Fermi Surfaces in

N=4 SYM

From calculations with Oliver DeWolfe and Steve Gubser

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Order of Events

About Fermi Surfaces

A few defining properties

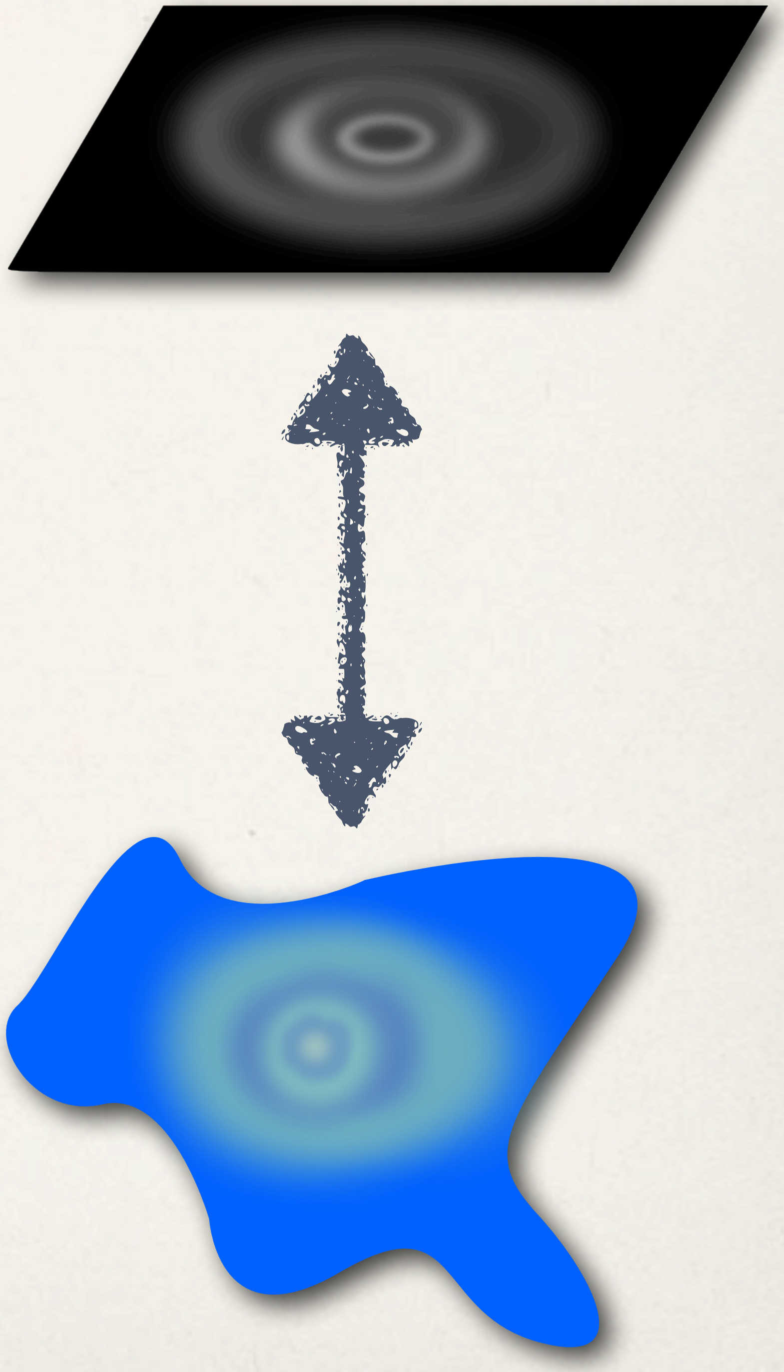
Fermi Surface Embeddings

Fermi surface physics in holography

Dirac equations with a prestigious pedigree

Survey of selected results

What's Next

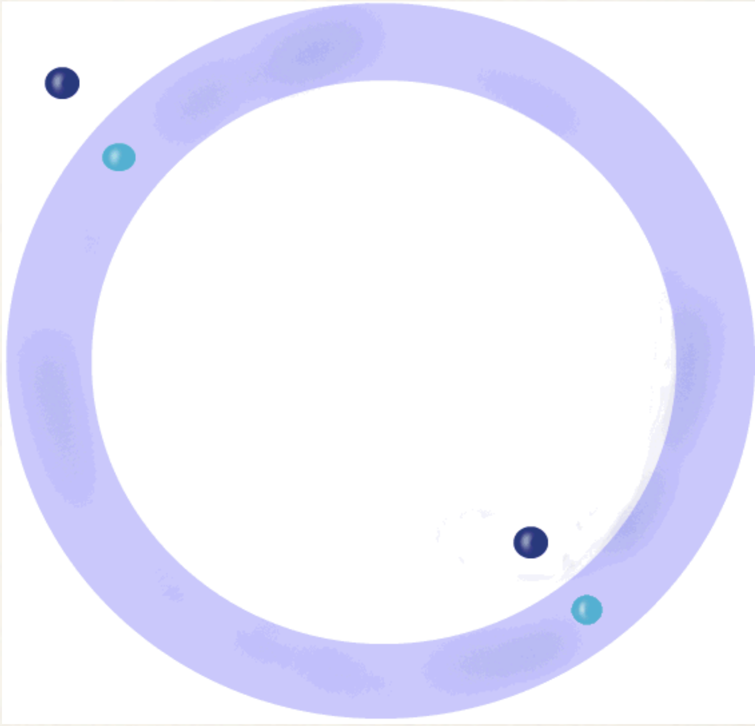


About Fermi Surfaces:

