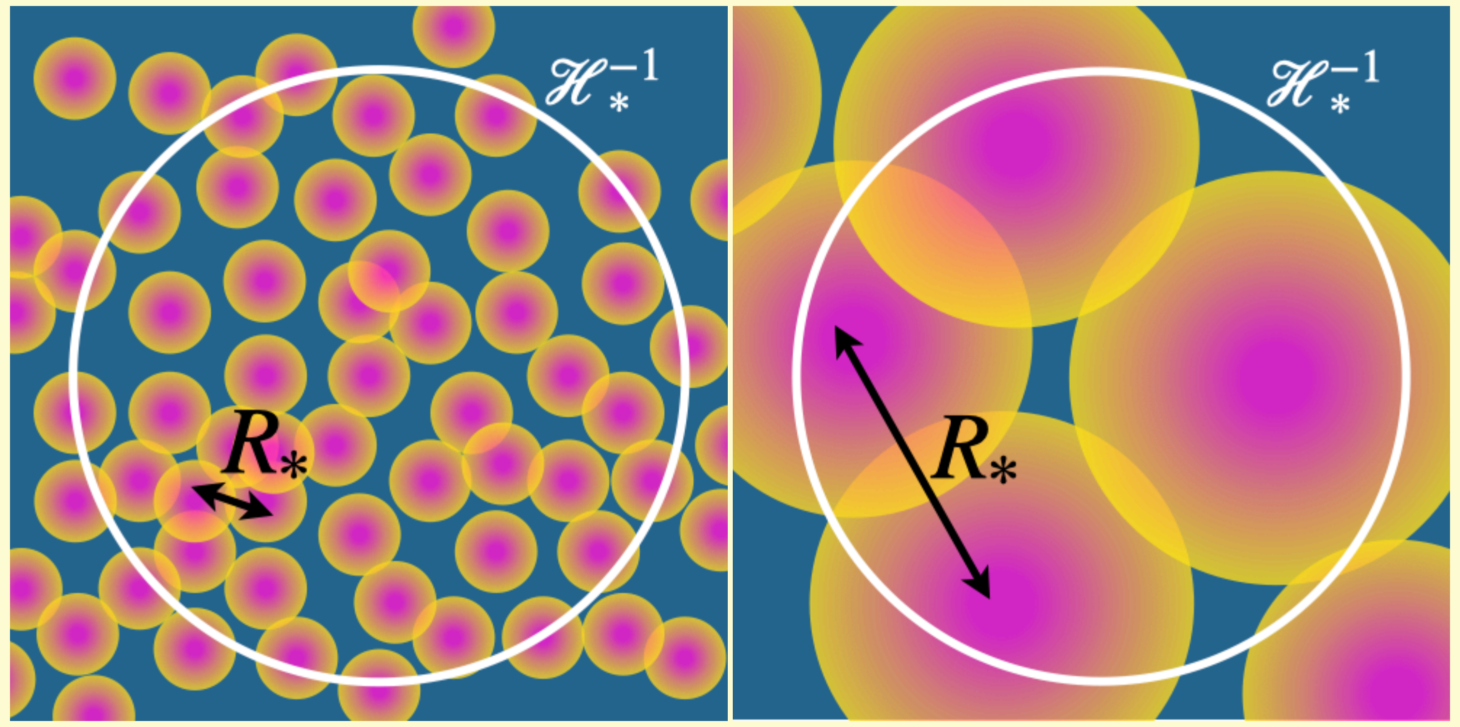
Guo et al. *JCAP* 01 (2021),
Roper Pol et al, *Phys.Rev.D* 109 (2024),
Sharma et al *JCAP* 12 (2023)

- Pressureless fluid $c_s = 0$

Guo et al. *JCAP* 01 (2021)

- **Non conformal fluid $0 \leq c_s \leq 1/\sqrt{3}$?**

Slow transitions or large bubbles
$H_* R_* \sim \mathcal{O}(1)$



- Loud signal regime

LISA CosmoWG, *JCAP* 10 (2024)

- **General relativistic effects beyond Universe expansion?**

II. Role of the speed of sound on the gravitational wave power spectrum

Macro photography of a soap bubble
Ingredients: detergent 80%, glycerine 20%, water to taste.
Picture from Canon Ireland

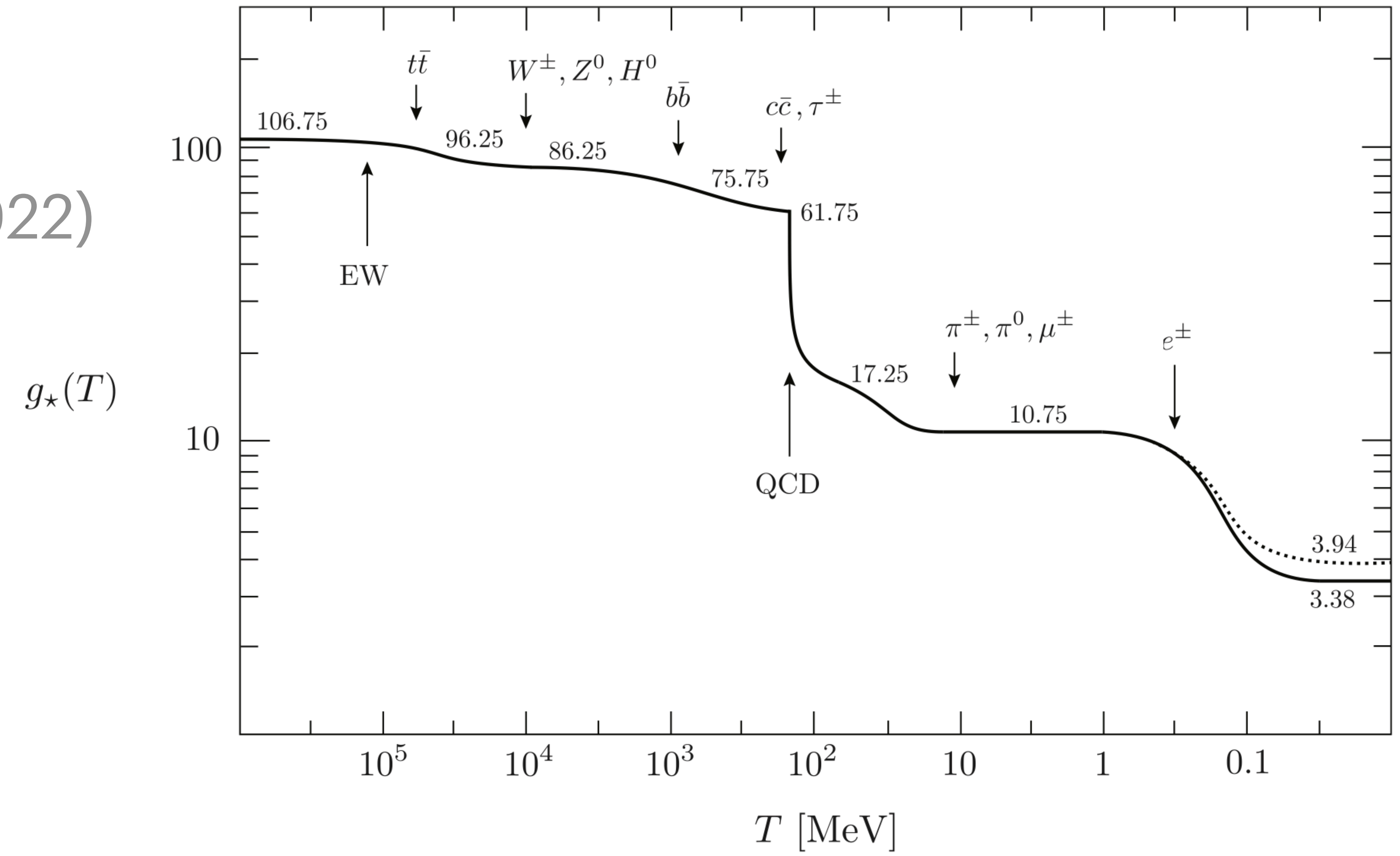
Softening of the equation of state

Leitao et al. Nucl.Phys.B 891 (2015), Tenkanen & van de Vis *JHEP* 08 (2022)

- Phase transition is accompanied by a change in the equation of state (EoS)

- **Bag EoS**: Ultra-relativistic plasma + vacuum energy $V_0(\phi)$

$$c_s = 1/\sqrt{3}$$



(Baumann, https://cmb.wintherscoming.no)

Universe at EW scale (clearly)

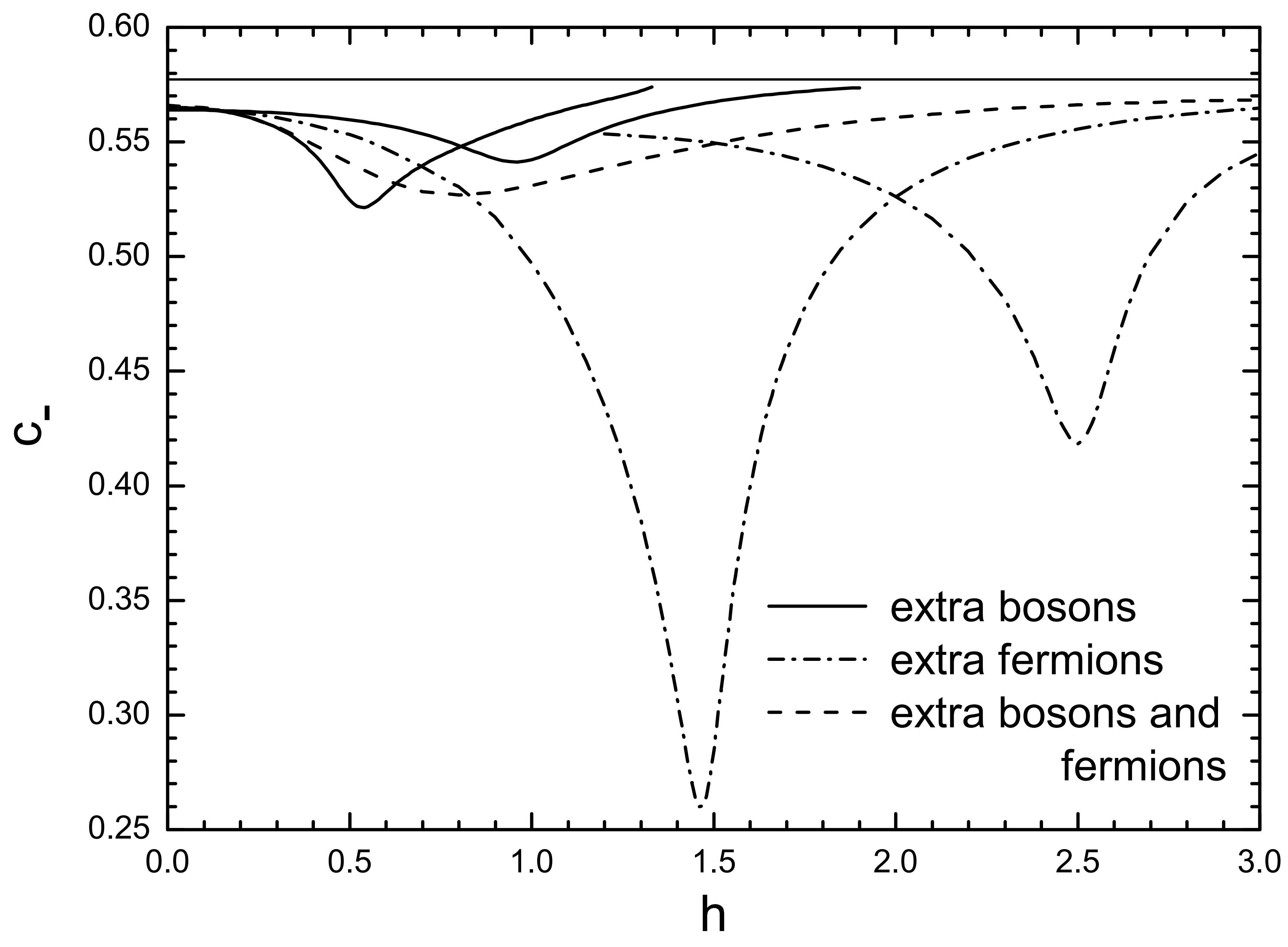
+ Relativistic particles
+ Vacuum energy
+ Massive particles $m \sim T$

Picture from pngtree

- Massive species $m \sim T$ ⟶ $c_s = c_s(T) \lesssim 1/\sqrt{3}$
- If $T \sim T_\mathrm{n}$ ⟶ **Constant speed of sound EoS**

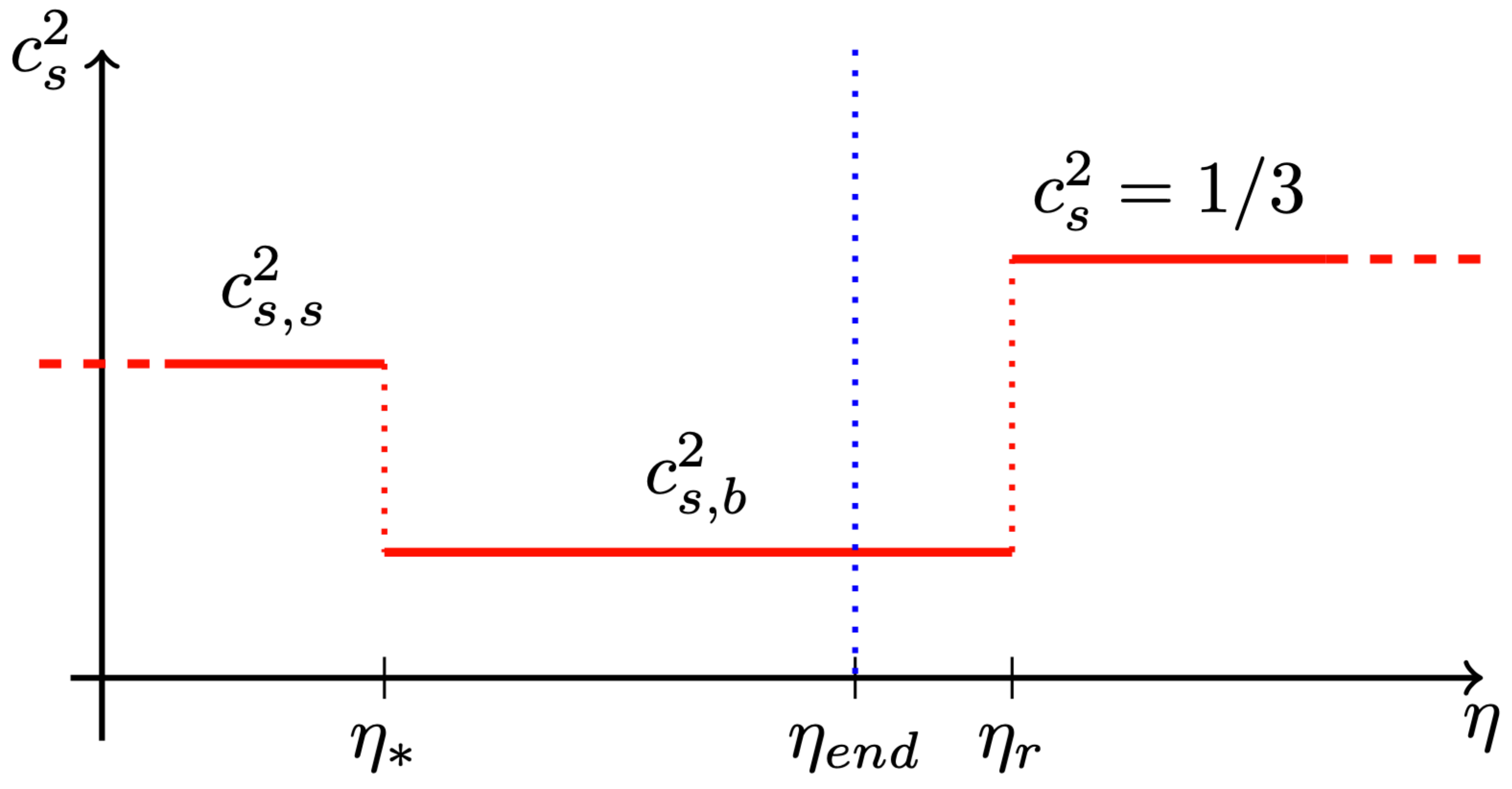
$$0 \leq c_s \leq 1/\sqrt{3}$$

Giese et al JCAP 01 (2021)



Leitao & Megevand *Nucl.Phys.B* 891 (2015) 159-199

Effects of a softer equation of state



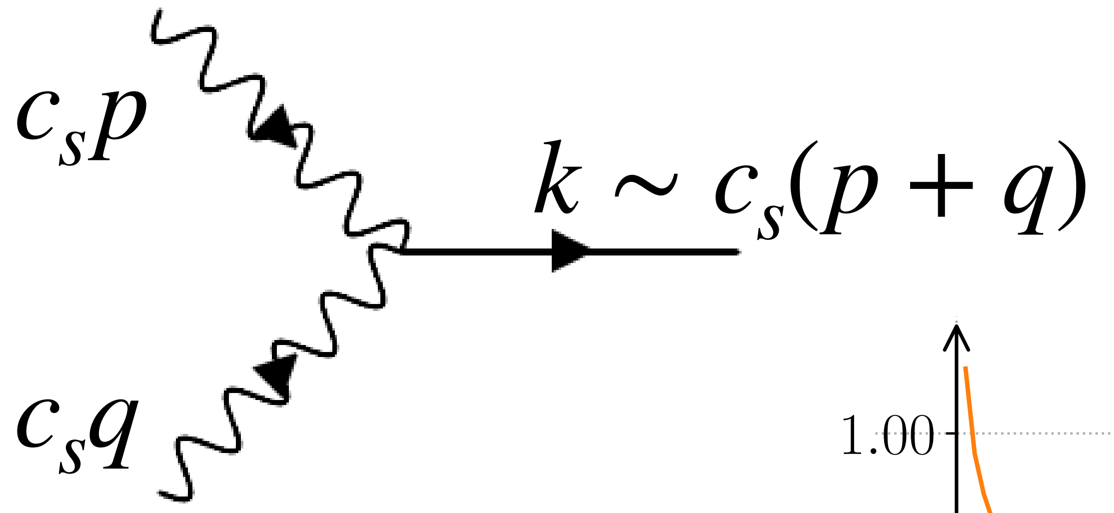
Consider $p = c_s^2 e$

$$\nu = \frac{1 - 3c_s^2}{1 + 3c_s^2}$$

η_* : begin acoustic phase

η_{end} : end acoustic phase

η_r : begin radiation domination

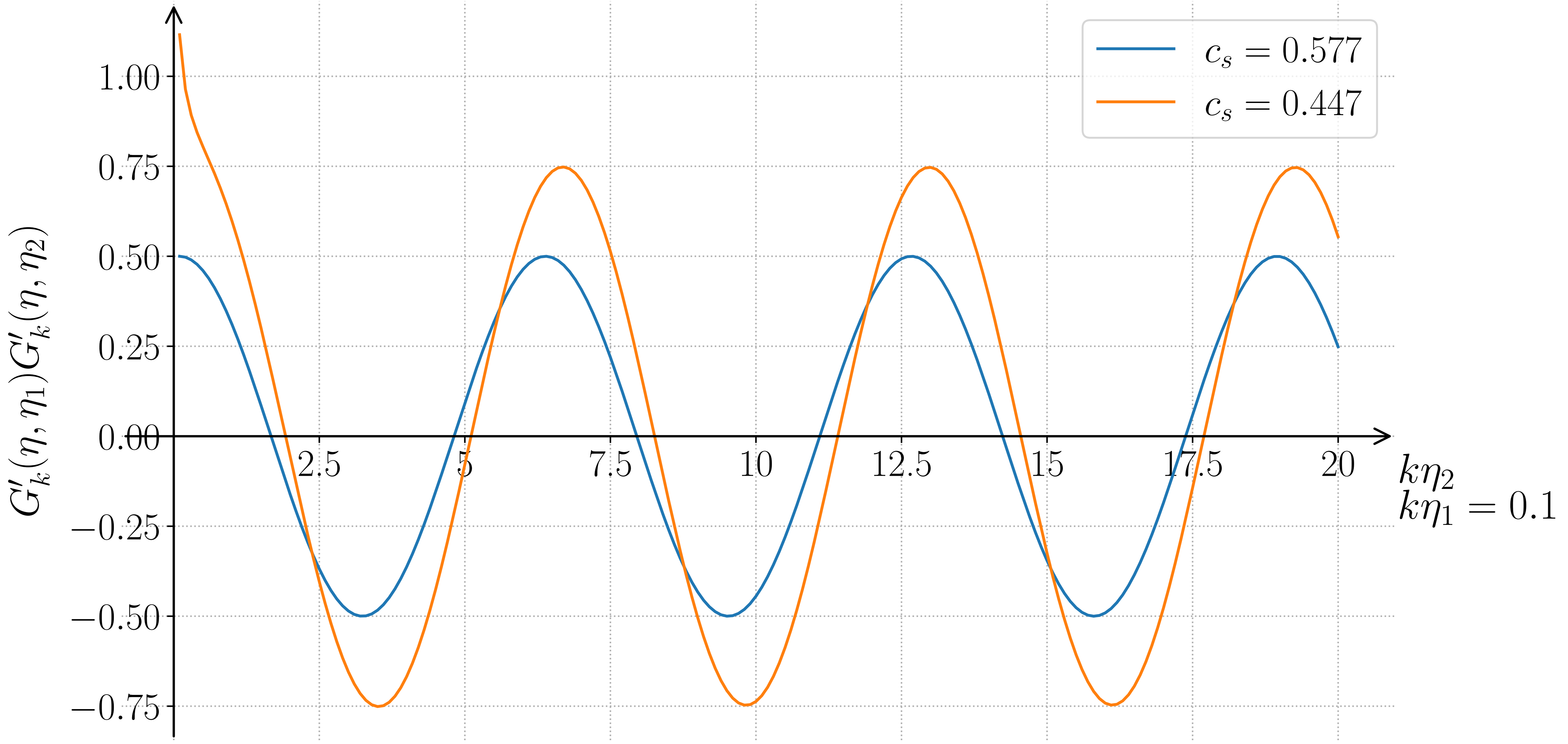


- Sound wave frequency $c_s p$

- Sound wave damping

$$\tilde{v}_p^i(\eta) = \left(\frac{\eta_*}{\eta}\right)^\nu \left[v_p^i e^{-ic_s p\eta} + v_p^{*i} e^{ic_s p\eta}\right]$$

