This research has been co-financed by the European Union (European Social Fund, ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF), under the grants schemes "Funding of proposals that have received a positive evaluation in the 3rd and 4th Call of ERC Grant Schemes" and the EU program "Thales" ESF/NSRF 2007-2013.

European Union
European Social Fund

OPERATIONAL PROGRAMME
EDUCATION AND LIFELONG LEARNING
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MINISTRY OF EDUCATION & RELIGIOUS AFFAIRS, CULTURE & SPORTS
MANAGING AUTHORITY

NSRF
2007-2013
programme for development
EUROPEAN SOCIAL FUND

Co-financed by Greece and the European Union

Holographic models for QCD in the Veneziano limit

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6 March 2013

Outline

- Holographic V-QCD models

  [MJ, Kiritsis arXiv:1112.1261]

- Spectra and the S-parameter

  [Arean, Iatrakis, MJ, Kiritsis, arXiv:1211.6125, arXiv:130n.xxxx]

- Turning on finite temperature: Next talk!

  [Alho, MJ, Kajantie, Kiritsis, Tuominen arXiv:1210.4516]

Motivation

QCD: $SU(N_c)$ gauge theory with N_f quark flavors (fundamental)

- Often useful: "quenched" or "probe" approximation, $N_f \ll N_c$
- Here Veneziano limit: large N_f, N_c with $x = N_f/N_c$ fixed $\Rightarrow$ backreaction

Important new features can be captured in the Veneziano limit:

- Phase diagram of QCD (at zero temperature, baryon density, and quark mass), varying $x = N_f/N_c$
- The QCD thermodynamics as a function of x
- Phase diagram as a function of baryon density

Holographic V-QCD: the fusion

Holographic bottom-up models (V-QCD) that describe QCD significantly well in the Veneziano limit

[MJ, Kiritsis arXiv:1112.1261]

The fusion:

1. IHQCD: model for glue by using dilaton gravity

   [Gursoy, Kiritsis, Nitti; Gubser, Nellore]

2. Adding flavor and chiral symmetry breaking via tachyon brane actions

   [Klebanov,Maldacena; Bigazzi,Casero,Cotrone,Iatrakis,Kiritsis,Paredes]

Consider 1 + 2 with full backreaction $\Rightarrow$ V-QCD models

Defining V-QCD

Degrees of freedom:

- $\tau \leftrightarrow \bar{q}q$; $\lambda \leftrightarrow \mathrm{Tr}F^2$
- $\lambda = e^{\phi}$ is identified as the 't Hooft coupling $g^2 N_c$

$$\mathcal{S}_{\mathrm{V-QCD}} = N_c^2 M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right] - N_f N_c M^3 \int d^5x \, V_f(\lambda,\tau) \sqrt{-\det(g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau)}$$

$$V_f(\lambda,\tau) = V_{f0}(\lambda) \exp(-a(\lambda)\tau^2) \; ; \qquad ds^2 = e^{2A}(dr^2 + \eta_{\mu\nu}x^\mu x^\nu)$$

The simplest and most reasonable potential choices do the job!

Matching to QCD

- In the UV ($\lambda \to 0$):
  - UV expansions of the various potentials can be matched with the perturbative QCD beta function and the anomalous dimension of the quark mass/chiral condensate
  - After this, a single undetermined parameter in the UV: W_0
- In the IR, the tachyon action $\propto e^{-a(\lambda)\tau^2}$ must become small
  - $V_g(\lambda)$ chosen as for Yang-Mills, so that a "good" IR singularity exists
  - $V_{f0}(\lambda)$, $a(\lambda)$, and $\kappa(\lambda)$ chosen to produce tachyon divergence: several possibilities ($\to$ Potentials I and II)
  - Extra constraints from the asymptotics of the meson spectra

Background analysis: zero temperature

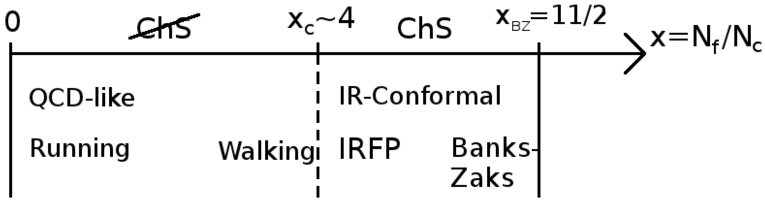
Analysis of the backgrounds (r-dependent solutions of EoMs) at zero temperature

- Expect two kinds of solutions, with
  1. Nontrivial tachyon profile (chirally broken)
  2. Identically vanishing tachyon (chirally symmetric)
- Fully backreacted system $\Rightarrow$ rich dynamics, complicated numerical analysis . . .

Phase diagram

At zero quark mass:

- Conformal window for $x_c < x < x_{BZ}$, ChSB for $0 < x < x_c$
- Critical value $x_c \sim 4$ arising from dynamics
- Walking backgrounds for x slightly below x_c

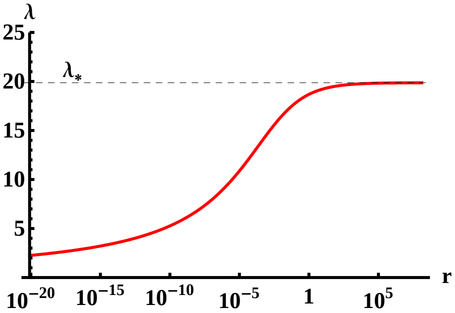


- Meets standard expectations from QCD!
- How does this diagram arise?

