Collaborative Research Center TRR 257
Particle Physics Phenomenology after the Higgs Discovery

Karlsruher Institut für Technologie

Scalar Field Dynamics and Dark Matter Production in Supercooled Phase Transitions

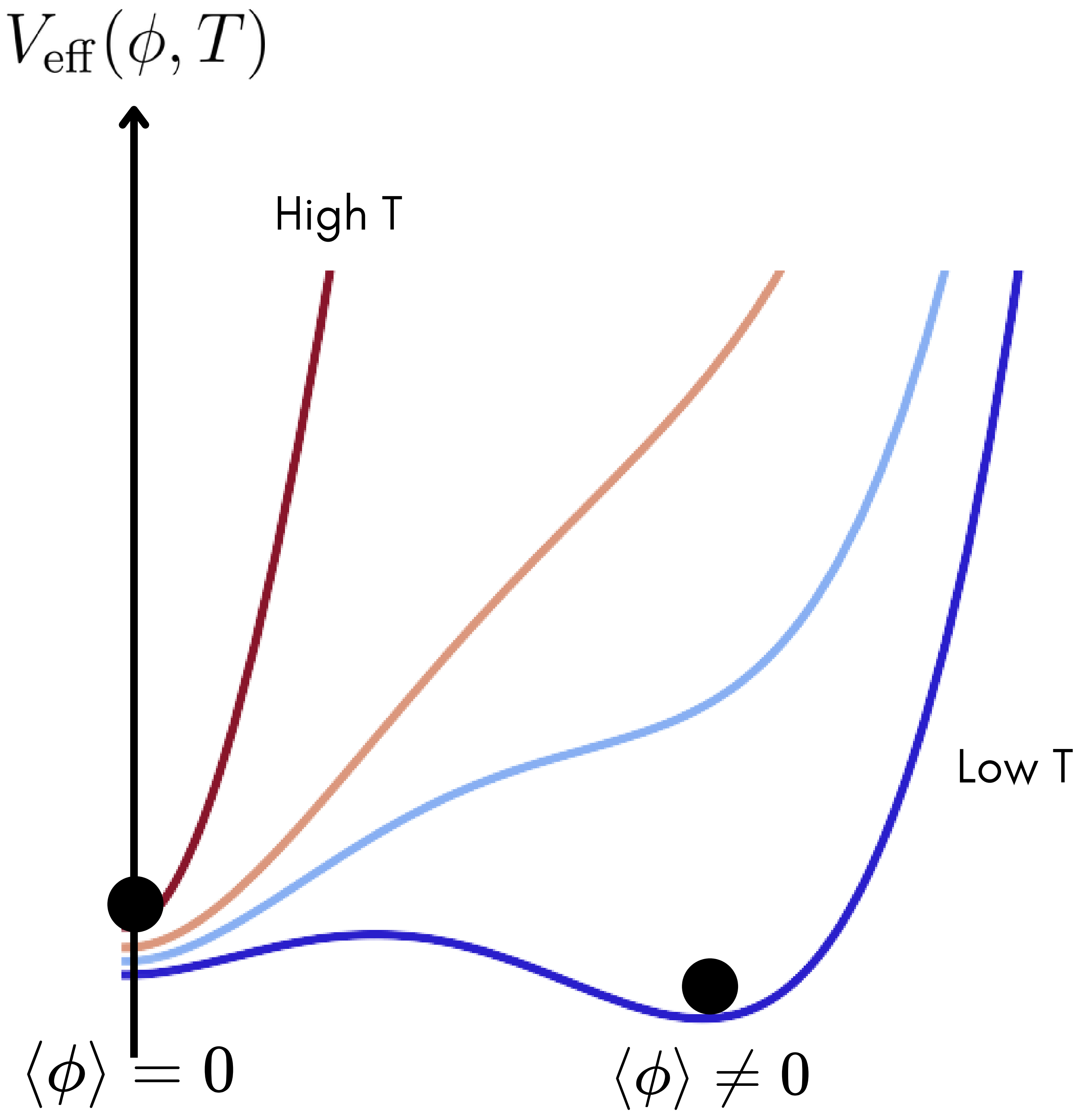
Speaker: Henda Mansour

Based on:
[2504.10593] with Cristina Benso and Felix Kahlhoefer
and [2511.xxxxx] with Yann Gouttenoire and Felix Kahlhoefer

Outline

- **Introduction :** Cosmological First-Order Phase Transitions
- **Part I :** The equation of state of the Universe after a supercooled phase transition
- **Part II :** Dark Matter Phase-In

First-Order Phase Transitions



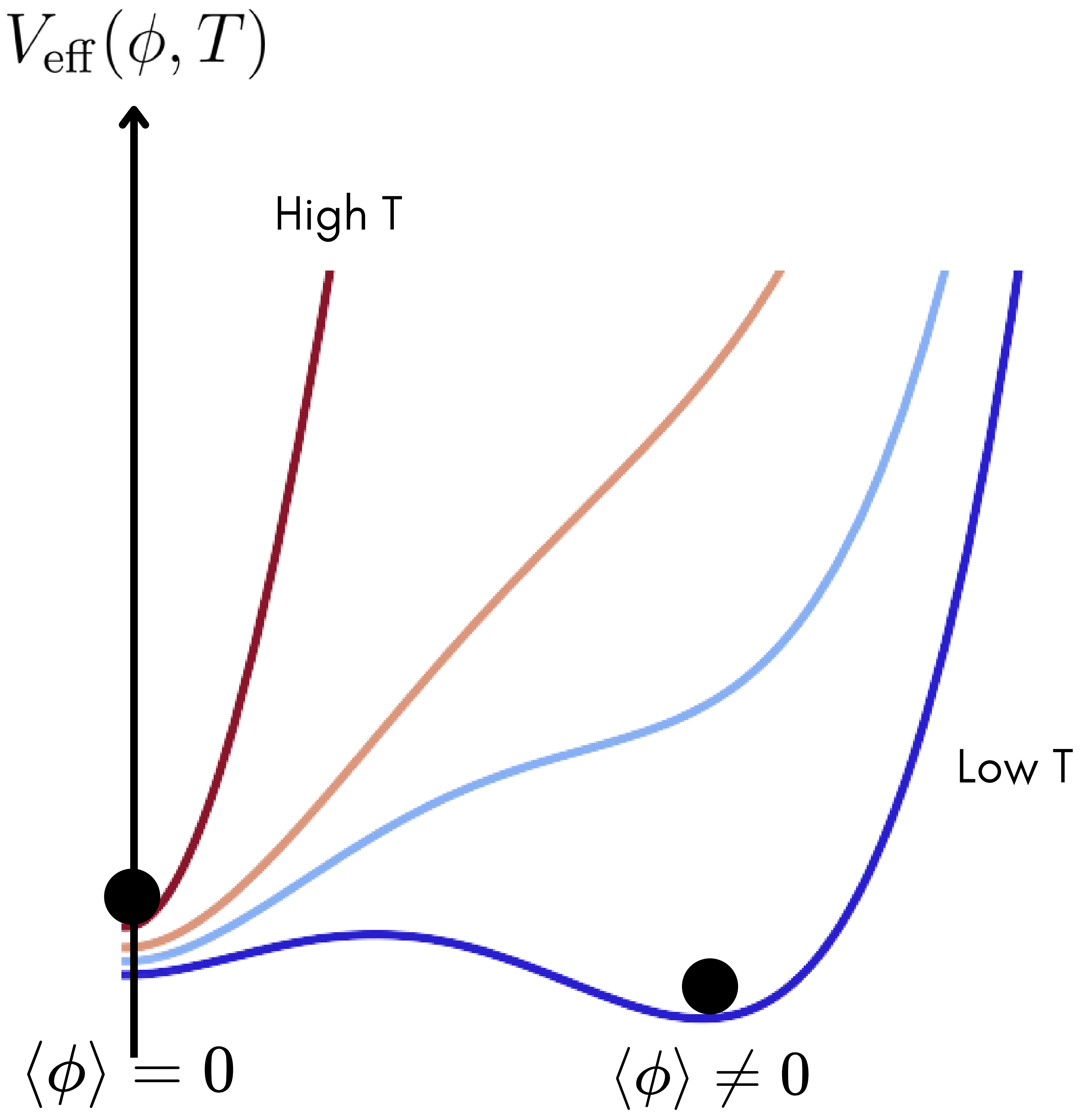
- Scalar potential + thermal corrections:

$$V_{\text{eff}}(\phi, T) = V(\phi) + \Delta V(\phi, T)$$

$$V_{\text{eff}}(\phi, T) = \left(-\lambda\nu^2 + \frac{\alpha}{24}T^2\right)\phi^2 - \gamma T\phi^3 + \lambda\phi^4$$

(Example of potential shape)

First-Order Phase Transitions



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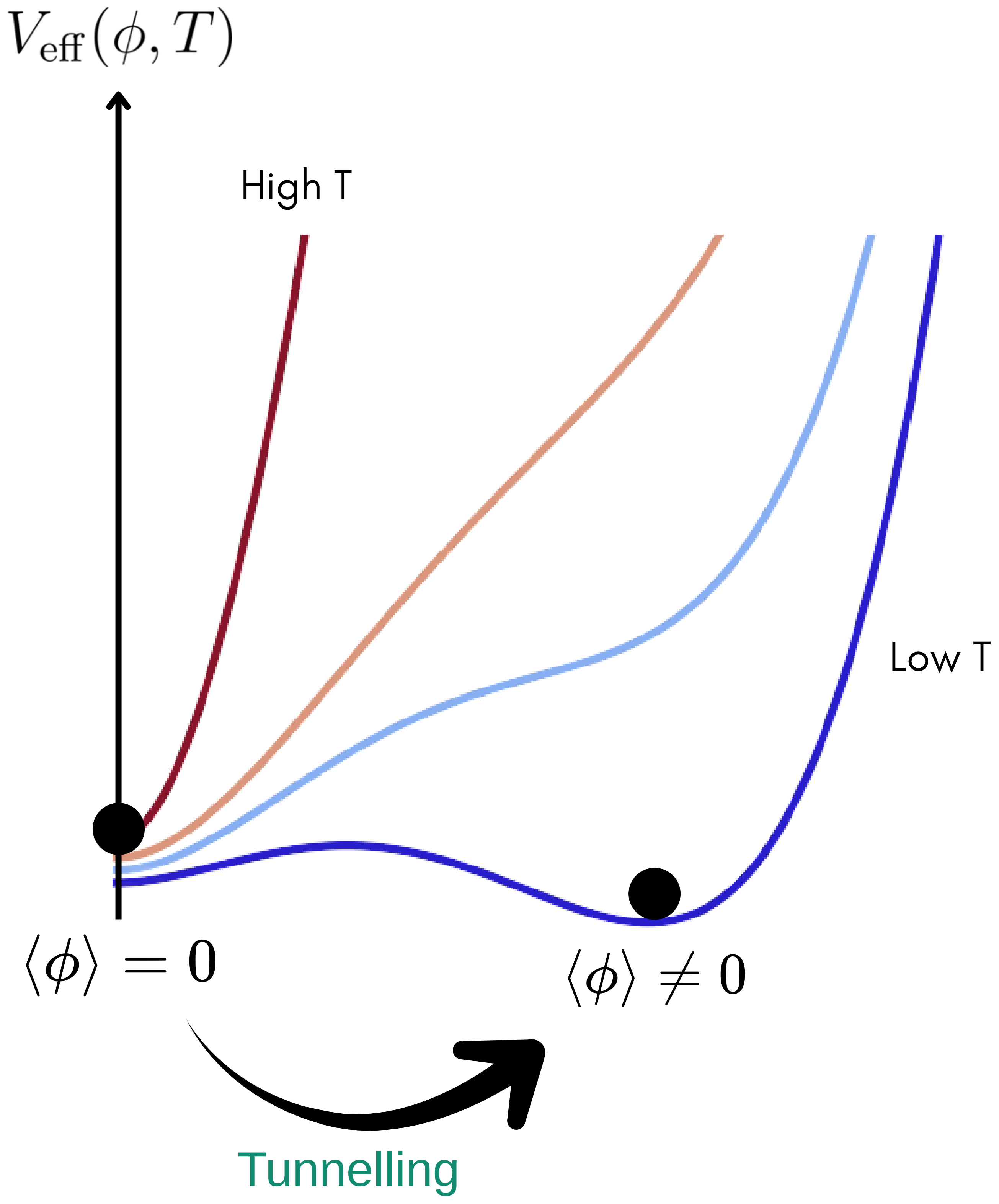
(Example of potential shape)

- Motivated in many extensions of the SM for a number of reasons :
  - Electroweak Baryogenesis
  - Gravitational Waves Production
  - Primordial Black Holes
  - Dark Matter Production

First-Order Phase Transitions



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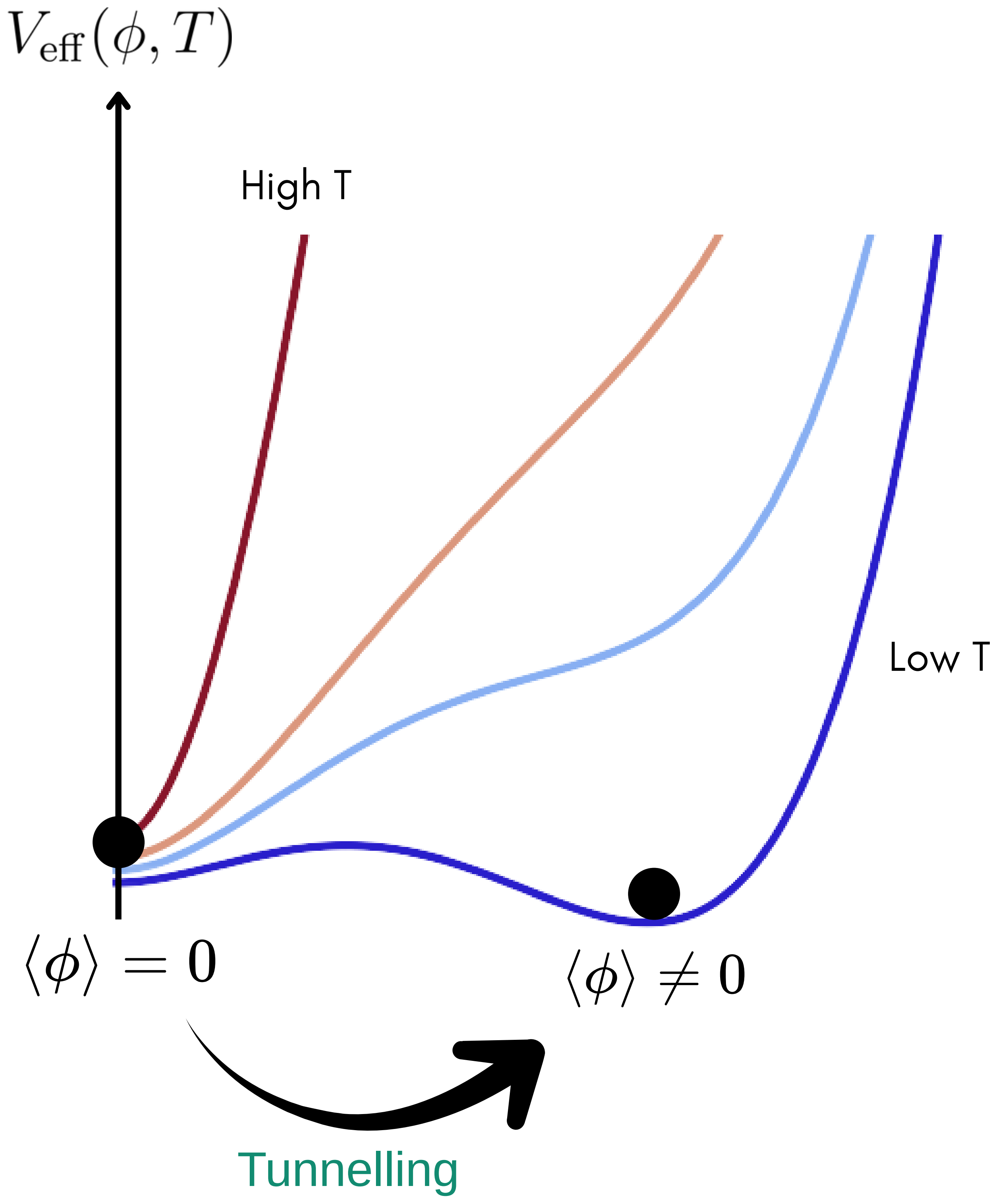
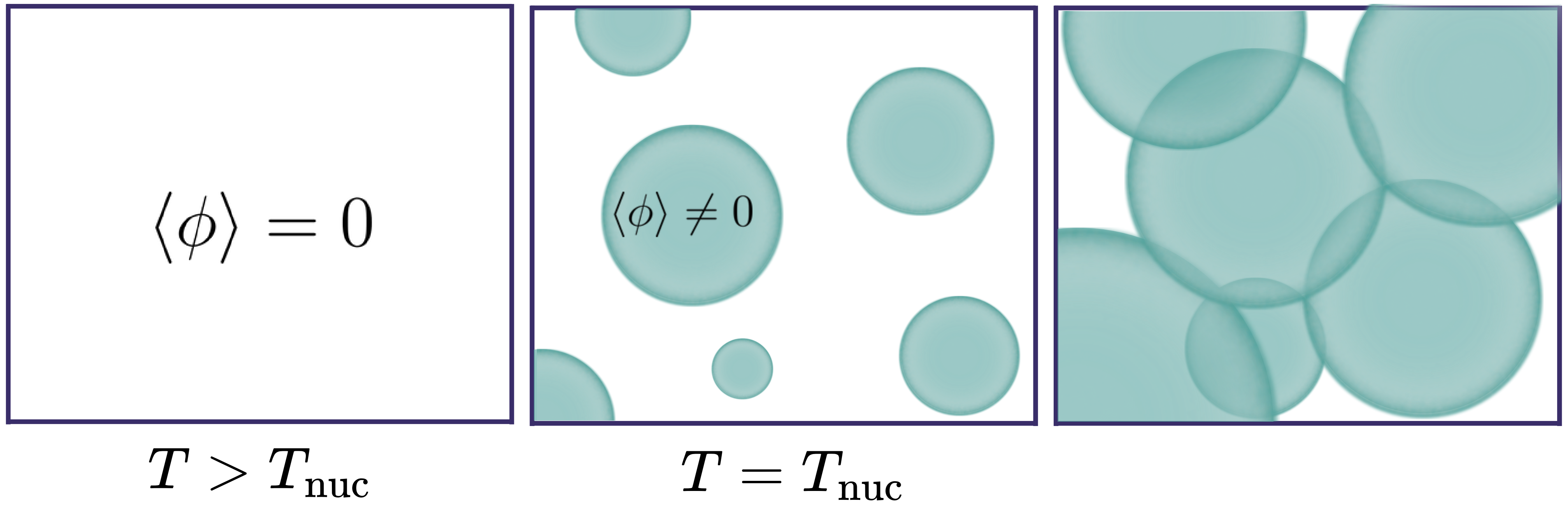


The transition proceeds through bubble nucleation:



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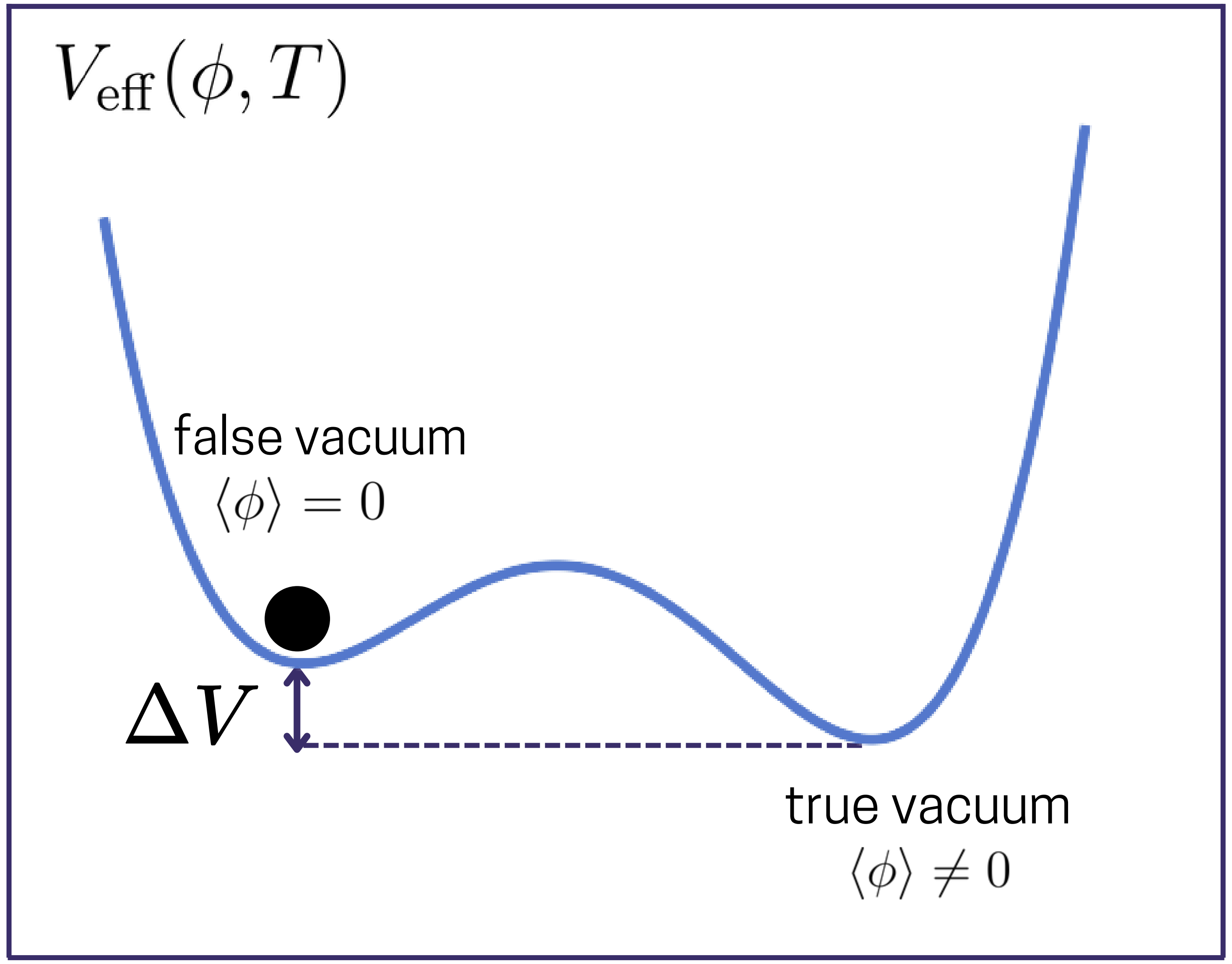

