Holographic Models for Theories with Hyperscaling Violation

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Hyperscaling Violating Lifshitz

- Recently a new class of space-times with potential applications to CMT has been introduced. These are the so-called hyperscaling violating Lifshitz (hvLif) space-times:

4D hvLif metric: $$ds^2 = r^\theta \left(-\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{dx^2 + dy^2}{r^2} \right) .$$

[Charmousis, Goutéraux, Kim, Kiritsis, Meyer, 2010], [Ogawa, Takayanagi, Ugajin, 2011], [Huijse, Sachdev, Swingle, 2011], [Dong, Harrison, Kachru, Torroba, Wang, 2011].

- These space-times are generically UV and IR singular and should be thought of as effective geometries for a certain energy range of the dual field theory.

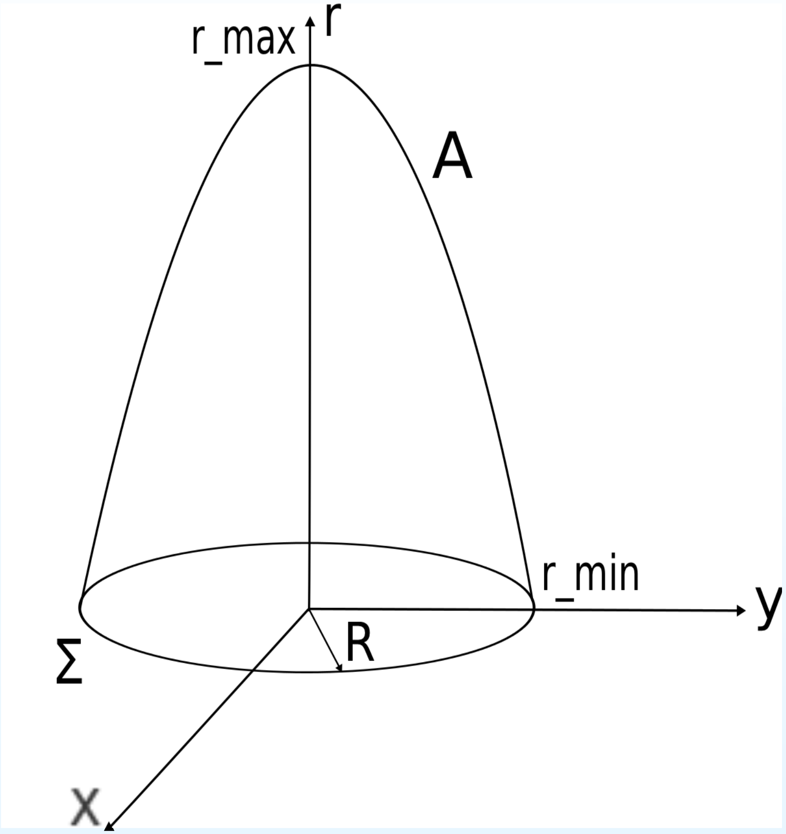
Outline of this talk

4D hvLif metric: $$ds^2 = r^\theta \left(-\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{dx^2 + dy^2}{r^2} \right)$$

For related work see [Goutéraux, Kiritsis, 2012] (previous talk by Elias Kiritsis).

- Motivation:
  - Entanglement entropy
  - Thermal entropy
  - Singularities
- Holographic models:
  - Einstein–Proca-dilaton model
  - Probe fields

Entanglement entropy



- Boundary entangling region with surface area $\Sigma \sim R$.
- Minimal bulk surface ending on boundary entangling region with surface A.
- Entanglement entropy $S_E \sim A$ [Ryu, Takayanagi, 2006].

- For $0 \leq \theta < 1$ we have an area law: $S_E \sim \Sigma$.
- For $\theta = 1$ we have $S_E \sim \Sigma \log \Sigma$.
- $1 < \theta < 2$: something between $\Sigma \log \Sigma$ and Σ^2.

