Holographic de Haas – van Alphen Oscillations

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HoloGrav 2013 Workshop
Helsinki
March 5, 2013

based on collaboration with V. Giangreco Puletti, S. Nowling and L. Thorlacius

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- physics behind the effect explained by Lars Onsager in 1952
- caused due to quantization into Landau orbits and the change in occupation numbers whenever one of these orbits coincides with an extremal cross-section of the Fermi surface
- has since proved a useful tool to e.g. image the Fermi surface or measure the carrier density

Electron Star

[Hartnoll,Tavanfar 2010]

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S &= S^{EM} + S^{fl} \\
S^{EM} &= \int d^4x \sqrt{-G}\,(R - 2\Lambda) - \frac{1}{4}\int d^4x \sqrt{-g} F_{\mu\nu}F^{\mu\nu} \\
S^{fl} &= \int d^4x \sqrt{-g}\,\mathcal{L}^{fl}
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- the Lagrangian $\mathcal{L}^{fl}$ describes a charged perfect fluid,

$$\mathcal{L}^{fl} = p[u^\nu, \underbrace{A_\nu - \psi_{,\nu}}_{=:\tilde{A}_\nu}]$$

Electron Star

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$$E^2 = m^2 + k^2$$

at Fermi level $\varepsilon_F = q\mu_{loc} =: \upsilon$ at vanishing temperature

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- particle density : $n = \beta\,\theta(\upsilon^2 - m^2)\,(\upsilon^2 - m^2)^{\frac{3}{2}}$
  charge density : $\sigma = q\,n$
  pressure : $\frac{\partial p}{\partial \mu_{loc}} = \sigma$

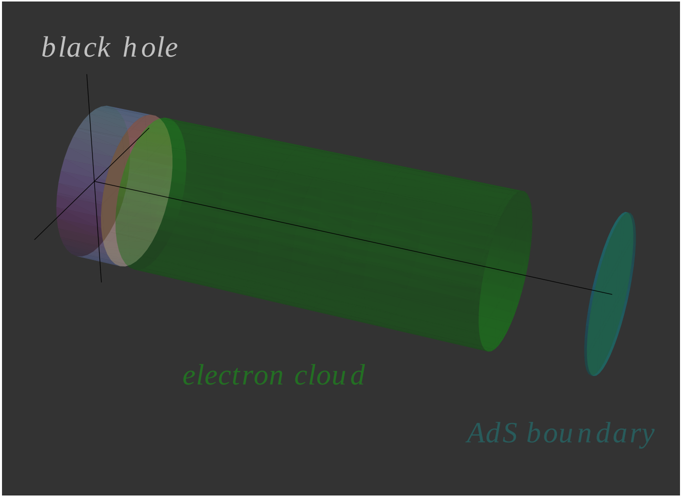
Electron Star : Equations of Motion

$$
\begin{aligned}
R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} &= T^{em}_{\mu\nu} + T^{fl}_{\mu\nu} \\
\nabla_\mu F^{\nu\mu} &= J^\nu
\end{aligned}
$$

$$
\begin{aligned}
T^{\mathrm{em}}_{\mu\nu} &= \frac{1}{2}\left(F_{\mu\lambda} F_\nu{}^\lambda - \frac{1}{4} g_{\mu\nu} F_{\lambda\sigma} F^{\lambda\sigma} \right) \\
T^{fl}_{\mu\nu} &= \sigma \mu_{loc} u_\mu u_\nu + p g_{\mu\nu}
\end{aligned}
$$