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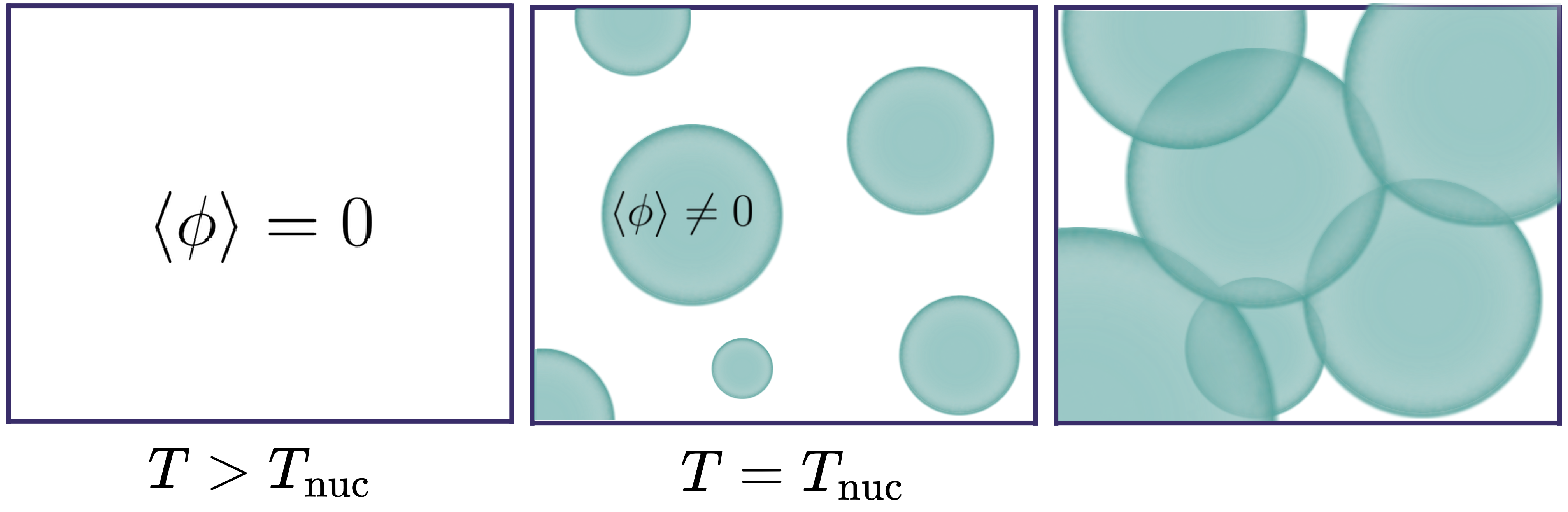
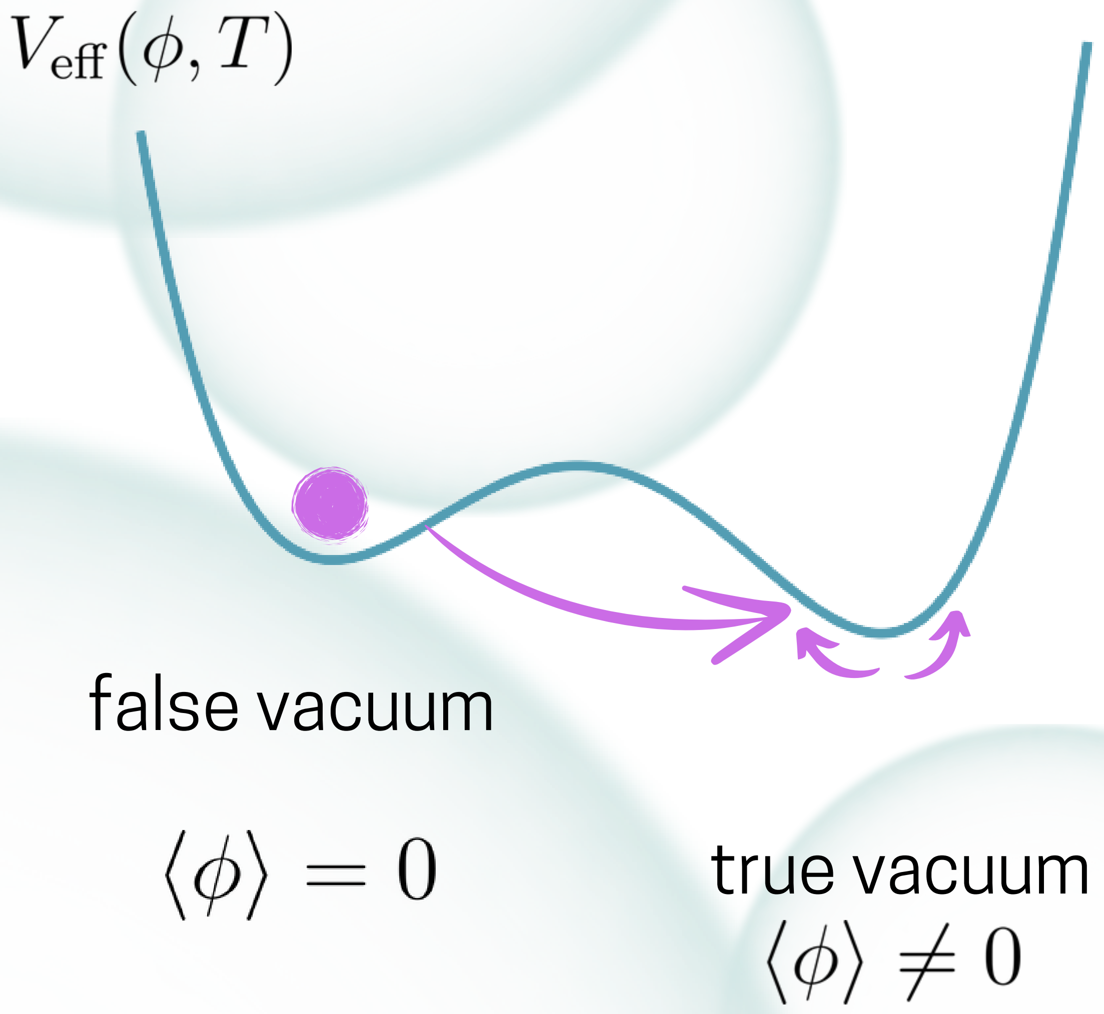


+ The scalar field acts like a cosmological constant before the transition: $\omega_\phi = -1$

+ For strong supercooling: $\Delta V > \rho_{\text{rad}}(T_{\text{PT}})$
+ If the scalar field decays slowly, there will be a period of **scalar field domination** after percolation

Scalar field domination after FOPT



If reheating is slow and the energy density in the scalar field dominates, such that we can focus only on the scalar dynamics:

After the transition, the field (on average) oscillates around the new minimum

→ **Matter domination?**

Scalar field domination after FOPT

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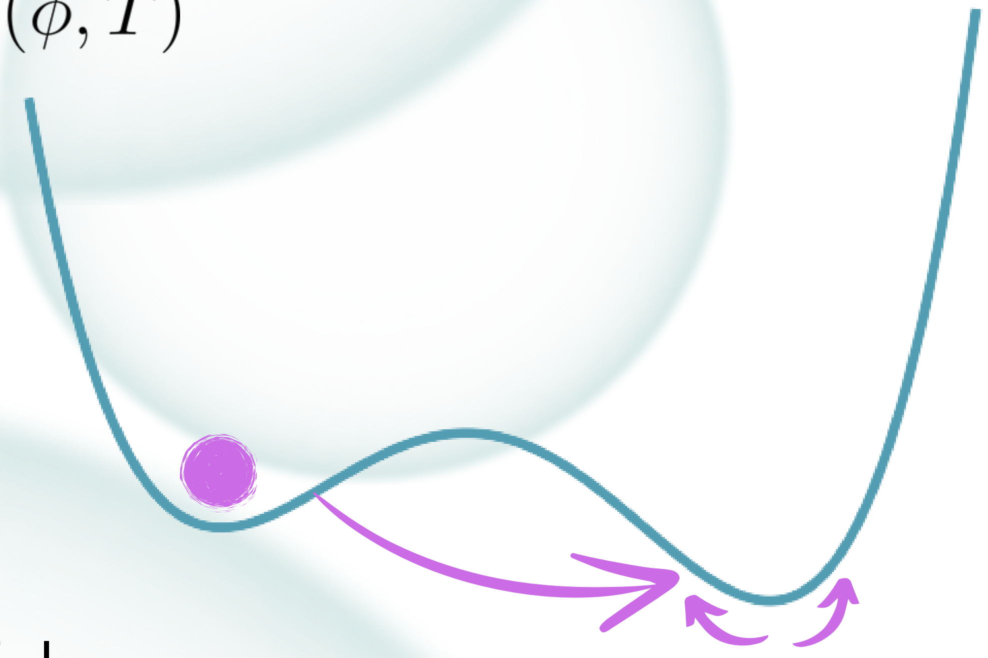
$V_{\text{eff}}(\phi, T)$



false vacuum

$\langle\phi\rangle = 0$

true vacuum

$\langle\phi\rangle \neq 0$

Focus only on the scalar dynamics:

After the transition, the field (on average) oscillates around the new minimum

→ **Matter domination?**

Equation of state: $\langle\omega\rangle = \frac{\langle p\rangle}{\langle\rho\rangle}$ pressure, energy density

In the case of homogeneous coherent oscillations:

$$\langle\omega\rangle = \frac{\langle p_\phi\rangle}{\langle\rho_\phi\rangle} = \frac{\frac{1}{2}\langle\dot{\phi}^2\rangle - \langle V(\phi)\rangle}{\frac{1}{2}\langle\dot{\phi}^2\rangle + \langle V(\phi)\rangle}$$

Around the minimum:

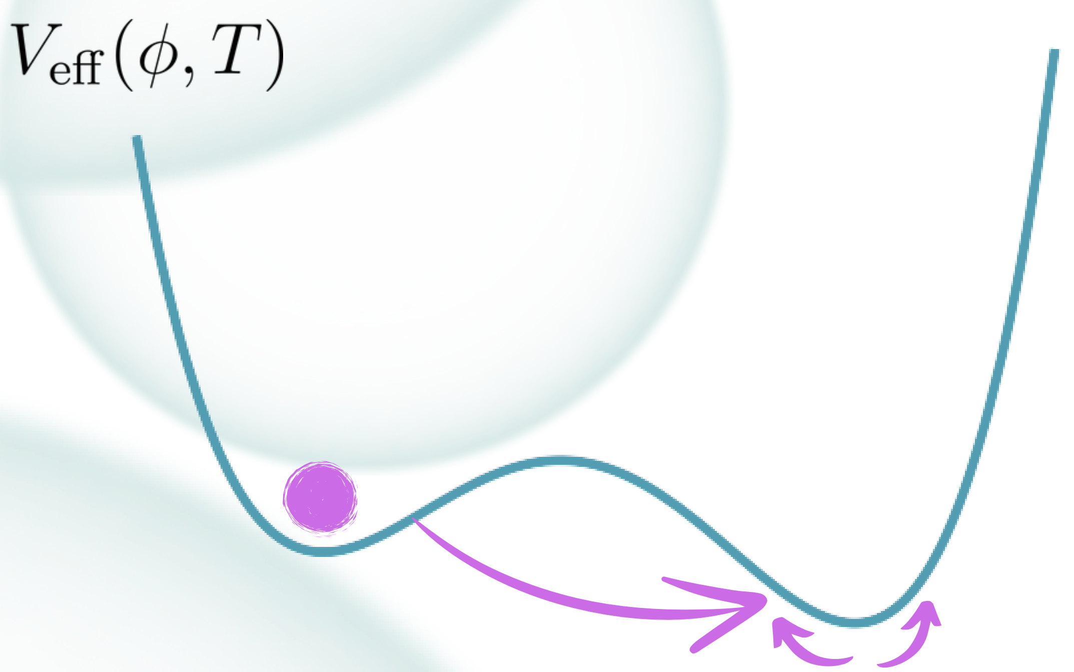
Virial Theorem

$$V(\phi) \sim \phi^k \quad \Rightarrow \quad \boxed{\langle E_{\text{kin}}\rangle = \frac{k}{2}\langle V(\phi)\rangle} \quad \Rightarrow \quad \text{For: } k = 2 \qquad \omega = 0$$

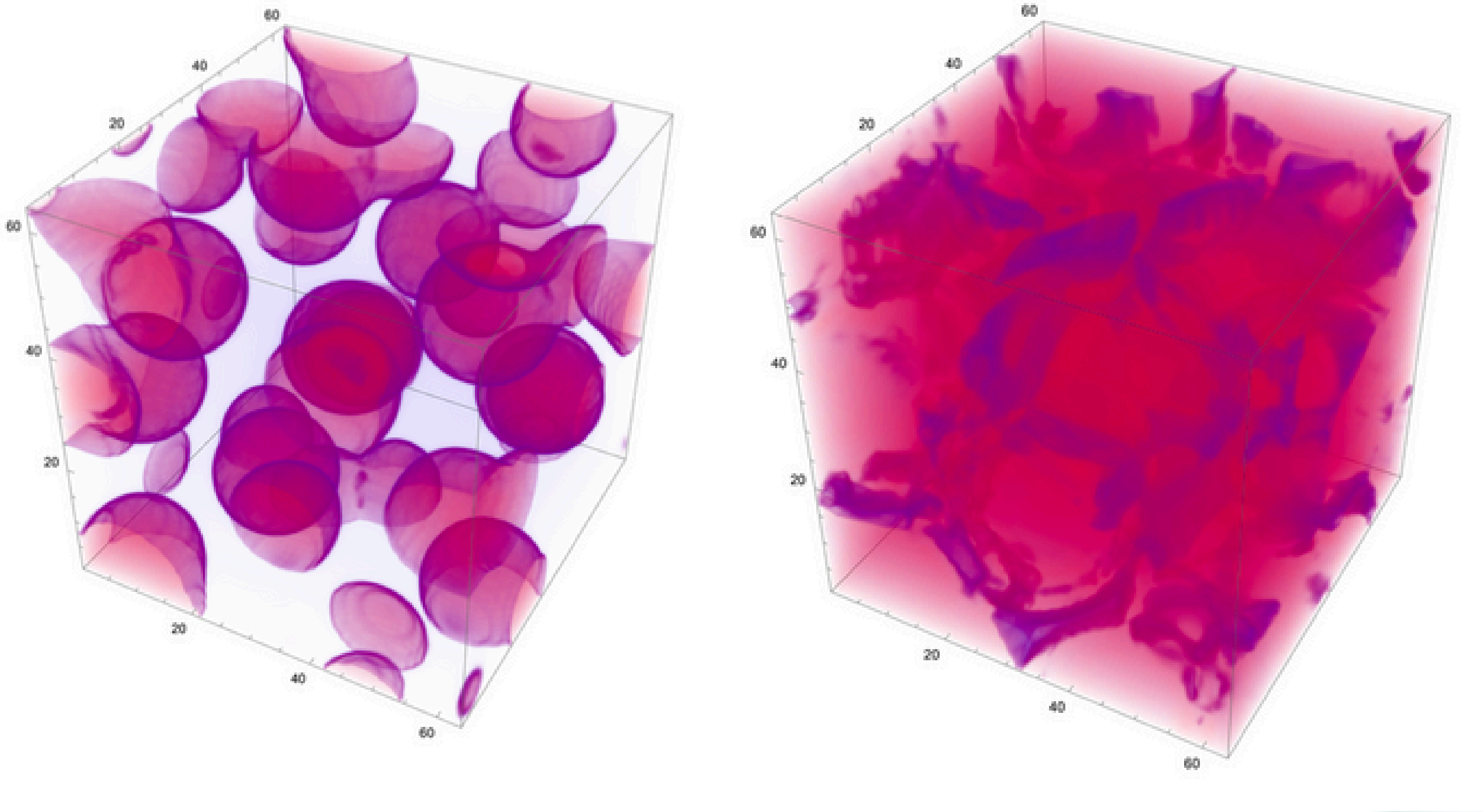
$$\Rightarrow \quad \boxed{\omega = \frac{k-2}{k+2}}$$

... It is however not as simple for a scalar field configuration post collision

Scalar field domination after FOPT



Bubble nucleation and percolation on the lattice:



1. Non-vanishing gradients have to be taken into consideration
2. Bubble walls are relativistic:
   → relativistic scalar waves?
   → radiation domination?

Spatially inhomogeneous

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - \frac{1}{6}(\nabla\phi)^2 - V(\phi)$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)$$

Why does it matter ?

Gravitational Waves:

Impacts the spectrum of GW expected

General impact of MD : (Kazunori Nakayama et al. (2008), Boyle and Steinhardt (2008), Seto and Yokoyama (2003), D'Eramo and Schmitz (2019)....)

- More specific to FOPTs: (Ellis et al. (2019) and (2020) , Gonstal et al. (2025))

Particle production :

Dilution of pre-existing abundances in the case of matter domination (MD)

(Co. et al. (2015), Cirelli et al. (2016), Bishara et al. (2024) and many others)

Production of primordial black holes :

Smaller overdensity threshhold for collapse during MD

(PBH production during matter domination (Harada et al. arXiv: 1609.01588),
PBH production during FOPTs: Y. Gouttenoire and T. Volansky, arXiv: 2305.04942)

Part I: What is the equation of state of the Universe after a supercooled phase transition?

Step I: Analytical understanding …

- Build some analytical understanding of the equation of state:

General EoS for a inhomogeneous scalar field:

$$\omega = \frac{p_\phi}{\rho_\phi} = \frac{\langle K \rangle - (d-2)/d\,\langle G \rangle - \langle V \rangle}{\underbrace{\langle K \rangle}_{\text{kinetic}} + \underbrace{\langle G \rangle}_{\text{gradient}} + \underbrace{\langle V \rangle}_{\text{potential}}}$$

with $d =$ # of spatial dimensions

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kinetic gradient potential

with $d =$ # of spatial dimensions

- Assume a generic polynomial potential and focus on the scalar field dynamics (no T dependence):

Equation of motion in expanding background

$$\ddot{\phi} - a^{-2}\vec{\nabla}^2\phi + dH\dot{\phi} = -V_{,\phi} \quad \text{with} \quad V(\phi) = \frac{m_\phi^2}{2}\phi^2 - \kappa\phi^3 + \alpha\phi^4$$

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- Consider the average field after percolation and apply the virial theorem to determine a relation between the averaged energy densities:

Approximate around the minimum

$$\langle K\rangle \simeq \langle G\rangle + \langle \phi V_{,\phi}/2\rangle + d\left\langle H\phi\dot{\phi}\right\rangle/2 \quad\Rightarrow\quad V(\phi) \sim \phi^2 \quad\Rightarrow\quad \langle K\rangle \simeq \langle G\rangle + \langle V\rangle$$

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- Apply the virial theorem to determine a relation between the averaged energy densities: After percolation, the solution can be approximated by oscillations of the field

Approximate around the minimum

$$\langle K \rangle \simeq \langle G \rangle + \langle \phi V_{,\phi}/2 \rangle + d \left\langle H\phi\dot{\phi} \right\rangle /2 \quad \Longrightarrow \quad V(\phi) \sim \phi^2 \quad \Longrightarrow \quad \langle K \rangle \simeq \underbrace{\langle G \rangle}_{\text{gradient}} + \underbrace{\langle V \rangle}_{\text{potential}}$$

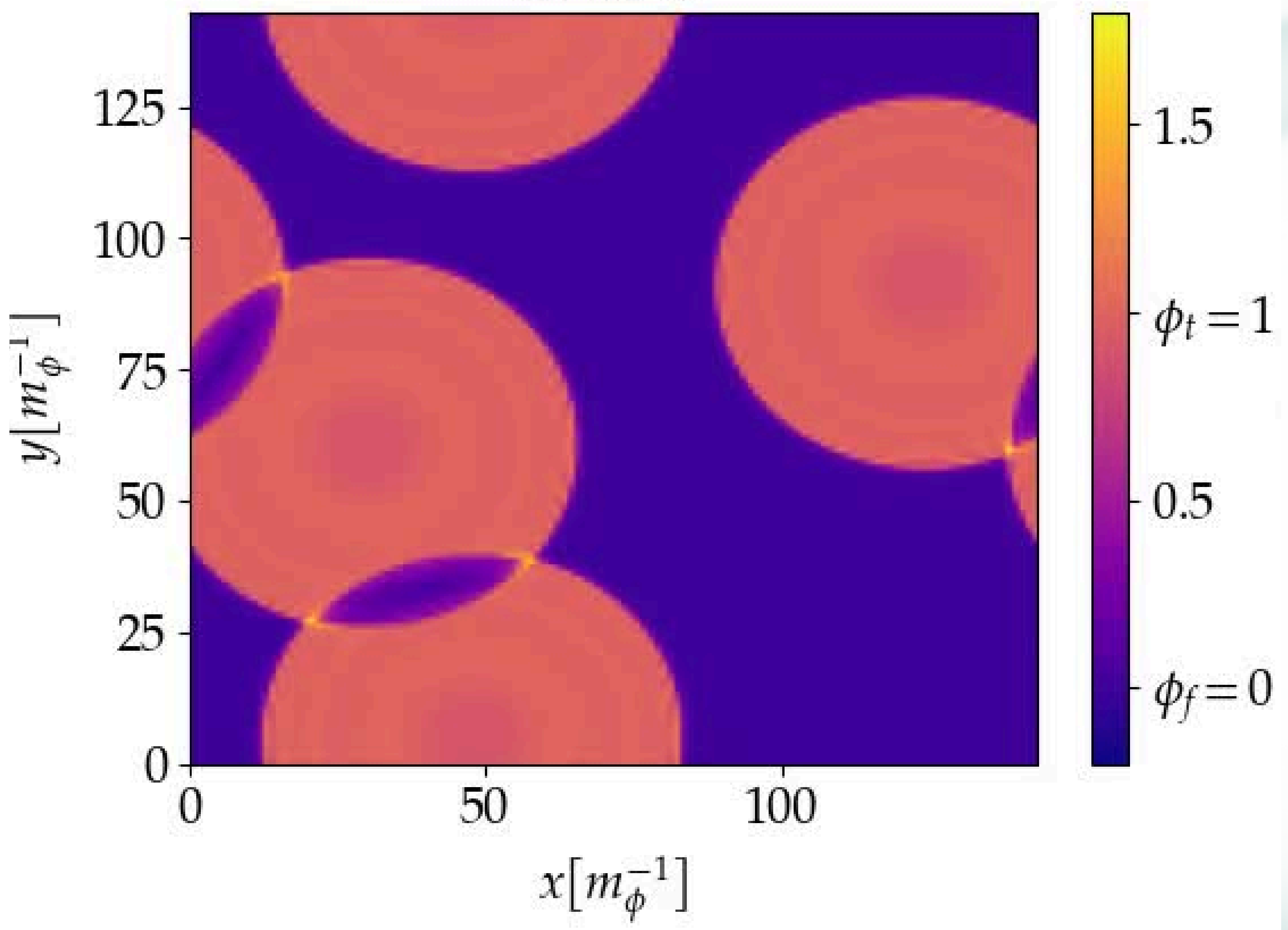
- We find a simplified expression: $\boxed{\omega = d^{-1}\dfrac{\langle G \rangle}{\langle G \rangle + \langle V \rangle}}$

