Modifications of Gravity vs. Dark Matter/Energy

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MARIE CURIE ACTIONS

Outline

1. Gravity
   - Problems with General Relativity
2. Galactic rotation curves
   - Statement of the problem
   - Dark matter?
   - Alternatives
3. General relativistic description
   - Motivation
   - Axially symmetric stationary solutions
   - Toy model for a galaxy

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Theoretical problems

“Standard folklore”

GR = beautiful and experimentally established
only “a few details” are missing

- No comprehensive theory of quantum gravity
- Several experimental “anomalies”

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Experimental anomalies

- Dark Energy
- Dark Matter
- Pioneer anomaly
- other anomalies?

Note intriguing coincidences:

$$\underbrace{a_\Lambda}_{\text{Dark Energy}} \approx \underbrace{a_{\text{MOND}}}_{\text{Dark Matter}} \approx \underbrace{a_{\text{P}}}_{\text{Pioneer anomaly}} \approx \underbrace{a_{\text{H}}}_{\text{Hubble}} \approx 10^{-9} - 10^{-10} m/s^2$$

in natural units: $a \approx 10^{-61} - 10^{-62}$

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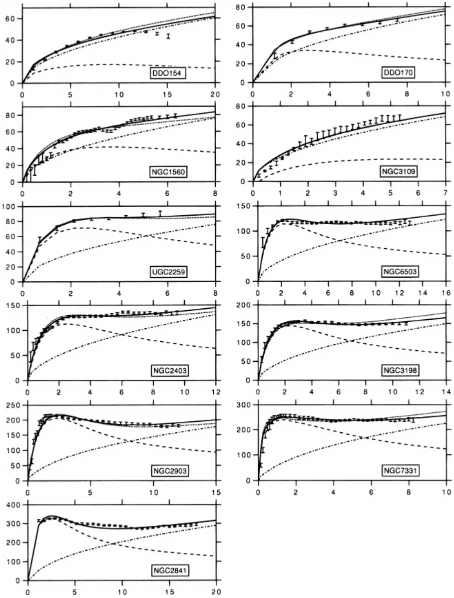

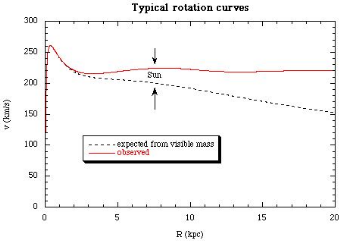

Rotation curve of our Galaxy

Inconsistent with Newton!

(or Kepler...)

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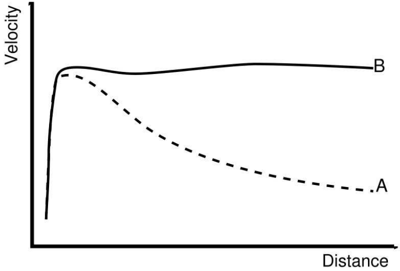
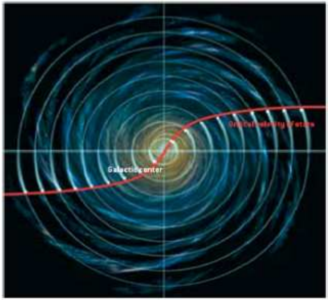
Sketch of experimental data

A typical non-Keplerian galactic rotation curve





- Curve A: Newtonian prediction
- Curve B: Observed velocity profile

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Newtonian calculation

$$\underbrace{v^2}_{\text{kinetic}} \propto \underbrace{\frac{\int \rho dV}{r}}_{\text{potential}} \propto \frac{M}{r}$$

- Regime with $\rho = \text{const.}$: $v \propto r$
- Regime with $M = \text{const.}$: $v \propto 1/\sqrt{r}$

Conclusion

Flat rotation curves not described well by Newton

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Saving Newton

- Postulate existence of Dark Matter
- Fit Dark Matter density as to “explain” rotation curves

Note: other hints for Dark Matter, e.g. gravitational lensing!

Exciting prospect for near future

Dark Matter might be discovered next year at LHC!

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$$\vec{F} = m\vec{a} \cdot \mu(a/a_{\mathrm{MOND}})$$

with

- $\mu(x) \to 1$ for $x \gg 1$
- $\mu(x) \to x$ for $x \ll 1$
- critical acceleration: $a_{\mathrm{MOND}} \approx 10^{-10} m/s^2$

Phenomenologically rather successful, but no deeper theoretical understanding

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Main issues

Two important questions:

- Do we have the correct theory of gravity?
- Are we applying it correctly?

Possible answers:

- Regarding 1: e.g. MOND, IR modifications of GR, Yukawa-type corrections, ...
- Regarding 2: Newtonian limit justified?

Consider the second point

Study appropriate General Relativistic (exact) solutions

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Validity of Newtonian approximation

- Locally: can use Newton almost everywhere in galaxy (except for central galactic Black Hole region and near jet axis)
- But: GR is a non-linear theory!
- Not granted that Newton is applicable globally

Thus, use GR instead of Newton!

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The Cooperstock-Tieu attempt

Ansatz:

- Axial symmetry
- Reflection symmetry
- Stationarity
- Pressurless perfect fluid sources
- Corotating perfect fluid
- Weak field limit

Claim of Cooperstock-Tieu:

GR differs essentially from Newton
However, technically incorrect!