Backgrounds at zero quark mass

Sketch of behavior in the conformal window ($x > x_c$):

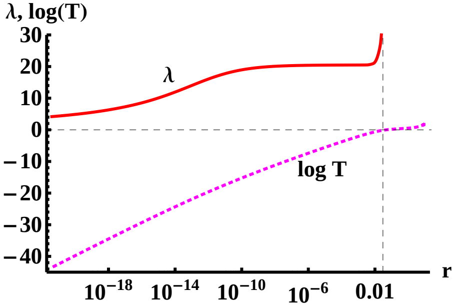
- Tachyon vanishes (no ChSB)
- Similar to IHQCD, different potential $\Rightarrow$ IR fixed point
- Dilaton flows between UV/IR fixed points



Here UV: $r \to 0$, IR: $r \to \infty$

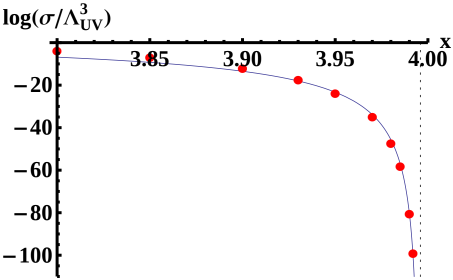
Right below the conformal window ($x < x_c$; $|x - x_c| \ll 1$)

- Dilaton flows very close to the IR fixed point
- "Small" nonzero tachyon induces an IR singularity



Result: "walking"

Important features



1. Miransky scaling as $x \to x_c$ from below
   - The ratio of the IR and UV scales behaves as expected
   - E.g. $\langle \bar{q} q \rangle \propto \sigma \sim \exp(-\kappa/\sqrt{x_c - x})$, with calculable κ
2. Unstable Efimov vacua observed for $x < x_c$
3. Turning on the quark mass modifies the dynamics in a natural way

Fluctuation analysis

1. Meson spectra (at zero temperature and quark mass)
   - Four towers: scalars, pseudoscalars, vectors, and axial vectors
   - Flavor singlet ($U(1)$) and nonsinglet ($SU(N_f)$) states
2. The S-parameter

$$S \sim \frac{d}{dq^2} \left[\langle VV \rangle - \langle AA \rangle \right]_{q^2=0}$$

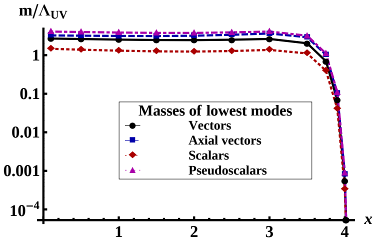
Open questions in the region relevant for technicolor ($x \to x_c$ from below):

- The S-parameter might be reduced
- Possibly a light "dilaton" (flavor singlet scalar): Goldstone mode due to almost unbroken conformal symmetry. The 125 GeV state seen at the LHC?

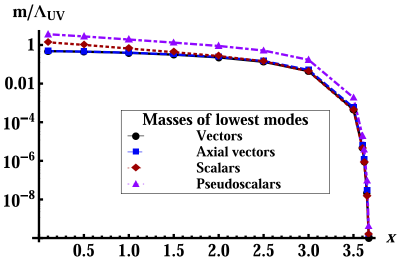
Meson masses

Flavor nonsinglet masses (two choices of potentials):

PotI $W_0 = 3/11$

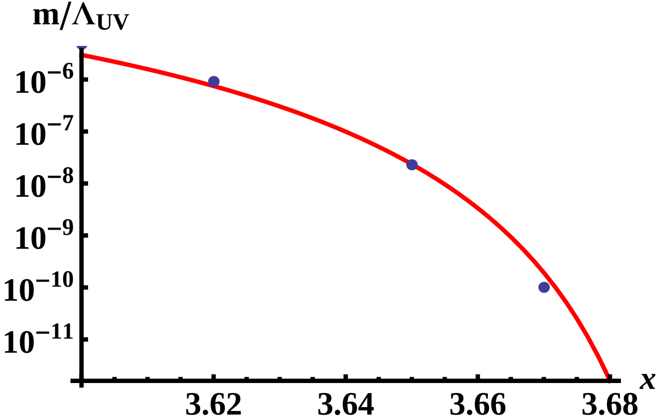


PotII W_0 SB



- All masses show Miransky scaling as $x \to x_c$
- $m_n^2 \sim n$ or $m_n^2 \sim n^2$ depending on potentials

Fit of ρ mass to Miransky scaling (PotII W_0 SB)

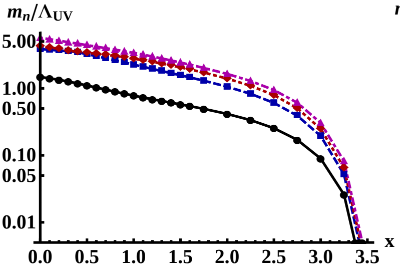


$$m_\rho/\Lambda_{\mathrm{UV}} \sim \exp\left(-\frac{\kappa}{\sqrt{x_c - x}}\right)$$

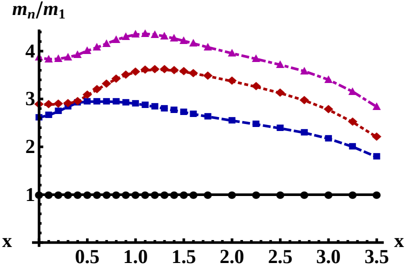
Scalar singlet masses

PotII W_0 SB:

In log scale



Normalized to the lowest state

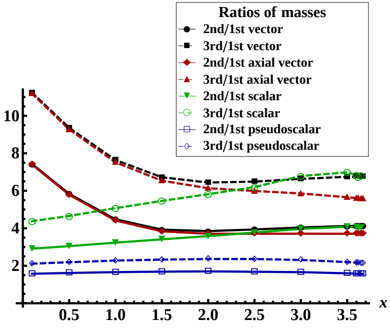


No light dilaton?

