Little bang, big bang and gravity duals

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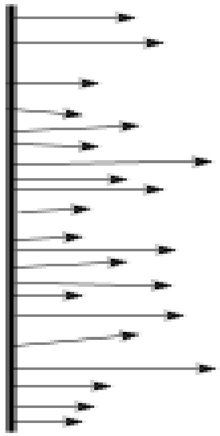
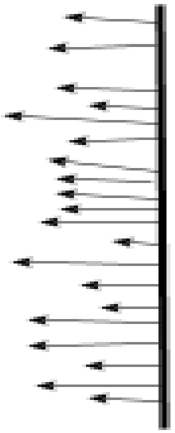
Work with T. Tahkokallio

Janik, Peschanski; Nakamura, Sin, Kim

Pre-little bang



cloud of gluons

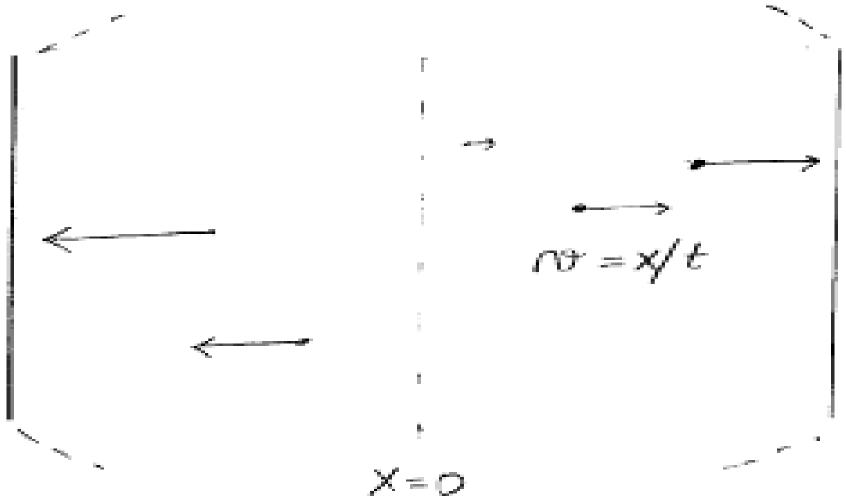


cloud of gluons

$$x = \frac{p_{\text{gluon}}}{p_{\text{nucleus}}/A} \sim \frac{1\,\text{GeV}}{1000\,\text{GeV}} = 10^{-3}$$

"small-x physics"

Little bang, $\eta_{\mu\nu} = (-1, 1, 1, 1)$:



1+1d similarity flow in 1+3 dimensions, $x^\mu = (t, x)$, $\tau = \sqrt{t^2 - x^2}$:

$$v_x(t,x,y,z) = v(t,x) = \frac{x}{t} \qquad v_y = v_z = 0, \qquad u^\mu = (\gamma, \gamma v) = \frac{x^\mu}{\tau}$$

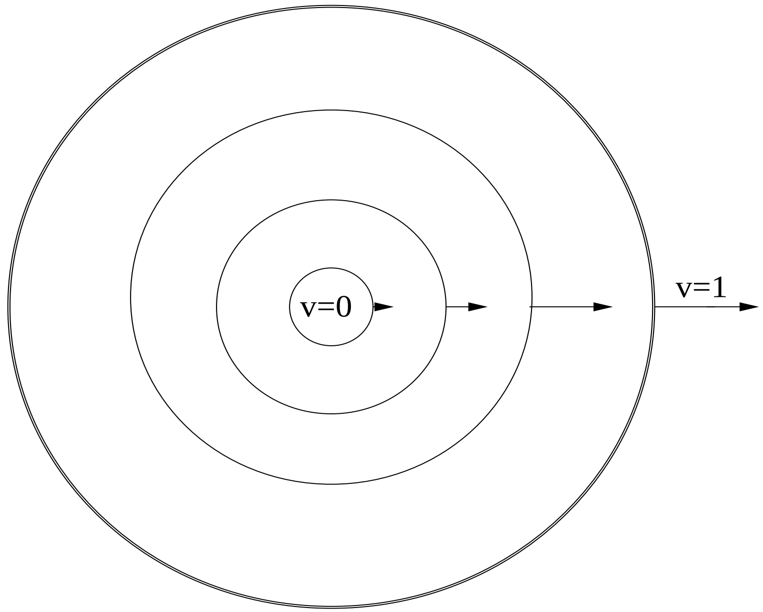
$$\epsilon'(\tau) + \frac{1}{\tau}(\epsilon \underbrace{+p}_{pdV\ \text{work}}) - \frac{4}{3}\frac{\eta}{\tau^2} = 0 \ \Rightarrow \ \epsilon(\tau) = \frac{\epsilon_0}{\tau^{4/3}}$$