In Field Theory

A Fermi surface is the place, k_F, where

$$G_R{}^{-1}(\omega = 0, k = k_F) = 0$$



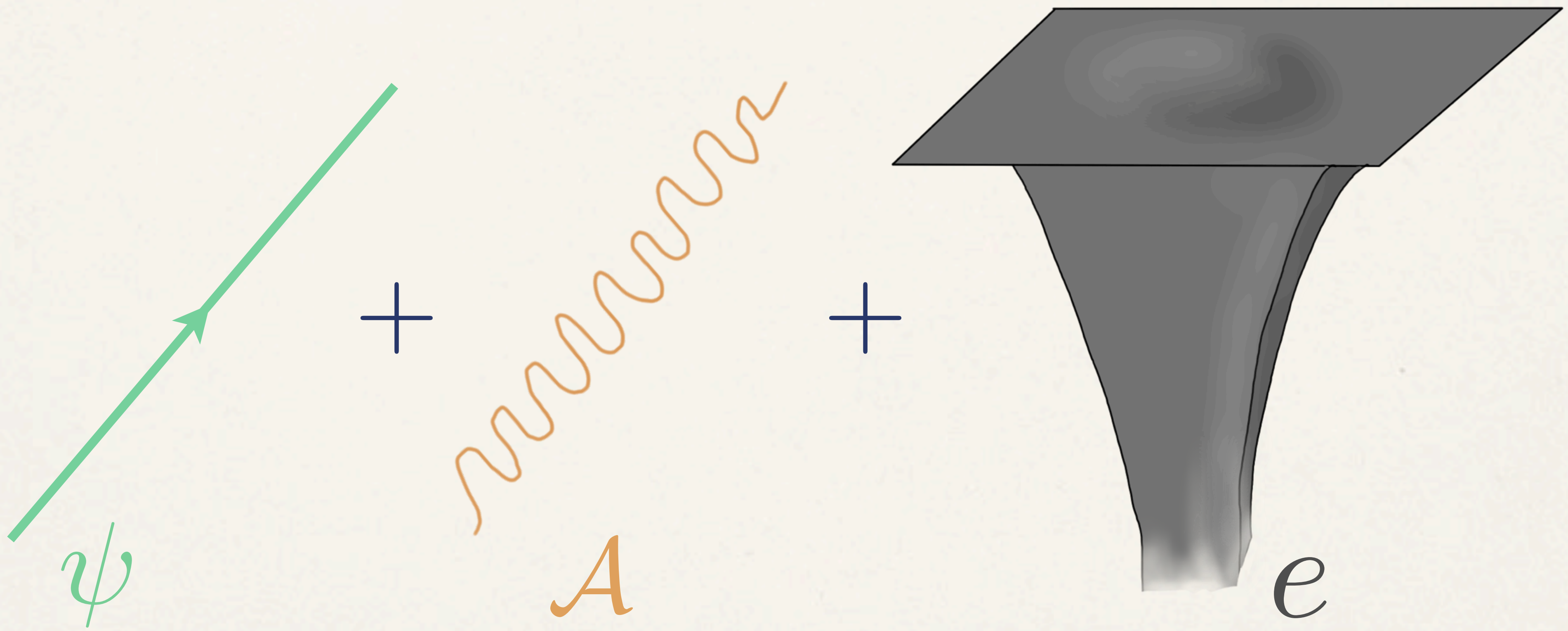
Low energy excitations about this surface can be parametrized by

$$G_R = \frac{Z}{\omega - v_F(k - k_F) + \Sigma(\omega, k)}$$

About Fermi Surfaces:

In Gravity Theories

Bottom up finite density physics

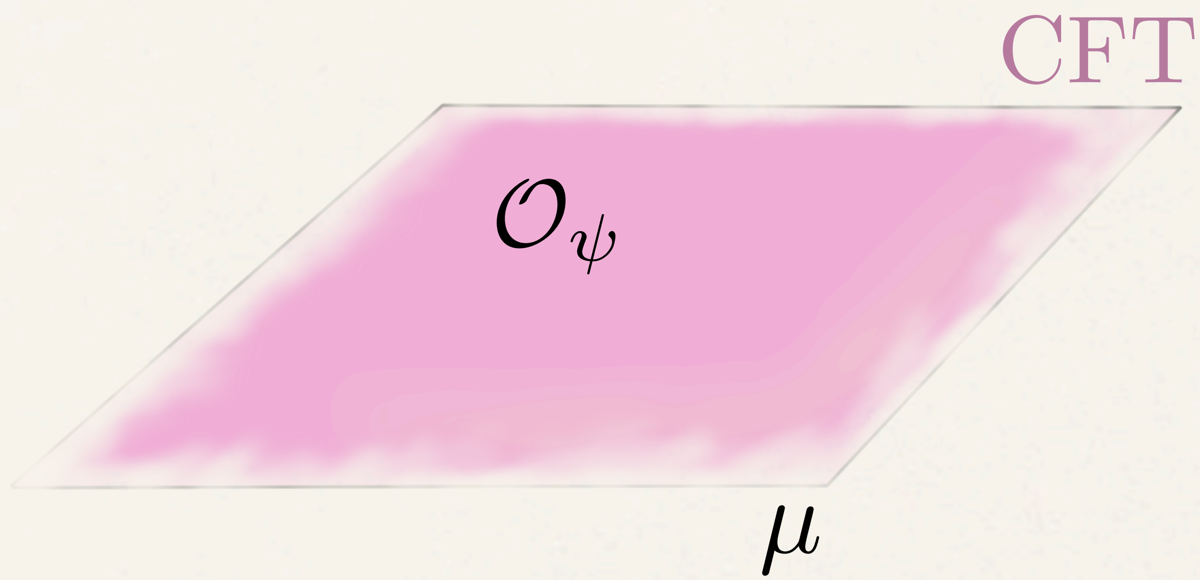


About Fermi Surfaces:

In Gravity Theories

Bottom up finite density physics

$\stackrel{?}{=}$

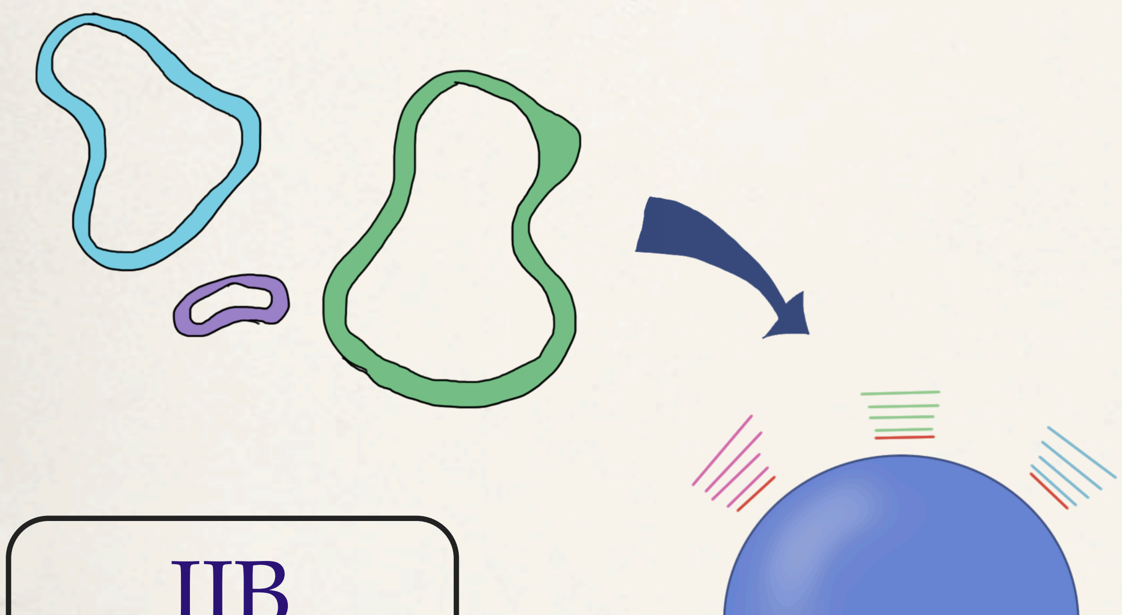


$$(i\Gamma^\mu \nabla_\mu + q\,\Gamma^\mu \mathcal{A}_\mu - m + \ldots)\,\psi = 0$$

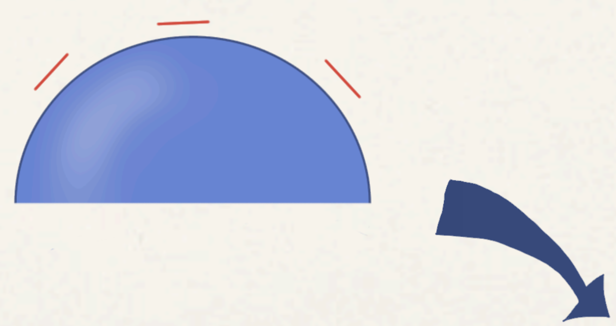
About Fermi Surfaces:

In Gravity Theories

The top down alternative



IIB
SUGRA
in D=10



N=8
gauged
SUGRA in
D=5

$$\mathcal{L}_B\left[e, \mathcal{A}_i, \varphi_j\right]$$

Fermi Surface Embeddings

In Gravity Theories

Which Background?

Try the "2+1 Q" BBs ⟷ N=4 SYM at (2x) finite density, T

$$ds^2 = e^{2A(r)}\left(-h(r)\,\mathrm{d}t^2 + \mathrm{d}\vec{x}^2\right) - \frac{e^{2B(r)}}{h(r)}\mathrm{d}r^2$$

$$a = \Phi_1(r)\mathrm{d}t \qquad \mathcal{A} = \Phi_2(r)\mathrm{d}t$$

$$\varphi = \phi(r)$$

The functions A, h, B, $\Phi 1$, $\Phi 2$, and ϕ are cumbersome but explicitly known

About Fermi Surfaces:

In Gravity Theories

What about the fermions?

Fix background, study spin-1/2 fluctuations...

$$\left(i\Gamma^\mu \nabla_\mu + q_j \Gamma^\mu \mathcal{A}^j_\mu - m(\varphi) + i p_j(\varphi) \mathcal{F}^j_{\mu\nu} \Gamma^{\mu\nu}\right) \psi = 0$$

Important: fermion properties are no longer arbitrary

Fermi Surface Embeddings

In Gravity Theories

Bulk fermion fields of interest and their dual SUGRA operators:

SUGRA Fermion	AdS Mass	SO(6) Rep	SYM Op
ψ	$\frac{1}{2L}$	20	$\mathrm{Tr}\,\lambda Z$
Ψ	$\frac{3}{2L}$	4	$\mathrm{Tr}\,F_+\lambda$