- Hubble friction on propagation of h_{ij}



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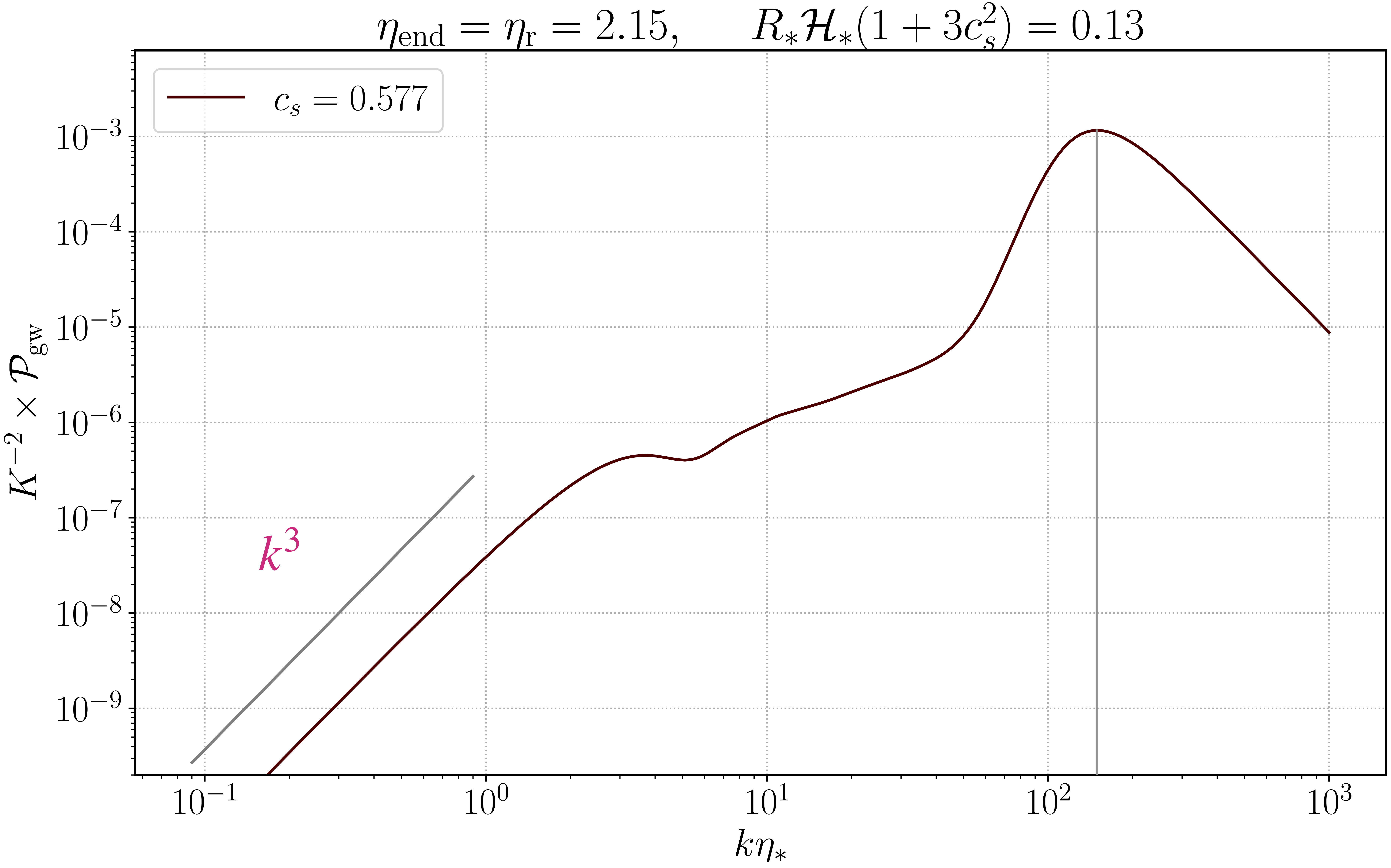

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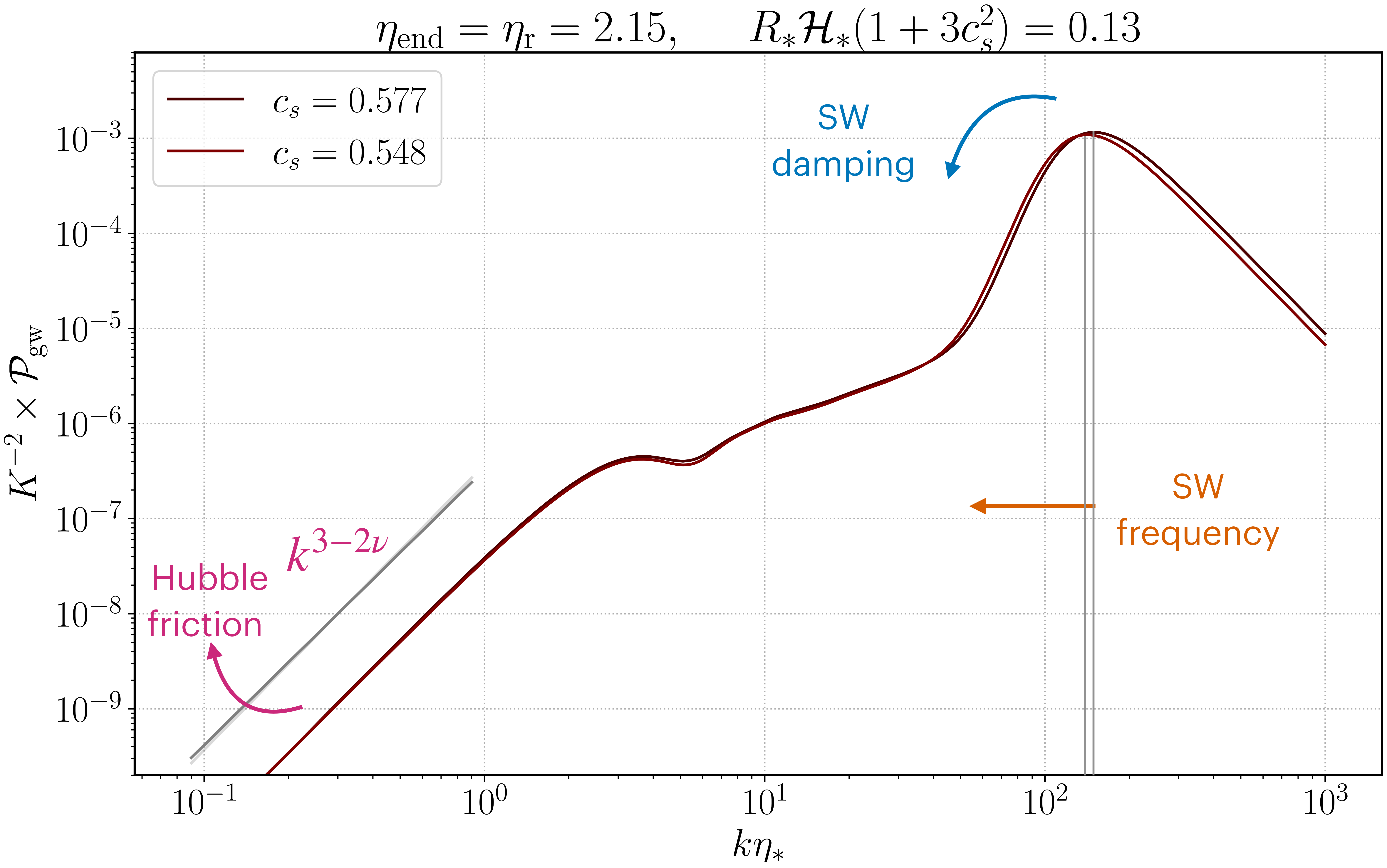
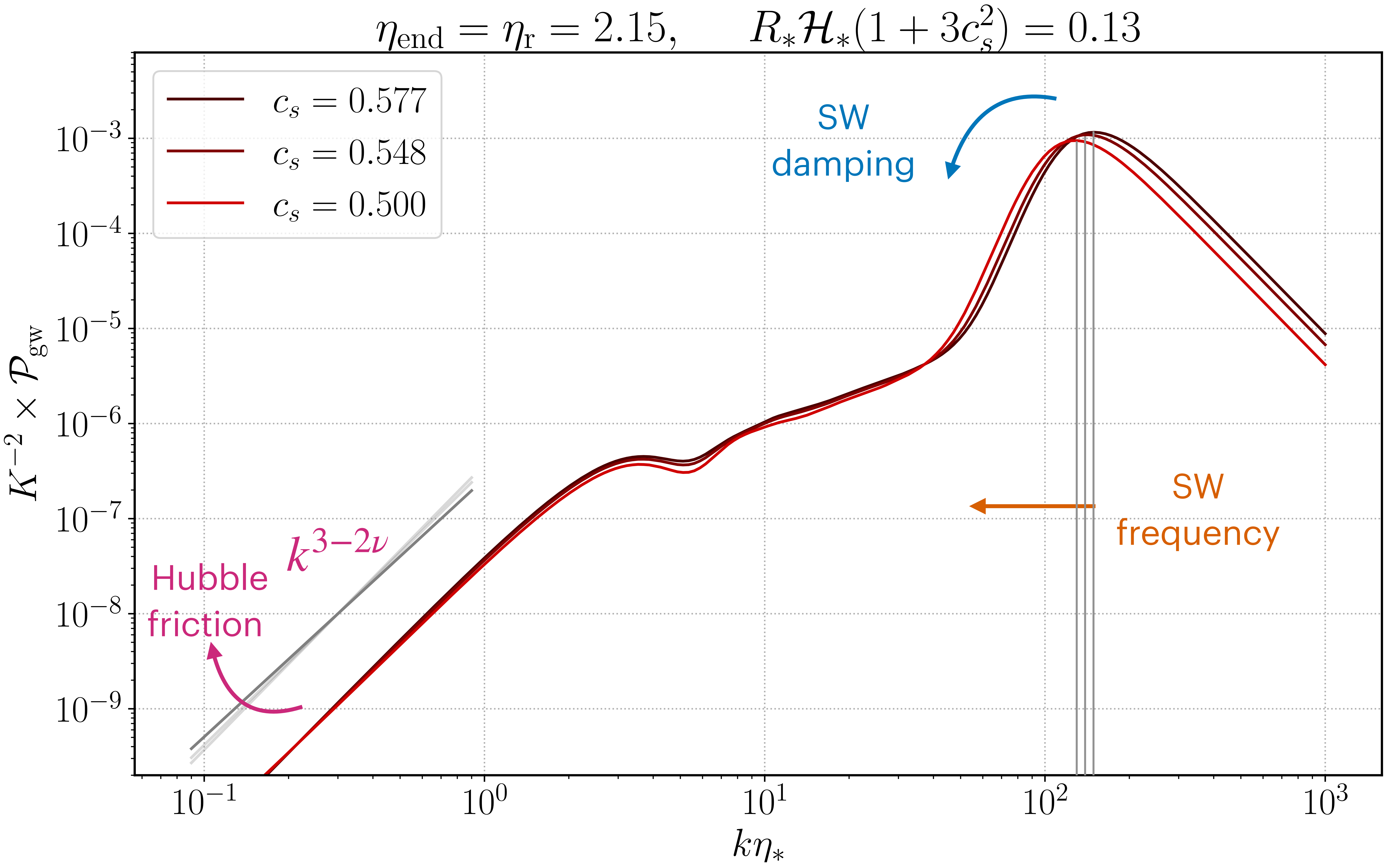


