Probe Brane Descriptions of Dynamical Gaps

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Introduction

Probe brane duals of relativistic 3+1d gauge theories

Chiral symmetry breaking dynamics

T & μ phase diagrams – chiral + density formation transitions

The holography of the chiral transition and the conformal window

2+1d gauge theories

Graphene, bilayers, Hall states

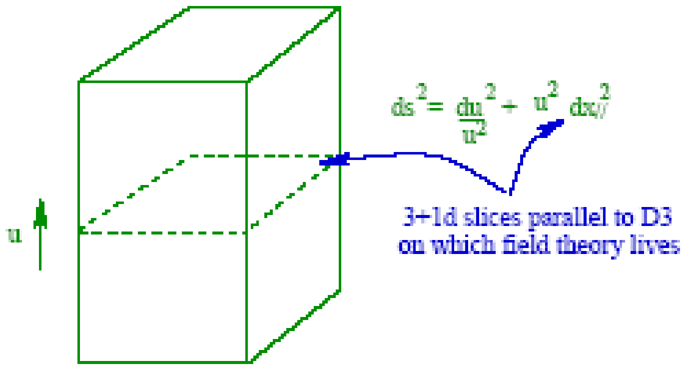
(Out of equilibrium computations)

AdS/CFT Correspondence

Maldacena, Witten...

4d strongly coupled $\mathcal{N}$=4 SYM = IIB strings on $AdS_5 \times S^5$

Pretty well established by this point!



u corresponds to energy (RG) scale in field theory

The SUGRA fields act as sources

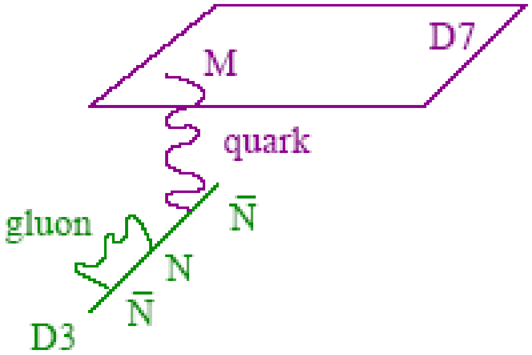
$$\int d^4x \; \Phi_{SUGRA}(u_0)\lambda\lambda$$

eg asymptotic solution ($u \to \infty$) of scalar

$$\varphi \simeq \frac{m}{u} + \frac{\langle\lambda\lambda\rangle}{u^3}$$

Adding Quarks

Bertolini, DiVecchia...; Polchinski, Grana; Karch, Katz...



Quarks can be introduced via D7 branes in AdS

The brane set up is

	0	1	2	3	4	5	6	7	8	9
D3	—	—	—	—	•	•	•	•	•	•
D7	—	—	—	—	—	—	—	—	•	•

fills radial dirn of AdS S^3 embedded in S^5 (directions 4–7); $U(1)_A$ (directions 8–9)

We will treat D7 as a probe - quenching in the gauge theory.

Minimize D7 world volume with DBI action

$$S_{D7} = -T_7 \int d\xi^8 \sqrt{P[G_{ab}]}, \qquad P[G_{ab}] = G_{MN} \frac{dx^M}{d\xi^a} \frac{dx^N}{d\xi^b}$$

Quarks In AdS

Myers et al

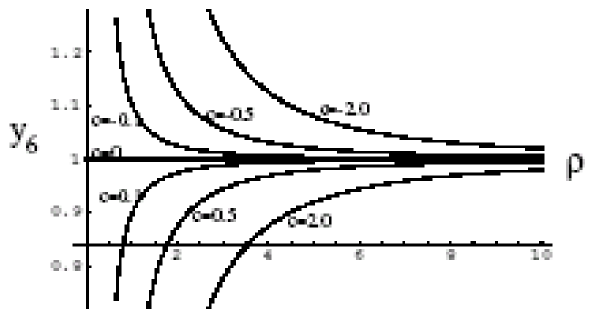
$$S_{D7} = -T_7 \int d^8\xi \, \epsilon_3 \, \rho^3 \sqrt{1 + \frac{g^{ab}}{\rho^2 + u_5^2 + u_6^2} (\partial_a u_5 \partial_b u_5 + \partial_a u_6 \partial_b u_6)}$$

EoM is: $$\frac{d}{d\rho}\left[\frac{\rho^3}{\sqrt{1+\left(\frac{du_6}{d\rho}\right)^2}} \frac{du_6}{d\rho}\right] = 0$$

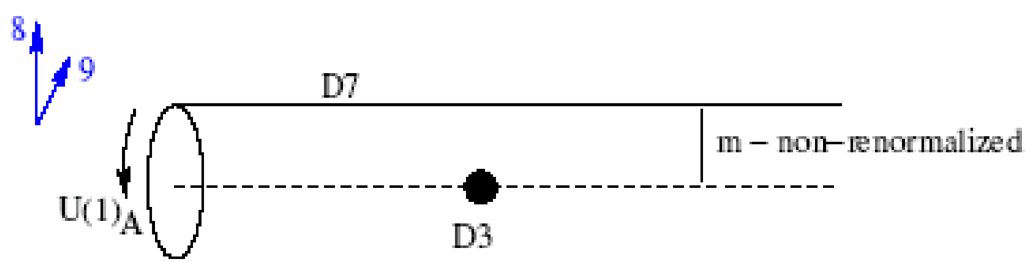
UV asymptotic solution is

$$u_6 = m + \frac{c}{\rho^2} + \dots$$

m is the quark mass, c the $\langle \bar{q} q \rangle$ condensate



In AdS regular D7 solution is flat brane



Mesons in AdS$_5$

Kruczenski, Myers, Mateos, Winters

The D7 lie flat in AdS. We can consider fluctuations that describe R-chargeless mesons

$$w_6 + i w_5 = d + \delta(\rho) e^{ik.x}$$

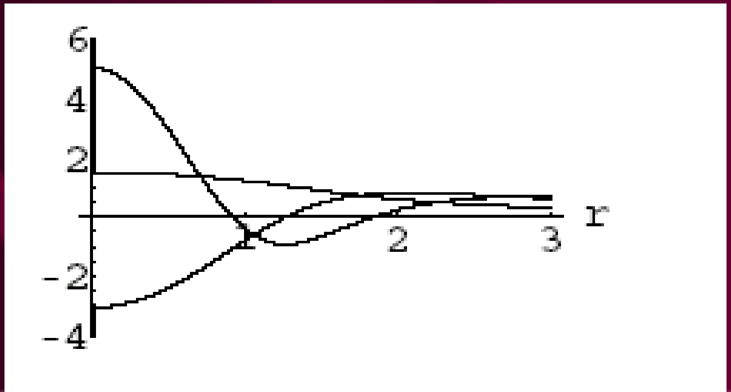
δ satisfies a linearized EoM

$$\partial_\rho^2 \delta + \frac{3}{\rho} \partial_\rho \delta + \frac{M^2}{(\rho^2+1)^2} \delta = 0$$

and the mass spectrum is

$$M = \frac{2d}{R^2} \sqrt{(n+1)(n+2)} \sim \frac{2m}{\sqrt{\lambda_{YM}}}$$

Tightly bound - meson masses suppressed relative to quark mass



Orthonormal wave functions

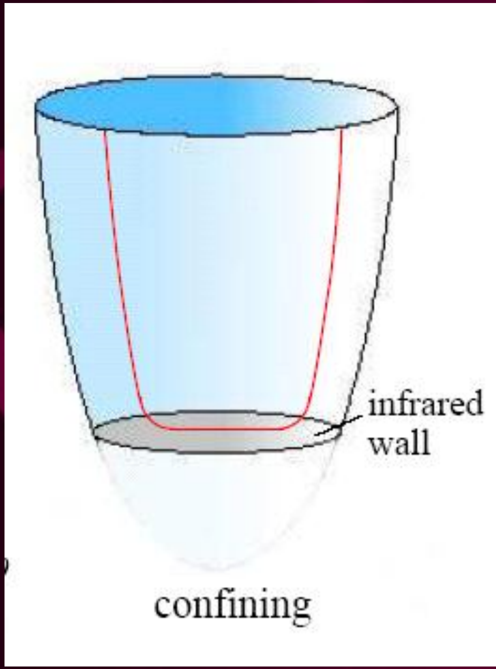
Add Confinement and Chiral Symmetry Breaking

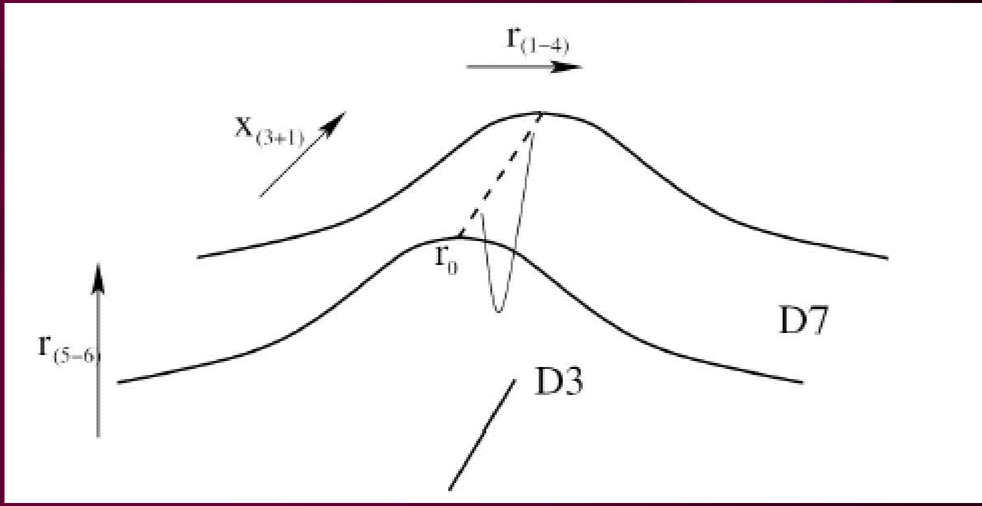
$$ds^2 = \frac{r^2}{R^2}A^2(r)dx_{3+1}^2 + \frac{R^2}{r^2}dr_6^2,$$

$$A(r) = \left(1 - (\frac{r_w}{r})^8\right)^{1/4}, \qquad e^{\phi} = \left(\frac{1+(r_w/r)^4}{1-(r_w/r)^4}\right)^{\sqrt{3/2}}$$

Dilaton Flow Geometry: Gubser, Sfetsos

Here, this is just a simple, back reacted, repulsive, hard wall….





