Holographic thermalization for expanding plasmas

Michał P. Heller m.p.heller@uva.nl
University of Amsterdam, The Netherlands
&
National Centre for Nuclear Research, Poland (on leave)

HoloGrav 2013
Helsinki, Finland

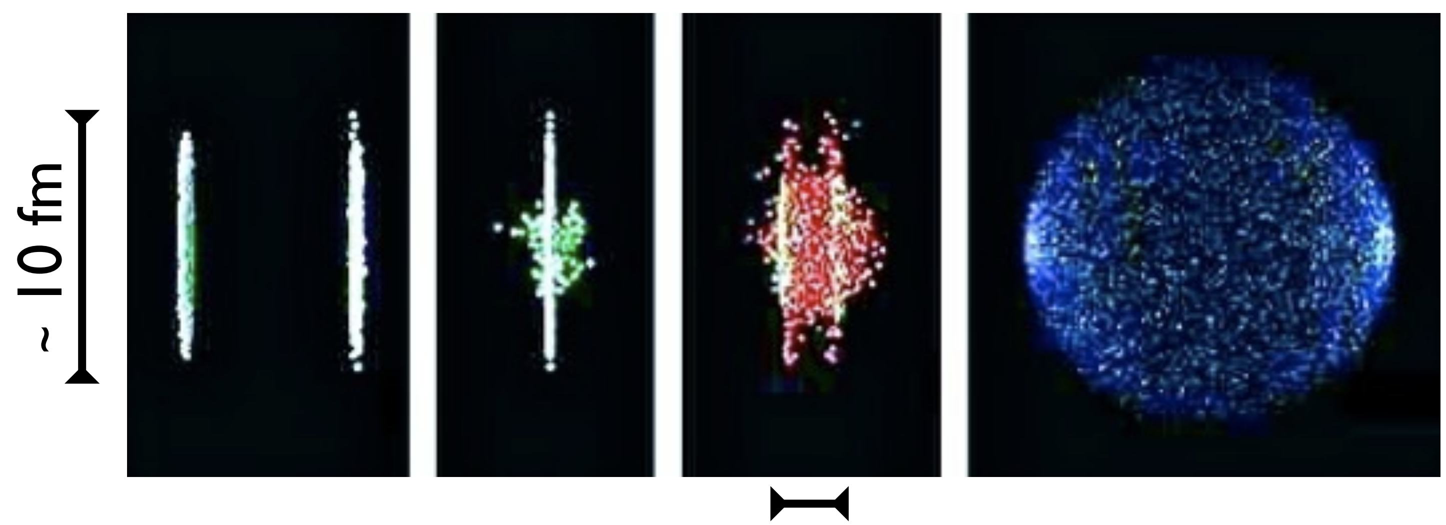
Netherlands Organisation for Scientific Research

Motivation

„Thermalization” puzzle Heinz [nucl-th/0407067]

There are overwhelming evidences that relativistic heavy ion collision programs at RHIC and LHC created strongly coupled quark-gluon plasma (sQGP)

Successful description of experimental data is based on hydrodynamic simulations of an almost perfect fluid of $\eta/s = O(1/4\pi)$ starting on very early (< 1 fm/c)



described by hydro after < 1 fm/c

Explaining *ab initio* this very quick applicability of hydro is a fascinating puzzle

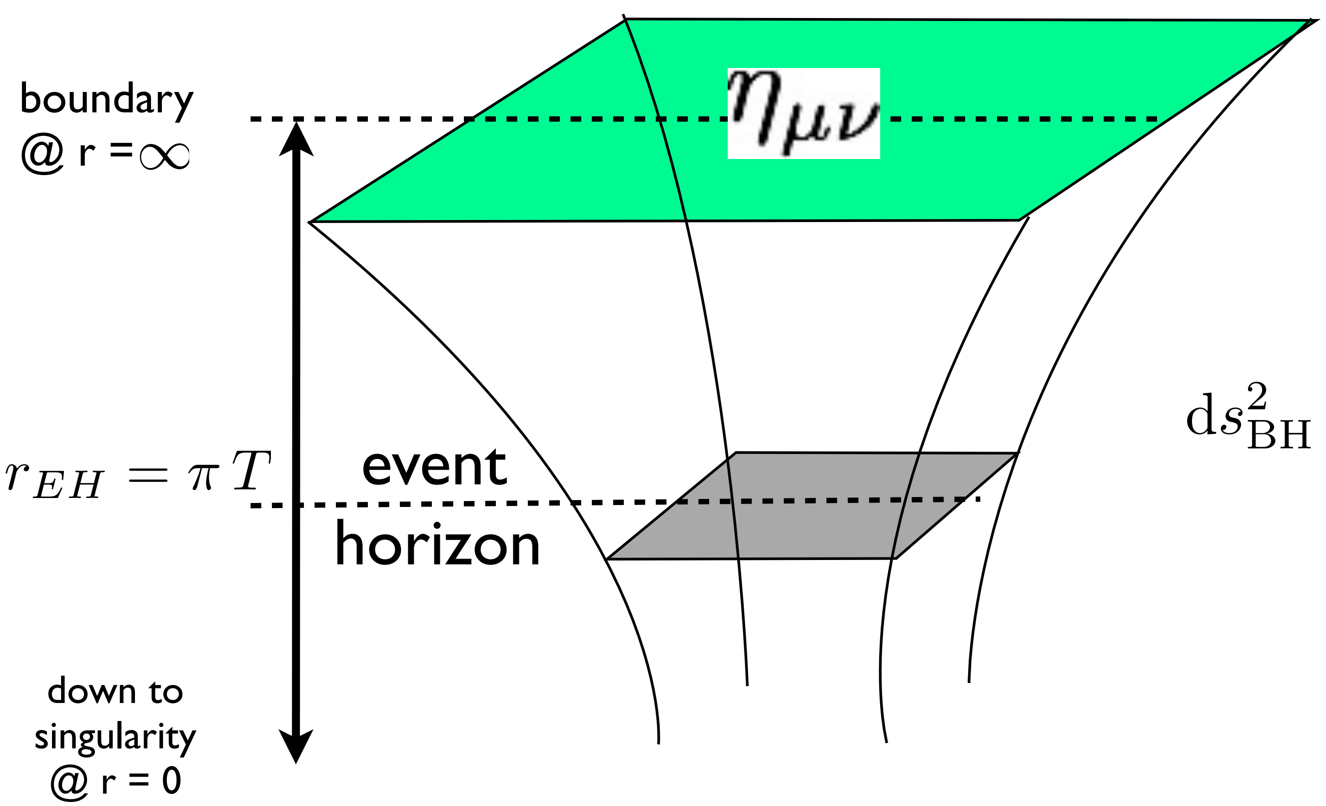
What can the holography teach us about equilibration in similar models?

Brief intro to the near-equilibrium holography

Global equilibrium

Thermal deconfined hQFT = **Bulk black hole**

AdS-Schwarzschild black hole is described by the metric



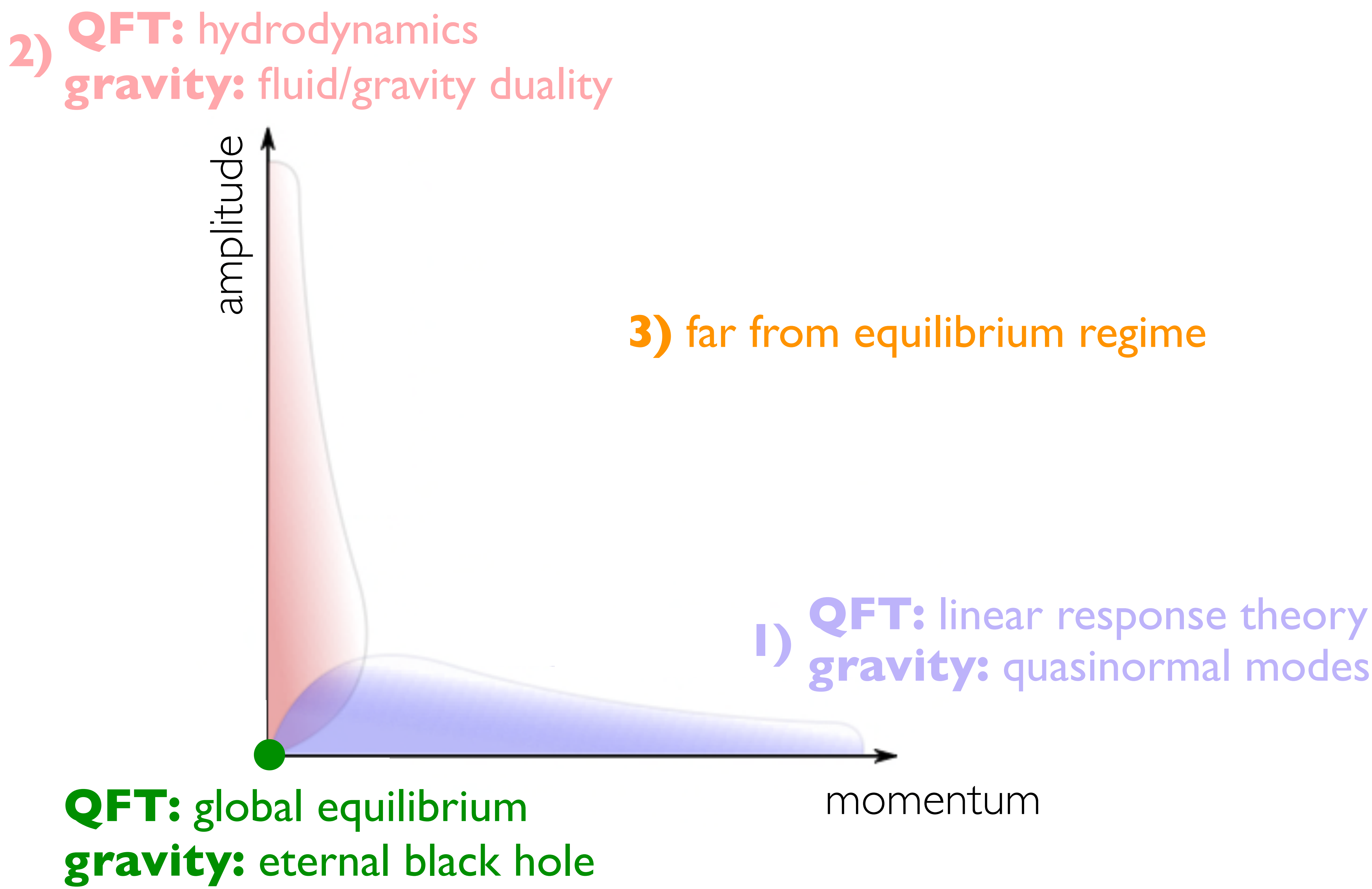
$$ds^2_{\rm BH} = 2{\rm d}t{\rm d}r - r^2\left(1 - \frac{\pi^4 T^4}{r^4}\right){\rm d}t^2 + r^2 {\rm d}\vec{x}^2$$

The plasma/black hole thermodynamics is given by

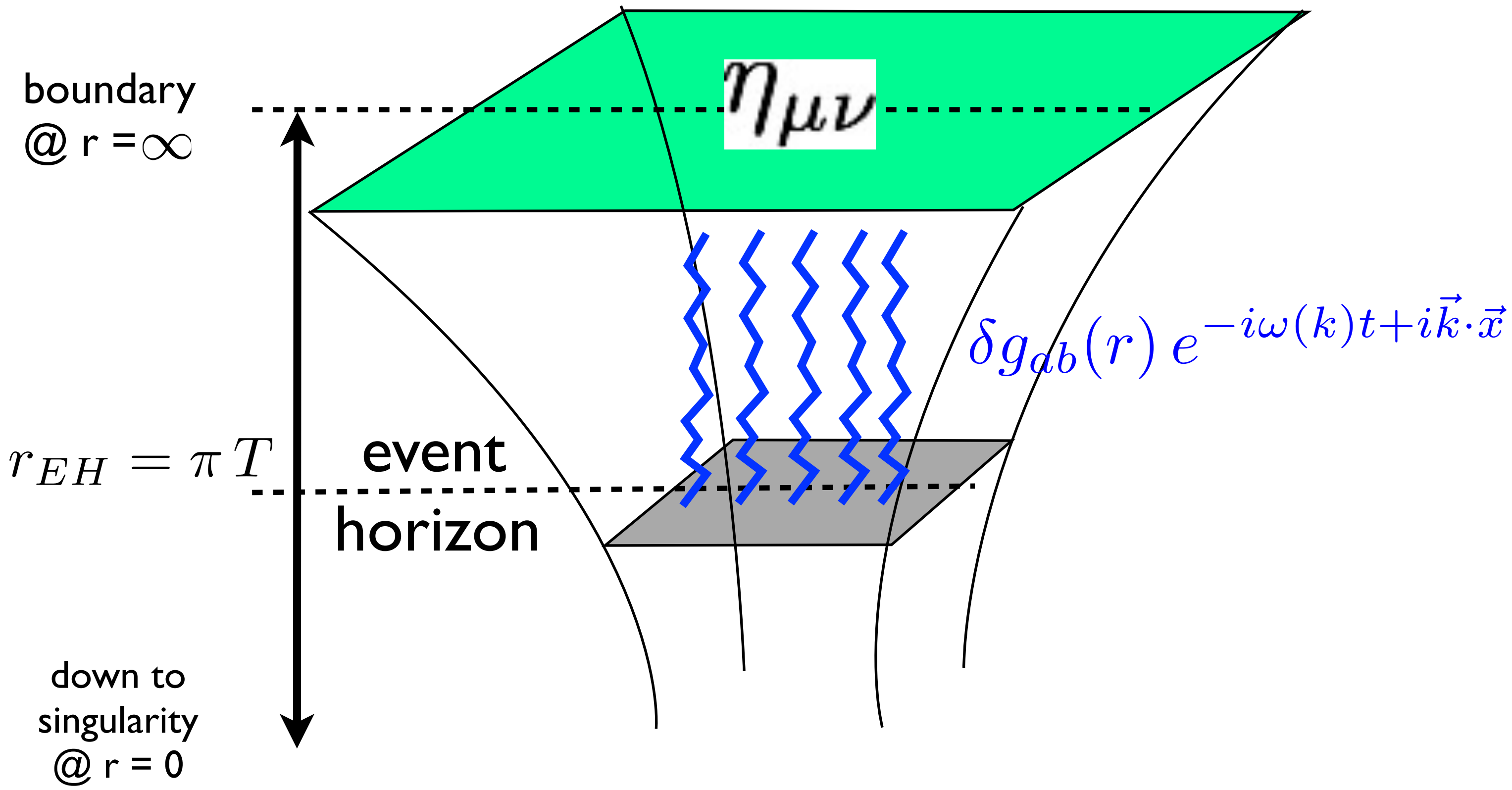
$$T_{\mu\nu} = \frac{1}{8}\pi^2 N_c^2 T^4\, {\rm diag}\,(3,1,1,1)_{\mu\nu},\; s = {\rm Area}/4l_P^2 = \frac{1}{2}N_c^2\pi^2 V\, T^3$$

Going away from equilibrium

review: Hubeny & Rangamani
1006.3675 [hep-th]