Entanglement entropy

- The entanglement entropy between a region A and its complement in a quantum field theory in its ground state generally scales as the area of ∂A [Bombelli, Koul, Lee, Sorkin, 1986], [Srednicki, 1993].
- Systems with a co-dimension one Fermi surface display logarithmic violations of the area law [Swingle, 2009].

Thermal entropy

- A typical finite temperature deformation looks like:

$$ds^2 = r^\theta \left(-f \frac{dt^2}{r^{2z}} + \frac{dr^2}{f r^2} + \frac{dx^2 + dy^2}{r^2} \right) , \qquad f = 1 - \left(\frac{r}{r_h} \right)^{2+z-\theta}$$

- Depending on the Lagrangian the metric may be more complicated.
- The thermal entropy density scales with temperature as

$$S \sim r_h^{\theta-2} \sim t^{(\theta-2)/z} \sim T^{(2-\theta)/z} .$$

Thermal entropy

- Naive scaling for a system with d spatial dimensions is $S \sim T^{d/z}$ (known as hyperscaling). Here $d = 2$ but $S \sim T^{(2-\theta)/z}$ and so $\theta \neq 0$ violates hyperscaling.
- If $\theta = 1$ the thermal entropy scales with temperature as $S \sim T^{1/z}$.
- Hence it sees an effective one-dimensional space of thermal excitations with dynamical critical exponent z.
- This ties in well with the fact that for $\theta = 1$ the entanglement entropy scales like that of a co-dimension one Fermi surface.
- Third law of thermodynamics: $2 - \theta > 0$.

Singularities

- Consider the metric with $\theta > 0$:

$$ds^2 = r^\theta \left(-\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{dx^2 + dy^2}{r^2} \right)$$

- Curvature scalars blow up as $r \to 0$ (UV) and go to zero as $r \to \infty$ (IR).
- Tidal forces blow up as $r \to \infty$ (IR) unless $z = 3/2$ and $\theta = 1$ [Copsey, Mann, 2012].
- This singles out $\theta = 1$ and $z = 3/2$ as an IR regular metric whose entanglement and thermal entropies agree with that of a co-dimension one Fermi surface.

Singularities

- $\theta = 1$ and $z > 3/2$ (NEC) can still be interesting as we could imagine replacing the IR with something else that is regular.
- The UV for $\theta = 1$ is for all values of z singular so we cannot trust this metric for r very small.
- We imagine the $\theta = 1$ hvLif metric as being an effective low energy description of some asymptotically AdS/Lifshitz space-time.

The EPD model

- A toy model (as in not necessarily low energy string theory) Lagrangian supporting hvLif space-times:

$$\mathcal{L} = \sqrt{-g}\left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4}e^{a\phi}F^2 - \frac{m^2}{2}e^{b\phi}A^2 - V_0 e^{(b-a)\phi}\right) .$$

  - For $m^2 = 0$ this is called the Einstein–Maxwell-dilaton (EMD) model.
  - For $m^2 \neq 0$ we will call this the Einstein–Proca-dilaton (EPD) model.
  - For $m^2 > 0$ we can write it as a Maxwell field coupled to a charged complex scalar [Goutéraux, Kiritsis, 2012].
  - We will allow $m^2 < 0$ (more later).

Solution $(\theta, z, \alpha, \phi_0)$

$$\begin{aligned} ds^2 &= r^\theta \left(-\frac{dt^2}{r^{2z}} + \frac{dr^2}{r^2} + \frac{dx^2 + dy^2}{r^2} \right) , \\ \phi &= \phi_0 + \alpha \log r \, , \\ A &= \pm e^{-a\phi_0/2} \left(\frac{2(z-1)}{z + \frac{b\alpha}{2}} \right)^{1/2} r^{-b\alpha/2} \frac{dt}{r^z} \, . \end{aligned}$$

