Towards a holographic realization of the quarkyonic phase

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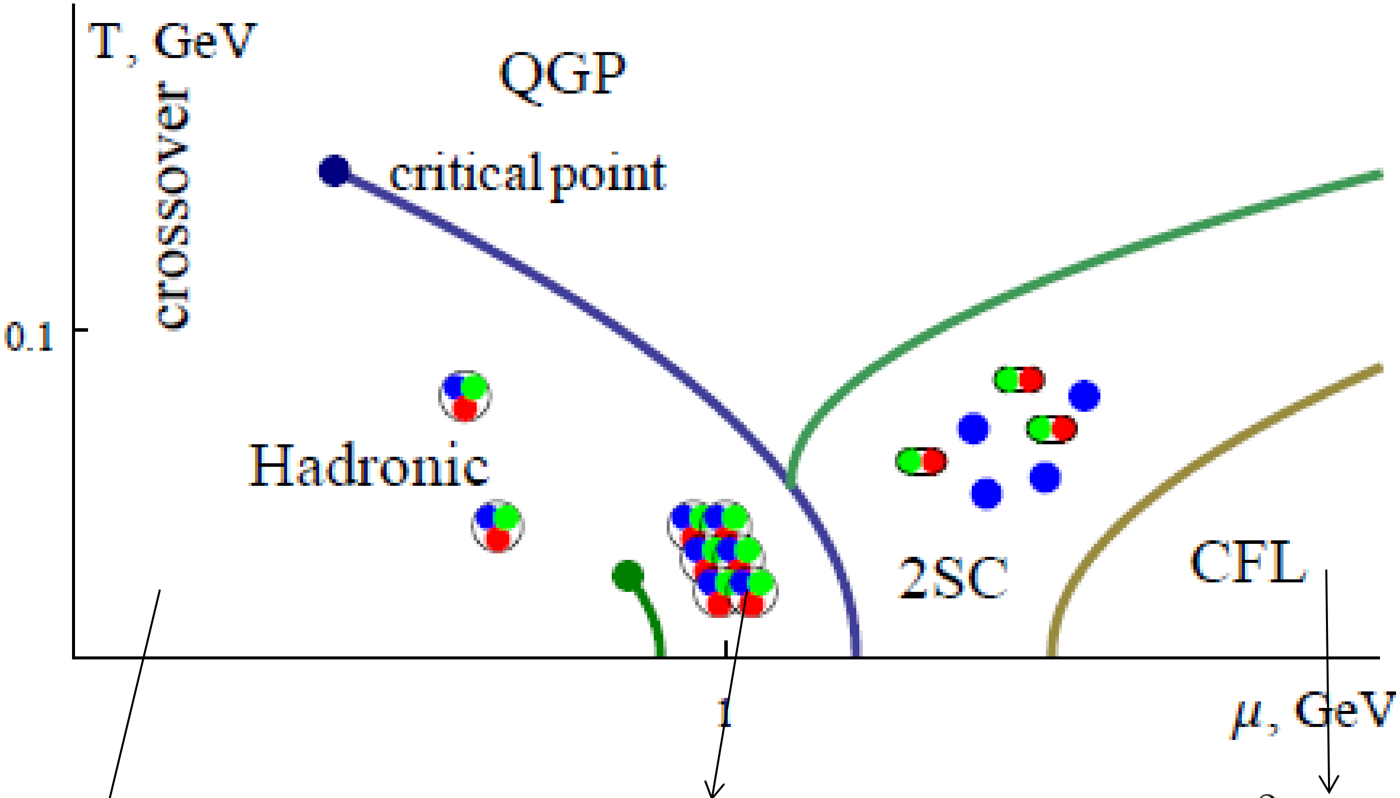
Helsinki, March 5, 2013

