Smooth geometries and their CFT duals

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   - D1-D5 system
   - Dual CFT
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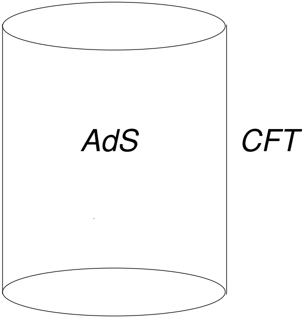
Motivation: Extending AdS/CFT dictionary

AdS/CFT maps geometry to CFT

- For CFT, focus on vacuum: pure AdS space
- For quantum gravity, need to consider asymptotically AdS spaces
- Excited states in CFT
- BH: thermal ensemble
  Horizon, singularity difficult to understand
- Solitons: explicit id with pure states



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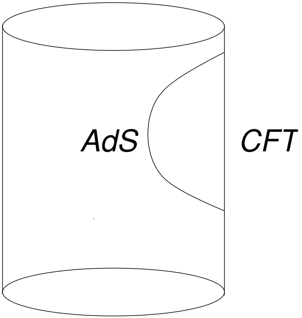

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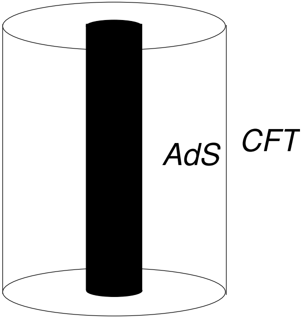


D1-D5 solutions

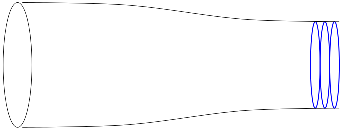
Type IIB supergravity on $T^4 \times S^1 \times \mathbb{R}^{4,1}$:

- coordinates z^i, $y \sim y + 2\pi R$, $(t, r, \theta, \phi, \psi)$

SUSY: fermions periodic around asymptotic S^1.

Rotating D1-D5 system:

- Q_1 D1s along t, y, Q_5 D5s along t, y, z^i.
- Momentum P_y, angular momenta J_ψ, J_ϕ in $\mathbb{R}^4$.



Field theory on D1-D5 decouples at low energies.

D1-D5 solutions

Cvetic & Youm, Cvetic & Larsen

Construct a black string solution with these charges:

- Symmetries $\mathbb{R}_t \times U(1)_y \times U(1)_\psi \times U(1)_\phi$
- Seven parameters: $M, \delta_1, \delta_5, \delta_p, a_1, a_2, R$.
- Topology $\mathbb{R}^2_{t,r} \times S^1 \times S^3 \times T^4$.
- Extremal limit BMPV black hole, entropy $S = 2\pi\sqrt{Q_1 Q_5 P_y - J^2}$.
- Reproduced in CFT describing massless modes of D1-D5 system.

AdS/CFT

- Near-extremal limit: $Q_1, Q_5 \gg M, a_1^2, a_2^2$.
- Near-horizon limit: focus on $r^2 \sim M \ll Q_1 Q_5$.

⋆ Near-horizon geometry locally $\mathrm{AdS}_3 \times S^3 \times T^4$. $\ell^2 = \sqrt{Q_1 Q_5}$.
Dual to a 1+1 CFT with $c = 6 Q_1 Q_5$.

Geometry BTZ black hole,

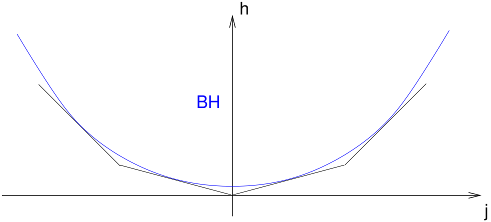
$$M_3 = \frac{R^2}{\ell^4}[(M - a_1^2 - a_2^2)\cosh 2\delta_p + 2a_1 a_2 \sinh 2\delta_p],$$

$$J_3 = \frac{R^2}{\ell^3}[(M - a_1^2 - a_2^2)\sinh 2\delta_p + 2a_1 a_2 \cosh 2\delta_p].$$

D1-D5 CFT

Dual description in terms of $1+1$ CFT on $\mathbb{R} \times S^1$:

- CFT deformation of σ-model on $(T^4)^{Q_1 Q_5}/S_{Q_1 Q_5}$, $c = 6 Q_1 Q_5$.
- $SO(2,2) \times SO(4)_R = SL(2,\mathbb{R}) \times SL(2,\mathbb{R}) \times SU(2) \times SU(2)$ symmetry. Charges (h, j), $(\bar{h}, \bar{j})$.
- NS, R sectors. Chiral primaries in NS sector related to R vacua by spectral flow: $h' = h + \alpha j + \alpha^2 \frac{c}{24}, j' = j + \alpha \frac{c}{12}$.