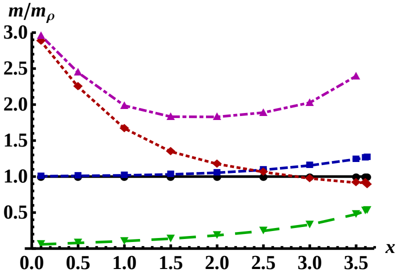
Meson mass ratios

PotII W_0 SB:

Within towers



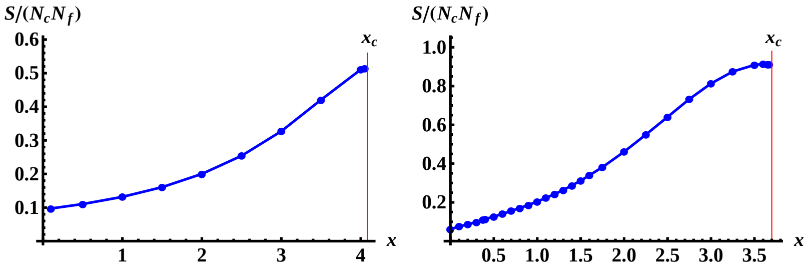
Lowest states normalized to ρ



All ratios tend to constants as $x \to x_c$: indeed no dilaton

S-parameter

PotI $W_0 = 3/11$ PotII W_0 SB



The S-parameter increases with x: no expected suppression

Conclusion

- A class of holographic bottom-up models (V-QCD) was obtained by a fusion of IhQCD with tachyonic brane action
- A subclass of V-QCD models meets expectations from QCD at qualitative level
- V-QCD has no light dilaton or suppression of the S-parameter, which might be an issue for some technicolor models

Extra slides

Extra slides . . .

Potentials I

$$
\begin{aligned}
V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))} \\
V_f(\lambda,\tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\
V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473-2726x+92x^2}{42768\pi^4}\lambda^2 \\
a(\lambda) &= \frac{3}{22}(11-x) \\
\kappa(\lambda) &= \frac{1}{\left(1+\frac{115-16x}{288\pi^2}\lambda\right)^{4/3}}
\end{aligned}
$$

In this case the tachyon diverges exponentially:

$$
\tau(r) \sim \tau_0 \exp\left[\frac{81\ 3^{5/6}(115-16x)^{4/3}(11-x)}{812944\ 2^{1/6}}\frac{r}{R}\right]
$$

Potentials II

$$
\begin{aligned}
V_g(\lambda) &= 12 + \frac{44}{9\pi^2}\lambda + \frac{4619}{3888\pi^4}\frac{\lambda^2}{(1+\lambda/(8\pi^2))^{2/3}}\sqrt{1+\log(1+\lambda/(8\pi^2))} \\
V_f(\lambda,\tau) &= V_{f0}(\lambda)e^{-a(\lambda)\tau^2} \\
V_{f0}(\lambda) &= \frac{12}{11} + \frac{4(33-2x)}{99\pi^2}\lambda + \frac{23473-2726x+92x^2}{42768\pi^4}\lambda^2 \\
a(\lambda) &= \frac{3}{22}(11-x)\frac{1+\frac{115-16x}{216\pi^2}\lambda + \lambda^2/(8\pi^2)^2}{(1+\lambda/(8\pi^2))^{4/3}} \\
\kappa(\lambda) &= \frac{1}{(1+\lambda/(8\pi^2))^{4/3}}
\end{aligned}
$$

In this case the tachyon diverges as

$$
\tau(r) \sim \frac{27\ 2^{3/4}3^{1/4}}{\sqrt{4619}}\sqrt{\frac{r-r_1}{R}}
$$

Effective potential

For solutions with $\tau = \tau_* = \mathrm{const}$

$$\mathcal{S} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) - xV_f(\lambda, \tau_*) \right]$$

IhQCD with an effective potential

$$V_{\mathrm{eff}}(\lambda) = V_g(\lambda) - xV_f(\lambda, \tau_*) = V_g(\lambda) - xV_{f0}(\lambda) \exp(-a(\lambda)\tau_*^2)$$

Minimizing for τ_* we obtain $\tau_* = 0$ and $\tau_* = \infty$

- $\tau_* = 0$: $V_{\mathrm{eff}}(\lambda) = V_g(\lambda) - xV_{f0}(\lambda)$;
  fixed point with $V'_{\mathrm{eff}}(\lambda_*) = 0$
- $\tau_* \to \infty$: $V_{\mathrm{eff}}(\lambda) = V_g(\lambda)$ (like YM, no fixed points)

Numerical solutions for backgrounds

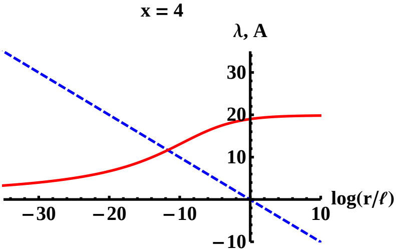
Color code:

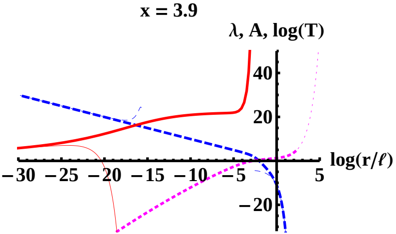
λ, A, τ ($= T$ here)