We study the fermions that do not mix with the gravitini

Fermi Surface Embeddings

In Gravity Theories

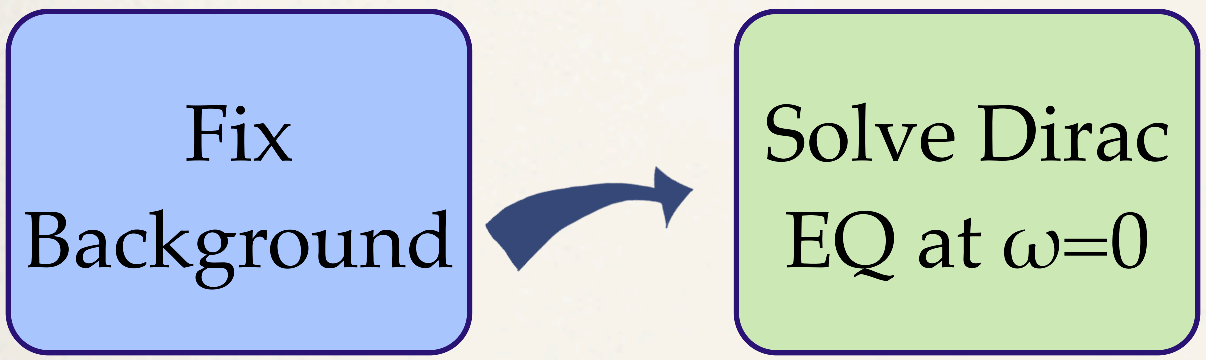
Workflow

Fix Background

Fermi Surface Embeddings

In Gravity Theories

Workflow



$$\psi_{r\to\infty} \sim A(k)\sqrt{r} + B(k)r^{-3/2}$$

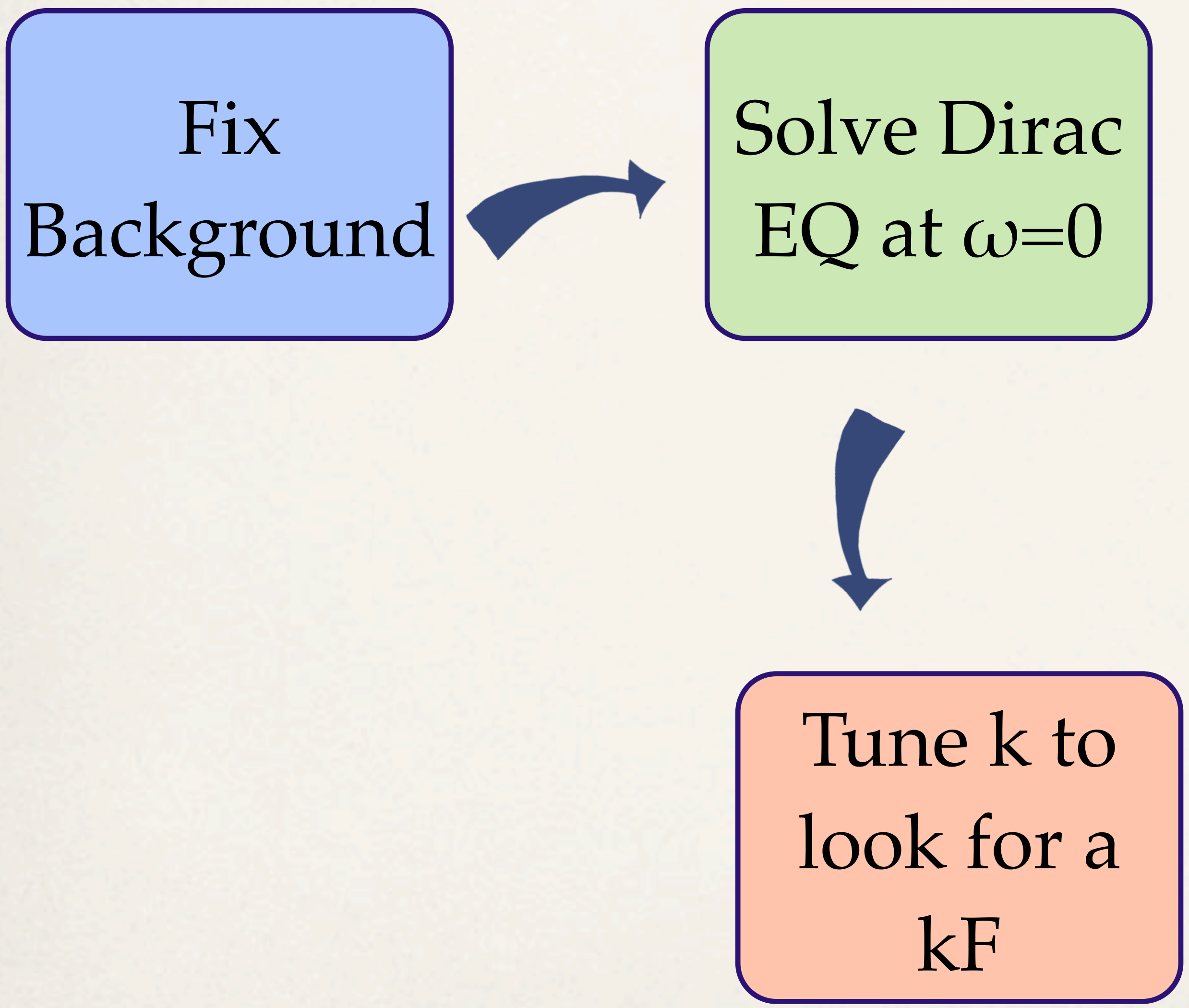
where

$$\delta S_{\mathrm{CFT}} = \int \mathrm{d}^4x\, A(x)\mathcal{O}_\psi(x)$$

Fermi Surface Embeddings

In Gravity Theories

Workflow



$$\psi_{r\to\infty} \sim A(k)\sqrt{r} + B(k)r^{-3/2}$$

where

$$\delta S_{\rm CFT} = \int \mathrm{d}^4x\, A(x)\mathcal{O}_\psi(x)$$

$$G_R(\omega = 0, k) \sim \frac{B(k)}{A(k)}$$

so

$$A(k_F) = 0$$

Fermi Surface Embeddings

In Gravity Theories

Finite frequency fluctuations

$$G_R = \frac{Z}{\omega - v_F(k - k_F) + \Sigma(\omega, k)}$$

In the extremal 2+1 system, controlled by IR AdS2:

$$\Sigma(\omega, k) \sim e^{i\gamma_{k_F}} \omega^{2\nu_{k_F}} \qquad \text{with} \qquad [\mathcal{O}]_{\text{IR}} = \frac{1}{2} + \nu_k$$

IR dimension dictates dispersion relation, characterizes medium

If:

$\nu_{k_F} < \frac{1}{2}$ IR CFT operator is relevant, non-Fermi liquid

$\nu_{k_F} > \frac{1}{2}$ IR CFT operator is irrelevant, stable qp's

$\nu_{k_F} = \frac{1}{2}$ IR CFT operator is marginal, like "optimally doped cuprates"