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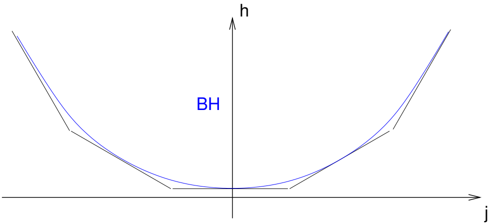
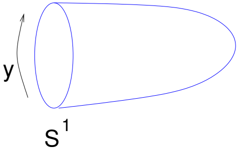


Supersymmetric solitons

Balasubramanian, de Boer, Keski-Vakkuri & Ross; Maldacena & Maoz

Special cases of D1-D5 solution with smooth 6d geometry.
Twisted circle shrinks smoothly to zero in interior.

First example:
$S^1 \to 0$ is $R\partial_y - \partial_\phi$.
$M = 0, a_2 = 0, a_1 = \frac{\sqrt{Q_1 Q_5}}{R}$.
Topologically
$\mathbb{R}^2_{r,y} \times \mathbb{R}_t \times S^3_{(\theta,\tilde{\phi},\psi)} \times T^4$.



'Near-core' region *global* $\mathrm{AdS}_3 \times S^3 \times T^4$: $M_3 = -1$, $J_3 = 0$.

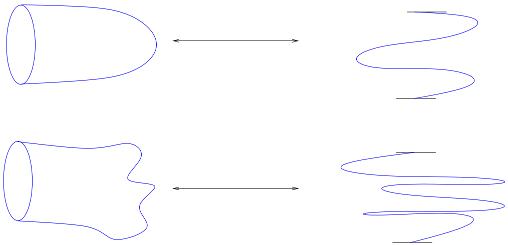
Dual CFT:

Global $\mathrm{AdS}_3 \times S^3 \leftrightarrow$ NS vacuum state
Twist $\tilde{\phi} = \phi + y/R$ corresponds to spectral flow.
Soliton identified with RR ground state of maximal R-charge.

Supersymmetric solitons

Lunin & Mathur, Lunin, Maldacena & Maoz

More general solutions: D1-D5 dual to F1-P.



Solutions determined by arbitrary profile $\vec{F}(y) \in \mathbb{R}^4$.

$$ds^2 = H^{-1}(-(dt - A)^2 + (dy + B)^2) + Hd\vec{x}^2$$

Supertubes

Mateos & Townsend

F1-D0 bound states expand into a D2-brane tube: similar arb profile. Here, D1-D5 bound states expand into KK monopole tube.

Relation to CFT

Lunin & Mathur, Lunin, Maldacena & Maoz

- Near-core limit smooth, SUSY, asymp $AdS_3 \times S^3$ geom.
- In NS sector, identify with chiral primary—read off map from F1-P picture:

$$F^i(v) = \sum_k \delta^i_{i_k} m_k e^{i n_k v} \leftrightarrow [\alpha^{i_1}_{-n_1}]^{m_1} \dots [\alpha^{i_k}_{-n_k}]^{m_k} |0\rangle \leftrightarrow [\sigma^{\pm\pm}_{n_1}]^{m_1} \dots [\sigma^{\pm\pm}_{n_k}]^{m_k} |0\rangle_{NS}$$

$\sigma^{\pm\pm}_{n_i}$ are twist ops: join n_i components to form a single long string. Allowing also oscillations in T^4, build up general chiral primary.

Orbifolds

If $\vec{F}(v) = \vec{a} e^{ikv}$, profile again S^1, but traversed k times:
SUSY $(AdS_3 \times S^3)/\mathbb{Z}_k$ orbifold geometry.
Corresponding state has $n_1 n_5 / k$ component strings of length k.

Mathur proposal

Mathur, . . . (see hep-th/0502050)

CFT microstates $\leftrightarrow$ smooth geometries

- "Horizon" arises by coarse graining
- Microstates desc directly in geometric terms
- Evidence: probe scattering, counting states in supertube picture.

No real black hole - no information loss problem

I don't advocate this proposal

Problems:

- Finding enough solitons
- Curvatures large for typical states
- Perturbations mix states (geometries?).
- Dynamical formation in grav collapse difficult: topology, horizon teleological.