- The Lagrangian parameters are:

$$\begin{aligned} V_0 &= -\frac{1}{2} \left(\theta^2 - 6\theta + 8 + 2z^2 - 2\theta z + 2z + \alpha^2 \right) e^{(a-b)\phi_0} , \\ m^2 &= \frac{z + \frac{b\alpha}{2}}{2(z-1)} \left[(\theta - 2)(\theta - 2z + 2) - \alpha^2 \right] e^{(a-b)\phi_0} , \\ b &= \frac{\alpha^2 - \theta(\theta - 2)}{(z-1)\alpha} , \\ a &= \frac{\alpha^2 - \theta(\theta - z - 1)}{(z-1)\alpha} \, . \end{aligned}$$

Scale transformations

- Under the Lifshitz scaling

$$r \to \lambda r\,, \qquad t \to \lambda^z t\,, \qquad \vec{x} \to \lambda \vec{x}\,,$$

  we have

$$ds^2 \to \lambda^\theta ds^2\,, \qquad F \to \lambda^{-b\alpha/2} F\,, \qquad \phi \to \phi + \alpha \log \lambda\,.$$

- In the $\theta = 0$ case the Lifshitz scaling becomes a symmetry. A natural requirement is to demand that for $\theta = 0$ the matter fields respect this symmetry. In order that this symmetry is restored for θ going to zero we need that for small θ we have

$$\alpha^2 = c_1\theta + c_2\theta^2 + \mathcal{O}(\theta^3)\,, \qquad c_1 \neq 0\,.$$

The EMD vs. the EPD model

- General features of the EMD model:
  - Easy to write down black hole solutions (thermodynamics the same as for $m^2 \neq 0$).
  - Matter fields break the scale symmetry when $\theta = 0$.
  - Does not describe $\theta = 1$ and $z = 3/2$.
- General features of the EPD model:
  - θ controls the breaking of scale invariance also for the matter fields.
  - Can account for all (θ, z) satisfying the NEC if we allow $m^2 < 0$.
  - Difficult to construct analytic black hole solutions.

$\theta = 1$ and $z = 3/2$

- The mass parameter is

$$m^2 = \frac{z + \frac{b\alpha}{2}}{2(z-1)} \left[(\theta - 2)(\theta - 2z + 2) - \alpha^2 \right] e^{(a-b)\phi_0} \, .$$

- For $\theta = 1$ and $z = 3/2$ we get $m^2 < 0$.
- This does not need to be a problem as e.g. on AdS we have the Breitenlohner–Freedman bound.
- To find out whether it is admissible to have $m^2 < 0$ we look at linearized perturbations around an hvLif space-time.

Perturbations

$$g_{\mu\nu} = \bar{g}_{\mu\rho}\left(\delta^\rho_\nu + \epsilon h^\rho{}_\nu\right) ,$$
$$A_\mu = \bar{A}_t\left(\delta^t_\mu + \epsilon a_\mu\right) ,$$
$$\phi = \bar{\phi} + \epsilon\varphi .$$

where the barred fields refer to the hvLif background.

- We will employ radial gauge, i.e. $h_{r\mu} = 0$.
- We consider purely radial perturbations. These satisfy scale invariant equations.
- Types of perturbations:
  - tensor perturbations: h_{xy}, $h_{xx} - h_{yy}$,
  - vector perturbations: h_{ti}, a_i with $i = x, y$,
  - scalar perturbations: $h_{xx} + h_{yy}$, h_{tt}, φ, a_t, a_r,

  with respect to the isometry group of the $x - y$ plane.