$$
J^\nu = \sigma u^\nu
$$

Electron Star : Schematics



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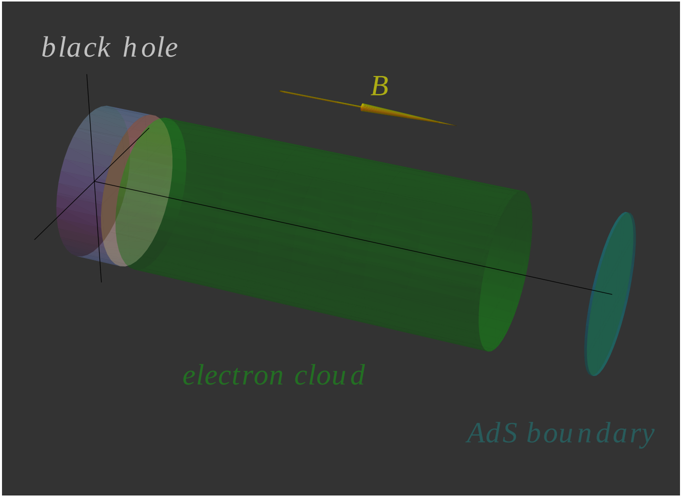

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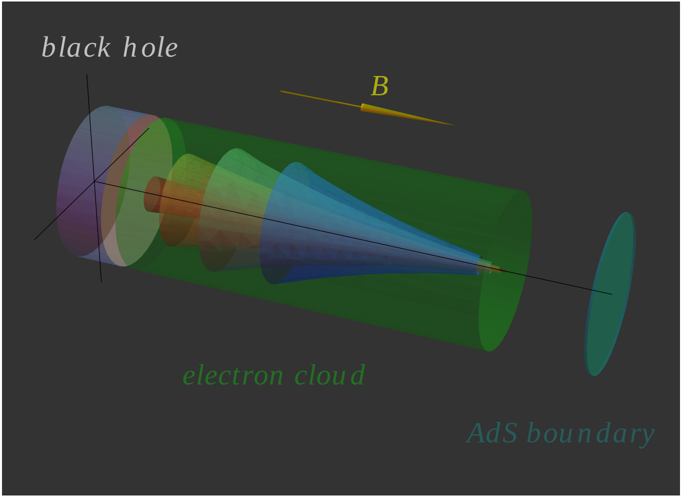

Anisotropic Electron Star

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- at each point, the fluid sees an internal frame and couples to $F_{\mu\nu}$

$$\mathcal{L}^{fl} = \mathcal{L}^{fl}[u^\nu, \tilde{A}_\nu, e_a^\nu, \dot{e}_a^\nu, F_{\mu\nu}]$$

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$\longrightarrow$ cf. spin fluids

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- electric current : $J^\nu = -\frac{\delta \mathcal{L}^{fl}}{\delta \tilde{A}_\nu}$

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  angular momentum tensor : $\Omega_{\mu\nu} = \dot{e}_{A\mu} e^A_\nu$

Anisotropic Electron Star : Equations of Motion

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = T^{em}_{\mu\nu} + T^{J}_{\mu\nu} + T^{M}_{\mu\nu} + T^{fl}_{\mu\nu}$$

$$\nabla_\mu \left(F^{\nu\mu} - M^{\nu\mu}\right) = J^\nu$$

$$\begin{aligned}
T^{em}_{\mu\nu} &= \frac{1}{2}\left(F_{\mu\lambda} F_{\nu}{}^{\lambda} - \frac{1}{4} g_{\mu\nu} F_{\lambda\sigma} F^{\lambda\sigma}\right) \\
T^{J}_{\mu\nu} &= -J_{(\mu}\tilde{A}_{\nu)} + u^\lambda \tilde{A}_\lambda u_{(\mu} J_{\nu)} - u^\lambda J_\lambda u_{(\mu}\tilde{A}_{\nu)} \\
T^{M}_{\mu\nu} &= \frac{1}{2}\left(M_{\lambda(\mu} F_{\nu)}{}^{\lambda} + u^\lambda F_{\lambda}{}^{\rho} u_{(\mu} M_{\nu)\rho} - u^\lambda M_{\lambda\rho} u_{(\mu} F_{\nu)}{}^{\rho}\right) \\
T^{fl}_{\mu\nu} &= \mathcal{L}^{fl} g_{\mu\nu} - \Omega_{\kappa\lambda}\Sigma^{\kappa\lambda}\, u_\mu u_\nu + \nabla_\lambda \left(u_{(\mu}\Sigma_{\nu)}{}^{\lambda}\right)
\end{aligned}$$

$$h_{\mu\kappa} h_{\nu\lambda} \dot{\Sigma}^{\kappa\lambda} = -h_{\mu\kappa} h_{\nu\lambda} \left(J^{[\kappa}\widetilde{A}^{\lambda]} + M^{[\kappa}{}_{\sigma} F^{\lambda]\sigma} - div u\, \Sigma^{\kappa\lambda}\right)$$

