Instabilities of Near-Extremal Smeared Branes and the Correlated Stability Conjecture

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(with T. Harmark and V. Niarchos)

and 0407094 (JHEP 09 (2004)) (with T. Harmark)

Introduction

gauge/gravity correspondence

near-horizon brane backgrounds ⟷ dual non-gravitational theories

deep connection between thermodynamics of the two sides

↑ classically unstable brane background ⇒ ↑ phase transition in dual gauge theory

classical stability

↕

thermodynamic stability

Gregory-Laflamme instability

Gregory,Laflamme (1993)

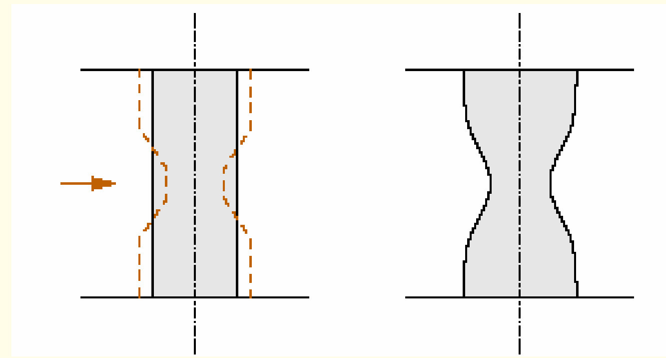
GL instablity of neutral black strings: perturbations oscillating in direction
In which string extends $\lambda \gtrsim r_{\text{hor}}$

$M < M_{\text{GL}}$

M

classically unstable

classically stable



threshold mode $(\Omega = 0)$

non-uniform static solution emerges

$$\delta g_{\mu\nu} \sim e^{i\mu z} h_{\mu\nu} \ , \quad h_{\mu\nu} \propto e^{\Omega t}$$

$$\Delta_{\text{Lich}} \ \delta g_{\mu\nu} = 0$$

thermodynamic argument for instability: $S_{\text{bh}} > S_{\text{bs}}$ for mall masses

interpretation: black string decays to black holes

important observation:
classical Lorentzian threshold unstable mode
corresponds to Euclidean negative mode

$$\tilde{\Delta}_{\text{Lich}} h_{\mu\nu} = -\mu^2 h_{\mu\nu}$$

Correlated Stability Conjecture

- CSC: Gubser,Mitra:
brane system with translational symmetry and infinite extent,
GL instablity occurs when system has local thermodynamic instability

positivity of Hessian matrix of second derivatives of mass w.r.t. entropy and any charges that can be redistributed over direction in which instability is supposed to occur

- semiclassical proof (for magnetically charged Dp-branes) Reall/Gregory,Ross:
Based on Euclidean path-integral formulation of gravity

$$Z[g] = \int d[g] e^{-I[g]} \sim (\mathrm{det}[\Delta^{\mathrm{E}}_{\mathrm{Lich}}])^{-1/2} e^{-I_0[g_0]}$$

$$g = g_0 + \delta g$$

key ingredient: relation between threshold mode of classical instability and a Euclidean negative mode in semi-classical path integral

⇒ classical instability occurs precisely when specific heat becomes negative

Further recent work on CSC

bound states of branes
(D0-D2,D0-D4,…)
+ consequences for NCSYM/NCOS

Gubser/Ross,Wiseman/
Friess,Gubser

counter-examples to CSC

Friess,Gubser,Mitra

magnetically charged branes with ``scalar hair''
(+ usual conserved quantum numbers)
perturbatively unstable mode (at linear level) involves only scalar field

Smeared Dp-branes

electrically charged brane is smeared:
direction with translational symmetry along which brane is not charged
(magnetically charged: opposite)

T-dual

Smeared Dp-brane compactified on isometric direction ⟺ D(p+1)-brane wrapped on T-dual circle

↓ Near-extremal limit

(p+2)-dim SYM on $\mathbb{R}^{p+2} \times \mathbb{T}^2$

stability of smeared branes has
non-trivial implications for stability of dual SYM

From Neutral Strings To Smeared Dp-branes

solutions of pure gravity with event horizon asymptoting to d-dimensional Minkowski-space times a circle (d = 9- p)

for recent reviews: Harmark,NO/Kol

static and neutral
Kaluza-Klein black holes

boost/
U-duality

Harmark,NO/Bostock,Ross/
Aharony,Marsano,Minwalla,Wiseman

non- and near-extremal
Dp-branes
on a transverse circle

neutral black strings

GL point

smeared Dp-branes

critical mass/energy point

unstable mode at threshold:
new static phase of non-uniform strings emerges

new phase of non- and near-extremal branes non-uniformly distributed on circle emerges