"strongly interacting quark-gluon plasma" Gravity dual = ?

Spherical bang, $\eta_{\mu\nu} = (-1, 1, ..., 1)$: matter expands

$T_{\mu\nu} = (\epsilon + p)\frac{x_\mu x_\nu}{\tau^2} + p g_{\mu\nu}$

Fixed time t:



Similarity flow in $1+3$: $\mathbf{v} = \frac{\mathbf{x}}{t}\theta(t - |\mathbf{x}|)$, $u^\mu = (\gamma, \gamma\mathbf{v}) = \frac{x^\mu}{\tau}$, $\tau = \sqrt{t^2 - \mathbf{x}^2}$

$$\epsilon'(\tau) + \frac{3}{\tau}(\epsilon + p) = 0 \underset{p=\epsilon/3}{\Rightarrow} \quad \epsilon(\tau) = \frac{\epsilon_0}{\tau^4} = \frac{\epsilon_0}{(t^2 - \mathbf{x}^2)^2}$$

Big bang, $g_{\mu\nu} = (-1, r(t), r(t), r(t))$: space expands

$$\epsilon'(t) + \frac{3\,\dot{r}(t)}{r(t)}(\epsilon + p) = 0 \qquad r(t) = \frac{t}{t_0} \text{ gives the above}$$

What is a gravity dual?

and

What is the gravity dual of these expanding systems?

and

What is the use of them?

4d gauge field theory ⇔ 5d classical gravity

$$\langle \exp\left[\int d^4x\, O(x)\phi(x,0)\right]\rangle_{\rm FT} = \exp\left\{-\int d^4x \int_0^{z_0} dz\, \mathcal{L}_{\rm class}[\phi(x,z)]\right\}$$

$$x^\mu = (t, x^1, x^2, x^3) \qquad\qquad x^M = (t, x^1, x^2, x^3, z)$$

$$\frac{\delta^2 \mathrm{LHS}}{\delta\phi(x,0)\delta\phi(y,0)} = \langle O(x) O(y)\rangle_{\rm FT}$$

Simplest case: Hot SYM matter in thermal equilibrium,

$$p = \frac{3}{4}\frac{\pi^2 N_c^2}{6} T^4 = \frac{1}{3}\epsilon \qquad\qquad \frac{\eta}{s} = \frac{\hbar}{4\pi}$$

Gravity dual of hot SYM matter: AdS_5 black hole

$$\text{4d Black Hole :} \quad ds^2 = -\left(1-\frac{r_s}{r}\right)dt^2 + \frac{1}{1-r_s/r}dr^2 + r^2 d\Omega^2$$

$$\text{Solves} \quad R_{\mu\nu} = 0, \quad T_{\text{Hawk}} = \frac{1}{4\pi r_s}$$

$$\text{5d AdS} = \frac{\mathcal{L}^2}{z^2}(-dt^2 + d\mathbf{x}^2 + dz^2) \quad z = 0 \text{ is } \textbf{boundary}, \; z > 0 \text{ is } \textbf{bulk}$$