$$\nu = \frac{1 - 3c_s^2}{1 + 3c_s^2}$$

$$\nu \approx 0.052$$

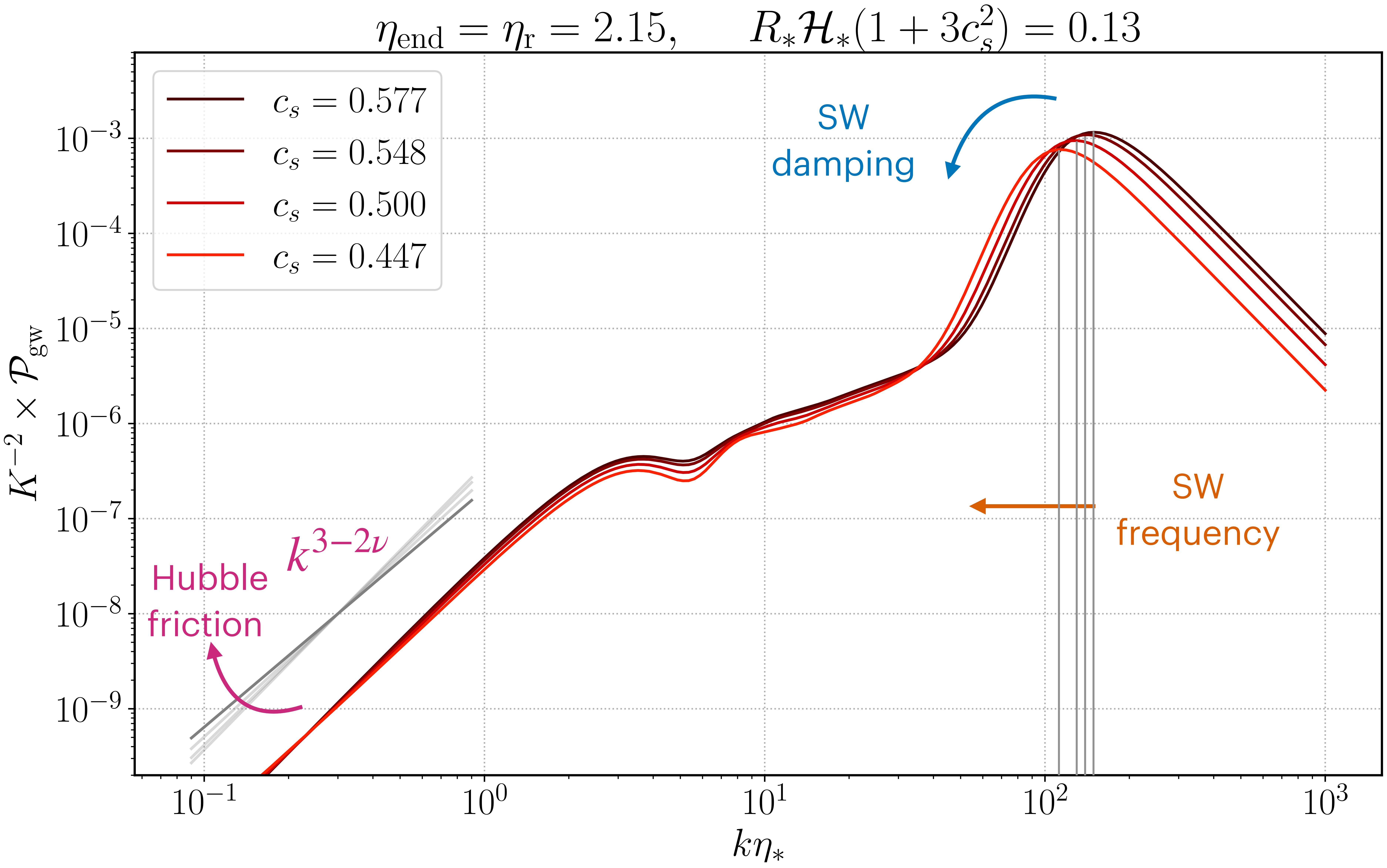
Effects of a softer equation of state



$$\nu = \frac{1-3c_s^2}{1+3c_s^2}$$

$$\nu \approx 0.143$$

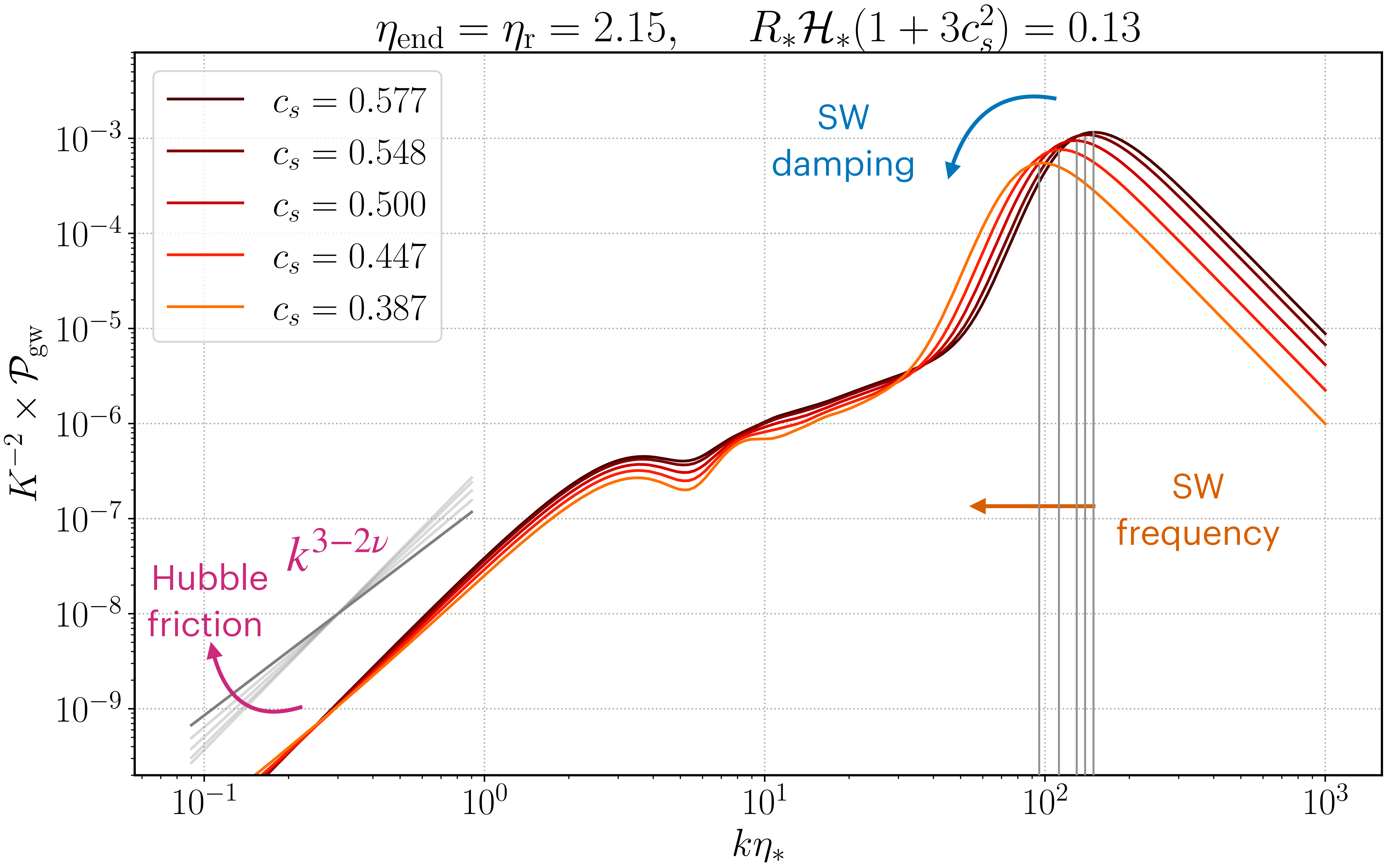
Effects of a softer equation of state



$$\nu = \frac{1-3c_s^2}{1+3c_s^2}$$

$$\nu \approx 0.250$$

Effects of a softer equation of state



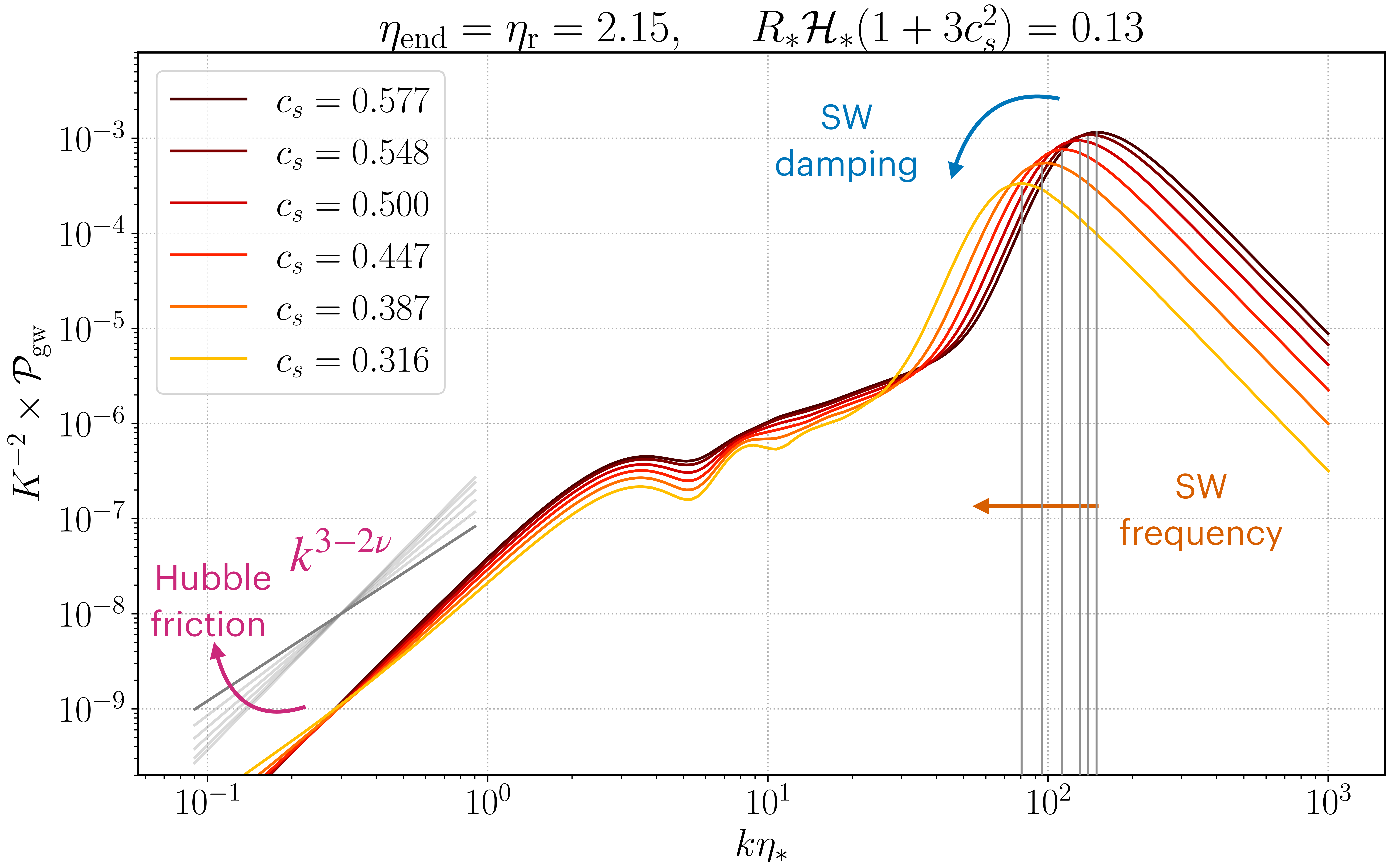
$$\nu = \frac{1 - 3c_s^2}{1 + 3c_s^2}$$

$$\nu \approx 0.380$$

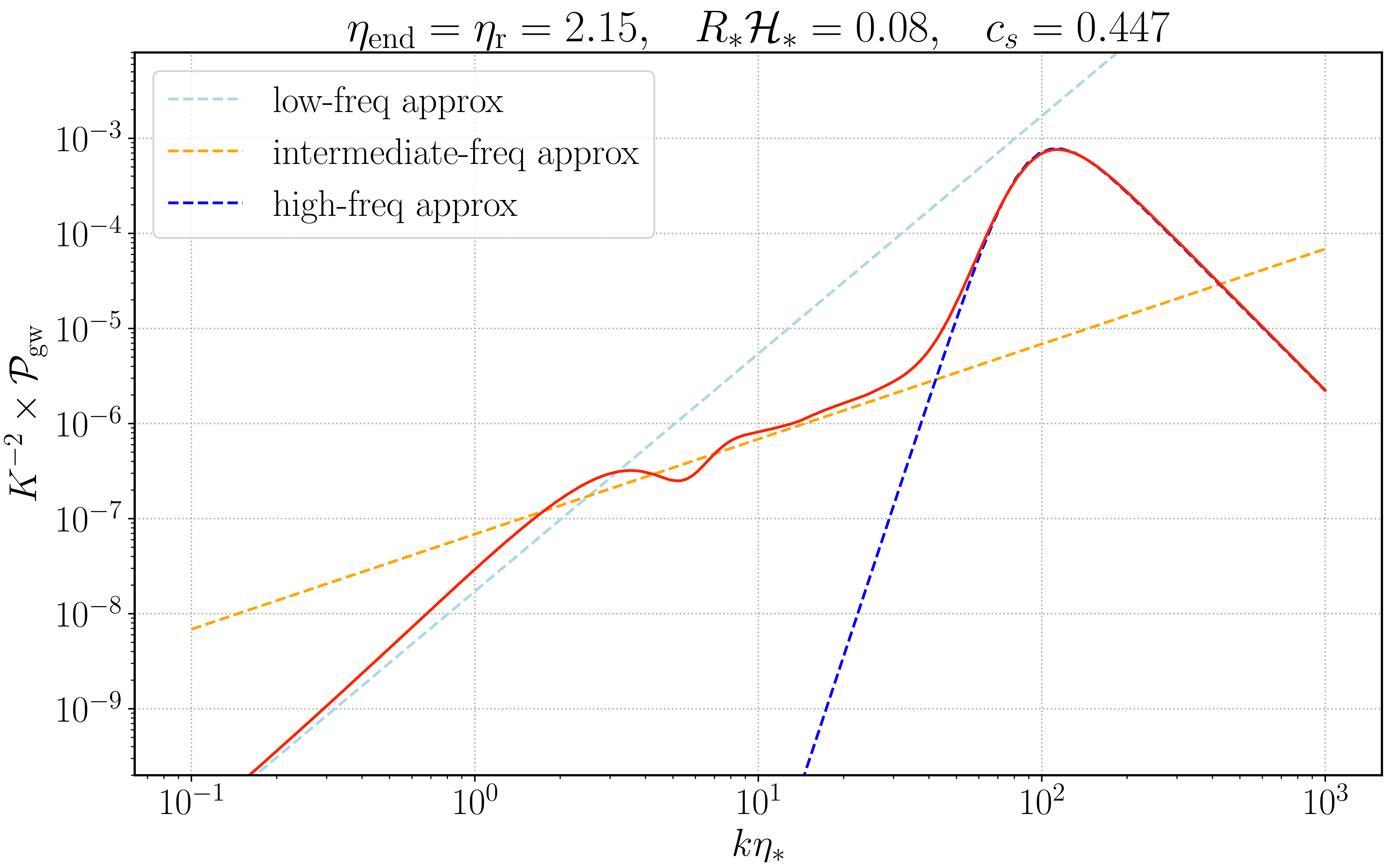
Effects of a softer equation of state

$$\nu = \frac{1 - 3c_s^2}{1 + 3c_s^2}$$

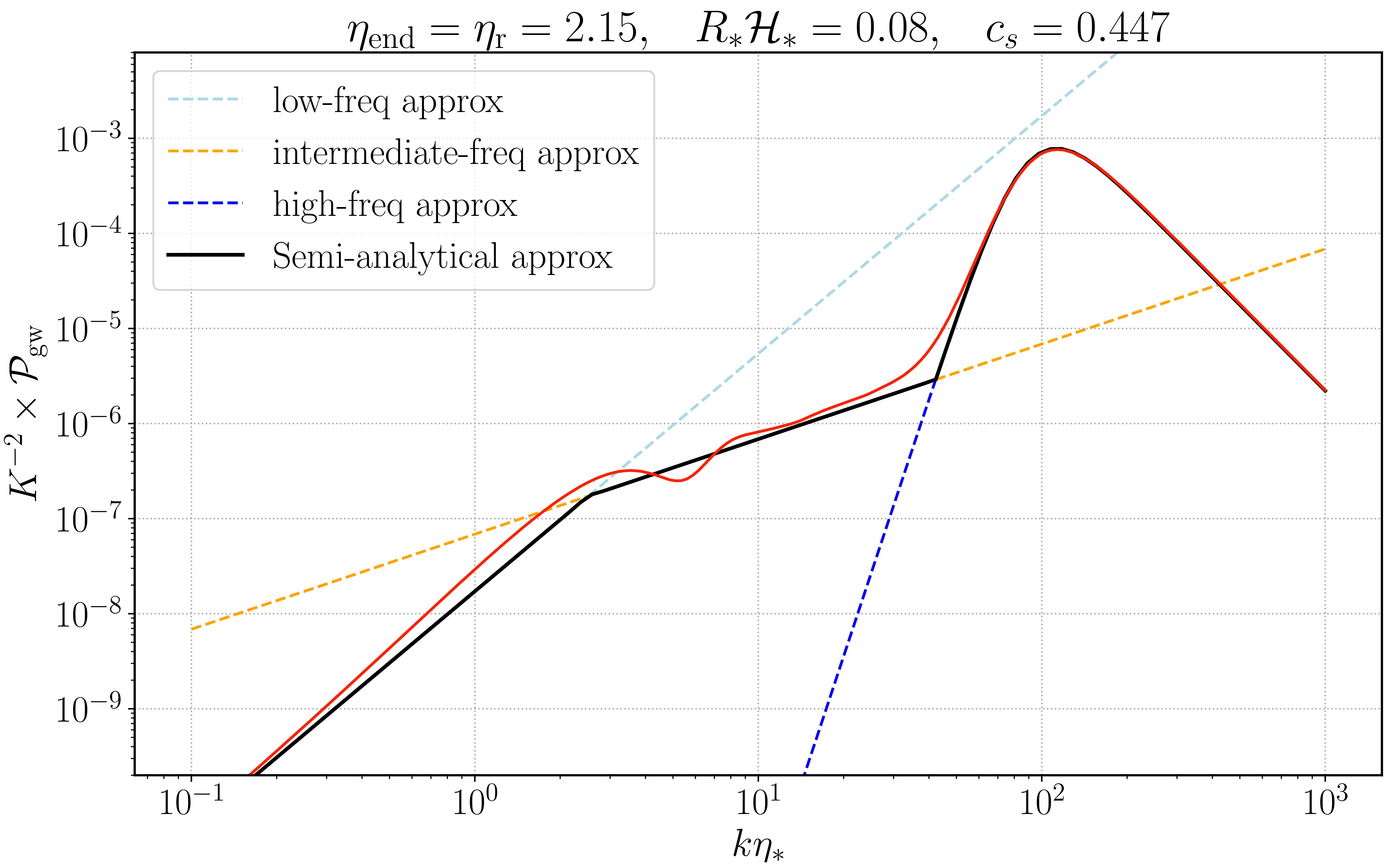
$$\nu \approx 0.540$$



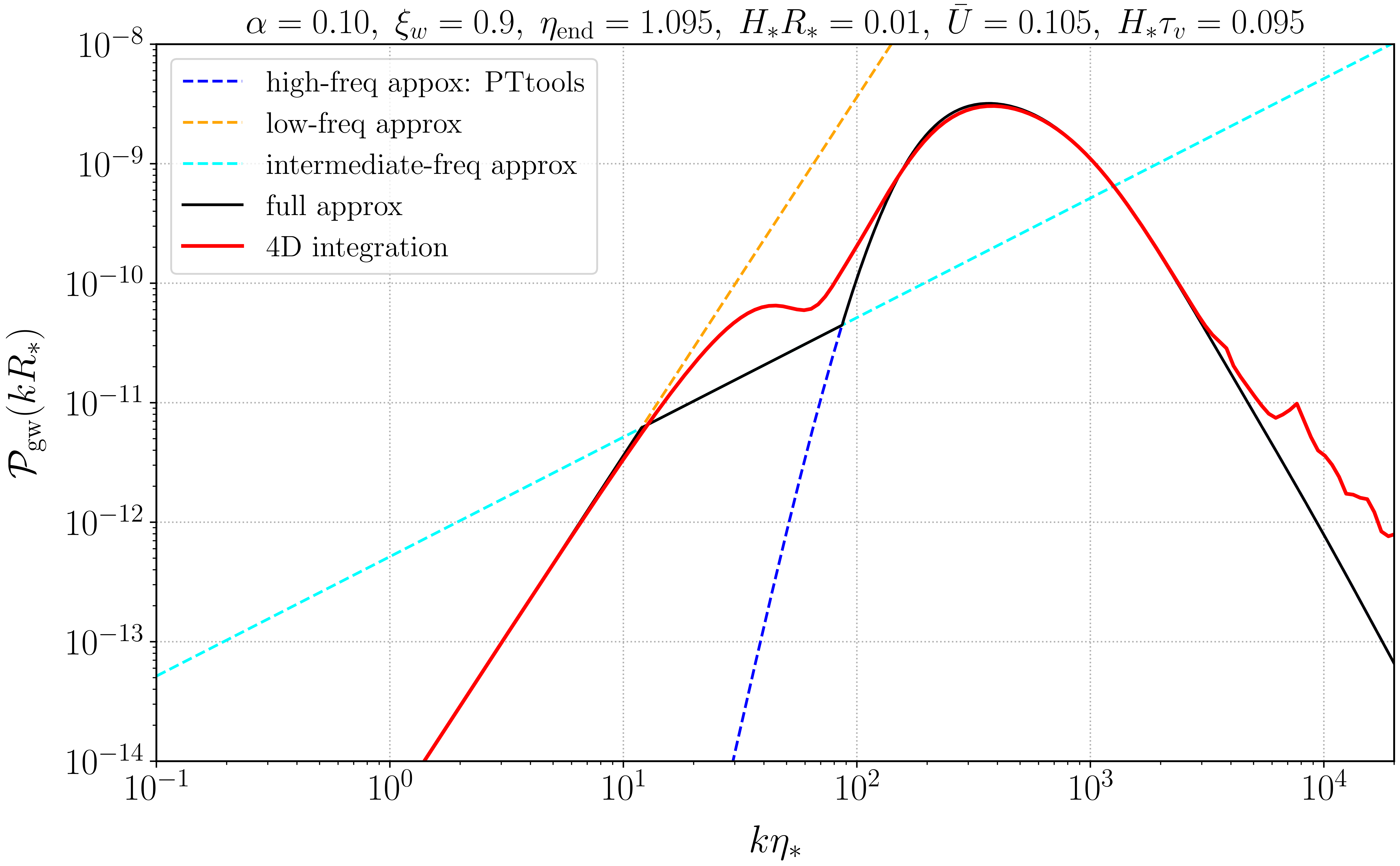
Effects of a softer equation of state: analytical approximations