Step II : Let a computer do it

Solve on the lattice: $\ddot{\phi} - a^{-2}\vec{\nabla}^2\phi + dH\dot{\phi} = -V_{,\phi}$ with $V(\phi) = \frac{m_\phi^2}{2}\phi^2 - \kappa\phi^3 + \alpha\phi^4$

Example: 2D simulation with random nucleation and static background

Initial configuration :
randomly nucleated critical bubbles



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The EoS can be determined from the evolution of the energy densities:

$$\omega = \frac{p_\phi}{\rho_\phi} = \frac{\langle K\rangle - (d-2)/d\,\langle G\rangle - \langle V\rangle}{\langle K\rangle + \langle G\rangle + \langle V\rangle}$$

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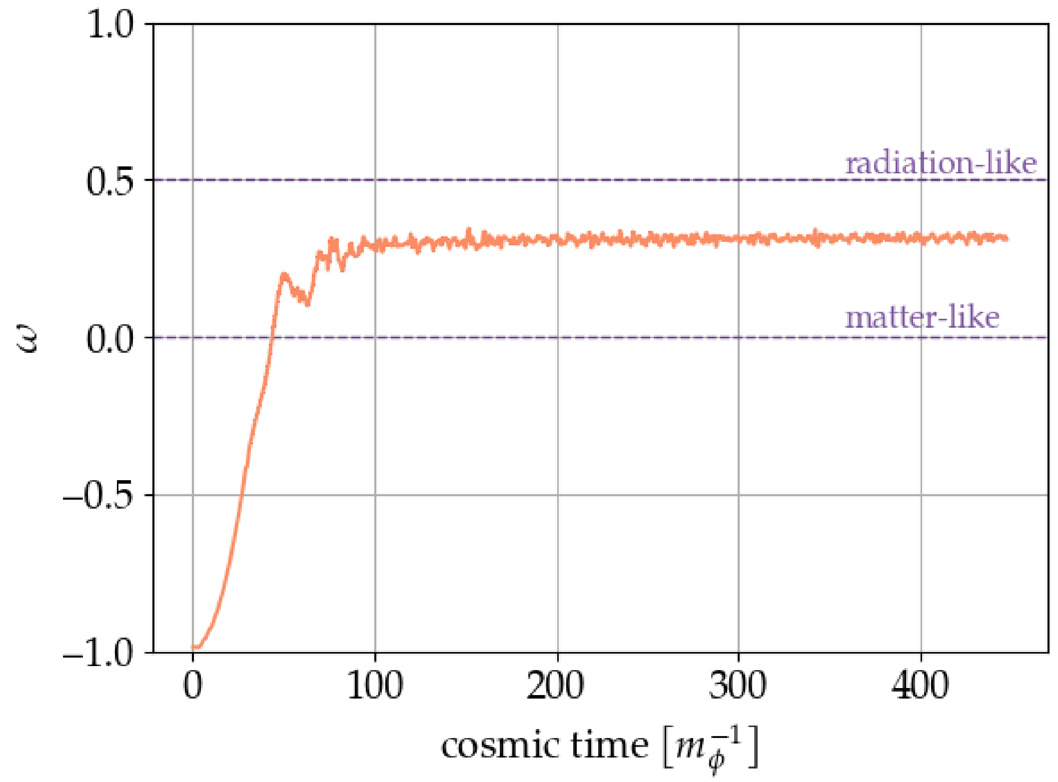

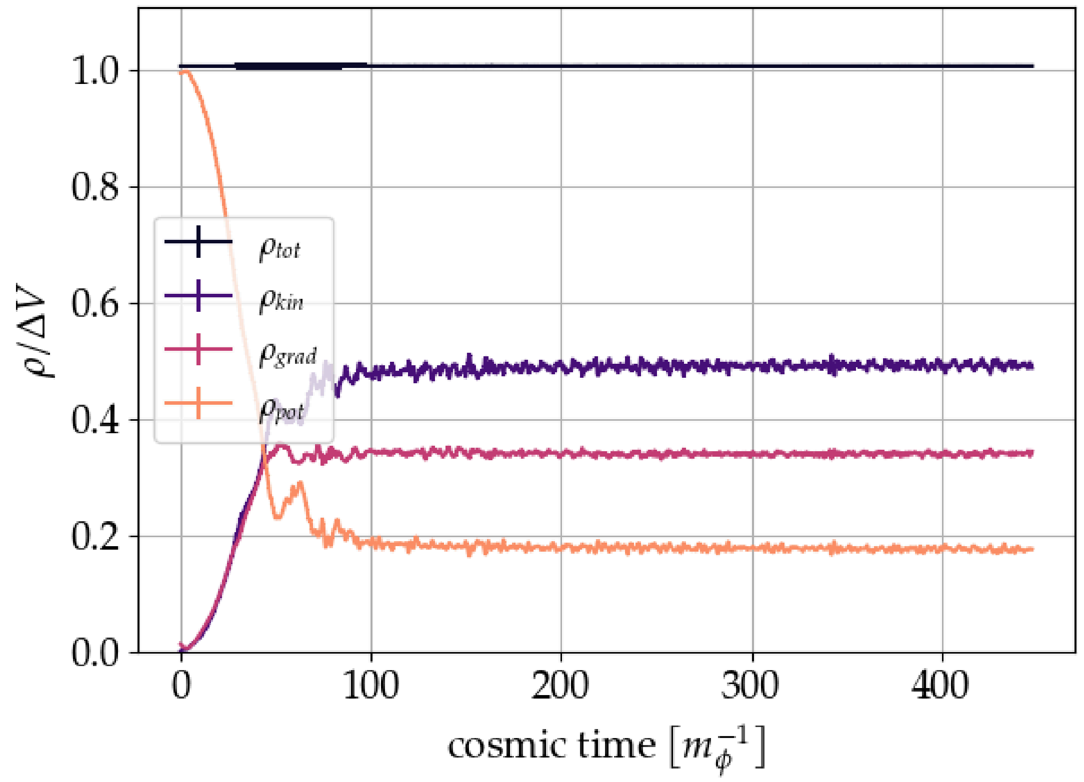

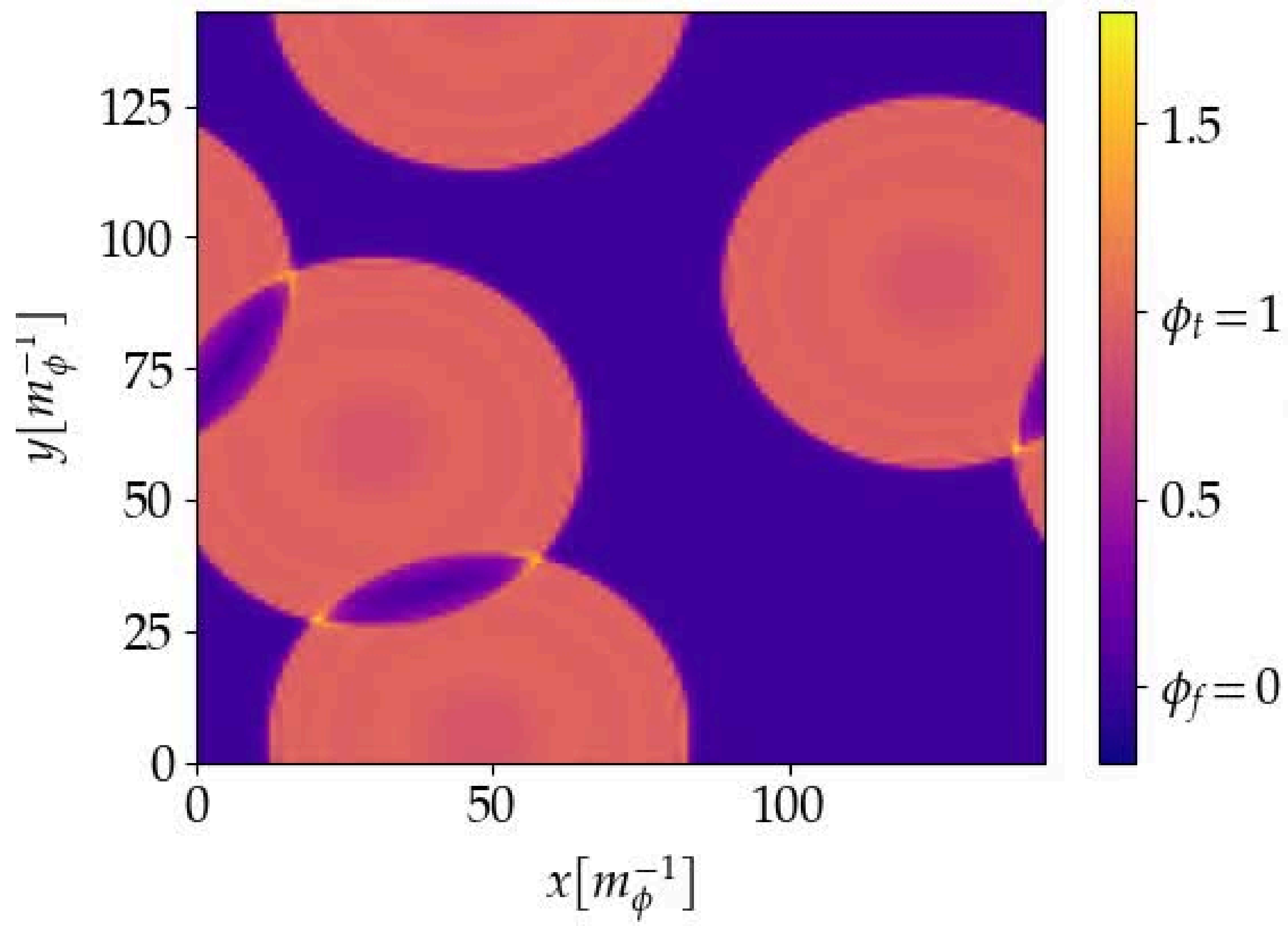

Virial Theorem

$\boxed{\langle K\rangle \simeq \langle G\rangle + \langle V\rangle}$ is fullfilled!

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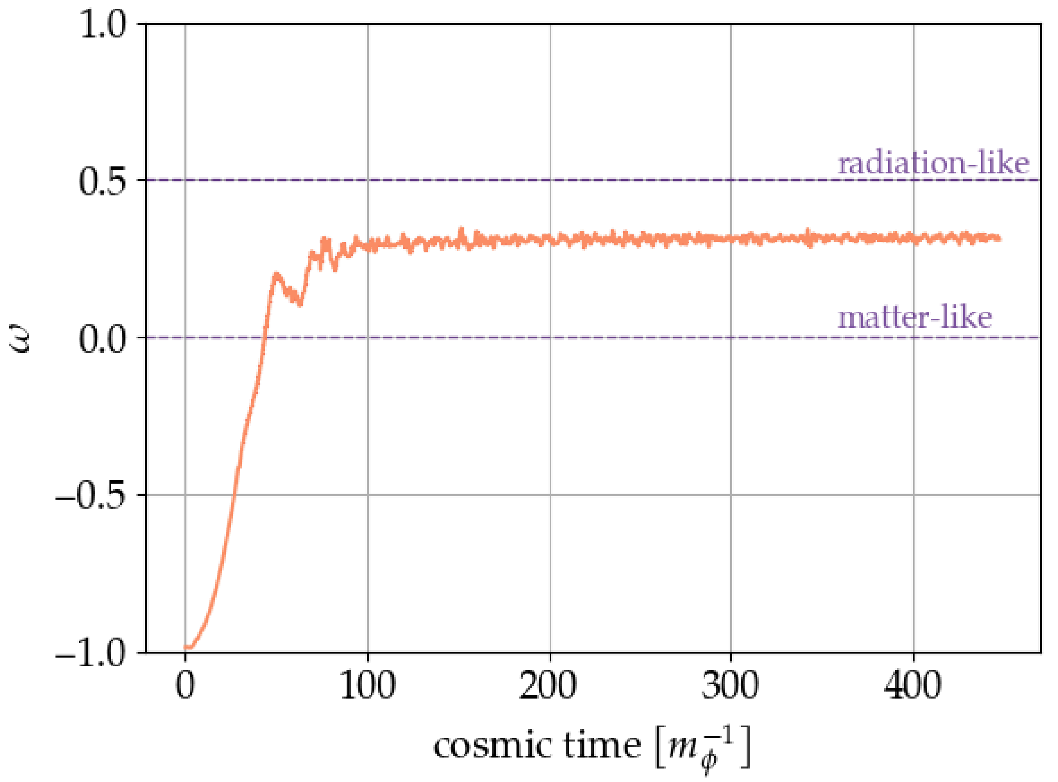

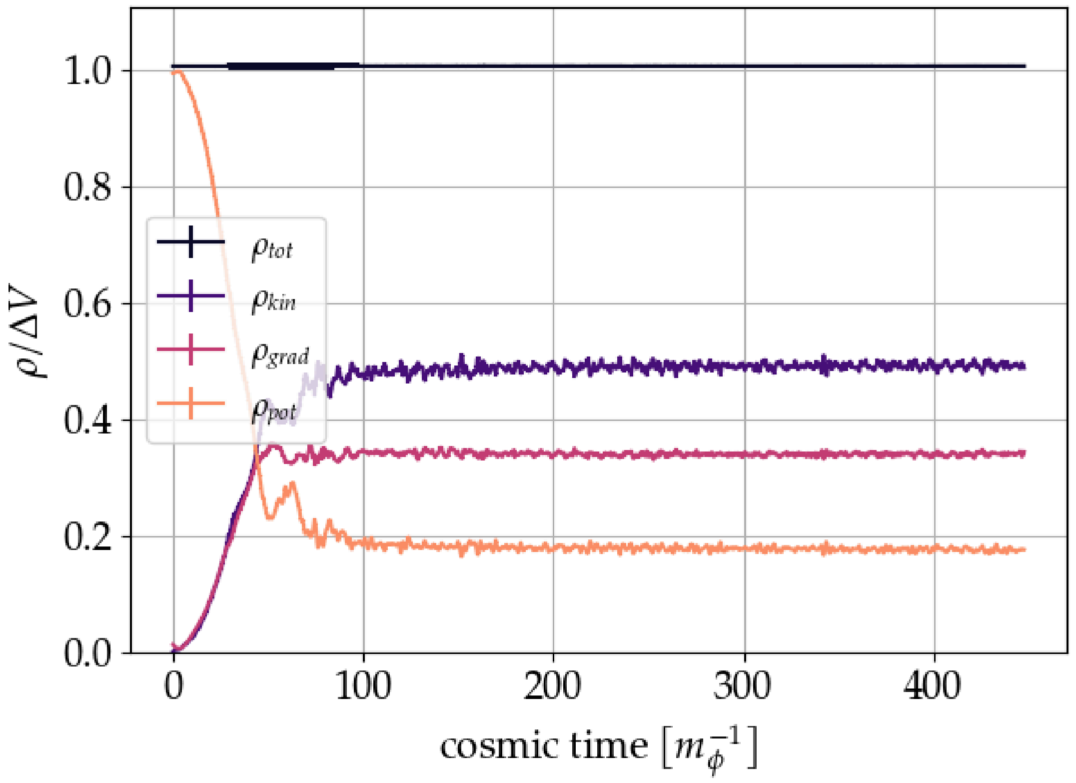

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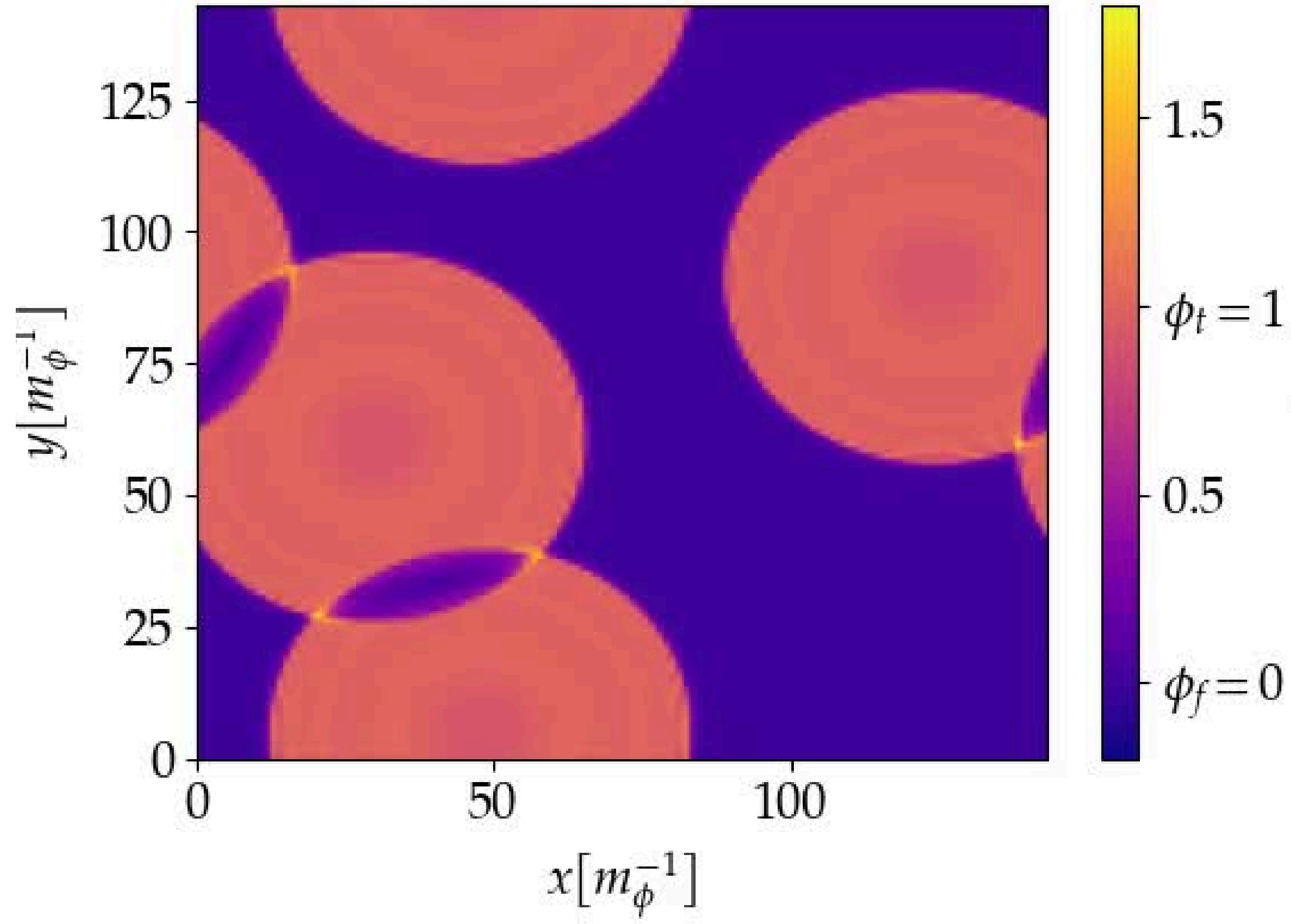
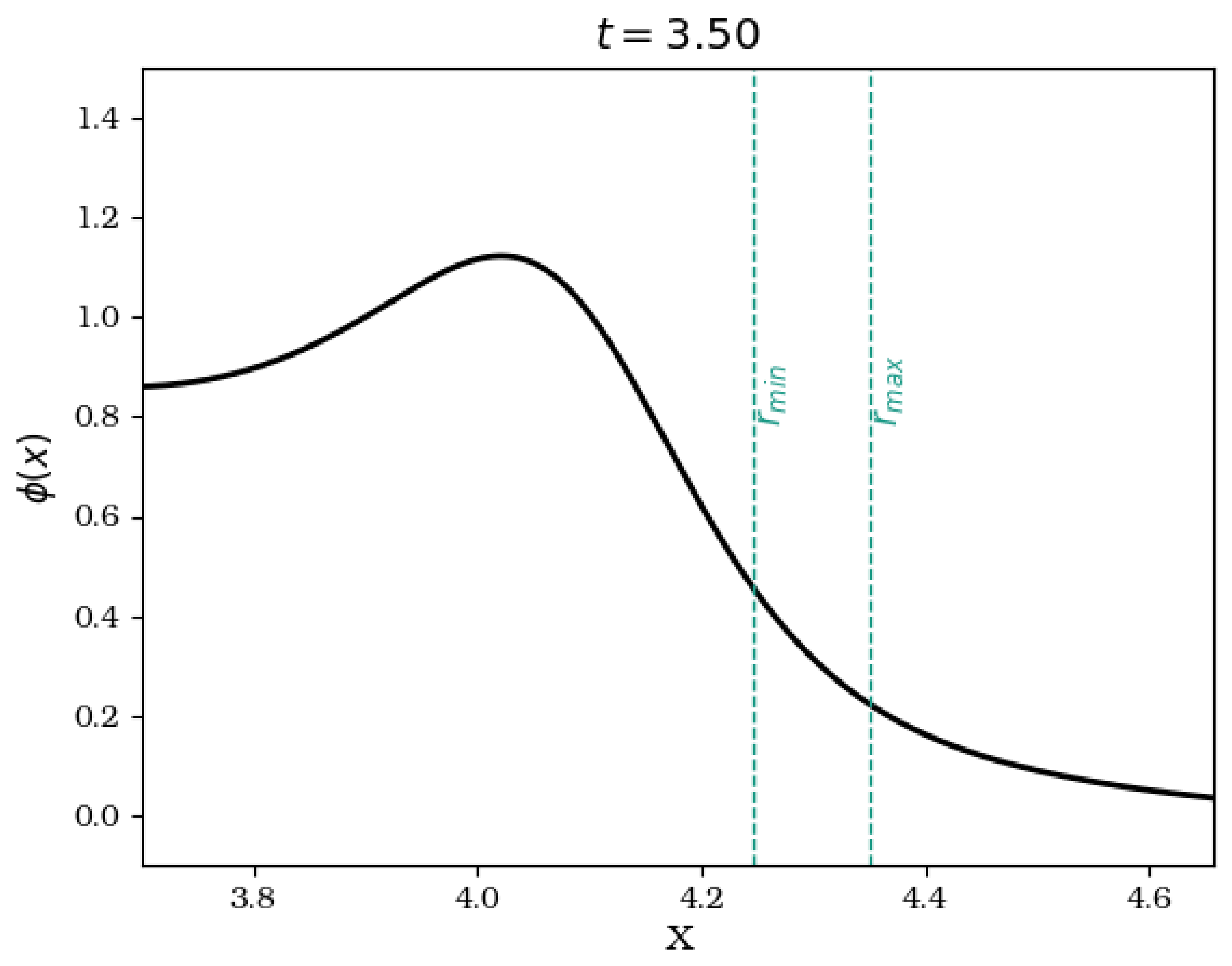
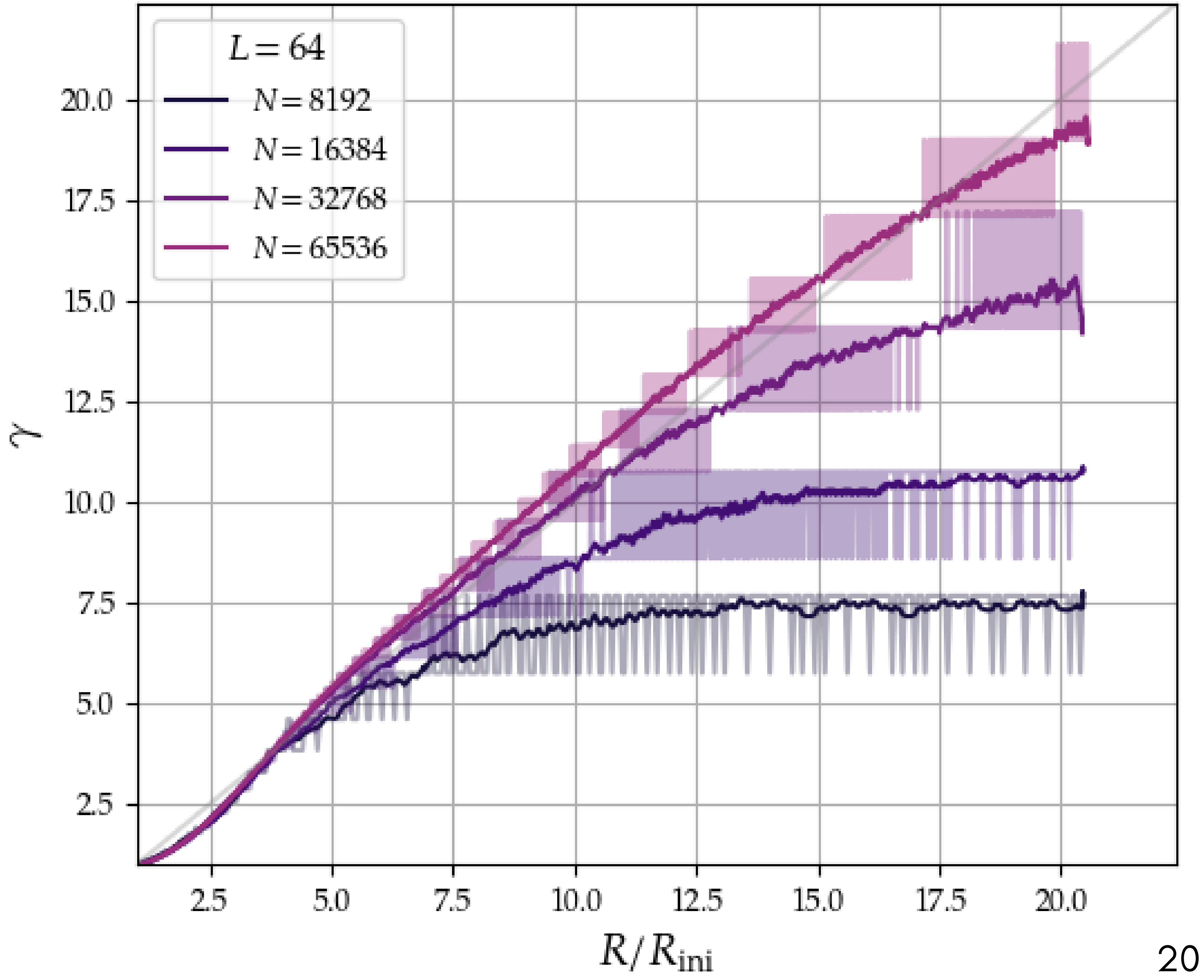