Based on arXiv:1209.5915 with Borun Chowdhury, Michal Heller and Jakub Jankowski

Phase diagram of QCD (from 1201.1331)



$$\mu^2 = m_B^2 + k_F^2$$

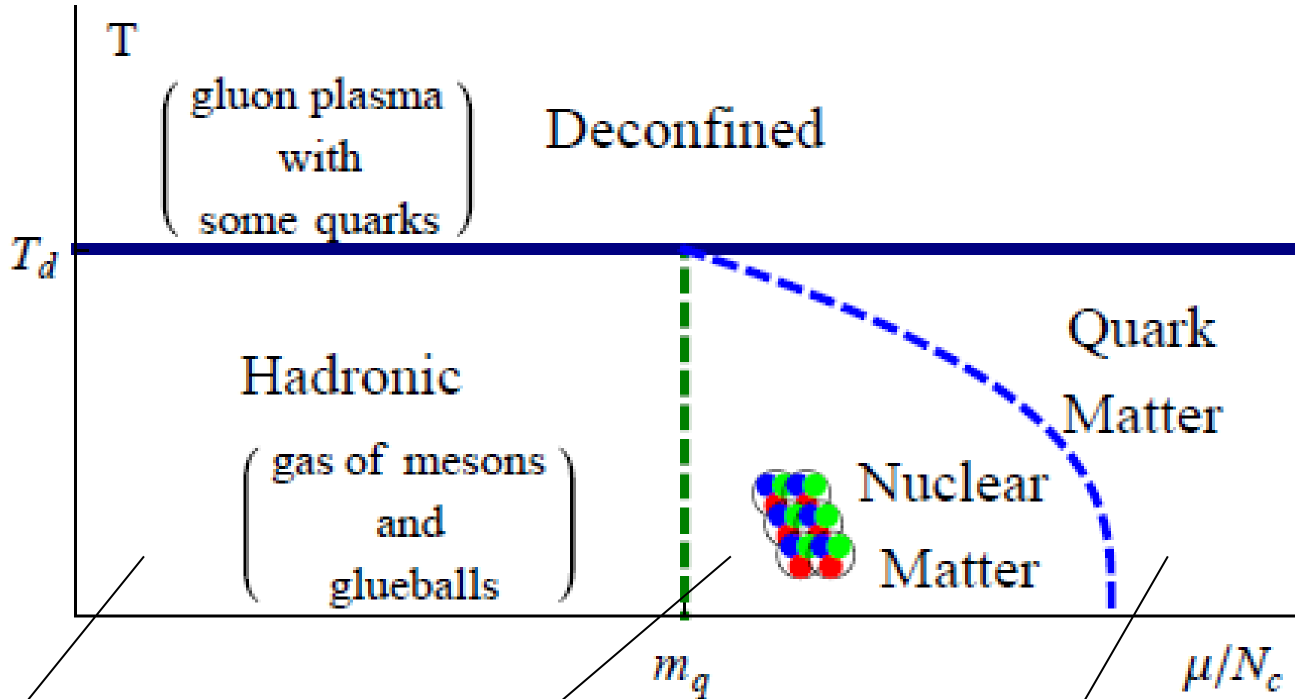
$$\langle \bar{\psi}_L \psi_R \rangle \neq 0$$

Chiral
symmetry
breaking

Nuclear matter
Difficult
Baryons condense
Probably liquid (Coulomb)
Existence solid phase?

As. Freedom
Pert theory
Condensates form

Phase diagram of large N QCD (from 1201.1331)



$\langle \bar{\psi}_L \psi_R \rangle \neq 0$

Chiral symmetry breaking
Confinement

Nuclear matter
Small region
Baryons condense
Crystal
Baryons do not overlap

Baryons do overlap

$\mu^2 = m_B^2 + k_F^2$

As. Freedom
Pert theory
Condensates form
Fermi surface

Conjecture: quark matter is in a “quarkyonic” phase (Pisarski and McLerran (arXiv:0706.2191)

- Chiral Density Waves (CDW’s): instabilities of the form $\langle \bar{\psi}_L \psi_R \rangle \neq 0, \quad \langle \bar{\psi}_L \gamma^0 \gamma^i \psi_R \rangle \neq 0$
- Quark Fermi surface, pressure of order Nc (due to overlap of baryons)
- Excitations are still confined
- Chiral symmetry is possibly restored: the modes that make up the chiral condensate are no longer available (Pauli blocking)

Deryagin, Grigoriev and Rubakov in 1972 already found evidence for instabilities in large Nc QCD at weak coupling.