Anisotropic Electron Star : Isotropic Limit

when the dependence on $F_{\mu\nu}$ and e_a^ν vanishes

$$\begin{aligned} \mathcal{L}^{fl} &\longrightarrow p[u^\nu, \tilde{A}_\nu] \\ J^\nu &\longrightarrow \sigma u^\nu \\ M^{\mu\nu} &\longrightarrow 0 \\ \Sigma^{\mu\nu} &\longrightarrow 0 \end{aligned}$$

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hence

$$\begin{aligned} T^J_{\mu\nu} &\longrightarrow \mu_{loc}\sigma u_\mu u_\nu \\ T^M_{\mu\nu} &\longrightarrow 0 \\ T^{fl}_{\mu\nu} &\longrightarrow p g_{\mu\nu} \end{aligned}$$

Anisotropic Electron Star : Bulk Fluid

- in addition to μ_{loc}, the fluid quantities also depend on a local magnetic field

$$H_{loc} = e_1^\kappa e_2^\lambda F_{\kappa\lambda}$$

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- particle density

$$n_{\uparrow,\downarrow} = \frac{\beta q H_{loc}}{2} \sum_{l=0}^{l_{filled}} \sqrt{\upsilon^2 - m^2 - [(2l+1)q \pm \nu]\, H_{loc}}$$

$$l_{filled} = \left\lfloor \frac{\upsilon^2 - m^2 - qH_{loc} \mp \nu H_{loc}}{2qH_{loc}} \right\rfloor$$

Parameterization

static stationary ansatz :

$$
\begin{aligned}
e_0 &= \frac{c(r)}{r\,g(r)}\mathrm{d}t \\
e_1 &= \frac{1}{r}\mathrm{d}x \\
e_2 &= \frac{1}{r}\mathrm{d}y \\
e_3 &= \frac{g(r)}{r}\mathrm{d}r \\
A &= a(r)e_0 + Bx\,\mathrm{d}y \\
F &= b(r)e_0 \wedge e_r + B\mathrm{d}x \wedge \mathrm{d}y
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$\longrightarrow$ $\mu_{loc} = a(r)$, $\quad H_{loc} = Br^2$

$\longrightarrow$ $\sigma_{\uparrow,\downarrow} = qn_{\uparrow,\downarrow}$, $\quad \frac{\partial p_{\uparrow,\downarrow}}{\partial \mu_{loc}} = \sigma_{\uparrow,\downarrow}$, $\quad \mathfrak{m}_{\uparrow,\downarrow} = \frac{\partial p_{\uparrow,\downarrow}}{\partial H_{loc}}$

Equations of Motion

$$
\begin{aligned}
rc' &= -\frac{ag^2 c\sigma}{4r} \\
rg' &= -\frac{3g}{2} - \frac{g^3}{8}\left(B^2 r^4 - 12 + r^4 b^2 - 2p + 2a\sigma\right) \\
ra' &= -\frac{a}{2} + bg - \frac{ag^2}{8}\left(B^2 r^4 - 12 + r^4 b^2 - 2p\right) \\
rb' &= \frac{g\sigma}{r^2} \\
r\mathfrak{M}' &= -Br - \frac{\mathfrak{m}}{r} + \frac{ag^2\sigma\mathfrak{M}}{4r}
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⟶ conserved quantity :

$$\mathcal{Y} = c\left[-\frac{3}{r^3 g^2} + \frac{ab}{rg} - B\mathfrak{M} - \frac{B^2 r^4 - 12 + r^4 b^2 - 2p}{4r^3}\right],$$

