Entanglement entropy of holographic semi-local quantum liquids

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Overview

- Applying AdS/CFT to real-world physics: QCD, CMP...
- One bottom-up approach for studying AdS/CMT: the Effective Holographic Theory (EHT) (Charmousis, Gouteraux, Kim, Kiritsis and Meyer, '10).
- The central point: to truncate a string theory to a finite spectrum of low-lying states, in analogy with effective field theory.
- One concrete realization: the Einstein-Maxwell-Dilaton (EMD) system.
- The exact solutions of the EMD describe the IR asymptotic geometry.

Overview cont'd

- The EMD theory admits $(d+2)-$dimensional solution with hyperscaling violation symmetry (Huijse, Sachdev and Swingle, '11).

$$
\begin{aligned}
ds^2_{d+2} &= L^2 \left(-f(r)dt^2 + g(r)dr^2 + \frac{1}{r^2}\sum_{i=1}^{d} dx_i^2 \right), \\
f(r) &= f_0 r^{-2-2d(z-1)/(d-\theta)}, \\
g(r) &= g_0 r^{-2+2\theta/(d-\theta)},
\end{aligned}
\tag{1}
$$

  z-the dynamical exponent, θ-the hyperscaling violation parameter.

- $\theta = d-1$, the holographic entanglement entropy (HEE) exhibits a logarithmic violation of the area law.

Overview cont'd

- The log violation of the area law signifies the existence of Fermi surfaces, although the EMD does not contain explicit fermionic degrees of freedom.
- An interesting limit: $z \to \infty, \theta \to -\infty$, $\eta \equiv -\theta/z > 0$ fixed (Hartnoll and Shaghoulian, '12).
- The resulting geometry: conformal to $AdS_2 \times \mathbf{R}^d$.
- Extremal RN-AdS, near horizon $AdS_2 \Leftrightarrow$ semi-local quantum liquids (SLQL) (Iqbal, Liu and Mezei, '11).
- Our background $\to$ more general SLQL.

What will happen for the holographic entanglement entropy in the more general semi-local background?

Outline

The general semi-local background

- Recall the background with hyperscaling violation symmetry (1) and take the limit $z \to \infty, \theta \to -\infty$, $\eta \equiv -\theta/z > 0$ fixed,

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{\frac{2d}{\eta}}} + \frac{dr^2}{r^2} + \sum_{i=1}^{d} dx_i^2\right), \qquad (2)$$

- Taking a new coordinate $\xi = r^{d/\eta} \to$,

$$ds^2 = \frac{1}{\xi^{\frac{2\eta}{d}}}\left(-\frac{dt^2}{\xi^2} + \frac{\eta^2 d\xi^2}{d^2\xi^2} + \sum_{i=1}^{d} dx_i^2\right), \qquad (3)$$

conformal to $AdS_2 \times \mathbf{R}^d$.

Previous results on HEE

- 4D bulk, the boundary separation length $l = l_{\rm crit}$ is constant for a strip and the minimal surface area diverges (Hartnoll and Shaghoulian, '12).
- Connected minimal surface only exists for separations $l < l_{\rm crit}$. When $l > l_{\rm crit}$, the disconnected minimal surface–two parallel hypersurfaces falling into the IR at constant separation, dominates.
- reminiscent of the HEE in confining backgrounds (Klebanov, Kutasov and Murugan, '07).
- When l is sufficiently small, the minimal hypersurface should probe the UV regime of the full geometry (Kulaxizi, Parnachev and Schalm, '12).

Why UV completion?

- Recall the spirit of EHT: it just describes the IR geometry.
- The UV regime of the full geometry is needed when l is small.
- UV completion: to obtain solutions which are asymptotically AdS and conformal to $AdS_2 \times \mathbf{R}^d$ in the IR.
- Physical constraints should be imposed (Ogawa, Takayanagi and Ugajin, '11).

The ansatz for the configuration

- The action

$$S=\int d^{d+2}x\sqrt{-g}[R-\frac{1}{2}(\nabla\Phi)^2-V(\Phi)-\frac{1}{4}Z(\Phi)F_{\mu\nu}F^{\mu\nu}], \quad (4)$$

- The ansatz for the metric and gauge field

$$ds^2_{d+2}=\frac{L^2}{z^2}[-f(z)dt^2+g(z)dz^2+\sum_{i=1}^{d}dx_i^2],\ A_t=A_t(z), \quad (5)$$

- Asymptotically AdS_{d+2} solutions with boundary $z=0,\rightarrow$ $f(0)=g(0)=1$.
- Introducing the scale z_F, $z\gg z_F\rightarrow$ IR, $z\ll z_F\rightarrow$ UV.