Three-charge solutions

Giusto, Saxena & Mathur; Berglund, Gimon & Levi; Bena & Warner

For three-charge solutions, $\mathbb{R}^4$ replaced by Gibbons-Hawking space: hyper-Kähler space, S^1 fibred over $\mathbb{R}^3$.

$$ds^2_{GH} = V(d\psi + \alpha) + \frac{1}{V}(dx^2 + dy^2 + dz^2).$$

- Preserve $U(1)$ symmetry associated with S^1 fibre.
- Preserve half SUSY of two-charge cases.
- No supertube picture: geometry specified by sources in $\mathbb{R}^3$.
- KK reduction gives smooth 5d geometry.
- Can pass to different duality frames: M2 $\perp$ M2 $\perp$ M2 desc.

CFT interpretation

Not yet understood in general.
Two-centre case preserves $U(1) \times U(1)$. Near-core limit again global $AdS_3 \times S^3$. Interp as more general spectral flow of NS vacuum.

Non-supersymmetric solitons

Jejjala, Madden, Ross & Titchener

Look for soliton solutions in general "black string" metric.
As in SUSY two-charge case, $S^1 \to 0$ smoothly.

- Metric involves two harmonic functions $H_{1,5}$, $g(r) = (r^2 + a_1^2)(r^2 + a_2^2) - Mr^2 = (r^2 - r_+^2)(r^2 - r_-^2)$. Coordinate singularities at $H_{1,5} = 0$, $r^2 = r_\pm^2$.
- If $r_+^2 > 0$, horizon; if $r_+^2 < 0$, conical singularity.
- Require $H_{1,5}(r_+) > 0$.
- Make $r^2 = r_+^2$ smooth origin:
  - Need $||\xi||^2 = 0$ at $r^2 = r_+^2$ for some

$$\xi = R\partial_y + n\partial_\psi - m\partial_\phi .$$

  - ξ fixes $\tilde{\phi} = \phi + my$, $\tilde{\psi} = \psi - ny$.
  - Need $m, n \in \mathbb{Z}$, so ξ has closed orbits
  - Need an appropriate period to get smooth solution

Non-supersymmetric solitons

For $n = 0$: $\delta_p = 0$, $a_2 = 0$,

$$m = \frac{R a_1}{M \sinh \delta_1 \sinh \delta_5} = \frac{a_1}{\sqrt{a_1^2 - M}}.$$

Similarly for $n \neq 0$.
All parameters fixed in terms of Q_1, Q_5, R, m, n.

- Completely smooth non-BPS solutions.
- t is a global time function, so no CTCs.
- Can also consider $\mathbb{Z}_k$ orbifolds, rather than just smooth solutions:
  Take $y \sim y + 2\pi R k$ at fixed $\tilde{\phi}, \tilde{\psi}$ closed cycle.
- $m + n$ odd to have periodic fermions on asymptotic S^1_y.

Dual CFT intepretation

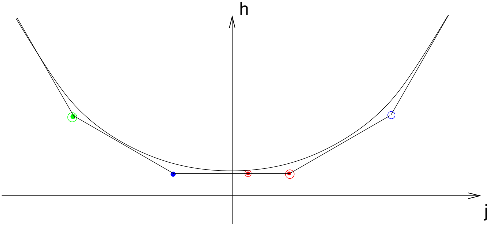
Near-core AdS region: $AdS_{3(t,y,r)} \times S^3_{(\theta,\tilde{\phi},\tilde{\psi})} \times T^4$

$\tilde{\phi} = \phi + ym/R$, $\tilde{\psi} = \psi - yn/R$.

Read off CFT charges from asymptotics:

$$h = \frac{c}{24}(m+n)^2, \quad j = \frac{c}{12}(m+n),$$

$$\bar{h} = \frac{c}{24}(m-n)^2, \quad \bar{j} = \frac{c}{12}(m-n),$$



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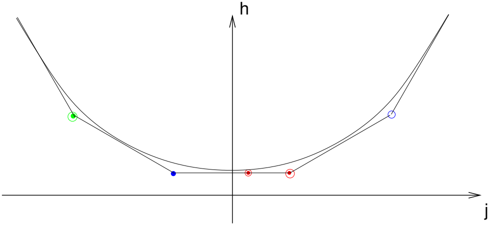
$\tilde\phi = \phi + ym/R$, $\tilde\psi = \psi - yn/R$.