Boundary Conditions

at $r = r_i$ matching to

$$
\begin{aligned}
ds^2 &= -\frac{f(r)dt^2}{r^2} + \frac{dx^2}{r^2} + \frac{dy^2}{r^2} + \frac{dr^2}{r^2 f(r)} \\
f(r) &= 1 - \frac{4 + q^2 + B^2}{4} r^3 + \frac{q^2 + B^2}{4} r^4 \\
A &= q(1 - r)dt + B\mathrm{d}x \wedge \mathrm{d}y \,.
\end{aligned}
$$

at $r = r_e$ matching to

$$
\begin{aligned}
ds^2 &= -\frac{\mathfrak{c}^2 \tilde{f}(r)dt^2}{r^2} + \frac{dx^2}{r^2} + \frac{dy^2}{r^2} + \frac{dr^2}{r^2 \tilde{f}(r)} \\
\tilde{f}(r) &= 1 - 2\hat{\mathcal{E}} r^3 + \frac{\hat{\mathcal{Q}}^2 + \hat{\mathcal{B}}^2}{4} r^4 \\
A &= \mathfrak{c}(\hat{\mu} - \hat{\mathcal{Q}} r)dt + \hat{\mathcal{B}}\mathrm{d}x \wedge \mathrm{d}y
\end{aligned}
$$

Thermodynamics

$$
\begin{aligned}
\hat{\mathcal{T}} &= \frac{12 - q^2 - B^2}{16\pi\mathfrak{c}} \\
\hat{\mathcal{S}} &= 4\pi \\
\hat{\mathcal{B}} &= B \\
\hat{\mathcal{M}} &= -Br_e - \mathfrak{M}(r_e)
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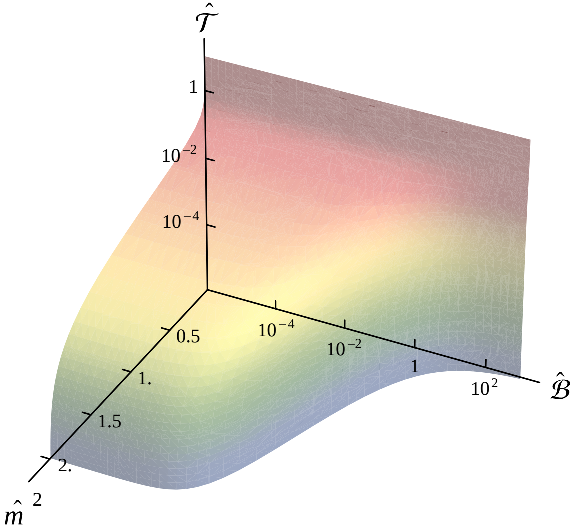
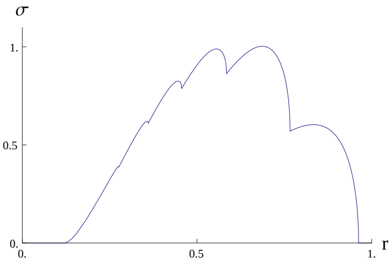
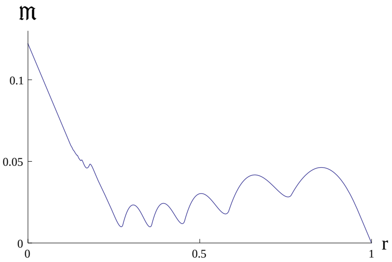
- free energy : $\hat{\mathcal{F}} = \hat{\mathcal{E}} + \hat{\mathcal{S}}\hat{\mathcal{T}} + \hat{\mu}\hat{\mathcal{Q}}$
- equation of state : $\frac{3}{2}\hat{\mathcal{E}} = \hat{\mathcal{S}}\hat{\mathcal{T}} + \hat{\mu}\hat{\mathcal{Q}} - \hat{\mathcal{M}}\hat{\mathcal{B}}$