The physical constraints

- Plugging in the configuration into the e.o.m.s and solving for $A_t, \Phi'^2, V(\Phi), Z(\Phi)$.
- Requiring that $\Phi'^2 > 0, Z(\Phi) > 0 \Rightarrow$

$$g(z)f'(z) + g'(z)f(z) \leq 0, \tag{6}$$

$$\begin{aligned} & zg(z)f'^2(z) + f(z)(zf'(z)g'(z) \\ & +g(z)\left(2df'(z) - 2zf''(z)\right) \leq 0, \end{aligned} \tag{7}$$

- (6, 7) are equivalent to the null energy condition (NEC) $T_{\mu\nu}N^\mu N^\nu \geq 0$. N^μ-unit normal vector.

The IR solution

$$f(z) = kz^{-p}, \;\; g(z) = \frac{z_F^2}{z^2}, \;\; p \equiv 2d/\eta, \;\; k > 0, \tag{8}$$

the solution

$$\Phi = \sqrt{d(p+2)}\log z, \;\; A_t'(z) = \frac{A}{Z(\Phi)}\sqrt{f(z)g(z)}z^{d-2},$$
$$V(\Phi) = -\frac{(p+2d)^2 z^2}{4L^2 z_F^2}, \;\; Z(\Phi) = \frac{2A^2 z_F^2 z^{2d-2}}{L^2 p(p+2d)}, \tag{9}$$

written in terms of Φ,

$$V(\Phi) \sim e^{\frac{2\Phi}{\sqrt{d(p+2)}}}, \;\; Z(\Phi) \sim e^{\frac{2(d-1)\Phi}{\sqrt{d(p+2)}}}, \tag{10}$$

The IR solution cont'd

- The black hole solution

$$g(z) = \frac{z_F^2}{z^2 h(z)}, \;\; f(z) = \frac{k}{z^p} h(z), \;\; h(z) = 1 - (\frac{z}{z_H})^{d+p/2}, \tag{11}$$

while the other field configurations remain the same.

- The temperature and the entropy density,

$$T \sim z_H^{-d/\eta}, \;\; s = z_H^{-d}, \;\; \Rightarrow \;\; s \sim T^\eta, \tag{12}$$

- The entropy density is vanishing at extremality.

The UV completion

- Choose the following functions

$$f(z) = \frac{k}{k + z^p}, \quad g(z) = \frac{z_F^2}{z^2 + z_F^2}, \tag{13}$$

- $f(0) = g(0) = 1$, $f(\infty), g(\infty)$ reduce to the IR results.
- $\Phi'^2 > 0$ is always satisfied.
- A sufficient but not necessary condition for $1/Z(\Phi) > 0$ is $2kd - 2kp > 0 \ \rightarrow \ p < d$.

The plot for $V(\Phi)$

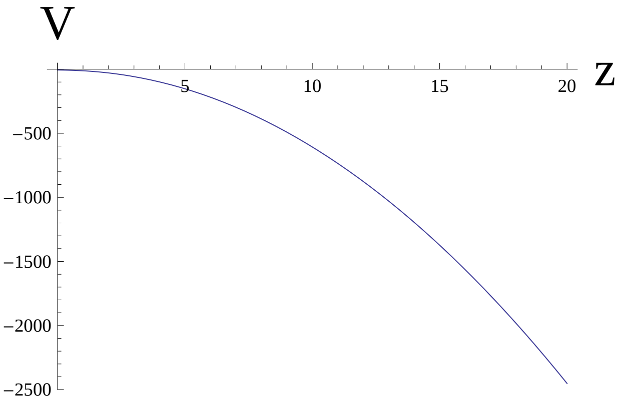

Figure: $V(\Phi)$ with $d = 2$. $k = L = z_F = 1$.

