Holographic Models of Strongly Coupled Anisotropic Plasmas

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Outline

Anisotropic systems of interest in condensed matter theory
(p-wave superfluids, liquid crystals, . . .)

and (here):

anisotropic (pre-equilibrium) quark-gluon-plasma

Two top-down models for $N = 4$ super-Yang-Mills plasma with fixed anisotropy

- Singular AdS_5 [Janik & Witaszczyk (2008)]
- Regular axion-dilaton-gravity [Mateos & Trancanelli (2011)]

♣ Study observables of potential interest to heavy-ion physics:

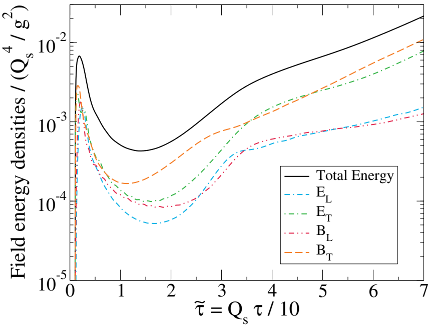
- Electromagnetic spectral functions, conductivities
- Hydrodynamic transport: shear viscosity
- Jet quenching
- Heavy quark potential

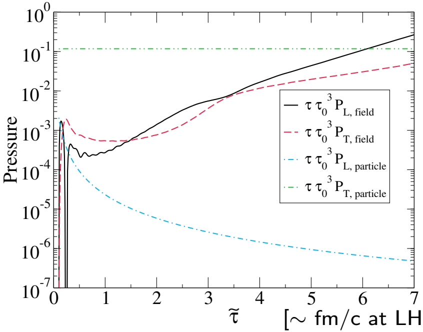
Anisotropy and heavy ion collisions

Weak coupling ("hard anisotropic loops"):
increasing anisotropy after collision counteracted by
nonabelian plasma instabilities (leading to anomalous viscosity [Asakawa, Bass, Müller '06])

Numerical studies with *fixed* anisotropy: AR, Romatschke, Strickland; Arnold, Moore; Bödeker, Rummukainen

Recently: Real-time lattice simulations of nonabelian Boltzmann-Vlasov equations in *Bjorken expansion*:
Attems, AR, Strickland, PRD87 (2013)



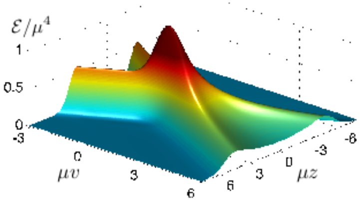


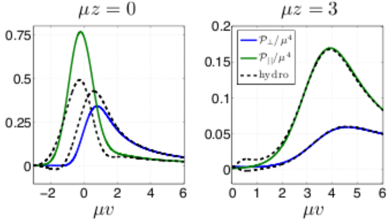
[$\sim$ fm/c at LHC]

$\rightarrow$ large anisotropies over lifetime of quark-gluon plasma

Anisotropy and heavy ion collisions

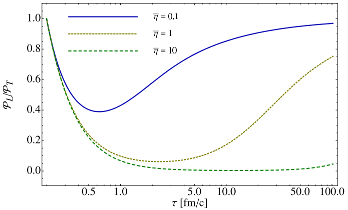
Shock waves in AdS$_5$ [Chesler, Yaffe '10] – strong pressure anisotropies (not only initial)





see also: Heller, Mateos, van der Schee, Trancanelli; $\longrightarrow$ **talk by Michał Heller**

Florkowski, Martinez, Ryblewski, Strickland 2012: *anisotropic hydro*
modifications to model instrinsic anisotropies (resumming larger viscous corrections) throughout lifetime of plasma



Dual geometry of (anisotropic) N=4 SYM plasma

Looking for simpler (holographic) model: stationary anisotropic plasma
(should be good for observables on sufficiently small time scales)