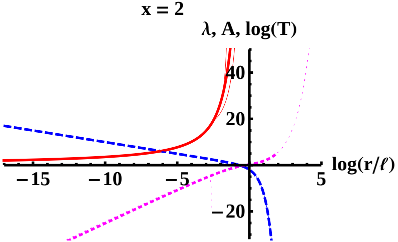
UV: $r = 0$

IR: $r = \infty$

$A \sim \log \mu \sim -\log r$



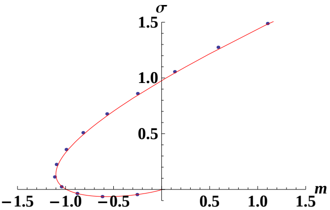


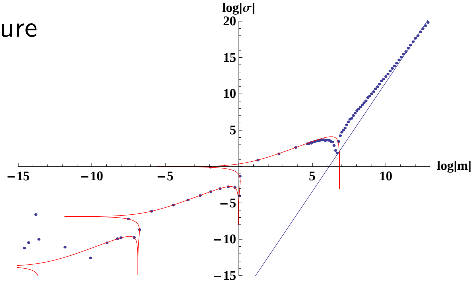


Efimov spiral

Ongoing work: the dependence $\sigma(m)$ of the chiral condensate on the quark mass

- For $x < x_c$ spiral structure



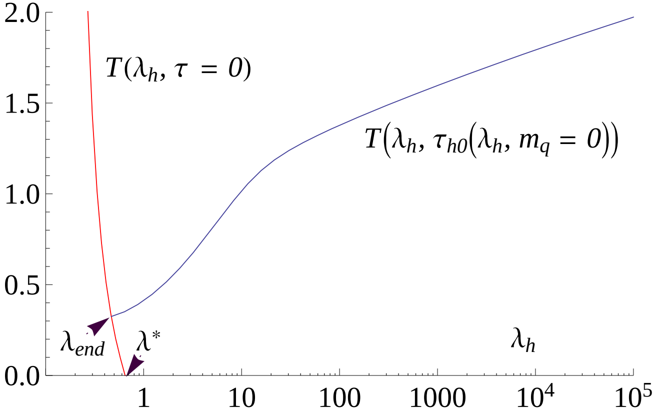


- Dots: numerical data
- Continuous line: (semi-)analytic prediction

Allows to study the effect of double-trace deformations

Black hole branches

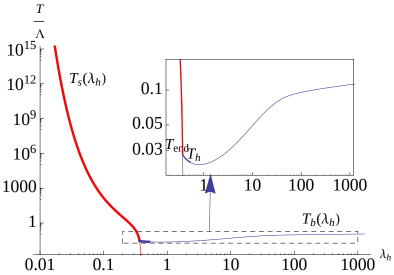
Example: PotII at $x = 3$, $W_0 = 12/11$



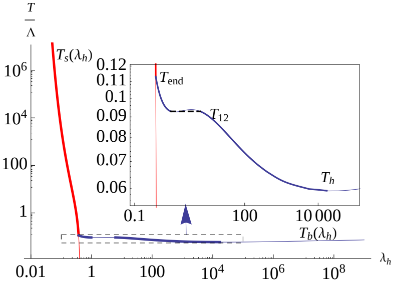
Simple phase structure: 1st order transition at $T = T_h$ from thermal gas to (chirally symmetric) BH

More complicated cases:

PotII at $x = 3$, W_0 SB



PotI at $x = 3.5$, $W_0 = 12/11$

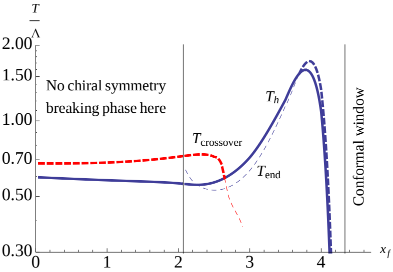


- Left: chiral symmetry restored at 2nd order transition with $T = T_{\rm end} > T_h$
- Right: Additional first order transition between BH phases with broken chiral symmetry

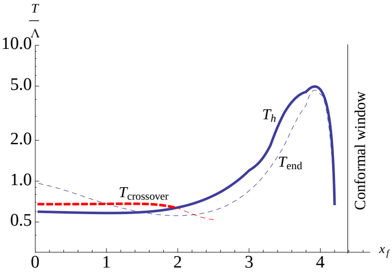
Also other cases ...

Phase diagrams on the (x, T)-plane

PotI$_*$ W_0 SB



PotII$_*$ W_0 SB



A step back: Glue – 5D dilaton gravity

For YM, "improved holographic QCD" (IhQCD): well-tested string-inspired bottom-up model

[Gursoy, Kiritsis, Nitti arXiv:0707.1324, 0707.1349]
[Gubser, Nellore arXiv:0804.0434]

$$\mathcal{S}_{\mathrm{g}} = M^3 N_c^2 \int d^5x \sqrt{g} \left[R - \frac{4}{3}(\partial\phi)^2 + V_g(\phi) \right]$$

with Poincaré invariant metric

$$ds^2 = e^{2A}(dr^2 + \eta_{\mu\nu}x^\mu x^\nu)$$

- Potential $V_g \leftrightarrow$ QCD β-function
  - $A \to \log\mu$ energy scale
  - $e^\phi \to \lambda$ 't Hooft coupling $g^2 N_c$

$$V_g = \frac{12}{\ell^2}(1 + c_1\lambda + \cdots), \quad \lambda \to 0, \qquad V_g \sim \lambda^{4/3}\sqrt{\log\lambda}, \quad \lambda \to \infty$$

Agrees well with pure YM, both a zero and finite temperature

[Gursoy, Kiritsis, Mazzanti, Nitti; Panero; . . .]