The plot for $Z(\Phi)$

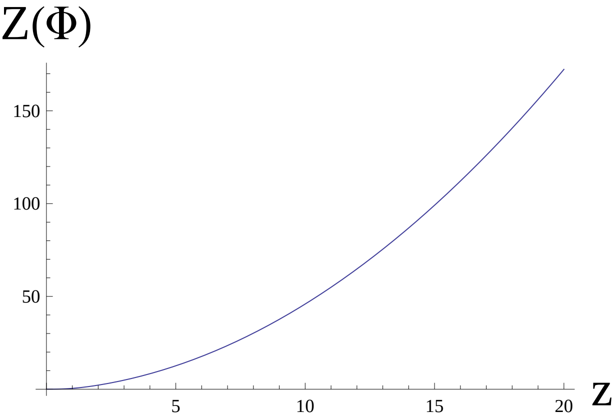

Figure: $Z(\Phi)$ with $d = 2$. $k = L = z_F = A = 1$.

The UV behavior

- The UV behavior ($z \to 0$)

$$\Phi \sim z, \;\; Z(\Phi) \sim \Phi^{2d-p}, \tag{14}$$

- The scalar potential

$$V(\Phi) = -\frac{d(d+1)}{L^2} - \frac{d}{2L^2}\Phi^2, \tag{15}$$

with mass square $m^2 = -d$.

- BF bound in AdS_{d+2} is $m^2 \geq -(d+1)^2/4 \;\Rightarrow$ the BF bound is not violated.

The entangling regions

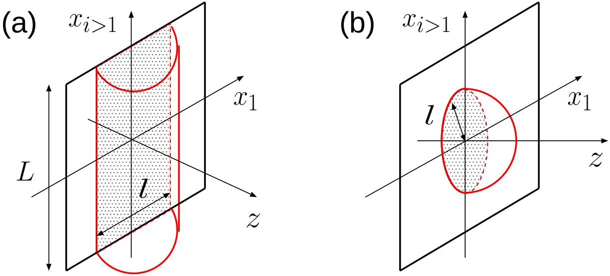

Figure: Left: strip; right: sphere. Taken from Ryu, Takayanagi, hep-th/0603001.

Only plots with $d = 2$ will be shown.

The strip

Consider the strip,

$$x_1 \equiv x \in [-\frac{l}{2}, \frac{l}{2}], \quad x_i \in [0, L_x], i = 2, \cdots, d, \tag{16}$$

$l \ll L_x$, the minimal surface area

$$A(\gamma) = 2L^d L_x^{d-1} \int \frac{dz}{z^d} \sqrt{g(z) + x'^2}, \tag{17}$$

we have

$$x' = \frac{\sqrt{g(z)}(\frac{z}{z_*})^d}{\sqrt{1 - (\frac{z}{z_*})^{2d}}}, \tag{18}$$

z_*- the turning point where $x' \to \infty$.

The boundary separation length

- The boundary separation length

$$\frac{l}{2} = \int_0^{z_*} dz \frac{\sqrt{g(z)}(\frac{z}{z_*})^d}{\sqrt{1-(\frac{z}{z_*})^{2d}}}, \tag{19}$$

- Plugging in the IR solution $g(z) = z_F^2/z^2$,

$$l \equiv l_{\text{crit}} = \frac{\pi z_F}{d} = \text{const}, \tag{20}$$

- For the UV-completed metric $g(z) = z_F^2/(z^2 + z_F^2)$, see the plot

The difference

- The equation of motion also admits the trivial solution $x' = 0 \to$ disconnected hypersurface.
- The holographic entanglement entropy (Ryu and Takayanagi, '06)

$$S = \frac{A(\gamma)}{4G_N^{(d+2)}}. \tag{21}$$

- Plot the differences between the finite parts of the connected minimal surface and the disconnected one $\Delta A = A_{\text{finite}} - A_{\text{dis,finite}}$.

Which configuration will dominate?

Plot of the boundary separation length

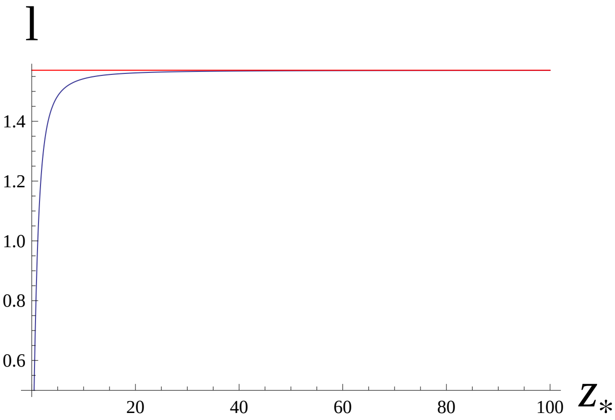

Figure: blue–UV completed; red–IR.