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Drop weak field limit, but keep rest:

- Axial symmetry $\rightarrow$ spacelike Killing
- Reflection symmetry (around galactic plane)
- Stationarity $\rightarrow$ timelike Killing $\xi^a = (\partial_t)^a$
- Pressurless perfect fluid sources $\rightarrow T^{ab} = \rho u^a u^b$
- Corotating perfect fluid $\rightarrow u^a = (u^0(r,z), 0, 0, 0)$

Consequently, line element can be brought into adapted form:

$$ds^2 = -(dt - N d\phi)^2 + r^2 d\phi^2 + \exp(\nu)(dr^2 + dz^2)$$

functions ρ, N, ν depend solely on r, z

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Einstein equations

$$r\nu_z + N_r N_z = 0\,, \tag{1}$$

$$2r\nu_r + N_r^2 - N_z^2 = 0\,, \tag{2}$$

$$\nu_{rr} + \nu_{zz} + \frac{1}{2r^2}\left(N_r^2 + N_z^2\right) = 0\,, \tag{3}$$

$$N_{rr} - \frac{1}{r}N_r + N_{zz} = 0\,, \tag{4}$$

$$\frac{1}{r^2}\left(N_r^2 + N_z^2\right) = \kappa\rho e^{\nu}\,. \tag{5}$$

Important notes:

- (4) is a *linear* PDE for N!
- For known N (1)-(3) yield ν (integration constant!)
- For known N, ν (5) establishes ρ

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Physical meaning of N:

$$V(r,z) = \frac{N(r,z)}{r}$$

V is 3-velocity as seen by asymptotic observer at rest with respect to center of galaxy

"Inverse problem":

- Take V as experimental input (rotation curve)
- Calculate mass density ρ
- Compare ρ with experimental data
- Compare with Newtonian prediction for ρ

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Boundary conditions and asymptotics

Solution mathematically trivial (separation Ansatz)
Physical input:

- Modes bounded for $|z| \to \infty$
- Modes bounded for $r \to \infty$
- No exotic δ-sources in galactic plane!

General solution for $r > 0$:

$$N(r,z) = r^2 \int_0^\infty dx C(x) \sum_\pm ((z \pm x)^2 + r^2)^{-3/2}$$

Remaining task:

Choose spectral density $C(x)$ appropriately!

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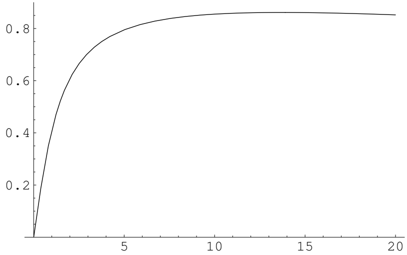
Choice of spectral density

Take simple 3-parameter family of $C(x)$:

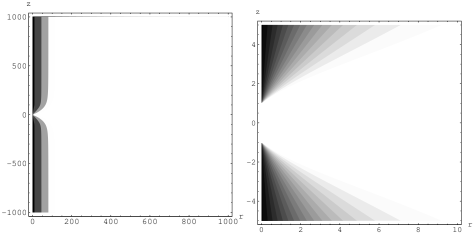
- Parameter V_0: flat region velocity (about $200 km/s$)
- Parameter r_0: bulge radius (about $1 kpc$)
- Parameter R: choose about $100 kpc$

With these choices $V(r,0)$ looks as follows:



Crucial observation:

Solutions (necessarily) singular close to axis $r = 0$ for $|z| \geq r_0$



Cut out this region and paste there a different solution of GR

Solutions are *not* asymptotically flat!

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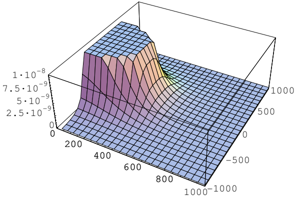
Prediction for mass density

Mass density

$$\rho = \underbrace{\beta}_{\text{int. const.}} \frac{N_r^2 + N_z^2}{r^2}$$

on large scales:



Integrating ρ over "white" region yields $\approx 10^{11}$ solar masses

Comparison with Newton

For any given velocity profile V Newton predicts:

$$\rho_{\text{Newton}} \propto \frac{V^2 + 2rVV'}{r^2}$$

Ratio of GR/Newton:

$$\frac{\rho}{\rho_{\text{Newton}}} = \tilde{\beta}\left(1 + \frac{r^2(V')^2}{V^2 + 2rVV'}\right)$$

Assume a spectral density which yields a linear regime $V \propto r$ and a flat regime $V = \text{const.}$

Important remaining task:

Have to fix free constant $\tilde{\beta}$!

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Fix $\tilde{\beta}$ such that

Newton and GR coincide in the linear regime $V \propto r$

- This yields $\tilde{\beta} = 3/4$.
- Calculate the ratio in the flat regime $V = \text{const.}$

$$\frac{\rho}{\rho_{\text{Newton}}}\Big|_{\text{flat}} = \tilde{\beta} = \frac{3}{4}$$

Conclusion

Newton predicts 133% of the mass density as compared to GR

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- Problem of flat galactic rotation curves
- May be solved by Dark Matter
- *GR attempts to avoid Dark Matter have failed*
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  - Perform patching
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Literature I

- C. Lammerzahl, O. Preuss and H. Dittus, "Is the physics within the Solar system really understood?," gr-qc/0604052.
- F. I. Cooperstock and S. Tieu, astro-ph/0507619.
- M. Korzynski, astro-ph/0508377; D. Vogt and P. S. Letelier, astro-ph/0510750.
- H. Balasin and D. Grumiller, astro-ph/0602519.

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