Small amplitude perturbations and dissipation



Quasinormal modes are small amplitude perturbations on top of BH that obey

- Dirichlet bdry conditions at the bdry
- Ingoing bdry conditions at the horizon

The latter lead to complex frequencies ω and hence dissipation

$$T_{\mu\nu} = \frac{1}{8}\pi^2 N_c^2 T^4 \,\mathrm{diag}\,(3,1,1,1)_{\mu\nu} + \delta T_{\mu\nu}\, e^{-i\omega(k)t+\vec{k}\cdot\vec{x}}$$

exponential decay with time

Quasinormal mode spectrum

[hep-th/0506184]
Kovtun & Starinets

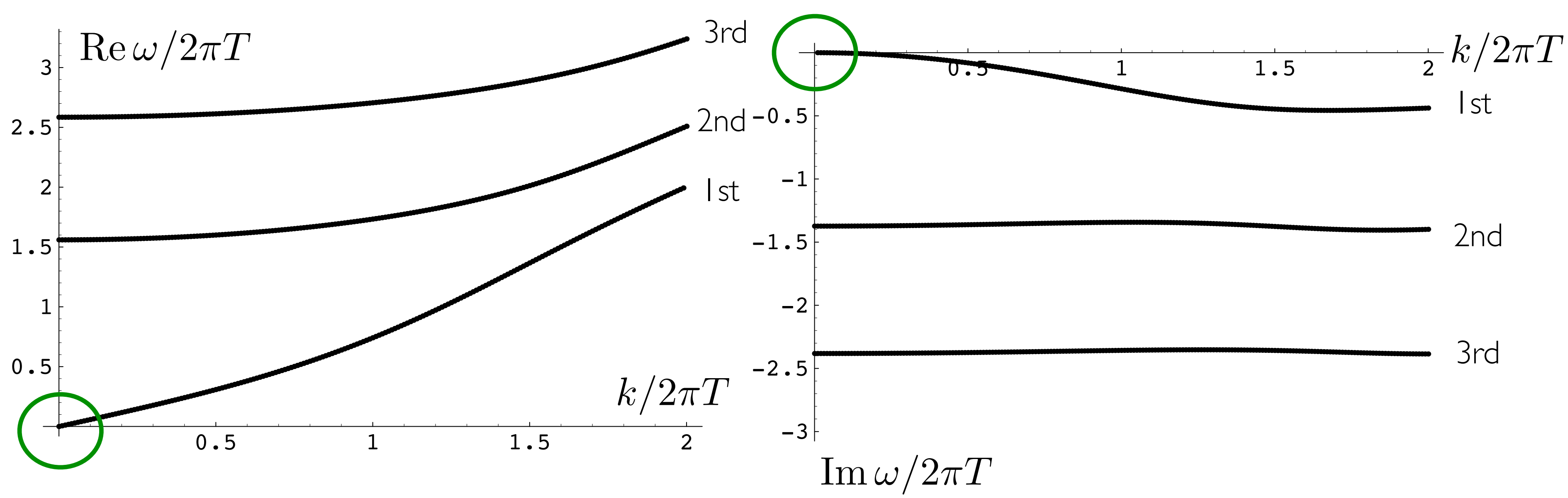
Consider small amplitude perturbations ($\delta T_{\mu\nu}/{N_c}^2 \ll T^4$) on top of a holographic plasma

$$T_{\mu\nu} = \frac{1}{8}\pi^2 N_c^2 T^4 \,\mathrm{diag}\,(3,1,1,1)_{\mu\nu} + \delta T_{\mu\nu} \quad (\sim e^{-i\,\omega(k)\,t + i\,\vec{k}\cdot\vec{x}})$$

Due to $\lambda = g_{YM}^2 N_c \to \infty$ (and $N_c \to \infty$?) the temperature T is the only microscopic scale

Complex $\omega(k)$ in the sound channel look like

$$\left.\frac{\partial\omega}{\partial k}\right|_{k\to 0} = c_{sound}$$



$\omega(k) \to 0$ as $k \to 0$: slowly evolving and dissipating modes (hydrodynamic sound waves)

all the rest: far from equilibrium (QNM) modes dampened over $t_{therm} = \mathcal{O}(1)/T$

This is also the meaning in which t_{hydro}^{RHIC} is fast: *0.5 fm/c x 350 MeV = T t_{therm} = 0.63* !!!

Modern relativistic (uncharged) hydrodynamics

hydrodynamics is an EFT of the slow evolution of conserved currents in collective media close to equilibrium

As any EFT it is based on the idea of the gradient expansion

DOFs: always local energy density ϵ and local flow velocity u^μ $(u_\nu u^\nu = -1)$

EOMs: conservation eqns $\nabla_\mu T^{\mu\nu} = 0$ for $T^{\mu\nu}$ *systematically* expanded in gradients

↑

gravity reminded us that **all terms allowed by symmetries can enter**

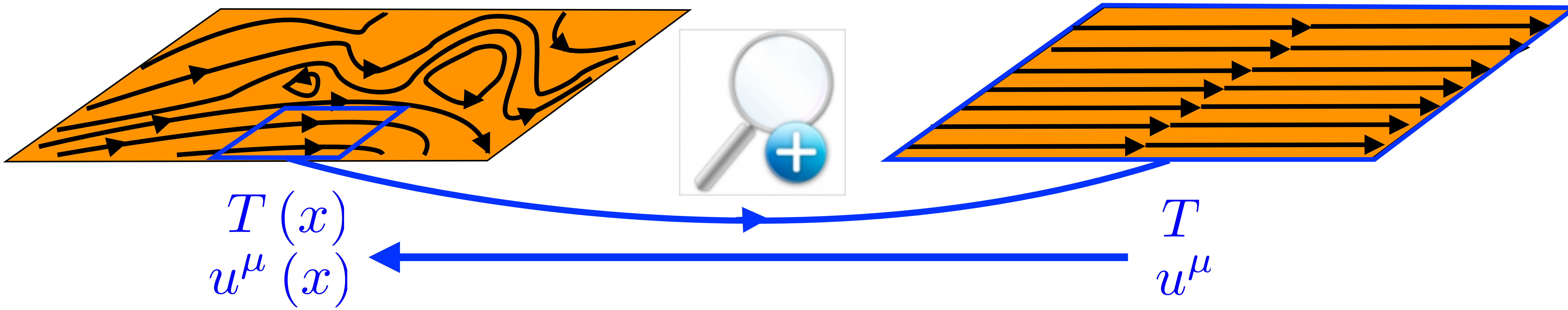
$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + P(\epsilon)\{\, g^{\mu\nu} + u^\mu u^\nu \,\} - \eta(\epsilon)\, \sigma^{\mu\nu} - \zeta(\epsilon)\{\, g^{\mu\nu} + u^\mu u^\nu \,\}(\nabla \cdot u) + \ldots$$

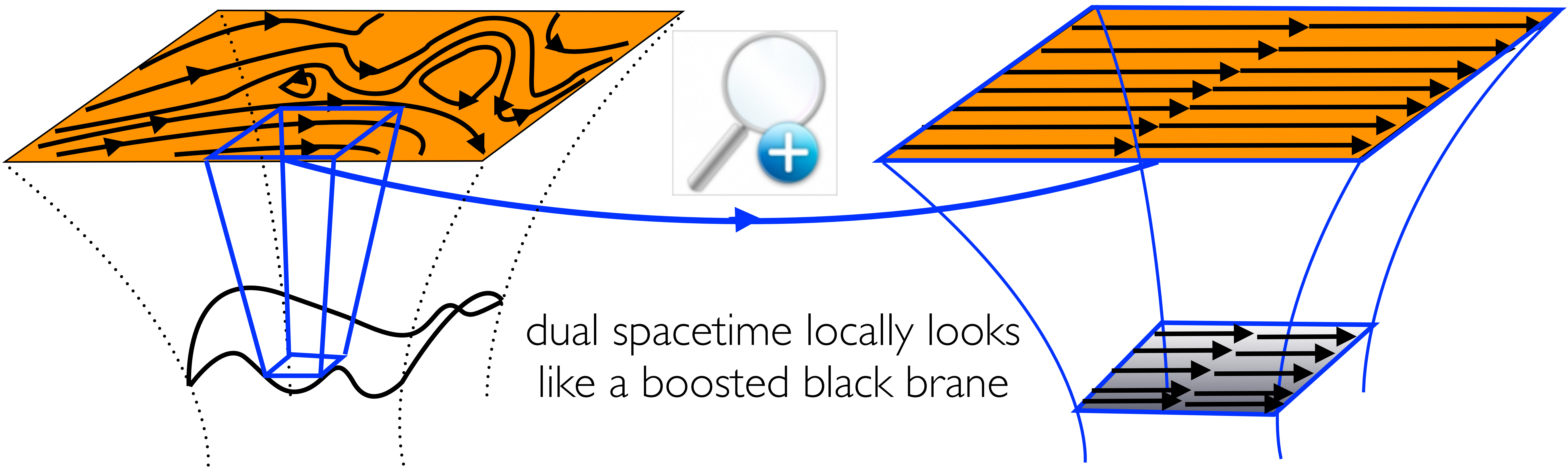
perfect fluid stress tensor

microscopic input: EoS; (famous) shear viscosity; bulk viscosity (vanishes for CFTs)

Fluid-gravity duality

0712.2456 [hep-th]
Bhattacharyya Hubeny Minwalla Rangamani





$$ds^2 = -2u_\mu dx^\mu dr - r^2\left(1-\frac{\pi^4 T^4}{r^4}\right)u_\mu u_\nu dx^\mu dx^\nu + r^2\left(\eta_{\mu\nu}+u_\mu u_\nu\right)dx^\mu dx^\nu$$ + gradient terms.

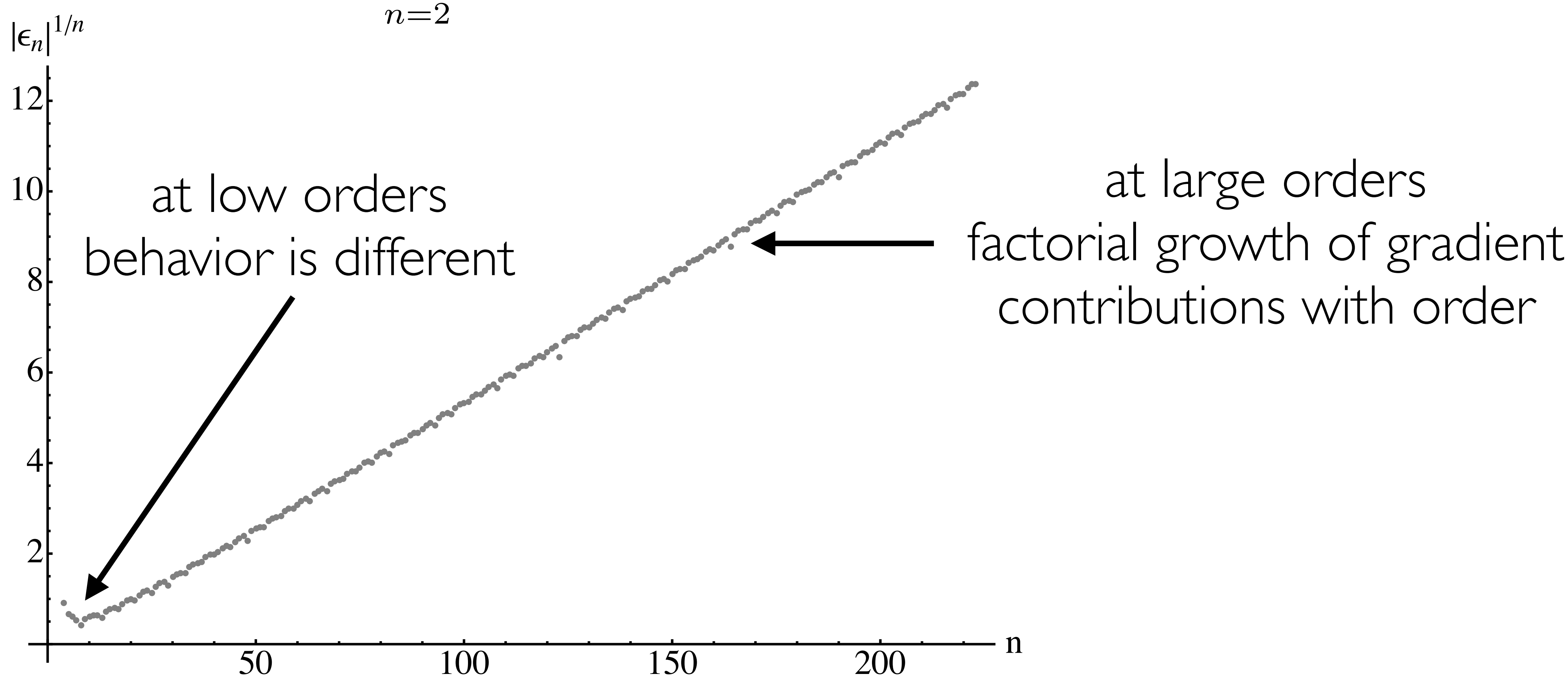
Fluid-gravity duality redux:
hydrodynamics is an asymptotic series

1302.0697 [hep-th]
MPH, R. A. Janik & P. Witaszczyk

So far nothing has been known about the character of hydrodynamic expansion

Idea: take a simple flow (here the boost-invariant flow) and using the fluid-gravity duality generate the on-shell form of its hydrodynamic stress tensor at high orders

$$T^{00} = \epsilon(\tau) \sim \sum_{n=2}^{\infty} \epsilon_n (\tau^{-2/3})^n \qquad (T^{-1}\nabla_\mu u^\nu \sim \tau^{-2/3})$$



First evidence that hydrodynamic expansion has zero radius of convergence!

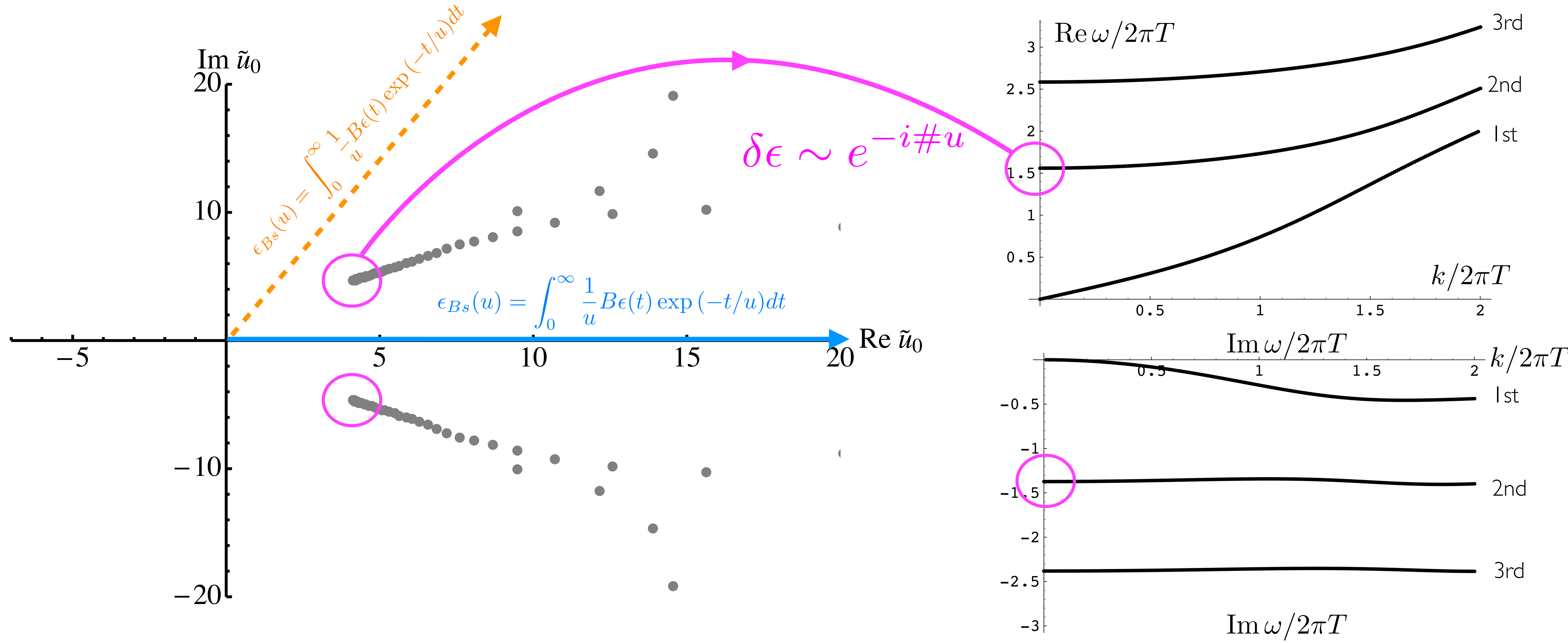
What controls the fast growth of hydroS coeffs?