A step back: Adding flavor

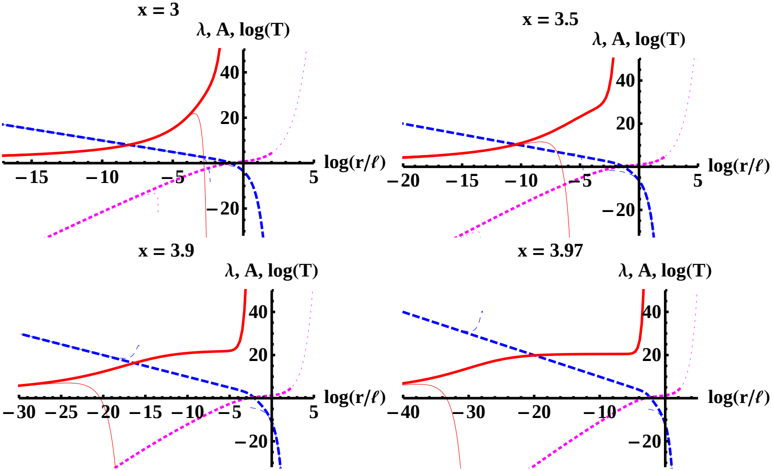
- Fundamental quarks $\rightarrow$ probe $D4 - \bar{D}4$ branes in 5D
- For the vacuum structure only the tachyon is relevant
- A tachyon action motivated by the Sen action
  - Confining asymptotics of the geometry trigger ChSB
  - Gell-Mann-Oakes-Renner relation
  - Linear Regge trajectories for mesons
  - A very good fit of the light meson masses

[Klebanov,Maldacena]

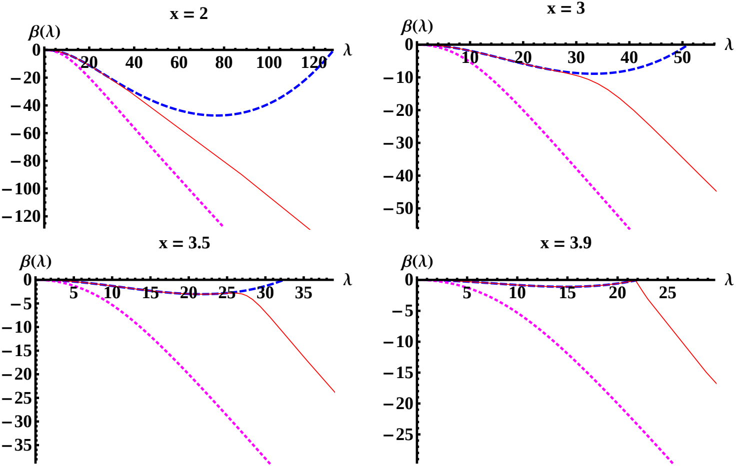
[Bigazzi,Casero,Cotrone,Iatrakis,Kiritsis,Paredes hep-th/0505140,0702155; arXiv:1003.2377,1010.1364]

Backgrounds in the walking region

Backgrounds with zero quark mass, $x < x_c \simeq 3.9959$ (λ, A, τ)



Beta functions along the RG flow (evaluated on the background), zero tachyon, YM $x_c \simeq 3.9959$



Holographic beta functions

Generalization of the holographic RG flow of IhQCD

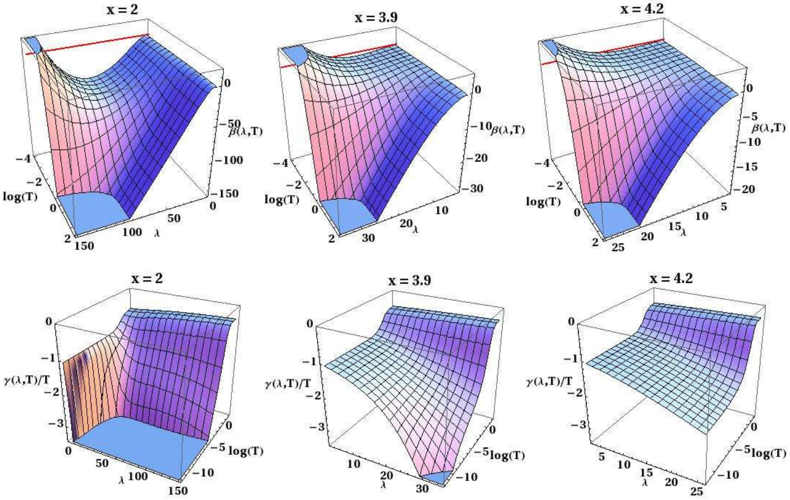
$$\beta(\lambda,\tau) \equiv \frac{d\lambda}{dA} \; ; \qquad \gamma(\lambda,\tau) \equiv \frac{d\tau}{dA}$$

linked to

$$\frac{dg_{\rm QCD}}{d\log\mu} \; ; \qquad \frac{dm}{d\log\mu}$$

The full equations of motion boil down to two first order partial non-linear differential equations for β and γ

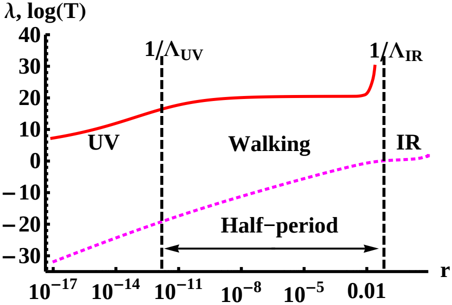
“Good” solutions numerically (unique)



Miransky/BKT scaling

As $x \to x_c$ from below: walking, dominant solution

- BF-bound for the tachyon violated at the IRFP
- x_c fixed by the BF bound: $\Delta = 2$ & $\gamma_* = 1$ at the edge of the conformal window