BEEGK, Ghoroku..

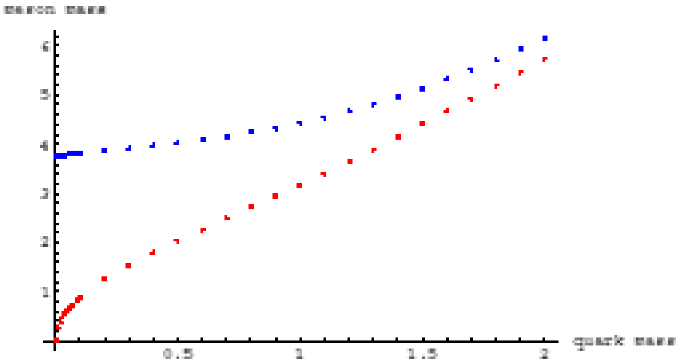
Pion Physics

Seek pion solutions of the form

$$\pi(x,r) = f(\rho)e^{ikx}, \quad k^2 = -M^2$$

$f(\rho)$ must be smooth - normalizable - at all ρ

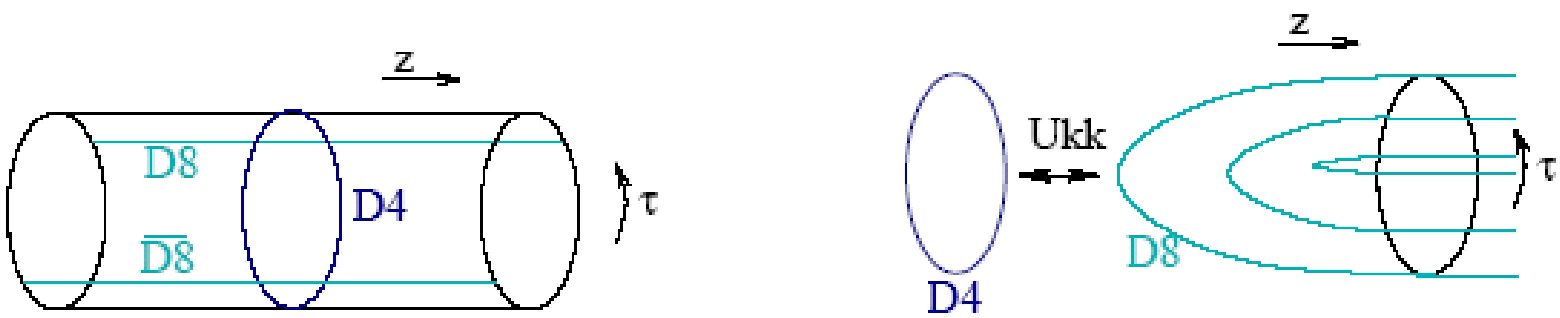
The pion and sigma masses can thus be computed as a function of quark mass



There is a Goldstone in the massless limit.

Expected $\sqrt{m}$ behaviour

Sakai Sugimoto



	0	1	2	3	(4)	5	6	7	8	9
D4	X	X	X	X	X					
D8 - $\overline{D8}$	X	X	X	X		X	X	X	X	X

- Displays non-abelian flavour symmetry

- UV 4+1d
- UV non-conformal & strongly coupled
- No description of the $\bar{q}q = 0$ state

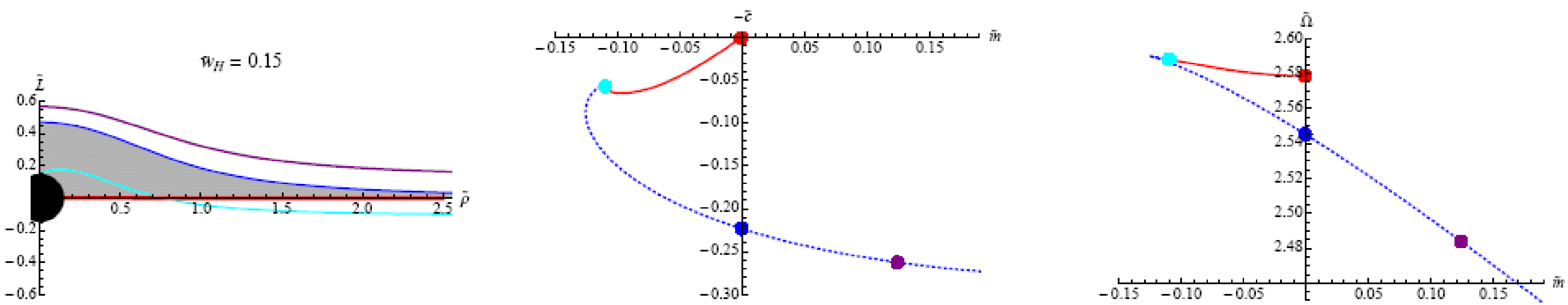
Magnetic Field Induced Chiral Symmetry Breaking

Johnson, Filev, Kundu….

$$P[G]+F = \begin{pmatrix} -\frac{r^2}{R^2} & & & & & & & \\ & \frac{r^2}{R^2} & B^z & & & & & \\ & -B^z & \frac{r^2}{R^2} & & & & & \\ & & & \frac{r^2}{R^2} & & & & \\ & & & & \frac{R^2}{r^2}\left(1+(\partial_\rho w_6)^2\right) & & & \\ & & & & & \frac{R^2}{r^2}\rho^2 & & \\ & & & & & & \frac{R^2}{r^2}\rho^2.. & \\ & & & & & & & \frac{R^2}{r^2}.. \end{pmatrix}$$

$$\mathcal{L} = \rho^3\sqrt{1+(\partial_\rho w_6)^2}\sqrt{1+\frac{B^z R^4}{r^4}}$$

B from surface gauge field describing vector mesons and U(1) source…



(a)Low temperature - $\tilde{w}_H = 0.15$. Here we see chiral symmetry breaking with the blue embedding thermodynamically preferred over the red at $\tilde{m} = 0$.

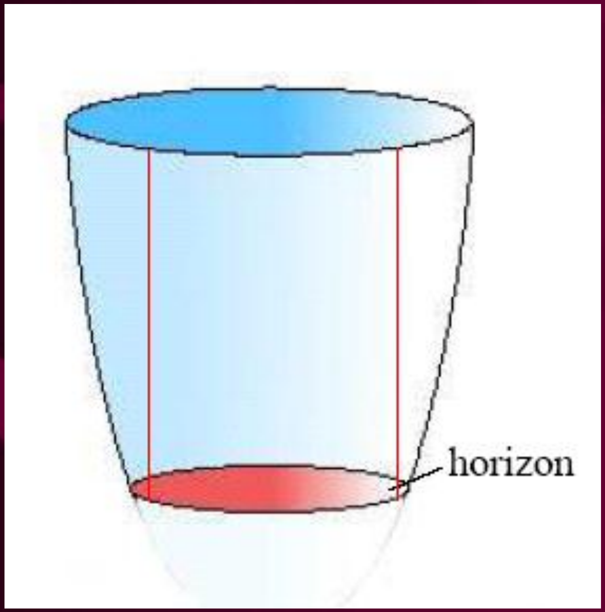
Finite T - AdS-Schwarzschild

$$ds^2 = \frac{r^2}{R^2}(-f dt^2 + d\vec{x}^2) + \frac{R^2}{r^2 f} dr^2 + R^2 d\Omega_5^2$$

where $R^4 = 4\pi g_s N \alpha'^2$ and

$$f := 1 - \frac{r_H^4}{r^4}\ , \qquad r_H := \pi R^2 T\ .$$

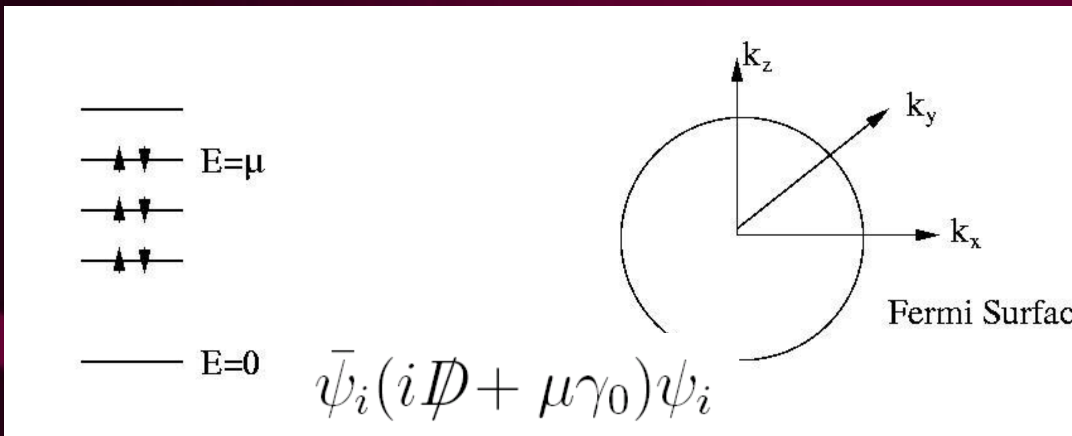
Asymptotically AdS, SO(6) invariant at all scales… horizon swallows information at rH …. Witten interpreted as finite temperature… black hole… has right thermodynamic properties…