Plot of the difference

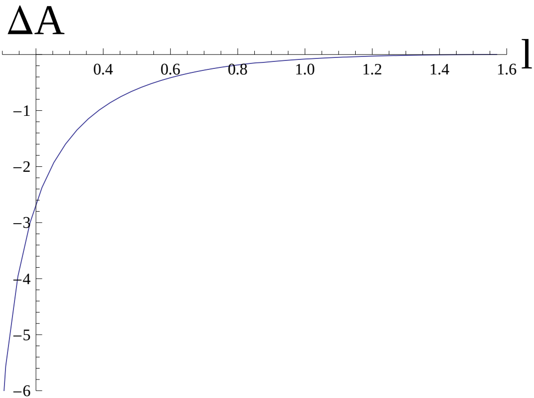

Figure: The difference between the connected minimal surface and the disconnected one.

Remarks

- For the boundary separation length, small z_*, l and l_{crit} have significant differences; large z_*, they almost coincide.
- For ΔA, when $l < l_{\text{crit}}$: the connected solution dominates;
- When $l \to l_{\text{crit}}$: the difference tends to zero; the disconnected solution takes over;
- *A phase transition occurs*.

The sphere

- The minimal surface area

$$A_{\rm sphere} = L^d \mathrm{Vol}(\Omega_{d-1}) \int \frac{dz}{z^d} \rho^{d-1} \sqrt{g(z) + \rho'^2}, \qquad (22)$$

- The equation of motion

$$\partial_z \left(\frac{\rho^{d-1} \rho'}{z^d \sqrt{g(z) + \rho'^2}} \right) = \frac{(d-1)\rho^{d-2}}{z^d \sqrt{g(z) + \rho'^2}}, \qquad (23)$$

  boundary conditions $\rho(0) = l, \rho(z_*) = 0$,
  z_* the turning point.

- No trivial solution $\rho' = 0$.

Plot of the finite part

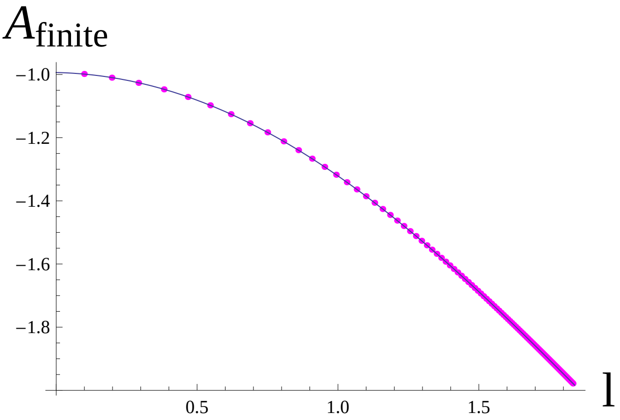

Figure: The finite part of the entropy for spherical entangling region.

Remarks

- Fitting the finite part,

$$A_{\text{finite}} = -0.993627 - 0.0203008 l - 0.303885 l^2 - 0.021952 l^3, \quad (24)$$

- The leading order behavior of the HEE is still governed by the area law.
- There is NO phase transition.

Summary

- We study the HEE of holographic semi-local quantum liquids.
- The IR geometry is conformal to $AdS_2 \times \mathbf{R}^d$, insufficient for studying the HEE.
- The UV completed geometry is constructed.
- For the strip case, the HEE exhibits a phase transition.
- For the spherical case, no such phase transition occurs.
- Similar behavior can be observed in $d \geq 3$.

Outlook

- How to understand the behavior of HEE with different entangling regions?
- A third scale, supplied by the anisotropy of the strip, may play a role. (Kulaxizi, Parnachev and Schalm, '12).
- A geometric setup to check this argument: considering the annulus.
- When the inner radius vanishes $\rightarrow$ sphere; both radii large, the difference small $\rightarrow$ strip.
- Mutual information? Field theory realization?