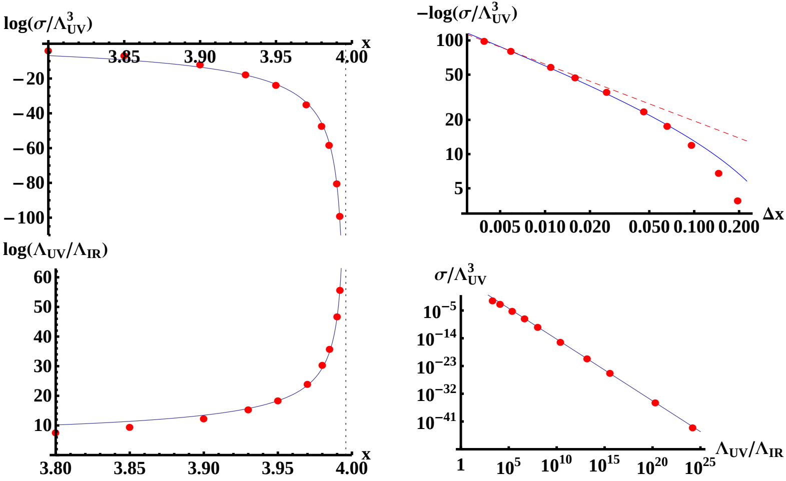
- $T(r) \sim r^2 \sin(\kappa\sqrt{x_c - x} \log r + \phi)$ in the walking region
- "0.5 oscillations" $\Rightarrow$ Miransky/BKT scaling, amount of walking $\Lambda_{\rm UV}/\Lambda_{\rm IR} \sim \exp(\pi/(\kappa\sqrt{x_c - x}))$

As $x \to x_c$

with known κ

$$\langle \bar{q} q \rangle \sim \sigma \sim \exp(-2\pi/(\kappa\sqrt{x_c - x}))$$

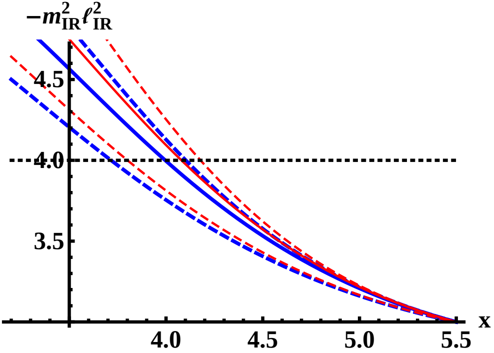
$$\Lambda_{\mathrm{UV}}/\Lambda_{\mathrm{IR}} \sim \exp(\pi/(\kappa\sqrt{x_c - x}))$$



Prediction for x_c

Dependence on the UV parameter W_0 and (reasonable) "IR choices" for the potentials

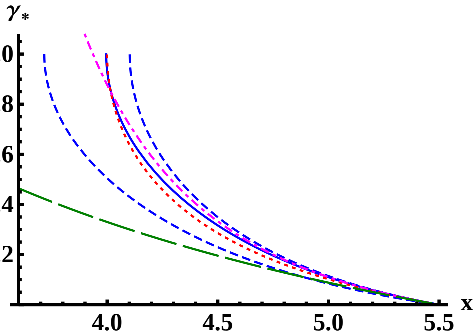
Resulting variation of the edge of conformal window $x_c = 3.7 \ldots 4.2$



Comparison to other guesses

V-QCD (dashed: variation due to W_0)
Dyson-Schwinger
2-loop PQCD
All-orders β

[Pica, Sannino arXiv:1011.3832]

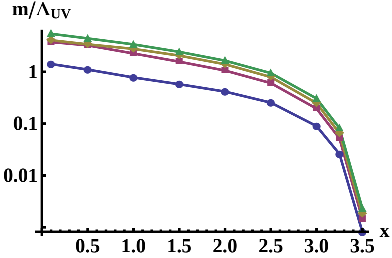


Mass spectra

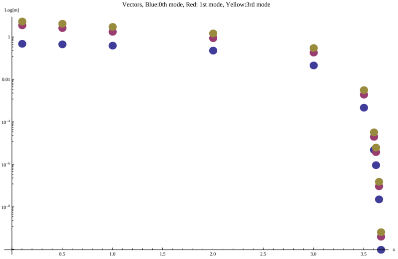
Full fluctuation analysis

- Miransky scaling
- Ratios depend mildly on x

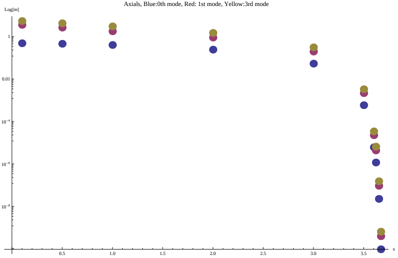
Scalars



Vectors



Axial vectors



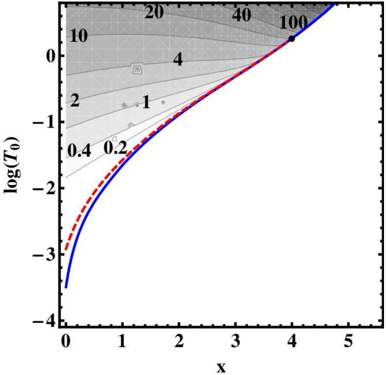
Parameters

Understanding the solutions for generic quark masses requires discussing parameters

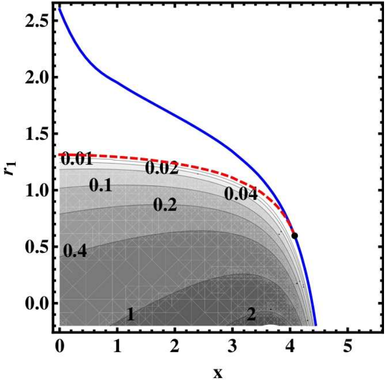
- YM or QCD with massless quarks: no parameters
- QCD with flavor-independent mass m: a single (dimensionless) parameter m/Λ_{QCD}
- In this model, after rescalings, this parameter can be mapped to a parameter (τ_0 or r_1) that controls the diverging tachyon in the IR
- x has become continuous in the Veneziano limit