Quarks are screened by plasma

Chemical Potential

At finite density the Fermi-sea of quarks fills up to an energy called the chemical potential

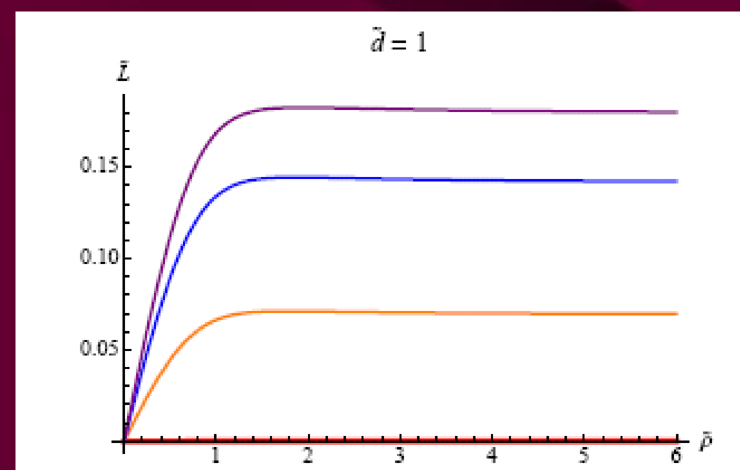


$$\bar{\psi}_i (i \not{D} + \mu \gamma_0) \psi_i$$

$$\bar{\psi} i (-i A^t \gamma_0) \psi \quad \rightarrow \quad \bar{\psi} \mu \gamma_0 \psi$$

We can think of μ as a background vev for the temporal component of the photon…

$$P[G]+F = \begin{pmatrix} -\frac{r^2}{R^2} & & & & \partial_\rho A_0 & & & \\ & \frac{r^2}{R^2} & & & & & & \\ & & \frac{r^2}{R^2} & & & & & \\ & & & \frac{r^2}{R^2} & & & & \\ -\partial_\rho A_0 & & & & \frac{R^2}{r^2}\left(1+(\partial_\rho w_6)^2\right) & & & \\ & & & & & \frac{R^2}{r^2}\rho^2 & & \\ & & & & & & \frac{R^2}{r^2}\rho^2.. & \\ & & & & & & & \frac{R^2}{r^2}.. \end{pmatrix}$$



Myers, Mateos,..

Does this System have a Quark Fermi Surface?

Liza Huijse,[1,2] Subir Sachdev, and Brian Swingle

Field theory	Holography
A gauge-dependent Fermi surface of overdamped gapless fermions.	Fermi surface is hidden.
Thermal entropy density $S \sim T^{1/z}$ in $d = 2$, where z is the dynamic critical exponent.	Thermal entropy density $S \sim T^{1/z}$ in all d for hyperscaling violation exponent $\theta = d - 1$, and z the dynamic critical exponent.
Logarithmic violation of area law of entanglement entropy, with prefactor proportional to the product of $\mathcal{Q}^{(d-1)/d}$ and the boundary area of the entangling region.	Logarithmic violation of area law of entanglement entropy for $\theta = d - 1$, with prefactor proportional to the product of $\mathcal{Q}^{(d-1)/d}$ and the boundary area of the entangling region.

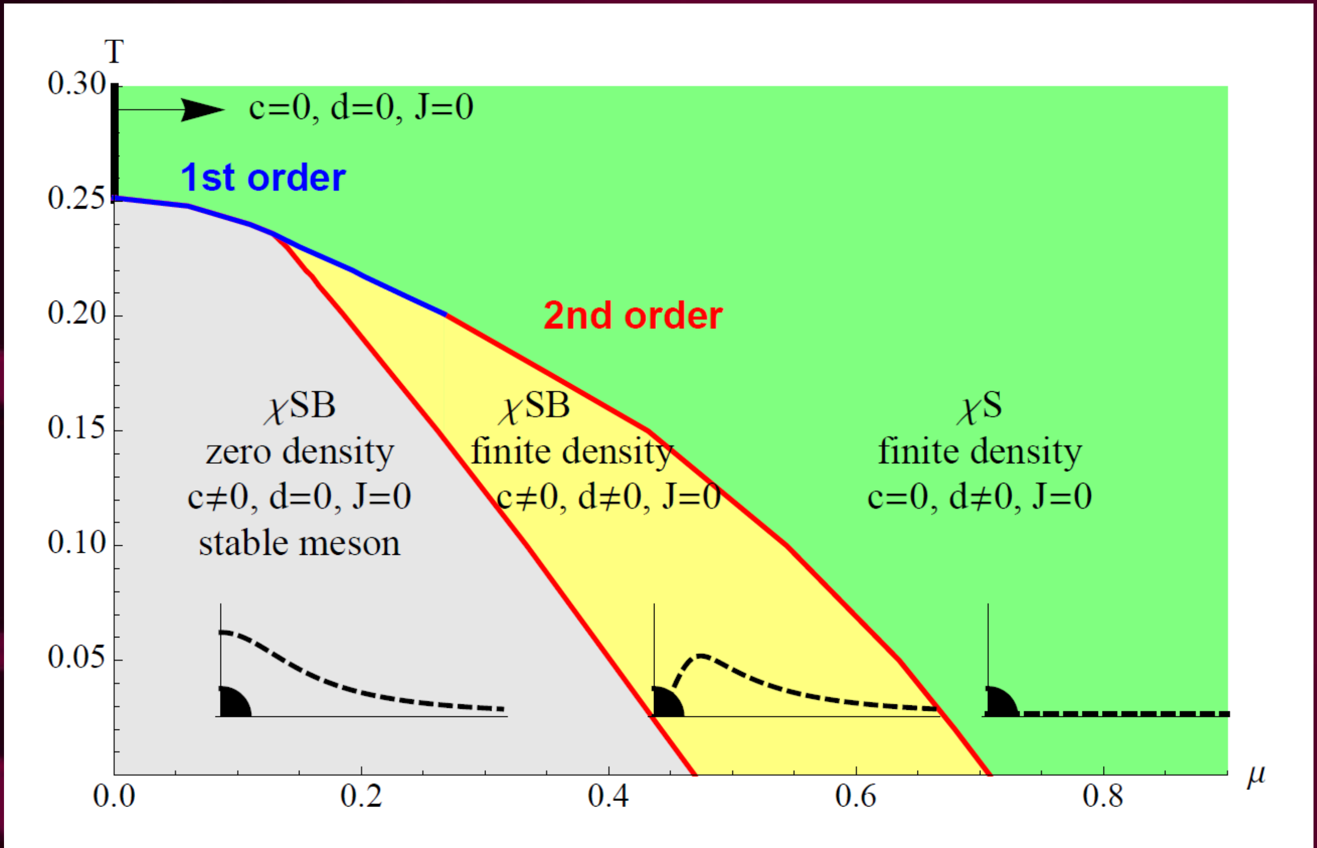
$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

Martin Ammon[a], Matthias Kaminski[b], Andreas Karch[b]

$$z = 2\,, \qquad \theta = 1\,.$$

See suitable scalings but not the IR geometry when μ ~ m_q…

Phase Diagram for B Field Theory, m=0

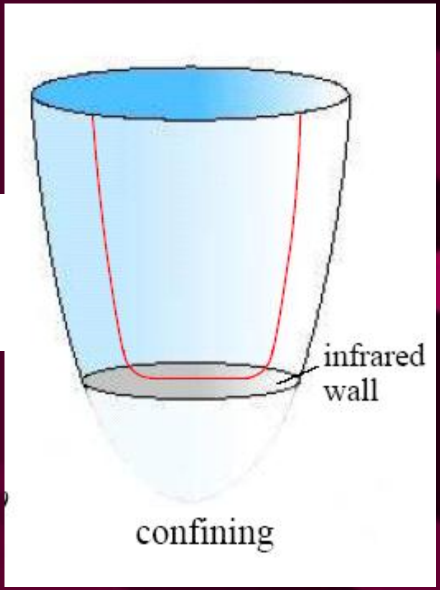


NE, KY Kim...