Glozman (2007,2008) found evidence (disputed) for chiral symmetry restoration

Earlier, some of these features have been seen in the Skyrme model.

Massless pion $U \in \mathrm{SU}(N_f) \times \mathrm{SU}(N_f)/\mathrm{SU}(N_f)$

Low energy effective action

$$\mathcal{L} \sim \mathrm{tr}\left(\partial_\mu U \partial^\mu U^\dagger + [\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2\right)$$

Baryons are solitons in this theory
Nuclear matter = skyrme lattice
See transitions from skyrmions to half-skyrmions?

<U> averaged over many lattice cells is zero: evidence for approximate chiral symmetry restoration.

Question:

Is the quarkyonic phased realized in holographic QCD?

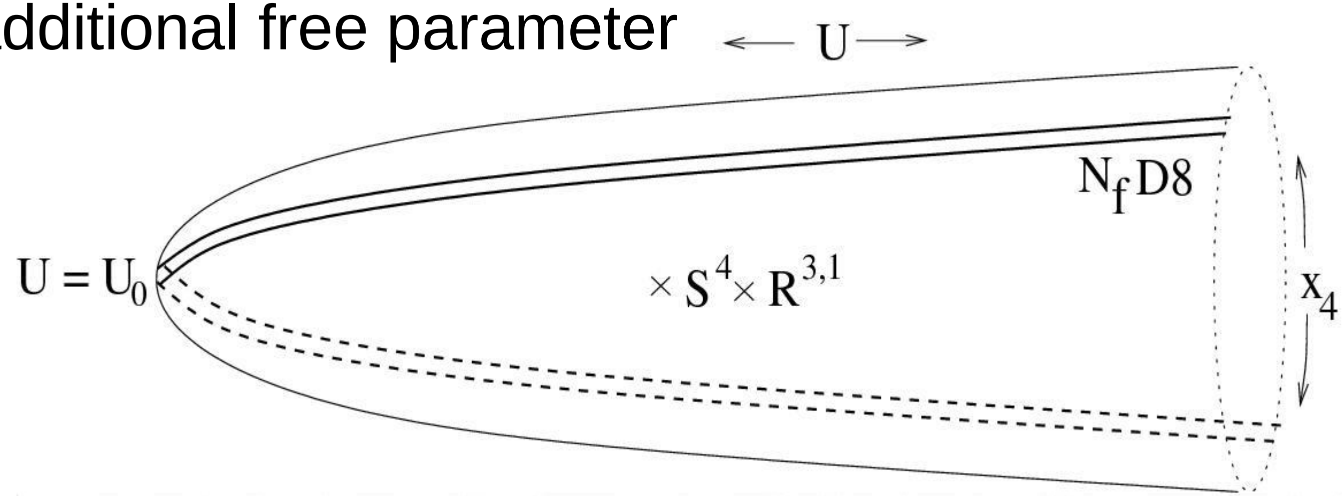
Is so, what are the instabilities and mechanisms for chiral symmetry restoration? We have not seen anything like this in string theory..

Will use the Sakai-Sugimoto construction to investigate this.

Put Nc D4 branes on a circle with susy breaking boundary conditions – yields QCD at low energies

Add Nf D8 and anti D8-branes – yields massless quarks localized at the intersection with the D4 branes

D8 and anti D8 have to reconnect – distance between intersection is additional free parameter



$$ds^2/R_4^2 = u^{\frac{3}{2}}(-d\tau^2 + d\vec{x}^2 + f(u)dx_4^2) + u^{-\frac{3}{2}}(\frac{du^2}{f(u)} + u^2 d\Omega_4^2)$$