In Fefferman-Graham coordinates of asymptotically AdS (boundary at $z = 0$)

$$ds^2 = \frac{\gamma_{\mu\nu}(x^\sigma, z) dx^\mu dx^\nu + dz^2}{z^2},$$

energy-momentum tensor contained in

$$\gamma_{\mu\nu}(x^\sigma, z) = \eta_{\mu\nu} + z^4 \gamma^{(4)}_{\mu\nu}(x^\sigma) + \mathcal{O}(z^6)$$

as

$$\langle T_{\mu\nu}(x^\sigma)\rangle = \frac{N_c^2}{2\pi} \gamma^{(4)}_{\mu\nu}(x^\sigma)$$

Janik&Peschanski 2005: construct geometry for given profile $\langle T_{\mu\nu}(x^\sigma)$ and select physical solutions from requirement of regularity of solutions of Einstein equations $R_{MN} = -4g_{MN}$

Singular anisotropic gravity dual

Dual geometry for *isotropic* traceless energy momentum tensor:
the AdS *black hole* (black brane) – Hawking temperature is dual temperature

Dual geometry for *static anisotropic* $\langle T_{\mu\nu}(x^\sigma)\rangle = \mathrm{diag}(\epsilon, P_L, P_T, P_T)$
contains *naked singularity*: [Janik & Witaszczyk 2008]

$$ds^2 = g_{tt}(u)dt^2 + g_{LL}(u)dx_L^2 + g_{TT}(u)d\mathbf{x}_T^2 + \frac{1}{4u^2}du^2, \qquad u \equiv z^2$$

$$\begin{aligned}
g_{tt}(u) &= -\frac{1}{u}(1+A^2u^2)^{1/2-\sqrt{36-2B^2}/4}(1-A^2u^2)^{1/2+\sqrt{36-2B^2}/4} \\
g_{LL}(u) &= \frac{1}{u}(1+A^2u^2)^{1/2-B/3+\sqrt{36-2B^2}/12}(1-A^2u^2)^{1/2+B/3-\sqrt{36-2B^2}/12} \\
g_{TT}(u) &= \frac{1}{u}(1+A^2u^2)^{1/2+B/6+\sqrt{36-2B^2}/12}(1-A^2u^2)^{1/2-B/6-\sqrt{36-2B^2}/12}
\end{aligned}$$

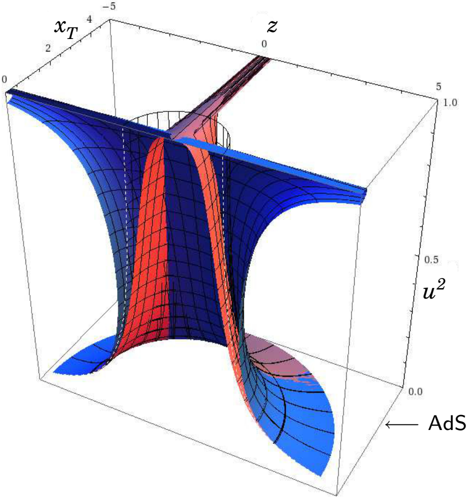
with $\epsilon = \frac{A^2}{2}\sqrt{36-2B^2}, \quad P_L = \frac{A^2}{6}\sqrt{36-2B^2} - \frac{2A^2B}{3}, \quad P_T = \frac{A^2}{6}\sqrt{36-2B^2} + \frac{A^2B}{3}$

$B = -\sqrt{6}\ldots\sqrt{2}$ delimited by $P_T > 0$ and $P_L > 0$, resp. (otherwise $B = -\sqrt{18}\ldots\sqrt{18}$)

$B \neq 0$: horizon at $u = 1/A$ becomes **naked singularity**

(induced metric at $t = const.$, $u = 1/A$ is degenerate: $g_{LL}g_{TT}^2 \propto (1-A^2u^2)^{\left[6-\sqrt{36-2B^2}\right]/4}$

Singular anisotropic gravity dual



⟵ naked singularity

⟵ AdS boundary

Asymptotically spherical congruences of (holographically) radial light-like geodesics which get deformed into ellipsoids as they approach the singularity at $u = 1$ in units where $A = 1$.
Blue: prolate with $B = -\sqrt{6}$; Red: oblate with $B = \sqrt{2}$

Spectral function of current-current correlator

$\chi_{\mu\nu}(K) = -2\,\mathrm{Im}\, C^{ret}_{\mu\nu}(K) = -2\mathrm{Im} \int d^4X\, e^{-iK\cdot X}\, \langle J^{EM}_\mu(0) J^{EM}_\nu(X)\rangle^{ret.}$

AdS/CFT:

$C^{ret}_{\mu\nu}$ determined by asymptotic behavior of solutions of 5D Maxwell equations

$$\partial_A(\sqrt{-g} g^{AC} g^{BD} F_{CD}) = 0$$

(A_C bulk gauge field dual to conserved U(1) R-current, not the electromagnetic field!)