Fermi Surface Embeddings

In Gravity Theories

Finite frequency fluctuations

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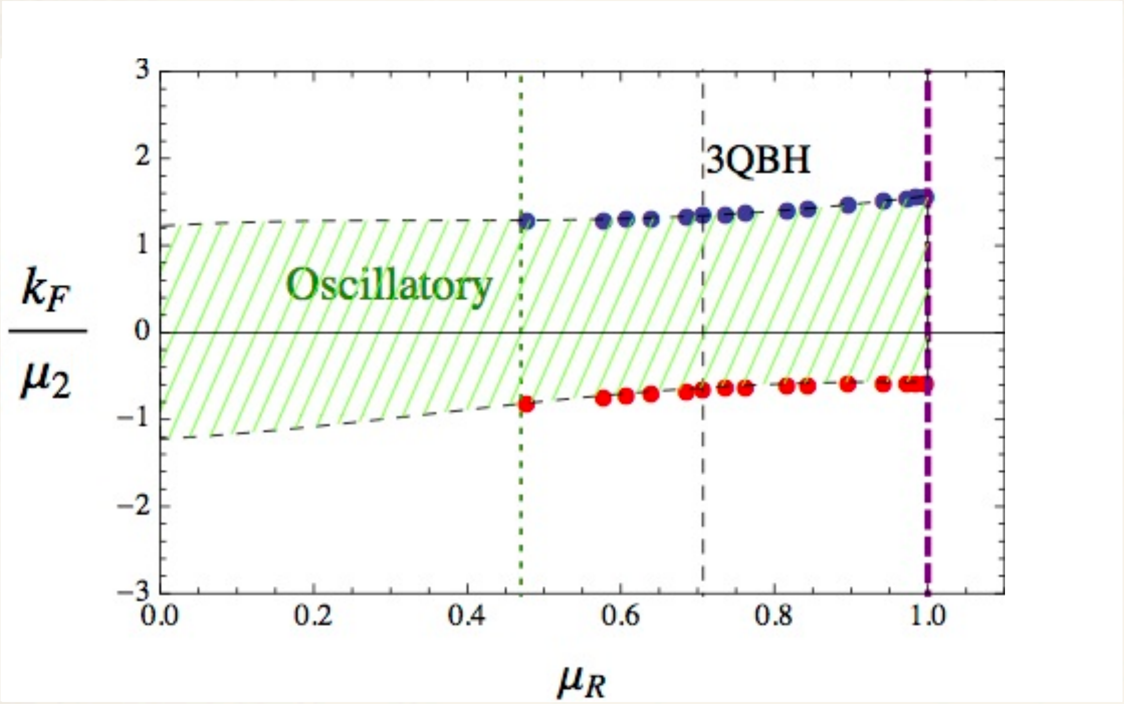
IR dimension dictates dispersion relation, characterizes medium

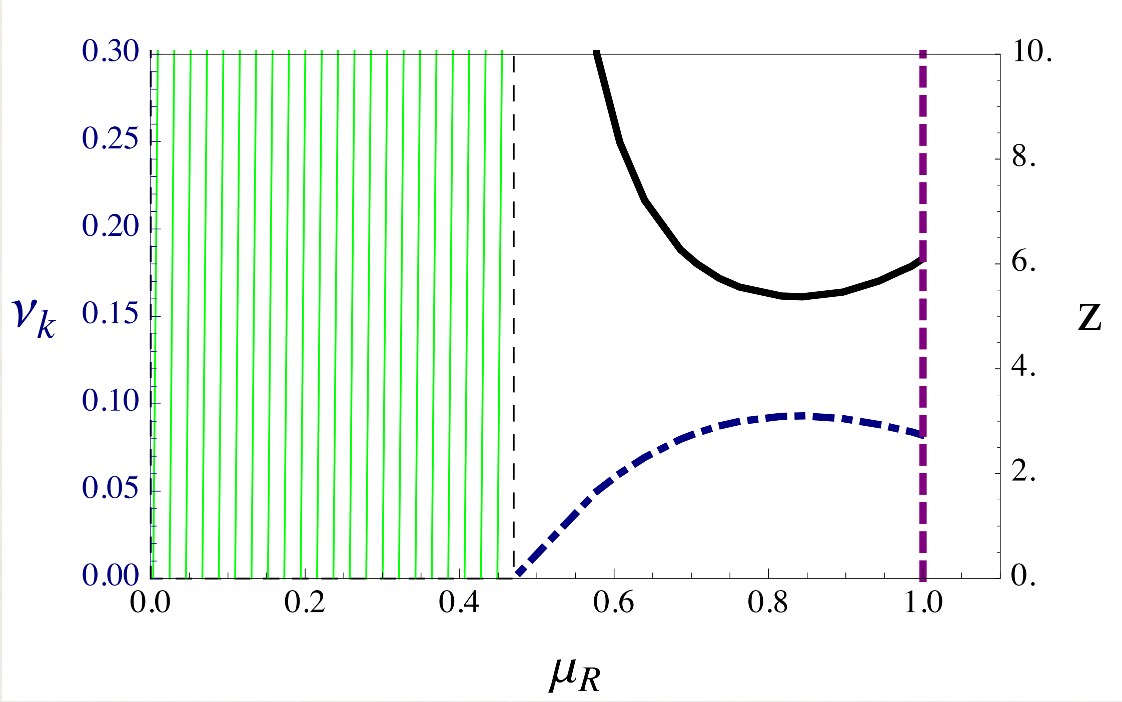
In these embeddings, ν_{k_F} is less than 1/2, and the self energy dominates:

$$\omega_* \sim (k-k_F)^z \qquad \text{where} \qquad z \equiv \frac{1}{2\nu_{k_F}}$$