Effects of a softer equation of state: analytical approximations

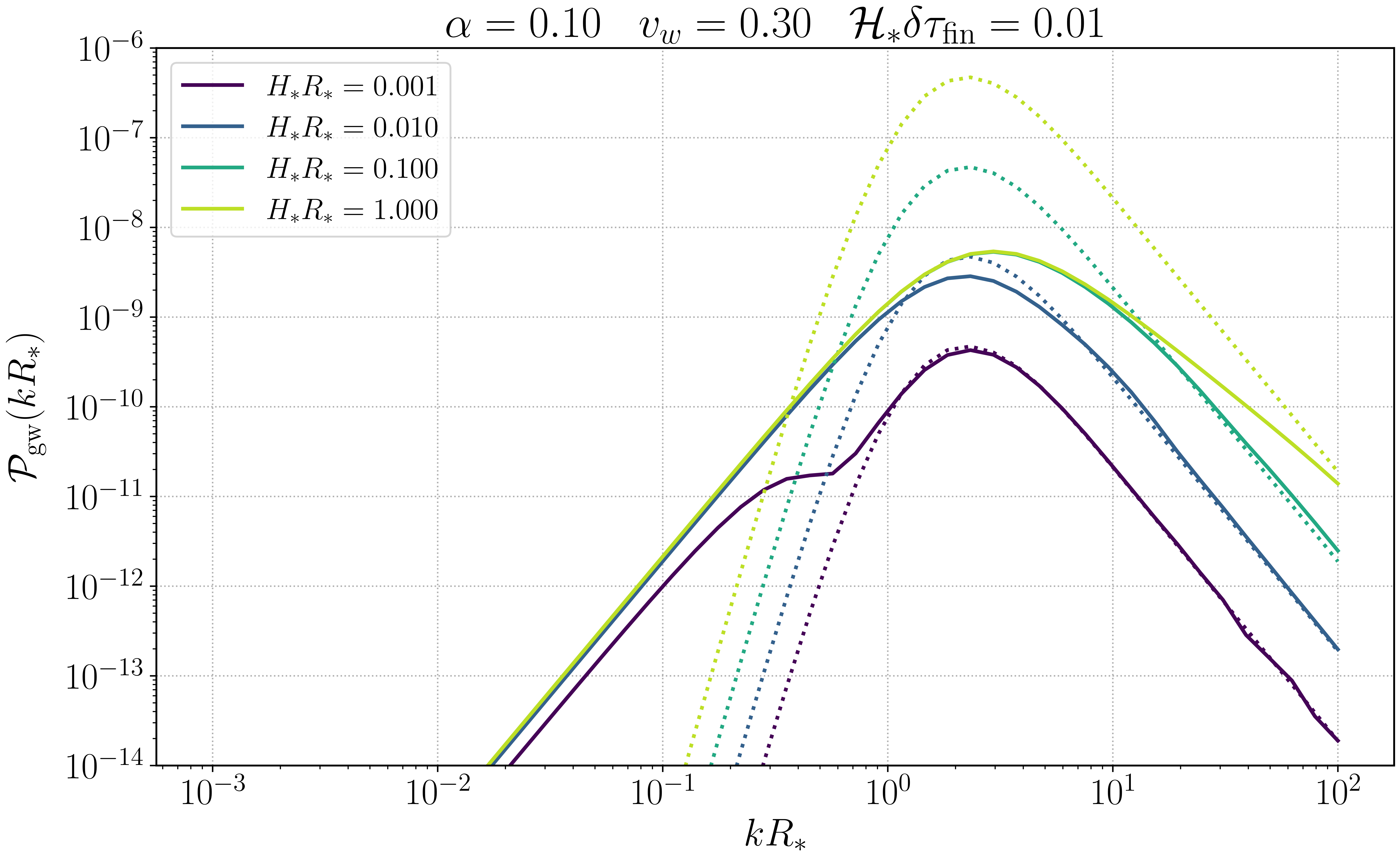


GW power spectrum in the Sound Shell Model

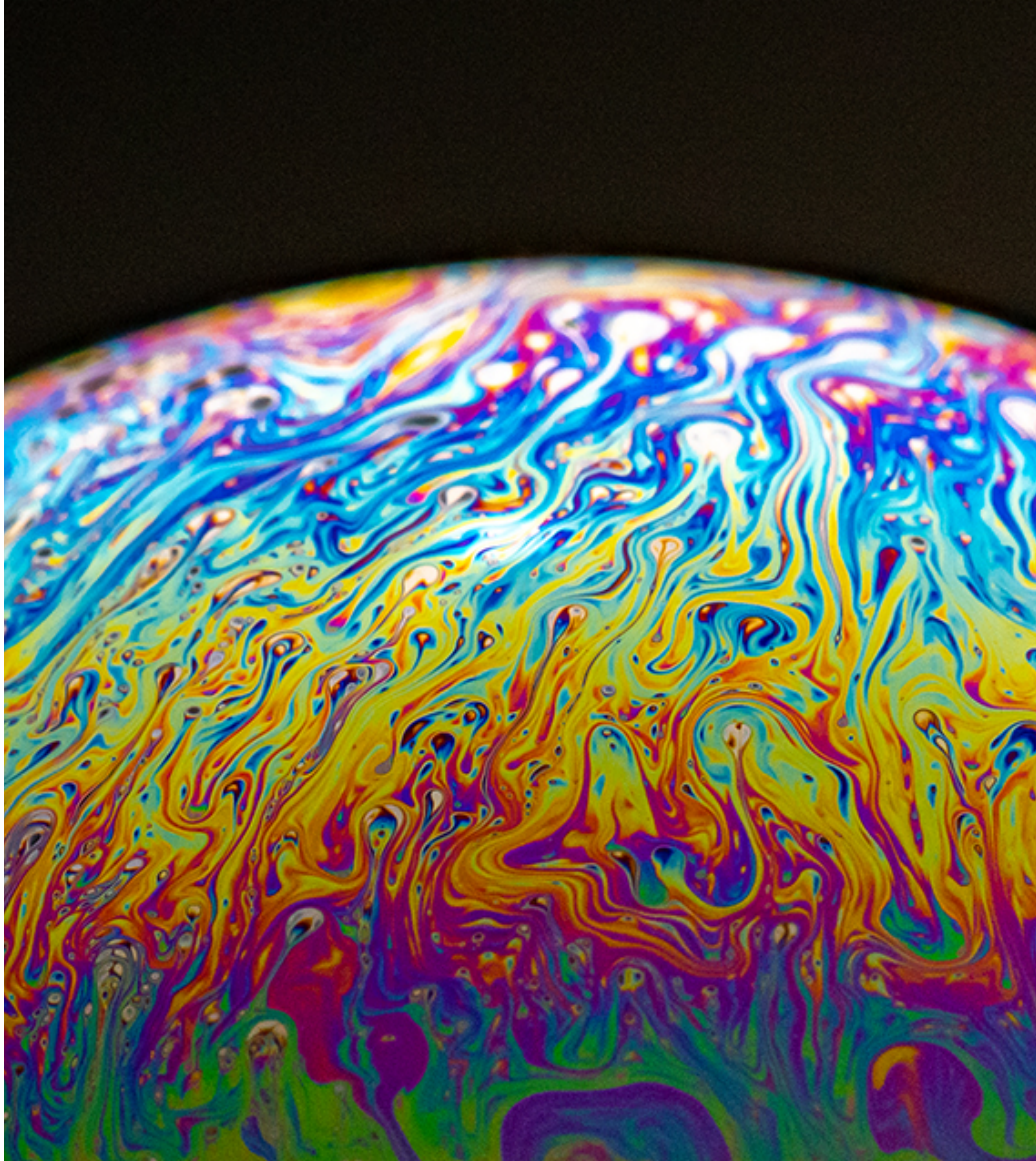


GW power spectrum in the Sound Shell Model

A. Roper Pol et al, *Phys.Rev.D* 109 (2024)



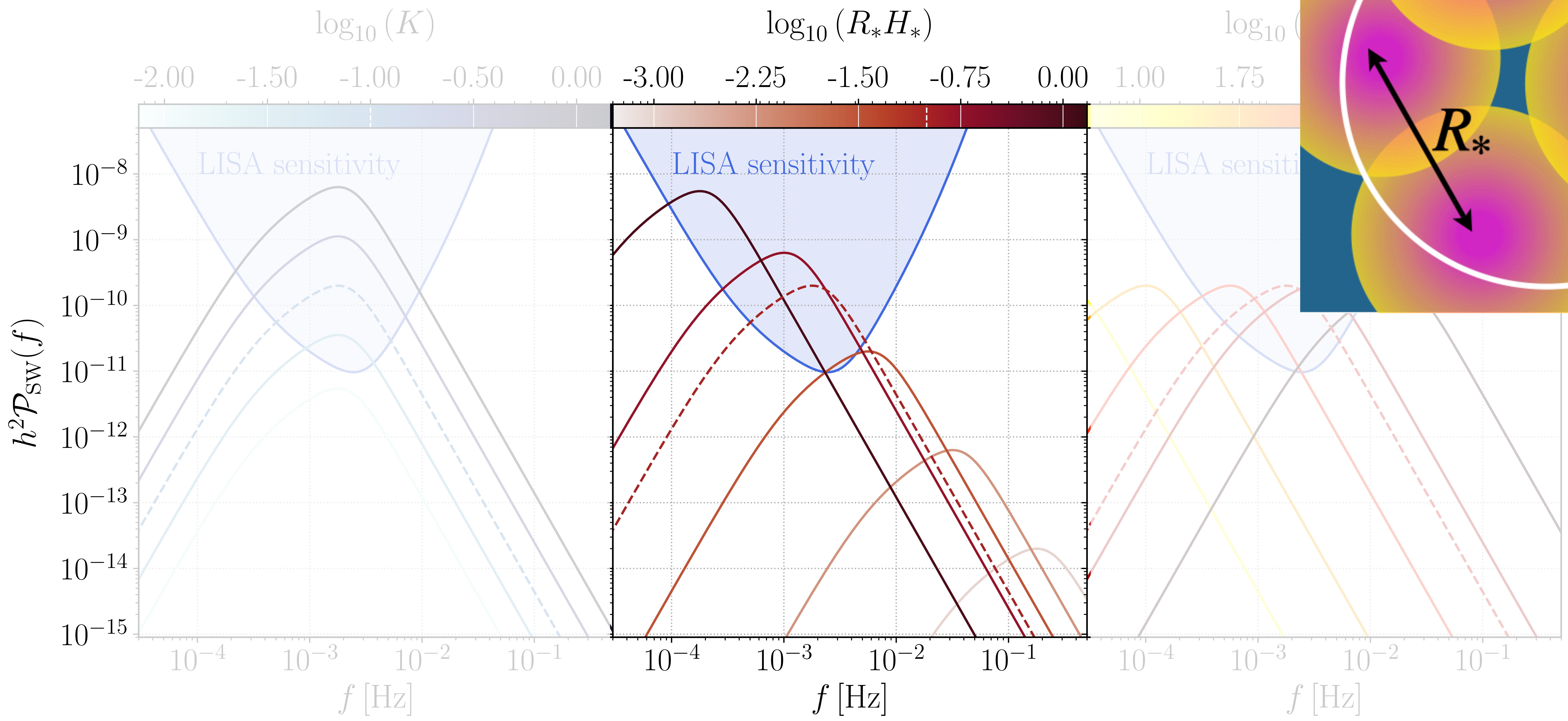
III. Gravitational effects on the production of gravitational wave

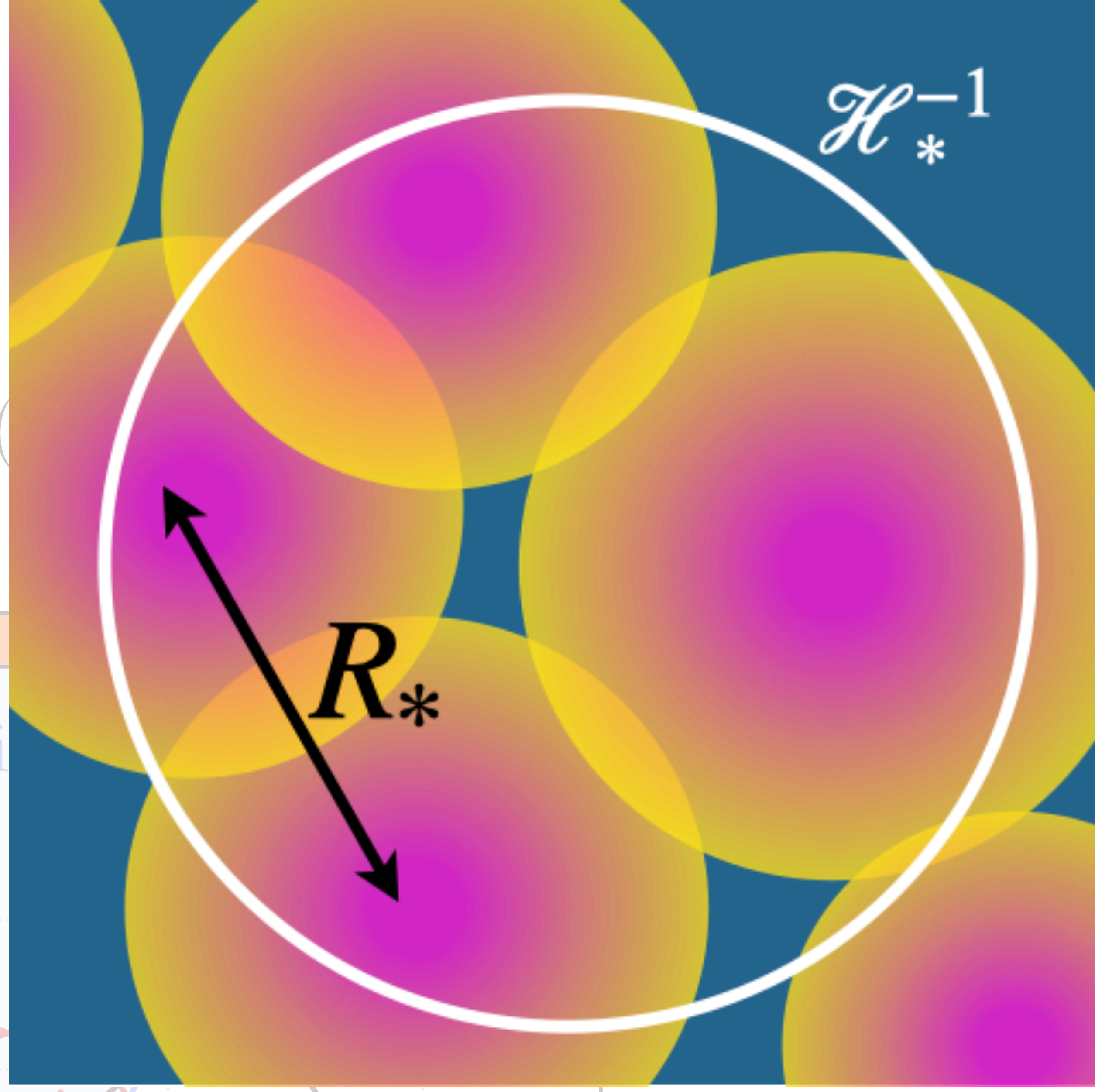


Macro photography of a soap bubble
Ingredients: 3 parts water, 1 part soap, 1 ts glycerine.
Picture from Tom Bol Photography

The larger the better

Caprini et al. LISA CosmoWG, *JCAP* 10 (2024)



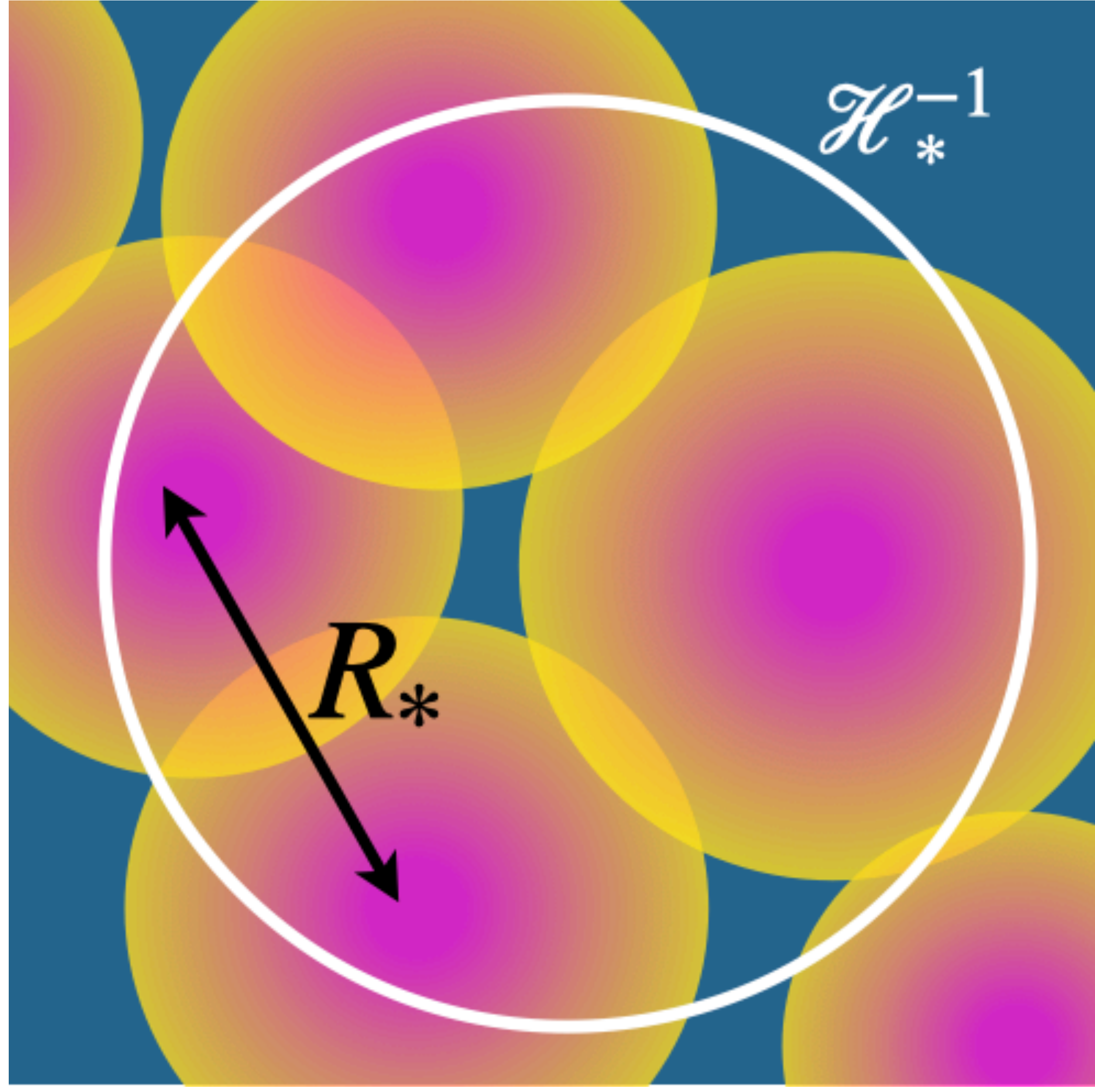


- Large bubble models will be the first one to be tested by LISA
- How large can we get?

Bubble mean center spacing: Large bubbles

$$R_* H_* = (8\pi)^{1/3} v_w \left(\frac{\beta}{H_*}\right)^{-1}$$

β : Inverse phase transition duration



Theoretical bounds

- Primordial black hole abundance

  Lewicki et al *JHEP* 09 (2023), Phys.Rev.Lett. 133 (2024), arXiv:2412.10366

  Franciolini et al. arxiv:2503.01962

$\beta/H_* \lesssim 3.8$

$\beta/H_* \gtrsim 3.8$
$R_* H_* \lesssim 0.77\, v_w$

Large bubbles are most likely in

- Strongly supercooled detonations $v_w \approx 1$
- Subsonic deflagrations (thermal suppression of nucleation rate)

(Ajmi & Hindmarsh 2022)

Are current models ready for large bubbles?

Gravitational effects

- Fluid self-gravity?
- Universe expansion ($c_s^2 \neq 1/3$)?
- Curvature perturbations?

Semi-analytic model of acoustic gravitational wave production

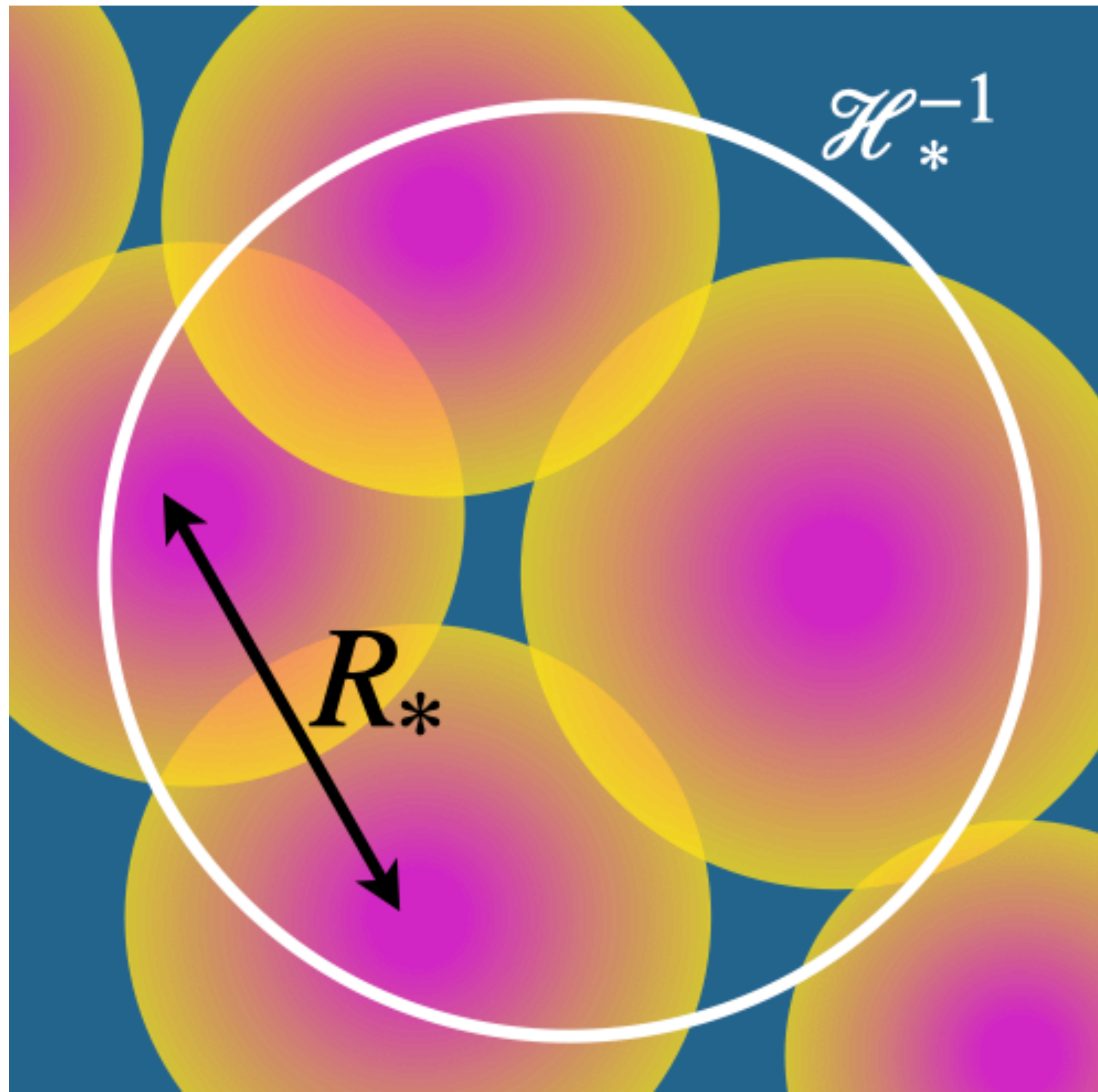
Compressional modes around bubbles generate scalar and tensor perturbations

$$ds^2 = a^2(\eta)\Big\{ -\big(1 + 2\Phi^{(1)} + 2\Phi^{(2)}\big)d\eta^2 + \Big[\big(1 - 2\Psi^{(1)} - 2\Psi^{(2)}\big)\delta_{ij} + h_{ij}\Big]\, dx^i dx^j \Big\}$$

$$h''_{ij} + 2\mathscr{H} h'_{ij} - \nabla^2 h_{ij} = \mathscr{S}_{ij} \longrightarrow$$