[Son&Starinets:]

retarded correlator obtained by infalling boundary conditions (complex)

Anisotropic case:

different for wave vector $\mathbf{k}$ parallel or orthogonal to direction of anisotropy $\mathbf{e}_L$:

$C^{ret}_{\mu\nu} = \sum P^a_{\mu\nu} \Pi_a(K)$ with orthogonal $P^a_{\mu\nu}$

$a = T, L$ when $\mathbf{k} \parallel \mathbf{e}_L$

$a = 1, 2, L$ when $\mathbf{k} \parallel \mathbf{e}_1 \perp \mathbf{e}_L$

$$\Pi_a(K) = -\frac{2}{g_B^2} \lim_{u\to 0} \frac{E'_a(K,u)}{E_a(K,u)} \quad \text{with } g_B = 16\pi^2 R/N_c^2$$

Spectral function of current-current correlator

JW-model:

$E_a(K, u)$ described by 2nd order ODE's in u

$$\frac{d^2}{du^2}\phi + \frac{C_1}{(1-u)}\frac{d}{du}\phi + \frac{\omega^2 C_2}{(1-u)^\alpha}\phi = 0 \qquad \text{with } \alpha = (2 + \sqrt{36 - 2B^2})/4 \leq 2$$

Isotropic: $\alpha = 2$ allows Frobenius ansatz at singular point $u = 1$ (horizon) with characteristic exponent $\pm i\omega/\sqrt{8}$ (ingoing/outgoing b.c.)

Anisotropic: $B \neq 0 \to \alpha < 2 \Rightarrow$ different character

coordinate transform $= (1-u)^{(2-\alpha)}$ gives

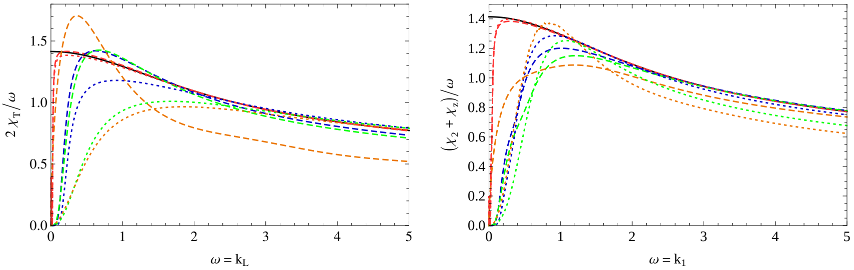
$$\frac{d^2}{dx^2}\phi + \frac{\beta}{x}\frac{d}{dx}\phi + \frac{\gamma^2}{x}\phi = 0 \qquad \text{with some } \beta, \gamma \ (\to \infty \text{ as } \alpha \to 2)$$

Solution $\quad \phi(u) \sim (1-u)^{(2-\alpha)(1-\beta)/2} \mathrm{H}^{(1,2)}_{1-\beta}(2\gamma(1-u)^{(2-\alpha)/2})$

where the Hankel function of the second kind $\mathrm{H}^{(2)}_\nu$ corresponds to ingoing boundary conditions – used in numerical evaluation

NB: JW-naked singularity rather benign! Still allows purely ingoing b.c.!