Gregory,Laflamme/Gubser/Wiseman

suggests that non-and near-extremal smeared branes exhibit classical instabilities

Classical stability and CSC for smeared branes

- How does charge affect GL instability of neutral black strings
- What happens when taking near-extremal limit

use as important tool: boost/U-duality map

main points:

• when applying CSC to smeared branes:
consider thermodynamic stability in grand-canonical ensemble
(charged can redistribute itself in the direction in which brane is smeared)
non-extremal branes are thermodynamically unstable in this ensemble

⇒ according to CSC: classical instability

• proof of CSC for smeared branes
(involves explicit construction of appropriate off-shell two-parameter family of Euclidean backgrounds)

explicit construction of GL unstable mode for non-extremal branes

time-dependent unstable GL mode of neutral black strings ⇒ time-dependent unstable mode of non-extr. smeared branes

Near-extremal limit and extremal smeared branes

• detailed analysis of near-extremal limit and validity of CSC

demonstrate that near-extremal limit of unstable GL mode is well-defined
so near-extremal. smeared branes classically unstable

⇒ according to CSC:
near extr. smeared branes are thermodynamically unstable
in grand-canonical ensemble

- how to define this ensemble in near-extremal limit ? (charge cannot vary anymore)

→ show that natural definition of such ensemble exists
+ near-extr. smeared branes are indeed thermodynamically unstable

• examine connection between GL mode for charged branes and marginal modes for extremal smeared branes:

GL mode in extremal limit = marginal modes of extremal smeared branes

• issues related to T-duality of smeared Dp-branes + properties of dual gauge theories

Non-extremal smeared Dp-branes

- start with metric of uniform black string in 10-p dimensions;
- add p+1 flat directions to go to 11 dimensions
- perform Lorentz boost in M-theory direction
- S-dualize to type IIA and T-dualize in the p directions

$$ds^2 = H^{-1/2}\left(-f dt^2 + \sum_{i=1}^{p} dx_i^2\right) + H^{1/2}\left(f^{-1}dr^2 + dz^2 + r^2 d\Omega_{7-p}^2\right)$$

$$H = 1 + \frac{r_0^{6-p}\sinh^2\alpha}{r^{6-p}}\ ,\quad e^{2\phi} = H^{\frac{3-p}{2}}\ ,\quad A_{01\ldots p} = \coth\alpha\,(H^{-1} - 1)$$

thermodynamic quantities (L is circumference of transverse circle)

$$\frac{M}{L} = \frac{\Omega_{7-p}}{16\pi G} V_p r_0^{6-p}[7-p+(6-p)\sinh^2\alpha]\ ,\quad \frac{S}{L} = 4\pi\frac{\Omega_{7-p}}{16\pi G} V_p r_0^{7-p}\cosh\alpha$$

$$\frac{Q}{L} = \frac{\Omega_{7-p}}{16\pi G} V_p r_0^{6-p}(6-p)\sinh\alpha\cosh\alpha\ ,\quad T = \frac{6-p}{4\pi r_0\cosh\alpha}\ ,\quad \nu = \tanh\alpha$$

Thermodynamic stability and ensembles

- canonical ensemble $F(T,Q) = M - TS\ , \quad dF = -SdT + \nu dQ$

$\longrightarrow$ positive specific heat

- grand canonical ensemble

$$G(T,\nu) = M - TS - \nu Q\ , \quad dG = -SdT - Qd\nu$$

$\longrightarrow$ positive specific heat and isothermal electric permittivity

(Hessian $H_G = -H_M^{-1}$ is negative definite)

thermodynamic stability

$$C_Q \equiv \left(\frac{\partial M}{\partial T}\right)_Q > 0 \qquad \Leftrightarrow \qquad \alpha > \mathrm{arcsinh}(1/\sqrt{4-p})$$

$$c \equiv \left(\frac{\partial \nu}{\partial Q}\right)_T > 0 \qquad 0 < \alpha < \mathrm{arcsinh}(1/\sqrt{4-p})$$

⇨ smeared Dp-branes are thermodynamically unstable in grand-canonical ensemble

CSC for charged branes

gravitational system with non-compact symmetry is classically stable
iff it is locally thermodynamically stable

which ensemble to employ:
grand-canonical ensemble with respect to charge Q when brane is smeared along at least one of the isometric directions

e.g.

smeared D0-brane: grand-canonical ensemble
D1-brane : canonical ensemble

T-dual to each other → Legendre transformation (at supergravity level)

in full string theory: momentum and winding modes interchanged

⇒ momentum instability for smeared brane ↔ winding instability for wrapping brane

other example: D0-D2 bound state: mixed ensemble (grand-can. w.r.t D0)

CSC for smeared branes

following proof of Reall for magnetically charged branes:

•demonstrate existence of two-parameter family of Euclidean backgrounds with action

$$I(x,y;\beta,\nu) = \beta(E(x,y) - \nu Q(x,y)) - S(x,y)$$

generic point in famility is off-shell background (satisfying Hamiltonian constraints)

extrema of action (wrt varying x,y) gives $(x,y) = (x(T,\nu), y(T,\nu))$

then background is exact solution of EOMs

⇨ quadratic perturbations of action I along (x,y) surface involves Hessian matrix

$$(I_{(2)}) = -M H_G M^{\top}$$

Jacobian of (x,y) to (T,ν)

Hessian of Gibbs free energy: negative definite for thermodyn stability

positive definite for classical stability

Conditions to fullfil

•construct the two-parameter family satisfying Hamiltonian constraints and appropriate boundary conditions:

→ is possible (technical)

•verify that norm of the on-shell perturbations on the space of theories is positive (i.e. normalizable on-shell perturbations)

static GL mode $\Psi^I = \mathrm{Re}(\psi^I e^{ikz})$ $\Delta I \simeq -\frac{k^2}{2}||\psi||^2$

→ follows from positivity of the neutral GL perturbations + fact that norm is invariant under boosts and U-duality transformations

•demonstrate sufficient overlap between off-shell deformations (x,y) and actual on-shell perturbations ψ^I

path in off-shell geometries = linear combo of eigenfunctions of Lichnerowicz operator

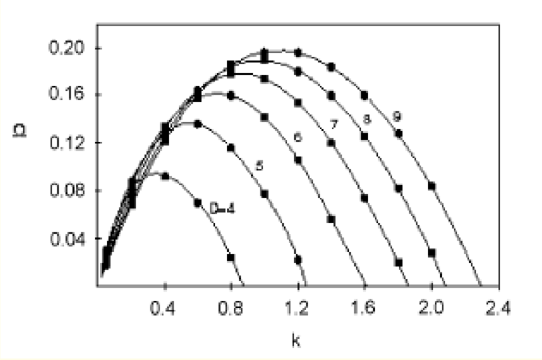
subtle point: (in recent counter-example of CSC: backgrounds with scalar hair extra directions in which the instability can take place)

GL mode for smeared branes

GL perturbation for neutral black string

$$h_{\mu\nu} = \mathrm{Re}\left\{\exp\left(\frac{\Omega t}{r_0} + i\frac{kz}{r_0}\right) P_{\mu\nu}\right\}$$

$$P_{tt} = -f\psi\ , \quad P_{tr} = \eta\ , \quad P_{rr} = f^{-1}\chi\ , \quad P_{\mathrm{sphere}} = r^2\kappa$$



cannot act naively with boost/U-duality transformation on the perturbation:
- non-normalizable exponential dependence on 11th direction y

trick: find complex transformation (``twisted'' boost/U-duality) that
i) gets rid of exponential dependence
ii) has same effect on zeroth order metric (i.e. on neutral black string)

→ at end take real part of the transformed perturbation
(works because of linearity of the perturbed EOMs)

⇒ in this way we can prove explicitly that non-extremal smeared branes are classically unstable in accord with the CSC

by careful analysis of the near-extremal limit: same for near-extremal smeared branes

Connection to marginal modes of extremal smeared branes

extremal Dp-brane distributed along single flat direction z

$$ds^2 = H^{-1/2}\left(-dt^2+\sum_{i=1}^{p} dx_i^2\right)+H^{1/2}\left(dr^2+dz^2+r^2 d\Omega_{7-p}^2\right)$$

$$H(r,z) = 1+\int_{-\infty}^{\infty} dz' \frac{\rho(z')}{(r^2+(z-z')^2)^{(7-p)/2}}$$

arbitrary charge distribution

extremal smeared brane has ρ constant but can add a single mode of any wave number q

$$H = 1+\frac{K l_s^{8-2p}}{r^{6-p}} + m(qr)\cos(qz)$$

satisfies same DE as is obtained by taking the extremal limit of the GL mode of near-extremal smeared branes

⇒ the marginal mode of extremal smeared brane becomes the near-extremal GL mode when we switch on temperature

extremal smeared branes are arbitrarily close to being unstable

any disturbance that makes it non-BPS causes it to decay (unlikely formation in cosmo)