Fermi Surface Embeddings

Lessons from the 2+1 Charge Zoo



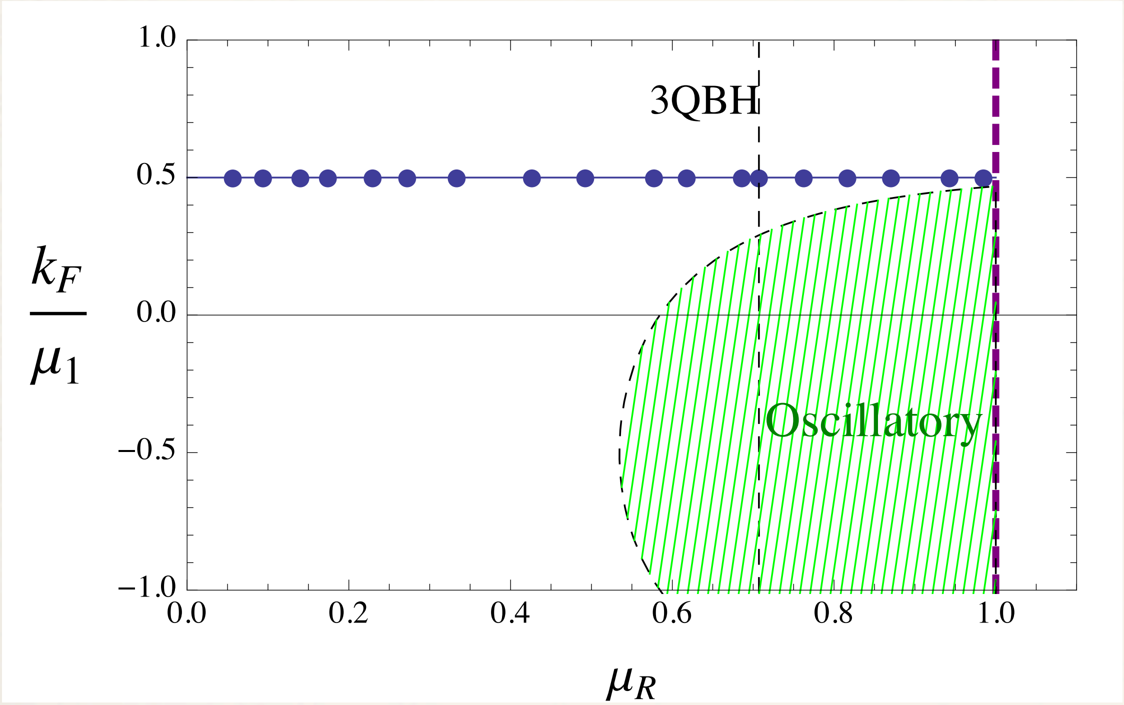


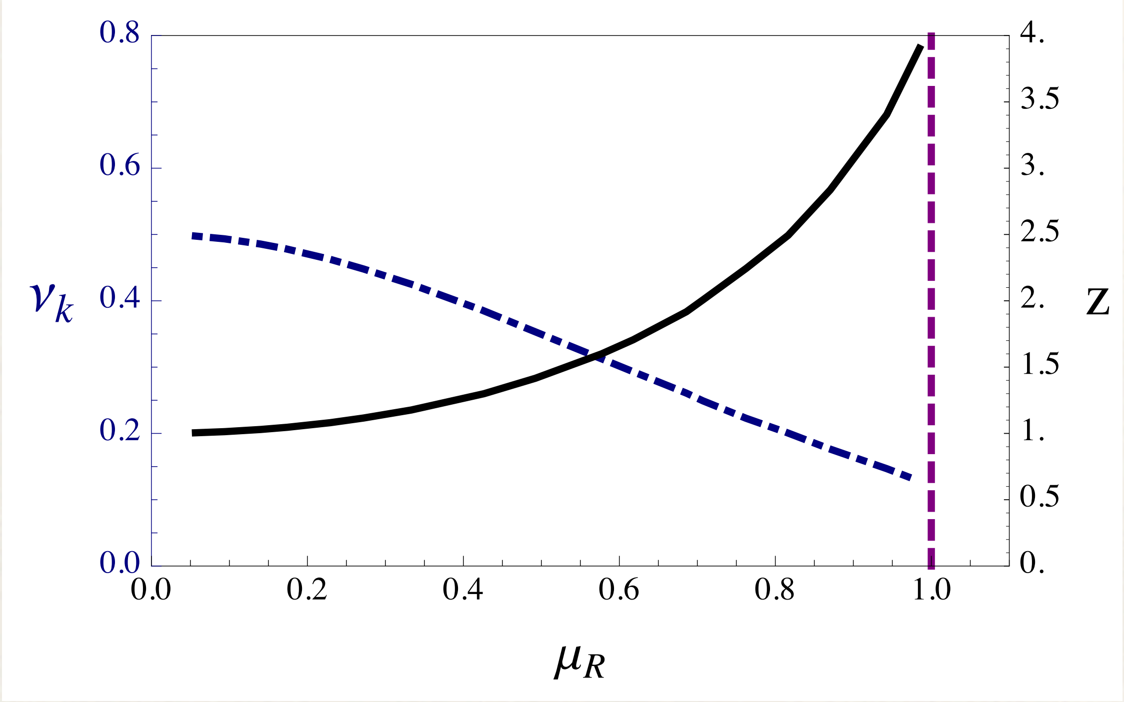
- Fermion modes dual to Tr Fλ have no Fermi surface

- Fermion modes dual to Tr λZ may have 0 or 1 or 2 Fermi surfaces, depending on their charge

Fermi Surface Embeddings

Lessons from the 2+1 Charge Zoo





- These systems are almost all non-Fermi liquids, but there exists one case resembling a MFL

Fermi Surface Embeddings

Lessons from the 2+1 Charge Zoo

The 2+1-Q black holes backgrounds have finite entropy at zero temperature...

...Can we repeat this analysis in a more phenomenologically favorable background?

Fermi Surface Embeddings

The extremal 2-Charge Solution

Background

$$A(r) = \log\frac{r}{L} + \frac{1}{3}\log\left(1 + \frac{Q^2}{r^2}\right)$$

$$B(r) = -\log\frac{r}{L} - \frac{2}{3}\log\left(1 + \frac{Q^2}{r^2}\right)$$

$$h(r) = 1 - \frac{Q^4}{(r^2+Q^2)^2}$$

$$\phi(r) = \sqrt{\frac{2}{3}}\log\left(1 + \frac{Q^2}{r^2}\right)$$

This is important!

$$\Phi(r) = \frac{Q}{2L}\left(1 - \frac{Q^2}{r^2+Q^2}\right)$$

Fermi Surface Embeddings

The extremal 2-Charge Solution

Fermi surfaces exist

So do novel features at finite ω...