The lattice results show an equation of state value between matter and radiation domination. What does the EoS depend on?

Wall thickness/ Lorentz factor

We track the wall thickness during the expansion of 1D bubbles to confirm the relation between γ and R_{coll}/R_{ini} .





What determines the EoS?

(Step III: Let a computer do it again and again)

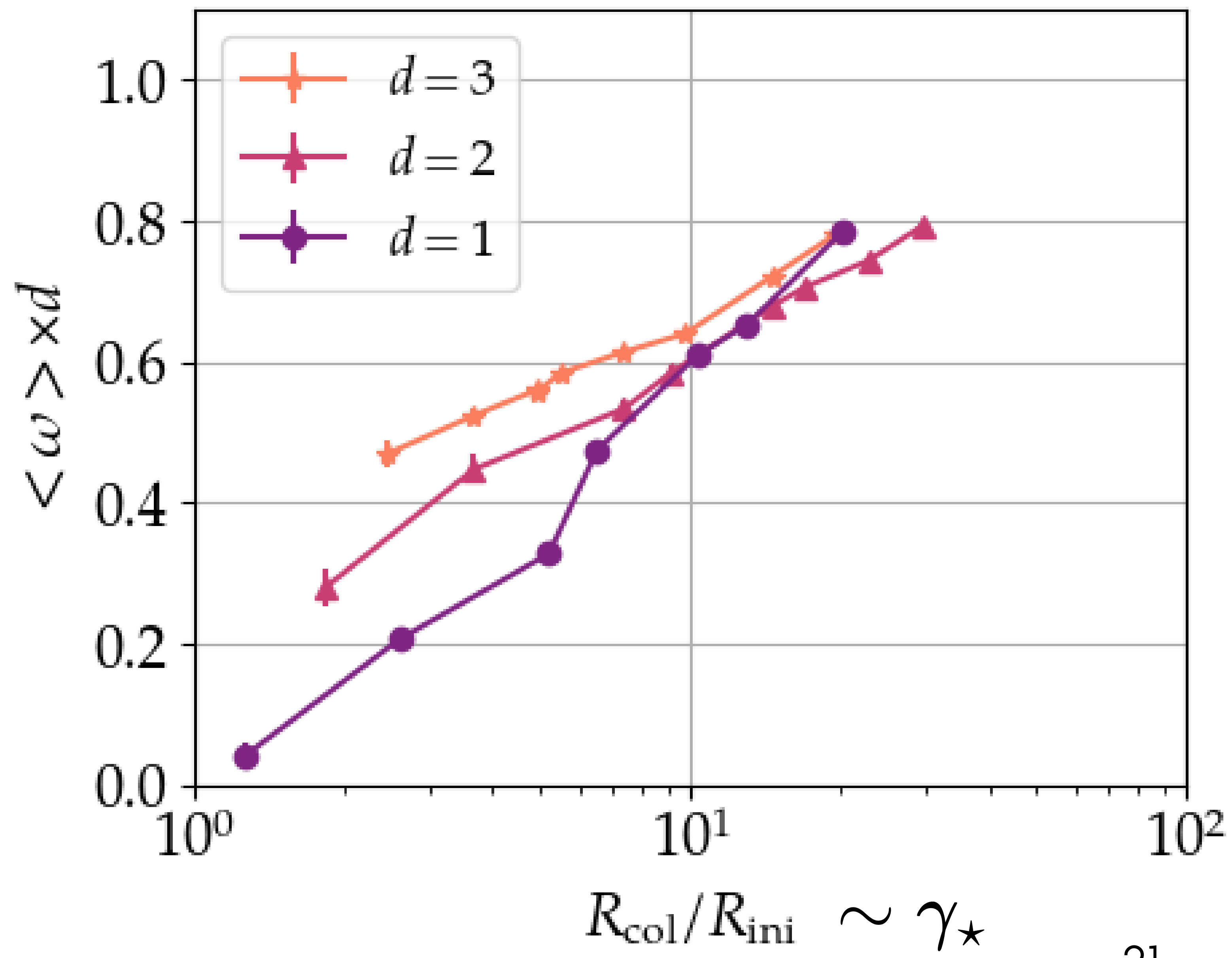
Systematic study of the simulation results for bubble collisions in 1, 2 and 3 spatial dimensions

Varying the initial bubble separation to reach different bubble wall velocities:

Larger bubble separation
→ more relativistic bubble walls
→ more energy in gradients
→ closer to radiation domination

What about predictivity? Is it possible to determine a function $\omega(\gamma_\star)$?

Summarized results for the EoS after percolation (1+1 , 1+2 and 1+3 simulations)



Step IV: develop more analytical understanding...

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Move to fourier space and re-write the previous expressions in terms of the power spectrum

$$\langle \phi_{\mathbf{k}'} \phi_{\mathbf{k}} \rangle = P_\phi(k)(2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}')$$

We introduce the dimensionless power spectrum

$$\Delta_\phi(k) = \frac{k^d}{(2\pi)^d} S_{d-1} P_\phi(k), \quad \text{with} \quad S_{d-1} = 2\pi^{d/2}/\Gamma\left(d/2\right)$$

With some simplifications, one can re-express the energy densities in terms of the power spectrum:

$$\omega = d^{-1} \frac{\langle G \rangle}{\langle K \rangle} = d^{-1} \frac{\int d \ln k (k/a)^2 \Delta_\phi(k)}{\int d \ln k \left(m_\phi^2 + (k/a)^2 \right) \Delta_\phi(k)}$$

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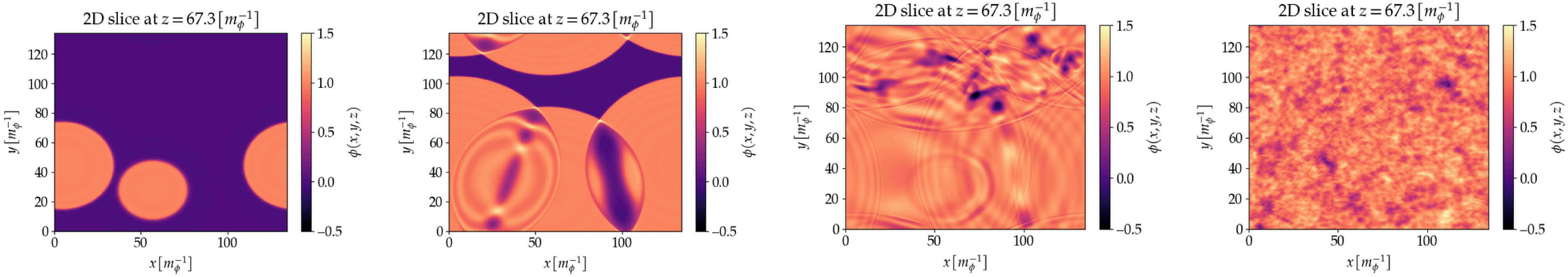
Is it possible to determine a function $\omega(\gamma_\star)$?

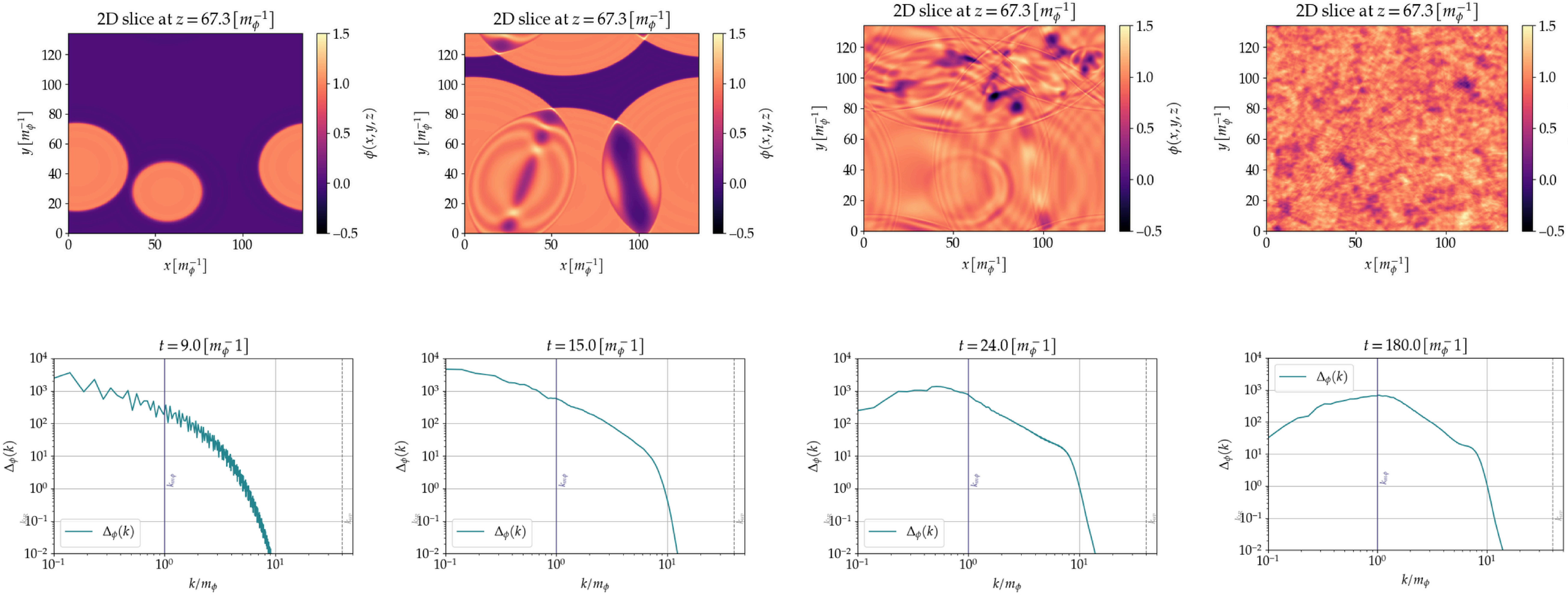
Maybe if we find determine an analytic form for the power spectrum in terms of the lorentz factor of the wall $\gamma_\star$

Step V: Study the power spectrum



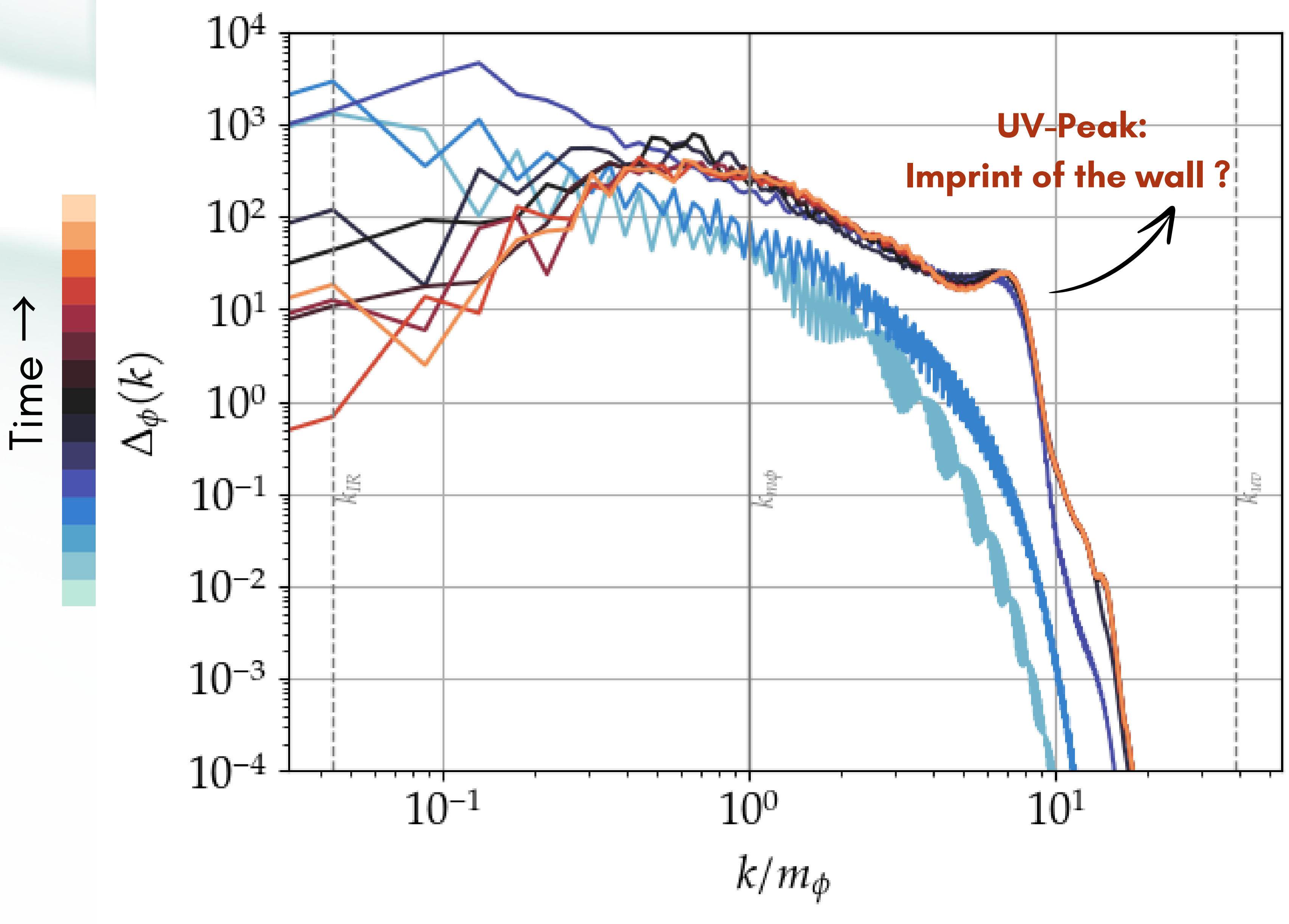
Compute the power spectrum from the Fourier transform at different steps of the evolution

Step V: Study the power spectrum



Step V: Study the power spectrum

Evolution of the power spectrum before and after collision/percolation (results of 3D simulation)



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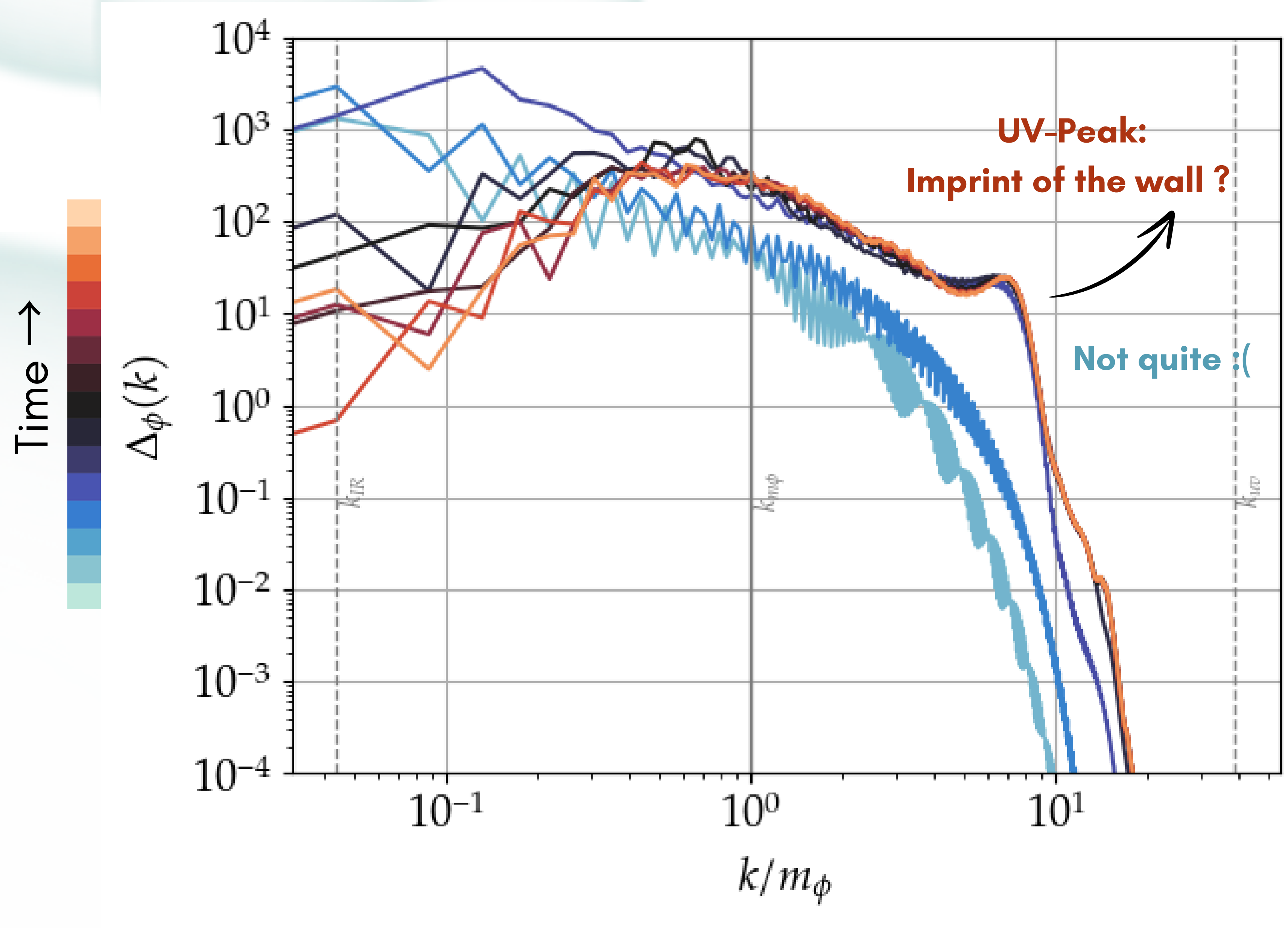
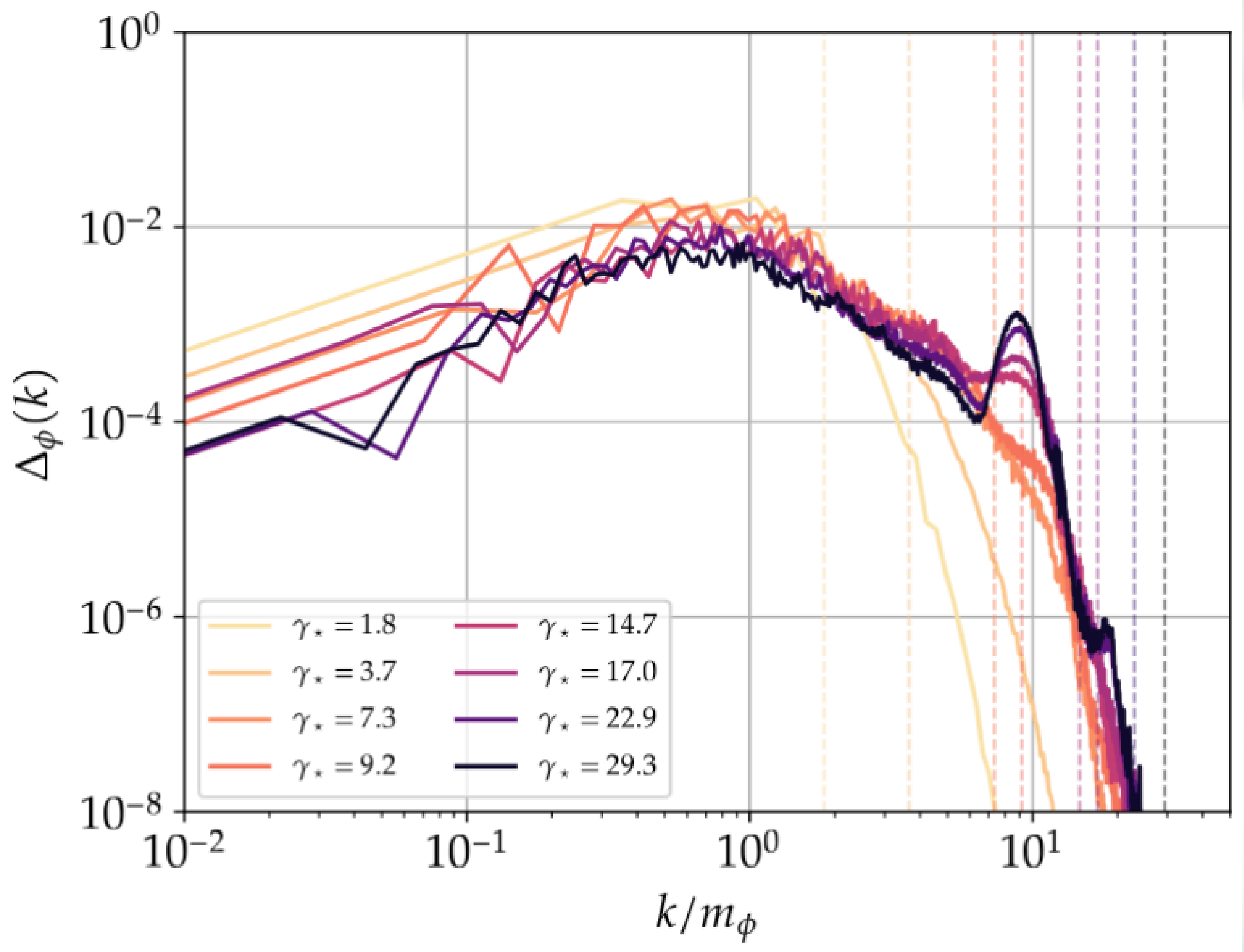