1302.0697 [hep-th] MPH, R. A. Janik & P. Witaszczyk

A standard method for asymptotic series is Borel transform and Borel summation

$$\epsilon(u) \sim \sum_{n=2}^{\infty} \epsilon_n u^n \quad (u = \tau^{-2/3}), \quad B\epsilon(\tilde{u}) \sim \sum_{n=2}^{\infty} \frac{1}{n!} \epsilon_n \tilde{u}^n, \quad \text{Borel sum}: \quad \epsilon_{Bs}(u) = \int_0^{\infty} \frac{1}{u} B\epsilon(t) \exp(-t/u) dt$$

$B\epsilon(\tilde{u})$ reveals singularities leading to 0 radius of convergence

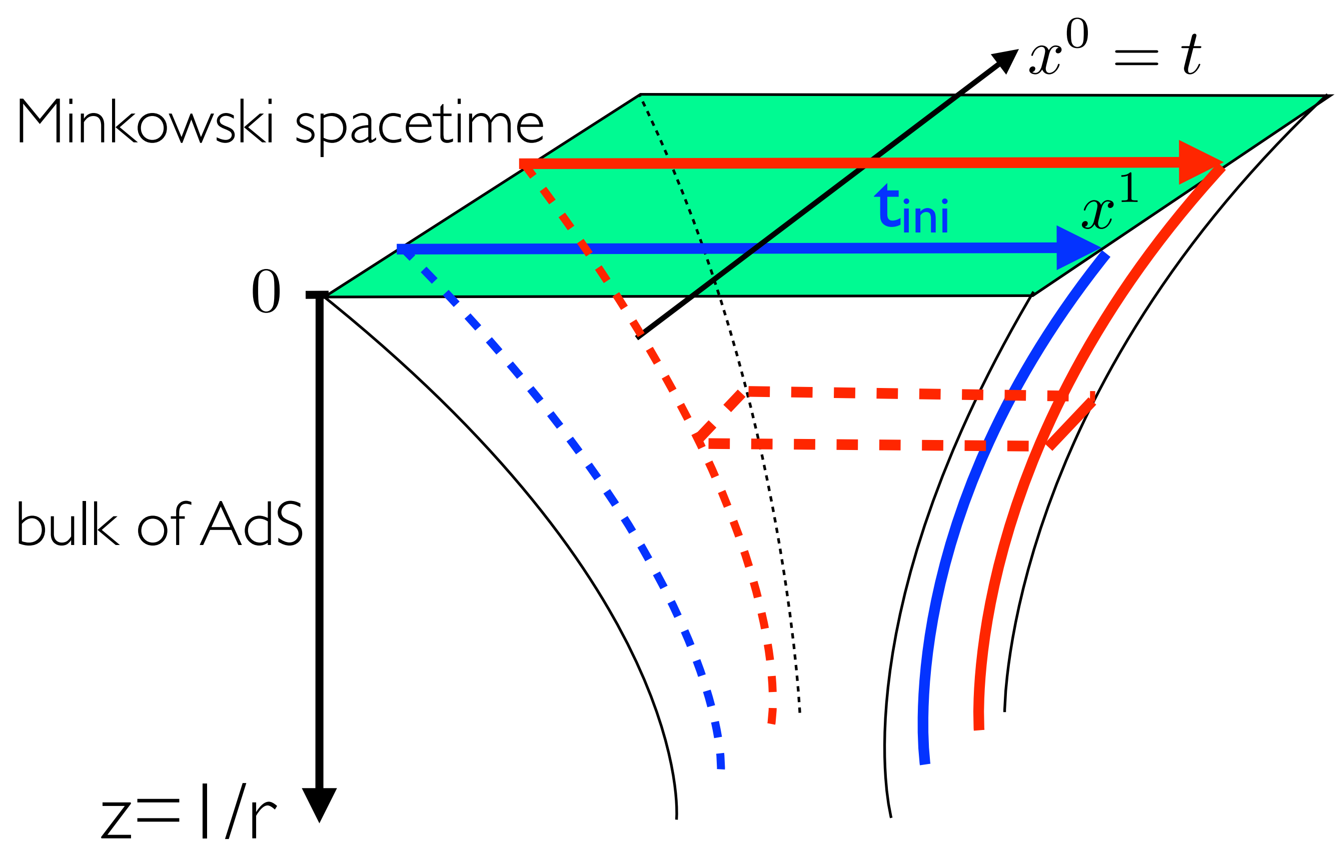


Closer inspection reveals that the closest one to 0 is the lowest non-hydro QNM!

Holographic thermalization: a primer

General idea behind the non-equilibrium holography

Of interest are geometries which interpolate between far-from-equilibrium states at the boundary at initial time t_{ini} and thermalized ones at (some) larger time t_{iso}



The stress tensor is read off from near-boundary expansion of dual solution

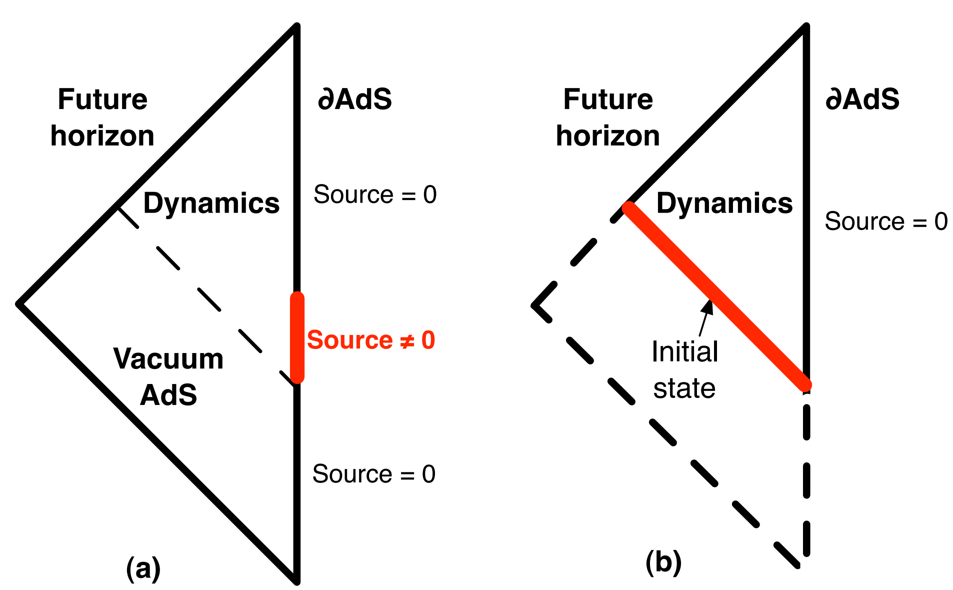
Skenderis et al. (2000)

The criterium for (local) „thermalization" is that the stress tensor is to a good accuracy described by hydrodynamics

There are two ways of defining n-eq. states:

- shaking equilibrium via QFT sources
- defining them without invoking their origin

Let's investigate the outcomes of the both!

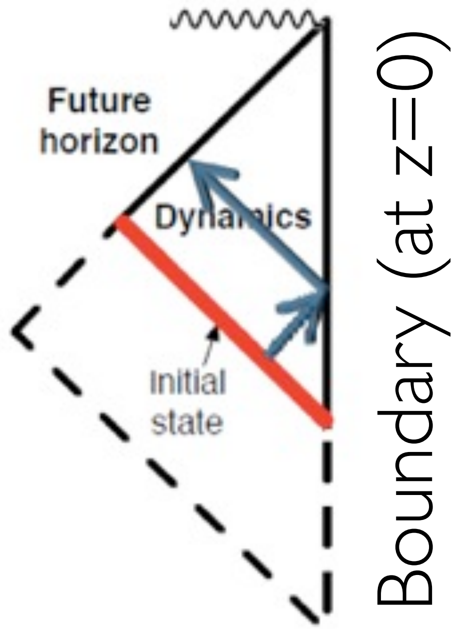


A typical holographic thermalization process

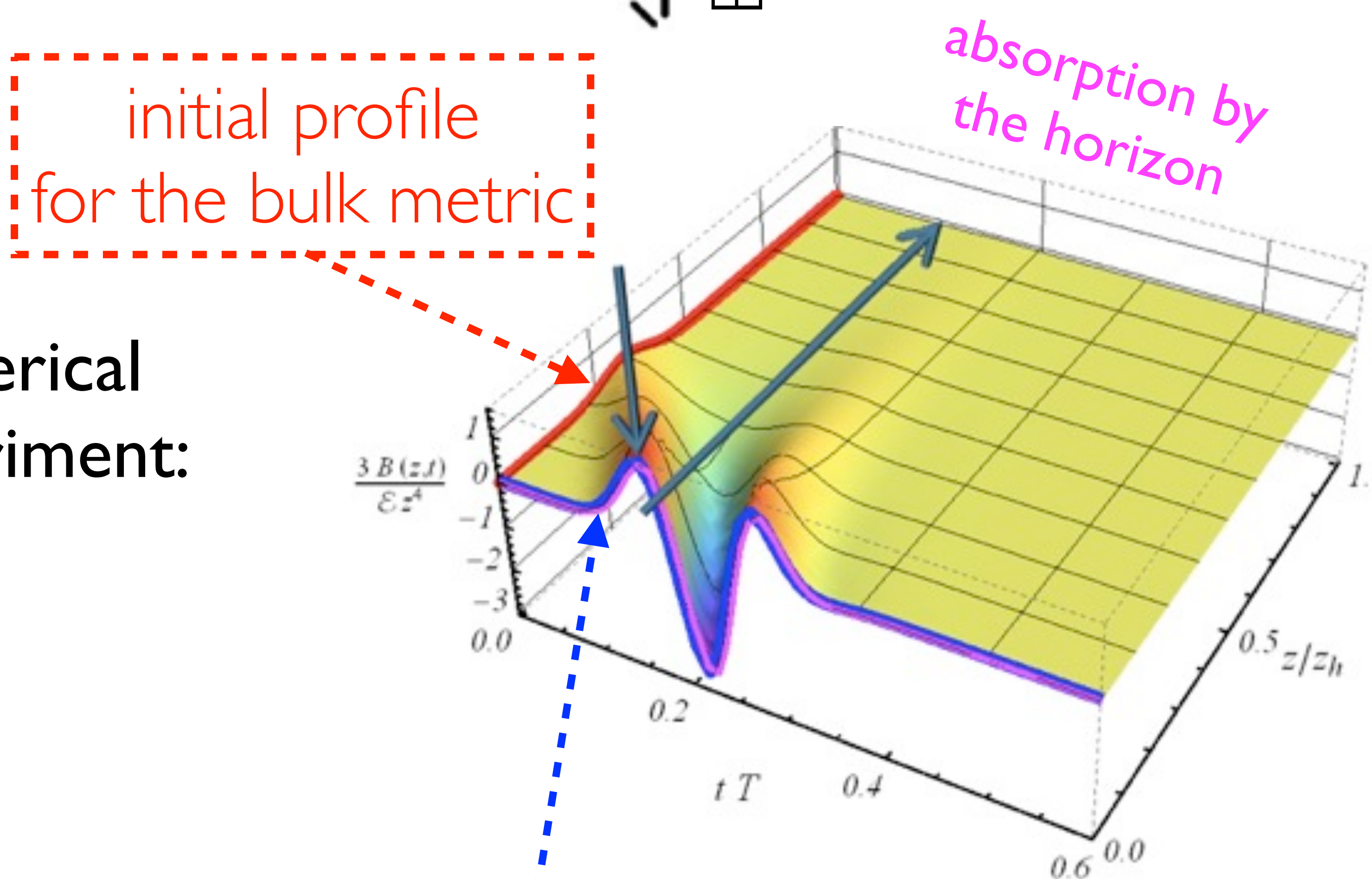
1202.0981 [hep-th]

MPH, D. Mateos, W. van der Schee, D. Trancanelli

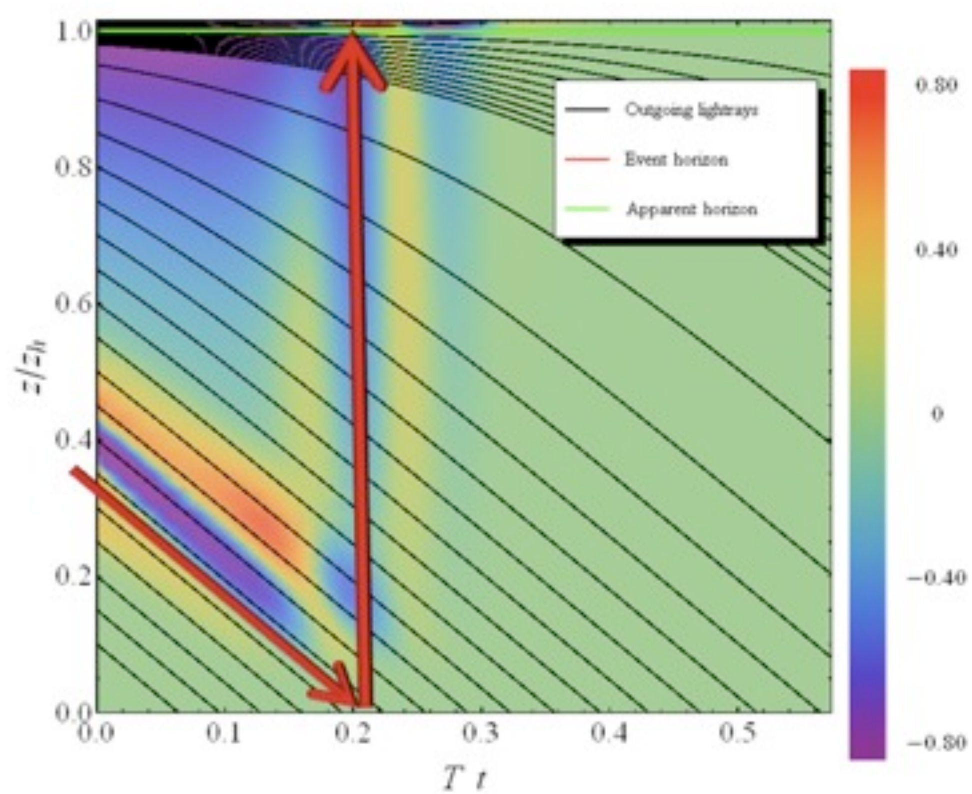
Theory:



Numerical experiment:



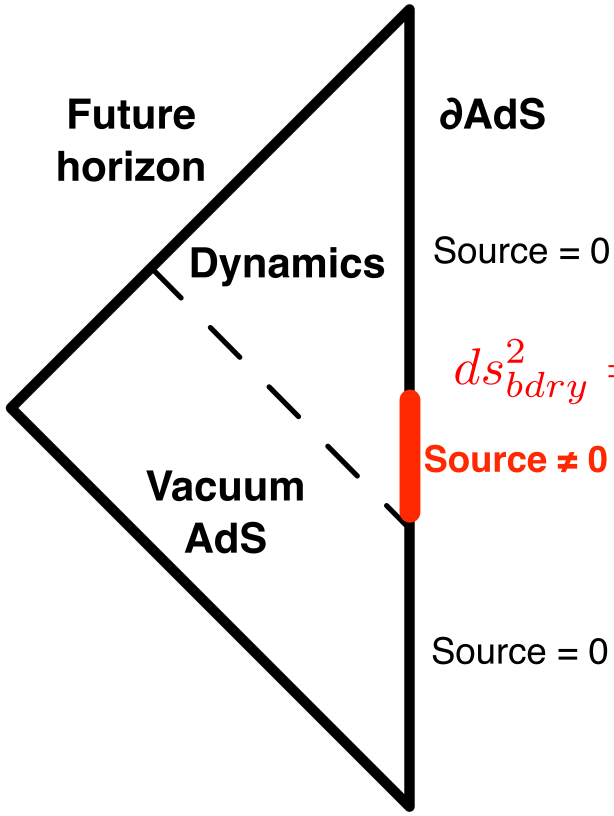
Curvature (BH subtracted)



$$\langle T_{\mu\nu}\rangle = \mathrm{diag}\left\{\epsilon,\ \frac{1}{3}\epsilon - \frac{2}{3}\Delta P(t),\ \frac{1}{3}\epsilon + \frac{1}{3}\Delta P(t),\ \frac{1}{3}\epsilon + \frac{1}{3}\Delta P(t)\right\}$$

Holographic quench

0812.2053 [hep-th]
Chesler&Yaffe



$$ds^2_{bdry} = -dt^2 + e^{B_0(t)} d\mathbf{x}_\perp^2 + e^{-2B_0(t)} dx_\parallel^2$$

$$B_0(t) = \frac{1}{2} c \left[1 - \tanh(t/\tau)\right]$$

„horizon formation”

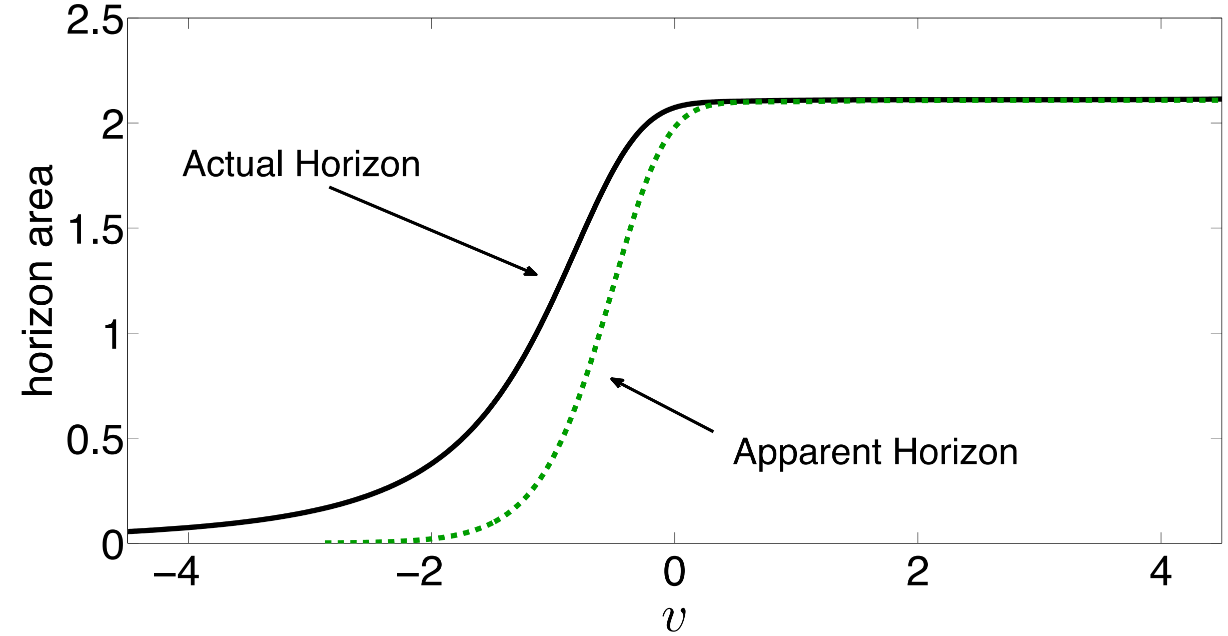


FIG. 3: Area elements of the true event horizon and the apparent horizon as a function of time.

$\|c\|$	1	1.5	2	2.5	3	3.5	4
τT	0.23	0.31	0.41	0.52	0.65	0.79	0.94
$\tau_{\rm iso} T$	0.67	0.68	0.71	0.92	1.2	1.5	1.8
$\tau_{\rm iso}/\tau$	3.0	2.2	1.7	1.8	1.8	1.9	1.9

TABLE I: Final equilibrium temperature T and isotropization time $\tau_{\rm iso}$ (in units of T^{-1} or τ), for various values of c. The isotropization time $\tau_{\rm iso}$ is the time at which the pressures deviate from their equilibrium values by less than 10%.

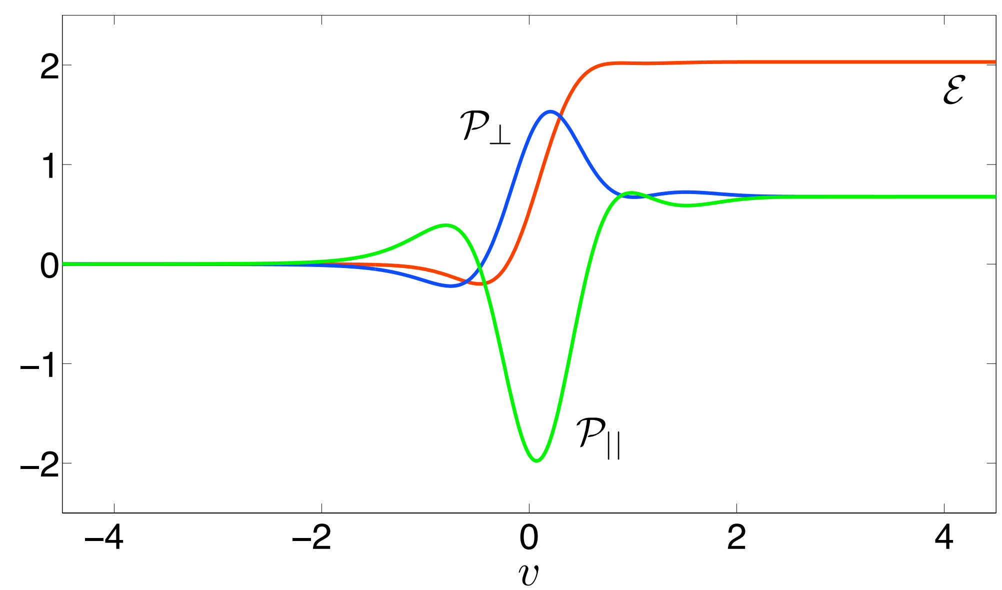


FIG. 1: Energy density, longitudinal and transverse pressure, all divided by $N_c^2/2\pi^2$, as a function of time for $c = 2$.

Holographic Bjorken flow

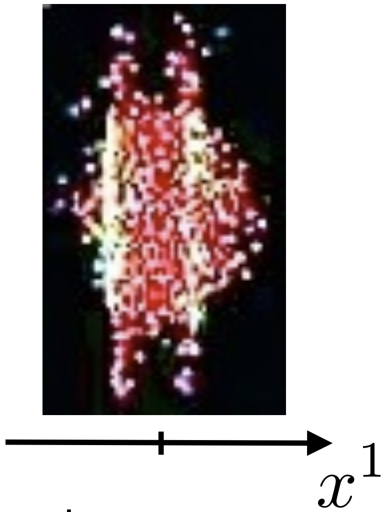
started in **[hep-th/0512162]** by Janik & Peschanski, part mostly based on

0906.4423 [hep-th] Beuf, MPH, Janik, Peschanski

1103.3452 [hep-th]
1203.0755 [hep-th] MPH, Janik, Witaszczyk

1211.2218 [hep-th] van der Schee

Model: boost-invariant flow [Bjorken 1982]



The simplest, yet phenomenologically interesting field theory dynamics is the **boost-invariant flow** with **no transverse expansion.**

|| relevant for central rapidity region

|| no elliptic flow (~ central collision)

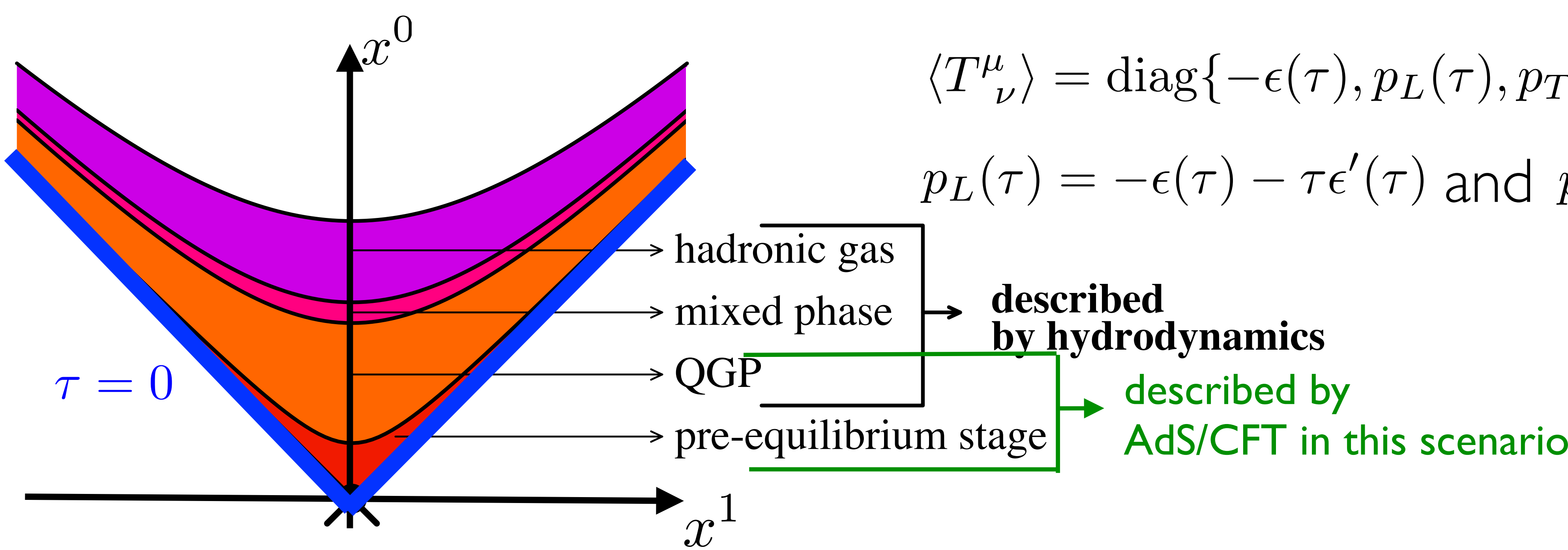
In Bjorken scenario dynamics depends only on proper time $\tau = \sqrt{(x^0)^2 - (x^1)^2}$

$$ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_1^2 + dx_2^2$$

and stress tensor (in conformal case) is entirely expressed in terms of energy density

$$\langle T^\mu{}_\nu \rangle = \text{diag}\{-\epsilon(\tau), p_L(\tau), p_T(\tau), p_T(\tau)\} \text{ with}$$

$$p_L(\tau) = -\epsilon(\tau) - \tau\epsilon'(\tau) \text{ and } p_T(\tau) = \epsilon(\tau) + \frac{1}{2}\tau\epsilon'(\tau)$$



We set strongly coupled n-eq states at $\tau = 0$ and tracked their relaxation to hydro.

Boost-invariant hydrodynamics

Hydrodynamics: $\nabla_\mu \langle T^{\mu\nu} \rangle = 0$ and $\langle T^{\mu\nu} \rangle = \{\epsilon(T) + P(T)\} u^\mu u^\nu + P(T)\eta^{\mu\nu} + \ldots$

In the conformal hydrodynamics $\ldots$ have gradients of u^μ only

But here due to symmetries $u^\mu \partial_\mu = \partial_\tau$, so its gradients are trivial (Christoffels)

Because of this $\nabla_\mu T^{\mu\nu} = 0$ in the boost-invariant hydro is a 1st order ODE for $\epsilon(\tau)$!

We define T_{eff} by $\epsilon(\tau) = \frac{3}{8} N_c^2 \pi^2 T_{eff}(\tau)^4$ and use dimensionless qty $w = \tau T_{eff}$

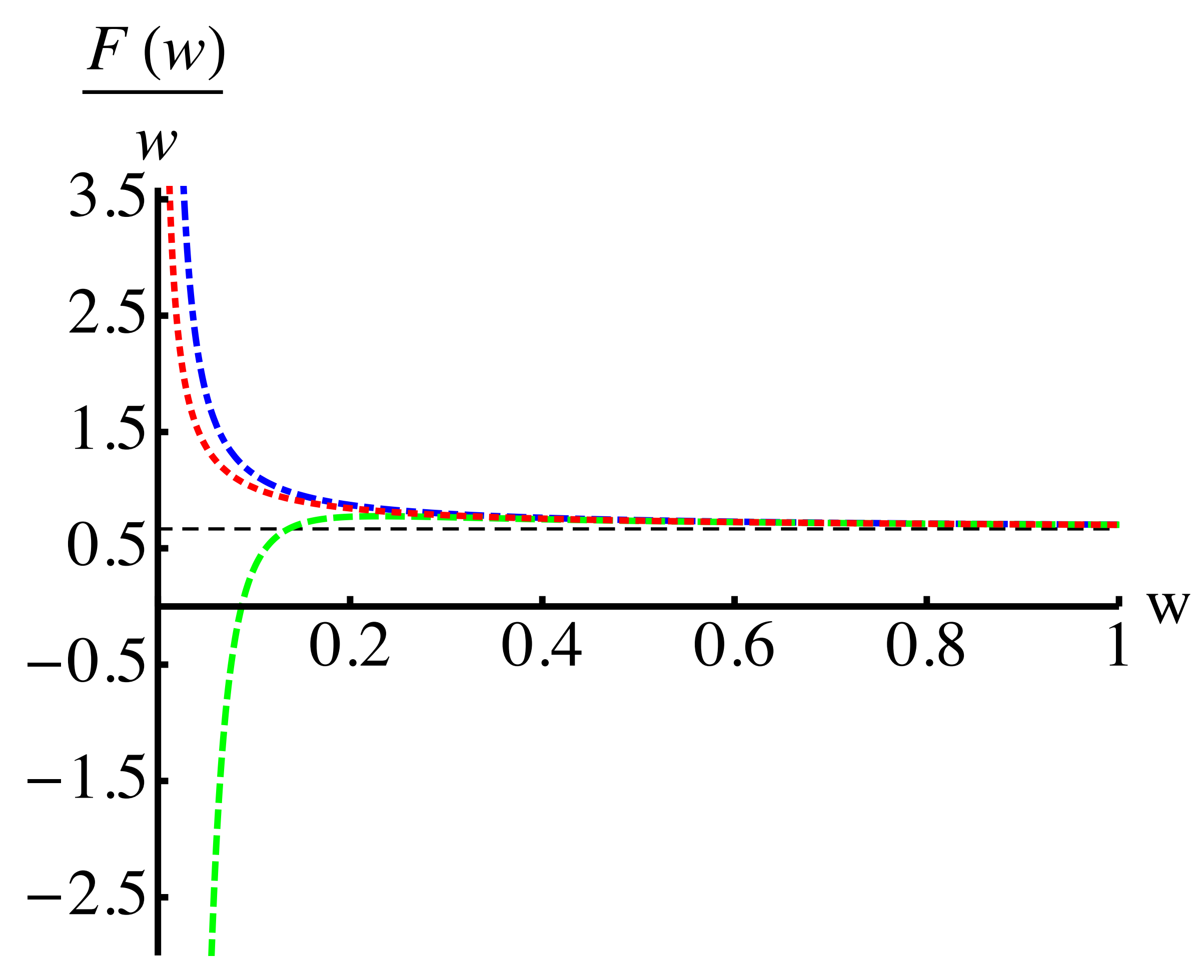
Equations of hydro: $\frac{\tau}{w}\frac{d}{d\tau} w = \frac{F_{hydro}(w)}{w}$

||

perfect fluid

$$\frac{2}{3} + \frac{1}{9\pi w} + \frac{1 - \log 2}{27\pi^2 w^2} + \frac{15 - 2\pi^2 - 45\log 2 + 24\log^2 2}{972\pi^3 w^3} + \ldots$$