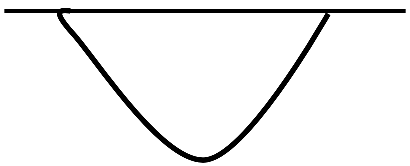
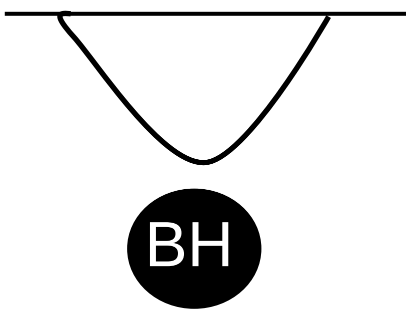
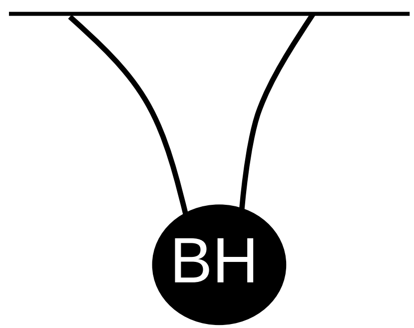
$$f(u) = 1 - \frac{u_{KK}^3}{u^3}$$

Field theory directions — Compact direction — Radial direction — 4D sphere

Some features:

- Fluctuations: $\delta A_\mu^L, A_\mu^R$ when symmetric: vector mesons, when anti-symmetric: axial mesons
- Spectrum is different: chiral symmetry is broken (which is clear from the geometry)
- Pion is related to boundary behavior. In gauge $A_u = 0$ the pion appears as $A_\mu^{L,R} \overset{u\to\infty}{\longrightarrow} \pm g^{-1}\partial_\mu g$
- The theory is confining.
- Pion can be related to the Skyrme field. Skyrme Lagrangian can be obtained from Sakai-Sugimoto model.

At finite T, the interior develops a black hole, and the theory deconfines. One has the following possibilities

	?quarkyonic?	confinement
		deconfinement
chiral symmetry breaking	chiral symmetry unbroken	

To study this, we should consider the Sakai-Sugimoto model at finite chemical potential and zero temperature.

$$\mathcal{L} \to \mathcal{L} - \mu(\psi_L^\dagger \psi_L + \psi_R^\dagger \psi_R)$$

This is realized with the boundary conditions

$$A_t^{L,R} \longrightarrow C \pm \frac{2E}{u}$$

Here E is the electric field. This field requires a source.

$$S_{\mathrm{D8}} = S_{\mathrm{DBI}} + \int C_3 \wedge F \wedge F \wedge F$$

$\downarrow$ reduce over S^4

$$S_{\mathrm{D8}} = S_{\mathrm{DBI}} + \int A \wedge F \wedge F$$

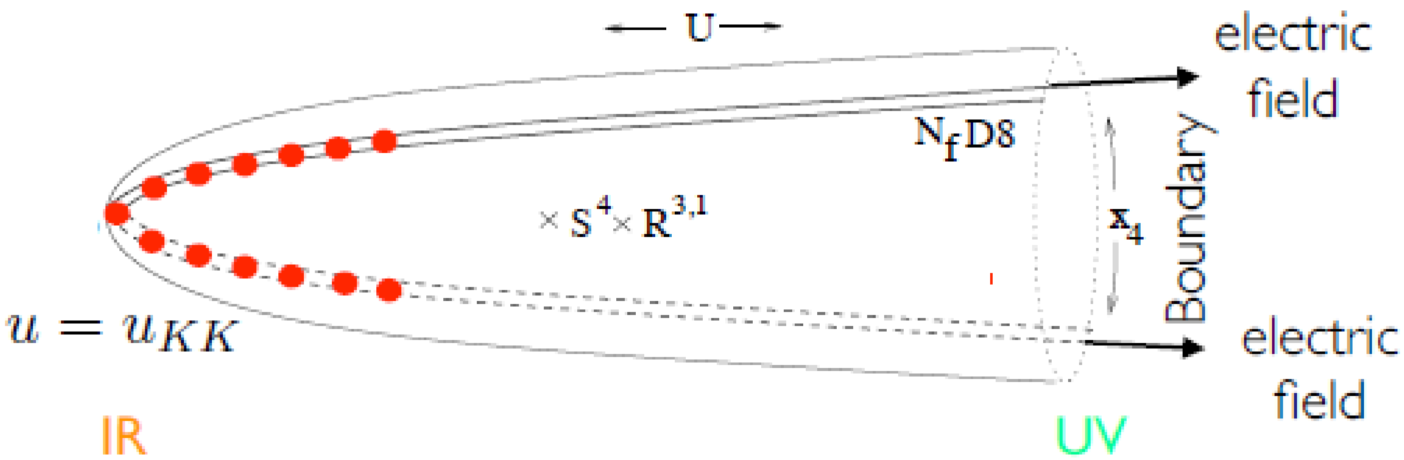
$\downarrow$ insert instanton

$$q \int dt \, A$$

“Instantons” (=wrapped D4-branes in the D8) behave like charged particles – need a complicated instanton configuration on the D8 brane to produce electric field.