Tests performed for 2D simulations. For fixed resolution the peaks appears at the same location and does not depend on the wall thickness.



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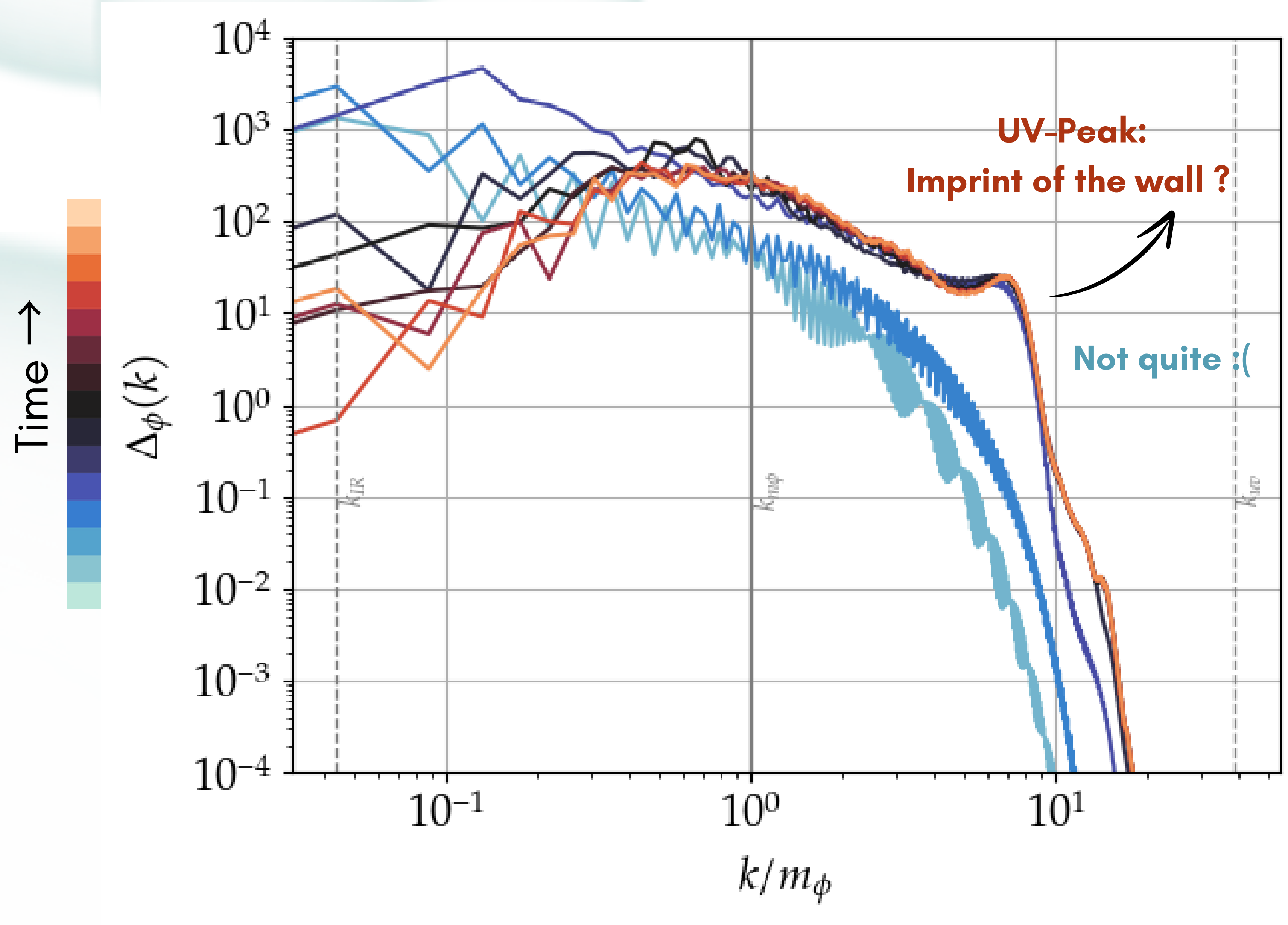
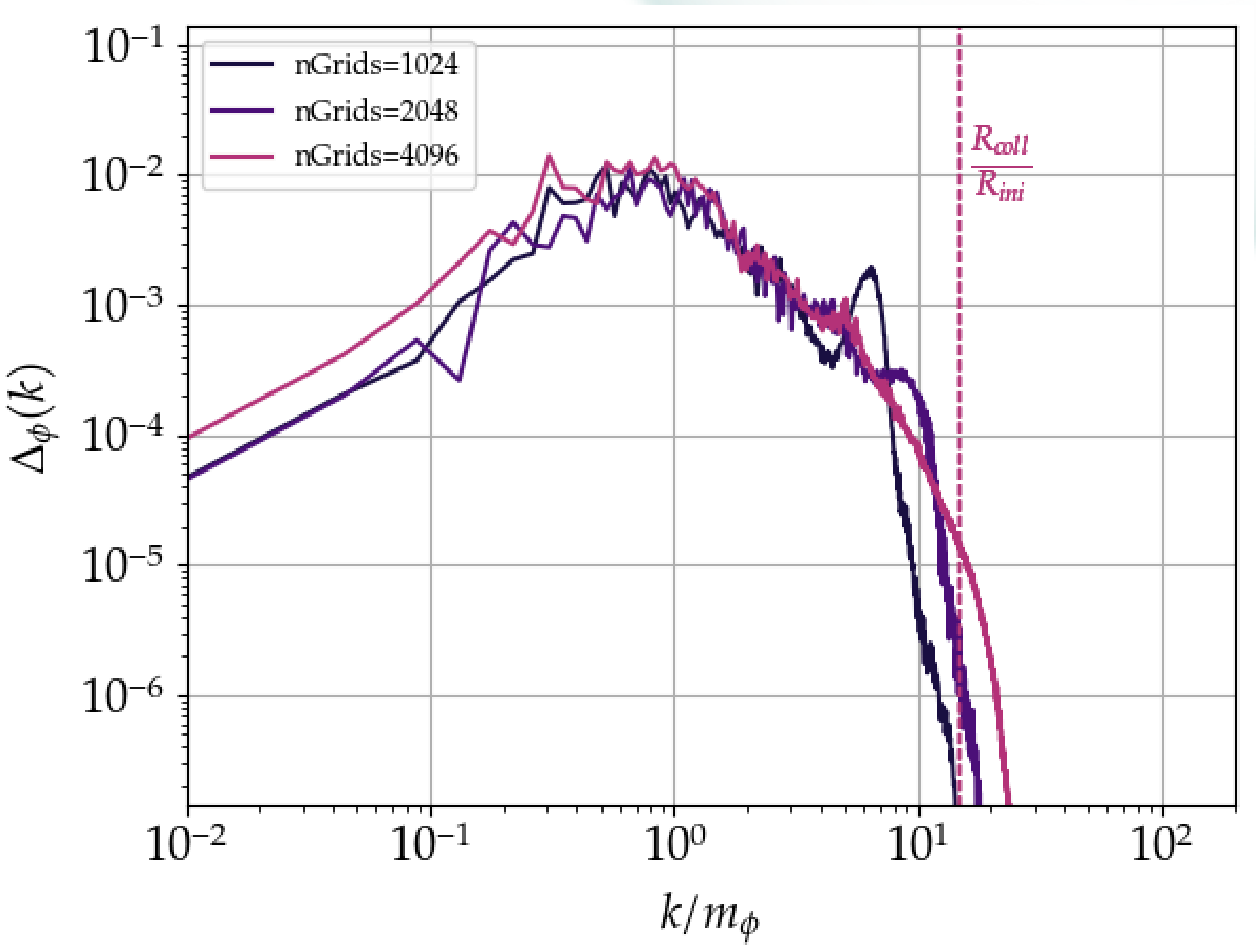


We have performed checks using 2D simulations. These show that the peak dissapears with better resolution.



The peak is a lattice artifact. The power spectrum follows instead a broken power law with a peak around $k_p \sim m_\phi$ **.**

Step VI: Modelling the power spectrum

Equation of state as a function of the power spectrum:

$$\omega = d^{-1}\frac{\langle G\rangle}{\langle K\rangle} = d^{-1}\frac{\int d\ln k (k/a)^2 \Delta_\phi(k)}{\int d\ln k \left(m_\phi^2 + (k/a)^2\right)\Delta_\phi(k)}$$

split the integral : contributions from

$k < k_0$ and $k_0 < k < \gamma_\star m_\phi$

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Our simulations suggest that the shape of the power spectrum for low-k is independent of $\gamma_\star$. The corresponding contributions to the EoS are constant and the exact shape is for $k < k_0$ therefore not important.

Under these assumptions and with some approximations:

$$\omega\left(\gamma_\star\right) = d^{-1}\frac{a + \kappa(\gamma_\star)}{b + \kappa(\gamma_\star)} \; .$$

The equation of state depends strongly on the UV-tail of the spectrum. This determines the functional dependence on $\gamma_\star$.

Step VI: Modelling the power spectrum

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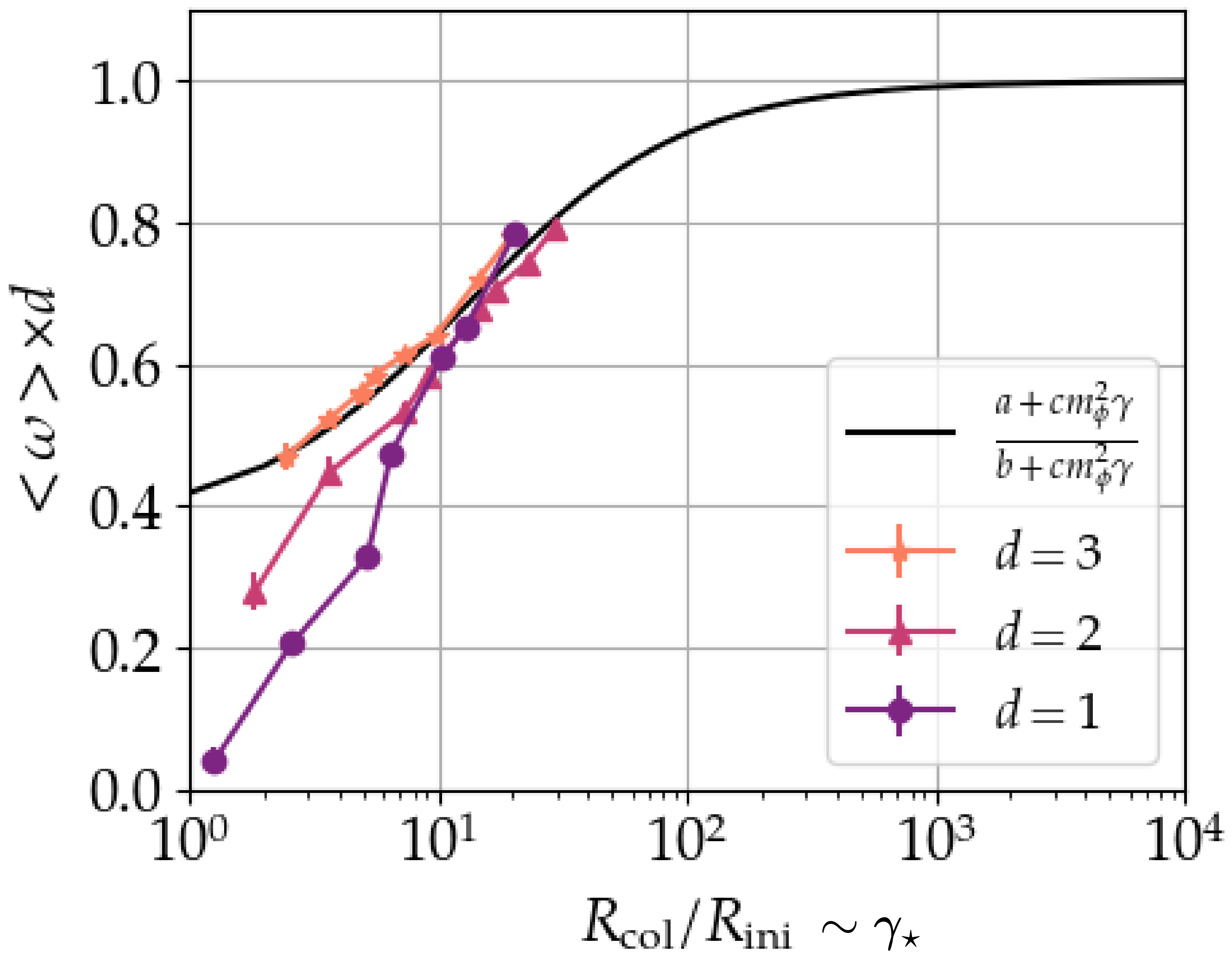
[PRELIMENARY]



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The equation of state depends strongly on the UV-tail of the spectrum. This determines the functional dependence on $\gamma_\star$.

Our results for the EoS in 3D seem to go the best with $k \propto k^{-1}$ in the high-k range.

Conclusions
(Part I)

Immediately after a first order phase transition we find that:
Larger bubble separation
→ more relativistic bubble walls
→ more energy in gradients
→ closer to radiation domination

This makes the realisation of early matter domination after relativistic bubble collisions questionable.

If thermalization is very slow, the expansion will eventually suppress the gradients and might still allow for a purely matter-dominated epoch.
Detailed results on the evolution of the EoS with expansion are upcoming.

Part II:
How does a strong phase transition impact dark matter freeze-in ?

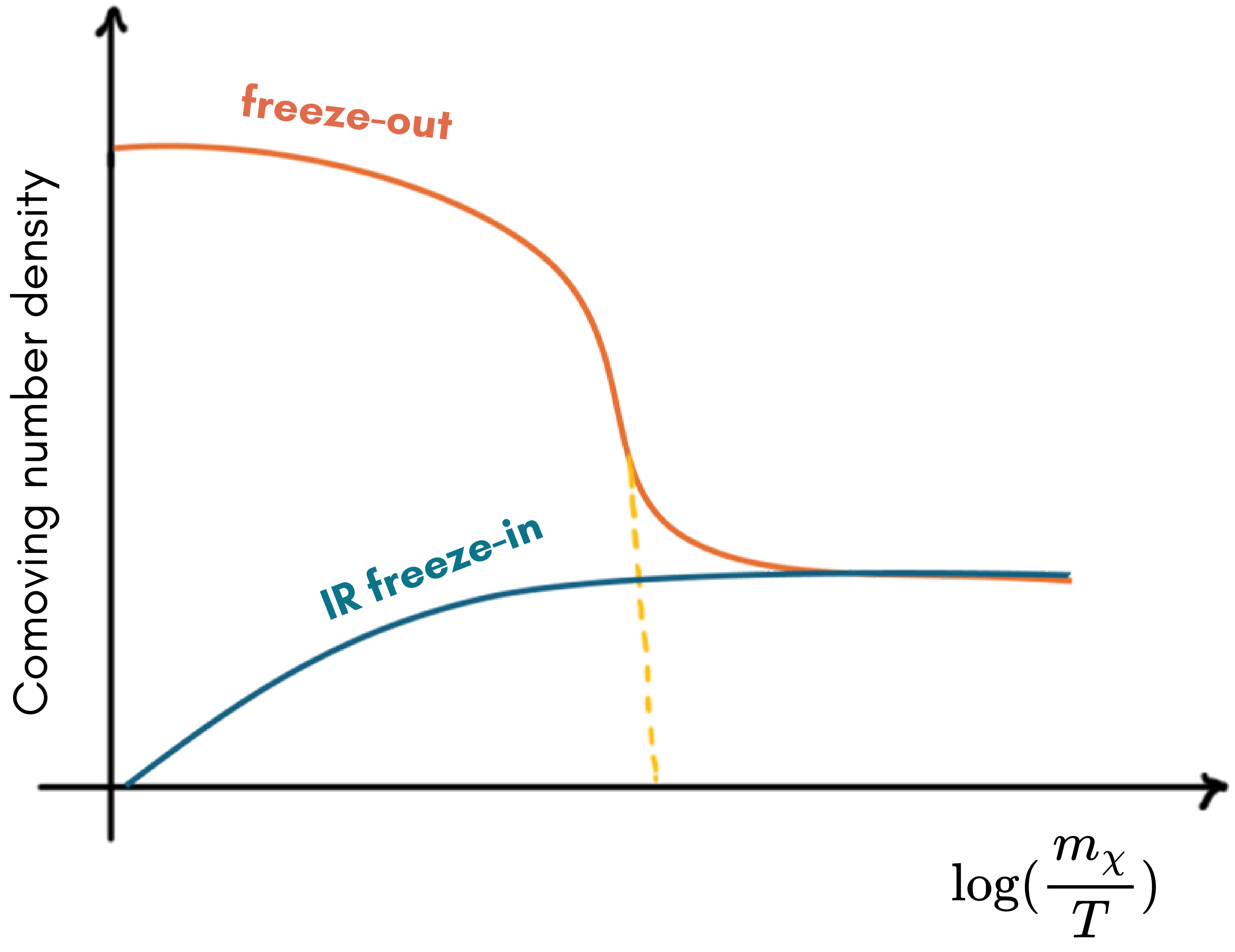
Non-Thermal Dark Matter Production

Strong bounds from direct detection experiments on WIMPs → **non-thermal production ?**

Interactions so feeble that DM and SM were never in thermal equilibrium
→ DM abundance builds up

IR freeze-in demands extremely small couplings

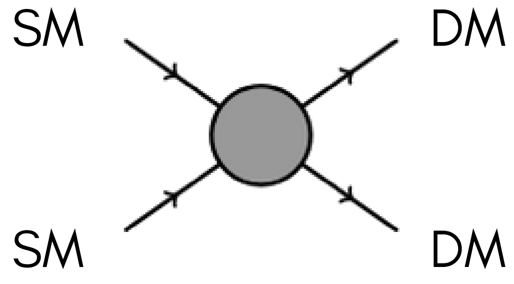
[Hall et al. 0911.1120]



Non-Thermal Dark Matter Production

Another realisation of freeze-in:

Interactions via non-renormalizable operators with dimension n+4:

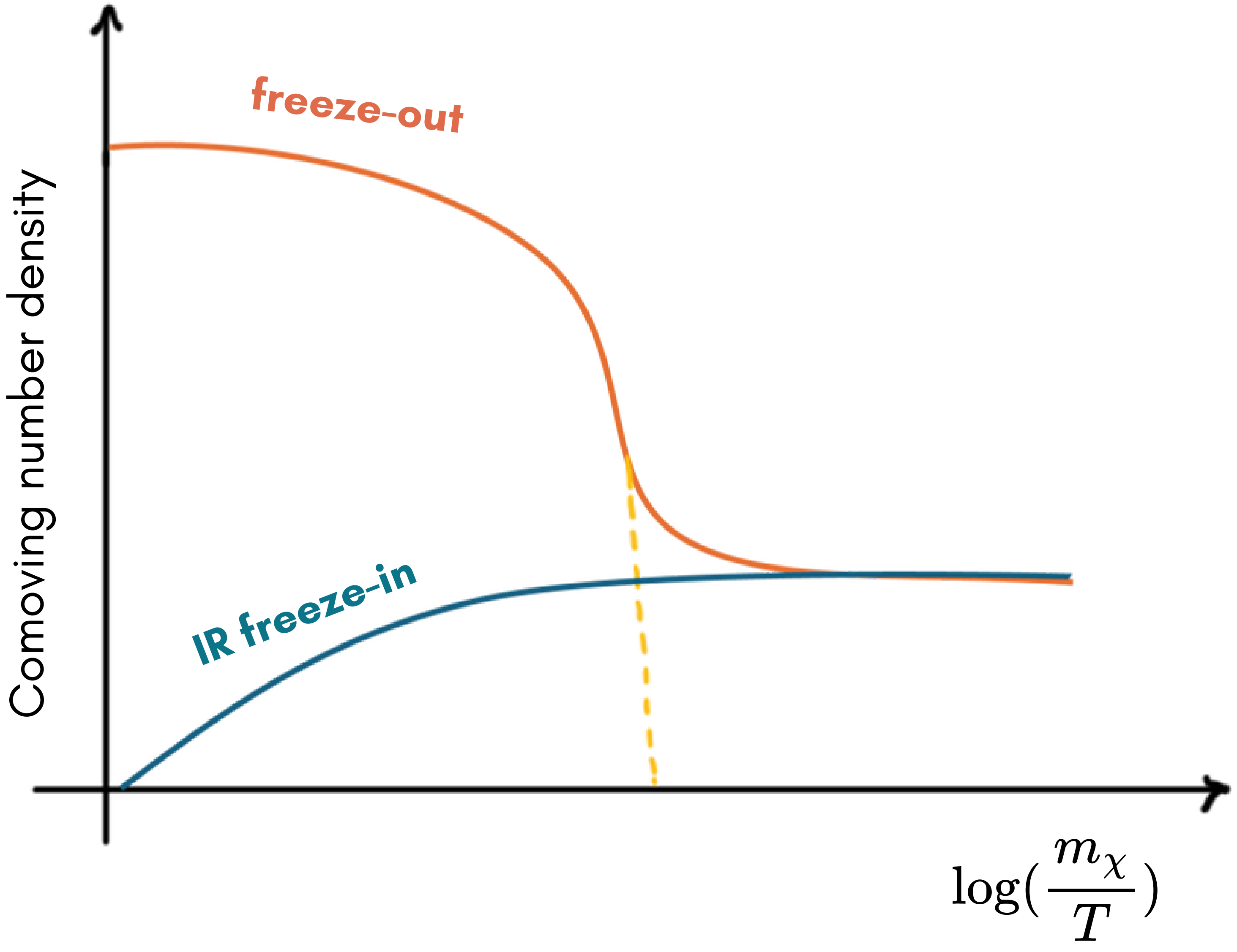


and thermally averaged crosssection:

$$\langle \sigma v \rangle \propto \frac{T^{2(n-1)}}{\Lambda^{2n}}$$

Temperature of SM radiation bath

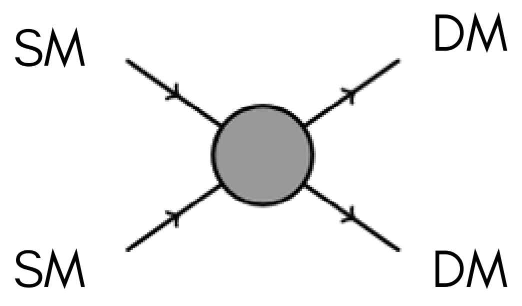
UV-suppression scale



Non-Thermal Dark Matter Production

Another realisation of freeze-in:

Interactions via higher dimensional operators:

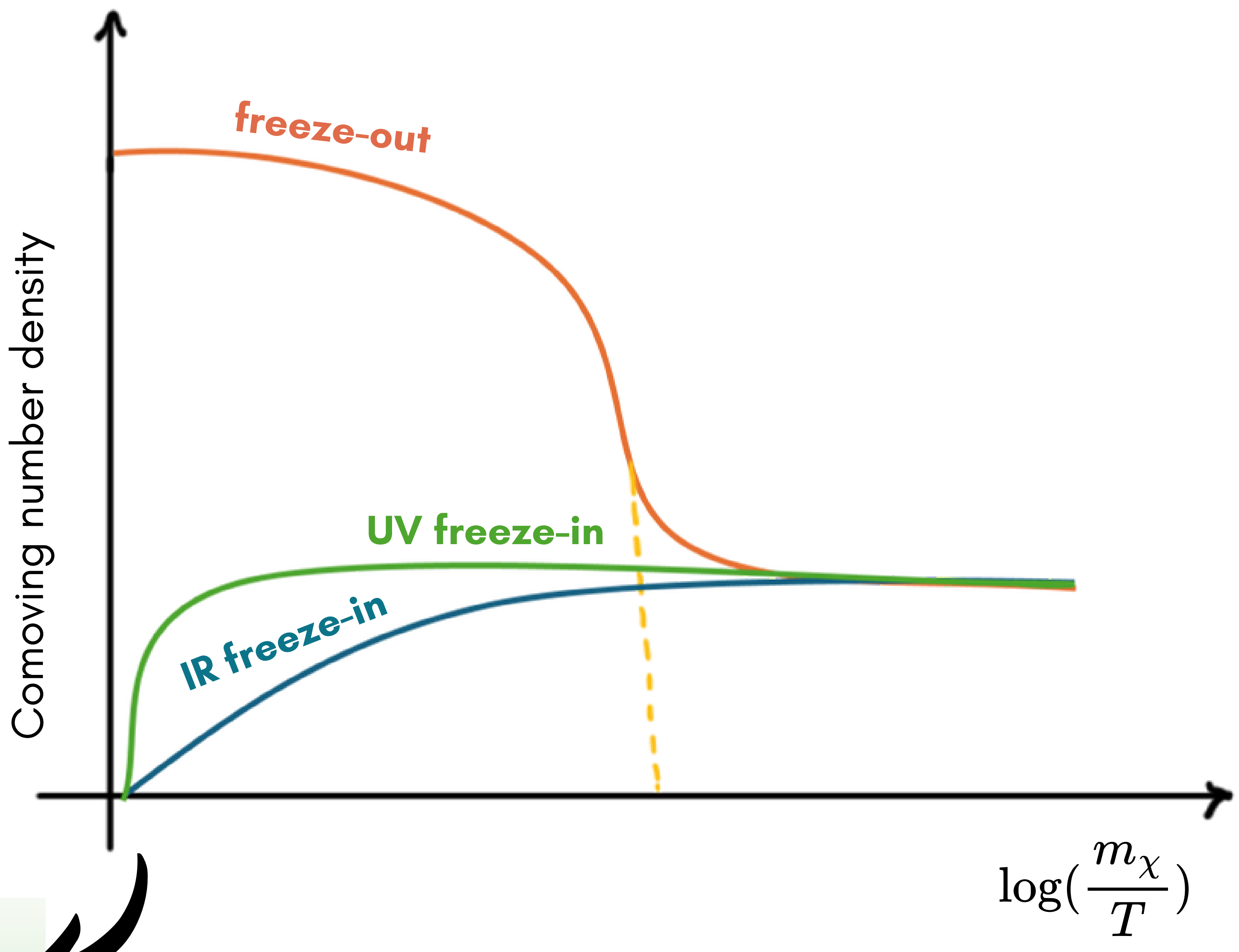


$$\langle \sigma v \rangle \propto \frac{T^{2(n-1)}}{\Lambda^{2n}}$$

Temperature of SM radiation bath

UV-suppression scale

→ **UV-dominated freeze-in**

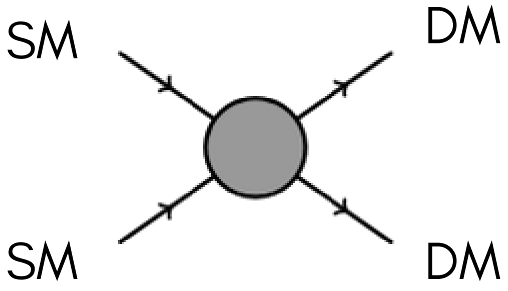


The bulk of DM production is at primordial reheating : $T \sim T_{\mathrm{RH}}$

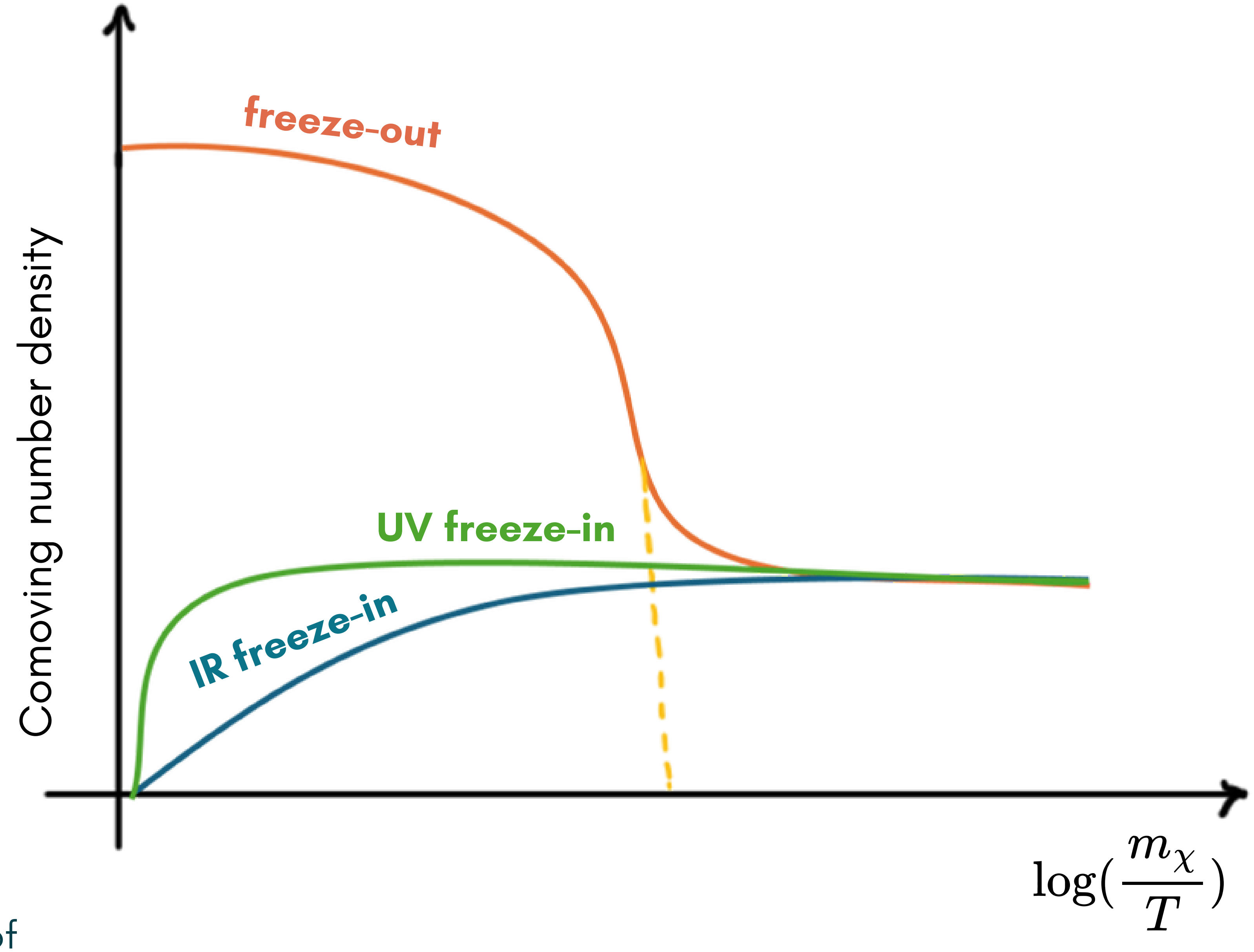
Non-Thermal Dark Matter Production

UV-dominated freeze-in:

Interactions via higher dimensional operators:



$$\langle \sigma v \rangle \propto \frac{T^{2(n-1)}}{\Lambda^{2n}}$$



Problem: sensitivity of the DM abundance to the reheating and maximal temperature of SM radiation bath:

[Bernal et al. 1909.07992]

[Elahi et al. 1410.6157]

$$Y_{\mathrm{DM}} \propto \frac{M_{\mathrm{pl}} T_{\mathrm{RH}}^{2n-1}}{\Lambda^{2n}}$$

Reheating Temperature of SM radiation bath

UV- freeze-in and First Order Phase Transitions

UV freeze-in :

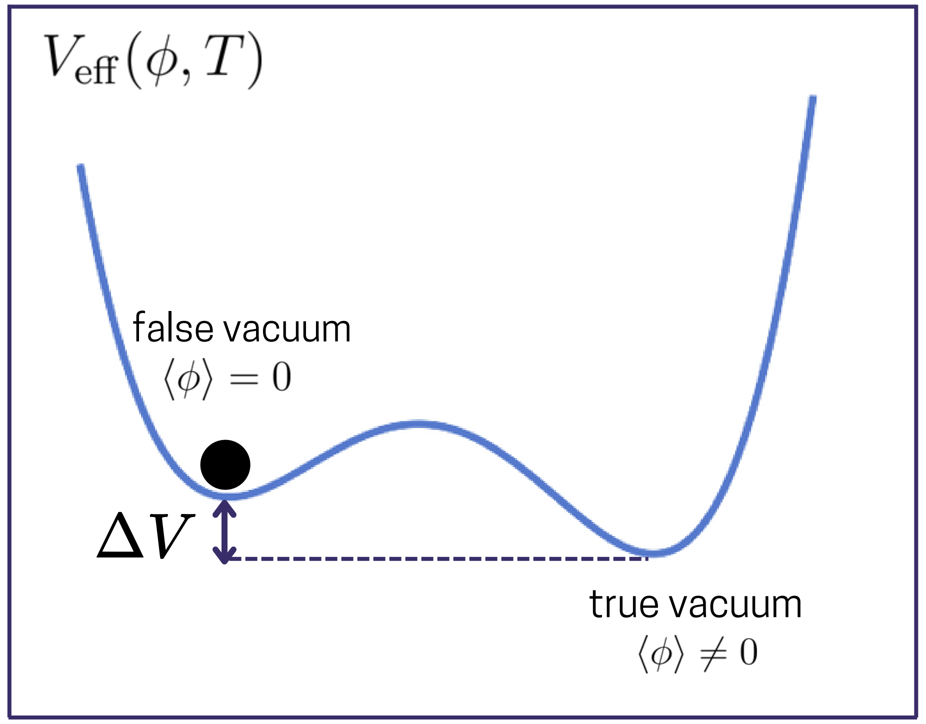
DM relic density is determined by the reheating / maximal temperature

$$Y_{\rm DM} \propto \frac{M_{\rm pl} T_{\rm RH}^{2n-1}}{\Lambda^{2n}}$$

[Elahi et al. 1410.6157]
[Bernal et al. 1909.07992]

First-Order Phase Transition (FOPT):

- The scalar field acts like a cosmological constant before the transition.
- Energy injection to the radiation bath after the phase transition : Can dilute pre-existing relics if supercooled.
- Relevant temperature scale is $T_{\rm PT}$



Question: Under which conditions does $T_{\rm PT}$ become the relevant scale that determines the relic density?

The DM phase-in scenario

Scalar Field ϕ $\xrightarrow[\text{decays afterwards to SM bath}]{\text{undergoes FOPT}}$ SM radiation $\xrightarrow[\text{non-renormalizable interaction}]{\text{produces DM via}}$ DM

$$\langle \sigma v \rangle = \frac{T^{2(n-1)}}{\Lambda^{2n}}$$

Operator of dim n+4

SM +SM → DM+DM

The DM phase-in scenario

Scalar Field ϕ $\xrightarrow{\text{undergoes FOPT, decays afterwards to SM bath}}$ SM radiation $\xrightarrow{\text{produces DM via non-renormalizable interaction}}$ DM

$$\langle \sigma v \rangle = \frac{T^{2(n-1)}}{\Lambda^{2n}}$$

Boltzmann equations for energy/number densities:

$$\frac{\mathrm{d}\rho_\phi}{\mathrm{d}a} = -\frac{3(1+\omega)}{a}\rho_\phi - \frac{\Gamma}{aH}\rho_\phi$$

$$\frac{\mathrm{d}\rho_{\rm SM}}{\mathrm{d}a} = -\frac{4}{a}\rho_{\rm SM} + \frac{\Gamma}{aH}\rho_\phi$$

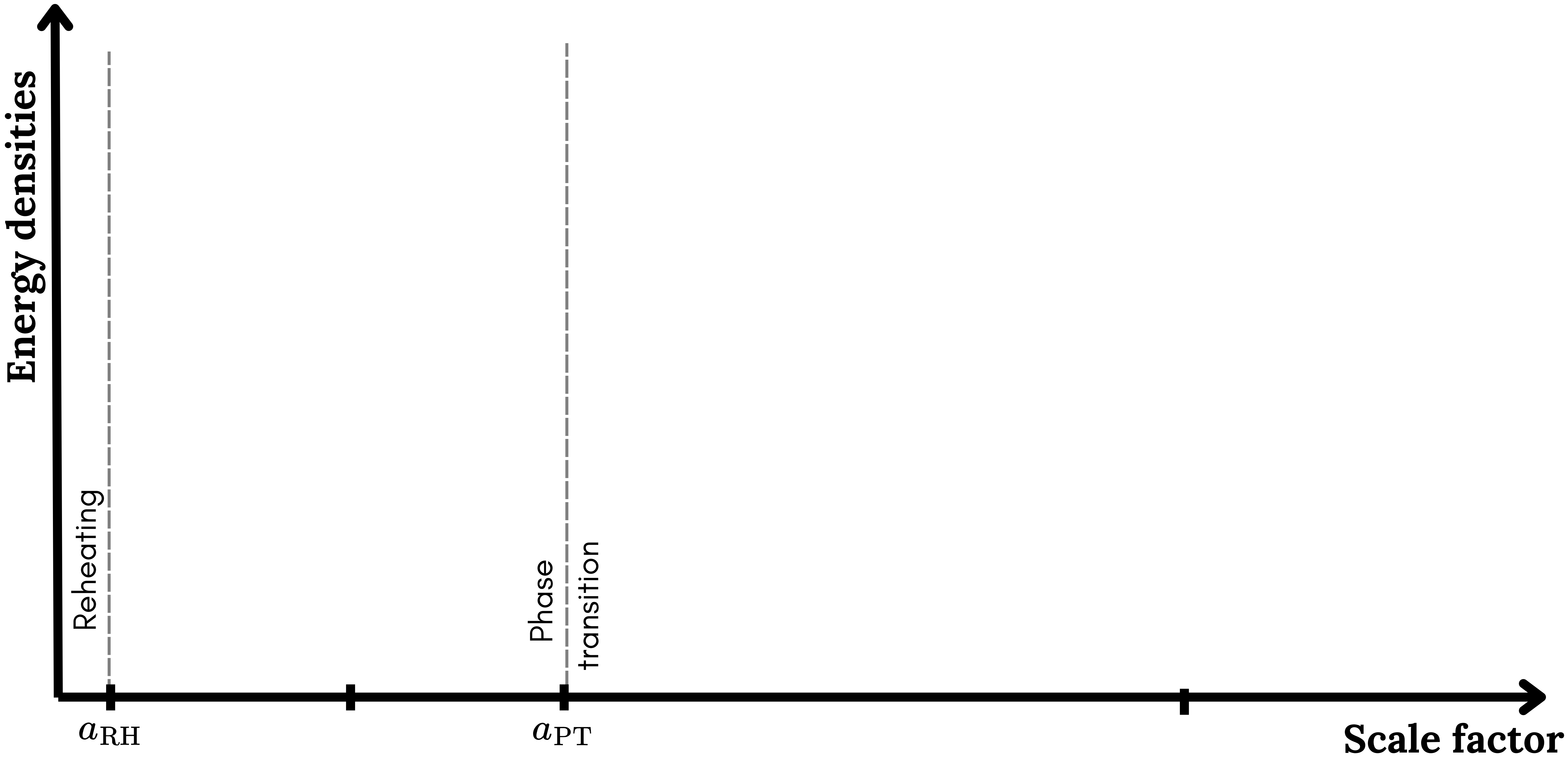
$$\frac{\mathrm{d}n_{\rm DM}}{\mathrm{d}a} = -\frac{3}{a}n_{\rm DM} + \frac{\langle \sigma v \rangle}{aH}n_{\rm SM}^2$$