Map of all solutions

All "good" solutions ($\tau \neq 0$) obtained varying x and τ_0 or r_1
Contouring: quark mass (zero mass is the red contour)

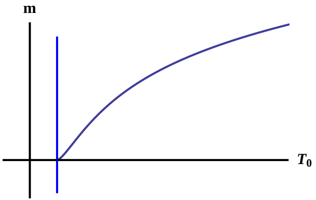


"Potentials I" $\leftrightarrow$ T_0



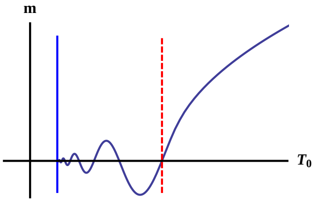
"Potentials II" $\leftrightarrow$ r_1

Mass dependence and Efimov vacua



Conformal window ($x > x_c$)

- For $m = 0$, unique solution with $\tau \equiv 0$
- For $m > 0$, unique "standard" solution with $\tau \neq 0$

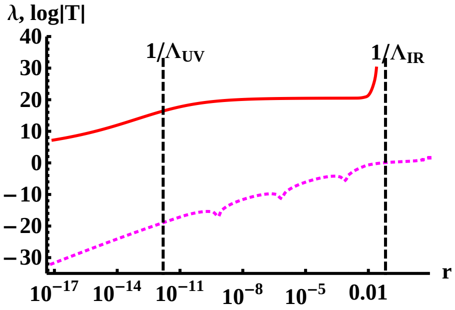


Low $0 < x < x_c$: Efimov vacua

- Unstable solution with $\tau \equiv 0$ and $m = 0$
- "Standard" stable solution, with $\tau \neq 0$, for all $m \geq 0$
- Tower of unstable Efimov vacua (small $|m|$)

Efimov solutions

- Tachyon oscillates over the walking regime
- $\Lambda_{UV}/\Lambda_{IR}$ increased wrt. "standard" solution

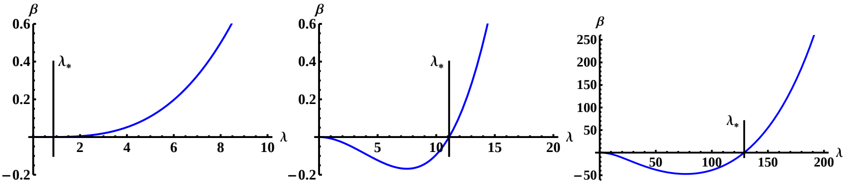


Effective potential: zero tachyon

Start from Banks-Zaks region, $\tau_* = 0$, chiral symmetry conserved $(\tau \leftrightarrow \bar{q}q)$, $V_{\text{eff}}(\lambda) = V_g(\lambda) - xV_{f0}(\lambda)$

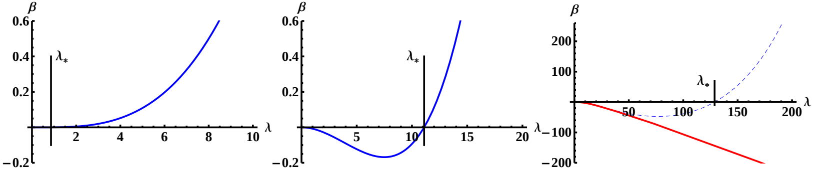
- V_{eff} defines a β-function as in IhQCD – Fixed point guaranteed in the BZ region, moves to higher λ with decreasing x
- Fixed point λ_* runs to ∞ either at finite $x(<x_c)$ or as $x \to 0$

Banks-Zaks $x \to 11/2$ | Conformal Window $x > x_c$ | $x < x_c$??



Effective potential: what actually happens

Banks-Zaks $x \to 11/2$ | Conformal Window $x > x_c$ | $x < x_c$



$\tau \equiv 0$ | $\tau \equiv 0$ | $\tau \neq 0$

- For $x < x_c$ vacuum has nonzero tachyon (checked by calculating free energies)
- The tachyon screens the fixed point
- In the deep IR τ diverges, $V_{\text{eff}} \to V_g \Rightarrow$ dynamics is YM-like

Where is x_c?

How is the edge of the conformal window stabilized?
Tachyon IR mass at $\lambda = \lambda_*$ $\leftrightarrow$ quark mass dimension

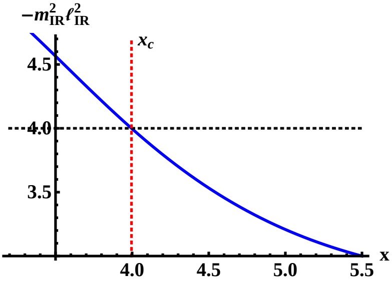
$$-m_{\rm IR}^2 \ell_{\rm IR}^2 = \Delta_{\rm IR}(4 - \Delta_{\rm IR}) = \frac{24a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - xV_0(\lambda_*))}$$

$$\gamma_* = \Delta_{\rm IR} - 1$$

Breitenlohner-Freedman (BF) bound (horizontal line)

$$-m_{\rm IR}^2 \ell_{\rm IR}^2 = 4 \;\Leftrightarrow\; \gamma_* = 1$$

defines x_c



Why $\gamma_* = 1$ at $x = x_c$?