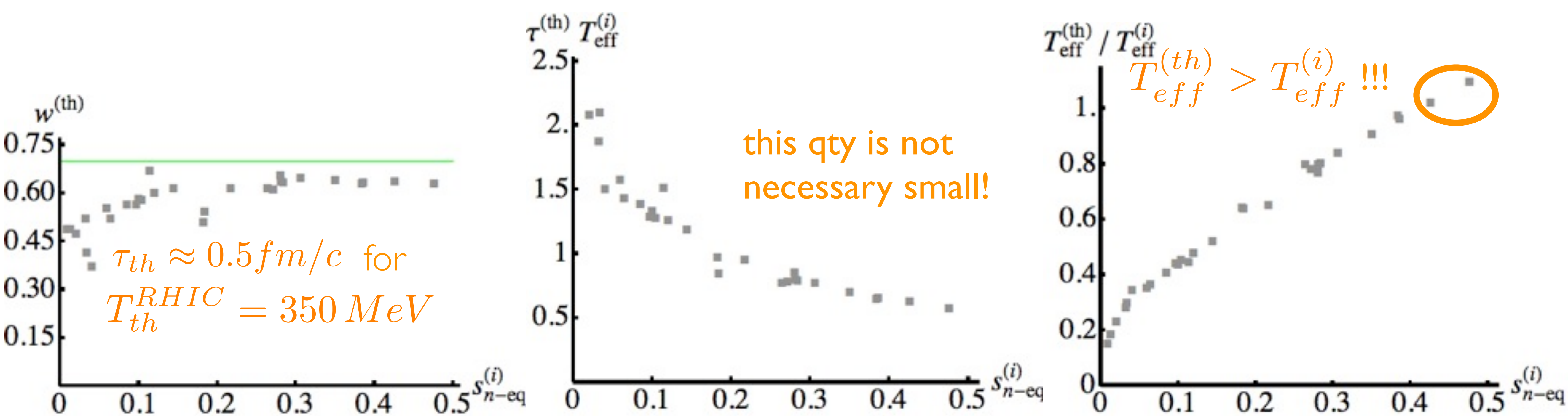
1st 2nd 3rd order hydro



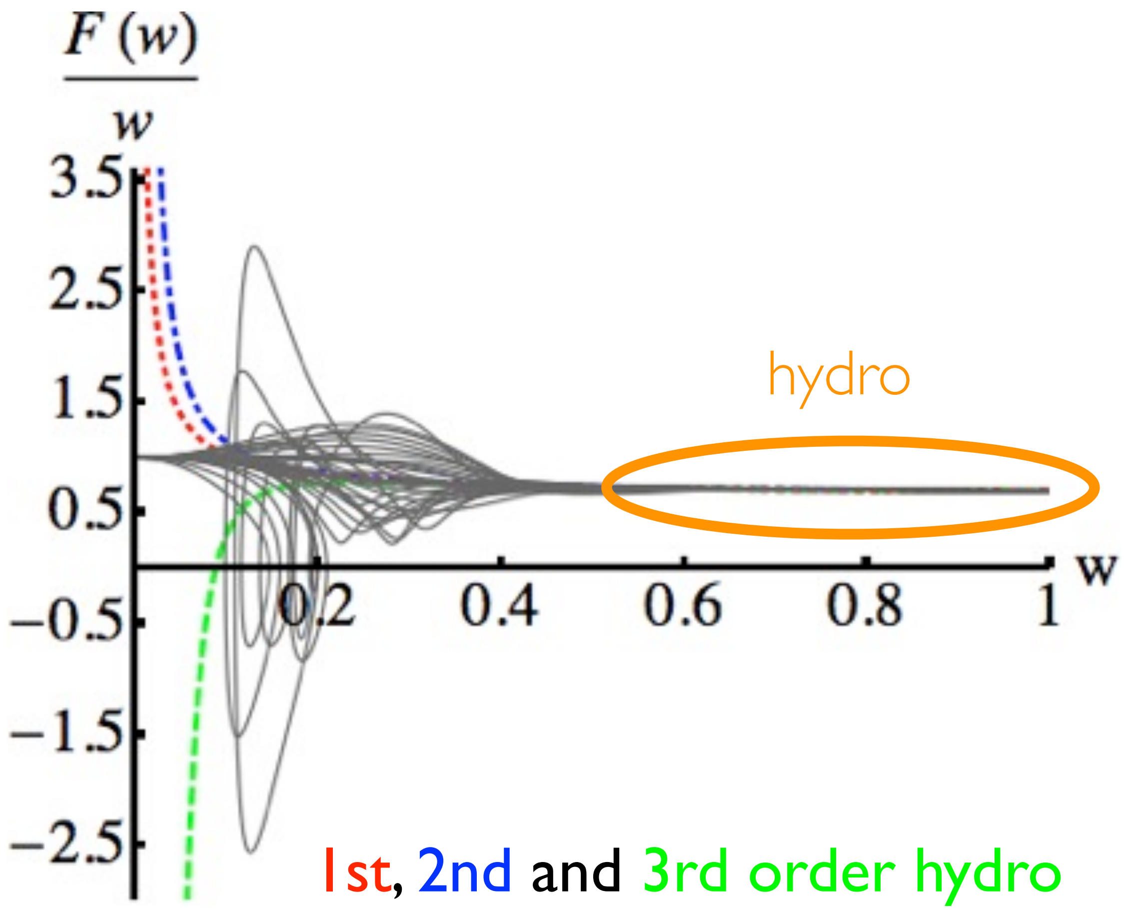
Characteristics of hydrodynamization

We choose $\left\| \frac{\tau \frac{d}{d\tau} w}{F^{3^{rd}\,order}_{hydro}(w)} - 1 \right\| < 0.5\%$ as a criterium for hydrodynamization.

Below are the plots of various non-equilibrium characteristics of plasma as a function of dimensionless entropy density defined by $S \cdot T_{eff}(0)^{-2} = N_c^2 \cdot \frac{1}{2}\pi^2 \cdot s$



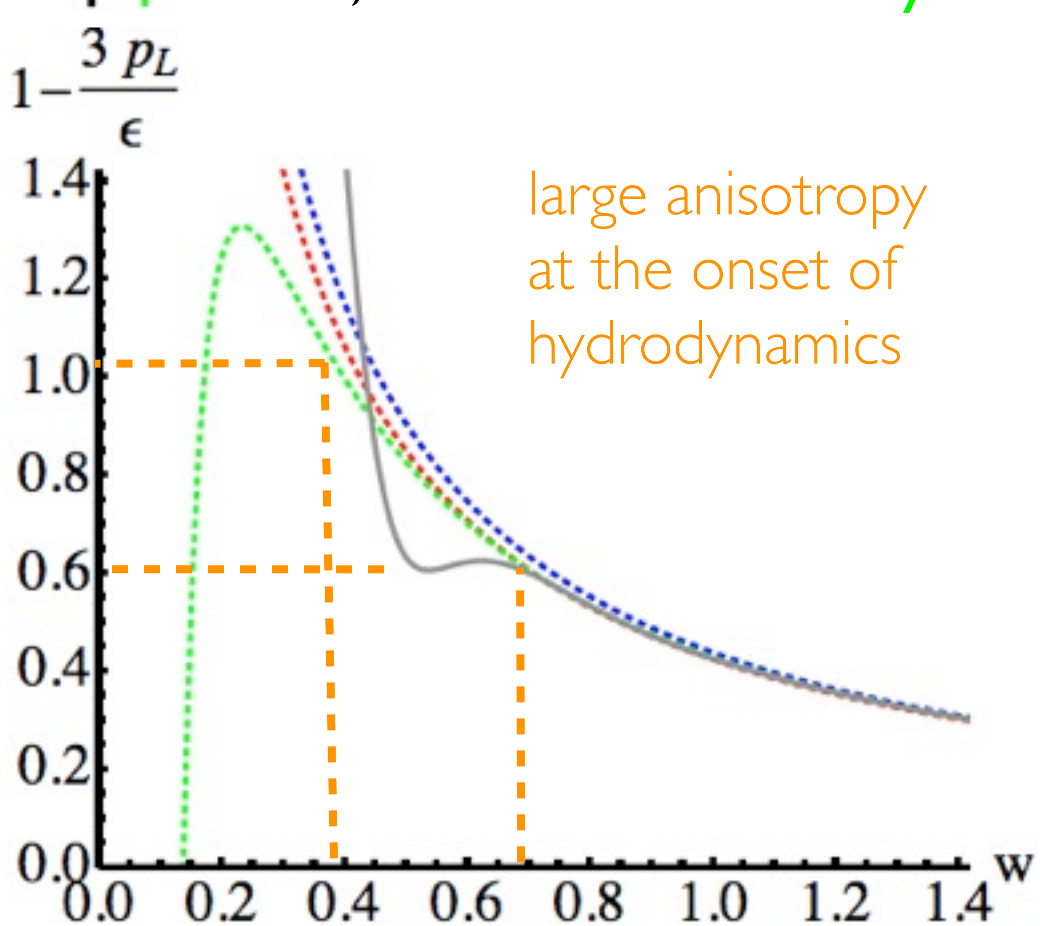
Hydronization vs thermalization/isotropization



Rewriting equations of hydrodynamics in a form

$$\frac{\tau}{w}\frac{d}{d\tau}w = \frac{F_{hydro}(w)}{w}.$$

allows to explicitly see whether non-hydro modes already relaxed when curves coincide!



The single most interesting result is that **hydrodynamization occurs well before isotropization!**

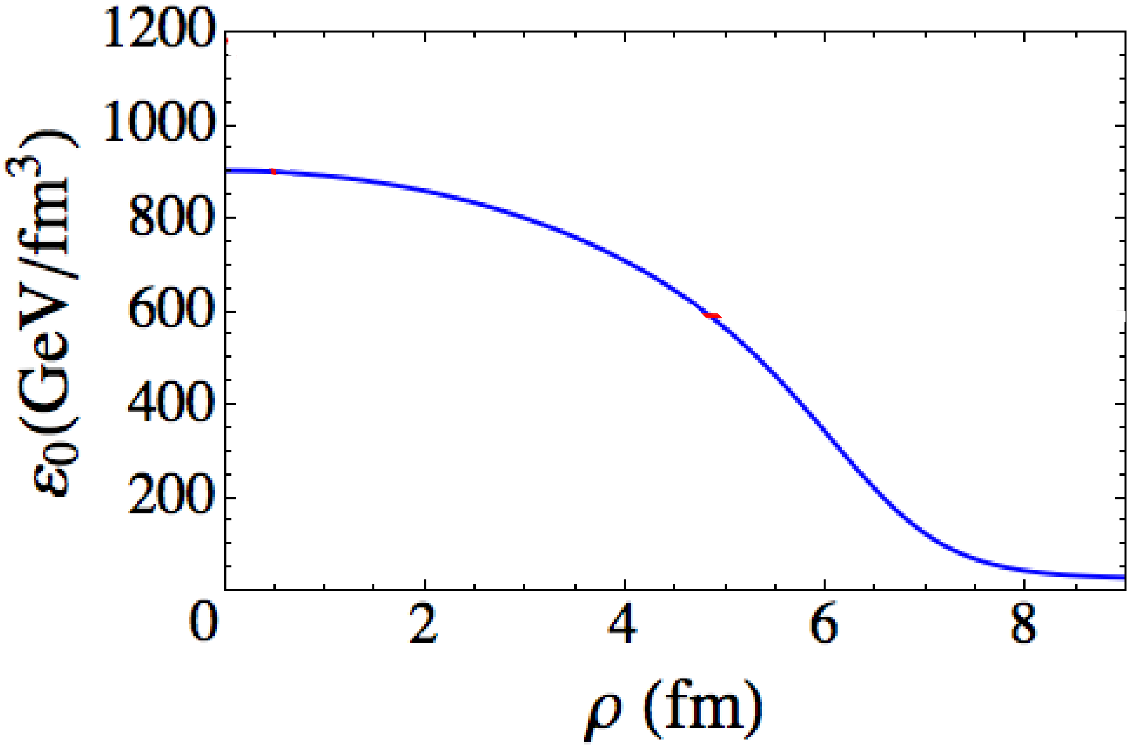
Pressure anisotropy is observed to be between

$$1 - \frac{3p_L}{\epsilon} \approx 0.6 \text{ to } 1.0$$

with hydrodynamics already being a valid description of the stress tensor dynamics.