- Primary source of gravitational waves v_i
- Secondary source of gravitational waves $\Phi^{(1)} = \Psi^{(1)}$

$$\mathscr{S}_{ij} = v_i^{(1)} v_j^{(1)} + \frac{1}{4\pi G a^2 \bar{w}} \partial_i \Psi^{(1)} \partial_j \Psi^{(1)} \quad \sim \quad v^2 + (R_* \mathscr{H}_*)^2 \left(\frac{\delta e}{\bar{e}}\right)^2$$



- We include curvature fluctuations Ψ perturbatively

  → Short sound wave wavelength expansion $R_* \mathscr{H}_* \lesssim \mathcal{O}(1)$

General relativistic corrections at NLO in $R_*\mathcal{H}_* \lesssim \mathcal{O}(1)$

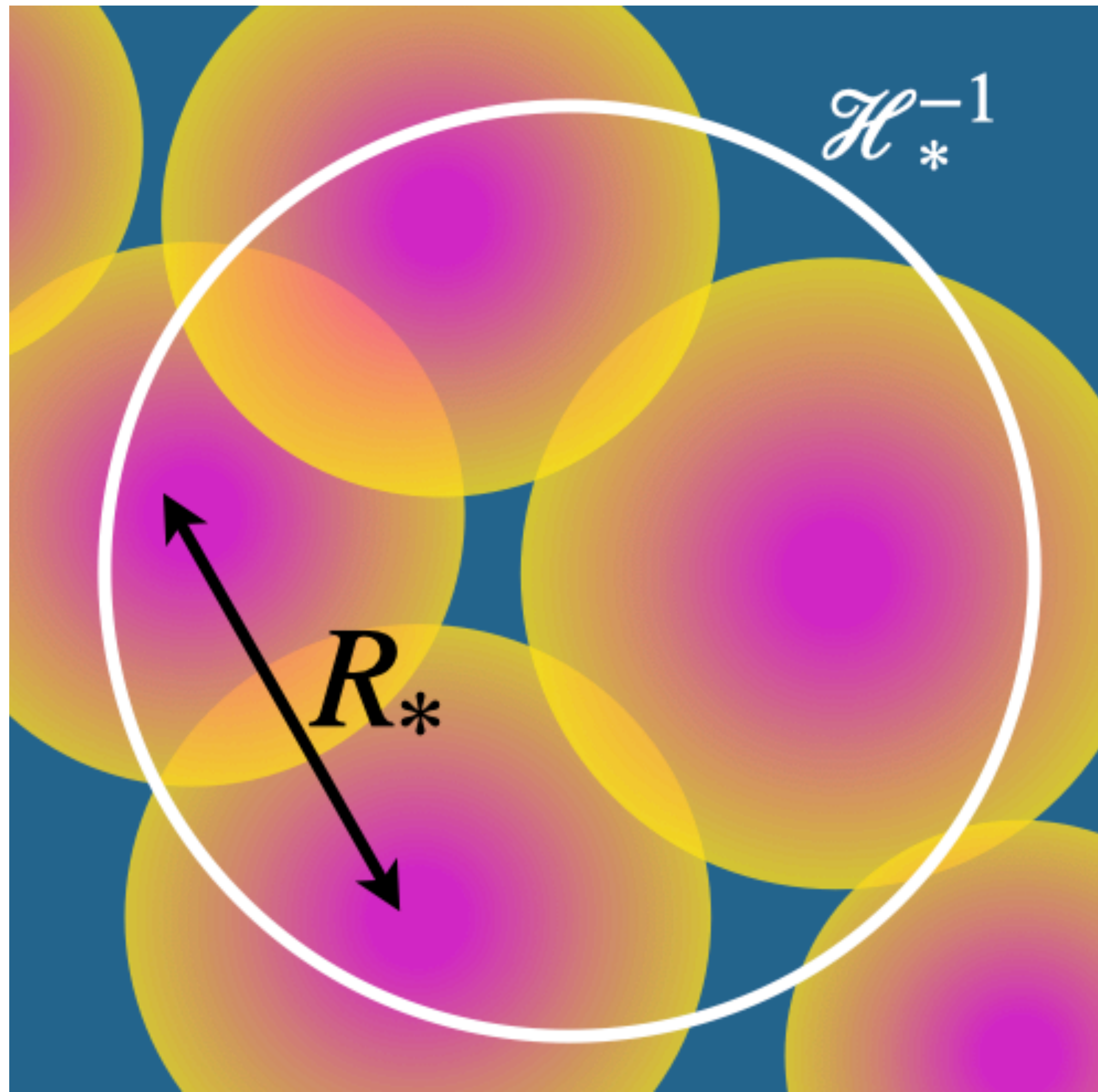
$$e_{\rm gw} \sim \langle h'_{ij} h'_{ij} \rangle \sim \iint d\eta_1 d\eta_2 \ \ldots \ \langle \tilde{S}_{ij}(k,\eta_1) \tilde{S}_{ij}(k,\eta_2) \rangle$$

$$\tilde{S}_{ij} \sim \int_p \tilde{v}^i_p \tilde{v}^j_q + \hat{p}^i \hat{q}^j (R_*\mathcal{H}_*)^2 \tilde{\delta}_p \tilde{\delta}_q \Big|_{q=p-k}$$

$$\tilde{v}^i_p(\eta) = v^i_p \left[1 + \mathcal{O}(R_*\mathcal{H}_*)\right] \ e^{-ic_s p\eta} + h.c.$$

$$\langle \tilde{v}_p \tilde{v}_q \rangle \langle \tilde{v}_p \tilde{v}_q \rangle + (R_*\mathcal{H}_*)^2 \langle \tilde{v}_p \tilde{\delta}_q \rangle \langle \tilde{v}_q \tilde{\delta}_p \rangle + (R_*\mathcal{H}_*)^2 \langle \tilde{v}_p \tilde{v}_q \rangle \langle \tilde{v}_p \tilde{v}_q \rangle + (R_*\mathcal{H}_*)^4 \langle \tilde{\delta}_p \tilde{\delta}_q \rangle \langle \tilde{\delta}_q \tilde{\delta}_p \rangle$$

LO (sound waves) — NLO (Curvature) — NLO (sound waves) — NNLO (curvature)



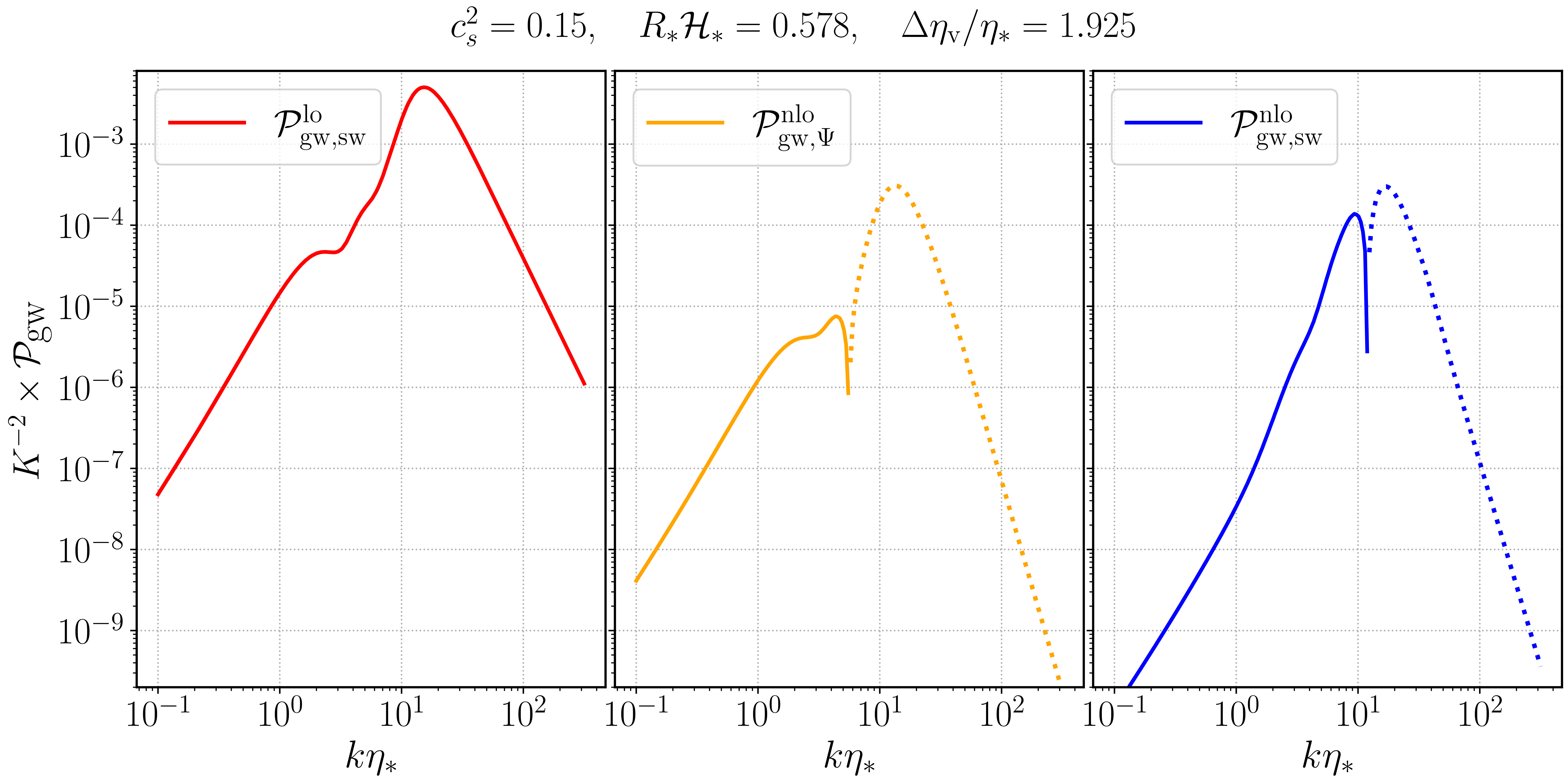
General relativistic corrections at NLO in $R_* \mathscr{H}_* \lesssim \mathcal{O}(1)$

$$e_{\rm gw} \sim \langle h'_{ij} h'_{ij} \rangle \sim \iint d\eta_1 d\eta_2 \ \dots \ \langle \tilde{S}_{ij}(k,\eta_1) \tilde{S}_{ij}(k,\eta_2) \rangle$$

$$\mathscr{P}_{\rm gw} = \frac{1}{\bar{e}} \frac{{\rm d}e_{\rm gw}}{{\rm d}\ln k}$$

$$\langle \tilde{v}_p \tilde{v}_q \rangle \langle \tilde{v}_p \tilde{v}_q \rangle + (R_* \mathscr{H}_*)^2 \langle \tilde{v}_p \tilde{\delta}_q \rangle \langle \tilde{v}_q \tilde{\delta}_p \rangle + (R_* \mathscr{H}_*)^2 \langle \tilde{v}_p \tilde{v}_q \rangle \langle \tilde{v}_p \tilde{v}_q \rangle$$

LO (sound waves) — NLO (Curvature) — NLO (sound waves)



General relativistic corrections at NLO in $R_*\mathcal{H}_* \lesssim \mathcal{O}(1)$

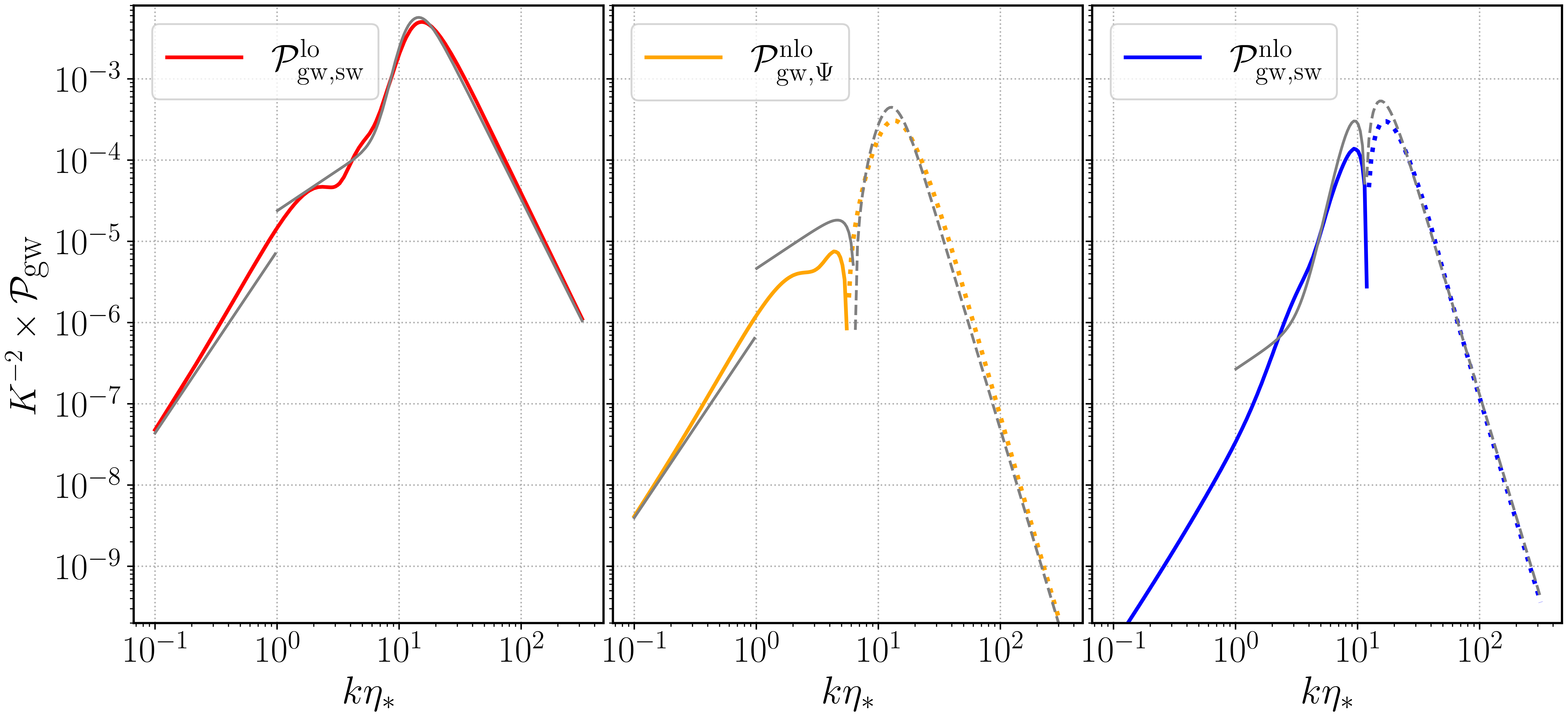
$$e_{\rm gw} \sim \langle h'_{ij} h'_{ij} \rangle \sim \iint d\eta_1 d\eta_2 \ \dots \ \langle \tilde{S}_{ij}(k,\eta_1) \tilde{S}_{ij}(k,\eta_2) \rangle$$

$$\mathcal{P}_{\rm gw} = \frac{1}{\bar{e}} \frac{{\rm d}e_{\rm gw}}{{\rm d}\ln k}$$

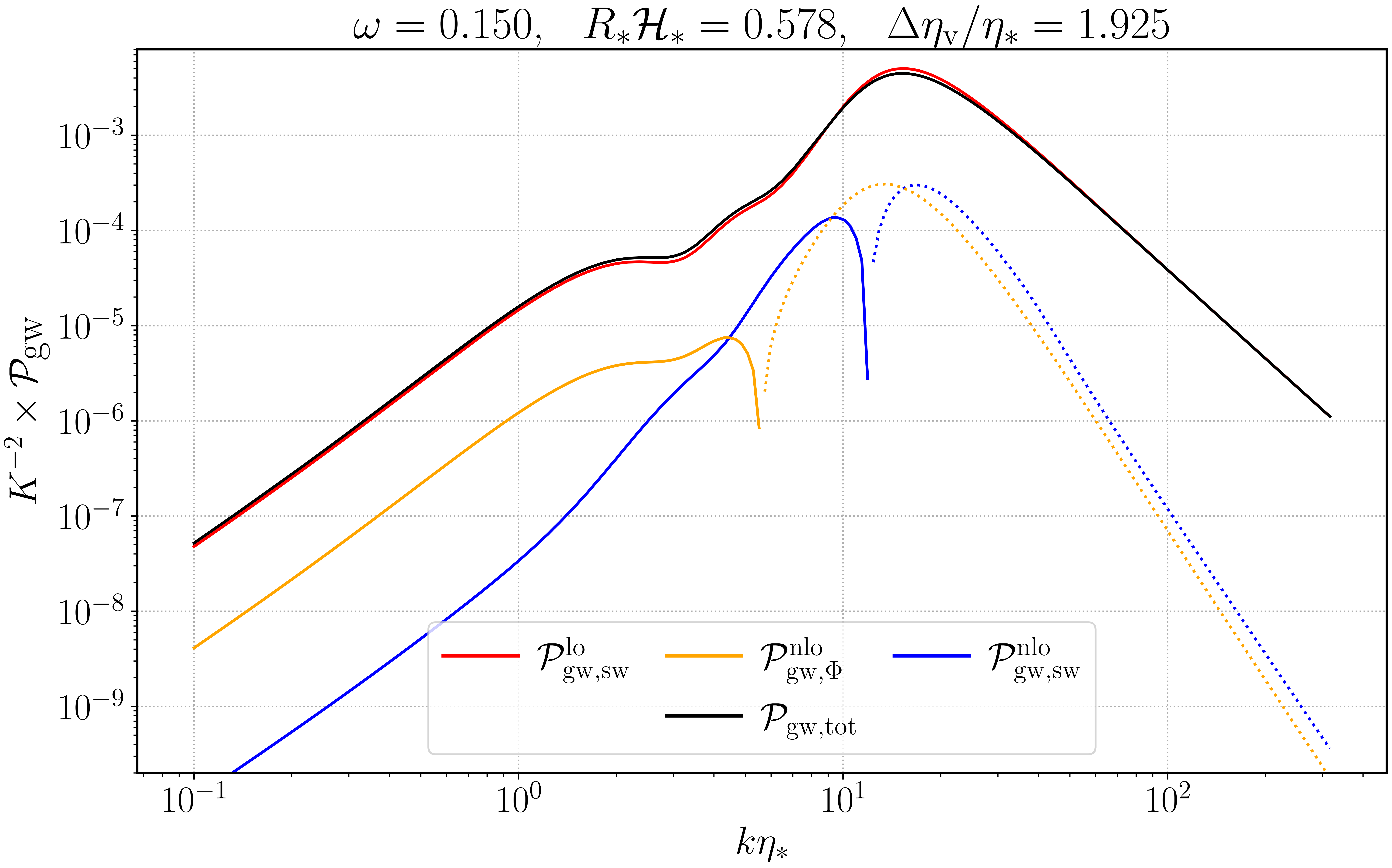
$$\langle \tilde{v}_p \tilde{v}_q \rangle \langle \tilde{v}_p \tilde{v}_q \rangle + (R_*\mathcal{H}_*)^2 \langle \tilde{v}_p \tilde{\delta}_q \rangle \langle \tilde{v}_q \tilde{\delta}_p \rangle + (R_*\mathcal{H}_*)^2 \langle \tilde{v}_p \tilde{v}_q \rangle \langle \tilde{v}_p \tilde{v}_q \rangle$$

LO (sound waves) | NLO (Curvature) | NLO (sound waves)