Spectral function of current-current correlator



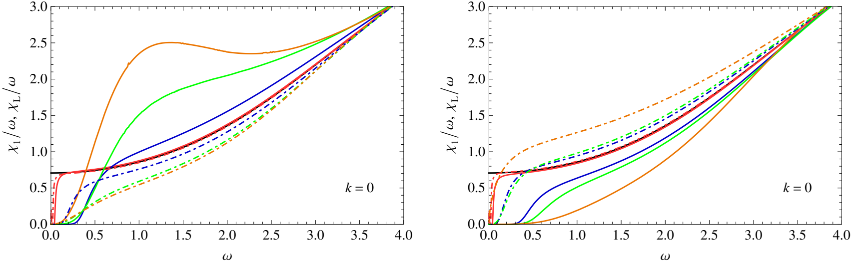
dashed lines: oblate, and dotted lines: prolate anisotropies

$B = 0$ (black $\equiv$ result of Huot, Kovtun, Starinets, Moore & Yaffe 2006),
$B = 0.1$ (red, dashed), $B = -0.1$ (red, dotted),
$B = 1$ (blue, dashed), $B = -1$ (blue, dotted),
$B = \sqrt{2}$ (green, dashed), $B = -\sqrt{6}$ (green,dotted),
$B = \pm 3$ (orange – involving negative pressures)

Anisotropic AC conductivities

prolate vs. oblated anisotropies
black: $B = 0$
red: $B = \mp 0.1$
blue: $B = \mp 1$
green: $B = -\sqrt{6}, +\sqrt{2}$

orange: $B = \mp 3$



full lines: longitudinal conductivity
dashed lines: transverse conductivity

- DC conductivities zero $\leftrightarrow$ hydrodynamic limit singular

Regular top-down model: Anisotropic axion-dilaton gravity

[Mateos, Trancanelli '11]

Boundary

$$S = S_{\mathcal{N}=4} + \frac{1}{8\pi^2}\int \theta(z)\mathrm{Tr}\; F \wedge F$$

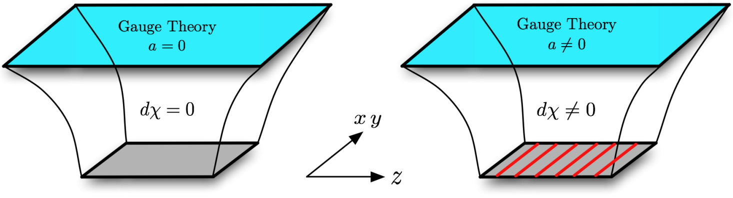
with $\theta(z) = 2\pi a z$

Bulk

$$S_{bulk} = \frac{1}{2\kappa^2}\int \sqrt{-g}\Big(R + 12 - \frac{(\partial\phi)^2}{2} - \frac{e^{2\phi}(\partial\chi)^2}{2}\Big)$$

with axion $\chi = az$

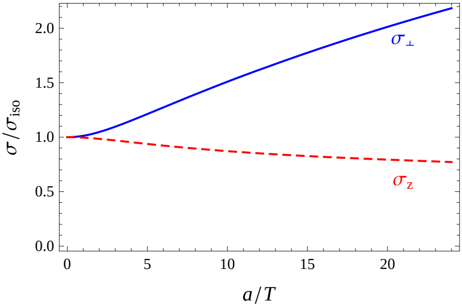
anisotropy parameter $a = \frac{\lambda n_{\mathrm{D7}}}{4\pi N_c}$ with n_{D7} density of D7 branes (magnetic source for χ)
extended along t, x, y but not z, u
homogeneously distributed along z-direction with $n_{\mathrm{D7}} = dN_{\mathrm{D7}}/dz$



anisotropic bulk geometry, anisotropic horizon

Electrical DC conductivity

Because of regular (albeit anisotropic) horizon: $\exists$ hydrodynamic limit



NB: $\sigma_\perp > \sigma_z$ independently of whether plasma oblate or prolate!