Near the horizon, bulk fermions are "gapped":

$$\psi_{r\to 0} \sim e^{-\frac{1}{2r}\sqrt{\frac{Q^2}{2}-\omega^2}}$$

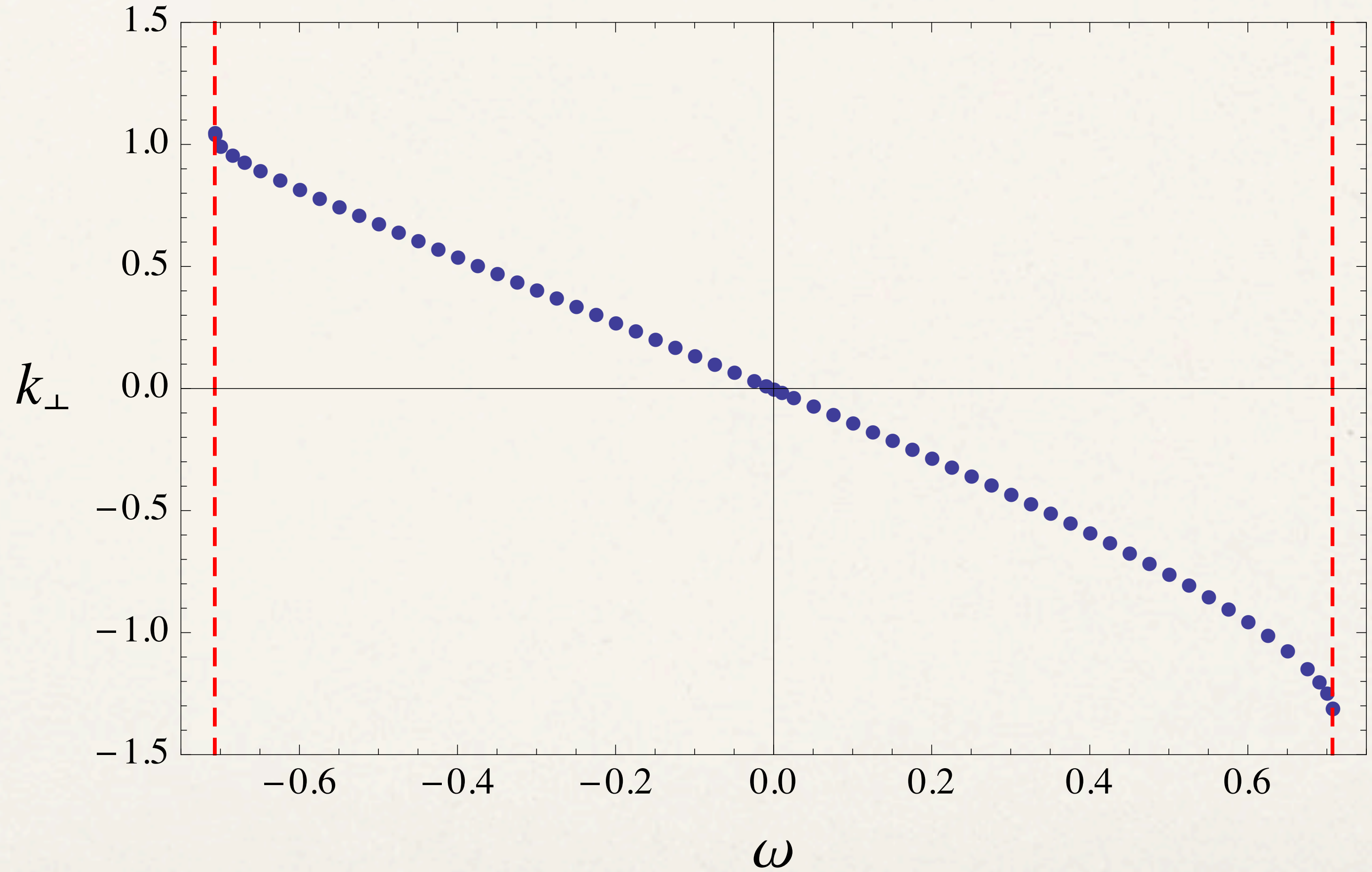
for $\omega < \omega^*$ bulk modes damped

