The origin of primordial non-gaussianity

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- **How may we compute the non-gaussianity?**

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- Constant value $\zeta(\mathbf{x})$ at $T \sim 10\,$keV provides the initial condition for adiabatic perturbations.

The correlators

Spectrum $\mathcal{P}_\zeta$, bispectrum† $f_{\rm NL}$, trispectrum†† $\tau_{\rm NL}$:

$$\begin{aligned}
\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle &= (2\pi)^3 \delta(\mathbf{k}+\mathbf{k}') K_1 \mathcal{P}_\zeta \\
\frac{5}{3} \langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \zeta_{\mathbf{k}''} \rangle &= (2\pi)^3 \delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}'') K_2 \mathcal{P}_\zeta^2 f_{\rm NL} \\
\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \zeta_{\mathbf{k}''} \zeta_{\mathbf{k}'''} \rangle_{\rm c} &= (2\pi)^3 \delta(\mathbf{k}+\mathbf{k}'+\mathbf{k}''+\mathbf{k}''') K_3 \mathcal{P}_\zeta^3 \tau_{\rm NL}
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where the kinematic factors depend on the wave-vectors:

$$\begin{aligned}K_1 &\equiv 2\pi^2/k^3 \\ K_2 &\equiv K_1(k)K_1(k') + \text{ 5perms} \\ K_3 &\equiv K_2K_1(|\mathbf{k}+\mathbf{k}''|) + 23\text{perms}\end{aligned}$$

† Komatsu/Spergel 2000; Maldacena 2003 †† Boubekeur/DHL 2005

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\begin{aligned}
\zeta(\mathbf{x}, t) &= N_i(\phi_i(\mathbf{x}), \rho(t)) - N(\phi_i, \rho(t)) \\
&= N_i(t)\delta\phi_i(\mathbf{x}) + \frac{1}{2}N_{ij}(t)\delta\phi_i(\mathbf{x})\delta\phi_j(\mathbf{x}) + \cdots
\end{aligned}
$$

DHL, Malik & Sasaki 05; DHL & Rodriguez 05 (non-perturbative)

Calculating f_{NL}

Simplest case

$$\zeta(\mathbf{x}) = b\phi(\mathbf{x}) + \left(\phi(\mathbf{x}) - \bar{\phi}\right)^2$$

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&= \frac{b^2}{(2\pi)^3} \int \langle \phi_{\mathbf{k}_1} \phi_{\mathbf{k}_2} \phi_{\mathbf{q}} \phi_{\mathbf{k}_3 - \mathbf{q}} \rangle d^3q + \text{ cyclic} \\
&= \frac{b^2}{(2\pi)^3} \int \langle \phi_{\mathbf{k}_1} \phi_{\mathbf{q}} \rangle \langle \phi_{\mathbf{k}_2} \phi_{\mathbf{k}_3 - \mathbf{q}} \rangle d^3q + \text{ 5 perms.}
\end{aligned}$$

Gives $\boxed{(3/5) f_{\rm NL} = 1/b^2}$

Loop correction

Include quadratic term of ζ

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'} \rangle = b^2 \langle \phi_{\mathbf{k}} \phi_{\mathbf{k}'} \rangle + \langle \left[(\phi - \bar{\phi})^2 \right]_{\mathbf{k}} \left[(\phi - \bar{\phi})^2 \right]_{\mathbf{k}'} \rangle$$

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Infared divergent, use box size L then

$$\boxed{\mathcal{P}_\zeta = b^2 \mathcal{P}_\phi + 4 \mathcal{P}_\phi^2 \ln(kL)}$$

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Now go to box size $M \ll L$

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But $\overline{(\bar{\phi}_M - \bar{\phi})^2} = \mathcal{P}_\phi \ln L/M$ hence $\boxed{\overline{\mathcal{P}_{\zeta\, M}} = \mathcal{P}_\zeta}$

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so $\partial N/\partial\phi = M_{\rm P}^{-2} V/V'$ giving

$$\zeta = \left(\frac{\partial N}{\partial \phi}\delta\phi\right) + \left(\epsilon - \frac{1}{2}\eta\right)\left(\frac{\partial N}{\partial \phi}\delta\phi\right)^2$$

where $\epsilon \equiv (M_{\rm P}^2/2)(V'/V)^2$ and $\eta \equiv M_{\rm P}^2 V''/V$

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Keeping just tree level, total $f_{\rm NL}$ is [with $0 < y < 5/6$]

$$\boxed{\frac{3}{5} f_{\rm NL} = -\frac{1}{4} \left[n - 1 + (r/8) y(k_1, k_2, k_3) \right] \sim \pm 10^{-2}}$$

Maldacena 2003; Seery/Lidsey 2005

The curvaton model

Mollerach 90; Linde/Mukhanov 96; DHL/Wands 01; Moroi/Takahashi 2001

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- Curvaton model is not an epicycal!
  - Candidate sRHν discovered serendiptiously

    (Hamaguchi/Murayama/Yanagida 01)

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- If $\sigma_{\rm os}$ linear and $\Omega_\sigma \ll 1$ need $\Omega_\sigma \gtrsim 10^{-2}$
- If $\sigma_{\rm os}$ linear and $\Omega_\sigma \simeq 1$ then $f_{\rm NL} = -\frac{4}{5}$

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    - Exception ??: single-component inflation