JW model: although $\sigma_{\perp,z} \to 0$, similarly $\forall B$: $\sigma_\perp(\omega)/\sigma_z(\omega) > 1$ for small ω

Shear viscosity in anisotropic fluid

Kubo formula

$$\eta_{ijkl} = -\lim_{\omega\to 0}\frac{1}{\omega}\,\mathrm{Im}\,G^R_{ij,kl}(\omega,0)$$

with $G^R_{ij,kl}(\omega,0) = -i\int dt\,d\mathbf{x}\,e^{i\omega t}\,\theta(t)\,\langle[T_{ij}(t,\mathbf{x}),T_{kl}(0,\mathbf{0})]\rangle$

Axisymmetry around z-axis (direction of anisotropy):

$\rightarrow$ 2 different shear viscosities:

- $\eta_\perp = \eta_{xyxy}$
  (shear planes $\perp$ z-axis)
- $\eta_{\|} = \eta_{xzxz} = \eta_{yzyz}$
  (shear planes $\|$ z-axis)

Calculating η/s with gauge/gravity duality

Kubo formula

$$\eta_{ijkl} = -\lim_{\omega \to 0} \frac{1}{\omega} \, \mathrm{Im}\, G^R_{ij,kl}(\omega, 0)$$

with $G^R_{ij,kl}(\omega, 0) = -i \int dt\, d\mathbf{x}\, e^{i\omega t}\, \theta(t) \, \langle [T_{ij}(t, \mathbf{x}), T_{kl}(0, \mathbf{0})] \rangle$

Gauge/gravity duality

perturb metric by $\psi_a = h^i_j$ and expand action to second order in ψ_a
$\Rightarrow$ effective action for massless scalar ψ_a

$$G^R_a(q) = -\lim_{u \to 0} \frac{\Pi_a(u, q)}{\psi_a(u, q)} \qquad \text{with } \Pi_a = \frac{\partial \mathcal{L}^{(2)}}{\partial(\partial_u \psi_a)} \propto \partial_u \psi_a$$

retarded correlator $\leftrightarrow$ infalling boundary conditions at horizon

MT model:

Obtained numerically (on numerically given background!) and also from membrane paradigm

Calculating η/s from membrane paradigm

Membrane paradigm [Iqbal, Liu '08]

generic transport coefficient of boundary theory $\Rightarrow$ geometric quantities evaluated at horizon

at the horizon

$$\psi_a(t, u, \mathbf{x}) = \psi_a(v, \mathbf{x}) \qquad \text{where } dv = dt - \sqrt{\frac{g_{uu}}{-g_{tt}}}du$$

regularity in infalling coordinates implies $\Pi_a \propto \partial_t \psi_a$

shear viscosity

$$\eta_a(u_h) = \frac{\Pi_a(u_h, q)}{i\omega\psi_a(u_h, q)} \qquad \text{with } \Pi_a(u_h, q) \propto i\omega\psi_a$$

if $\partial_u \eta_a = 0$ in limit of zero momenta and frequency $\rightarrow$ trivial RG flow to boundary

η/s from membrane paradigm

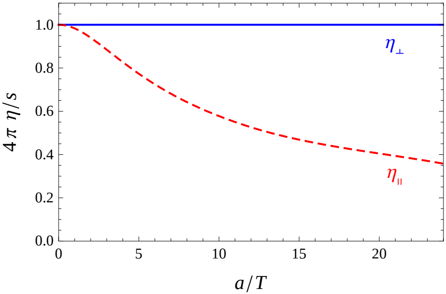
Two shear viscosity components with trivial RG flow
(coinciding with numerical result from Kubo formula/absorption calculation)

- purely transverse $\psi_\perp = h^x_y$

$$\eta_\perp = \frac{s}{4\pi}$$

- longitudinal $\psi_\parallel = h^x_z$

$$\eta_\parallel = \eta_\perp \frac{g_{xx}(u_h)}{g_{zz}(u_h)} = \frac{s}{4\pi\mathcal{H}(u_h)}$$



[AR, Dominik Steineder, PRL 2012]

Violation of the holographic viscosity bound!

unbounded: $\mathcal{H} \to \infty$ as $\frac{a}{T} \to \infty$,

but eventually breakdown of supergravity approximation (naked singularity at $T = 0$)

Third shear viscosity component in bulk (only)

3rd shear viscosity $\psi_{\tilde{L}} = h^z_x$

$$\eta^z{}_x{}^z{}_x = \eta_\perp \frac{g_{zz}(u_h)}{g_{xx}(u_h)} = \frac{s\mathcal{H}(u_h)}{4\pi} > \frac{s}{4\pi}$$