Phase Space


$\hat{\mathcal{T}}$
1
10^{-2}
10^{-4}
0.5
1.
1.5
2.
$\hat{m}^2$
10^{-4}
10^{-2}
1
10^{2}
$\hat{\mathcal{B}}$

Fluid Profile

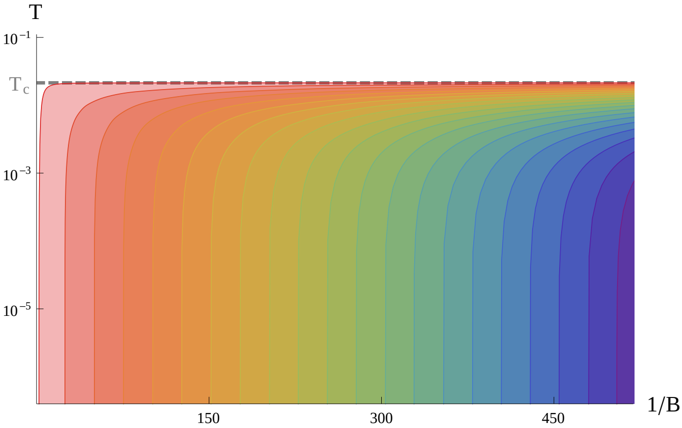




$$m = 0.5\ , \quad l_{max} = 5$$

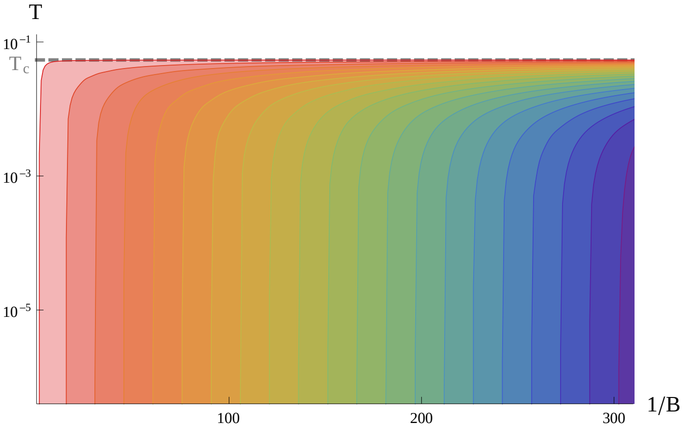
Phase Diagram

$m = 0.95$



Phase Diagram

$m = 0.7$



Phase Diagram

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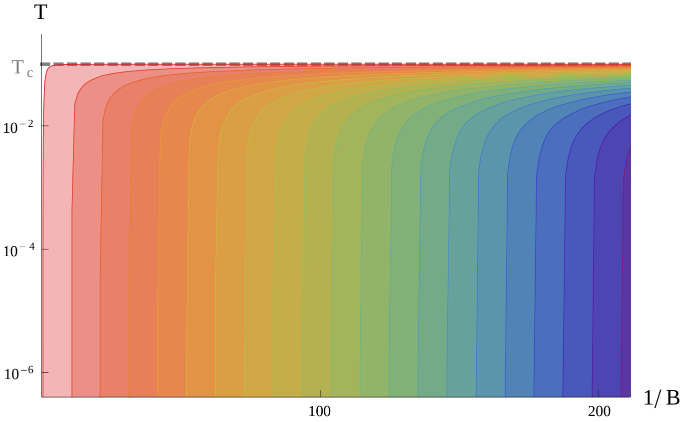
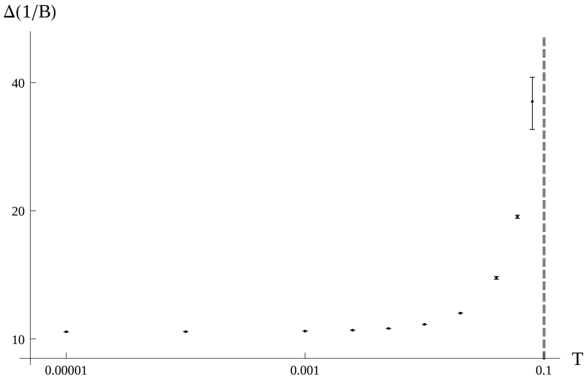