Before the PT: $\Gamma = 0$ and $\omega = -1$

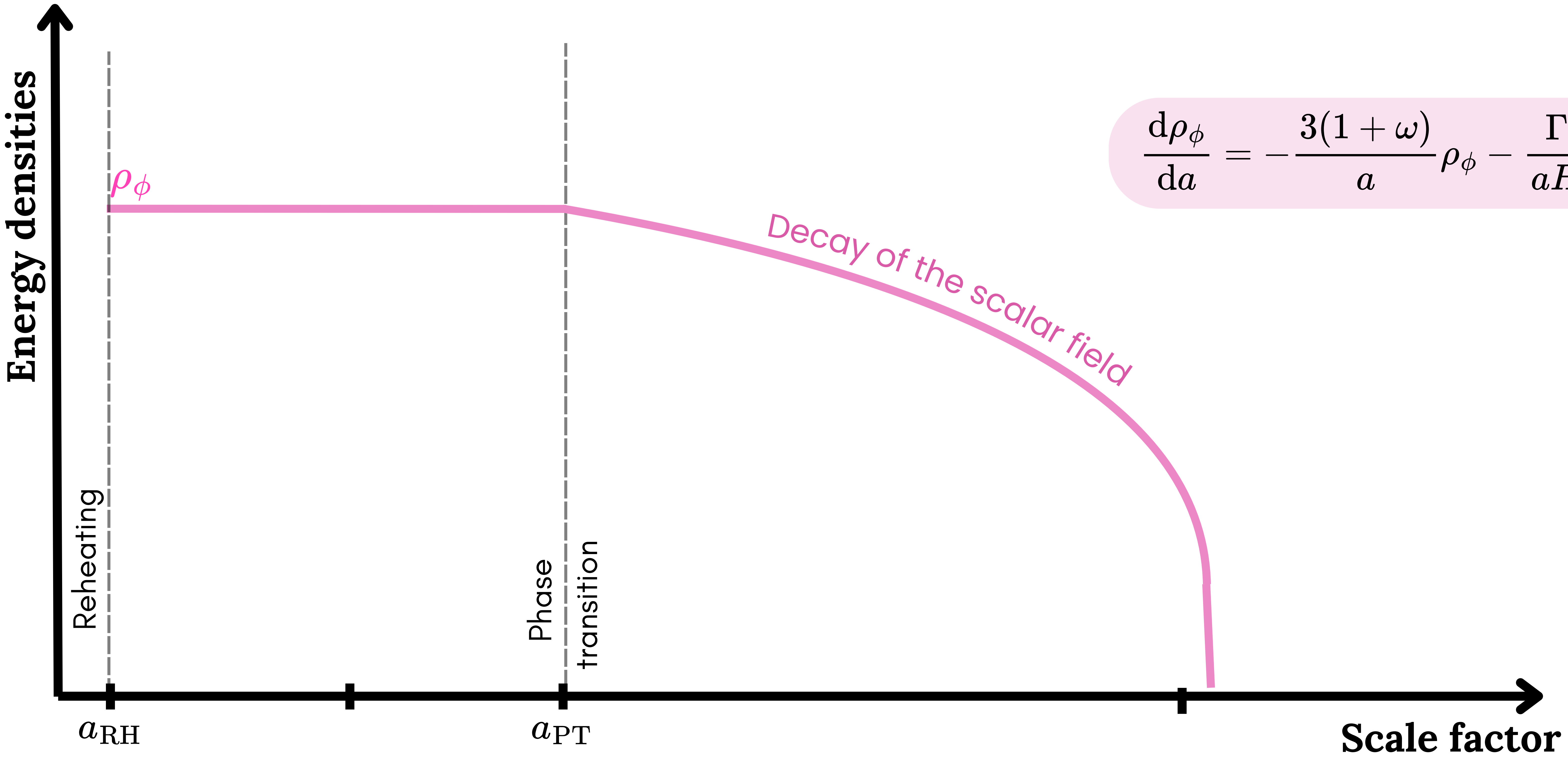
After the PT: $\Gamma = \text{const}$ and $0 \leq \omega \leq 1/3$

Friedmann eq: $H = \frac{\dot{a}}{a} = \sqrt{\frac{8\pi}{3M_{\rm Pl}^2}(\rho_{\rm SM} + \rho_\phi)}$

The DM phase-in scenario

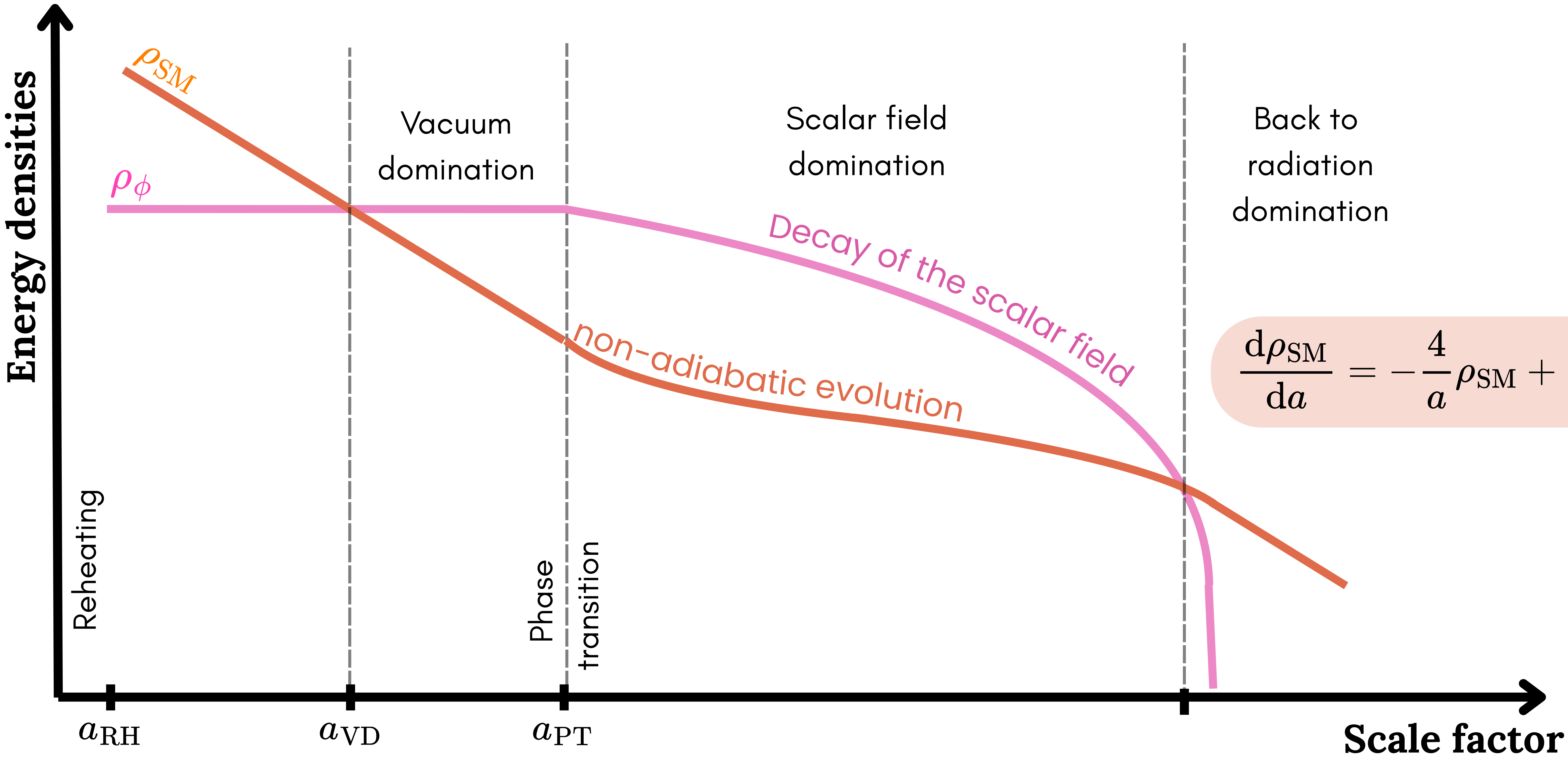


The DM phase-in scenario



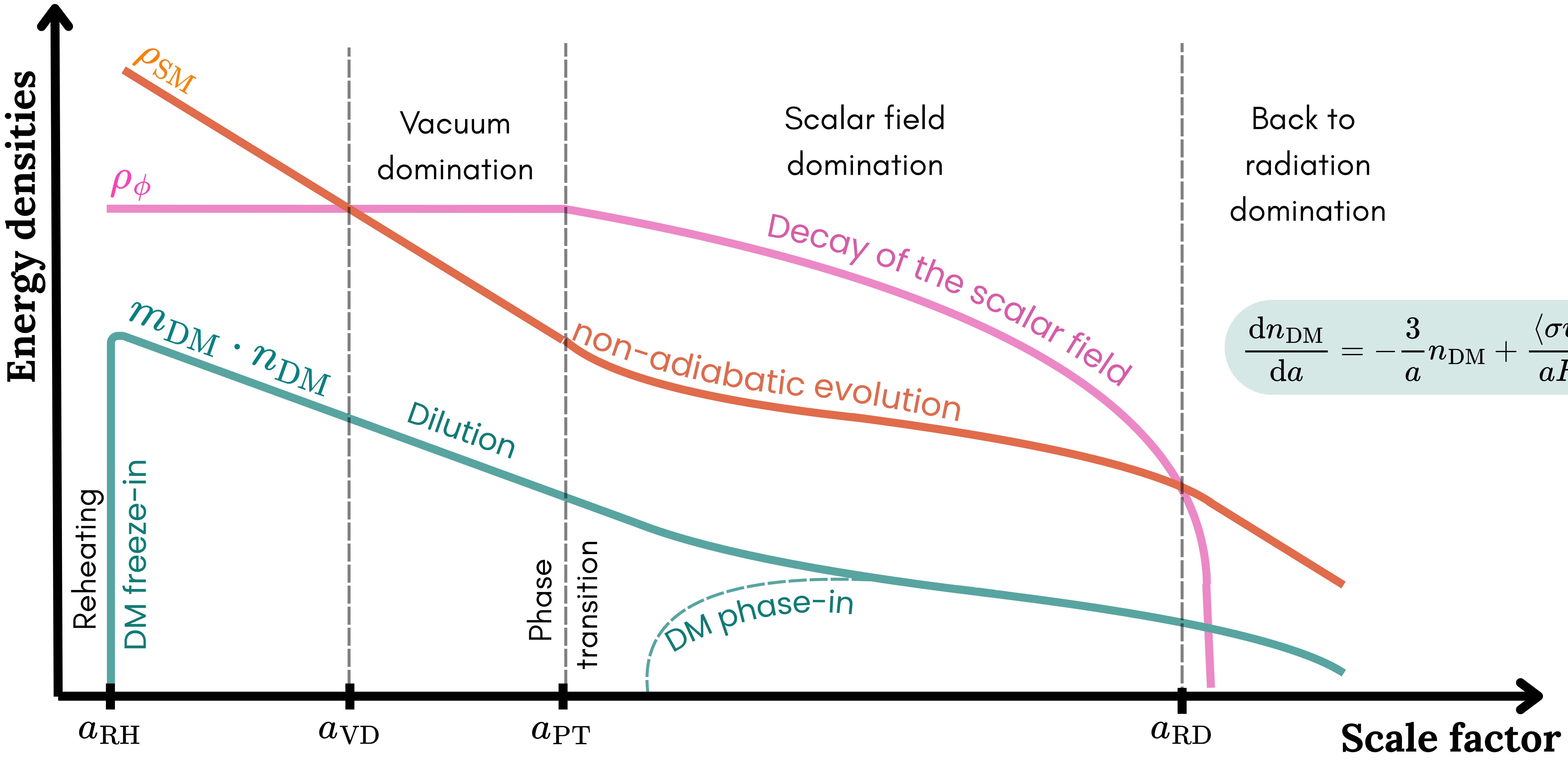
$$\frac{\mathrm{d}\rho_\phi}{\mathrm{d}a} = -\frac{3(1+\omega)}{a}\rho_\phi - \frac{\Gamma}{aH}\rho_\phi$$

The DM phase-in scenario



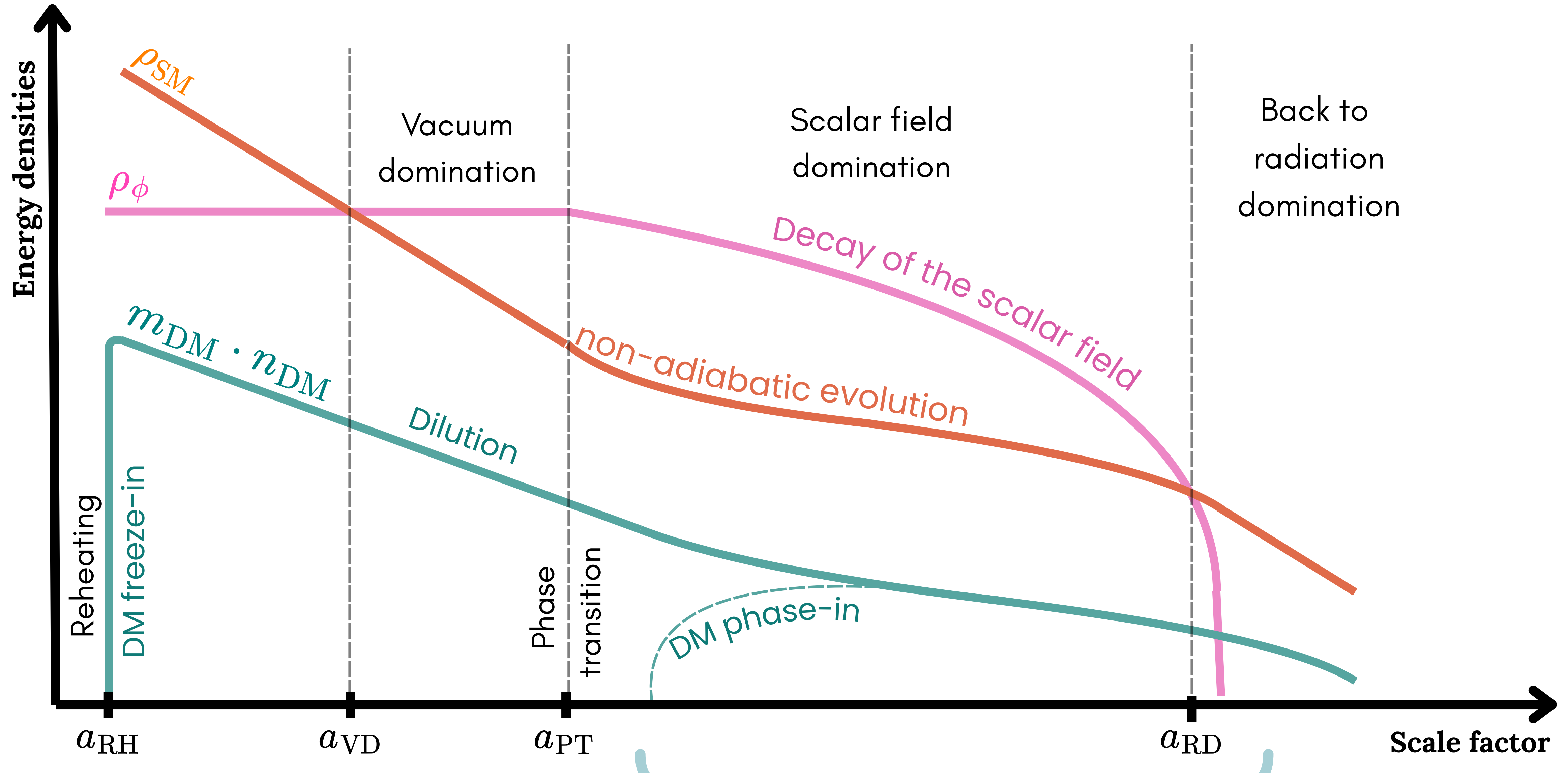
$$\frac{\mathrm{d}\rho_{\rm SM}}{\mathrm{d}a} = -\frac{4}{a}\rho_{\rm SM} + \frac{\Gamma}{aH}\rho_\phi$$

The DM phase-in scenario



$$\frac{dn_{DM}}{da} = -\frac{3}{a}n_{DM} + \frac{\langle\sigma v\rangle}{aH}n_{SM}^2$$

Phase-in condition



Dark matter is produced in different phases.

$\Omega_{\rm DM,FI}$

↓

Determined by reheating temperature

$n_{\rm DM,FI} \propto T_{\rm RH}^{2n-1}$

$\Omega_{\rm DM,PI}$

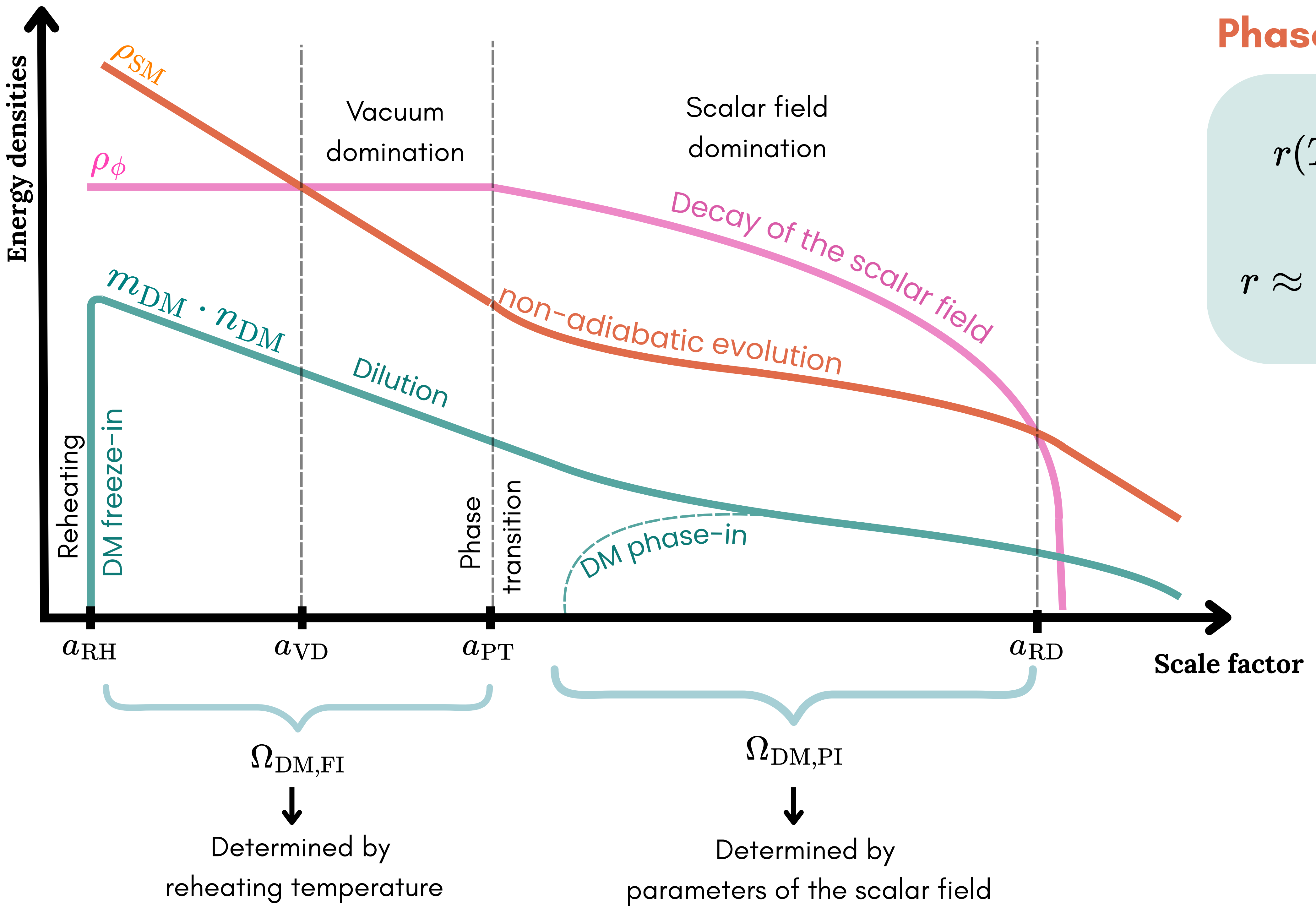
↓

Determined by parameters of the scalar field

Sensitive to the radiation bath temperature after the decay: $T_{\rm RD}$

$n_{\rm DM,PI} \propto T_{\rm RD}^{2n-1}$

Phase-in condition



Phase-in condition:

$$r(T_{\rm RH}, T_{\rm PT}, \Delta V, \Gamma, \omega, n) > 1 \quad \text{with: } r = \frac{\Omega_{\rm DM,PI}}{\Omega_{\rm DM,FI}} = \frac{\rm phase-in}{\rm freeze-in}$$

$$r \approx T_{\rm RH}^{-2n+1} T_{\rm PT}^{-3} \Delta V^{\frac{n+1}{2}} g_\star^{-(n+1)/2} \left(\frac{\sqrt{\Delta V}}{M_{\rm Pl}\Gamma} + \sqrt{\frac{3}{8\pi}} \right)^{\frac{2}{1+w}-1-n}$$

Parameters of the problem:

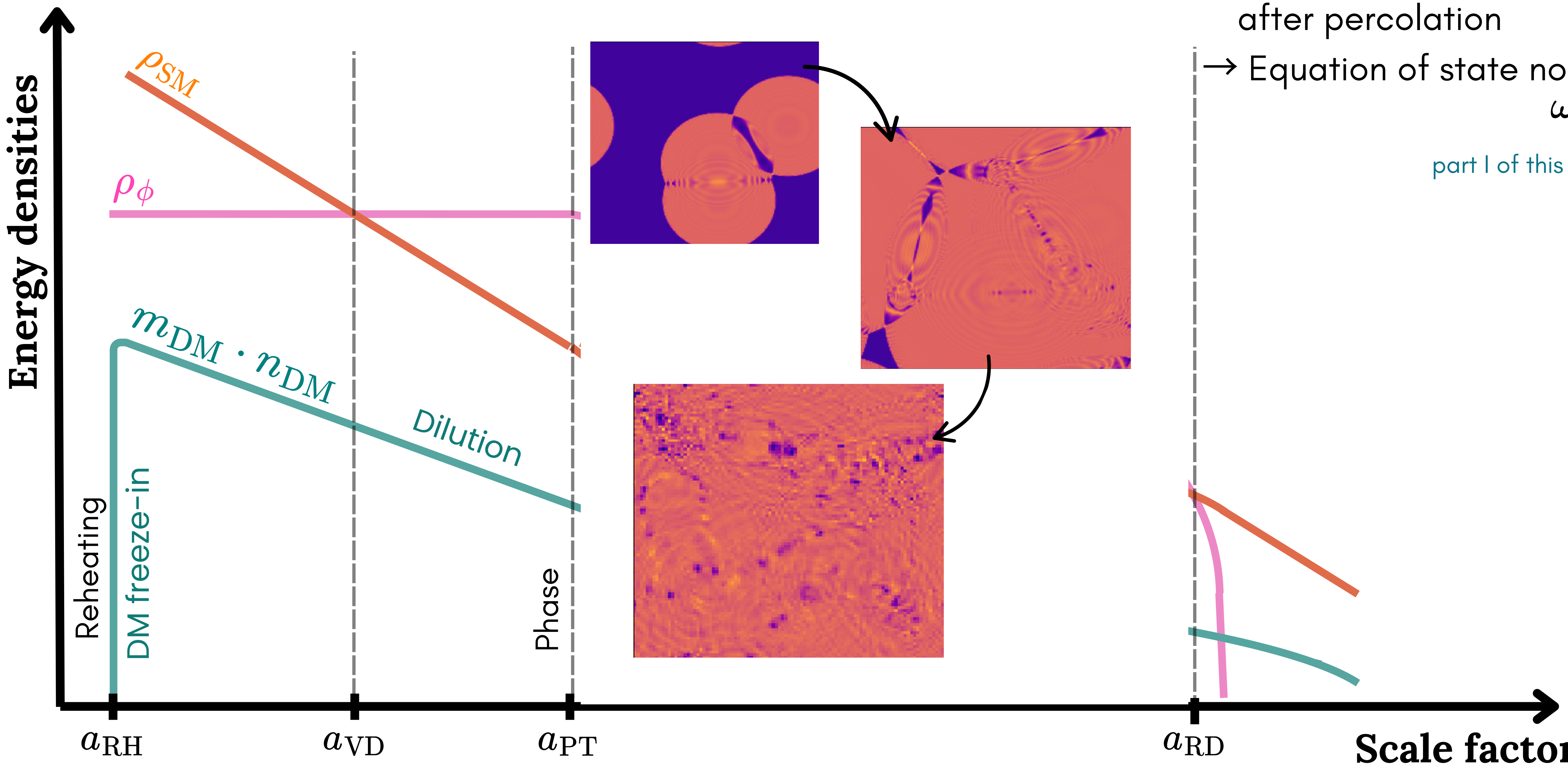
- $T_{\rm RH}$: Reheating temperature
- $T_{\rm PT}$: Phase transition temperature
- ΔV : Potential energy/ latent heat
- Γ : Decay rate of the scalar field
- ω : Equation of state parameter
- n : Dimensions of operator (-4)