Reason for 3rd viscosity component, while axi-symmetry should allow only 2: Wilsonian energy-momentum tensor away from the boundary is nonsymmetric! (cp. Adams, Balasubramanian, McGreevy JHEP 0811)

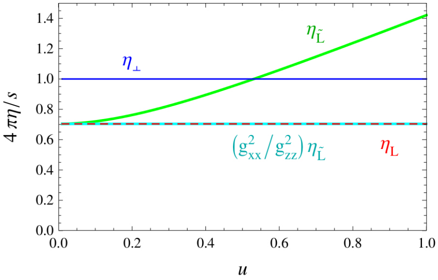
Nontrivial flow towards boundary!

Mamo JHEP1210: analytic check to order a^2;

Steineder (thesis 2012): to all orders numerically

$$\partial_u(\eta_{\tilde{L}}) \propto a^2 \quad \Rightarrow \quad \eta_{\tilde{L}} = \eta_{\tilde{L}}(u)$$

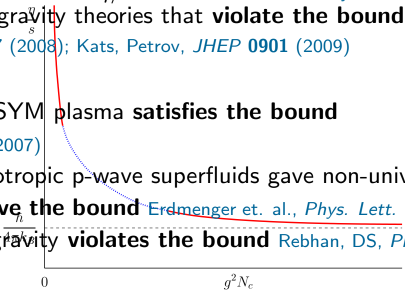
$\leftrightarrow$ **only 2 shear viscosities in boundary theory**



Other deviations from universal KSS result

Prior:

- higher derivative gravity!
  - finite coupling corrections **increase** η/s Buchel et. al., *Nucl. Phys.* **B707** (2005)
  - but also higher derivative gravity theories that **violate the bound** were found
    Brigante et. al., *Phys. Rev.* **D77** (2008); Kats, Petrov, *JHEP* **0901** (2009)
- spatial anisotropy:
  - non-commutative $\mathcal{N} = 4$ SYM plasma **satisfies the bound**
    Landsteiner, Mas, *JHEP* **0707** (2007)
  - bottom-up model for anisotropic p-wave superfluids gave non-universal **shear viscosity component above the bound** Erdmenger et. al., *Phys. Lett.* **B699** (2011); ...
  - anisotropic axion-dilaton gravity **violates the bound** Rebhan, DS, *Phys. Rev. Lett.* **108** (2012)



By now also:

- anisotropic top-down Einstein gravity model with 5+1d field theory found which **violates the bound**

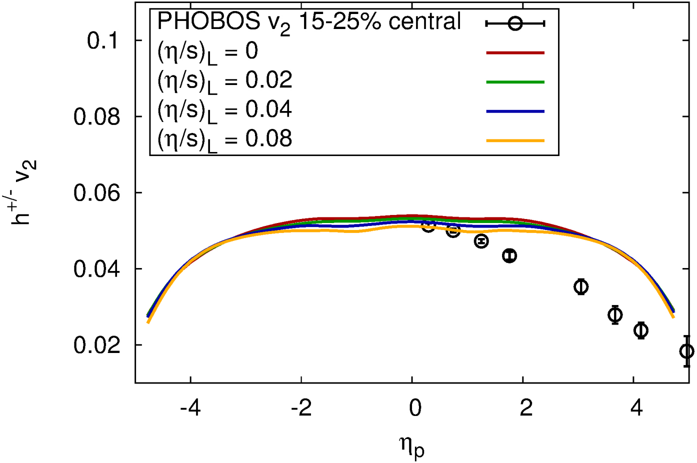
Polchinski, Silverstein, *Class. Quant. Grav.* **29** (2012)

Implications for QGP hydro simulations?

v_2 dominantly driven by $\eta_\perp$ which respects KSS bound
but (insignificant) effect for rapidity dependence:

B. Schenke: MUSIC code (private communication)

Hydro simulations with $\eta_L \neq \eta_\perp$:



Jet quenching

Chernicoff et. al. JHEP 1208; Rebhan, DS, JHEP 1208

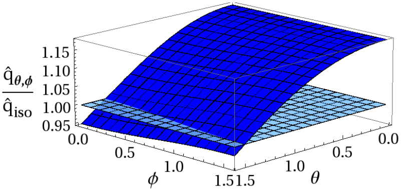
momentum broadening Δp of a hard parton moving at angle θ wrt z-axis

