Localization on the Landscape

D.Podolsky, K. Enqvist
Helsinki Institute of Physics

1. Landscape of vacua in string theory. Tunelling between vacua. Classification of parts of the landscape.
2. Stochastic inflation. Langevin and Fokker-Planck.
3. Anderson localization in disordered systems (insulators, glasses).
4. Localization and eternal inflation on the landscape. Recalling the initial state.

The landscape of vacua of string theory

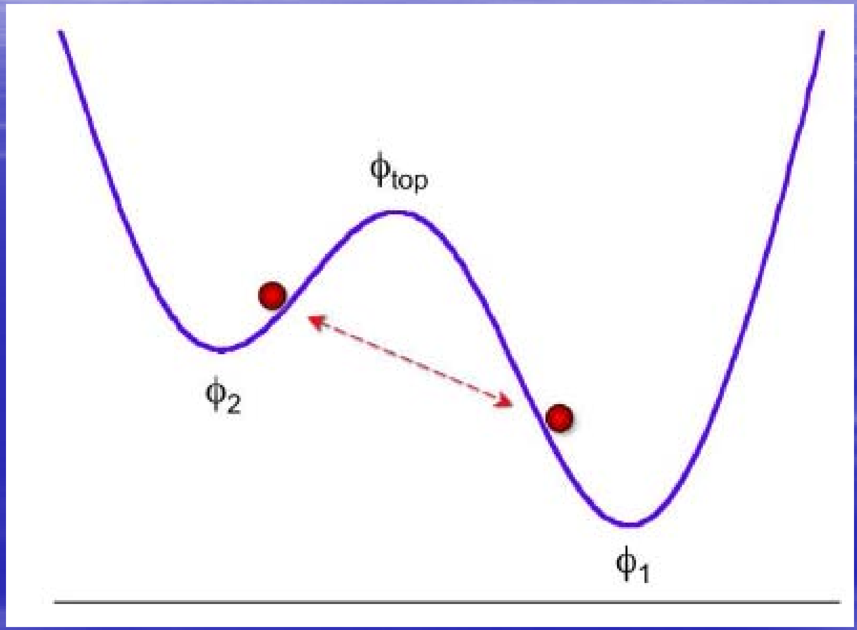
- Metastable vacua with positive effective cosmological constant (dS)
- True vacua with vanishing (Dine-Seiberg-Minkowski) or negative cosmological constant (AdS)
- The overall number is $10^{100} \div 10^{1000}$
- Possibility to tunnel from one vacuum to another
- AdS vacua are "sinks" – the bubbles of collapsing spacetime

Already the statistical problem of counting vacua on the landscape (or calculating distribution functions of vacua) is very complicated (NP hard). However, we want more than that - to understand **the dynamics** of fields on the landscape, in particular - how eternal inflation is realized in this setup.

Tunelling between dS vacua on the landscape 1

Suppose we have a single inflaton field. The tunneling rate between two ajacent vacua is defined by the action on the corresponding Hawking-Moss instanton:

$$\Gamma_{12} = e^{-S_{\rm top}+S_1} = \exp\left(-\frac{24\pi^2}{V(\phi_1)} + \frac{24\pi^2}{V(\phi_{\rm top})}\right)$$



Both tunnelings 1->2 and 2->1 are possible, so in the system of two minima

$$\frac{P_2}{P_1} = e^{-S_2+S_1} = \exp\left(-\frac{24\pi^2}{V_1} + \frac{24\pi^2}{V_2}\right)$$

For an arbitrary number of dS vacua (and AdS sinks) one has the ”vacuum dynamics” equations:

$$\frac{dP_i}{dt} = -\sum_{j\neq i}\Gamma_{ij}P_i + \sum_{j\neq i}\Gamma_{ji}P_j - \Gamma_{is}P_i$$

Tunneling between dS vacua on the landscape 2

The number of equations is $10^{100} \div 10^{1000}$!

If we are interested in time scales $t_{\rm AdS} \gg t \gg t_\tau \gg t_{\rm tunnel}$
then physical answers are given by the average over disorder on the landscape.

Classifying parts of the landscape according to Hausdorff dimension of the graph: quasi-one-dimensional, quasi-two-dimensional, etc.

Quasi-one-dimensional: two nearest neigbors

$$\partial_t P_i = -\Gamma_{i,i+1} P_i + \Gamma_{i+1,i} P_{i+1} - \Gamma_{i,i-1} P_i + \Gamma_{i-1,i} P_{i-1}$$