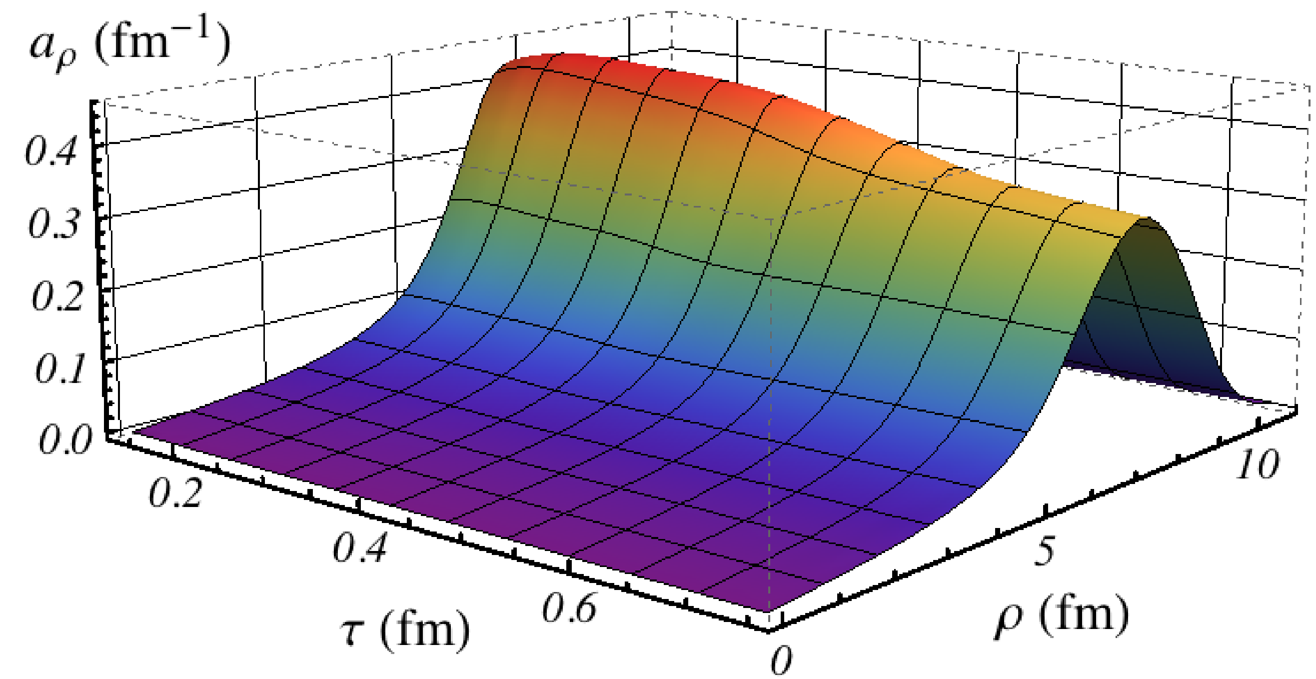
similar findings in Chesler & Yaffe 0906.4426 and 1011.3562

Including the radial flow 1211.2218 [hep-th] van der Schee

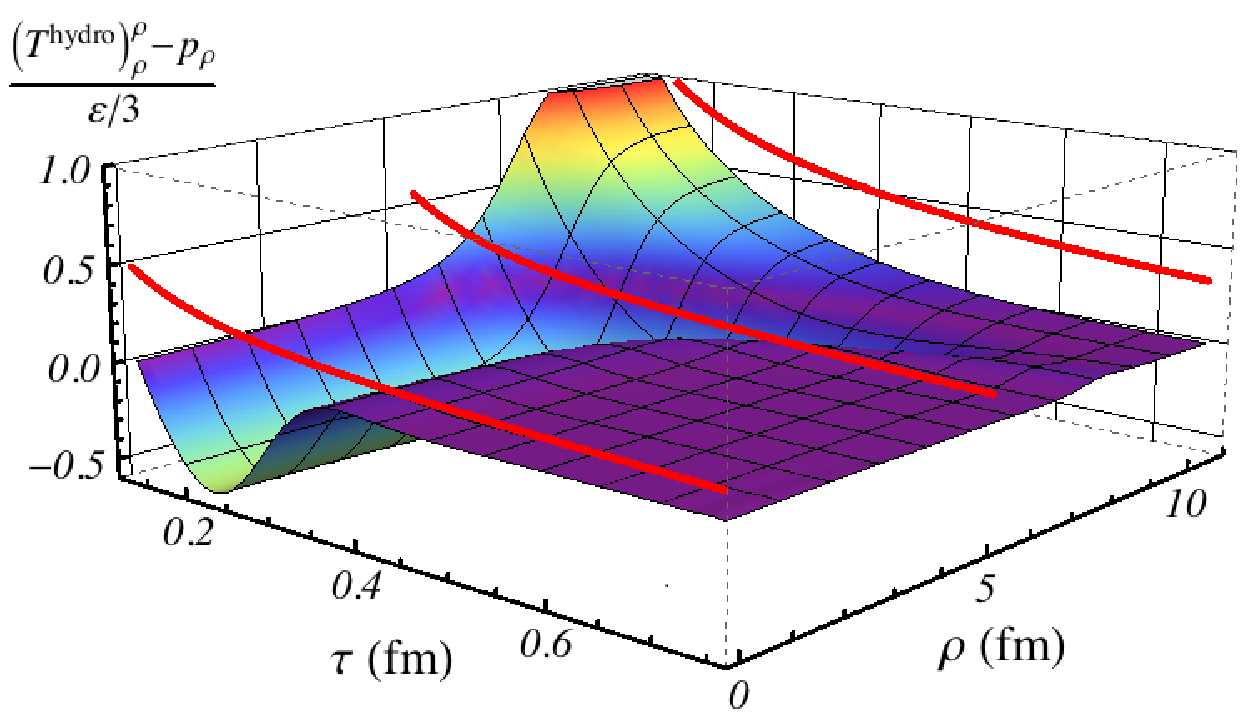


init. cond. (Glauber) at $\tau_{in} \approx 0.12\,\mathrm{fm/c}$

$(u^{\rho}_{in} = 0)$



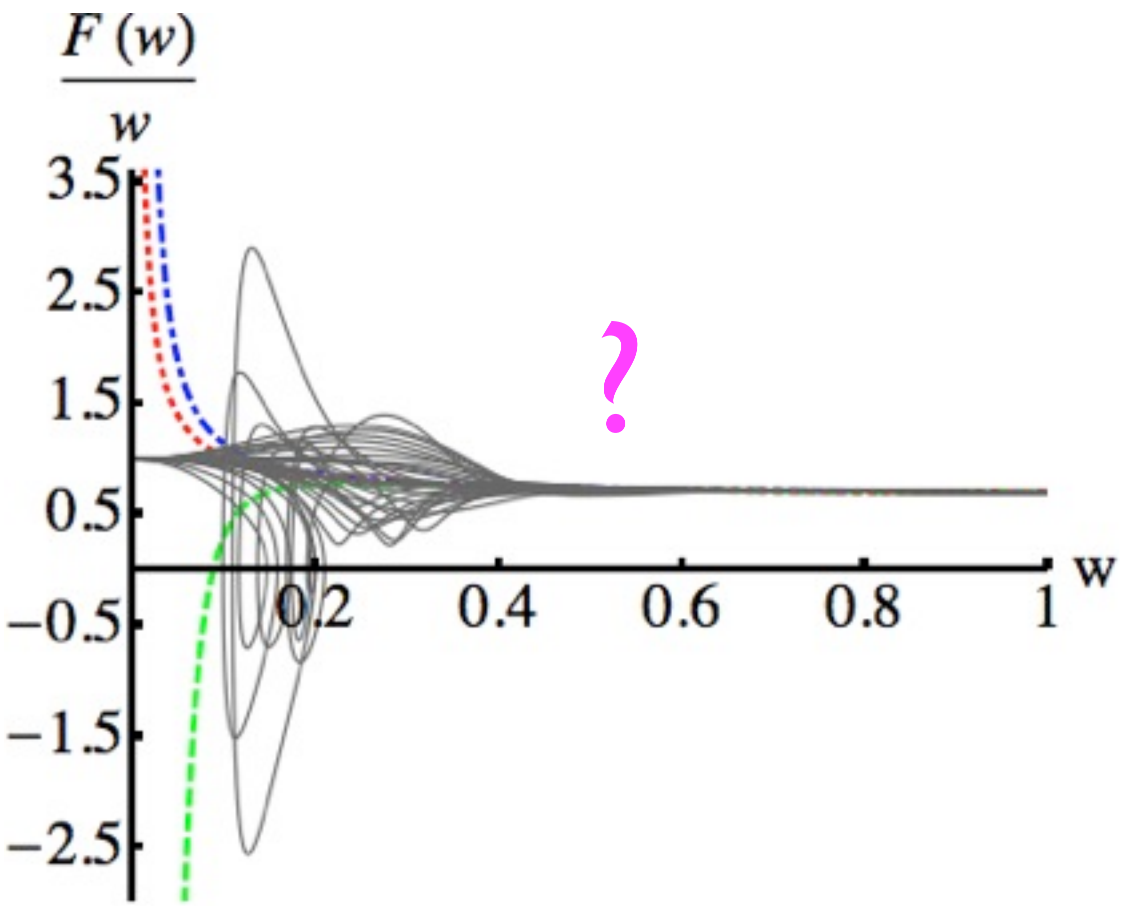
radial acceleration is mainly localized at the edge of the plasma

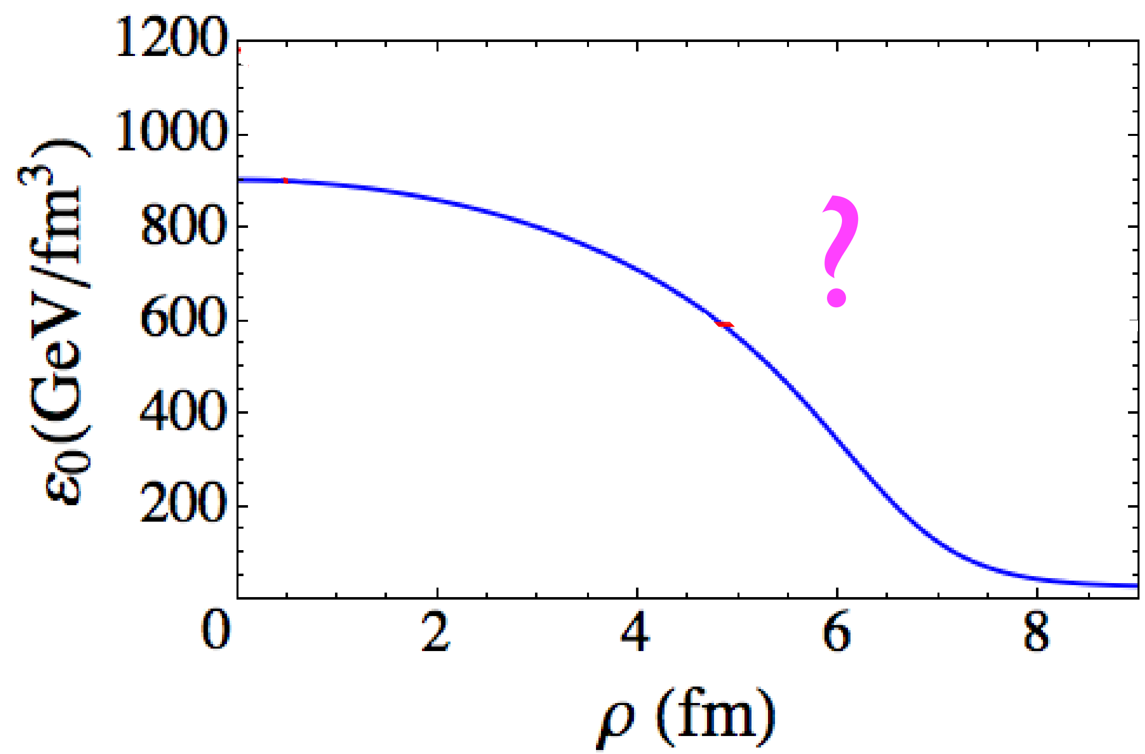


system hydrodynamizes around $\tau_{hydro} = 0.4\,\mathrm{fm/c}$, first at the center

for some initial conditions we would again expect sizable pressure anisotropy

The main problem





HUGE FREEDOM OF CHOICE

Which far from equilibrium initial condition corresponds to the experiment?

Towards holographic heavy ion collisions

1011.3562 [hep-th] Chesler, Yaffe

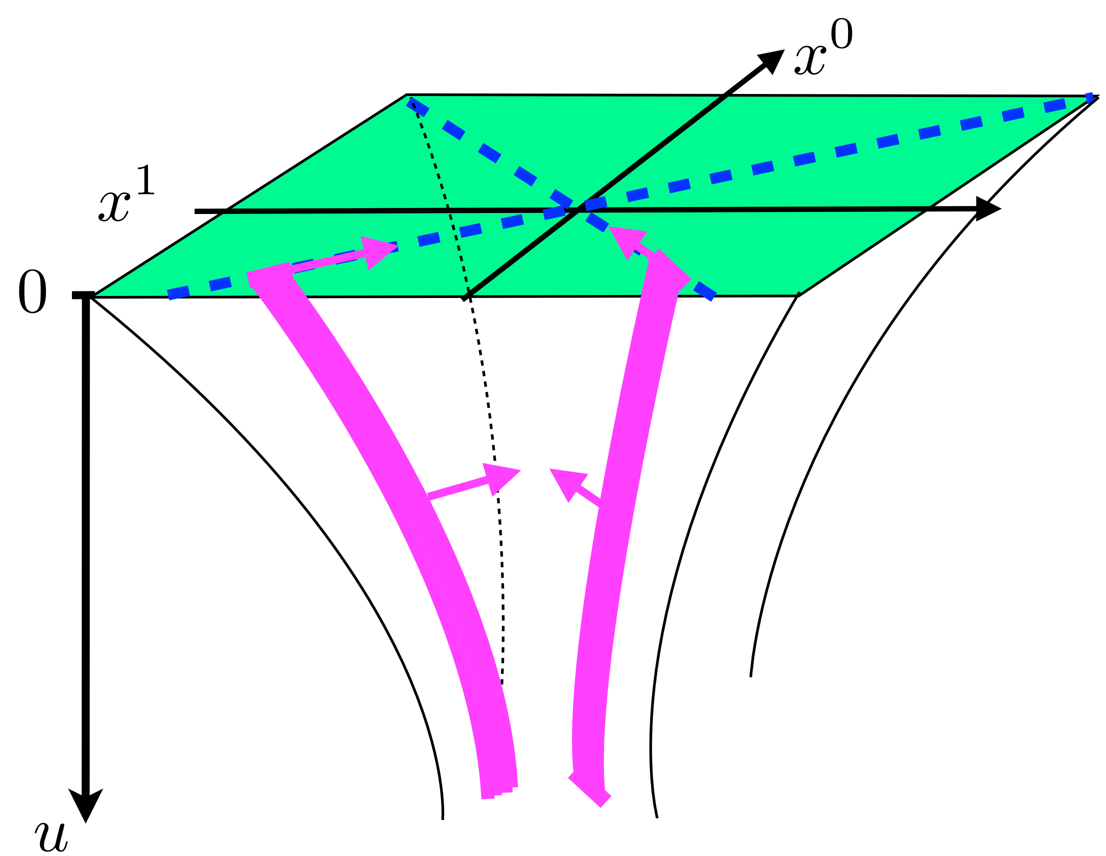
1203.xxxx [hep-th] Casalderrey-Solana, MPH, Mateos, van der Schee

Towards a holographic „heavy ion collision"

Operational view:

collide holographically two lumps of matter moving at relativistic speeds

unfortunately necessarily deconfined, i.e. with $\langle T^{\mu\nu} \rangle = \mathcal{O}(N_c^2)$



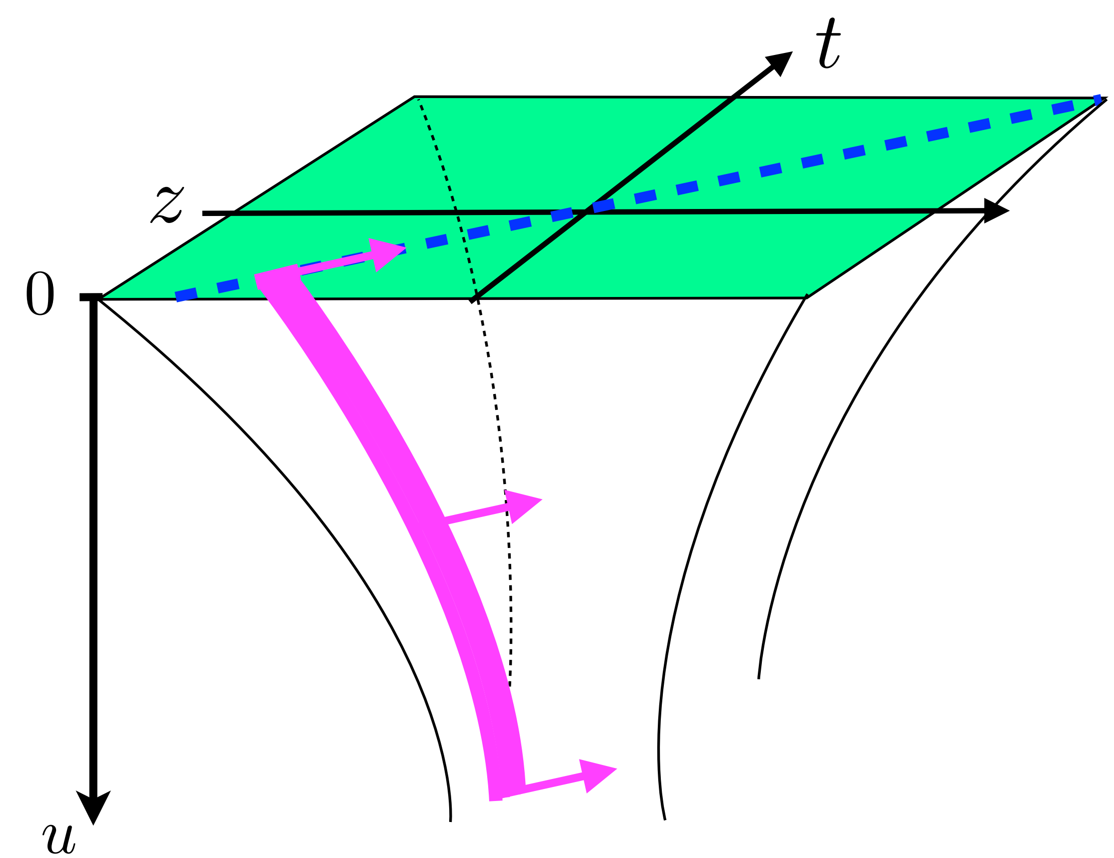
State of the art as of March 2013: colliding gravitational shock wave solutions