Read off CFT charges from asymptotics:

$$h = \left(1 + \frac{(m+n)^2 - 1}{k^2}\right), \quad j = \frac{c}{12}\frac{(m+n)}{k},$$

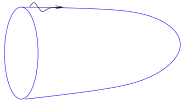
$$\bar h = \left(1 + \frac{(m-n)^2 - 1}{k^2}\right), \quad \bar j = \frac{c}{12}\frac{(m-n)}{k},$$

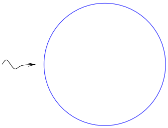


Probe calculations

Lunin & Mathur, ...

Consider scattering of closed string states in AF geometry:





$$\Delta t_{sugra} = \pi R k \eta \qquad \Delta t_{CFT} = \pi R k$$

Mismatch associated with redshift between AF & near-core coords.

Indicates we don't understand CFT desc of full AF geom.

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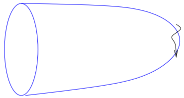

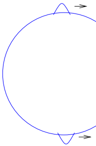

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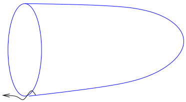

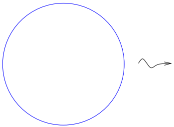

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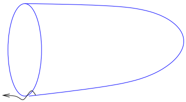

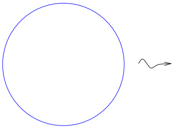

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Two-point functions of probe ops

Balasubramanian, Kraus & Shigemori

Non-twist ops probing *typical* R ground states: $\langle \sigma^\dagger \mathcal{A}(w_1)\mathcal{A}(w_2)\sigma \rangle$.
For light probes, universal behaviour for $t_1 - t_2 \ll \sqrt{Q_1 Q_5}$.
Reproduced by $M = 0$ BTZ black hole geometry.
(NB: calculations at free orbifold point of CFT)

Stability

For non-SUSY solitons, important to study stability

- 5d AF geometry has ergoregion - ∂_t spacelike.
- Negative-energy modes in ergoregion. $\Rightarrow$ instability?
- Should study linearized field equations

Dual to excited state in CFT: left and right-moving excitations. Expect they will decay into closed string modes

$\star$ Interesting to compare decays

1/2 BPS states in $AdS_5 \times S^5$

IIB on $AdS_5 \times S^5 \sim \mathcal{N} = 4$ SYM on $\mathbb{R} \times S^3$

Interesting subsector: 1/2 BPS primary operators:
R-charge J under $SO(2) \subset SO(6)_R$, $\Delta = J$.
Operators $\prod_i (TrZ^{n^i})^{r^i}$, where $Z = \phi_1 + i\phi_2$.

- For $J \ll N$, BPS modes propagating in $AdS_5 \times S^5$.
- BMN limit: $J \sim N^{1/2}$, $\Delta - J$ finite as $N \to \infty$.
- For $J \sim N$, branes in bulk - 'giant gravitons'
- For $J \sim N^2$, expect these operators become dual to a deformation of bulk geometry.

I will focus on IIB, but there is a very similar (less developed) picture for M theory.

1/2 BPS solitons: "Bubbling AdS"

Lin, Lunin & Maldacena

Consider general ansatz for geometry determined by symmetry:

- Operators of interest s-waves on S^3—unbroken $SO(4)$.
- $SO(4) \subset SO(6)_R$ also preserved
- $\mathbb{R}$ symmetry associated with $\Delta - J = 0$.

Assume only $F_{(5)}$ non-zero.

Require geometry preserves 1/2 SUSY

▷Fermion bilinears, satisfying algebraic & differential equations

⋆ Fixes form of metric up to one independent function:

$$ds^2 = -h^{-2}(dt + V_i dx^i)^2 + h^2(dy^2 + dx^i dx^i) + ye^G d\Omega_3^2 + ye^{-G} d\tilde{\Omega}_3^2,$$

where $i = 1, 2$, $h^{-2} = 2y \cosh G$, $y\partial_y V = *dz$, $*dV = y^{-1}\partial_y z$, and $z = \tanh G/2$.