JHEP 1003:132,2010.
e-Print:
arXiv:1002.1885
[hep-th]

Looks QCD-ish but no quark or glue confinement…

Phenomenological Models

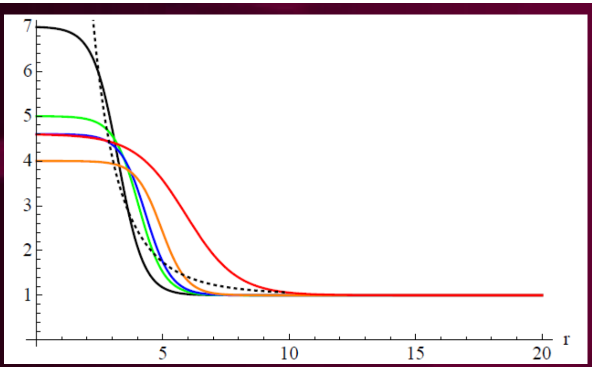


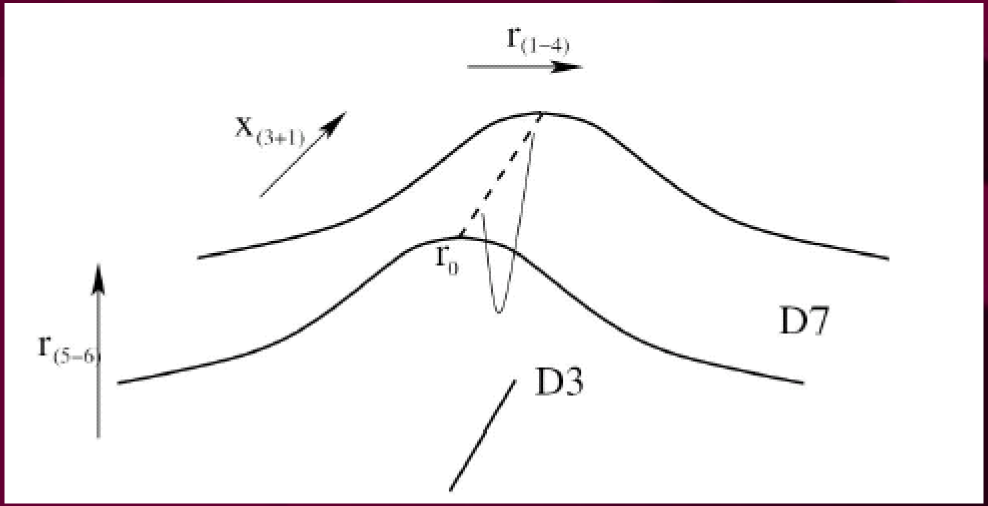
Top down – introduce a B field

$$e^{\Phi} = \sqrt{1 + \frac{B^2}{(\rho^2 + L^2)^2}}$$

Or phenomenologically

$$e^{\Phi} = g^2_{\rm YM}(r^2) = g^2_{\rm UV}\Big(A + 1 - A\tanh\left[\Gamma(r - \lambda)\right]\Big)$$



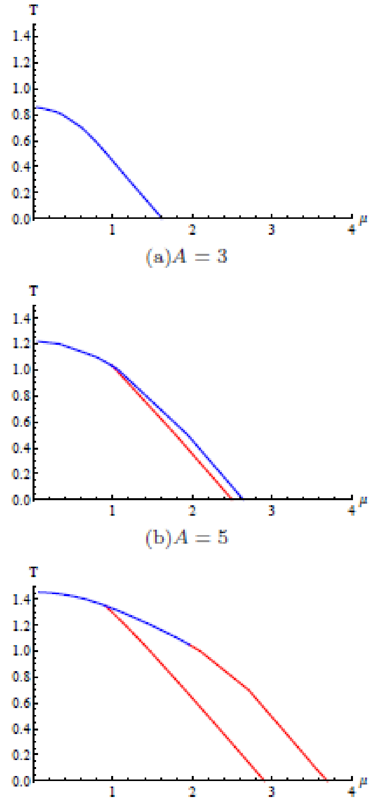


The dilaton interpolates between QCD like case and "walking" dynamics (black is B field induced chiral symmetry breaking)

λ is the scale of the problem..
A is height
Γ is width

More Phase Diagrams

NE, K-Y K, Gebauer, Magou

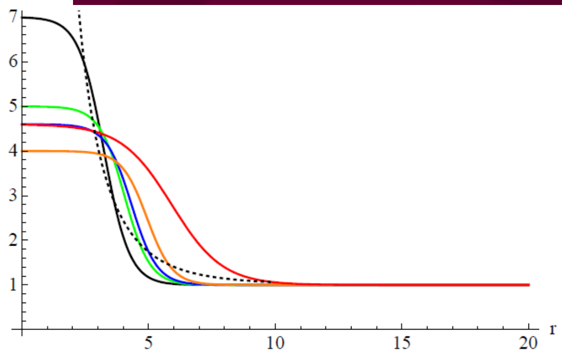


(a) $A = 3$

(b) $A = 5$

(c) $A = 8$

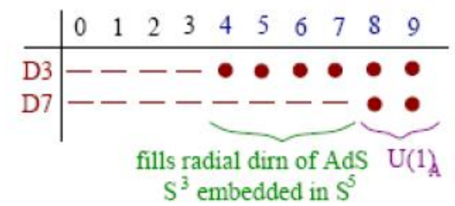
FIG. 6: Plots for three possible phase diagrams for the choices $A = 3, 5, 8$. Large (small) A gives second (first) order transition at low T. $\Gamma = 1, \lambda = 1.7$.



Walking encourages first order transition

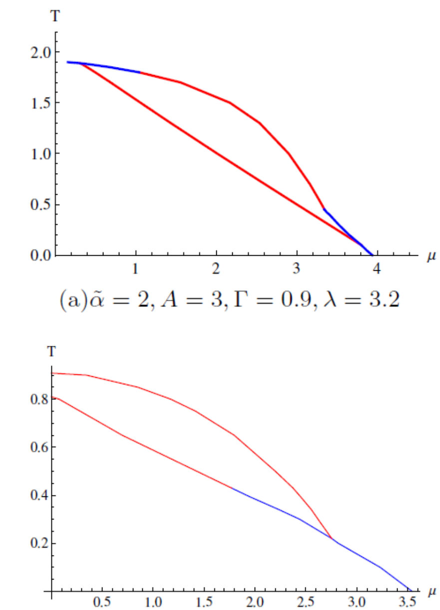
Breaking the ρ-L symmetry

QCD-like phase diagrams…



$$g_t = \frac{(w^4 - w_H^4)^2}{w^4(w^4 + w_H^4)}, \qquad g_x = \frac{w^4 + w_H^4}{w^4}$$

$$w^4 \to \rho^2 + \frac{1}{\tilde{\alpha}} L^2$$



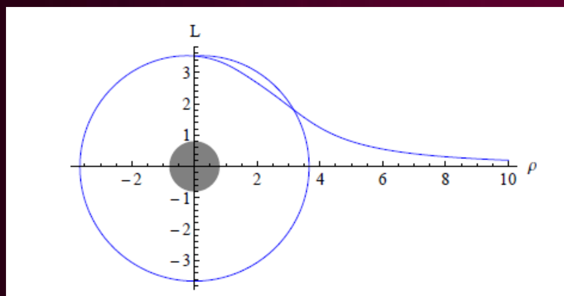
(a) $\tilde{\alpha} = 2, A = 3, \Gamma = 0.9, \lambda = 3.2$

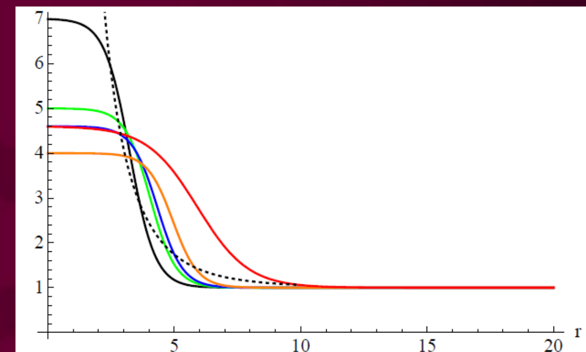
(b) $\tilde{\alpha} = 3, A = 3\Gamma = 1, \lambda = 1.715$