for $\omega > \omega^*$ bulk modes oscillatory, expect field theory dissipation

Fermi Surface Embeddings

The extremal 2-Charge Solution

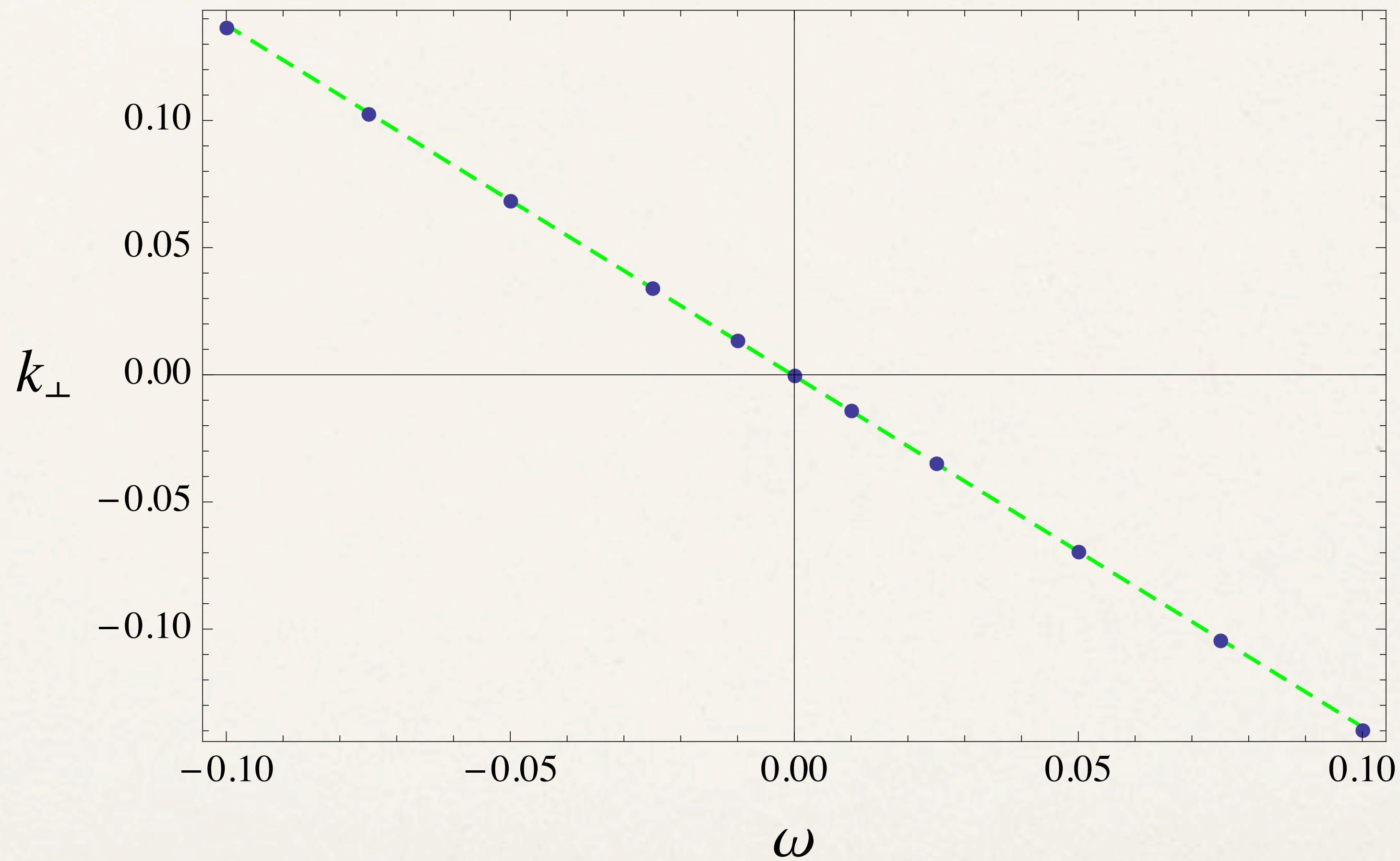
Low Energy Excitations



Fermi Surface Embeddings

The extremal 2-Charge Solution

Low Energy Excitations



More like a Fermi liquid?

Up and Coming

In the 2+1-Q BHs

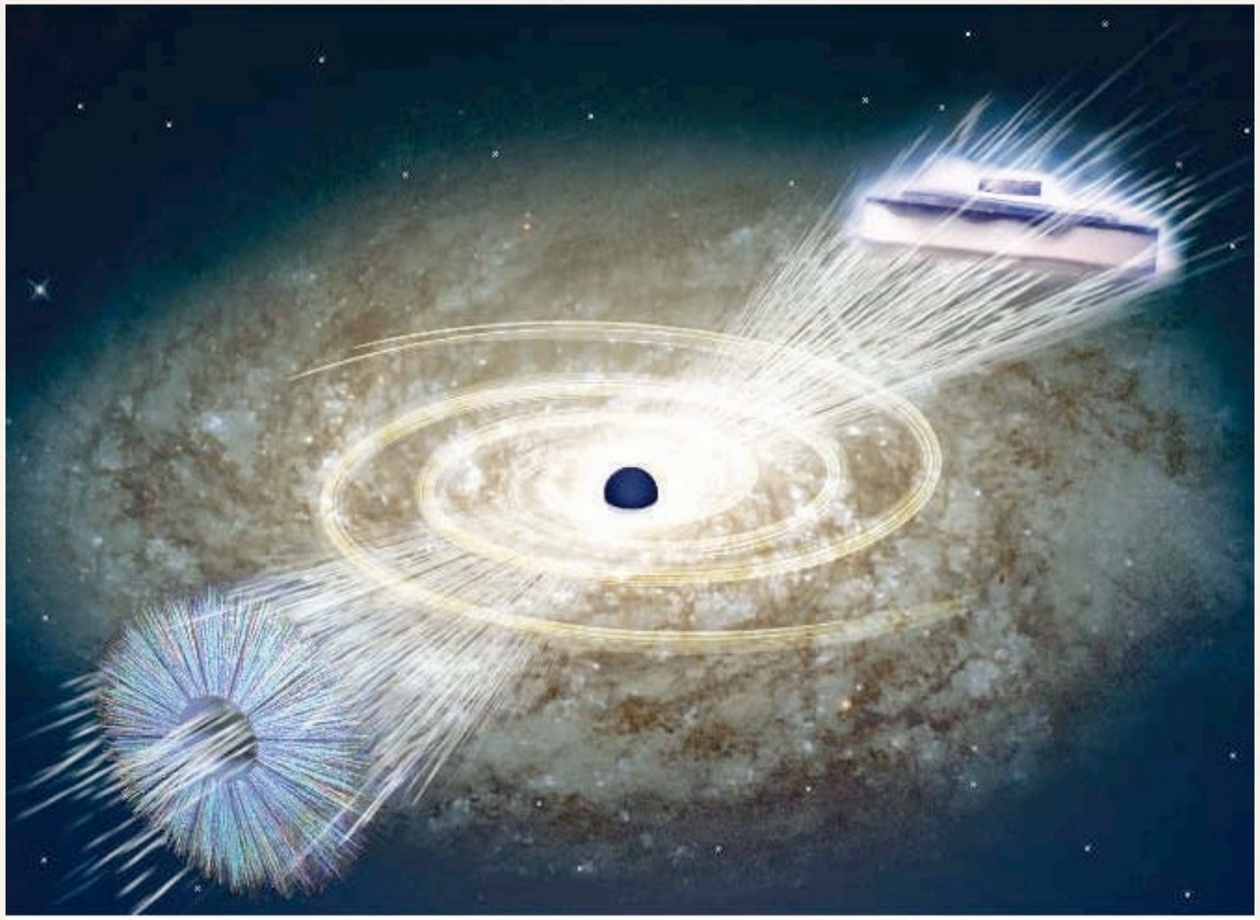
Fermi surface behavior is ubiquitous in strongly coupled N=4 SYM theory

Interesting physics abounds near the limits of these solutions (1 and 2-Q BHs, BTZ, etc.)

A better understanding of instabilities would be useful

Lots more to do...

Thanks to:



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