CSC for near-extremal branes and dual gauge theories

near-extr. smeared Dp-branes have positive specific heat (for $p \leq 3$)

but we have seen: classically unstable

⇒ puzzle with the CSC (how to define grand canonical ensemble)

define rescaled chemical potentia and charge

$$\hat{\nu} \equiv \frac{1}{g_{\mathrm{YM}}^2} \lim_{l_s \to 0} \frac{1}{l_s^4} (\nu - 1) \ , \quad \hat{Q} \equiv g_{\mathrm{YM}}^2 \lim_{l_s \to 0} l_s^4 Q$$

first law of thermo $dE = TdS + \hat{\nu} d\hat{Q}$

→ $\hat{c} = \left(\frac{\partial \hat{\nu}}{\partial \hat{Q}} \right)_T < 0$ so indeed thermodynamically unstable in agreement with CSC

in dual gauge theories: GL instabilities translate into new set of phase transitions parameterized by Wilson line around spatial circle Aharony,Marsano,Minwalla,Wiseman

from T-duality: near-extremal (wrapped) D(p+1)-brane has winding mode instability along longitudinal circle when T-dual radius sufficiently small

⇒ suggests new phase transitions in compactified SYM

Outlook

• general and complete proof of CSC
  -wider class of examples
  -universal argument (negative specific heat ↔ imaginary speed of sound
Buchel

- sharpening of conjecture to account for counter examples

• T-duality smeared branes ⟺ wrapped branes

unstable mode has non-vanishing winding

→ winding state expected to become tachyonic when (T-dual) radius smaller than critical radius

explore situations in string theory where dynamics of mode can be analyzed explicitly Adams et al/Horowitz/Ross

• better understanding of processes in the gauge theories dual to the brane instabilities
(recent examination in 0+1 and 1+1 dimensions) Aharony et al

Near-extremal smeared branes

near-extremal limit $l_s \to 0\ ,\quad u = \frac{r}{l_s^2}\ ,\quad \hat{z} = \frac{z}{l_s^2}\ ,\quad g_{\mathrm{YM}},\ \bar{L}$ fixed

$$l_s^{-2} ds^2 = \hat{H}^{-1/2}(-f dt^2 + \sum_{i=1}^{p} dx_i^2) + \hat{H}^{1/2}(f^{-1} du^2 + d\hat{z}^2 + u^2 d\Omega_{7-p}^2)$$

$$e^{2\phi} = \hat{H}^{\frac{3-p}{2}}\ ,\quad A_{01\ldots p} = \hat{H}^{-1}\ ,\quad \hat{H} = \frac{K}{u^{6-p}}\ ,\quad f = 1 - \frac{u_0^{6-p}}{u^{6-p}}$$

thermodynamics

$$E = \frac{1}{\mathcal{G}} u_0^{6-p} \frac{8-p}{2}\ ,\quad S = \frac{4\pi}{\mathcal{G}} u_0 (u_0^{6-p} K)^{1/2}\ ,\quad T = \frac{6-p}{4\pi u_0} \left(\frac{u_0^{6-p}}{K} \right)^{1/2}$$

specific heat $C = \frac{8-p}{4-p} S$ positive for $p \leq 3$

will see that near-extremal branes smeared branes are classically stable

- how to define a grand canonical ensemble ?
- should be thermodynamically unstable in such ensemble

Perturbed non-extremal smeared Dp-branes

boosted perturbation of the 11-dimensional metric

$$\tilde{h}_{\mu\nu} = \mathrm{Re}\left\{\exp\left(\frac{\tilde{\Omega}t}{r_0} + i\frac{\tilde{k}z}{r_0}\right)\tilde{P}_{\mu\nu}\right\} \qquad \tilde{P} = \Lambda^{-1}M^{-1}P(M^{-1})^T(\Lambda^{-1})^T$$

apply same U-dualities as on the boosted neutral black string

$$ds^2 = H_{\mathrm{c}}^{-1/2}\left[-f_{\mathrm{c}}dt^2 + \sum_{i=1}^{p} dx_i^2 + 2\eta\cosh\alpha\mathcal{E}dtdr\right]$$
$$+H_{\mathrm{c}}^{1/2}\left[f^{-1}(1+\chi\mathcal{E})dr^2+dz^2+r^2(1+\kappa\mathcal{E})d\Omega_{7-p}^2\right]$$

$$e^{2\phi} = H_{\mathrm{c}}^{(3-p)/2}, \quad A_{01\cdots p} = \coth\alpha(H_{\mathrm{c}}^{-1}-1), \quad A_{r1\cdots p} = -H^{-1}\eta\sinh\alpha\mathcal{E}$$

$$f = 1 - \frac{r_0^{6-p}}{r^{6-p}}, \quad H = 1 + \sinh^2\alpha(1-f)$$

$$\mathcal{E} = \cos\left(\tilde{k}r_0^{-1}z\right)\exp\left(\tilde{\Omega}r_0^{-1}t\right)$$

$$\tilde{k}^2 = k^2 + \Omega^2\tanh^2\alpha, \quad \tilde{\Omega} = \Omega/\cosh\alpha$$