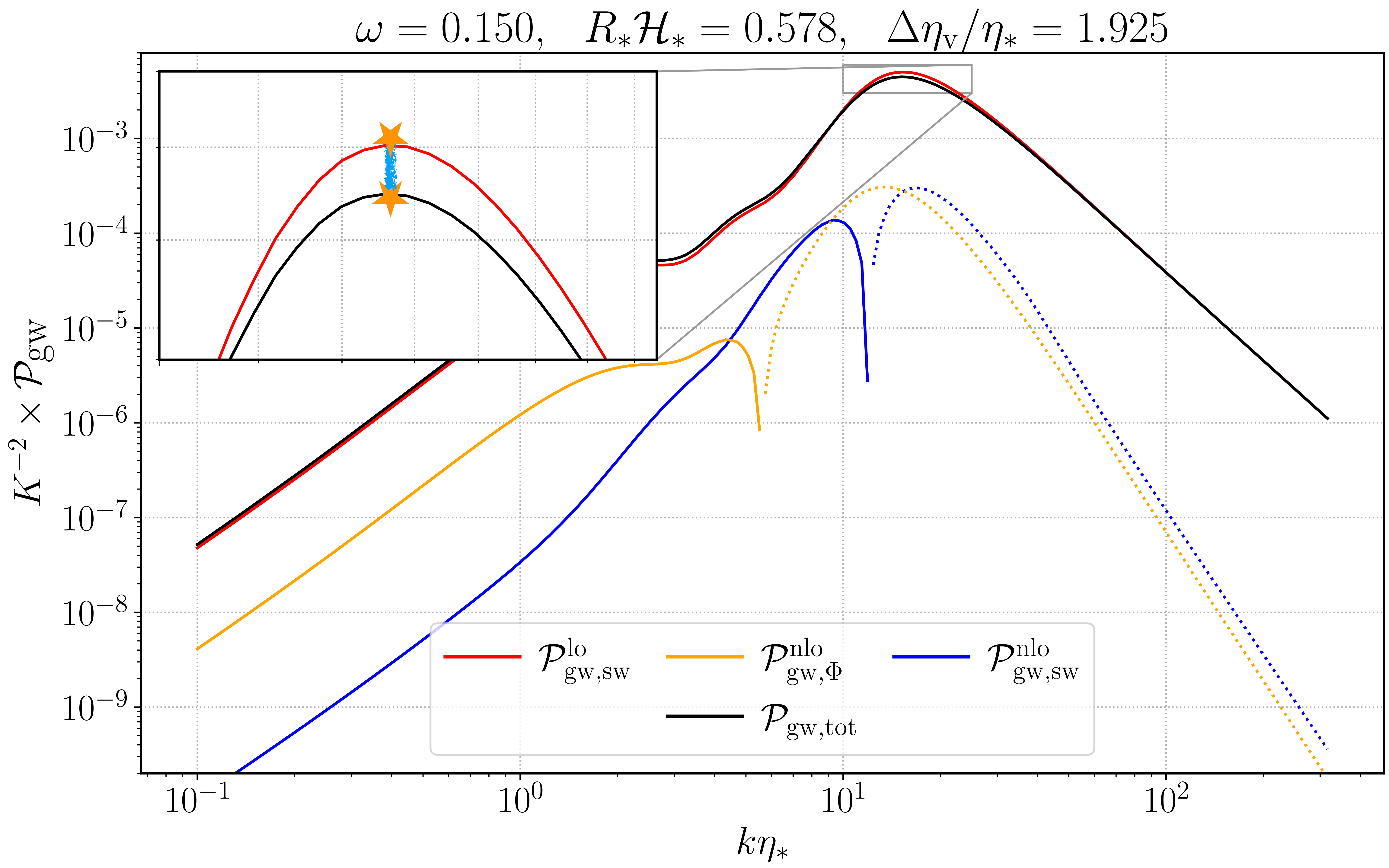
$c_s^2 = 0.15, \quad R_*\mathcal{H}_* = 0.578, \quad \Delta\eta_{\rm v}/\eta_* = 1.925$



Gravitational effects beyond Universe expansion



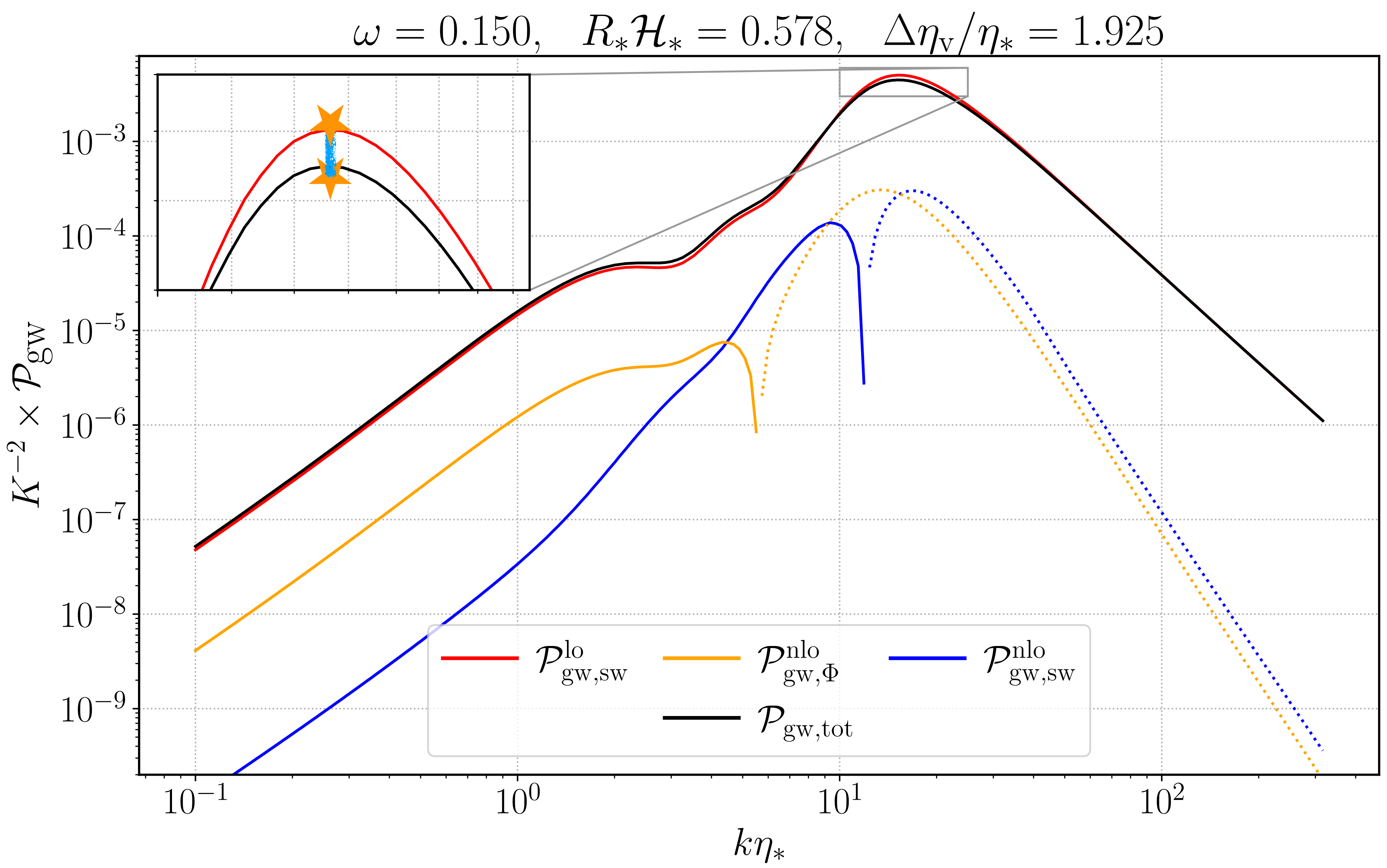
GR effects: should you care?

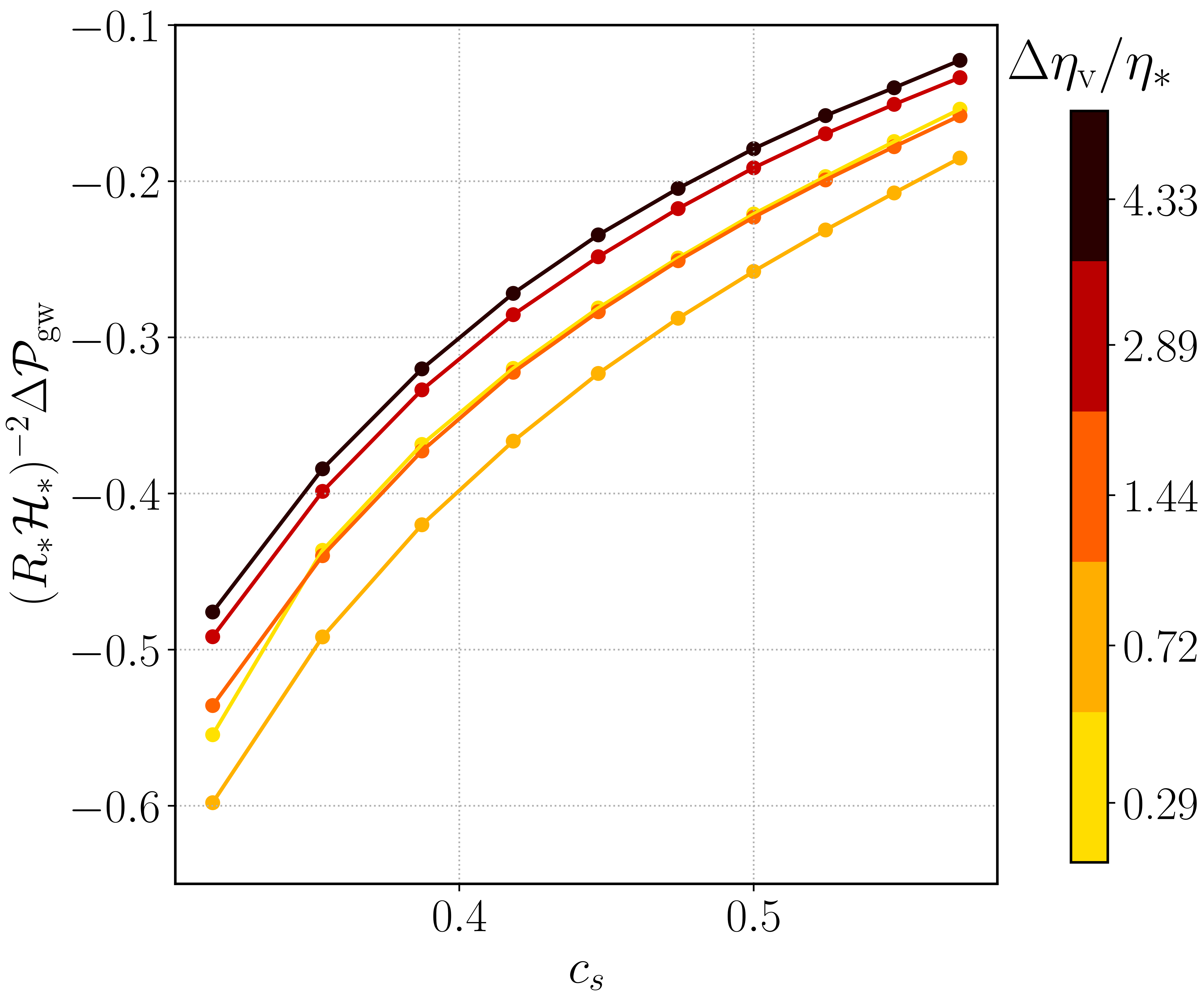


Suppression at energy injection scale

$$\Delta\mathscr{P}_{gw} = \left.\frac{\mathscr{P}_{tot} - \mathscr{P}^{lo}_{gw,sw}}{\mathscr{P}^{lo}_{gw,sw}}\right|_{k=k_p}$$

GR effects: should you care?





- Suppression at energy injection scale

$$\Delta\mathscr{P}_{gw} = \left.\frac{\mathscr{P}_{tot} - \mathscr{P}^{lo}_{gw,sw}}{\mathscr{P}^{lo}_{gw,sw}}\right|_{k=k_p}$$

$\Delta\mathscr{P}_{gw} \sim 5\,\%$ to $15\,\%$ if $R_*\mathscr{H}_* \approx 0.5$

SUMMARY (part 1)

Macro photography of a soap bubble
Picture from [Philippe's blog]

- Speed of sound changes the shape of the gravitational wave power spectrum

$$k_{\rm peak} \propto c_s 2\pi / R_*$$

Sound wave damping

Hubble friction $\mathscr{P}_{\rm gw}^{kR_* \ll 1} \propto k^{3-2\nu}$

- Gravitational effects beyond the Universe expansion are important for large bubbles

$$\mathscr{P}_{\rm gw}^{\rm nlo} / \mathscr{P}_{\rm gw}^{\rm lo} \propto (H_* R_*)^2$$

$$\Delta \mathscr{P}_{gw} \sim 15\,\%$$

when

$$R_* \mathscr{H}_* \approx 0.5$$

IV. Gravitational effects in the bubble hydrodynamics

Macro photography of a soap bubble
Ingredients: 1 part water, 1 part dishwashing soap,
1 part bubble liquid for kid, glycerine to taste.
Picture from Philippe's blog

Modelling the source with large bubbles

- Ansatz
  (fit simulations)

$$P_v \sim \frac{(kR_*)^2}{1+(kR_*)^6} \begin{cases} \propto k^2, & k \ll R_* \\ \propto k^{-4}, & k \gg R_* \end{cases}$$

causality

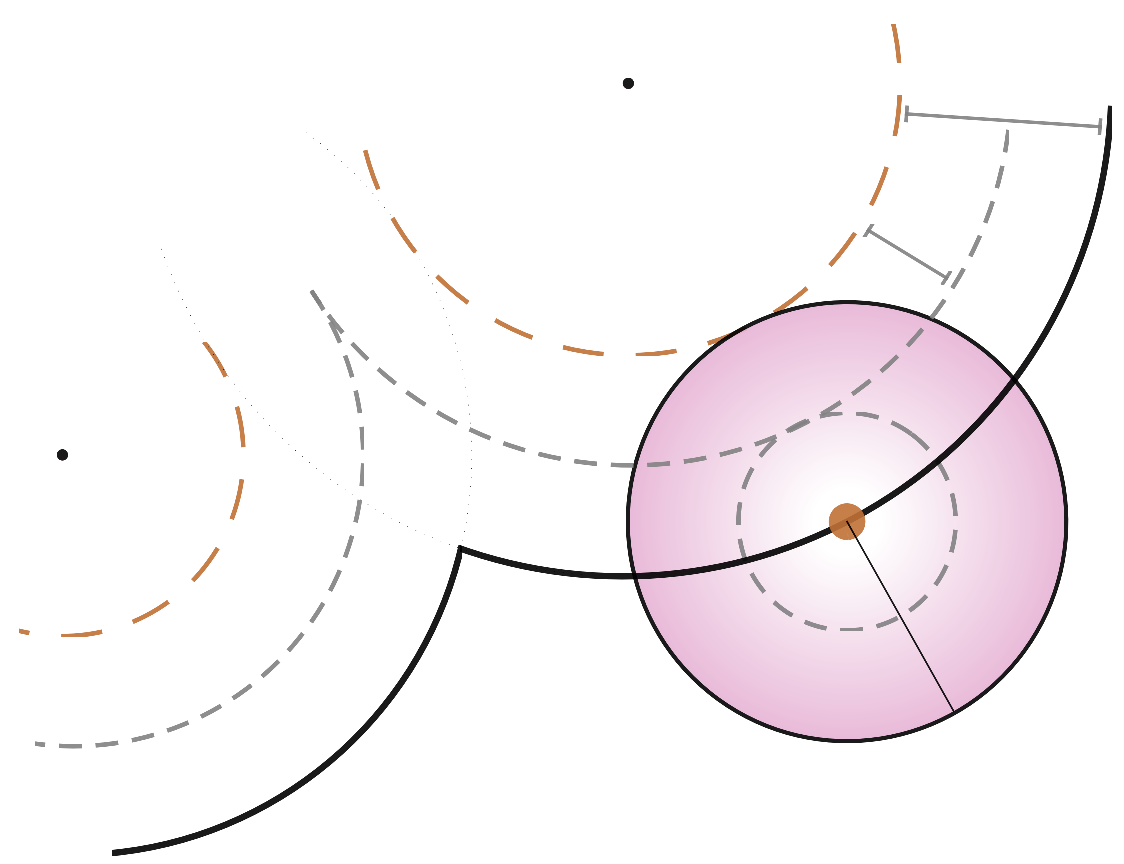
shocks

We do not have simulations on large bubbles

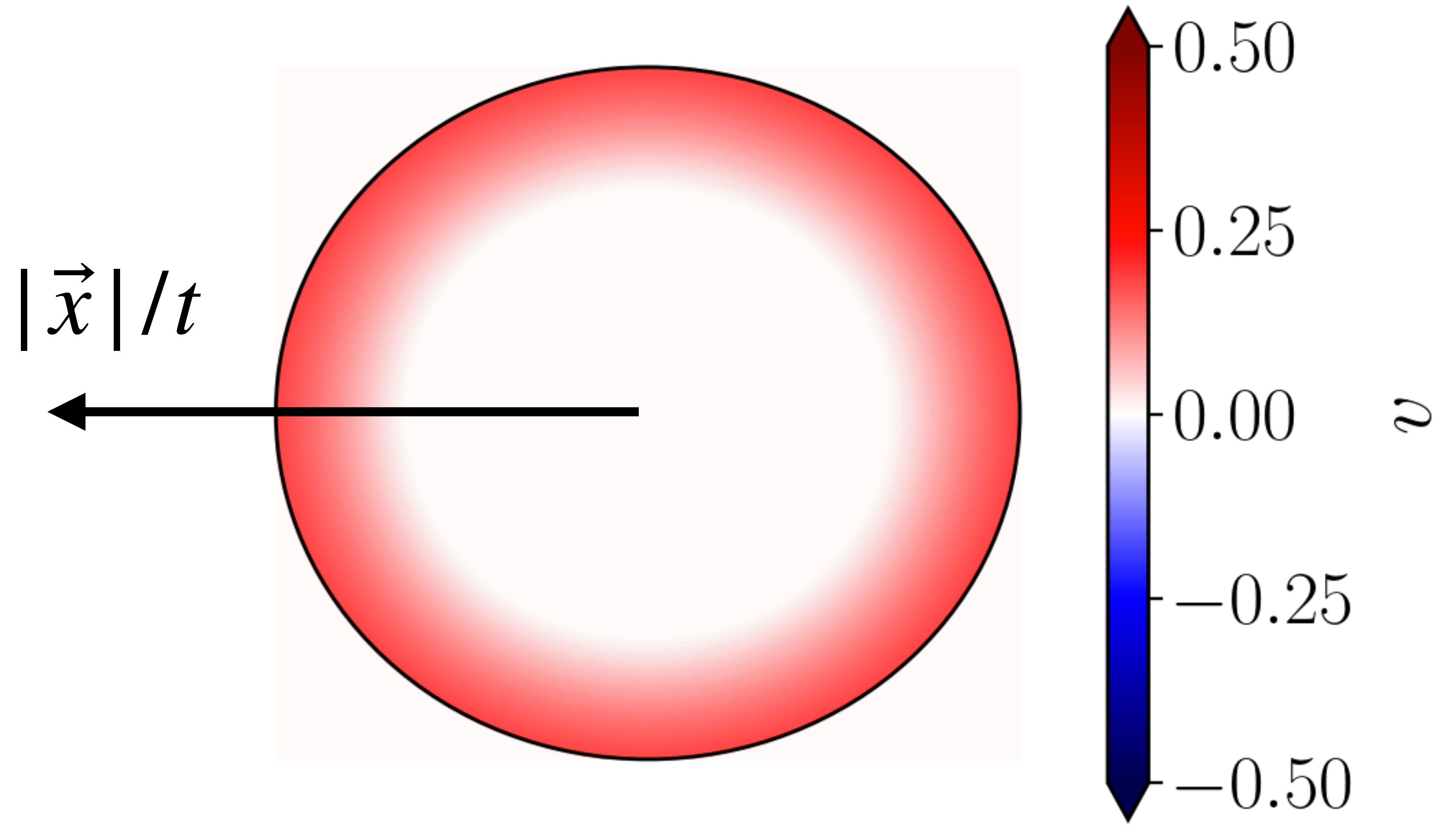
- **Sound shell model**: incoherent sum of contributions from each individual bubble