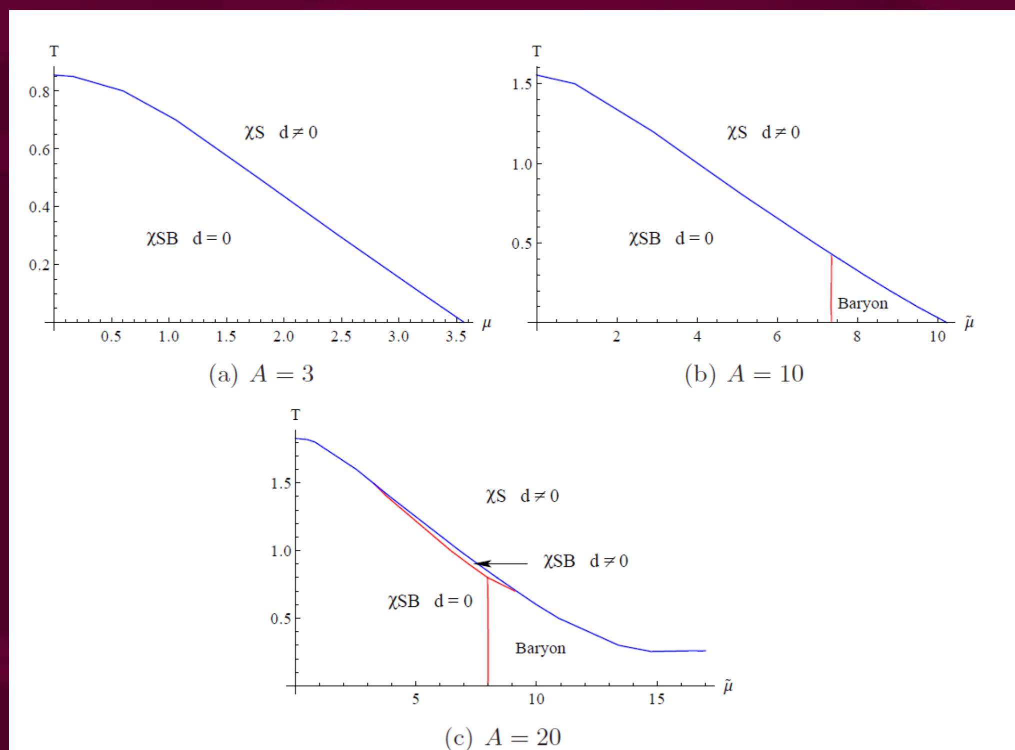
Baryonic Phase

NE, Kim, Seo, Sin.. **arXiv:1204.5640**

Linked D7/D5 systems describe a baryonic density







(a) $A = 3$

(b) $A = 10$

(c) $A = 20$

Stability Analysis

$$S = \int d\rho \lambda(r) \rho^3 \sqrt{1 + L'^2}$$

We expand for small L

$$S = \int d\rho \left(\frac{1}{2}\lambda(r)\Big|_{L=0} \rho^3 L'^2 + \rho^3 \frac{d\lambda}{dL^2}\Big|_{L=0} L^2 \right)$$

we can now make a coordinate transformation

$$\lambda(\rho)\rho^3 \frac{d}{d\rho} = \tilde{\rho}^3 \frac{d}{d\tilde{\rho}}, \qquad \tilde{\rho} = \sqrt{\frac{1}{2} \frac{1}{\int_\rho^\infty \frac{d\rho}{\lambda \rho^3}}}$$

$$L = \tilde{\rho}\phi$$

$$S = \int d\tilde{\rho} \frac{1}{2} \left(\tilde{\rho}^5 \phi'^2 - 3\tilde{\rho}^3 \phi^2 \right) + \int d\tilde{\rho} \frac{1}{2} \lambda \frac{\rho^5}{\tilde{\rho}} \frac{d\lambda}{d\rho} \phi^2$$

This is the action of a scalar in AdS with a mass squared of -3 + ρ dependent correction from the gradient of λ

$$m^2 = \Delta(\Delta - 4)$$

The mass is just a statement of the running of the dimension of $\bar{q}q$

The instability is due to a violation of the BF bound

$$m^2 \geq -4$$

ie when γ = 1 (an old result in gap eqns)

Brutally Simplified Model

NE, Kimmo Tuominen

Why not model the $\bar{q}q$ condensate by a scalar in AdS and input the running mass through Δm?

$$\mathcal{L} \sim \frac{1}{2}\left[\rho^3(\partial_\rho L)^2 + \rho \Delta m^2(\rho, L) L^2\right]$$

The Conformal Window

SU(Nc) gauge theory with Nf fundamental quarks

No AF

Nf=11/2 Nc

CFT

Nf = ? Nc

χ SB

$$\beta(g) = -\frac{g^3}{(4\pi)^2}\left\{\frac{11}{3}N_c - \frac{2}{3}N_f\right\} - \frac{g^5}{(4\pi)^4}\left\{\frac{34}{3}N_c^2 - \frac{N_f}{N_c}\left[\frac{13}{3}N_c^2 - 1\right]\right\} + \cdots$$

Using the 't Hooft coupling, and setting $\frac{N_f}{N_c} \to x$ we obtain

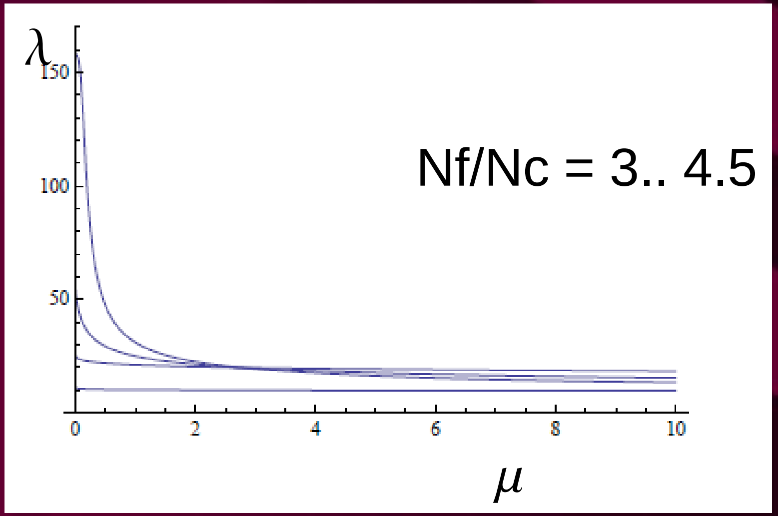
$$\lambda \equiv g^2 N_c \quad , \quad \dot{\lambda} = -b_0\lambda^2 + b_1\lambda^3 + \mathcal{O}(\lambda^4)$$

with

$$b_0 = \frac{2}{3}\frac{(11-2x)}{(4\pi)^2} \quad , \quad \frac{b_1}{b_0^2} = -\frac{3}{2}\frac{(34-13x)}{(11-2x)^2}$$

$m\ \bar{q}q$

$$\gamma_m^{(1)} = \mu\frac{d\ln m_q}{d\mu} = \frac{3\lambda}{(4\pi)^2}$$



If critical γ = 1…. Nf/Nc ~ 4

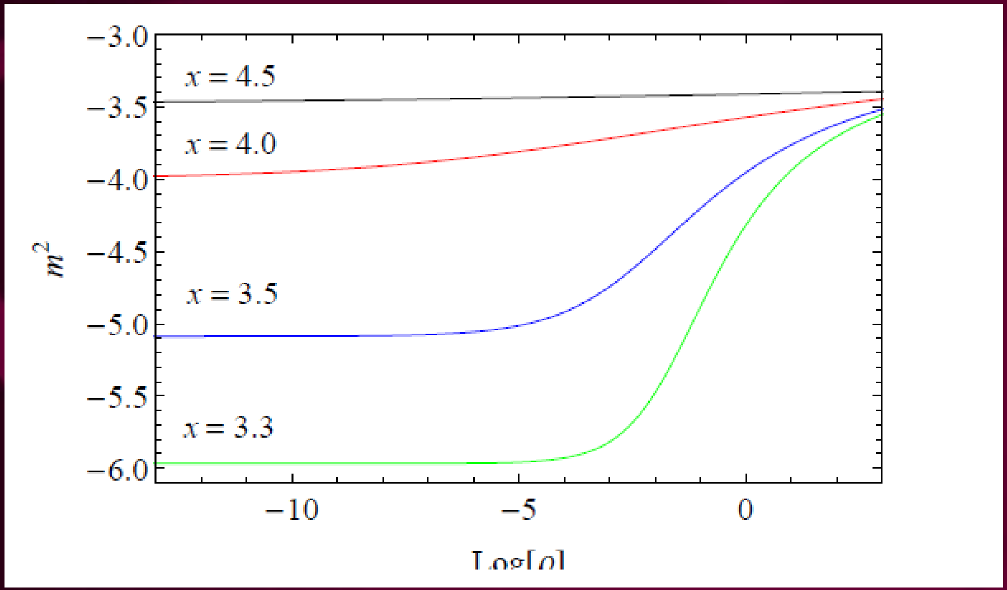
Appelquist, Terning, Sannino,Kiritsis,Jarvinen…

Brutally Simplified Model

We model the $\bar{q}q$ condensate by a scalar in AdS and input the running mass through Δm

$$\mathcal{L} \sim \frac{1}{2}\left[\rho^3(\partial_\rho L)^2 + \rho\Delta m^2(\rho, L)L^2\right]$$

EG $m^2 = \Delta(\Delta - 4)$ $\gamma_m^{(1)} = \mu\frac{d\ln m_q}{d\mu} = \frac{3\lambda}{(4\pi)^2}$ $\delta m_*^2 \sim 2\gamma_{m*}^{(1)} = \frac{6\lambda_*}{(4\pi)^2}$