$\theta = 0$: rotationally invariant broadening

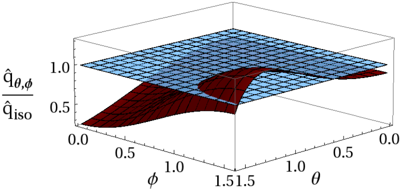
$\theta \neq 0$: dependence on directions orthogonal to partion trajectory

$\phi = 0 \ldots \frac{\pi}{2}$: Δp measured orthogonal to ... in plane $[\hat{v}\ \hat{z}]$

$\theta = \frac{\pi}{2}$: $\phi = 0 \rightarrow \hat{q}_\perp$, $\phi = \frac{\pi}{2} \rightarrow \hat{q}_L$ MT: always $\boxed{\hat{q}_L > \hat{q}_\perp}$



oblate



prolate plasma

• agrees with Hard Anisotropic Loop calculation [Romatschke '07; Baier, Mehtar-Tani '08] for oblate, but not for prolate

• inclusion of effects of chromomagnetic fields from plasma instabilities however also points to $\hat{q}_L > \hat{q}_\perp$ for both oblate and prolate plasma

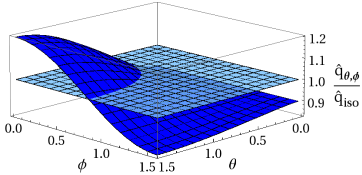
[Dumitru et al., 2006; Ipp, AR, Strickland, in preparation]

Jet quenching

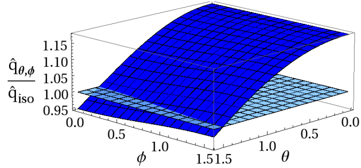
Qualitative differences:

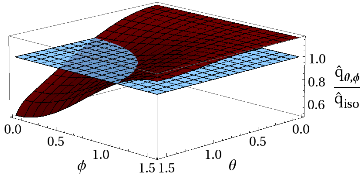
JW model ($\epsilon = const.$)

MT model ($s = const.$)

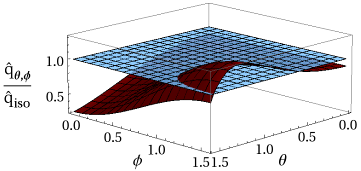


oblate





prolate

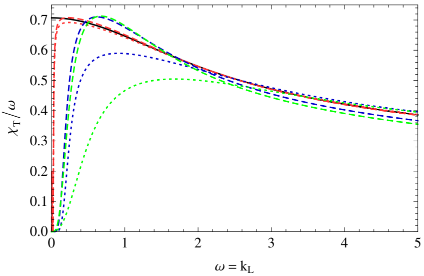


Opposite trend in JW model for oblate plasma (relevant for QGP)!

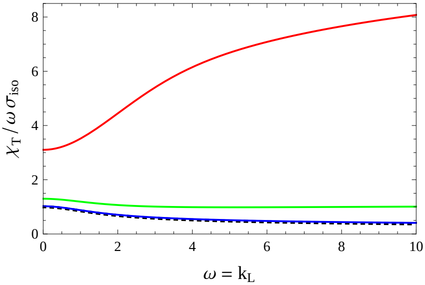
More differences between JW and MT models

Spectral densities for photons

AR, Steineder *JHEP* **1108** (2011); Patino, Trancanelli, `arXiv:1211.2199`



JW model

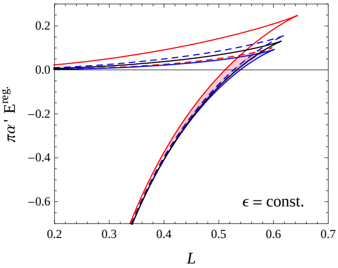


MT model

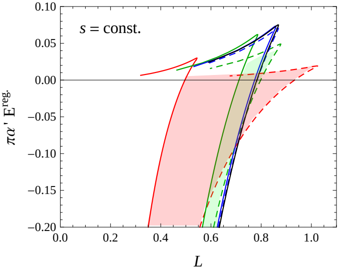
Stark differences, in particular for large momenta!