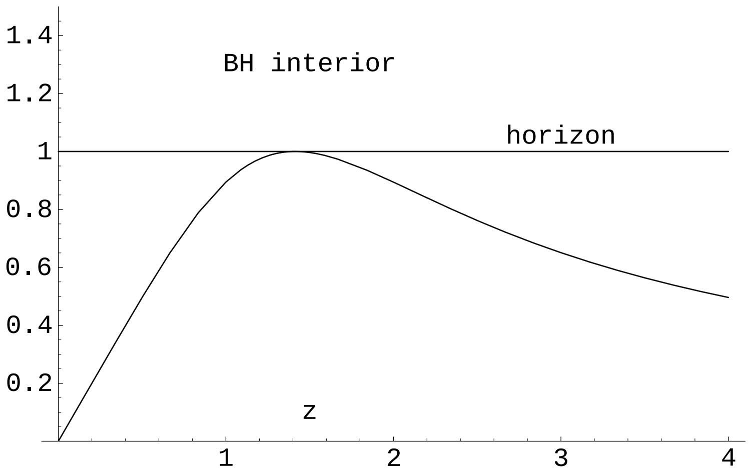
$$\text{5d AdS with BH in bulk} = \frac{\mathcal{L}^2}{z^2}\left[-\left(1-\frac{z^4}{z_0^4}\right)dt^2 + d\mathbf{x}^2 + \frac{dz^2}{1-z^4/z_0^4}\right] \quad T_{\text{Hawk}} = \frac{1}{\pi z_0}$$

$$\text{Both solve} \quad R_{MN} - \frac{1}{2}R g_{MN} = \frac{6}{\mathcal{L}^2} g_{MN} \sim T_{MN}^{\text{bulk}} \qquad \text{NO } T_{MN}^{\text{brane}}!$$

Pochinski, cosmicvariance.com/2006/12/07/: Physicists have found that some of the properties of this plasma are better modeled (via duality) as a tiny black hole in a space with extra dimensions than as the expected clump of elementary particles in the usual four dimensions of spacetime.

$$ds^2 = \frac{\mathcal{L}^2}{\tilde{z}^2}\left[-(1-\frac{\tilde{z}^4}{z_0^4})dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + \frac{1}{1-\tilde{z}^4/z_0^4}d\tilde{z}^2\right]$$

Transform $\tilde{z}^2 = z^2/(1+z^4/4z_0^4)$:



$$ds^2 = \frac{\mathcal{L}^2}{z^2}\left[-\frac{(1-z^4/(4z_0^4))^2}{1+z^4/(4z_0^4)}dt^2 + \left(1+\frac{z^4}{4z_0^4}\right)(dx_1^2 + dx_2^2 + dx_3^2) + dz^2\right]$$