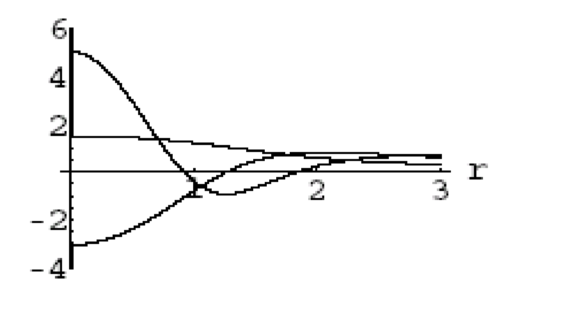


With the perturbative result for two loop running of $\bar{q}q$ dimension the transition occurs at

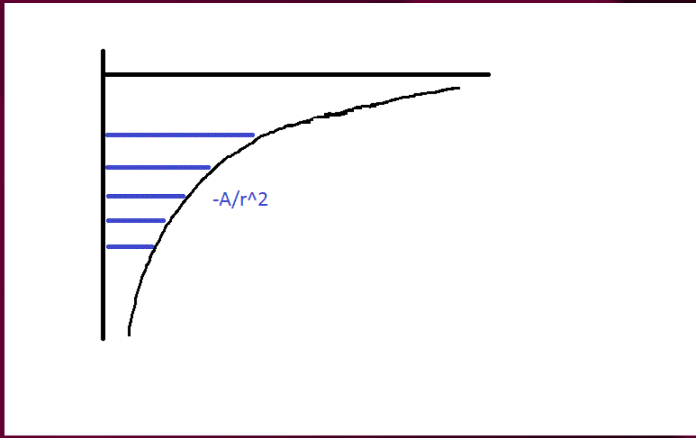
BKT transition

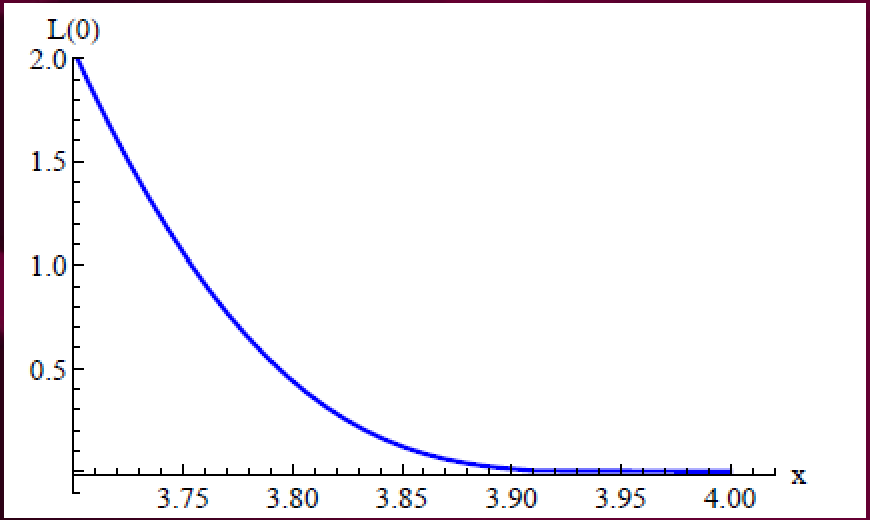
Son, Kaplan, Karch…

A transition due to a violation of the BF bound in the deep IR is of holographic BKT type… $L = L_0 + \delta(\rho) e^{ikx}$ $k^2 = -M^2$



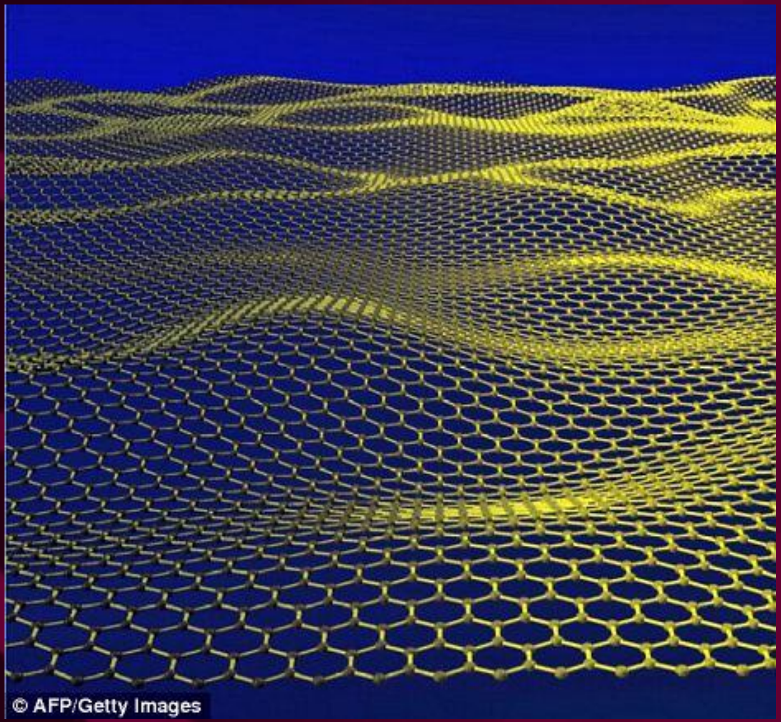
The Schroedinger equation for the mesonic fluctuations at $m^2 = -4$ has an infinite number of unstable modes…





(Miransky scaling)

2+1d systems



Graphene has relativistic fermions

Interacting with 3+1d EM

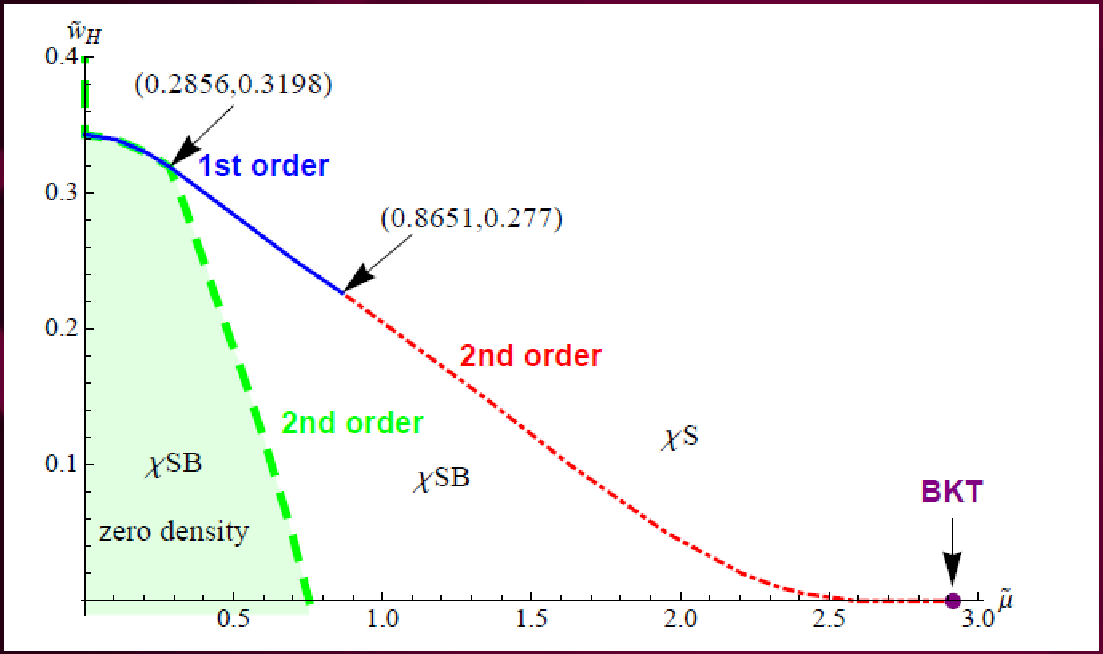
EM might be effectively strong due to c_eff <c

Then system looks like a probe brane(?)

Phase Diagram for D3/D5 + B

	0	1	2	3	4	5	6	7	8	9
$D3$	—	—	—	—	●	●	●	●	●	●
$D5$	—	—	—	●	—	—	—	●	●	●

2+1d N=2 quark defect theory



e-Print:
arXiv:1003.2694
[hep-th]