Period

$m = 0.5$



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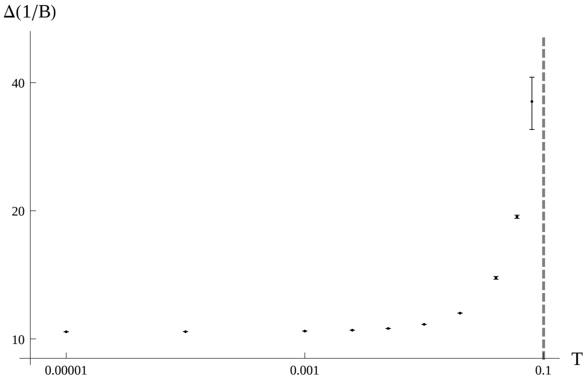
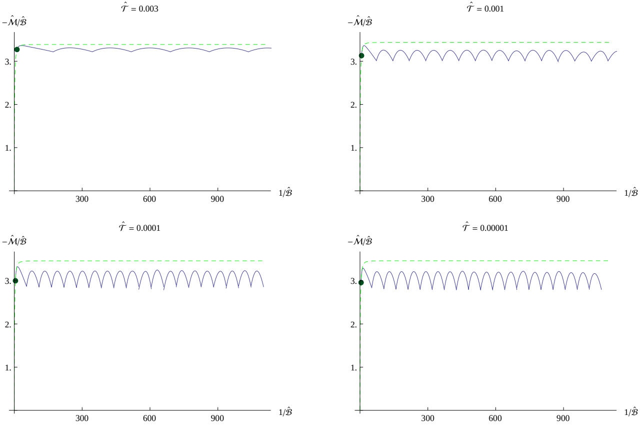


$\longrightarrow$ in agreement with observation in experiments

[Berlincourt, Steele 1954]

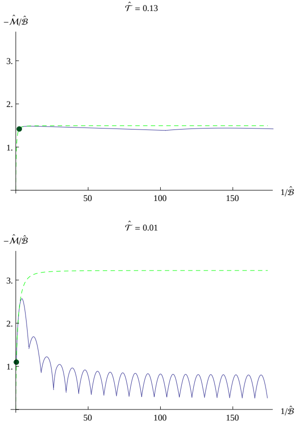
dHvA Oscillations in the Magnetization

$$m = 1.2$$



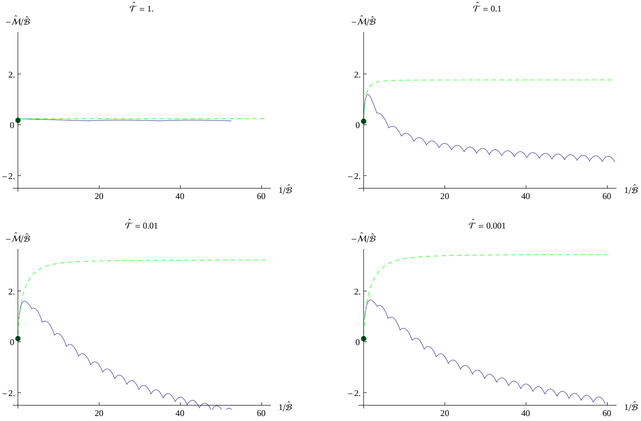
dHvA Oscillations in the Magnetization

$$m = 0.4$$



dHvA Oscillations in the Magnetization

$$m = 0.05$$



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Summary

- extended the electron star model to incorporate a direct coupling between $F_{\mu\nu}$ and the electron fluid by allowing the latter to become anisotropic
- crossover from a diamagnetic to a paramagnetic ground state depending on parameters
- appearance of dHvA oscillations when an external magnetic field is applied
- direct evidence of a Fermi surface in the dual field theory without the need of probe calculations or other (perturbative, WKB, $1/N$, etc.) corrections