More differences between JW and MT models

Heavy quark potentials AR, Steineder, *JHEP* **1208** (2012)



JW model



MT model

JW: quarks separately in z-direction have deeper (shallower) potential for oblate (prolate) anisotropy in qualitative agreement with weak coupling (hard anisotropic loop) results

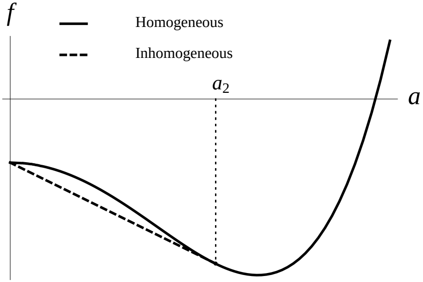
MT: always qualitatively like prolate JW

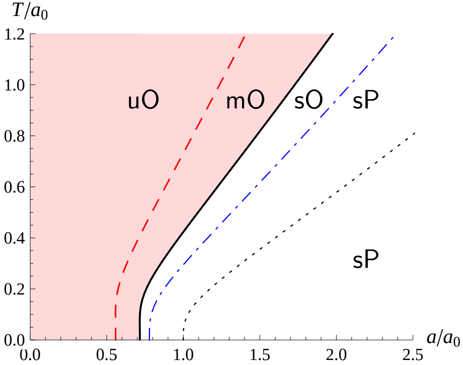
Thermodynamics of MT model (infinite coupling)

Mateos, Trancanelli, JHEP 1107; Gynther, AR, Steineder, JHEP 1210

Instability against redistribution of homogeneous anisotropy "charge" density a into inhomogeneous (lasagna) phase for $0 < a < a_2$

(vaguely reminiscent of filamentation instability in weakly coupled anisotropic plasma)





s/u/m: stable/unstable/metastable; O/P: oblate/prolate

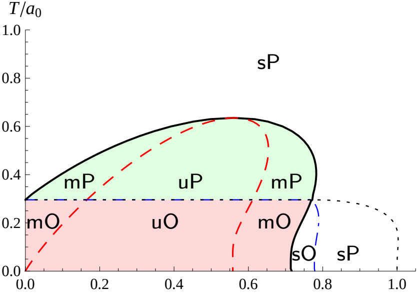
Thermodynamics of MT model (zero…weak coupling)

Gynther, AR, Steineder, JHEP 1210

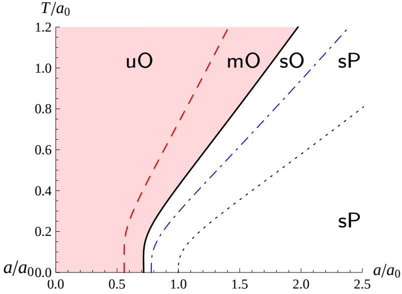
plasma of weakly (or non-) interacting vector bosons coupled to anisotropic Chern-Simons charge

• anisotropic dispersion laws, but no unstable modes (unlike hard anisotropic loop theory!)

• yet: even richer phase diagram with instabilities of homogeneous phase against redistribution of anisotropy charge



$(g = 0)$



$(g^2 N_c = \infty)$

s/u/m: stable/unstable/metastable; O/P: oblate/prolate

Conclusion

- Two interesting toy models for strongly coupled anisotropric SYM plasmas:
  JW model: simple singular geometry with rather benign naked singularity
  MT model: regular equilibrium geometry with anisotropy through linear axion
- MT model leads to longitudinal shear viscosity η_L
  below the KSS result universal to isotropic Einstein gravity duals!
  (unfortunately elliptic flow rather insensitive to η_L)
- Stark differences of heavy-ion physics observables in both models!
  Jet quenching: only MT model same trend as expected from weak-coupling plasma instabilities
  Heavy quark potential: only JW model same trend as weak coupling results
- Time-dependent nonequilibrium AdS with colliding shock waves could in principle decide whether those are good toy models!