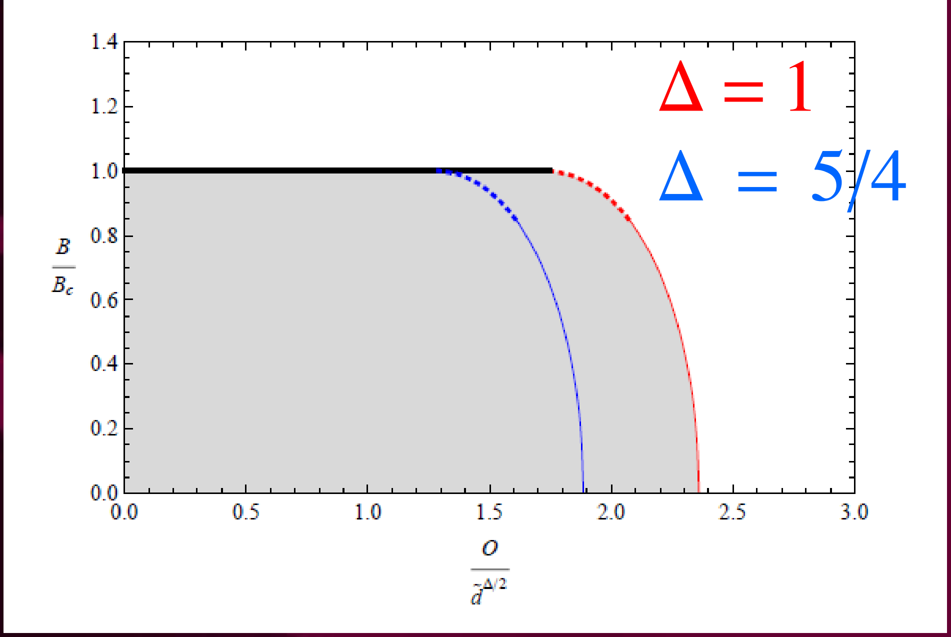
$$m^2 = -3O^2/(\tilde{d}^2 + O^2)$$

From Mean-Field 2nd Order to BKT

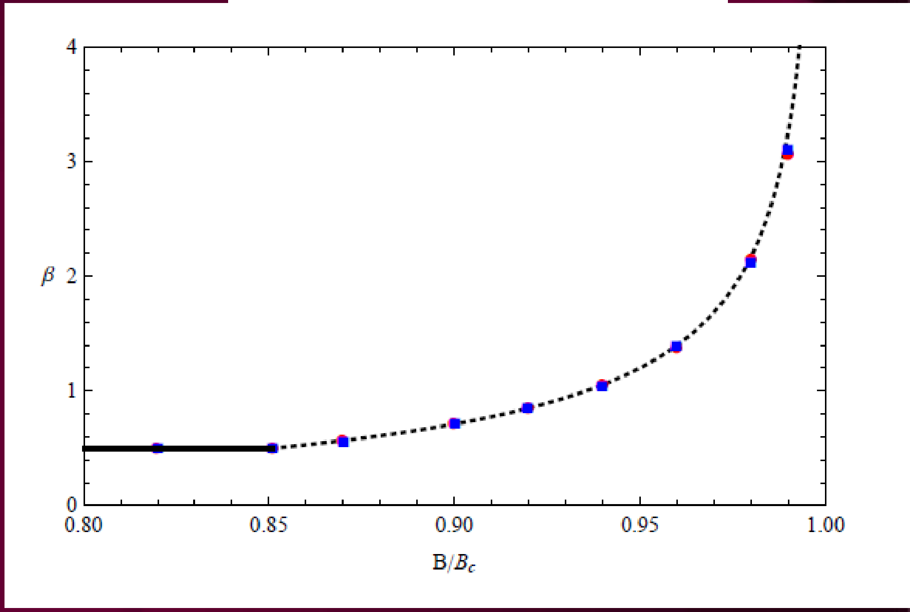
arXiv:1008.1889
NE, Kim, Jensen

$$\tilde{S}_5 = -\mathcal{N} \int d\rho \sqrt{1 + L'^2} \sqrt{\tilde{d}^2 + \rho^4 \left(1 + \frac{B^2}{w^4} + \frac{O^2}{w^{2\Delta}}\right)}$$

If we add a phenomenological operator O that causes symmetry breaking but is not dim 2… B+d triggers BKT…. O+d is second order mean-field… what about O+B+d:



$$c \sim (B - B_c)^{\beta}$$



cf Faulkner, Horowitz,Roberts - **arXiv:1008.1581** [hep-th]

Superconductivity in D3/D5

0812.3273

A chemical potential should destabilize any charged scalars in AdS

NE, Petrini – hep-th/0108052

$$|D^{\mu}\phi|^2 \rightarrow m^2|\phi|^2 = -A_t^2|\phi|^2$$

NB there are squark instabilities in these systems .. that we ignore

hep-th/0504151
1208.3197

We have baryon number chemical potential…

The D7 brane embedding is a scalar…

But does not cause scalar condensation – all D7 excitations have B=0.

Superconductivity in D3/D5

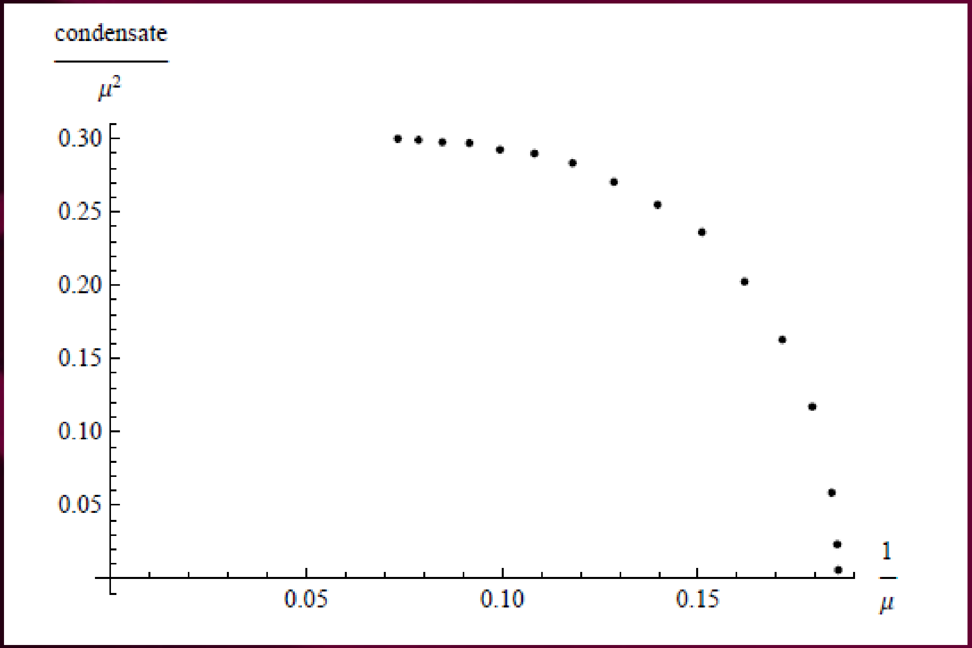
Top-down Gubser p-wave

M. Ammon, J. Erdmenger, M. Kaminski and P. Kerner, [arXiv:0810.2316 [hep-th]].
P. Basu, J. He, A. Mukherjee, H. Shieh, [arXiv:0810.3970 [hep-th]].

ISOSPIN CHEMICAL POTENTIAL

Truncated NADBI

$$S \sim T_5 \int d^6\xi \sqrt{-\det G} \left(1 - \frac{1}{4} Tr\left(F^2\right)\right)$$

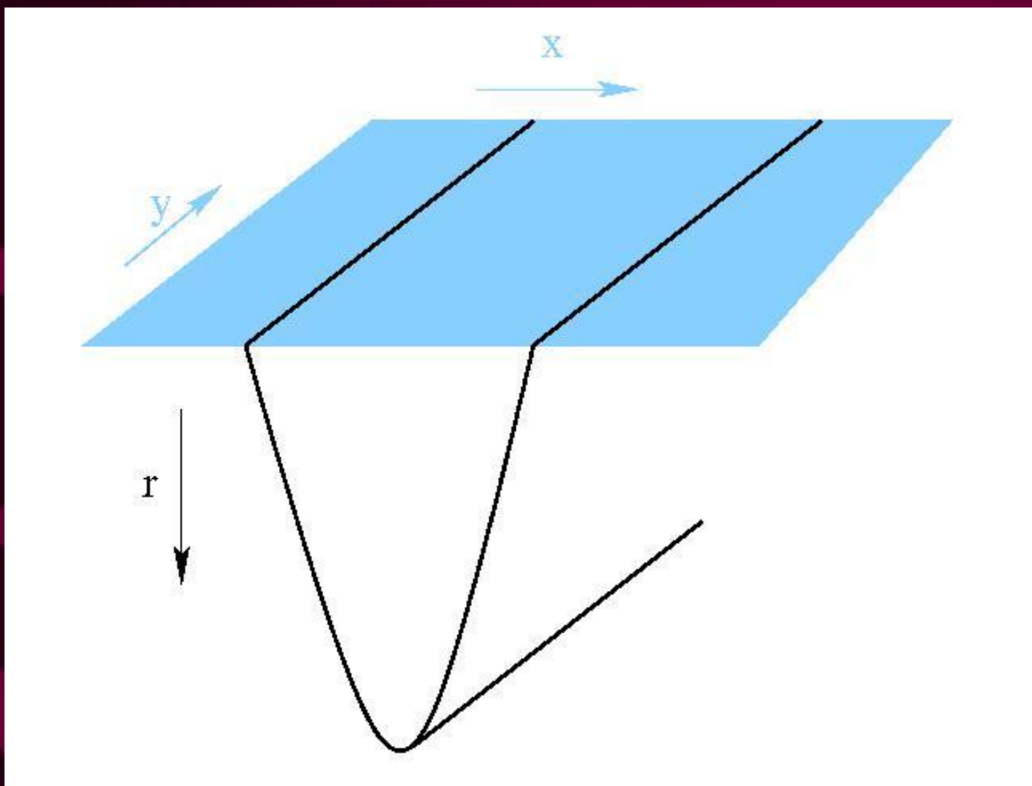


$$A = \Phi(\rho)\tau^3 dt + w(\rho)\tau^1 dx_1$$

$$\Phi \sim \mu - \frac{\rho}{r}$$
$$w \sim \mu' + \frac{c}{r}$$

Rho meson condensation…
“p-wave” condensation…

New Semenoff Configurations - 1



D5/$\overline{\text{D5}}$ configuration

Cf Wilson loop, Sakai Sugimoto

Graphene bilayers (not graphite) seem to display chiral symmetry breaking

“Exciton” condensation

New Semenoff Configurations - 2

See also Lippert

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
$D3$	×	×	×	×						
$D5$	×	×	×		×	×	×			
$D7$	×	×	×		×	×	×	×	×	