  Hindmarsh Phys.Rev.Lett. 120 (2018)

$$v_i(\vec{x},t) = \sum_{n=1}^{N_b} v_i^{(n)}(\vec{x},t)$$



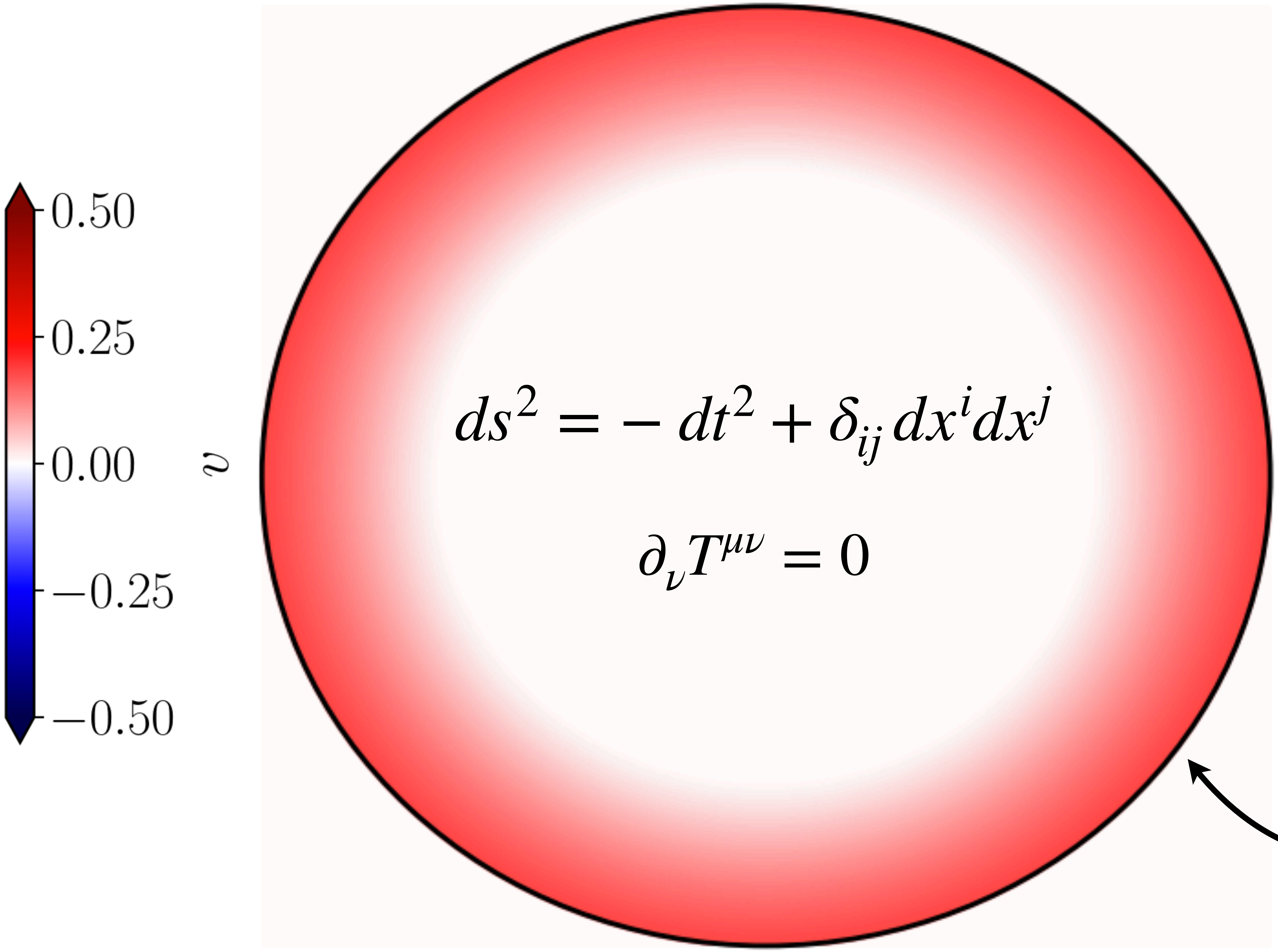
Assumes $R_* H_* \ll 1$



Can we do this with large bubbles?

Non-gravitating bubble

H. Kurki-Suonio and M. Laine, Phys.Rev.D 54 (1996)



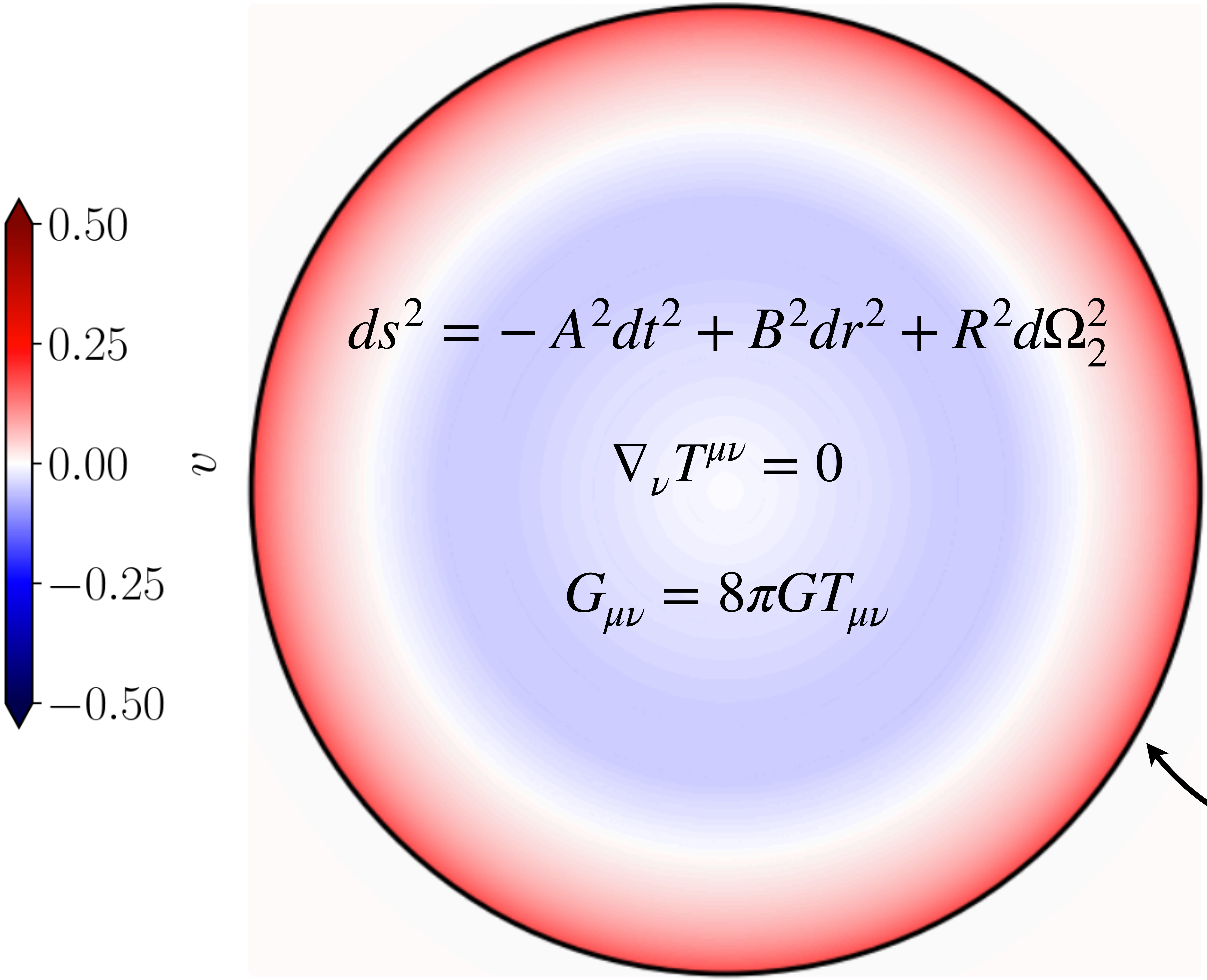
$$ds^2 = -dt^2 + \delta_{ij} dx^i dx^j$$

$$\partial_\nu T^{\mu\nu} = 0$$

Junction conditions

$$T^{\mu\nu}_{\text{in}} = T^{\mu\nu}_{\text{out}}$$

Self-gravitating bubble



$$ds^2 = -A^2dt^2 + B^2dr^2 + R^2d\Omega_2^2$$

$$\nabla_\nu T^{\mu\nu} = 0$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Friedmann Lemaître Robertson Walker

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2 d\Omega_2^2\right]$$

$$\nabla_\nu T^{\mu\nu} = 0$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Junction conditions:

Israel junction conditions

Modelling the source with large bubbles

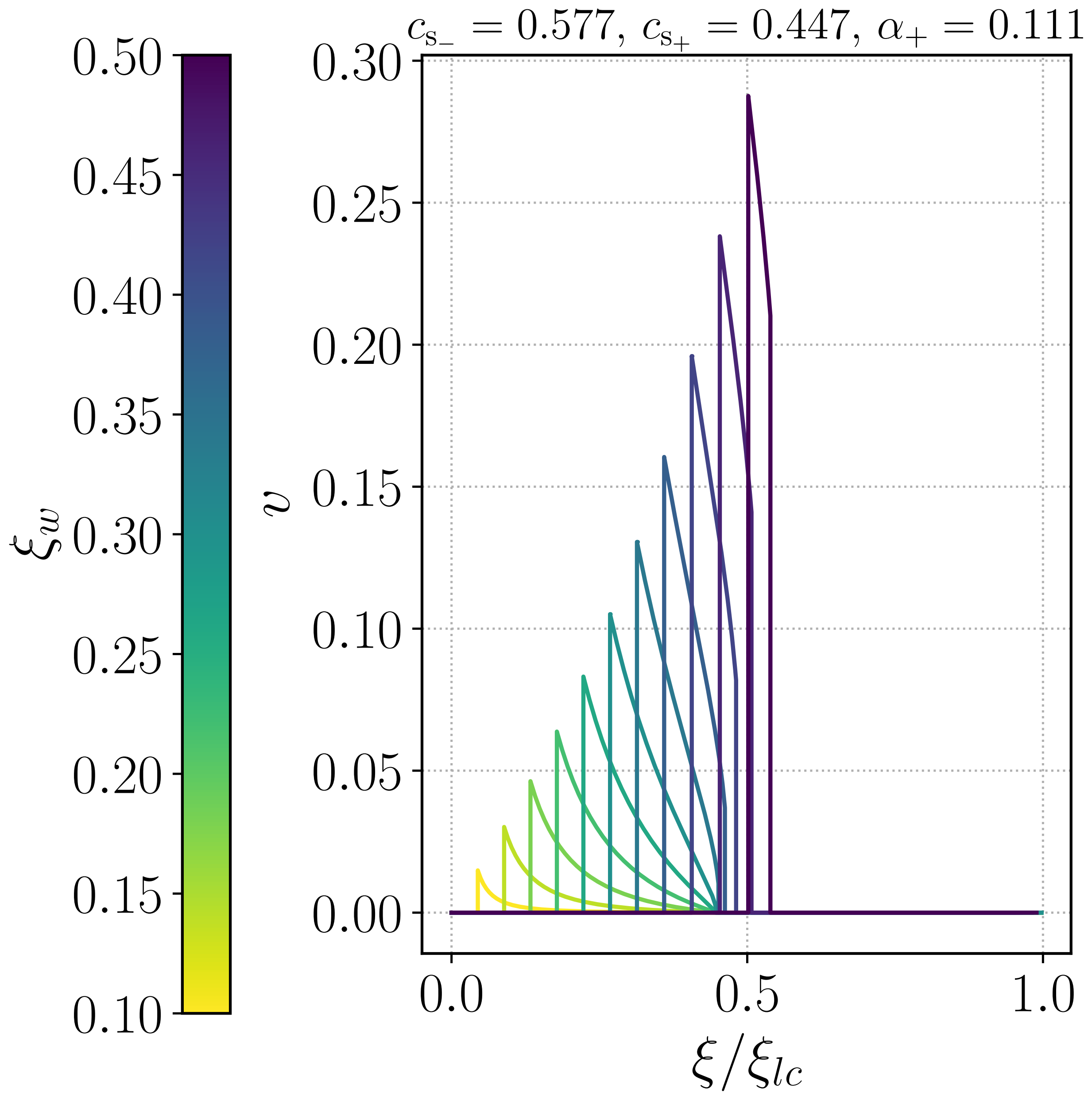
- **Non-perturbative self-gravitating fluid**

  LG & Hindmarsh *JCAP* 03 (2024) 059

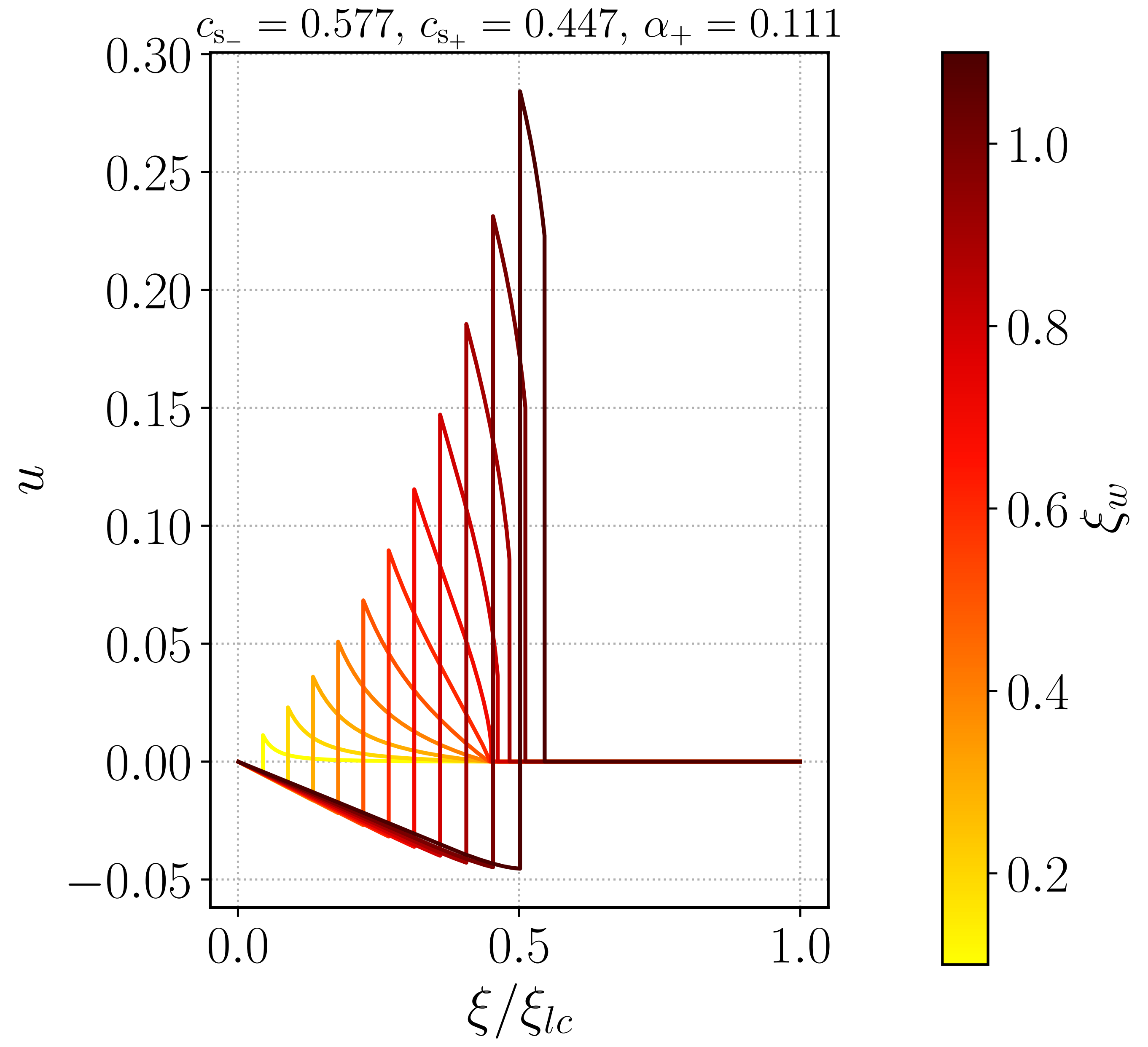
  \+ Fluid self gravity
  \+ Non perturbative

  — Difficult interpretation of velocity
  — Self similar

Giese et al. *JCAP* 01 (2021) 072
Non-gravitating solutions



LG & Hindmarsh *2024*
Gravitating solutions

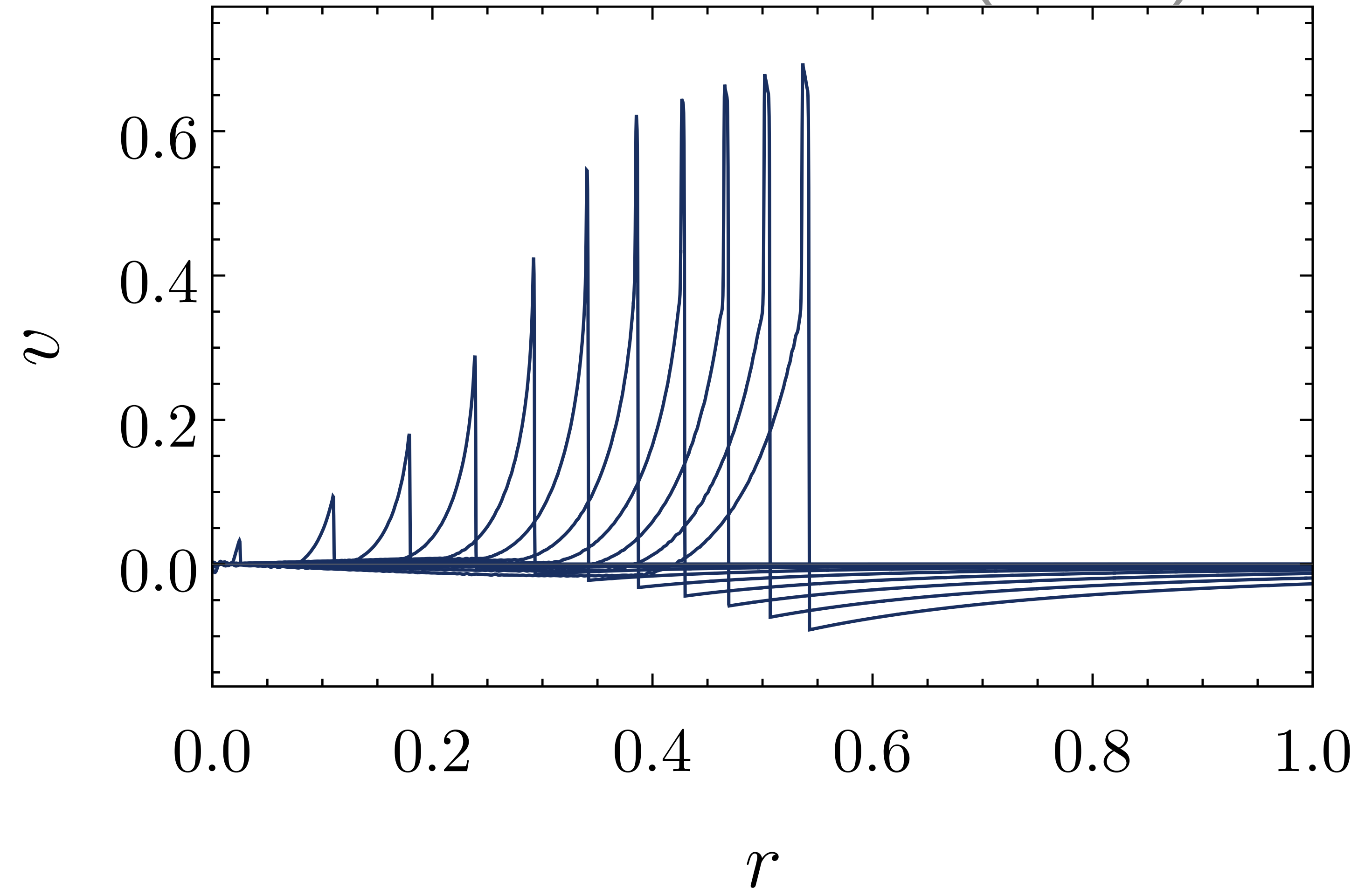


- **Cosmological perturbations around FLRW**

  Jinno & Kume *JCAP* 02 (2025) 057

  \+ v is the peculiar velocity for "uniformly-expanding" observers
  \+ Non self-similar
  — Small gravitational back-reaction $\Phi, \Psi \ll 1$

Jinno & Kume *JCAP* 02 (2025) 057



SUMMARY (part 2)

Macro photography of a soap bubble
Picture from [Philippe's blog]

$k_{\text{peak}} \propto c_s$

$R_* \mathcal{H}_* \approx 0.5$

- Self-gravity in the bubble hydrodynamics is far from understood

- Non-self similar solutions
- Non spherical solutions (many bubbles)