These instantons are also the baryons of the theory.

The gravitational potential pulls down the instantons to the bottom of the throat



This is extremely difficult to study analytically: one requires a multi-instanton configuration in non-abelian DBI.

One might expect that this collection of baryons/instantons is somehow responsible for disconnecting the two sides of the D8 and for (approximate) chiral symmetry restoration.

Bergman, Lifshytz and Lippert (arXiv:0708.0326) studied this system with all instantons localized at the tip as delta functions.

Rozali, Shieh, van Raamsdonk and Wu (arXiv:0708.1322) did a mean-field approximation for Nf=1 and found that the baryons do not stay at the tip – similar result in Ghoroku, Kubo, Tachibana, Taminato and Toyoda (arXiv:1211.2499).

Kaplunovsky, Melnikov and Sonnenschein (arXiv:1201.1331) used various approximations for the instantons and showed that first the instantons stay at the tip and at some point pop out in the extra dimension.

Our claim is that one needs a density of order $\mathcal{O}(\lambda^2)$ to see interesting new physics. This is where the baryons start to overlap from the boundary point of view and this is also the density at which they occupy the extra dimension.

For such densities:

- The DBI action is important (not just Maxwell)
- The Chern-Simons term is important
- The baryon/instanton condensate starts to effect the meson spectrum (in-medium effects)

To examine whether there is evidence of the quarkyonic phase, we looked for

(i) CDW type instabitilies

(ii) The spectrum of vector and axial mesons for signs of approximate chiral symmetry restoration

First we need a model to describe the instantons. We took Nf=1 and modeled the instantons in a mean-field spirit by pressureless dust.

$$S_{dust} = \int d^5x \sqrt{-\det(g_{ab})} \{-m_b(z) w^2 \beta + q_b w^2 u^a \left(A_a - \partial_a \phi\right) \gamma + \lambda(u_a u^a + 1)\},$$

- $m_b(z) w^2$ → density x mass
- $q_b w^2 u^a$ → current coupling
- $\partial_a \phi$ → current conservation and gauge invariance
- u_a → 4-velocity

β, γ --fudge factors

Result for instability:

$$\delta A \sim h(z)\left[\cos(kx^3)dx^1 + \sin(kx^3)dx^2\right]$$

Did not find any instability, unless we play with the fudge factors β, γ

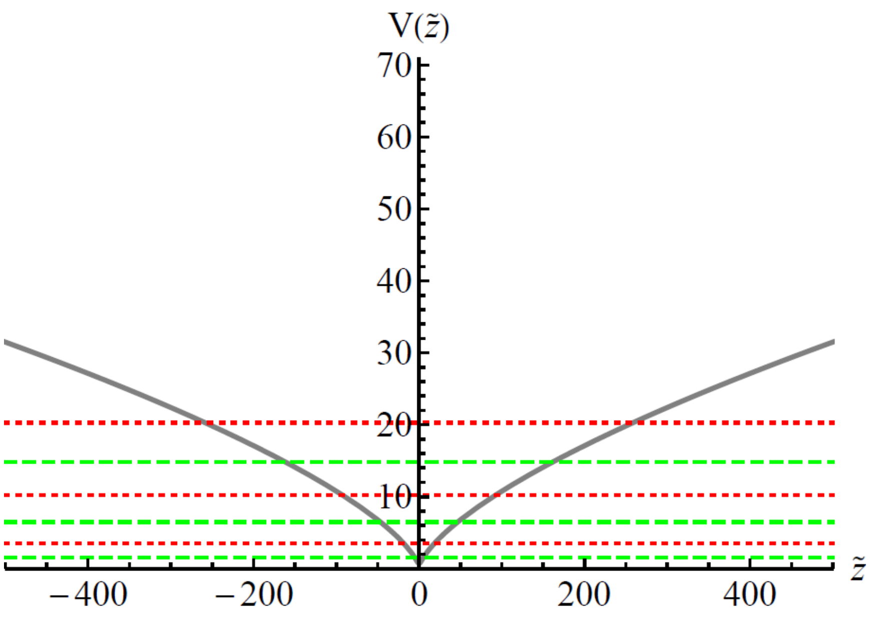
Notice that in any case these are no CDWs but condensates of the form