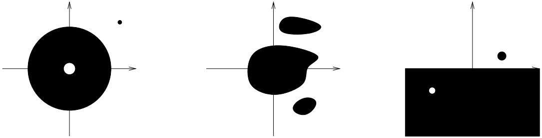
Everything determined by z, which satisfies

$$\partial_i \partial_i z + y\partial_y(y^{-1}\partial_y z) = 0.$$

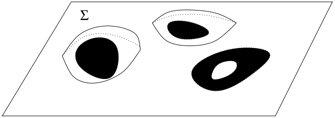
1/2 BPS solitons: "Bubbling AdS"

Lin, Lunin & Maldacena

Regularity: $y \geq 0$. Require $S^3 \to 0$ at $y = 0$ a smooth origin.
Implies $G \propto \pm \ln y$: $z \to \pm\frac{1}{2}$ at $y \to 0$.



"Bubbling" solutions: geometry specified by giving regions in the plane where $z = -\frac{1}{2}$. Weak curv if scales large compared to ℓ_{pl}.



Complicated topology: hemisphere Σ defines a non-trivial 5-cycle.
Flux $\int_{\Sigma \times S^3} F_{(5)}$ given by area of black region.

CFT: matrix description

Corley, Jevicki & Ramgoolam; Berenstein

States are described by matrix QM

$$\mathcal{L} = N \int dt \frac{1}{2} \mathrm{tr}[(D_t X)^2 - X^2].$$

Gauge field A_0 enforces Gauss constraint—states $SU(N)$ singlets.

▷Matrix/closed string description: choose $A = 0$.
N^2 free harmonic oscillators, gauge-invariant states.

- Normal multi-trace basis $\mathrm{tr}(a^\dagger)^{n_1} \ldots \mathrm{tr}(a^\dagger)^{n_k}|0\rangle$, $N \geq n_1 \geq n_2 \ldots \geq n_k$.
- Schur polynomial basis, $\mathrm{tr}_R(a^\dagger)|0\rangle$ in rep R of $SU(N)$.

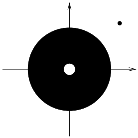
Both have natural corr to Young diagrams.

CFT: matrix description

Corley, Jevicki & Ramgoolam; Berenstein

▷Eigenvalue/open string description: choose X diagonal.
Van der Monde determinant $\longrightarrow$ eigenvalues λ_i fermions.

Basis of states Slater determinants:
$\psi = \det(H^{n_i}(\lambda_j))e^{-\lambda^2/2}$, where $n_1 > n_2 > \ldots > n_N \geq 0$.
Matches Schur poly basis in previous picture.
⋆ Describe general state by distribution in fermion phase space:



E.g., Wigner dist, $W(p,q) \sim \int dy \langle q - y|\hat{\rho}|q + y\rangle e^{2ip\cdot y/\hbar}$.

Correspondence

Natural corr between CFT states & 1/2 BPS geometries

Signs of phase space in SUGRA:

- Flux $\propto A_{\mathcal{D}}$ implies $A_{\mathcal{D}}$ quantized.
- Excitation energy $\Delta \propto \int_{\mathcal{D}} d^2x \, x^2 - A_{\mathcal{D}}^2/2\pi$.
- Symplectic form obtained by 'on-shell quantization' agrees.

  Maoz & Rychkov

More than matching spectrum: preferred coords on phase space.

Singular solutions:

- Superstar: n smeared giant gravitons. Grey disk $z = \frac{1}{2}\frac{n-N}{n+N}$.
- Solutions with z outside $[-1/2, 1/2]$ have CTCs.
- Also singular if $\partial\mathcal{D}$ self-intersects: topology changing transition.

Ensembles & typical states

Balasubramanian, de Boer, Jejjala & Simon

Expect to have a smooth semiclassical geom only for special states.
What corresponds to the typical state with $\Delta = J \sim N^2$?

- Consider canonical ensemble at $T \sim N$
- Limit shape for Young diagram:
    - Fixing N, Δ, complicated curve, $x \sim \log y$
    - Fixing N, Δ, N_C, triangle
- Corresponding phase space distribution gives singular geom:
    - Hyperstar
    - Superstar
- Argue correlation functions of light probes ($\Delta' = J' \sim \mathcal{O}(1)$) in a typical state exhibit universal behaviour.

▷Bulk geometry provides a coarse-grained desc of ensemble
▷Differences betw individual pure states & ensemble hard to see

Discussion

- Explicit correspondence between CFT states & smooth geometry.
- Well-controlled for SUSY states, but extends to some non-SUSY states.
- New tests of AdS/CFT.
- Laboratory for exploring description of spacetime in CFT.

Future issues:

- Understand correspondence better.
- More explicit & systematic description of D1-D5 states
- Understand relation of AF geometries to CFT
- Extend to cases with horizons, understand relation to bh