B+d

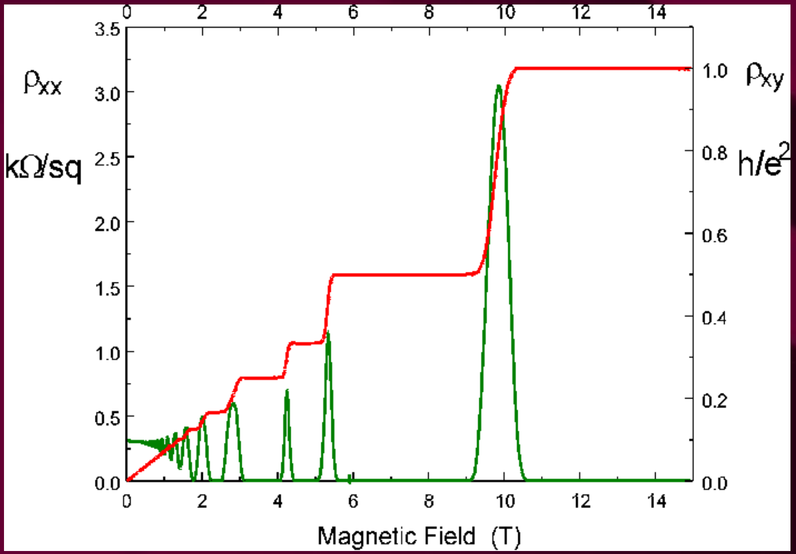
D5s can blow up into fuzzy ν D7…

$$2\pi\alpha'\mathcal{F} = \sqrt{\lambda}\alpha' \left(\frac{d}{dr} a(r) dr \wedge dt + b dx \wedge dy + \frac{f}{2} d\cos\tilde{\theta} \wedge d\tilde{\phi} \right)$$

Density ends on monopole configuration on D7 world volume..

New zero T, non-zero d vacuum configurations…

Semenoff identifies these with states with filled fermionic Landau levels… ie Integer Quantum Hall states…



Conclusion

Brane probes remain a key tool for studying gauge theory holographically…

Quantum critical points, first, second, BKT and non-mean field transitions…

New physics still emerging: Fermi surface dynamics

: Conformal Window dynamics

: Bi-layer configurations

: Hall states….

(Out of equilibrium dynamics without dynamical gravity…)

Out of Equilibrium Dynamics

$$S \sim \int d^8\xi \sqrt{-\det(P[G]_{ab})} \sim \int d\tau\, d\rho\, \tau \rho^3 \mathbb{A} \sqrt{1 + (\partial_\rho L)^2 - \mathbb{B}\frac{(\partial_\tau L)^2}{(\rho^2 + L^2)^2}},$$

Chiral Transition in Janik's Cooling Geometry

With KY Kim, Ingo Kirsch, Tigran Kalaydzhyan (DESY)

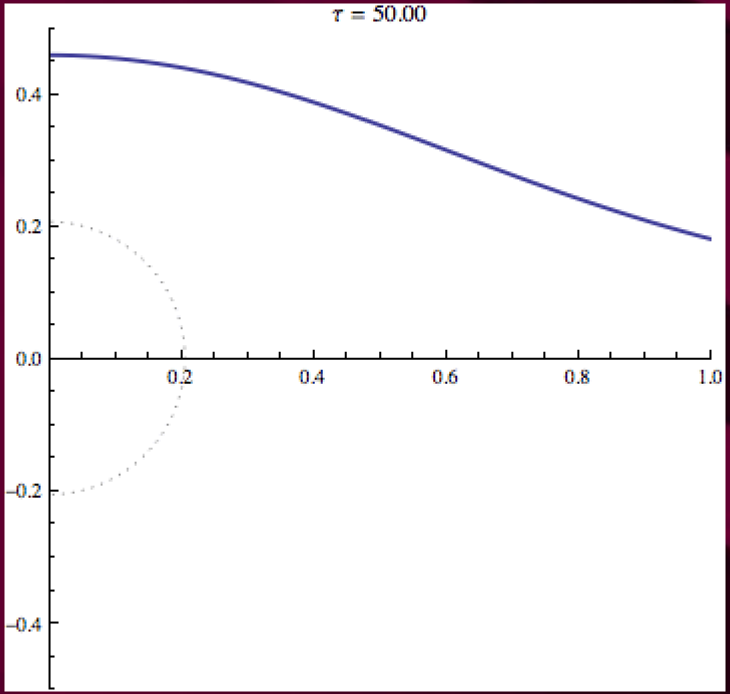
The black hole shrinks changing the effective potential…

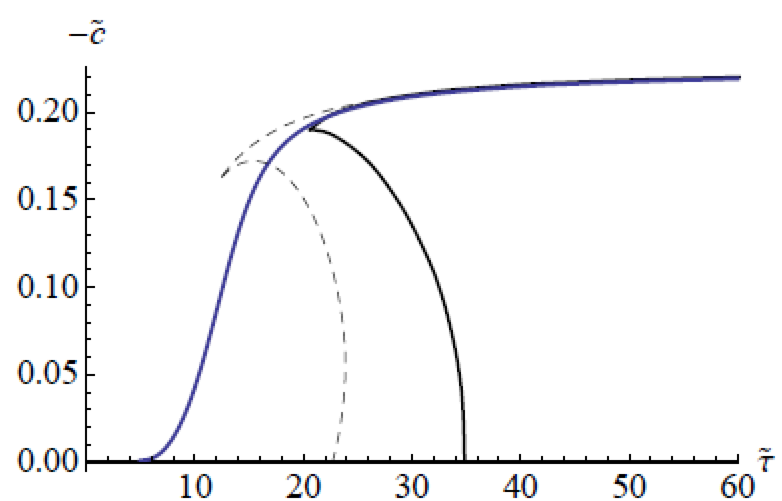
$$ds^2 = \frac{r^2}{R^2}(-e^{a(\tau,r)}d\tau^2 + e^{b(\tau,r)}\tau^2 dy^2 + e^{c(\tau,r)}dx_\perp^2) + \frac{R^2}{r^2}(d\rho^2 + \rho^2 d\Omega_3^2 + dL^2 + L^2 d\phi^2)$$

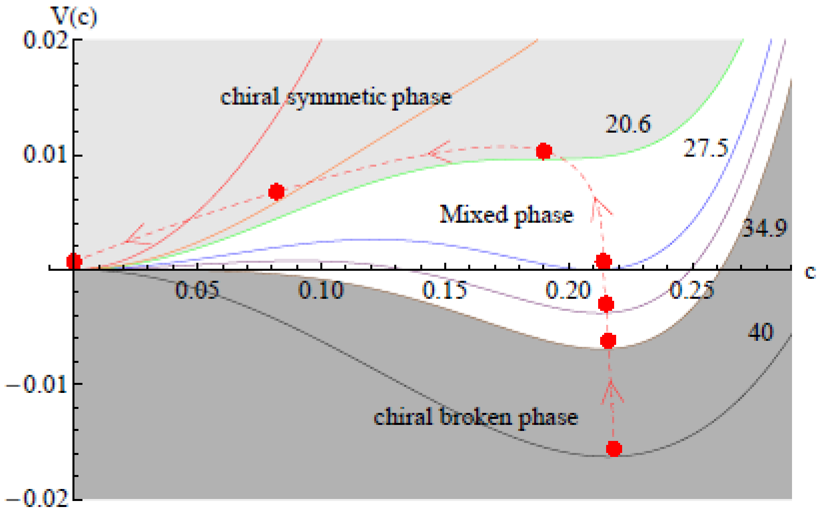
$$a(\tau, z) = \ln\left(\frac{(1 - v^4/3)^2}{1 + v^4/3}\right) + 2\eta_0 \frac{(9 + v^4)v^4}{9 - v^8}\left[\frac{1}{(\varepsilon_0^{3/8}\tau)^{2/3}}\right] + \mathcal{O}\left[\frac{1}{\tau^{4/3}}\right],$$

$$b(\tau, z) = \ln(1 + v^4/3) + \left(-2\eta_0 \frac{v^4}{3 + v^4} + 2\eta_0 \ln\frac{3 - v^4}{3 + v^4}\right)\left[\frac{1}{(\varepsilon_0^{3/8}\tau)^{2/3}}\right] + \mathcal{O}\left[\frac{1}{\tau^{4/3}}\right],$$

$$b(\tau, z) = \ln(1 + v^4/3) + \left(-2\eta_0 \frac{v^4}{3 + v^4} - \eta_0 \ln\frac{3 - v^4}{3 + v^4}\right)\left[\frac{1}{(\varepsilon_0^{3/8}\tau)^{2/3}}\right] + \mathcal{O}\left[\frac{1}{\tau^{4/3}}\right],$$







Equilibrium vs PDE solutions…

Bubble formation…