Gravitational shock wave solutions

Janik & Peschanski **[hep-th/0512162]** Chesler & Yaffe **1011.3562 [hep-th]**

dual stress tensor:

$$T^{tt} = T^{zz} = +T^{tz} = \frac{N_c^2}{2\pi^2} h(t-z)$$



shock wave disturbance
moving with the speed of light

$$ds^2 = \frac{1}{u^2}(du^2 + \eta_{\mu\nu} dx^\mu dx^\nu) + u^2 h(x_-) dx_-^2$$

Poincare patch
vacuum AdS

Solution of Einstein's equations with the negative CC
for <u>any</u> longitudinal profile $h(x_-)$

We will specialize to $h(t \pm z) = \mathcal{E}_0 \exp\left[-(t \pm z)^2/2\sigma^2\right]$. But we're in a CFT, so the only qty that matters is

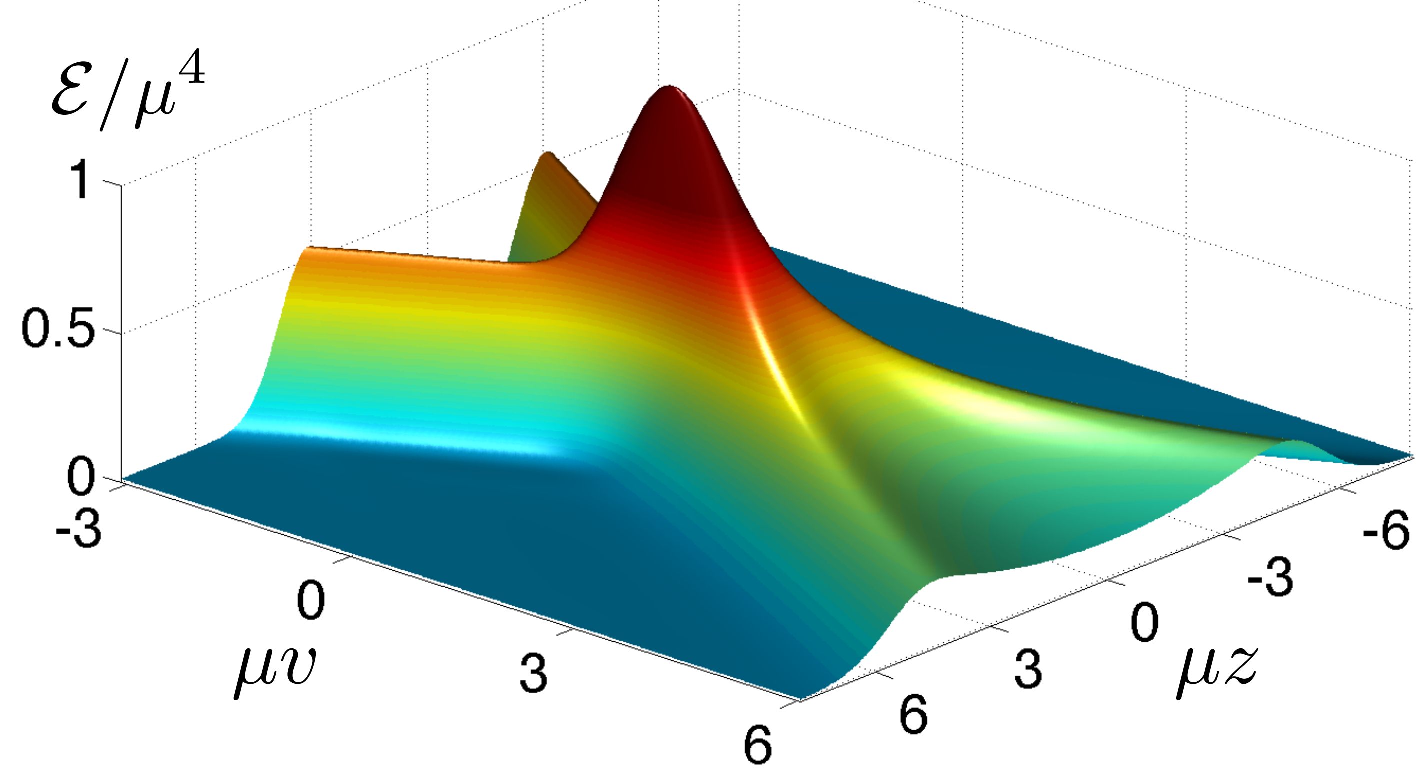
$$e = \mathcal{E}_0^{1/4} \sigma$$

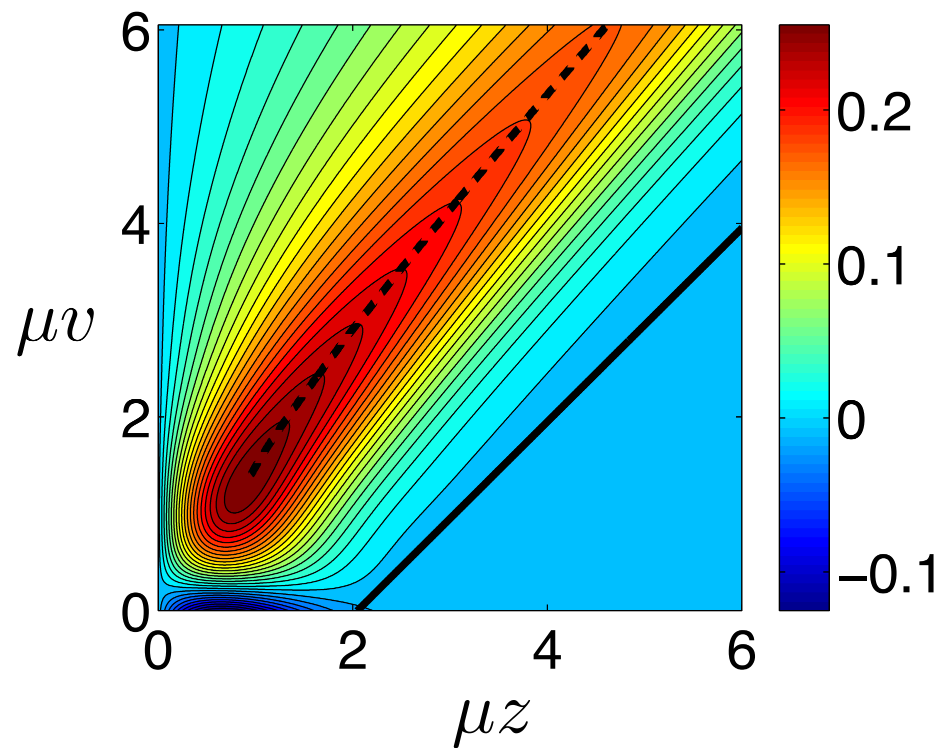
Colliding shocks at $e_{CY} \simeq 0.64$

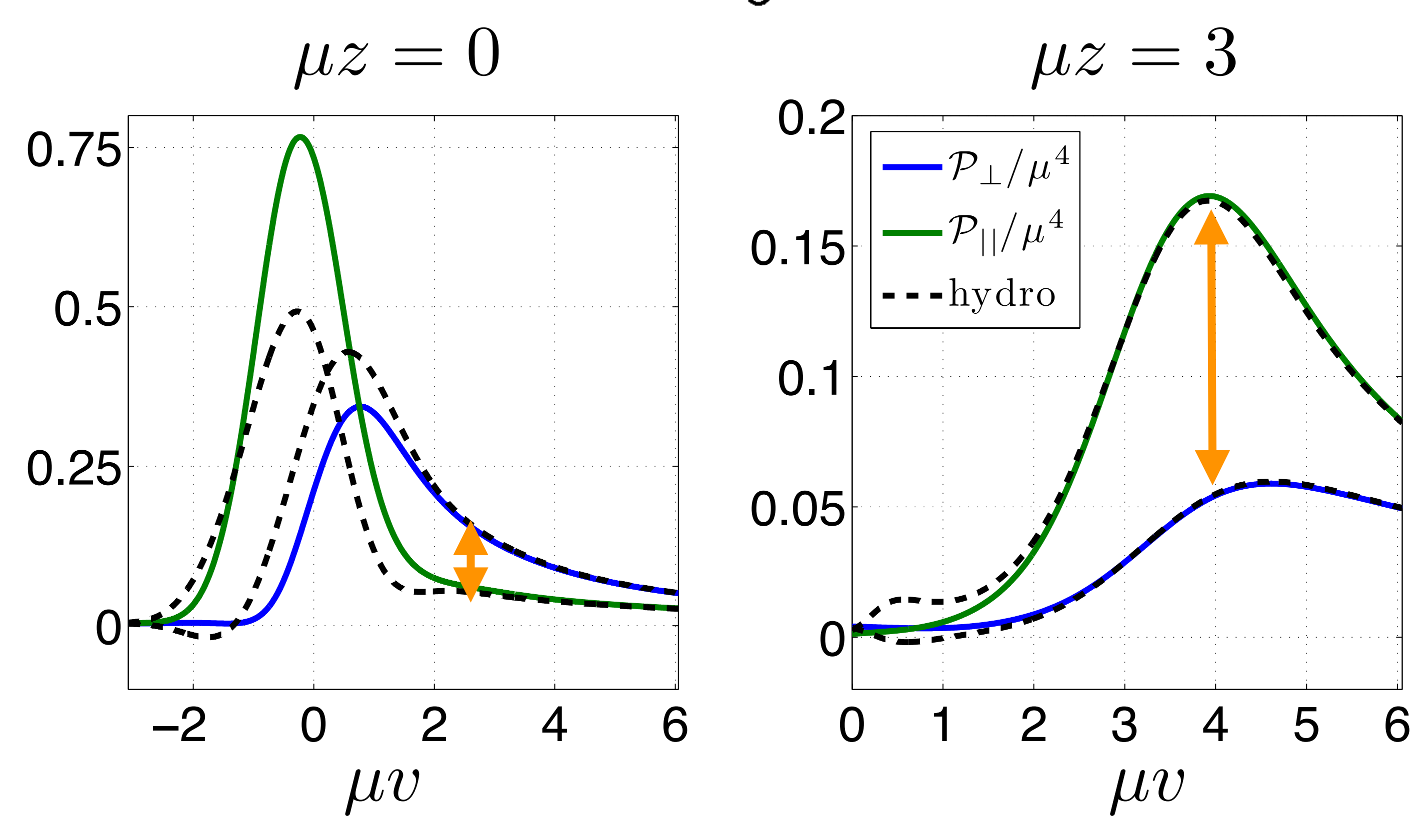
Chesler & Yaffe **1011.3562 [hep-th]**

$e_{CY} \simeq 0.64$ is roughly the value corresponding to Pb nuclei boosted to RHIC energies

$\mu \sim$ total energy*
$\mathcal{E} \sim T^{00}$







significant stopping:
15% slow-down of T^{00} maxima

again, anisotropy at hydrodynamization

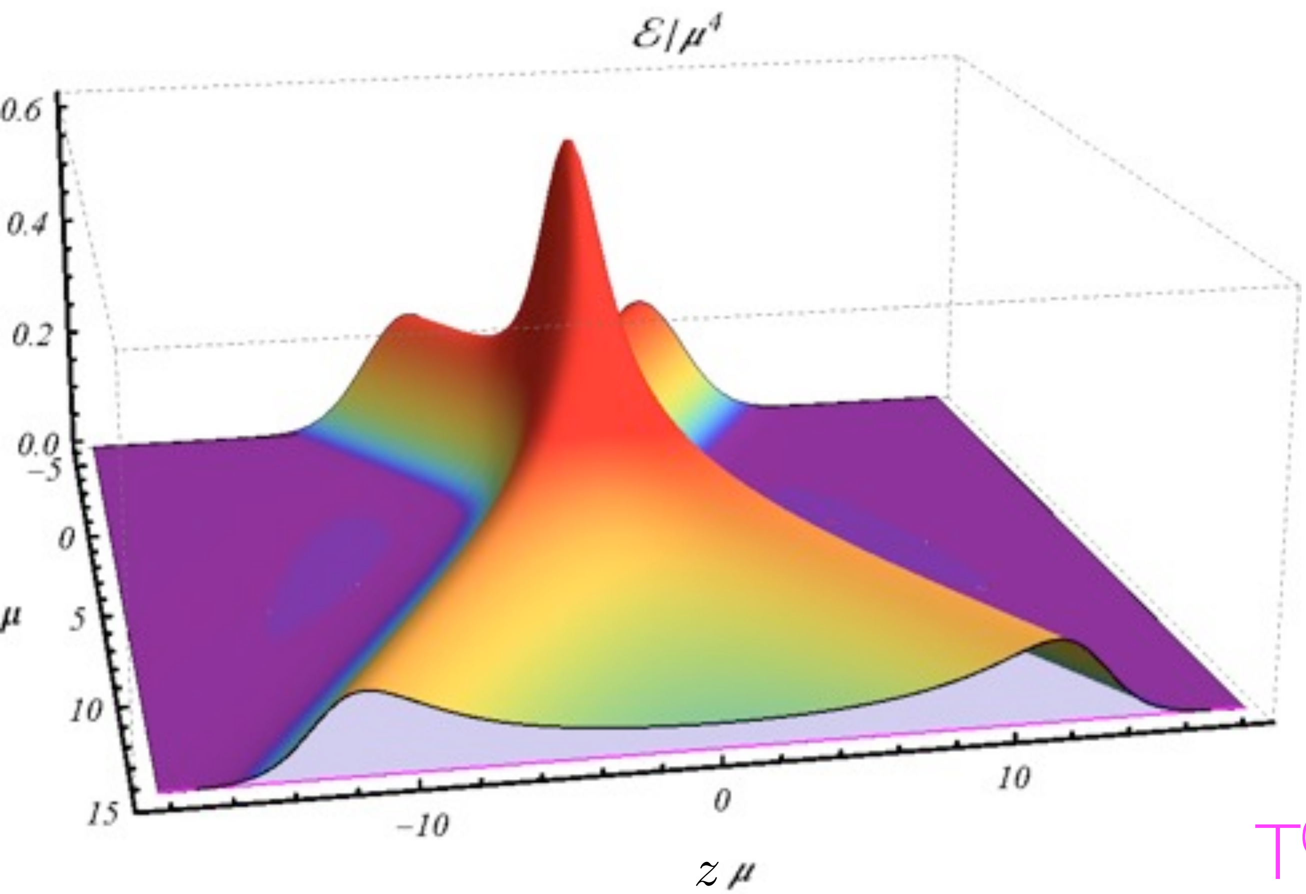
is it the full story?