Perturbations

- The radial perturbations are described by a set of coupled second order differential equations with constant coefficients. This can be written as a set of first order differential equations of the form

$$r \frac{d}{dr} h_i = M_i{}^j h_j \, .$$

- We demand that the eigenvalues of the matrix M are real.
- All eigenvalues except two are manifestly real. These are

$$\frac{1}{2} \left(2 + z - \theta \pm \frac{\sqrt{\sigma}}{\alpha(z-1)} \right) \, .$$

Perturbations

$$\begin{aligned}\sigma \;=\; & 4\alpha^6 + 4\alpha^4\left(-3\theta^2 + 3z^2 + 2\theta(z+2) - 7z + 4\right) \\ & +\alpha^2\left[12\theta^4 + 2\theta(z-1)\left(5z^2 + z - 14\right) - 16\theta^3(z+2)\right. \\ & \left.+\theta^2((62-11z)z-3) + (z-1)^2(z(9z-20)+20)\right] \\ & -4(\theta-2)\theta^2(-\theta+z+1)\left(-\theta^2+\theta+\theta z+(z-3)z+2\right)\end{aligned}$$

- Taking $0 \leq \theta \leq 1$ (area law entanglement with logarithmic violations for $\theta = 1$) and $z \geq 1 + \theta/2$ (NEC) does not guarantee that $\sigma > 0$. It is sufficient to impose furthermore one of the following three restrictions:

$$z \geq 2\,, \qquad \theta \geq 0.10558\,, \qquad \alpha^2 \geq 0.0058351\,.$$

- Hence $\sigma > 0$ for $\theta = 1$ and $z = 3/2$. We therefore do not expect a BF-type instability, which corresponds to a complex eigenvalue.

Scalar probe fields

- The linearized perturbations suggest a natural Lagrangian for a probe field. This is found by taking the couplings for a scalar perturbation to the background fields and setting the coupling constants free.
- Consider the following Lagrangian for a probe scalar field χ on the hvLif background

$$\mathcal{L} = \sqrt{-\bar{g}}\left(-\frac{1}{2}(\partial\chi)^2 + \left[c_R\bar{R} + \frac{c_F}{4}e^{a\bar{\phi}}\bar{F}^2 + \frac{c_A}{2}e^{b\bar{\phi}}\bar{A}^2 - \frac{c_\phi}{2}e^{(b-a)\bar{\phi}}\right]\chi^2\right)$$

- The equation of motion for the scalar field is

$$-r^{2z}\partial_t^2\chi + r^2\partial_x^2\chi + r^2\partial_y^2\chi + r^2\partial_r^2\chi - (1+z-\theta)r\partial_r\chi - m_\chi^2\chi = 0\,,$$

which is invariant under Lifshitz scaling.

Scalar probe fields

- The probe mass parameter is

$$m_\chi^2 = c_R \left(3(\theta-2)^2 + 4z^2 + (8-6\theta)z\right) + c_F\, 2(z-1)\left(z+\frac{b\alpha}{2}\right) + \left(c_A \frac{2(z-1)}{z+\frac{b\alpha}{2}} + c_\phi\right) e^{-\frac{\theta\phi_0}{\alpha}} .$$

- Keeping only the radial dependence, the solution to the equation of motion is

$$\chi(r) = C^+ r^{\frac{1}{2}\left(2+z-\theta+\sqrt{(2+z-\theta)^2+4m_\chi^2}\right)} + C^- r^{\frac{1}{2}\left(2+z-\theta-\sqrt{(2+z-\theta)^2+4m_\chi^2}\right)} .$$

- The BF bound for the scalar field is $m_\chi^2 \geq -\frac{(2+z-\theta)^2}{4}$.
- This is a direct generalization of the AdS result for $z=1$, $\theta=0$.

Summary & Outlook

- 4D hvLif space-times with $\theta = 1$: entanglement entropy shows logarithmic violations to the area law and thermal entropy sees an effective 1D system with dynamical exponent z: dual to a co-dimension one Fermi surface?
- For $z = 3/2$ these space-times are IR regular. They are supported by Lagrangians containing vector fields with negative m^2, but there are no BF type instabilities when $m^2 < 0$.
- A natural class of probe fields has been found (not the usual Klein–Gordon particles).
- Next: compute fermionic correlation functions and test the holographic Fermi surface idea further.