No time to go into details ... the question boils down to the linearized tachyon solution at the fixed point

- For $\Delta_{\rm IR}(4 - \Delta_{\rm IR}) < 4 \qquad (x > x_c)$:

$$\tau(r) \sim m_q r^{\Delta_{\rm IR}} + \sigma r^{4-\Delta_{\rm IR}}$$

- For $\Delta_{\rm IR}(4 - \Delta_{\rm IR}) > 4 \qquad (x < x_c)$:

$$\tau(r) \sim C r^2 \sin\left[(\mathrm{Im}\Delta_{\rm IR}) \log r + \phi\right]$$

Rough analogy:
Tachyon EoM $\leftrightarrow$ Gap equation in Dyson-Schwinger approach
Similar observations have been made in other holographic frameworks

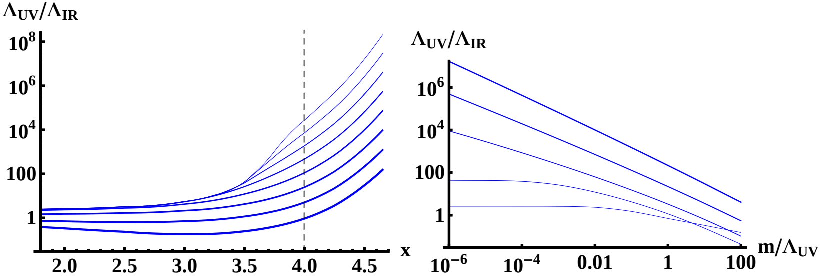
[Kutasov, Lin, Parnachev arXiv:1107.2324, 1201.4123]

Mass dependence

For $m > 0$ the conformal transition disappears

The ratio of typical UV/IR scales $\Lambda_{\mathrm{UV}}/\Lambda_{\mathrm{IR}}$ varies in a natural way

$m/\Lambda_{\mathrm{UV}} = 10^{-6}, 10^{-5}, \ldots, 10 \qquad x = 2, 3.5, 3.9, 4.25, 4.5$



sQCD phases

The case of $\mathcal{N} = 1$ $SU(N_c)$ superQCD with N_f quark multiplets is known and provides an interesting (and more complex) example for the nonsupersymmetric case. From Seiberg we have learned that:

- $x = 0$ the theory has confinement, a mass gap and N_c distinct vacua associated with a spontaneous breaking of the leftover R symmetry Z_{N_c}.
- At $0 < x < 1$, the theory has a runaway ground state.
- At $x = 1$, the theory has a quantum moduli space with no singularity. This reflects confinement with ChSB.
- At $x = 1 + 1/N_c$, the moduli space is classical (and singular). The theory confines, but there is no ChSB.
- At $1 + 2/N_c < x < 3/2$ the theory is in the non-abelian magnetic IR-free phase, with the magnetic gauge group $SU(N_f - N_c)$ IR free.
- At $3/2 < x < 3$, the theory flows to a CFT in the IR. Near $x = 3$ this is the Banks-Zaks region where the original theory has an IR fixed point at weak coupling. Moving to lower values, the coupling of the IR $SU(N_c)$ gauge theory grows. However near $x = 3/2$ the dual magnetic $SU(N_f - N_c)$ is in its Banks-Zaks region, and provides a weakly coupled description of the IR fixed point theory.
- At $x > 3$, the theory is IR free.

Saturating the BF bound (sketch)

Why is the BF bound saturated at the phase transition (massless quarks)??

$$\Delta_{\rm IR}(4-\Delta_{\rm IR}) = \frac{24a(\lambda_*)}{\kappa(\lambda_*)(V_g(\lambda_*) - xV_0(\lambda_*))}$$

- For $\Delta_{\rm IR}(4-\Delta_{\rm IR}) < 4$:
  $\tau(r) \sim m_q r^{4-\Delta_{\rm IR}} + \sigma r^{\Delta_{\rm IR}}$
- For $\Delta_{\rm IR}(4-\Delta_{\rm IR}) > 4$:
  $\tau(r) \sim C r^2 \sin\left[(\mathrm{Im}\Delta_{\rm IR}) \log r + \phi\right]$
- Saturating the BF bound, the tachyon solutions will engtangle $\rightarrow$ required to satisfy boundary conditions
- Nodes in the solution appear trough UV $\rightarrow$ massless solution

Saturating the BF bound (sketch)

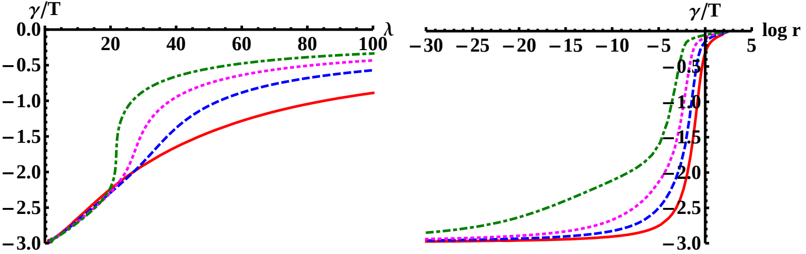
Does the nontrivial (ChSB) massless tachyon solution exist?
Two possibilities:

- $x > x_c$: BF bound satisfied at the fixed point $\Rightarrow$ only trivial massless solution ($\tau \equiv 0$, ChS intact, fixed point hit)
- $x < x_c$: BF bound violated at the fixed point $\Rightarrow$ a nontrivial massless solution exist, which drives the system away from the fixed point

Conclusion: phase transition at $x = x_c$
As $x \to x_c$ from below, need to approach the fixed point to satisfy the boundary conditions $\Rightarrow$ nearly conformal, "walking" dynamics

Gamma functions

Massless backgrounds: gamma functions $\frac{\gamma}{\tau} = \frac{d \log \tau}{dA}$



$x = 2, 3, 3.5, 3.9$