Let us supply this system with delta-function-like initial conditions:

$$P_i(0) = 1, \;\; P_{j\neq i}(0) = 0$$

Then, old result from the theory of diffusion on random lattices says that

the probability distribution spreads out with time much slower then diffusively:

$$\langle n^2(t) \rangle \sim \log^4 t$$

Stochastic inflation and selfreproducing universe 1

To understand what happens physically, let us use continuous model of the landscape. Suppose that the overall potential for the inflaton is

$$V(\phi) = V_0 + \delta V(\phi)$$ where $$|\delta V(\phi)| \ll V_0$$

The VEV of the inflaton in a given Hubble patch satisfies the Langevin equation

$$\dot{\phi} = -\frac{1}{3H_0}\frac{\partial \delta V}{\partial \phi} + f(t)$$

where the stochastic force f is distributed according to the gaussian law and has the correlation properties

$$\langle f(t) f(t') \rangle = \frac{H_0^3}{4\pi^2}\delta(t - t')$$

The corresp. Fokker-Planck equation

$$\frac{\partial \rho(\phi, t)}{\partial t} = \frac{H_0^3}{8\pi^2}\frac{\partial^2 \rho}{\partial \phi^2} + \frac{1}{3H_0}\frac{\partial}{\partial \phi}\left(\frac{\partial V}{\partial \phi}\rho\right)$$

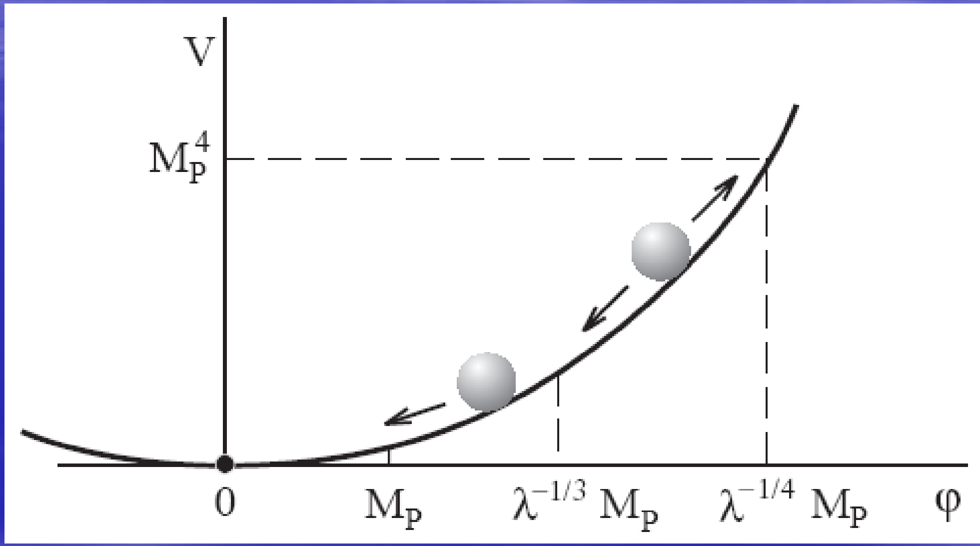
defines the probability distribution to have a given value of the inflaton within a given Hubble patch.

Stochastic inflation and selfreproducing universe 2

What happens physically?

During one Hubble time $\Delta t \sim H^{-1}$
the inflaton decreases by

$$\Delta\phi \sim \dot{\phi}\Delta t \sim \frac{1}{3H^2}\frac{\partial V}{\partial \phi} \sim \frac{M_P^2}{8\pi V}\frac{\partial V}{\partial \phi}$$



However, during the same time the fluctuations of the inflaton are generated with charact. wavelength

$$l \sim k^{-1} \sim H^{-1}$$

and typical amplitude

$$|\delta\phi| \sim \frac{H}{2\pi} \sim \sqrt{\frac{2V}{3\pi M_P^2}}$$

In different Hubble patches the VEVs of the inflaton measured by local observers are different. In some Hubble patches, due to the stochastic kicks produced by generated fluctuations, the VEV of the inflaton may grow.

Solution of Fokker-Planck equation

$$\rho = \exp\left(-\frac{4\pi^2 \delta V(\phi)}{3H_0^4}\right) \sum_n c_n \psi_n(\phi) \exp\left(-\frac{E_n H_0^3 (t - t_0)}{4\pi^2}\right)$$

where ψ's are eigenfunctions of the following Schrödinger equation:

$$\frac{1}{2}\frac{\partial^2 \psi_n}{\partial \phi^2} + (E_n - W(\phi))\psi_n = 0$$

and

$$W(\phi) = \frac{8\pi^4}{9H_0^8}\left(\frac{\partial \delta V}{\partial \phi}\right)^2 - \frac{2\pi^2}{3H_0^4}\frac{\partial^2 \delta V}{\partial \phi^2}$$

is the "superpotential".