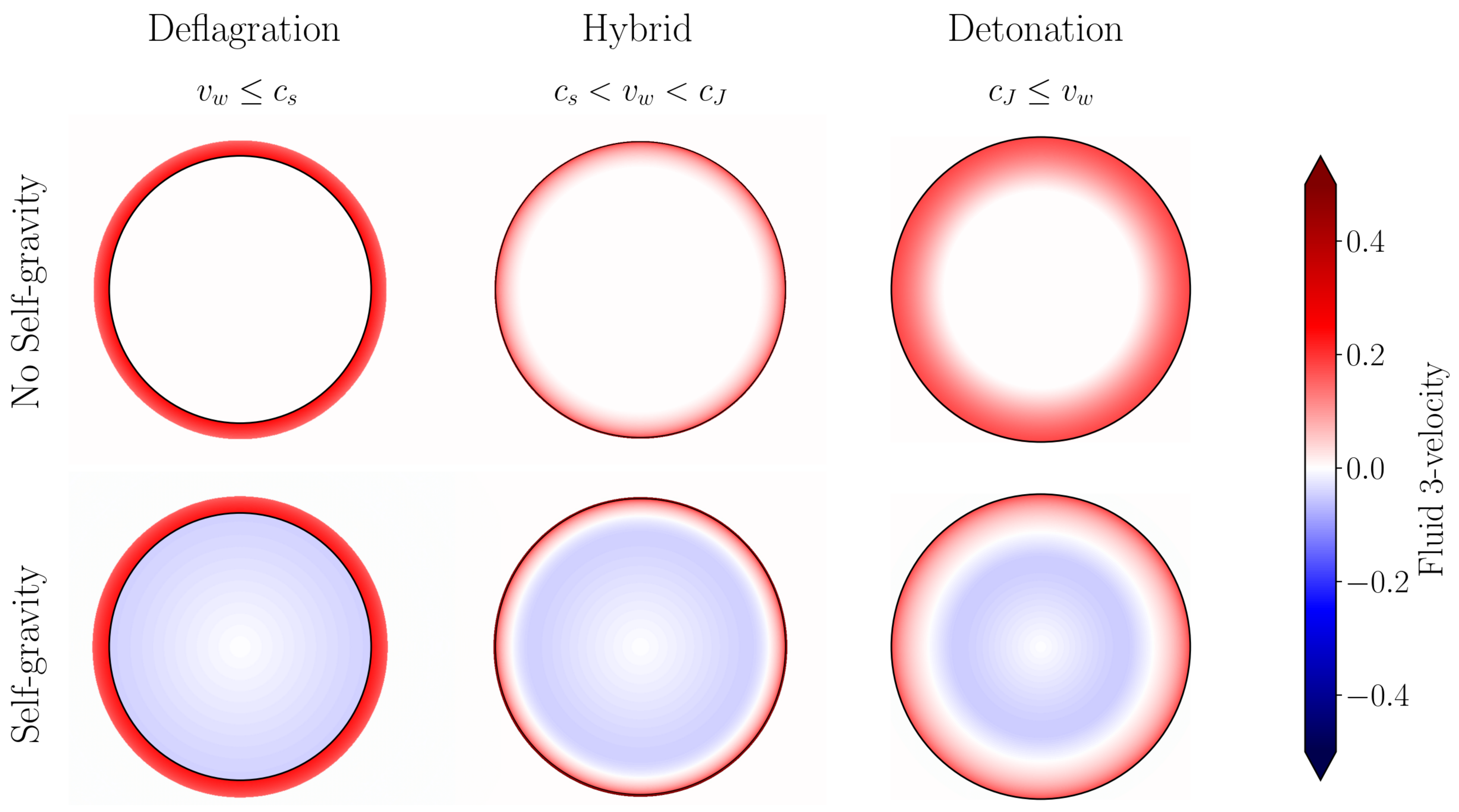
Consequences of self-gravitating bubble solutions

- Collapsing bubbles?
- Colliding bubbles?
- Black holes?

Backup slides

Comparison to gravitating solutions

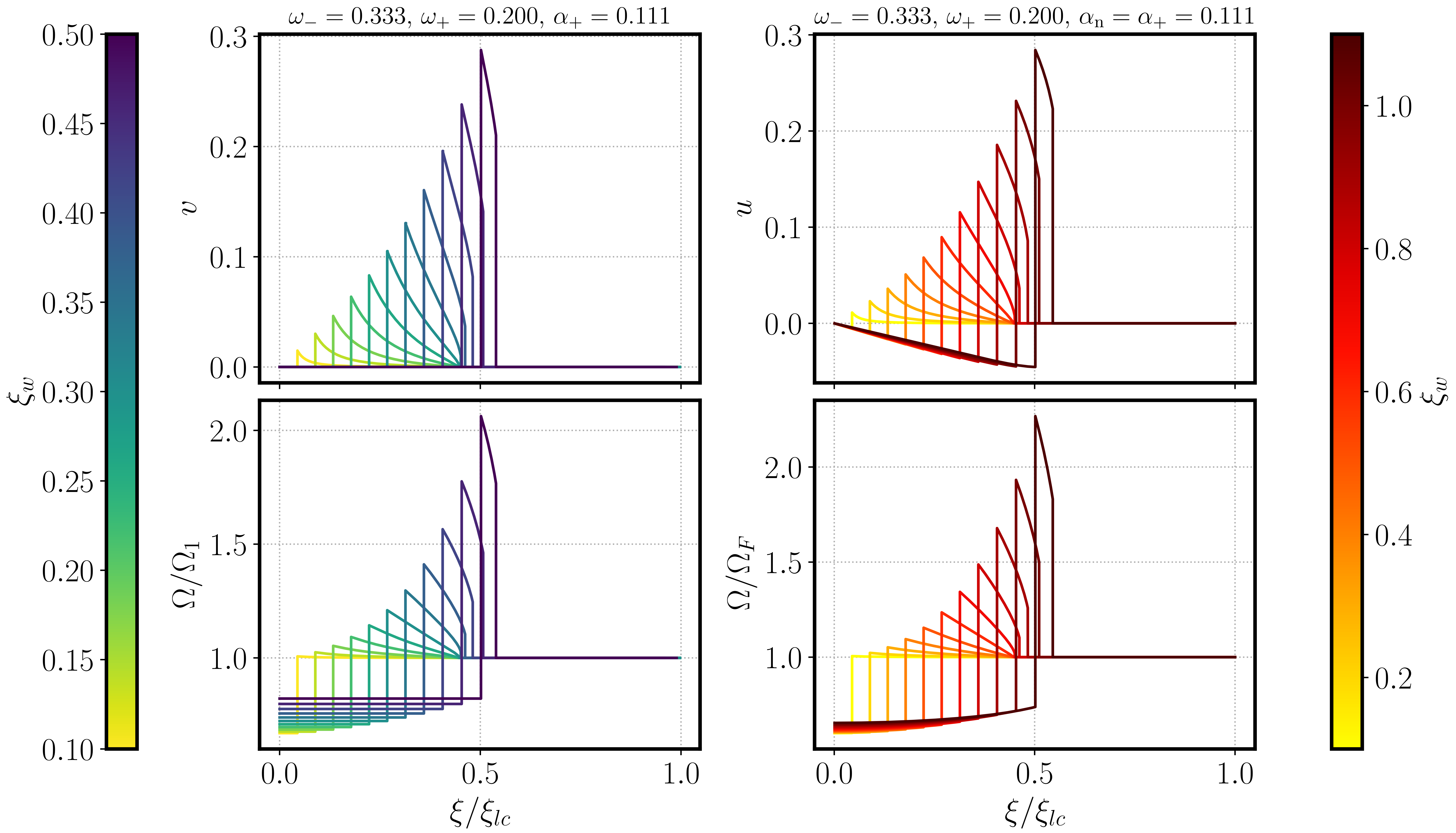
LG & Hindmarsh, JCAP 03 (2024) 059



$$\langle v^i_{q_1} v^{*j}_{q_2} \rangle \sim P_v(q_1) \ ?$$

Comparison to non-gravitating solutions $v(\xi_w)_- < c_{s_-}$

(Giese et al. 2010)

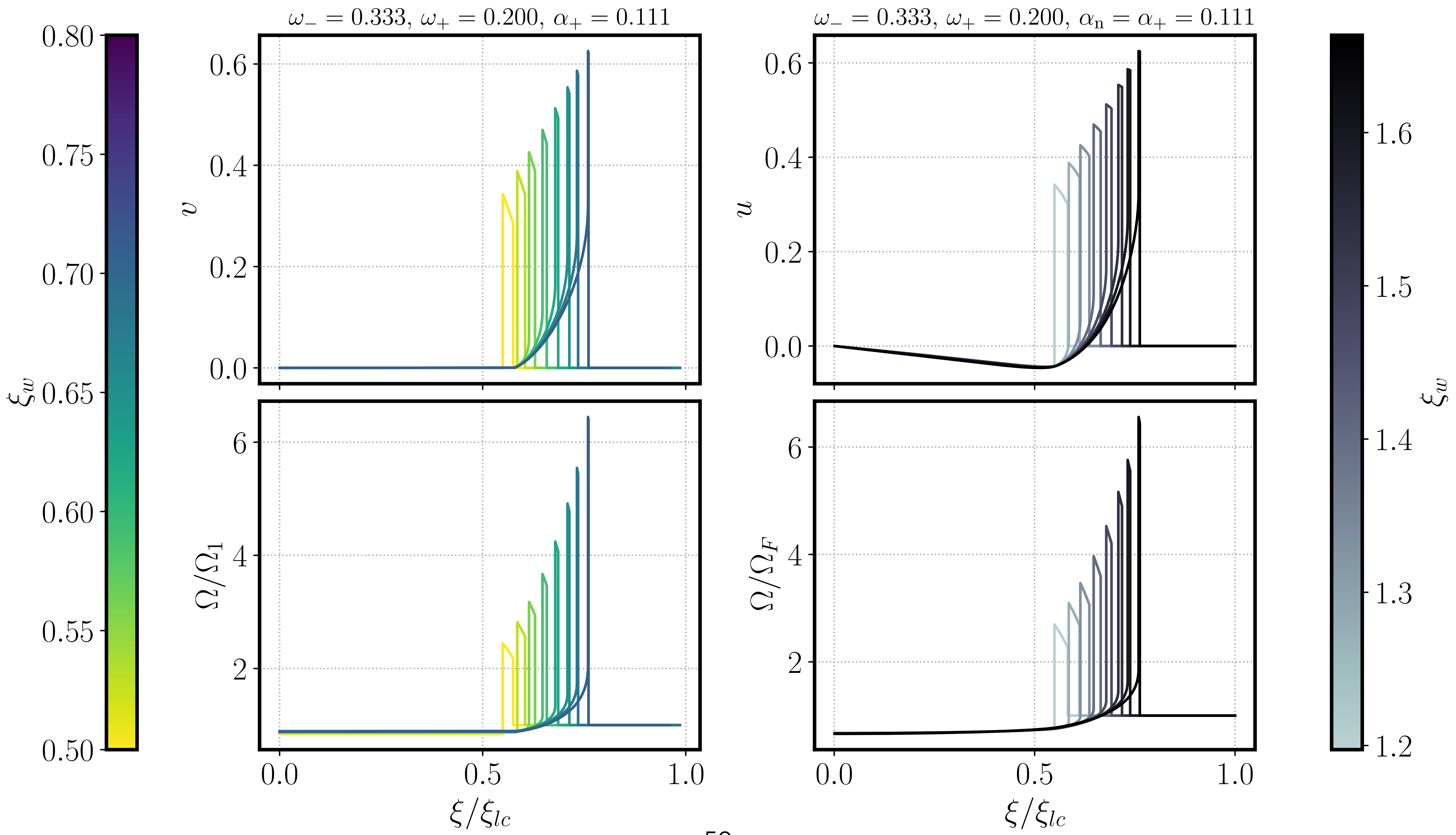


u

fluid speed measured in the rest frame of the outer FLRW fluid

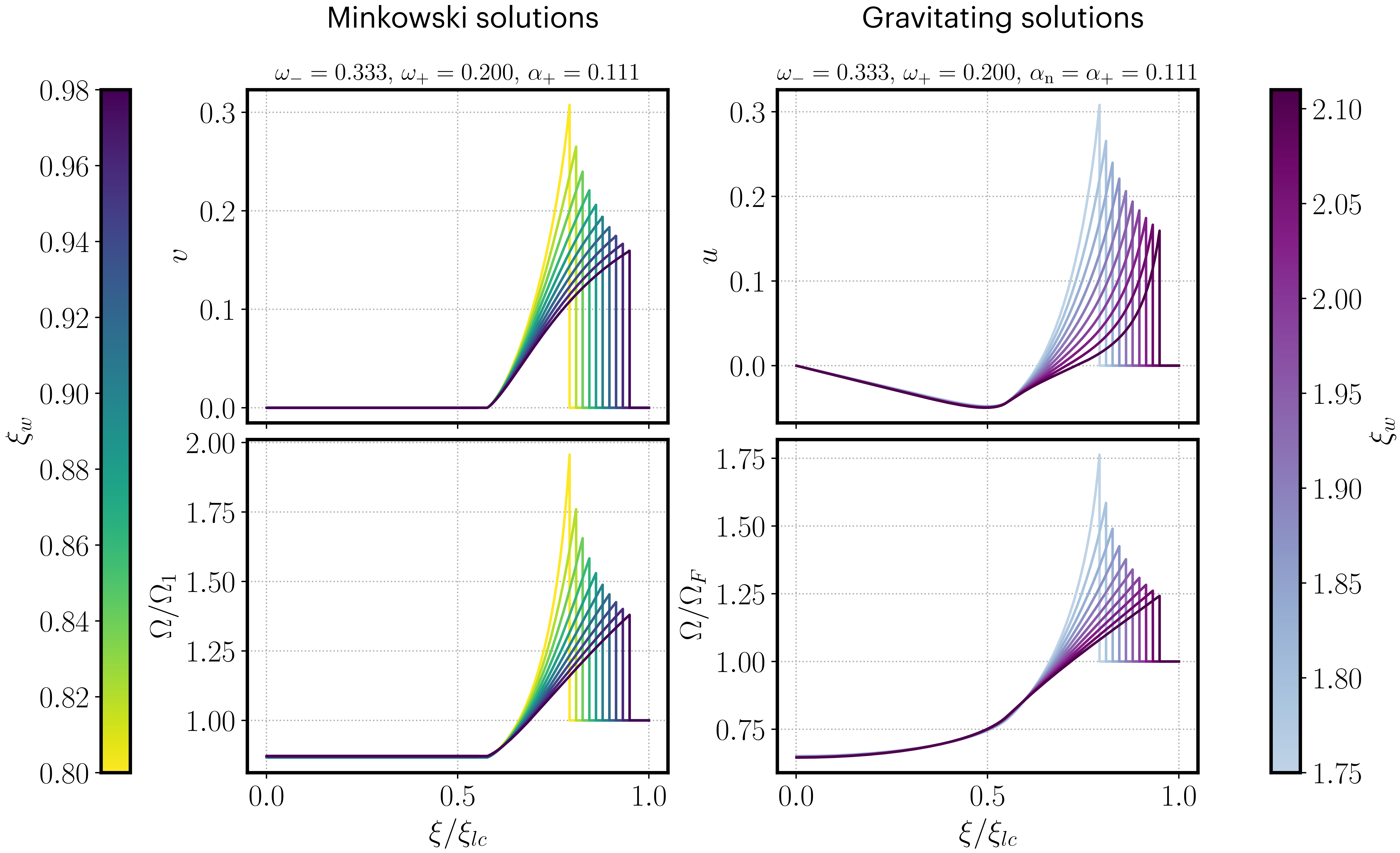
Comparison to non-gravitating solutions $v(\xi_w)_+ = c_{s_-}$

(Giese et al. 2010)



Comparison to non-gravitating solutions $v(\xi_w)_+ > c_{s_-}$

(Giese et al. 2010)



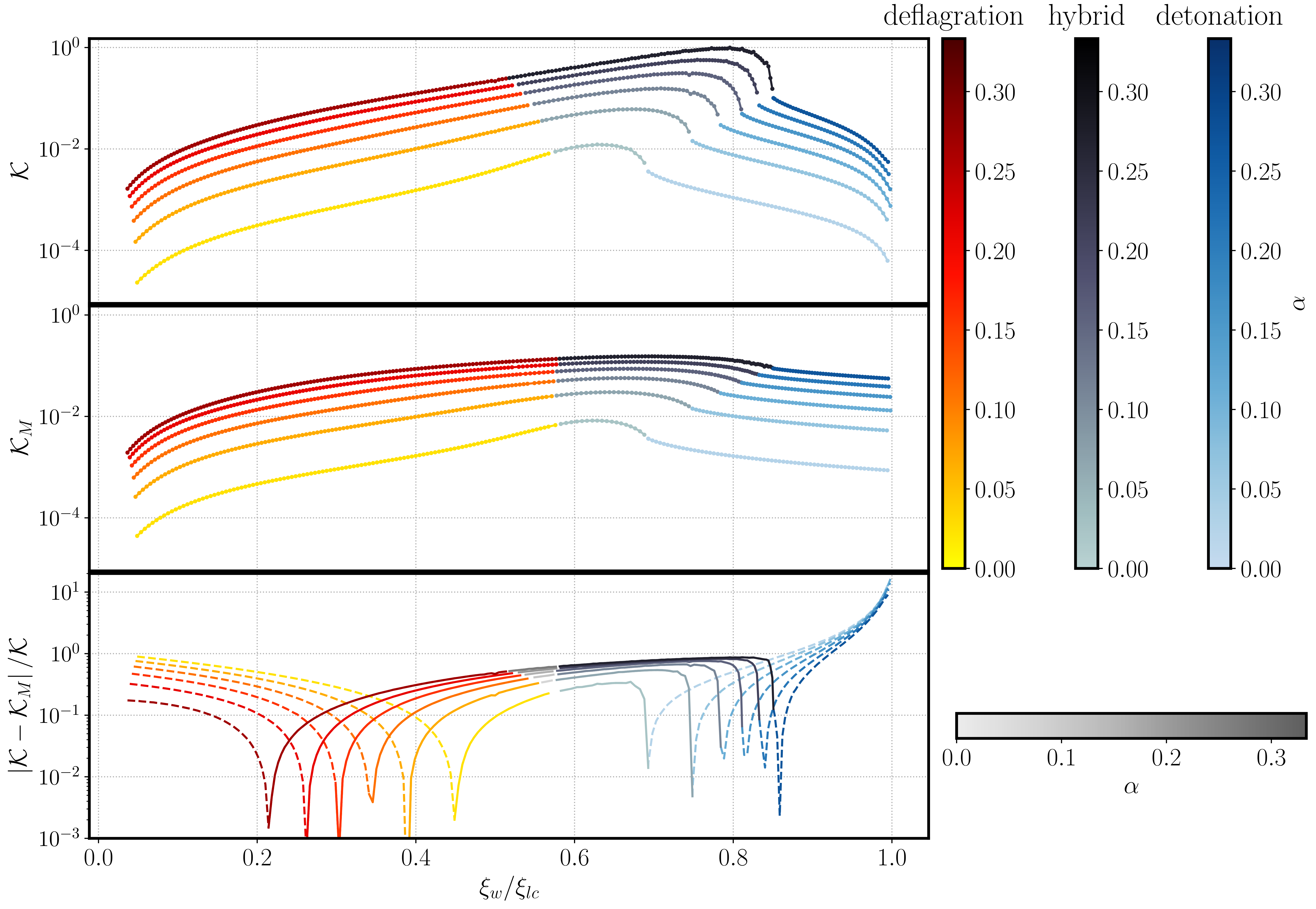
Kinetic energy fraction

$$\mathscr{P}_{\rm gw} = 3K^2 (\mathscr{H}_* R_*)(\mathscr{H}_* \tau_v) \frac{(kR_*)^3}{2\pi^2} \tilde{P}_{\rm gw}$$

$$K \equiv \frac{E_K}{E_b}$$

E_K : Kinetic energy of the fluid

E_b : Total energy in the bubble



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Newtonian potential

Projection tensor

$$\tilde{h}_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$$

Expansion tensor

$$\theta = \tilde{h}^{\alpha}{}_{\mu} \tilde{h}^{\beta\mu} \nabla_\alpha u_\beta$$

$$R_M^{(3)} \simeq 6H^2\delta$$

F. Giese et al., ArXiv:2010.09744