Stage III: scalar field domination

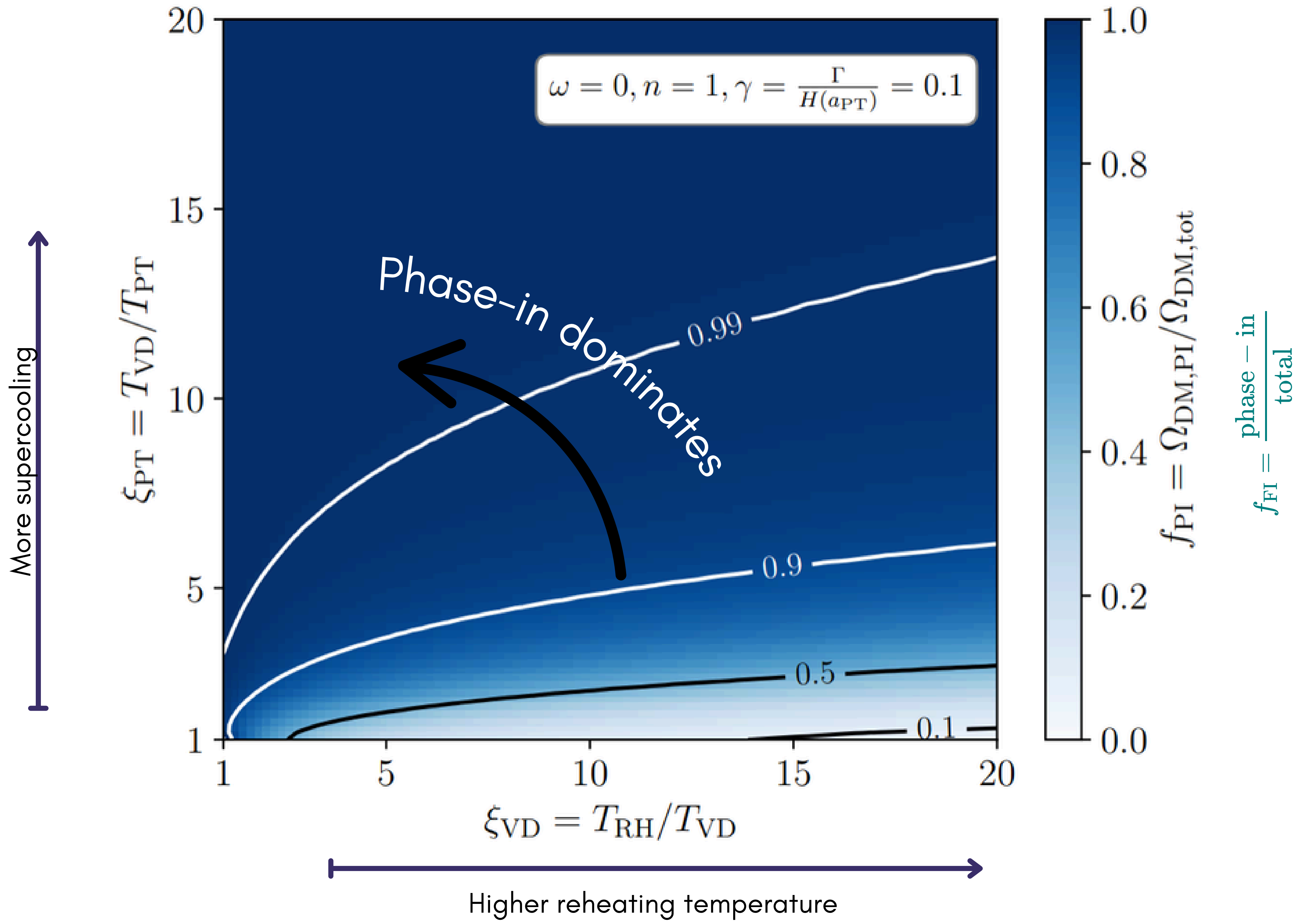
Highly inhomogeneous scalar field configurations after percolation

→ Equation of state non-trivial $\omega \neq 0$?

part I of this talk



Phase-in condition : results

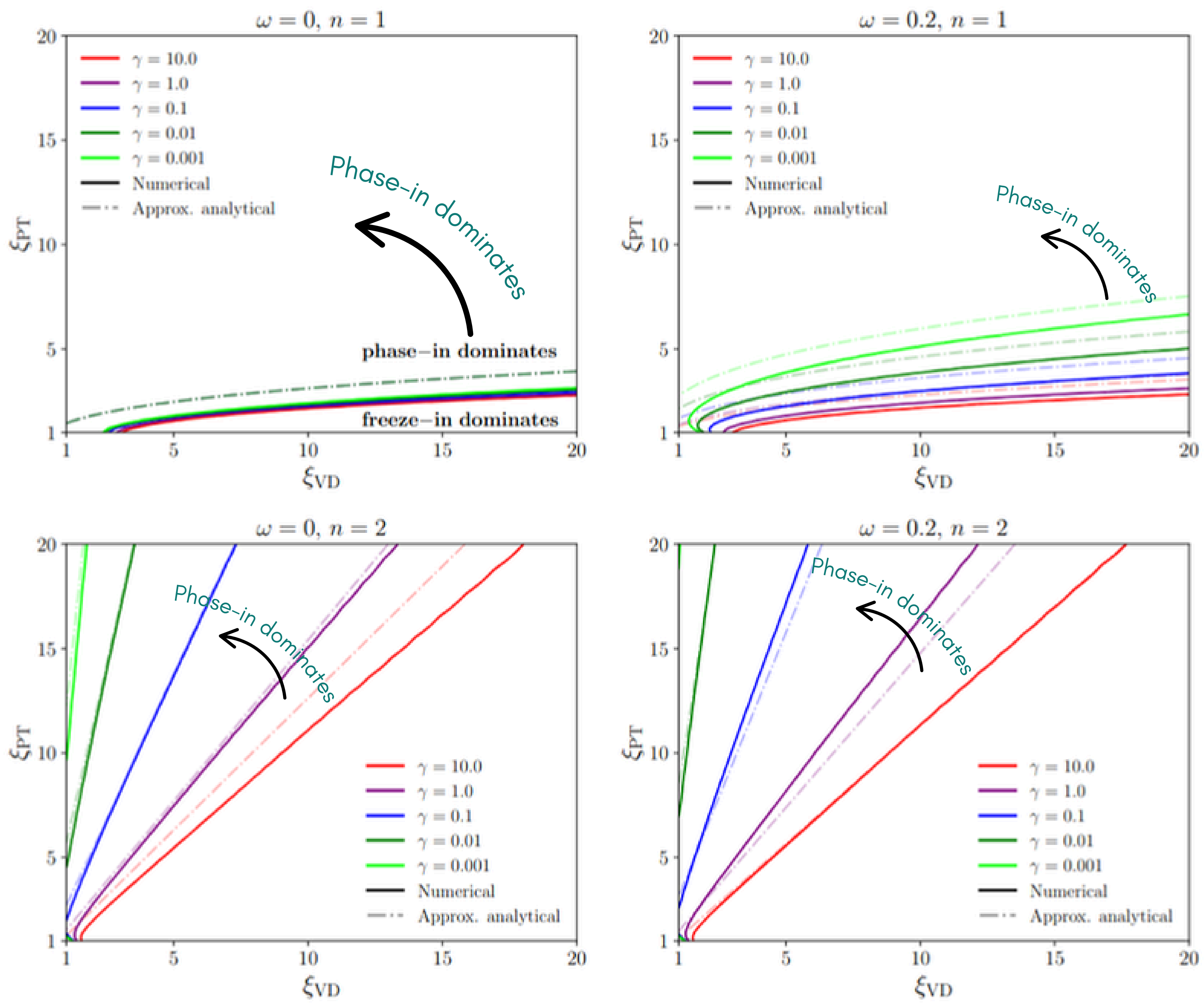


Phase-in condition : results

matter dom. | modified cosmology

Dim 5 operator

Dim 6 operator



with:

$$\xi_{PT} = \frac{T_{VD}}{T_{PT}}$$ (amount of supercooling)

$$\xi_{VD} = \frac{T_{RH}}{T_{VD}}$$ (high/low reheating temp.)

$$\gamma = \frac{\Gamma}{H(a_{\rm PT})}$$ (speed of the decay)

Phase-in is easier to achieve when the scalar field decays instantaneously.

Conclusions
(Part II)

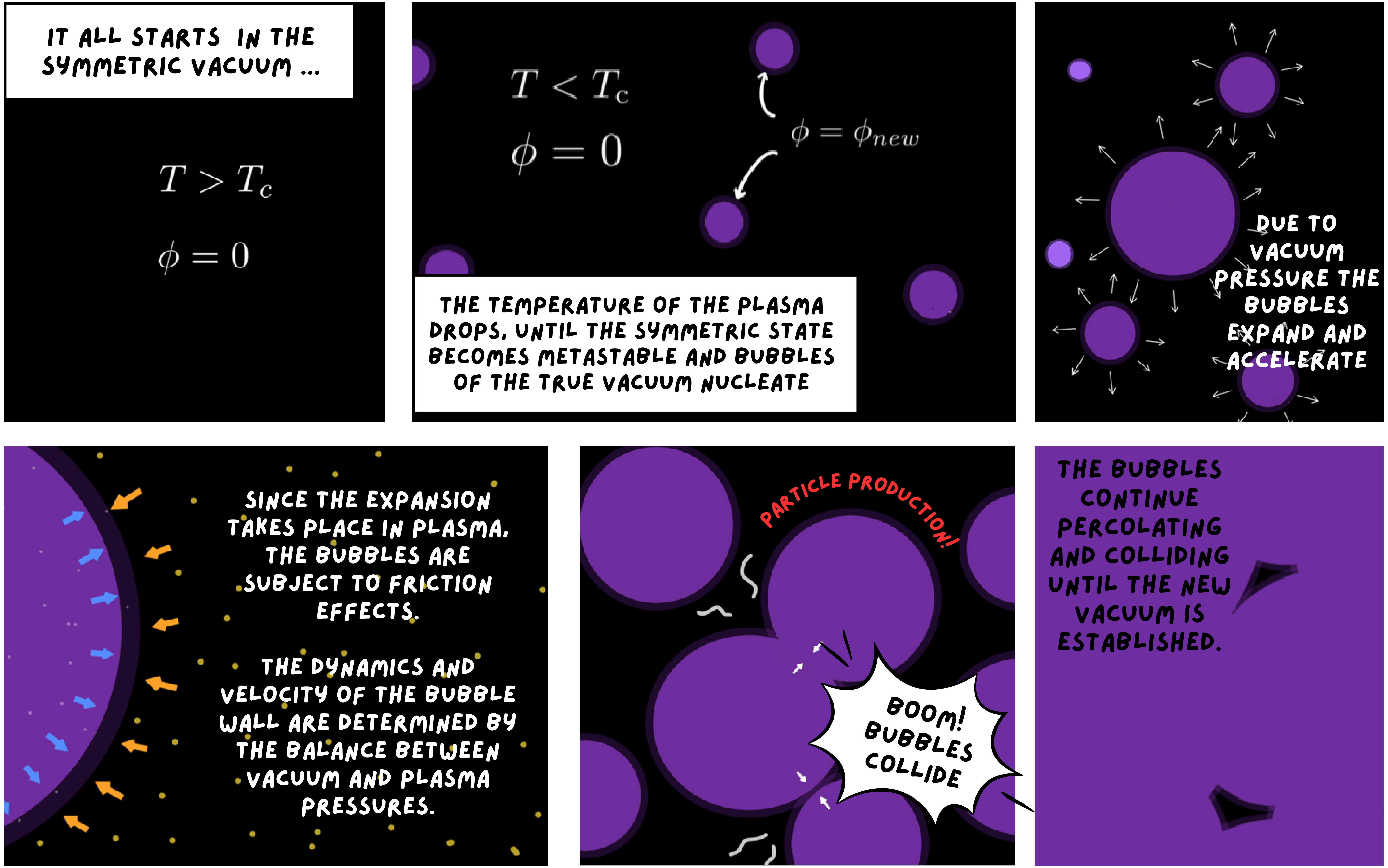
- Phase-in is feasible in many scenarios. In this case, the DM relic density becomes mostly sensitive to the temperature of the radiation after the PT and not as much to the reheating temperature.
- While the reheating temperature is challenging to determine from cosmological data, the temperature of the thermal bath after a strong cosmological 1rst order PT is more "accessible" through the expected gravitational waves background:

Peak frequency of GW signal $$f_{\text{peak}} \propto T_{\text{RD}}$$ Temperature after the PT

- Since, DM production would happen at different times in the evolution, the later produced DM could contribute via a WDM component

(more details in [2504.10593])

COSMOLOGICAL FIRST-ORDER PHASE TRANSITIONS IN A NUTSHELL



Back-up slides

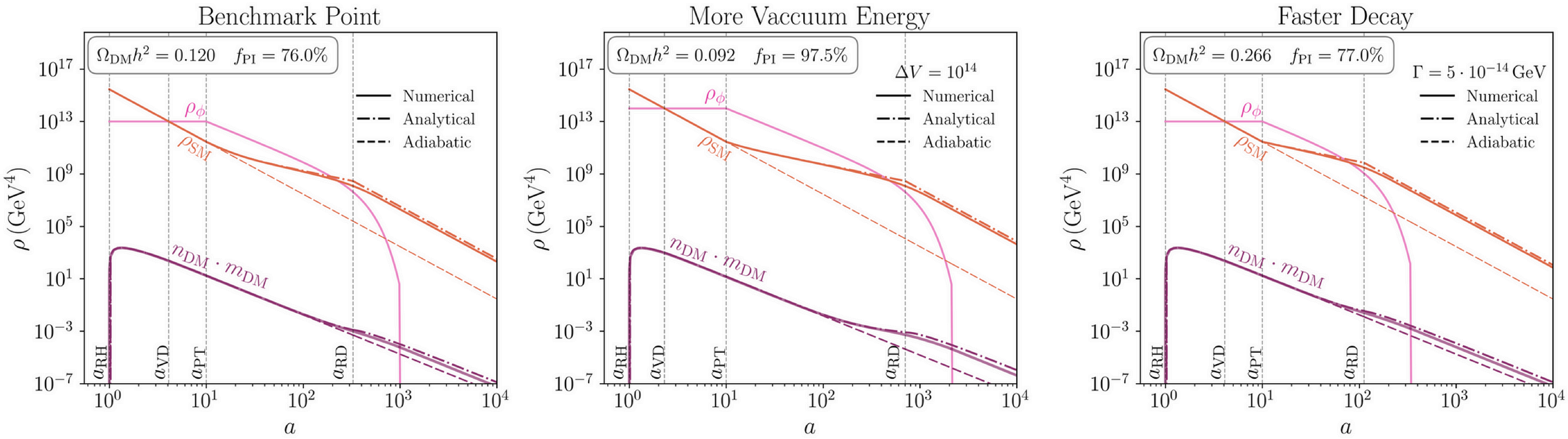
Some examples

Phase-in condition

$$r \approx T_{\rm RH}^{-2n+1} T_{\rm PT}^{-3} \Delta V^{\frac{n+1}{2}} g_\star^{-(n+1)/2} \left(\frac{\sqrt{\Delta V}}{M_{\rm Pl}\Gamma} + \sqrt{\frac{3}{8\pi}} \right)^{\frac{2}{1+w}-1-n} > 1$$

with: $r = \frac{\Omega_{\rm DM,PI}}{\Omega_{\rm DM,FI}} = \frac{\text{phase - in}}{\text{freeze - in}}$

For : $n = 1$ and $\omega = 0$ (i.e Dim 5 operator and assuming matter domination during the decay).

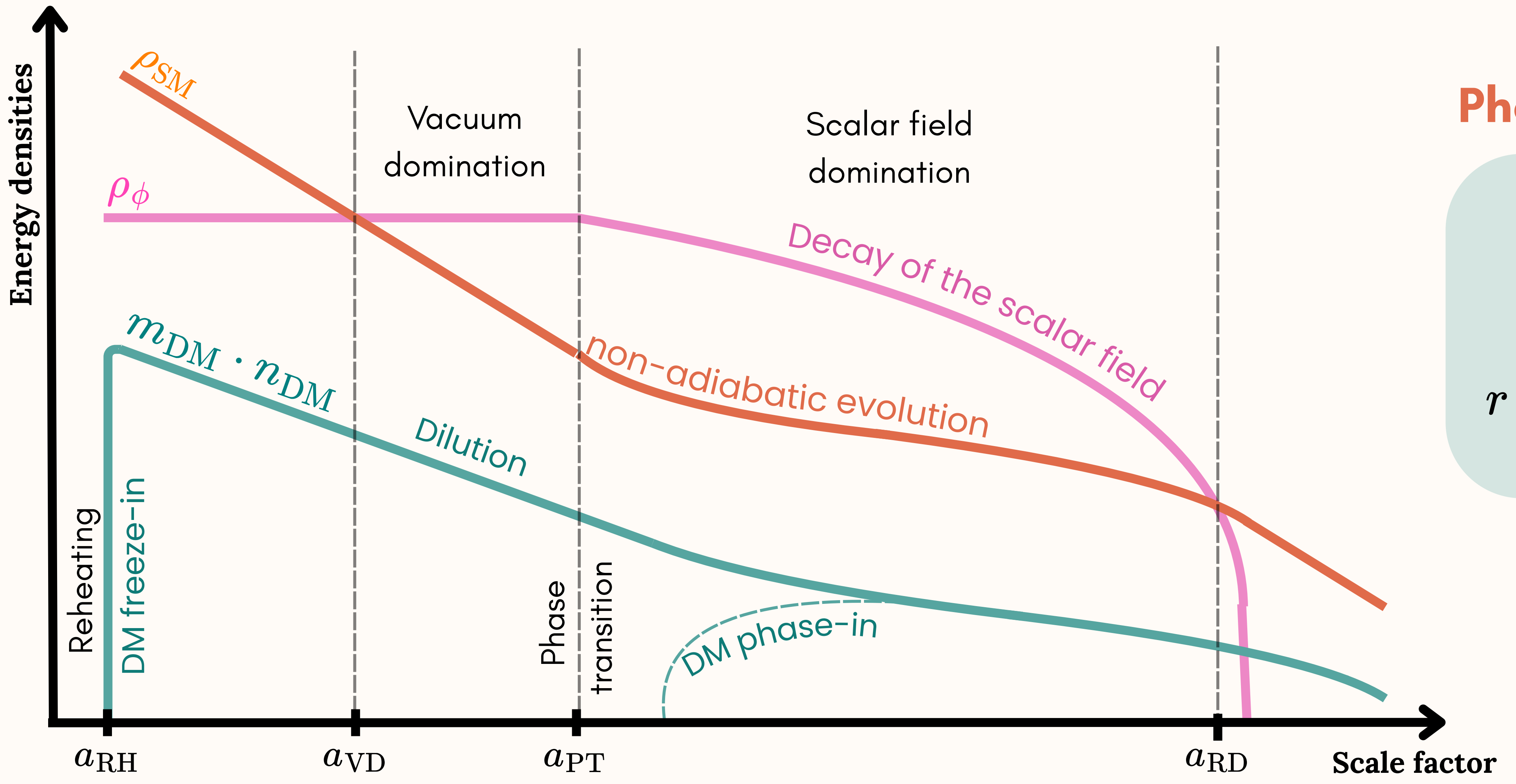


Benchmark values

$$m_{\rm DM} = 1\,\mathrm{MeV},\ T_{\rm RH} = 3 \cdot 10^3\,\mathrm{GeV}$$
$$T_{\rm PT} = 300\,\mathrm{GeV},\ \Delta V = 10^{13}\,\mathrm{GeV}^4$$
$$\Gamma = 10^{-14}\,\mathrm{GeV},\ \Lambda = 1.88 \cdot 10^{13}\,\mathrm{GeV}$$

with: $f_{\rm PI} = \frac{\Omega_{\rm DM,PI}}{\Omega_{\rm DM,tot}} = \frac{\text{phase - in}}{\text{total}}$

Phase-in condition



Phase-in condition:

$$r(T_{RH}, T_{PT}, \Delta V, \Gamma, \omega, n) > 1 \quad \text{with: } r = \frac{\Omega_{DM,PI}}{\Omega_{DM,FI}} = \frac{\text{phase}-\text{in}}{\text{freeze}-\text{in}}$$

$$r \approx T_{RH}^{-2n+1} T_{PT}^{-3} \Delta V^{\frac{n+1}{2}} g_\star^{-(n+1)/2} \left(\frac{\sqrt{\Delta V}}{M_{Pl}\Gamma} + \sqrt{\frac{3}{8\pi}} \right)^{\frac{2}{1+w}-1-n}$$

Parameters of the problem:

- T_{RH} : Reheating temperature
- T_{PT} : Phase transition temperature
- ΔV : Potential energy/ latent heat
- Γ : Decay rate of the scalar field
- ω : Equation of state parameter
- n : Dimensions of operator (-4)

Analytical estimate:

$$n_{DM}^{tot}(T) = \frac{1}{D}\left[n_{DM}^{I}(a_{VD}) \left(\frac{T}{T_{VD}}\right)^3 + n_{DM}^{II}(a_{PT}) \left(\frac{T}{T_{PT}}\right)^3 \right] + n_{DM}^{III}(a_{RD}) \left(\frac{T}{T_{RD}}\right)^3 + n_{DM}^{IV}(T)$$

Dilution factor: $D = \frac{S_{RD}}{S_{PT}} = \left(\frac{T_{RD} a_{RD}}{T_{PT} a_{PT}}\right)^3$