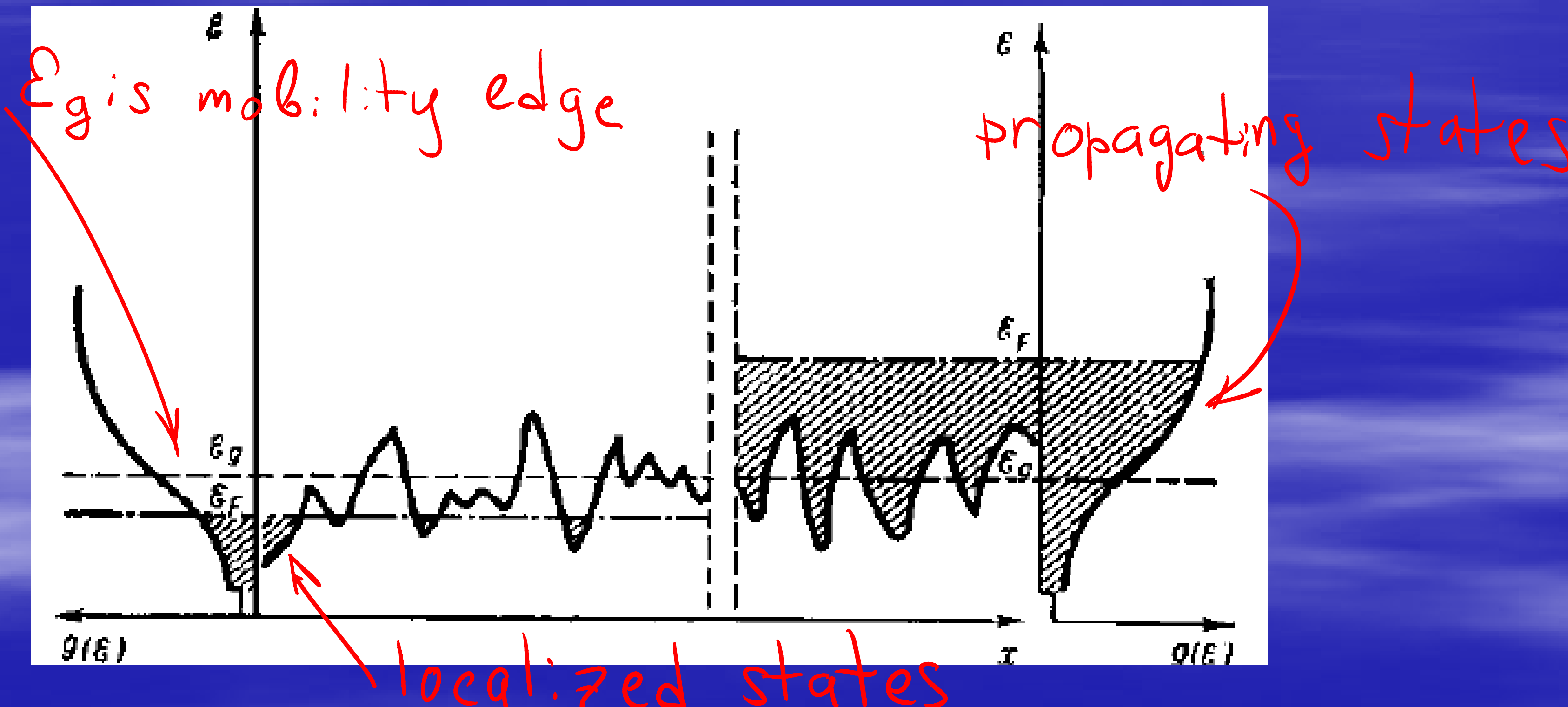
1. The eigenvalues E are all positive definite (supersymmetric form of the Hamiltonian). If the wavefunctions are normalizable, then the ground state corresponds to zero energy, i.e., is time-independent.
2. Steady state: contributions from higher eigenstates become negligible exponentially quickly. However, if the spectrum of the Hamiltonian is very dense, then higher eigenstates are also important at finite time scales. For example n first states are important for dynamics at $\Delta t \lesssim 1/E_n$
3. Anderson localization for random potential

Localization in disordered quantum systems 1

The motion of carriers (electrons) in random potential of impurities is governed by Schrödinger equation with the potential

$$\langle u(r)u(r')\rangle = \frac{1}{2\nu\tau}\delta(r-r'), \langle u(r)\rangle = 0$$

. Mean free time: $\tau \sim (na_0^2)^{-1}$



Localization in disordered quantum systems 2

If wave package of electron in ultra-pure metallic wire is located near the origin in the initial moment of time, then the probability density will spread out as

$\langle R^2(t) \rangle \sim t$ (usual diffusive behavior of the width of the probability density)

In the case of Anderson localization (metal with impurities) one has

$\rho(R) \sim \exp(-R/L)$ where L is the localization length (the same order of magnitude as the mean free path)

This happens in 1 dimension for **arbitrarily weak disorder.**

The reason for anomalous diffusion of the distribution function for eternal inflation is Anderson localization in the dual quantum problem: wave functions ψ behave as

$$\psi_n(\phi) \sim \exp\left(-\frac{|\phi - \phi_0|}{L}\right)$$

Instead of conclusion: some remarks

1. Quasi-one-dimensional islands: Strong dependence of eternal inflation history on initial conditions due to the effect analogous to Anderson localization. Weak logarithmic spreading of the distribution function:

$$\langle \phi^2(t) \rangle \sim \log^4 t$$

2. Quasi-two-dimensional islands: all states are localized but the localization length grows exponentially with energy. The result: subdominant corrections to the diffusive law

$$\langle \phi^2(t) \rangle \sim t \left(1 + c_1 \frac{1}{\log^\alpha t} + \cdots \right)$$

   Recalling initial conditions at later times.

3. Quasi-higher-dimensional parts: edge of mobility. Recalling initial conditions at later times: $\Delta t \gg E_g^{-1}$

One possibility to realize eternal inflation on the landscape: multiple KS throats