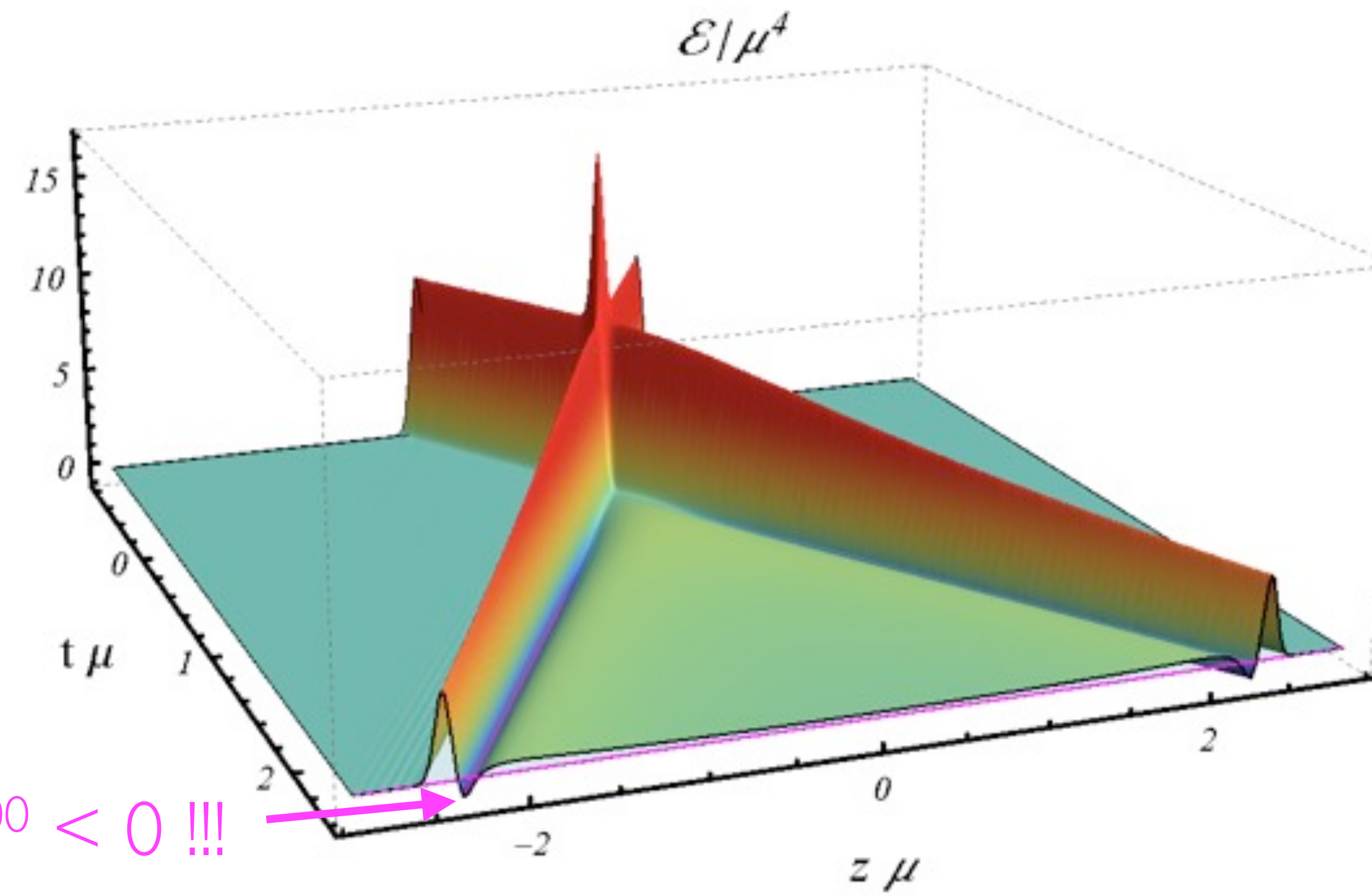
Dynamical crossover

1203.xxxx [hep-th] Casalderrey-Solana, MPH, Mateos, van der Schee

$e_{left} = 2\, e_{CY}$

$e_{right} = \frac{1}{8}\, e_{CY}$





$T^{00} < 0$!!!

again, significant stopping:
15% slow-down of T^{00} maxima ($v \approx 0.85$)

hydro kicks in soon after the
outer parts of incoming shocks meet

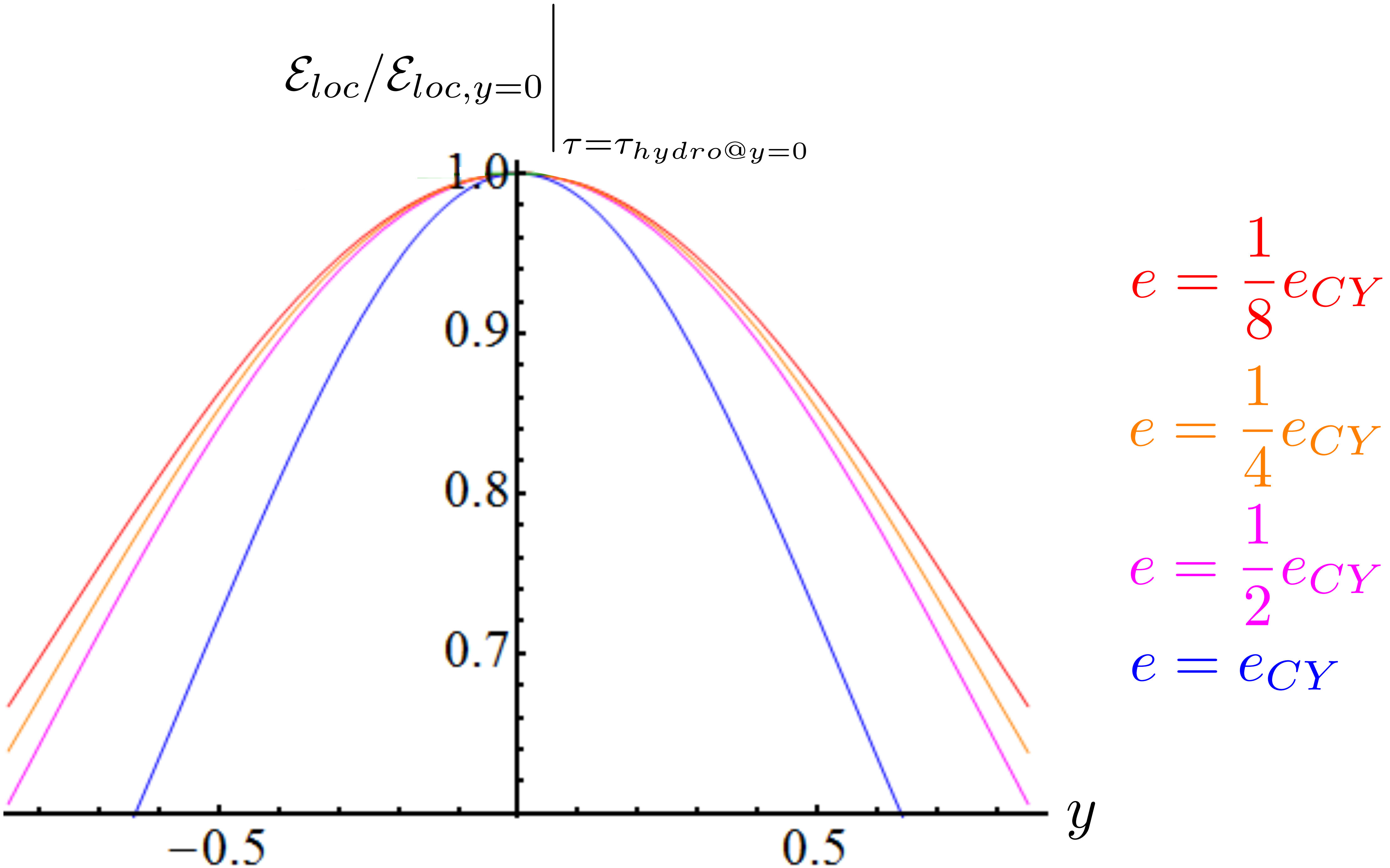
more like the old Landau picture

no stopping:
T^{00} maxima move with $v \approx 1$

hydro applicable only at mid-rapidities and late enough!!!

<u>more like what seems to be happening at RHIC and LHC</u>

Rapidity distribution



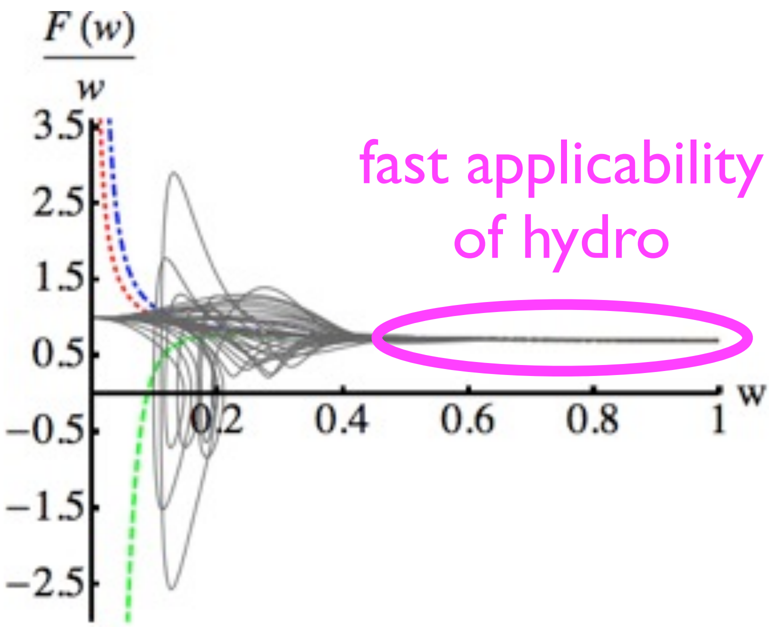
The dynamics is not boost-invariant in the sense introduced by Bjorken but nevertheless with decreasing e the rapidity distribution flattens out.

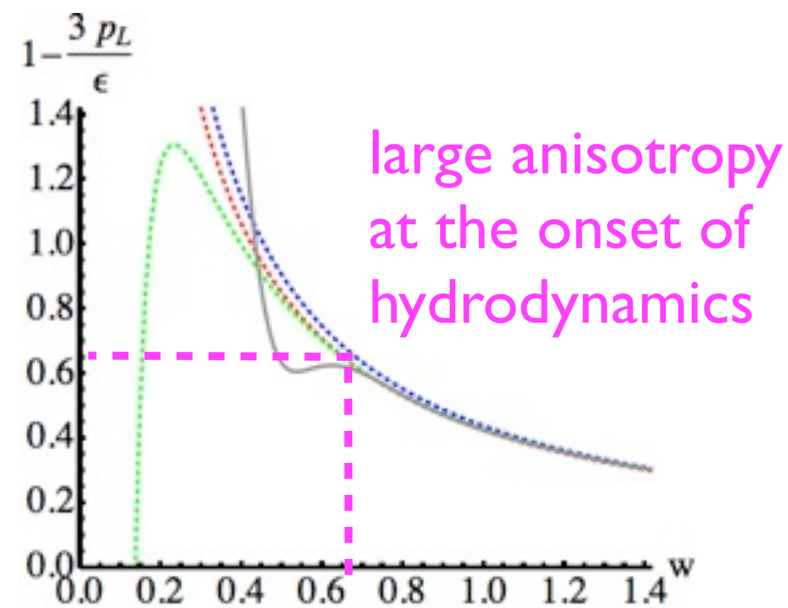
Summary and open problems

Summary

Interesting developments due to applying numerical GR techniques.

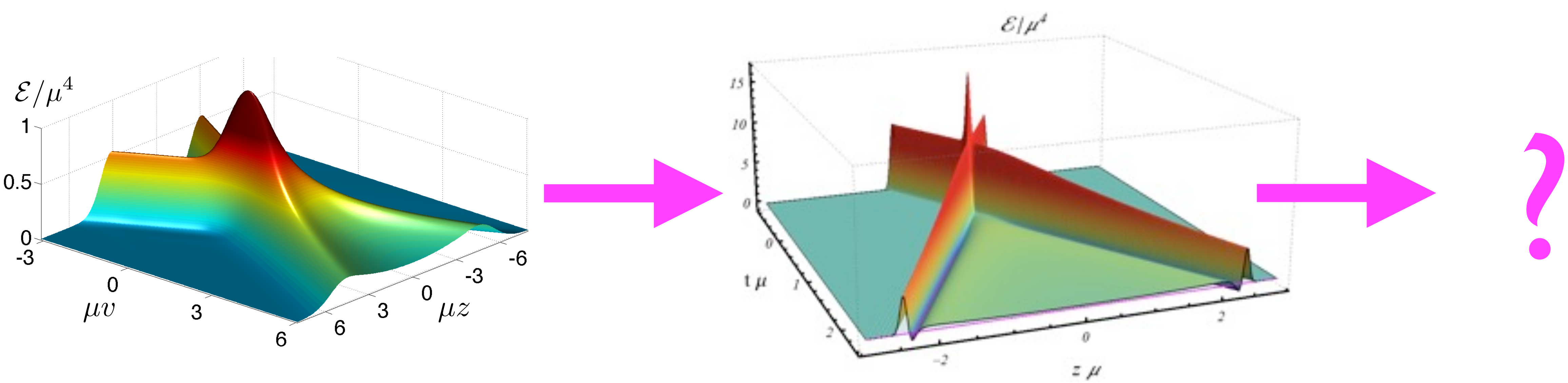
Two reoccurring themes





Discussed developments led to a new term: hydrodynamization!

Future: towards more realistic holographic heavy ion collisions...



Holography and QCD - Recent progress and challenges -

24-28 September 2013 *Kavli IPMU, the University of Tokyo*

Overview

Program

Registration

Accommodation

Access to IPMU

Links

Contact us

Overview

Over the last decade it has become apparent that the gauge/string duality is a powerful tool for unearthing various features of large classes of strongly coupled systems, enabling first principle calculations in the regimes unaccessible before. A very significant part of these developments was motivated by the physics of non-perturbative phases of QCD (spectrum of hadrons, chiral symmetry breaking, the phase diagram, real-time dynamics, etc.). The goal of the workshop is to discuss recent progress and challenges of holography with applications to QCD.

Dates

September 24 (Tue) - 28 (Sat), 2013

Venue

Lecture Hall (1F), Kavli IPMU main building

Organizers

Michal P. Heller (Amsterdam/Warsaw), Elias Kiritsis (APC/Crete), Mukund Rangamani (Durham), Jacob Sonnenschein (Tel Aviv), Shigeki Sugimoto (Kavli IPMU, LOC, chair), Taizan Watari (Kavli IPMU, LOC), Hirosi Ooguri (Caltech/Kavli IPMU, advisor)

Contact

Address: Kavli Institute for the Physics and Mathematics of the Universe (Kavli IPMU), the University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa City, Chiba 277-8583, Japan

Tel: +81-4-7136-5979 (Rie Ujita)

e-mail: hqcd2013@ipmu.jp (Shigeki Sugimoto (chair), Rie Ujita (secretary))

http://tinyurl.com/hqcd2013

extra