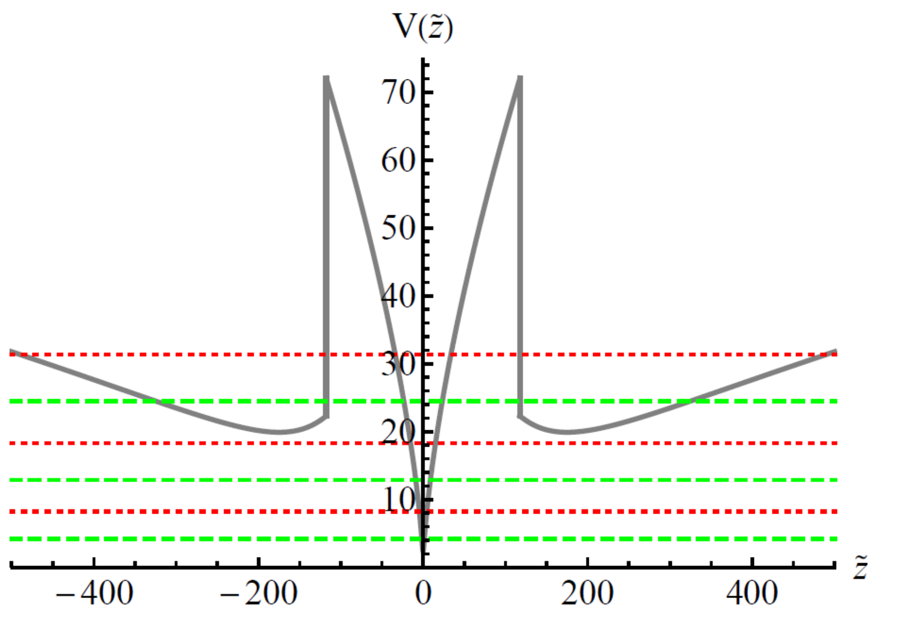
$$\langle \psi_L^\dagger \gamma_0 \gamma_1 \psi_L \rangle, \quad \langle \psi_R^\dagger \gamma_0 \gamma_2 \psi_R \rangle$$

Notice that instabilities appear due to the Chern-Simons term (Domokos, Harvey, arXiv:070.1604) either at finite temperature (Nakamura, Ooguri, Park, arXiv:0911.0679) and for axial chemical potential (Bayona, Peeters, Zamaklar, arXiv:1104.2291)

Study of perturbations:

Write as Schrödinger-type equation. As chemical potential increases, potential well near the tip of the D8 develops. Bound states in new well look like excitations of the medium.





We find that the splitting

$$\Delta m = 2\frac{m_A - m_V}{m_A + m_V}$$

becomes smaller as one increases the chemical potential, but it does not become zero.

Therefore, though this is a very interesting modification of the spectrum due to interactions with the medium, chiral symmetry is not restored.

Conclusions (much remains to be done!!)

- The edge of the baryon distribution is reminiscent of a bulk Fermi surface – but no Pauli principle and occupation is in coordinate space
- Spectrum depends rather strongly on L, distance between D8 and anti-D8 – L does not have a meaning in QCD
- We ignored many things and took Nf=1. In particular we ignored latttice effects. Perhaps some of these effects can be captured by modifying the fudge factors and instabilities do appear after all.
- The $\mathcal{O}(\lambda^2)$ density of states was crucial for all new physics – very interesting regime in its own right
- Not clear how to get CDW from string theory.
- Computation of chiral condensate via string world-sheet relates it directly to the pion vev – perhaps chiral symmetry is restored only after averaging a la Skyrme (so only at long distances)