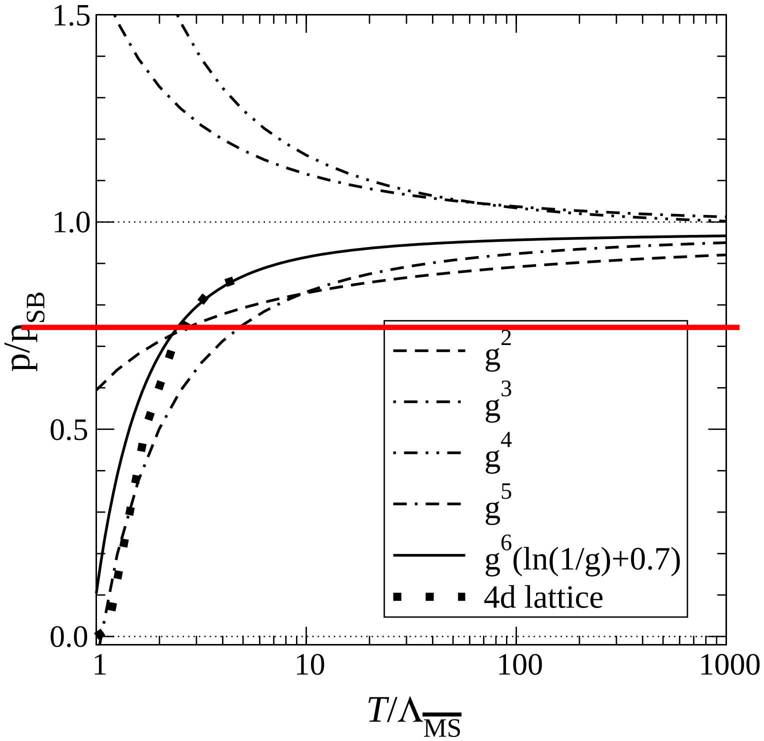
$$\equiv \frac{\mathcal{L}^2}{z^2}\left\{\left[g_{\mu\nu}(x,0) + \underbrace{g_{\mu\nu}^{(4)}(x)}_{\sim T_{\mu\nu}}z^4 + \ldots\right]dx^\mu dx^\nu + dz^2\right\}$$

Static boundary energy momentum tensor

$$T_{\mu\nu} = \frac{\mathcal{L}^3}{4\pi G_5} g^{(4)}_{\mu\nu} = \begin{pmatrix} 3aT^4 & 0 & 0 & 0 \\ 0 & aT^4 & 0 & 0 \\ 0 & 0 & aT^4 & 0 \\ 0 & 0 & 0 & aT^4 \end{pmatrix} \qquad a = \frac{\pi^2 N_c^2}{8}$$

Counting $N=4$ SYM dofs, 8 adjoint bosons, 8 fermions: $p_{\text{ideal}} = (N_c^2-1)\frac{\pi^2}{6}T^4$.

The famous factor 3/4



Gravity dual of spherical similarity expansion would be a time dependent solution of

$$R_{MN} + 4g_{MN} = 0$$

with the symmetries (coordinates $= t, r, \theta, \phi, z$)

$$ds^2 = \frac{\mathcal{L}^2}{z^2}\left[-a(t,r,z)dt^2 + b(t,r,z)dt\,dr + c(t,r,z)dr^2 + g(t,r,z)d\Omega_2^2 + dz^2\right],$$

which expanded near boundary

$$g_{\mu\nu}(x,z) = \eta_{\mu\nu} + ... + g^{(4)}_{\mu\nu}(x)z^4 + ...$$

leads to $T_{\mu\nu} = (\epsilon + p)x_\mu x_\nu/\tau^2 + pg_{\mu\nu}$, $\epsilon = 3p = 3aT^4(t)$:

$$T_{\mu\nu} = \frac{\mathcal{L}^3}{4\pi G_5} g^{(4)}_{\mu\nu} = \begin{pmatrix} 4p\frac{t^2}{\tau^2} - p & 4p\frac{tr}{\tau^2} & 0 & 0 \\ 4p\frac{tr}{\tau^2} & 4p\frac{r^2}{\tau^2} + p & 0 & 0 \\ 0 & 0 & r^2p & 0 \\ 0 & 0 & 0 & r^2p\sin^2\theta \end{pmatrix}$$

Can "only" do 1+1 dim or $r = 0$ in 1+3d, 2 functions, 2 variables:

$$ds^2 = \frac{\mathcal{L}^2}{z^2}[-a(t,z)dt^2 + b(t,z)d\mathbf{x}^2 + dz^2]$$

1+1 dimension

$$ds^2 = \frac{1}{z^2}\left[-(1-\frac{z^2}{z_0^2\tau^2})^2 d\tau^2 + (1+\frac{z^2}{z_0^2\tau^2})^2\tau^2 d\eta^2 + dz^2\right]$$

AdS_3 black hole with moving horizon at $z_H = z_0\tau$.

Boundary metric could be chosen to be Minkowski

1+3 dimensions: $r = r(t)$

$$ds^2 = [-a(t,z)dt^2 + b(t,z)d\mathbf{x}^2 + dz^2]/z^2, \quad R_{MN} + 4g_{MN} = 0$$

$$a(t,z) = \frac{\left[\left(1 - \frac{r''}{4r}z^2\right)^2 - \left(\frac{r''}{4r} - \frac{r'^2}{4r^2}\right)^2 z^4 - \frac{1}{4r^4 z_0^4} z^4\right]^2}{\left[\left(1 - \frac{r'^2}{4r^2}\, z^2\right)^2 + \frac{1}{4r^4 z_0^4} z^4\right]} \xrightarrow[r(t)=1]{} \frac{(1 - z^4/(4z_0^4))^2}{1 + z^4/(4z_0^4)}$$

$$b(t,z) = r^2 \left[\left(1 - \frac{r'^2}{4r^2}\, z^2\right)^2 + \frac{1}{4r^4 z_0^4} z^4\right] \xrightarrow[r(t)=1]{} 1 + \frac{z^4}{4z_0^4}$$

Boundary metric and $T_{\mu\nu}$: $\quad g_{\mu\nu}(x,0) = (-1, r^2(t), r^2(t), r^2(t)) =$ flat FRW

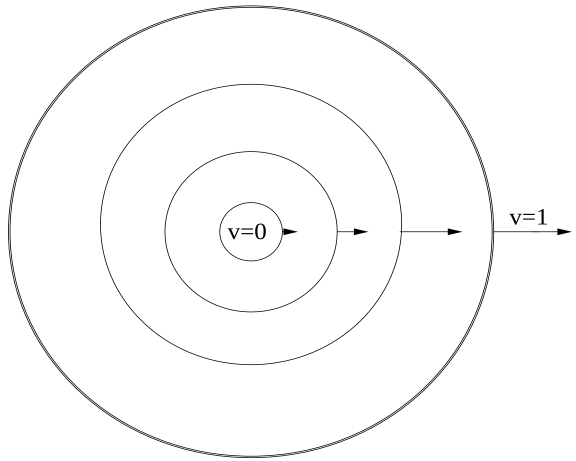
$$T_{tt} = \epsilon(t) = \frac{3N_c^2}{8\pi^2}\left(\frac{1}{z_0^4 r^4} + \frac{r'^4}{4r^4}\right) \sim \frac{T_0^4}{r^4(t)} + \frac{1}{t^4} \sim \underbrace{T^4(t)}_{\text{radiation}} + \underbrace{t^{-4}}_{\text{curvature}},$$

$$T_1^1(t) = \frac{1}{3}\,\epsilon(t) - \underbrace{\frac{N_c^2}{8\pi^2}\frac{r'^2 r''}{r^3}}_{\text{trace anomaly}/3}$$

Brane gravity: Introduce 4d Einstein gravity $\Rightarrow\ r(t)$.

$$\text{Choosing} \qquad r(t) = \frac{t}{t_0} \quad \Rightarrow \epsilon(t) = \frac{\epsilon_0}{t^4}$$

we have gravity dual of matter in the center of spherical bang:



Thermalisation condition: temperature can be defined for

$$t_0/z_0 = \pi T t_0 > 1 = \hbar$$

We are still missing flow with shear $\Rightarrow$ shear viscosity η.

Conclusions

- For hot equilibrium SYM systems gauge/gravity duality has made predictions (pressure, shear viscosity, quark energy loss) in qualitative agreement with experimental results on hot QCD matter
- For time dependent systems with flow, in only local thermal equilibrium, we do not (yet?) have the gravity dual for non-trivial configurations (1+1d or rest frame of 1+3d is very special)
- Brane cosmology is an example of what one can do in this set-up for a larger system: the whole universe